Massachusetts Institute of Technology

MIT $(^w^)$

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adapted from KACTL and MIT NULL

2020-02-14

Contest (1)

TemplateShort.cpp

d53b32, 32 lines

```
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef long double ld;
typedef pair<int,int> pi;
typedef vector<int> vi;
typedef vector<pi> vpi;

#define f first
#define s second
#define sz(x) (int)x.size()
#define all(x) begin(x), end(x)
#define rsz resize
#define bk back()
#define pb push_back

#define FOR(i,a,b) for (int i = (a); i < (b); ++i)
#define F0R(i,a) FOR(i,0,a)
#define ROF(i,a,b) for (int i = (b)-1; i >= (a); --i)
#define R0F(i,a) ROF(i,0,a)
#define trav(a,x) for (auto& a: x)

const int MOD = 1e9+7;
const ld PI = acos((ld)-1);

template<class T> bool ckmin(T& a, const T& b) {
  return b < a ? a = b, 1 : 0; }
template<class T> bool ckmax(T& a, const T& b) {
  return a < b ? a = b, 1 : 0; }

int main() { ios_base::sync_with_stdio(0); cin.tie(0); }
```

.bashrc

4 lines

```
alias clr="printf '\33c'"
# on mac, add -Wl,-stack_size -Wl,0x10000000 to co
co() { g++-9 -std=c++11 -O2 -Wall -Wextra -o $1 $1.cpp; }
run() { co $1 && ./$1; }
```

hash.sh

3 lines

```
# Hash file ignoring whitespace and comments. Verifies that
# code was correctly typed. Usage: sh hash.sh < A.cpp
cpp -dD -P -fpreprocessed|tr -d '[:space:]'|md5sum|cut -c-6
```

troubleshoot.txt

72 lines

```
General:
Write down most of your thoughts, even if you're not sure
whether they're useful.
Give your variables (and files) meaningful names.
Stay organized and don't leave papers all over the place!
You should know what your code is doing ...

Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Remove debug output.
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output as well.
Read the full problem statement again.
Have you understood the problem correctly?
Are you sure your algorithm works?
Try writing a slow (but correct) solution.
Can your algorithm handle the whole range of input?
Did you consider corner cases (ex. n=1)?
Is your output format correct? (including whitespace)
Are you clearing all data structures between test cases?
Any uninitialized variables?
Any undefined behavior (array out of bounds)?
Any overflows or NaNs (or shifting ll by >=64 bits)?
Confusing N and M, i and j, etc.?
Confusing ++i and i++?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some test cases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Rewrite your solution from the start or let a teammate do it.

Geometry:
Work with ints if possible.
Correctly account for numbers close to (but not) zero. Related:
for functions like acos make sure absolute val of input is not
(slightly) greater than one.
Correctly deal with vertices that are collinear, concyclic,
coplanar (in 3D), etc.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What's your complexity? Large TL does not mean that something
simple (like NlogN) isn't intended.
Are you copying a lot of unnecessary data? (References)
Avoid vector, map. (use arrays/unordered_map)
How big is the input and output? (consider FastIO)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
If using pointers try BumpAllocator.
```

Mathematics (2)

2.1 Equations

$$\begin{array}{c} ax+by=e \\ cx+dy=f \end{array} \Rightarrow \begin{array}{c} x=\dfrac{ed-bf}{ad-bc} \\ y=\dfrac{af-ec}{ad-bc} \end{array}$$

In general, given an equation $Ax=b$, the solution to a variable x_i is given by

$$x_i=\frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n=c_1a_{n-1}+\cdots+c_ka_{n-k}$, and $r_1,\ldots,r_k$ are distinct roots of $x^k+c_1x^{k-1}+\cdots+c_k$, there are $d_1,\ldots,d_k$ s.t.

$$a_n=d_1r_1^n+\cdots+d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n=(d_1n+d_2)r^n$.

2.3 Trigonometry

$$\sin(v+w)=\sin v\cos w+\cos v\sin w$$
$$\cos(v+w)=\cos v\cos w-\sin v\sin w$$
$$\tan(v+w)=\frac{\tan v+\tan w}{1-\tan v\tan w}$$
$$\sin v+\sin w=2\sin\frac{v+w}{2}\cos\frac{v-w}{2}$$
$$\cos v+\cos w=2\cos\frac{v+w}{2}\cos\frac{v-w}{2}$$

$$a\cos x + b\sin x = r\cos(x-\phi)$$
$$a\sin x + b\cos x = r\sin(x+\phi)$$

where $r = \sqrt{a^2+b^2}$, $\phi = \operatorname{atan2}(b,a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $s = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{s(s-a)(s-b)(s-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1-\left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2+c^2-2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$

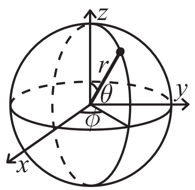
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2+d^2-a^2-c^2$:

$$4A = 2ef\cdot\sin\theta = F\tan\theta = \sqrt{4e^2f^2-F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac+bd$, and $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \operatorname{atan2}(y,x) \end{aligned}$$

2.5 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx}\arcsin x &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}\arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}\tan x &= 1+\tan^2 x & \frac{d}{dx}\arctan x &= \frac{1}{1+x^2} \\ \int\tan ax &= -\frac{\ln|\cos ax|}{a} & \int x\sin ax &= \frac{\sin ax - ax\cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) & \int xe^{ax}dx &= \frac{e^{ax}}{a^2}(ax-1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums/Series

$$1^4+2^4+3^4+\cdots+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$
$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots,\ (-\infty<x<\infty)$$
$$\ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots,\ (-1<x\le 1)$$
$$\sqrt{1+x} = 1+\frac{x}{2}-\frac{x^2}{8}+\frac{2x^3}{32}-\frac{5x^4}{128}+\ldots,\ (-1\le x\le 1)$$
$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots,\ (-\infty<x<\infty)$$
$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots,\ (-\infty<x<\infty)$$

2.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x-\mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation.

$$\mathbb{E}(aX+bY) = a\mathbb{E}(X)+b\mathbb{E}(Y)$$

If X, Y independent,

$$V(aX+bY) = a^2V(X)+b^2V(Y).$$

2.7.1 Discrete distributions

Binomial distribution

of successes in n independent yes/no experiments, each which yields success with probability p is $\operatorname{Bin}(n,p)$, $n = 1, 2, \ldots$, $0\le p\le 1$.

$$p(k) = \binom{n}{k}p^k(1-p)^{n-k}$$
$$\mu = np,\ \sigma^2 = np(1-p)$$

$\operatorname{Bin}(n,p)\approx\operatorname{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\operatorname{Fs}(p)$, $0\le p\le 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$
$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\operatorname{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$
$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.7.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\operatorname{U}(a,b)$, $a<b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a<x<b \\ 0 & \text{otherwise} \end{cases}$$
$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.8 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree. **(IMPORTANT)**

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

Data Structures (3)

3.1 STL

MapComparator.h

Description: example of function object for map or set

Usage: `set<int,cmp> s; map<int,int,cmp> m;`

5bfa6c, 1 lines

```
struct cmp{bool operator()(int l,int r)const{return l>r;}};
```

HashMap.h

Description: Hash map with the same API as unordered_map, but ~3x faster. Initial capacity must be a power of 2 if provided.

Usage: `ht<int,int> h({},{},{},{},{1<<16});`

<ext/pb_ds/assoc_container.hpp> a37e68, 10 lines

```
using namespace __gnu_pbds;
struct chash {
  const uint64_t C = ll(2e18*PI)+71; // large odd number
  const int RANDOM = rng();
  ll operator()(ll x) const {
    return __builtin_bswap64((x^RANDOM)*C); }
};
template<class K,class V> using ht = gp_hash_table<K,V,chash>;
template<class K,class V> V get(ht<K,V>& u, K x) {
  auto it = u.find(x); return it == end(u) ? 0 : it->s; }
```

PQ.h

Description: Priority queue w/ modification. Use for Dijkstra?

<bits/extc++.h> 879e4e, 9 lines

```
void pqExample() {
  __gnu_pbds::priority_queue<int> p;
  vi act; vector<decltype(p)::point_iterator> v;
  int n = 1000000;
  FOR(i,n) { int r = rand(); act.pb(r), v.pb(p.push(r)); }
  FOR(i,n) { int r = rand(); act[i] = r, p.modify(v[i],r); }
  sort(rall(act));
  FOR(i,n) { assert(act[i] == p.top()); p.pop(); }
}
```

IndexedSet.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. Change `null_type` for map.

Time: $\mathcal{O}(\log N)$

<ext/pb_ds/tree_policy.hpp>, <ext/pb_ds/assoc_container.hpp> 64d55b, 12 lines

```
using namespace __gnu_pbds;
template <class T> using Tree = tree<T, null_type, less<T>,
  rb_tree_tag, tree_order_statistics_node_update>;
#define ook order_of_key
#define fbo find_by_order

void treeExample() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).f; assert(it == t.lb(9));
  assert(t.ook(10) == 1 && t.ook(11) == 2 && *t.fbo(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

Rope.h

Description: insert element at i-th position, cut a substring and re-insert somewhere else

Time: $\mathcal{O}(\log N)$ per operation? not well tested

<ext/rope> 2ce450, 14 lines

```
using namespace __gnu_cxx;
void ropeExample() {
  rope<int> v(5, 0); // initialize with 5 zeroes
  FOR(i,sz(v)) v.mutable_reference_at(i) = i+1;
  FOR(i,5) v.pb(i+1); // constant time pb
  rope<int> cur = v.substr(1,2);
  v.erase(1,3); // erase 3 elements starting from 1st element
  for (rope<int>::iterator it = v.mutable_begin();
    it != v.mutable_end(); ++it) pr((int)*it,' ');
  ps(); // 1 5 1 2 3 4 5
  v.insert(v.mutable_begin()+2,cur); // index or const_iterator
  v += cur; FOR(i,sz(v)) pr(v[i],' ');
  ps(); // 1 5 2 3 1 2 3 4 5 2 3
}
```

LCold.h

Description: LineContainer; add lines of the form $kx+m$, compute greatest y-coordinate for any x.

Time: $\mathcal{O}(\log N)$

2ab606, 34 lines

```
bool Q;
struct Line {
  mutable ll k, m, p; // slope, y-intercept, last optimal x
  ll eval (ll x) { return k*x+m; }
  bool operator<(const Line& o) const { return Q?p<o.p:k<o.k; }
};

// for doubles, use inf = 1/.0, divi(a,b) = a/b
const ll inf = LLONG_MAX;
// floored div
ll divi(ll a, ll b) { return a/b-((a^b) < 0 && a%b); }
// last x such that first line is better
ll bet(const Line& x, const Line& y) {
  if (x.k == y.k) return x.m >= y.m ? inf : -inf;
  return divi(y.m-x.m,x.k-y.k); }

struct LC : multiset<Line> {
  // updates x->p, determines if y is unneeded
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return 0; }
    x->p = bet(*x,*y); return x->p >= y->p; }
  void add(ll k, ll m) {
    auto z = insert({k,m,0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    Q = 1; auto l = *lb({0,0,x}); Q = 0;
    return l.k*x+l.m;
  }
};
```

LCdeque.h

Description: LineContainer assuming both slopes and queries monotonic.

Time: $\mathcal{O}(1)$

"LCold.h" bdaf48, 33 lines

```
struct LCdeque : deque<Line> {
  void addBack(Line L) { // assume nonempty
    while (1) {
      auto a = bk; pop_back(); a.p = bet(a,L);
      if (size() && bk.p >= a.p) continue;
      pb(a); break;
    }
    L.p = inf; pb(L);
  }
  void addFront(Line L) {
    while (1) {
      if (!size()) { L.p = inf; break; }
```

```
      if ((L.p = bet(L,ft)) >= ft.p) pop_front();
      else break;
    }
    push_front(L);
  }
  void add(ll k, ll m) { // line goes to one end of deque
    if (!size() || k <= ft.k) addFront({k,m,0});
    else assert(k >= bk.k), addBack({k,m,0});
  }
  int ord = 0; // 1 = increasing, -1 = decreasing
  ll query(ll x) {
    assert(ord);
    if (ord == 1) {
      while (ft.p < x) pop_front();
      return ft.eval(x);
    } else {
      while(size()>1&&prev(prev(end()))->p>=x)pop_back();
      return bk.eval(x);
    }
  }
};
```

3.2 1D Range Queries

RMQ.h

Description: 1D range minimum query. Can also do queries for any associative operation in $O(1)$ with D&C
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(1)$

b1fe94, 18 lines

```
template<class T> struct RMQ { // floor(log_2(x))
  int level(int x) { return 31-__builtin_clz(x); }
  vector<T> v; vector<vi> jmp;
  int comb(int a, int b) { // index of min
    return v[a]==v[b]?min(a,b):(v[a]<v[b]?a:b); }
  void init(const vector<T>& _v) {
    v = _v; jmp = {vi(sz(v))}; iota(all(jmp[0]),0);
    for (int j = 1; 1<<j <= sz(v); ++j) {
      jmp.pb(vi(sz(v)-(1<<j)+1));
      F0R(i,sz(jmp[j])) jmp[j][i] = comb(jmp[j-1][i],
                  jmp[j-1][i+(1<<(j-1))]);
    }
  }
  int index(int l, int r) { // get index of min element
    int d = level(r-l+1);
    return comb(jmp[d][l],jmp[d][r-(1<<d)+1]); }
  T query(int l, int r) { return v[index(l,r)]; }
};
```

BIT.h

Description: range sum queries and point updates for D dimensions
Usage: {BIT<int,10,10>} gives 2D BIT
Time: $\mathcal{O}\left((\log N)^D\right)$

1cb741, 14 lines

```
template <class T, int ...Ns> struct BIT {
  T val = 0; void upd(T v) { val += v; }
  T query() { return val; }
};
template <class T, int N, int... Ns> struct BIT<T, N, Ns...> {
  BIT<T,Ns...> bit[N+1];
  template<typename... Args> void upd(int pos, Args... args) {
    for (; pos<=N; pos+=pos&-pos) bit[pos].upd(args...); }
  template<typename... Args> T sum(int r, Args... args) {
    T res=0; for (;r;r-=r&-r) res += bit[r].query(args...);
    return res; }
  template<typename... Args> T query(int l, int r, Args...
    args) { return sum(r,args...)-sum(l-1,args...); }
};
```

BITrange.h

Description: 1D range increment and sum query. Possible for higher dimensions.
Time: $\mathcal{O}(\log N)$

"BIT.h" 77a935, 13 lines

```
template<class T, int SZ> struct BITrange {
  BIT<T,SZ> bit[2]; // piecewise linear functions
  // let cum[x] = sum_{i=1}^{x}a[i]
  void upd(int hi, T val) { // add val to a[1..hi]
    // if x <= hi, cum[x] += val*x
    bit[1].upd(1,val), bit[1].upd(hi+1,-val);
    // if x > hi, cum[x] += val*hi
    bit[0].upd(hi+1,hi*val);
  }
  void upd(int lo,int hi,T val){upd(lo-1,-val),upd(hi,val);}
  T sum(int x) { return bit[1].sum(x)*x+bit[0].sum(x); }
  T query(int x, int y) { return sum(y)-sum(x-1); }
};
```

SegTree.h

Description: 1D point update, range query where comb is any associative operation. N doesn't have to be a power of 2 but then seg[1] != query(0,N-1).
Time: $\mathcal{O}(\log N)$

f597e1, 19 lines

```
template<class T> struct Seg {
  const T ID = 0; // comb(ID,b) must equal b
  T comb(T a, T b) { return a+b; }
  int n; vector<T> seg;
  void init(int _n) { n = _n; seg.assign(2*n,ID); }
  void pull(int p) { seg[p] = comb(seg[2*p],seg[2*p+1]); }
  void upd(int p, T value) {  // set value at position p
    seg[p += n] = value;
    for (p /= 2; p; p /= 2) pull(p);
  }
  T query(int l, int r) {  // sum on interval [l, r]
    T ra = ID, rb = ID;
    for (l += n, r += n+1; l < r; l /= 2, r /= 2) {
      if (l&1) ra = comb(ra,seg[l++]);
      if (r&1) rb = comb(seg[--r],rb);
    }
    return comb(ra,rb);
  }
};
```

Wavelet.h

Description: Segment tree on values instead of indices. Returns k-th largest number in 0-indexed interval [lo,hi). SZ should be a power of 2, and all values in a must lie in [0,SZ).
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(\log N)$ query

811b15, 21 lines

```
template<int SZ> struct Wavelet {
  vi nexl[SZ], nexr[SZ];
  void build(vi a, int ind = 1, int L = 0, int R = SZ-1) {
    if (L == R) return;
    nexl[ind] = nexr[ind] = {0};
    vi A[2]; int M = (L+R)/2;
    trav(t,a) {
      A[t>M].pb(t);
      nexl[ind].pb(sz(A[0])), nexr[ind].pb(sz(A[1]));
    }
    build(A[0],2*ind,L,M), build(A[1],2*ind+1,M+1,R);
  }
  int query(int lo,int hi,int k,int ind=1,int L=0,int R=SZ-1) {
    if (L == R) return L;
    int M = (L+R)/2, t = nexl[ind][hi]-nexl[ind][lo];
    if (t >= k) return query(nexl[ind][lo],
          nexl[ind][hi],k,2*ind,L,M);
    return query(nexr[ind][lo],
      nexr[ind][hi],k-t,2*ind+1,M+1,R);
  }
};
```

SegTreeBeats.h

Description: Lazy SegTree supports modifications of the form ckmin(a_i,t) for all $l \le i \le r$, range max and sum queries. SZ is power of 2.
Time: $\mathcal{O}(\log N)$

a473ba, 61 lines

```
template<int SZ> struct SegTreeBeats { // declare globally
  int N, mx[2*SZ][2], maxCnt[2*SZ];
  ll sum[2*SZ];
  void pull(int ind) {
    F0R(i,2) mx[ind][i] = max(mx[2*ind][i],mx[2*ind+1][i]);
    maxCnt[ind] = 0;
    F0R(i,2) {
      if (mx[2*ind+i][0] == mx[ind][0])
        maxCnt[ind] += maxCnt[2*ind+i];
      else ckmax(mx[ind][1],mx[2*ind+i][0]);
    }
    sum[ind] = sum[2*ind]+sum[2*ind+1];
  }
  void build(vi& a, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) { R = (N = sz(a))-1; }
    if (L == R) {
      mx[ind][0] = sum[ind] = a[L];
      maxCnt[ind] = 1; mx[ind][1] = -1;
      return;
    }
    int M = (L+R)/2;
    build(a,2*ind,L,M); build(a,2*ind+1,M+1,R); pull(ind);
  }
  void push(int ind, int L, int R) {
    if (L == R) return;
    F0R(i,2) if (mx[2*ind^i][0] > mx[ind][0]) {
      sum[2*ind^i] -= (ll)maxCnt[2*ind^i]*
              (mx[2*ind^i][0]-mx[ind][0]);
      mx[2*ind^i][0] = mx[ind][0];
    }
  }
  void upd(int x, int y, int t, int ind=1, int L=0, int R=-1) {
    if (R == -1) R += N;
    if (R < x || y < L || mx[ind][0] <= t) return;
    push(ind,L,R);
    if (x <= L && R <= y && mx[ind][1] < t) {
      sum[ind] -= (ll)maxCnt[ind]*(mx[ind][0]-t);
      mx[ind][0] = t;
      return;
    }
    if (L == R) return;
    int M = (L+R)/2;
    upd(x,y,t,2*ind,L,M); upd(x,y,t,2*ind+1,M+1,R); pull(ind);
  }
  ll qsum(int x, int y, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) R += N;
    if (R < x || y < L) return 0;
    push(ind,L,R);
    if (x <= L && R <= y) return sum[ind];
    int M = (L+R)/2;
    return qsum(x,y,2*ind,L,M)+qsum(x,y,2*ind+1,M+1,R);
  }
  int qmax(int x, int y, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) R += N;
    if (R < x || y < L) return -1;
```

```
    push(ind,L,R);
    if (x <= L && R <= y) return mx[ind][0];
    int M = (L+R)/2;
    return max(qmax(x,y,2*ind,L,M),qmax(x,y,2*ind+1,M+1,R));
  }
};
```

PSeg.h

Description: Persistent min segtree with lazy updates, no propagation. If making d a vector then save the results of upd and build in local variables first to avoid issues when vector resizes in C++14 or lower.
Memory: $\mathcal{O}(N + Q \log N)$

869f19, 46 lines

```
template<class T, int SZ> struct pseg {
  static const int LIM = 2e7;
  struct node {
    int l, r; T val = 0, lazy = 0;
    void inc(T x) { lazy += x; }
    T get() { return val+lazy; }
  };
  node d[LIM]; int nex = 0;
  int copy(int c) { d[nex] = d[c]; return nex++; }
  T comb(T a, T b) { return min(a,b); }
  void pull(int c) { d[c].val =
    comb(d[d[c].l].get(), d[d[c].r].get()); }
  //// MAIN FUNCTIONS
  T query(int c, int lo, int hi, int L, int R) {
    if (lo <= L && R <= hi) return d[c].get();
    if (R < lo || hi < L) return MOD;
    int M = (L+R)/2;
    return d[c].lazy+comb(query(d[c].l,lo,hi,L,M),
            query(d[c].r,lo,hi,M+1,R));
  }
  int upd(int c, int lo, int hi, T v, int L, int R) {
    if (R < lo || hi < L) return c;
    int x = copy(c);
    if (lo <= L && R <= hi) { d[x].inc(v); return x; }
    int M = (L+R)/2;
    d[x].l = upd(d[x].l,lo,hi,v,L,M);
    d[x].r = upd(d[x].r,lo,hi,v,M+1,R);
    pull(x); return x;
  }
  int build(const vector<T>& arr, int L, int R) {
    int c = nex++;
    if (L == R) {
      if (L < sz(arr)) d[c].val = arr[L];
      return c;
    }
    int M = (L+R)/2;
    d[c].l = build(arr,L,M), d[c].r = build(arr,M+1,R);
    pull(c); return c;
  }
  vi loc; //// PUBLIC
  void upd(int lo, int hi, T v) {
    loc.pb(upd(loc.bk,lo,hi,v,0,SZ-1)); }
  T query(int ti, int lo, int hi) {
    return query(loc[ti],lo,hi,0,SZ-1); }
  void build(const vector<T>&arr) {loc.pb(build(arr,0,SZ-1));}
};
```

Treap.h

Description: Easy BBST. Use split and merge to implement insert and delete.
Time: $\mathcal{O}(\log N)$

b2348e, 63 lines

```
typedef struct tnode* pt;
struct tnode {
  int pri, val; pt c[2]; // essential
  int sz; ll sum; // for range queries
  bool flip = 0; // lazy update
  tnode (int _val) {
    pri = rand()+(rand()<<15); sum = val = _val;
    sz = 1; c[0] = c[1] = NULL;
  }
};
int getsz(pt x) { return x?x->sz:0; }
ll getsum(pt x) { return x?x->sum:0; }
pt prop(pt x) {
  if (!x || !x->flip) return x;
  swap(x->c[0],x->c[1]);
  x->flip = 0; FOR(i,2) if (x->c[i]) x->c[i]->flip ^= 1;
  return x;
}
pt calc(pt x) {
  assert(!x->flip); prop(x->c[0]), prop(x->c[1]);
  x->sz = 1+getsz(x->c[0])+getsz(x->c[1]);
  x->sum = x->val+getsum(x->c[0])+getsum(x->c[1]);
  return x;
}
void tour(pt x, vi& v) {
  if (!x) return;
  prop(x); tour(x->c[0],v); v.pb(x->val); tour(x->c[1],v);
}
pair<pt,pt> split(pt t, int v) { // >= v goes to the right
  if (!t) return {t,t};
  prop(t);
  if (t->val >= v) {
    auto p = split(t->c[0], v); t->c[0] = p.s;
    return {p.f,calc(t)};
  } else {
    auto p = split(t->c[1], v); t->c[1] = p.f;
    return {calc(t),p.s};
  }
}
pair<pt,pt> splitsz(pt t, int sz) { // sz nodes go to left
  if (!t) return {t,t};
  prop(t);
  if (getsz(t->c[0]) >= sz) {
    auto p = splitsz(t->c[0],sz); t->c[0] = p.s;
    return {p.f,calc(t)};
  } else {
    auto p=splitsz(t->c[1],sz-getsz(t->c[0])-1); t->c[1]=p.f;
    return {calc(t),p.s};
  }
}
pt merge(pt l, pt r) {
  if (!l || !r) return l?:r;
  prop(l), prop(r); pt t;
  if (l->pri > r->pri) l->c[1] = merge(l->c[1],r), t = l;
  else r->c[0] = merge(l,r->c[0]), t = r;
  return calc(t);
}
pt ins(pt x, int v) { // insert v
  auto a = split(x,v), b = split(a.s,v+1);
  return merge(a.f,merge(new tnode(v),b.s)); }
pt del(pt x, int v) { // delete v
  auto a = split(x,v), b = split(a.s,v+1);
  return merge(a.f,b.s); }
```

3.3 2D Range Queries

BIT2DOff.h

Description: point add and rectangle sum with offline 2D BIT. $x \in (0, SZ)$.
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(N \log^2 N)$

9d5283, 43 lines

```
template<class T, int SZ> struct OffBIT2D {
  bool mode = 0; // mode = 1 -> initialized
  vpi todo;
  int cnt[SZ], st[SZ];
  vi val, bit;
  void init() {
    assert(!mode); mode = 1;
    int lst[SZ]; FOR(i,SZ) lst[i] = cnt[i] = 0;
    sort(all(todo),[](const pi& a, const pi& b) {
      return a.s < b.s; });
    trav(t,todo) for (int x = t.f; x < SZ; x += x&-x)
      if (lst[x] != t.s) lst[x] = t.s, cnt[x] ++;
    int sum = 0;
    FOR(i,SZ) { lst[i] = 0; st[i] = sum; sum += cnt[i]; }
    val.rsz(sum); bit.rsz(sum); // store BITs in single vector
    trav(t,todo) for (int x = t.f; x < SZ; x += x&-x)
      if (lst[x] != t.s) lst[x] = t.s, val[st[x]++] = t.s;
  }
  int rank(int y, int l, int r) {
    return ub(begin(val)+l,begin(val)+r,y)-begin(val)-l; }
  void UPD(int x, int y, int t) {
    int z = st[x]-cnt[x]; // x-BIT = range from z to st[x]-1
    for (y = rank(y,z,st[x]); y <= cnt[x]; y += y&-y)
      bit[z+y-1] += t;
  }
  void upd(int x, int y, int t) {
    if (!mode) todo.pb({x,y});
    else for (; x < SZ; x += x&-x) UPD(x,y,t);
  }
  int QUERY(int x, int y) {
    int z = st[x]-cnt[x], res = 0;
    for (y = rank(y,z,st[x]); y; y -= y&-y) res += bit[z+y-1];
    return res;
  }
  int query(int x, int y) {
    assert(mode);
    int res = 0; for (; x; x -= x&-x) res += QUERY(x,y);
    return res;
  }
  int query(int xl, int xr, int yl, int yr) {
    return query(xr,yr)-query(xl-1,yr)
      -query(xr,yl-1)+query(xl-1,yl-1); }
};
```

Number Theory (4)

4.1 Modular Arithmetic

ModIntShort.h

Description: Modular arithmetic operations. To make faster, change add and subtract so that they don't require %.

d13e25, 15 lines

```
struct mi {
  int v; explicit operator int() const { return v; }
  mi() { v = 0; }
  mi(ll _v) : v(_v%MOD) { v += (v<0)*MOD; }
};
mi operator+(mi a, mi b) { return mi(a.v+b.v); }
mi operator-(mi a, mi b) { return mi(a.v-b.v); }
mi operator*(mi a, mi b) { return mi((ll)a.v*b.v); }
mi pow(mi a, ll p) {
  mi ans = 1; assert(p >= 0);
  for (; p; p /= 2, a = a*a) if (p&1) ans = ans*a;
  return ans;
}
mi inv(const mi& a) { assert(a.v != 0); return pow(a,MOD-2); }
mi operator/(mi a, mi b) { return a*inv(b); }
```

ModFact.h

Description: pre-compute factorial mod inverses, assumes MOD is prime and $SZ < MOD$.
Time: $\mathcal{O}(SZ)$

6e2f94, 10 lines

```
vi invs, fac, ifac;
void genFac(int SZ) {
  invs.rsz(SZ), fac.rsz(SZ), ifac.rsz(SZ);
  invs[1] = fac[0] = ifac[0] = 1;
  FOR(i,2,SZ) invs[i] = MOD-(ll)MOD/i*invs[MOD%i]%MOD;
  FOR(i,1,SZ) {
    fac[i] = (ll)fac[i-1]*i%MOD;
    ifac[i] = (ll)ifac[i-1]*invs[i]%MOD;
  }
}
```

ModMulLL.h

Description: Multiply two 64-bit integers mod another if 128-bit is not available. modMul is equivalent to `(ul)(__int128(a)*b%mod)`. Works for $0 \le a, b < mod < 2^{63}$.

aef5ab, 9 lines

```
typedef unsigned long long ul;
ul modMul(ul a, ul b, const ul mod) {
  ll ret = a*b-mod*(ul)((ld)a*b/mod);
  return ret+((ret<0)-(ret>=(ll)mod))*mod; }
ul modPow(ul a, ul b, const ul mod) {
  if (b == 0) return 1;
  ul res = modPow(a,b/2,mod); res = modMul(res,res,mod);
  return b&1 ? modMul(res,a,mod) : res;
}
```

ModFast.h

Description: Unused. Barrett reduction computes $a\%b$ about 4 times faster than usual, where b is constant but not known at compile time. Fails for $b = 1$.

9fb741, 8 lines

```
typedef unsigned long long ul;
typedef __uint128_t L;
struct ModFast {
  ul b, m; FastMod(ul b) : b(b), m(ul((L(1)<<64)/b)) {}
  ul reduce(ul a) {
    ul q = (ul)((L(m)*a)>>64), r = a-q*b;
    return r>=b?r-b:r; }
};
```

ModSqrt.h

Description: square root of integer mod a prime
Time: $\mathcal{O}\left(\log^2(MOD)\right)$

"ModInt.h" f2cda6, 14 lines

```
T sqrt(mi a) {
  mi p = pow(a,(MOD-1)/2); if (p != 1) return p == 0 ? 0 : -1;
  T s = MOD-1; int e = 0; while (s % 2 == 0) s /= 2, e ++;
  // find non-square
  mi n = 1; while (pow(n,(MOD-1)/2) == 1) n = T(n)+1;
  mi x = pow(a,(s+1)/2), b = pow(a,s), g = pow(n,s);
  int r = e;
  while (1) {
    mi B = b; int m = 0; while (B != 1) B *= B, m ++;
    if (m == 0) return min((T)x,MOD-(T)x);
    F0R(i,r-m-1) g *= g;
    x *= g; g *= g; b *= g; r = m;
  }
}
```

ModSum.h

Description: divsum computes $\sum_{i=0}^{to-1} \left\lfloor \frac{ki+c}{m} \right\rfloor$, modsum defined similarly
Time: $\mathcal{O}(\log m)$

50ee96, 11 lines

```
typedef unsigned long long ul;
ul sumsq(ul to) { return (to-1)*to/2; } // sum of 0..to-1
ul divsum(ul to, ul c, ul k, ul m) {
  ul res = k/m*sumsq(to)+c/m*to;
  k %= m; c %= m; if (!k) return res;
  ul to2 = (to*k+c)/m;
  return res+(to-1)*to2-divsum(to2,m-1-c,m,k);
}
ll modsum(ul to, ll c, ll k, ll m) {
  c = (c%m+m)%m, k = (k%m+m)%m;
  return to*c+k*sumsq(to)-m*divsum(to,c,k,m); }
```

4.2 Primality

4.2.1 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

4.2.2 Divisors

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Let $s(x) = \sum_{i=1}^{x} \phi(i)$. Then

$$s(n) = \frac{n(n+1)}{2} - \sum_{i=2}^{n} s\left(\left\lfloor \frac{n}{i} \right\rfloor\right).$$

PrimeSieve.h

Description: Tests primality up to SZ. Runs faster if only odd indices are stored.
Time: $\mathcal{O}(SZ \log \log SZ)$ or $\mathcal{O}(SZ)$

67a9f0, 21 lines

```
template<int SZ> struct Sieve {
  bitset<SZ> pri; vi pr;
  Sieve() {
    pri.set(); pri[0] = pri[1] = 0;
    for (int i = 4; i < SZ; i += 2) pri[i] = 0;
    for (int i = 3; i*i < SZ; i += 2) if (pri[i])
      for (int j = i*i; j < SZ; j += i*2) pri[j] = 0;
    F0R(i,SZ) if (pri[i]) pr.pb(i);
  }
  int sp[SZ]; // smallest prime that divides
  void linear() { // linear time, but above is faster
    memset(sp,0,sizeof sp);
    FOR(i,2,SZ) {
      if (sp[i] == 0) sp[i] = i, pr.pb(i);
      trav(p,pr) {
        if (p > sp[i] || i*p >= SZ) break;
        sp[i*p] = p;
      }
    }
  }
};
```

MillerRabin.h

Description: Deterministic primality test, works up to 2^{64}. For larger numbers, extend A randomly.

"ModMulLL.h" 7fd07a, 11 lines

```
bool prime(ul n) { // not ll!
  if (n < 2 || n % 6 % 4 != 1) return n-2 < 2;
  ul A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n>>s;
  trav(a,A) {   // ^ count trailing zeroes
    ul p = modPow(a,d,n), i = s;
    while (p != 1 && p != n-1 && a%n && i--) p = modMul(p,p,n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

FactorFast.h

Description: Pollard's rho factors integers up to 2^{60}. Returns primes in sorted order.
Time: $\mathcal{O}\left(N^{1/4}\right)$ gcd calls, less for numbers with small factors

"PrimeSieve.h", "MillerRabin.h", "ModMulLL.h" 16fcfd, 26 lines

```
Sieve<1<<20> S; // primes up to N^{1/3}
ul pollard(ul n) {
  auto f = [n](ul x) { return (modMul(x,x,n)+1)%n; };
  if (!(n&1)) return 2;
  for (ul i = 2;;++i) {
    ul x = i, y = f(x), p;
    while ((p = __gcd(n+y-x,n)) == 1) x = f(x), y = f(f(y));
    if (p != n) return p;
  }
}
vpl factor(ll d) {
  vpl res;
  trav(t,S.pr) {
    if ((ul)t*t > d) break;
    if (d%t == 0) {
      res.pb({t,0});
      while (d%t == 0) d /= t, res.bk.s ++;
    }
  }
  if (prime(d)) res.pb({d,1}), d = 1;
  if (d == 1) return res; // now a product of at most 2 primes
  ll c = pollard(d); d /= c; if (d > c) swap(d,c);
  if (c == d) res.pb({c,2});
  else res.pb({c,1}), res.pb({d,1});
  return res;
}
```

4.3 Euclidean Algorithm

FracInterval.h

Description: Given fractions $a < b$ with non-negative numerators and denominators, finds fraction f with lowest denominator such that $a < f < b$. Should work with all numbers less than 2^{62}.

1860f3, 6 lines

```
pl bet(pl a, pl b) {
  ll num = a.f/a.s; a.f -= num*a.s, b.f -= num*b.s;
  if (b.f > b.s) return {1+num,1};
  auto x = bet({b.s,b.f},{a.s,a.f});
```

```
  return {x.s+num*x.f,x.f};
}
```

Euclid.h

Description: euclid finds $\{x, y\}$ such that $ax + by = \gcd(a, b)$ and $|ax|, |by| \le \frac{ab}{\gcd(a,b)}$. Should work for $a, b < 2^{62}$

Time: $\mathcal{O}(\log ab)$

838bb4, 6 lines

```
pl euclid(ll a, ll b) {
  if (!b) return {1,0};
  pl p = euclid(b,a%b); return {p.s,p.f-a/b*p.s}; }
ll invGen(ll a, ll b) {
  pl p = euclid(a,b); assert(p.f*a+p.s*b == 1); // gcd is 1
  return p.f+(p.f<0)*b; }
```

Euclid2.h

Description: finds smallest $x \ge 0$ such that $L \le Ax \pmod{P} \le R$

0b3047, 9 lines

```
ll cdiv(ll x, ll y) { return (x+y-1)/y; }
ll bet(ll P, ll A, ll L, ll R) {
  if (A == 0) return L == 0 ? 0 : -1;
  ll c = cdiv(L,A); if (A*c <= R) return c;
  ll B = P%A; // P = k*A+B, L <= A(x-Ky)-By <= R
  // => -R <= By % A <= -L
  auto y = bet(A,B,A-R%A,A-L%A);
  return y == -1 ? y : cdiv(L+B*y,A)+P/A*y;
}
```

CRT.h

Description: Chinese Remainder Theorem. $a.f \pmod{a.s}, b.f \pmod{b.s} \implies ? \pmod{\operatorname{lcm}(a.s, b.s)}$. Should work for $ab < 2^{62}$.

"Euclid.h" 574c0e, 10 lines

```
pl CRT(pl a, pl b) {
  if (a.s < b.s) swap(a,b);
  ll x,y; tie(x,y) = euclid(a.s,b.s);
  ll g = a.s*x+b.s*y, l = a.s/g*b.s;
  if ((b.f-a.f)%g) return {-1,-1}; // no solution
  // ?*a.s+a.f \equiv b.f \pmod{b.s}
  // ?=(b.f-a.f)/g*(a.s/g)^{-1} \pmod{b.s/g}
  x = (b.f-a.f)%b.s*x%b.s/g*a.s+a.f;
  return {x+(x<0)*l,l};
}
```

4.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ \ b = k \cdot (2mn),\ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

Combinatorial (5)

5.1 Permutations

5.1.1 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

5.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

5.1.3 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

IntPerm.h

Description: Unused. Convert permutation of $\{0, 1, ..., N-1\}$ to integer in $[0, N!)$ and back.

Usage: `assert(encode(decode(5,37)) == 37);`

Time: $\mathcal{O}(N)$

b5645b, 19 lines

```
vi decode(int n, int a) {
  vi el(n), b; iota(all(el),0);
  FOR(i,n) {
    int z = a%sz(el);
    b.pb(el[z]); a /= sz(el);
    swap(el[z],el.bk); el.pop_back();
  }
  return b;
}
int encode(vi b) {
  int n = sz(b), a = 0, mul = 1;
  vi pos(n); iota(all(pos),0); vi el = pos;
  FOR(i,n) {
    int z = pos[b[i]]; a += mul*z; mul *= sz(el);
    swap(pos[el[z]],pos[el.bk]);
    swap(el[z],el.bk); el.pop_back();
  }
  return a;
}
```

PermGroup.h

Description: Used only once. Schreier-Sims lets you add a permutation to a group, count number of permutations in a group, and test whether a permutation is a member of a group.

Time: ?

590e00, 48 lines

```
int n;
vi inv(vi v) { vi V(sz(v)); FOR(i,sz(v)) V[v[i]]=i; return V; }
vi id() { vi v(n); iota(all(v),0); return v; }
vi operator*(const vi& a, const vi& b) {
  vi c(sz(a)); FOR(i,sz(a)) c[i] = a[b[i]];
  return c;
}

const int N = 15;
struct Group {
  bool flag[N];
  vi sigma[N]; // sigma[t][k] = t, sigma[t][x] = x if x > k
  vector<vi> gen;
  void clear(int p) {
    memset(flag,0, sizeof flag);
    flag[p] = 1; sigma[p] = id();
    gen.clear();
  }
} g[N];
bool check(const vi& cur, int k) {
  if (!k) return 1;
  int t = cur[k];
  return g[k].flag[t] ? check(inv(g[k].sigma[t])*cur,k-1) : 0;
}
void updateX(const vi& cur, int k);
void ins(const vi& cur, int k) {
  if (check(cur,k)) return;
  g[k].gen.pb(cur);
  FOR(i,n) if (g[k].flag[i]) updateX(cur*g[k].sigma[i],k);
}
void updateX(const vi& cur, int k) {
  int t = cur[k]; // if flag, fixes k -> k
  if (g[k].flag[t]) ins(inv(g[k].sigma[t])*cur,k-1);
  else {
    g[k].flag[t] = 1, g[k].sigma[t] = cur;
    trav(x,g[k].gen) updateX(x*cur,k);
  }
}
ll order(vector<vi> gen) {
  assert(sz(gen)); n = sz(gen[0]); FOR(i,n) g[i].clear(i);
  trav(a,gen) ins(a,n-1); // insert perms into group one by one
  ll tot = 1;
  FOR(i,n) {
    int cnt = 0; FOR(j,i+1) cnt += g[i].flag[j];
    tot *= cnt;
  }
  return tot;
}
```

5.2 Partitions and subsets

5.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

5.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.

5.3 General purpose numbers

5.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able). $B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

5.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

5.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

5.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

5.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

5.3.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

5.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

5.4 Young Tableaux

Let a **Young diagram** have shape $\lambda = (\lambda_1 \geq \cdots \geq \lambda_k)$, where λ_i equals the number of cells in the i-th (left-justified) row from the top. A **Young tableau** of shape λ is a filling of the $n = \sum \lambda_i$ cells with a permutation of $1 \ldots n$ such that each row and column is increasing.

Hook-Length Formula: For the cell in position (i,j), let $h_\lambda(i,j) = |\{(I,J) | i \leq I, j \leq J, (I = i \text{ or } J = j)\}|$. The number of Young tableaux of shape λ is equal to $f^\lambda = \frac{n!}{\prod h_\lambda(i,j)}$.

Schensted's Algorithm: converts a permutation σ of length n into a pair of Young Tableaux $(S(\sigma), T(\sigma))$ of the same shape. When inserting $x = \sigma_i$,

1. Add x to the first row of S by inserting x in place of the largest y with $x < y$. If y doesn't exist, push x to the end of the row, set the value of T at that position to be i, and stop.
2. Add y to the second row using the same rule, keep repeating as necessary.

All pairs $(S(\sigma), T(\sigma))$ of the same shape correspond to a unique σ, so $n! = \sum (f^\lambda)^2$. Also, $S(\sigma^R) = S(\sigma)^T$.

Let $d_k(\sigma), a_k(\sigma)$ be the lengths of the longest subseqs which are a union of k decreasing/ascending subseqs, respectively. Then $a_k(\sigma) = \sum_{i=1}^{k} \lambda_i, d_k(\sigma) = \sum_{i=1}^{k} \lambda_i^*$, where λ_i^* is size of the i-th column.

RSK.h
Description: Computes $S(\sigma)$ in Schensted's algorithm. All elements of A should be distinct.
Time: $\mathcal{O}(N^2)$ with naive, $\mathcal{O}\left(N\sqrt{N}\log N\right)$ with `fastRsk`. d08e90, 30 lines

```
/*Ex. \sigma=(5,2,3,1,4)
S(\sigma) = 5 -> 2 -> 2 3 -> 1 3 -> 1 3 4
                 5 -> 5      2        2
                             5        5
T(\sigma) = 1 -> 1 -> 1 3 -> 1 3 -> 1 3 5
                 2    2      2        2
                             4        4
*/

vector<vi> boundedRsk(const vi& A, int k) {
```

```
  vector<vi> h(k);
  FOR(i,sz(A)) {
    int x = A[i];
    FOR(j,k) {
      int p = lb(all(h[j]),x)-begin(h[j]);
      if (p == sz(h[j])) { h[j].pb(x); break; }
      swap(x,h[j][p]);
    }
  }
  return h;
}
vector<vi> fastRsk(vi A) {
  int rtn = (int)ceil(sqrt(sz(A)));
  auto ha = boundedRsk(A, rtn);
  reverse(all(A)); auto hb = boundedRsk(A, rtn);
  ha.rsz(sz(hb[0]));
  FOR(i,rtn,sz(hb[0])) for (int j = 0; i < sz(hb[j]); j++)
    ha[i].pb(hb[j][i]);
  return ha;
}
```

RSKrecover.h

Description: Recovers k increasing disjoint subsequences that cover the maximum possible number of elements from A, which must be a permutation of $[0,N)$.
Time: $\mathcal{O}(MN)$, M equals sum of sizes of subseqs

fcdbce, 43 lines

```
vector<vi> RSKrecover(vi A, int k) {
  int N = sz(A); vector<vi> h(k); // current tableau
  vector<tuple<int,int,int>> swaps; // Run RSK algo
  FOR(i,N) {
    int x = A[i];
    FOR(j,k) { // type 3 swaps: (y,z,x) -> (y,x,z) where x<y<z
      if (!sz(h[j]) || h[j].bk < x) { h[j].pb(x); break; }
      for (int y = sz(h[j])-1; ; --y) {
        if (y==0 || h[j][y-1]<x) { swap(x,h[j][y]); break; }
        swaps.eb(x,h[j][y-1],h[j][y]);
      } // also type 2 swaps, but undoing them doesn't change
    } // anything so no use storing
  }
  while (!sz(h[k-1])) k --;
  vi nxt(N+1,-1), prv(N+1,-1); // Linked list with k increasing
  // subseqs, initially the canonical representation of A
  FOR(i,k) { // just take first k rows
    prv[h[i][0]] = N;
    FOR(j,1,sz(h[i])) {
      int a = h[i][j-1], b = h[i][j];
      prv[b] = a, nxt[a] = b;
    }
    nxt[h[i].bk] = N;
  } // Replay the swaps backwards and adjust subseqs
  ROF(i,sz(swaps)) { // type 1 swaps: x<y<z, yxz -> yzx
    int x,y,z; tie(x,y,z) = swaps[i];
    if (nxt[x] != z) continue; // x and y not in same subseq
    if (nxt[y] == -1) { // swap x,y
      prv[y] = prv[x]; nxt[prv[y]] = y;
      nxt[y] = z; prv[z] = y;
      prv[x] = nxt[x] = -1;
    } else { // Splice lists; a->y->b and c->x->z->d
      nxt[x] = nxt[y]; prv[nxt[x]] = x;
      nxt[y] = z; prv[z] = y;
    } // becomes a->y->z->d and c->x->b.
  } // Reconstruct actual subseqs from linked list
  int cnt = 0; vi seq(N,-1); vector<vi> res(k);
  FOR(i,N) if (prv[i] != -1) {
    seq[i] = prv[i] == N ? cnt++ : seq[prv[i]];
    res[seq[i]].pb(i); // start new or continue old seq
  }
  return res;
}
```

5.5 Other

DeBruijnSeq.h

Description: Recursive FKM, given alphabet $[0,k)$ constructs cyclic string of length k^n that contains every length n string as substr.

a7faa5, 13 lines

```
vi dseq(int k, int n) {
  if (k == 1) return {0};
  vi res, aux(n+1);
  function<void(int,int)> gen = [&](int t, int p) {
    if (t > n) { // consider lyndon word of len p
      if (n%p == 0) FOR(i,1,p+1) res.pb(aux[i]);
    } else {
      aux[t] = aux[t-p]; gen(t+1,p);
      FOR(i,aux[t-p]+1,k) aux[t] = i, gen(t+1,t);
    }
  };
  gen(1,1); return res;
}
```

NimProduct.h

Description: Product of nimbers is associative, commutative, and distributive over addition (xor). Forms finite field of size 2^{2^k}. Application: Given 1D coin turning games G_1, G_2 $G_1 \times G_2$ is the 2D coin turning game defined as follows. If turning coins at $x_1, x_2, \ldots, x_m$ is legal in G_1 and $y_1, y_2, \ldots, y_n$ is legal in G_2, then turning coins at all positions (x_i, y_j) is legal assuming that the coin at (x_m, y_n) goes from heads to tails. Then the grundy function $g(x,y)$ of $G_1 \times G_2$ is $g_1(x) \times g_2(y)$.
Time: 64^2 xors per multiplication, memorize to speed up.

c7770b, 25 lines

```
using ul = uint64_t;
ul _nimProd[64][64];
ul nimProd(int i, int j) { // nim prod of 2^i, 2^j
  ul& u =_nimProd[i][j]; if (u) return u;
  if (!(i&j)) return u = 1ULL<<(i|j);
  int a = (i&j)&-(i&j); // 2^{2^k}
  return u=nimProd(i^a,j)^nimProd((i^a)|(a-1),(j^a)|(i&(a-1)));
  // 2^{2^k}*2^{2^k} = 2^{2^k}+2^{2^k-1}
  // 2^{2^i}*2^{2^j} = 2^{2^i+2^j} if i<j
}
struct nb { // nimber
  ul x; nb() { x = 0; }
  nb(ul _x): x(_x) {}
  explicit operator ul() { return x; }
  nb operator+(nb y) { return nb(x^y.x); }
  nb operator*(nb y) {
    ul res = 0;
    FOR(i,64)if(x>>i&1)FOR(j,64)if(y.x>>j&1)res^=nimProd(i,j);
    return nb(res);
  }
  friend nb pow(nb b, ul p) {
    nb res = 1; for (;p;p/=2,b=b*b) if (p&1) res = res*b;
    return res; } // b^{2^{2^A}-1}=1 where 2^{2^A} > b
  friend nb inv(nb b) { return pow(b,-2); }
};
```

MatroidIsect.h

Description: Computes a set of maximum size which is independent in both graphic and colorful matroids, aka a spanning forest where no two edges are of the same color. In general, construct the exchange graph and find a shortest path.
Time: $\mathcal{O}\left(GI^{1.5}\right)$ calls to oracles, where G is the size of the ground set and I is the size of the independent set

"DSU.h" f9557e, 84 lines

```
map<int,int> m;
struct Element {
  pi ed; int col;
  bool indep = 0; int ipos; // independent set pos
  Element(int u, int v, int c) { ed = {u,v}; col = c; }
};
vi iset; // independent set
vector<Element> gset; // ground set
struct GBasis {
  DSU D;
  void reset() { D.init(sz(m)); }
  void add(pi v) { assert(D.unite(v.f,v.s)); }
  bool indep(pi v) { return !D.sameSet(v.f,v.s); }
};
GBasis basis;
vector<GBasis> basisWo; // basis without

bool graphOracle(int ins) { return basis.indep(gset[ins].ed); }
bool graphOracle(int ins, int rem) {
  return basisWo[gset[rem].ipos].indep(gset[ins].ed); }
void prepGraphOracle() {
  basis.reset(); FOR(i,sz(iset)) basisWo[i].reset();
  FOR(i,sz(iset)) {
    pi v = gset[iset[i]].ed; basis.add(v);
    FOR(j,sz(iset)) if (i != j) basisWo[j].add(v);
  }
}
vector<bool> colUsed;
bool colorOracle(int ins) {
  ins = gset[ins].col; return !colUsed[ins]; }
bool colorOracle(int ins, int rem) {
  ins = gset[ins].col, rem = gset[rem].col;
  return !colUsed[ins] || ins == rem;
}
void prepColorOracle() {
  colUsed = vector<bool>(sz(colUsed),0);
  trav(t,iset) colUsed[gset[t].col] = 1;
}

bool augment() {
  prepGraphOracle(); prepColorOracle();
  vi par(sz(gset),MOD); queue<int> q;
  FOR(i,sz(gset)) if (!gset[i].indep && colorOracle(i))
    par[i] = -1, q.push(i);
  int lst = -1;
  while (sz(q)) {
    int cur = q.ft; q.pop();
    if (gset[cur].indep) {
      FOR(to,sz(gset)) if (!gset[to].indep && par[to] == MOD) {
        if (!colorOracle(to,cur)) continue;
        par[to] = cur; q.push(to);
      }
    } else {
      if (graphOracle(cur)) { lst = cur; break; }
      trav(to,iset) if (par[to] == MOD) {
        if (!graphOracle(cur,to)) continue;
        par[to] = cur; q.push(to);
      }
    }
  }
  if (lst == -1) return 0;
  do {
    gset[lst].indep ^= 1;
    lst = par[lst];
  } while (lst != -1);
  iset.clear();
  FOR(i,sz(gset)) if (gset[i].indep)
    gset[i].ipos = sz(iset), iset.pb(i);
  return 1; // increased sz(iset) by 1
}
```

```
int solve() {
  m.clear(); gset.clear(); iset.clear();
  int R; cin >> R; if (!R) exit(0); // # edges
  colUsed.rsz(R); basisWo.rsz(R);
  FOR(i,R) { // edges (a,b) and (c,d) of same col
    int a,b,c,d; cin >> a >> b >> c >> d;
    gset.pb(Element(a,b,i)), gset.pb(Element(c,d,i));
    m[a] = m[b] = m[c] = m[d] = 0;
  }
  int co = 0; trav(t,m) t.s = co++;
  trav(t,gset) t.ed.f = m[t.ed.f], t.ed.s = m[t.ed.s];
  while (augment()); // keep increasing size of indep set
  return 2*sz(iset);
}
```

Numerical (6)

6.1 Matrix

Matrix.h

Description: 2D matrix operations. Change d to array if possible.

1a09a2, 27 lines

```
template<class T> struct Mat {
  int r,c; vector<vector<T>> d;
  Mat(int _r, int _c) : r(_r), c(_c) {
    d.assign(r,vector<T>(c)); }
  Mat() : Mat(0,0) {}
  Mat(const vector<vector<T>>&_d) :
    r(sz(_d)), c(sz(_d[0])) { d = _d; }
  Mat& operator+=(const Mat& m) {
    FOR(i,r) FOR(j,c) d[i][j] += m.d[i][j];
    return *this; }
  Mat& operator-=(const Mat& m) {
    FOR(i,r) FOR(j,c) d[i][j] -= m.d[i][j];
    return *this; }
  Mat operator*(const Mat& m) {
    assert(c == m.r); Mat x(r,m.c);
    FOR(i,r) FOR(j,c) FOR(k,m.c)
      x.d[i][k] += d[i][j]*m.d[j][k];
    return x; }
  Mat operator+(const Mat& m) { return Mat(*this)+=m; }
  Mat operator-(const Mat& m) { return Mat(*this)-=m; }
  Mat& operator*=(const Mat& m) { return *this = (*this)*m; }
  friend Mat pow(Mat m, ll p) {
    Mat res(m.r,m.c); FOR(i,m.r) res.d[i][i] = 1;
    for (; p; p /= 2, m *= m) if (p&1) res *= m;
    return res;
  }
};
```

MatrixInv.h

Description: Uses gaussian elimination to convert into reduced row echelon form and calculates determinant. For determinant via arbitrary modulos, use a modified form of the Euclidean algorithm because modular inverse may not exist. If you have computed $A^{-1} \pmod{p^k}$, then the inverse $\pmod{p^{2k}}$ is $A^{-1}(2I - AA^{-1})$.
Time: $\mathcal{O}(N^3)$, determinant of 1000×1000 matrix of modints in 1 second if you reduce # of operations by half

"Matrix.h" 879b16, 39 lines

```
const ld EPS = 1e-12;
int getRow(Mat<ld>& m, int n, int i, int nex) {
  pair<ld,int> bes = {0,-1};
  FOR(j,nex,n) ckmax(bes,{abs(m.d[j][i]),j});
  return bes.f < EPS ? -1 : bes.s;
}
int getRow(Mat<mi>& m, int n, int i, int nex) {
  FOR(j,nex,n) if (m.d[j][i] != 0) return j;
  return -1;
}
template<class T> pair<T,int> gauss(Mat<T>& m) {
  int n = m.r, rank = 0, nex = 0;
  T prod = 1; // determinant
  FOR(i,n) {
    int row = getRow(m,n,i,nex);
    if (row == -1) { prod = 0; continue; }
    if (row != nex) prod *= -1, swap(m.d[row],m.d[nex]);
    prod *= m.d[nex][i]; rank ++;
    auto x = 1/m.d[nex][i]; FOR(k,i,m.c) m.d[nex][k] *= x;
    FOR(j,n) if (j != nex) {
      auto v = m.d[j][i]; if (v == 0) continue;
      FOR(k,i,m.c) m.d[j][k] -= v*m.d[nex][k];
    }
    nex ++;
  }
  return {prod,rank};
}
template<class T> Mat<T> inv(Mat<T> m) {
  assert(m.r == m.c);
  int n = m.r; Mat<T> x(n,2*n);
  FOR(i,n) {
    x.d[i][i+n] = 1;
    FOR(j,n) x.d[i][j] = m.d[i][j];
  }
  if (gauss(x).s != n) return Mat<T>();
  Mat<T> res(n,n);
  FOR(i,n) FOR(j,n) res.d[i][j] = x.d[i][j+n];
  return res;
}
```

MatrixTree.h

Description: Kirchhoff's Matrix Tree Theorem. Given adjacency matrix, calculates # of spanning trees.

"MatrixInv.h", "ModInt.h" 5b0a26, 11 lines

```
mi numSpan(Mat<mi> m) {
  int n = m.r; Mat<mi> res(n-1,n-1);
  FOR(i,n) FOR(j,i+1,n) {
    mi ed = m.d[i][j]; res.d[i][i] += ed;
    if (j != n-1) {
      res.d[j][j] += ed;
      res.d[i][j] -= ed, res.d[j][i] -= ed;
    }
  }
  return gauss(res).f;
}
```

6.2 Polynomials

Karatsuba.h

Description: Multiply two polynomials. FFT almost always works instead.
Time: $\mathcal{O}\left(N^{\log_2 3}\right)$

21f372, 24 lines

```
int size(int s) { return s > 1 ? 32-__builtin_clz(s-1) : 0; }
void karatsuba(ll* a, ll* b, ll* c, ll* t, int n) {
  int ca = 0, cb = 0; FOR(i,n) ca += !!a[i], cb += !!b[i];
  if (min(ca, cb) <= 1500/n) { // few numbers to multiply
    if (ca > cb) swap(a, b);
    FOR(i,n) if (a[i]) FOR(j,n) c[i+j] += a[i]*b[j];
  } else {
    int h = n >> 1;
    karatsuba(a, b, c, t, h); // a0*b0
    karatsuba(a+h, b+h, c+n, t, h); // a1*b1
    FOR(i,h) a[i] += a[i+h], b[i] += b[i+h];
    karatsuba(a, b, t, t+n, h); // (a0+a1)*(b0+b1)
    FOR(i,h) a[i] -= a[i+h], b[i] -= b[i+h];
    FOR(i,n) t[i] -= c[i]+c[i+n];
    FOR(i,n) c[i+h] += t[i], t[i] = 0;
  }
}
vl conv(vl a, vl b) {
  int sa = sz(a), sb = sz(b); if (!sa || !sb) return {};
  int n = 1<<size(max(sa,sb)); a.rsz(n), b.rsz(n);
  vl c(2*n), t(2*n); FOR(i,2*n) t[i] = 0;
  karatsuba(&a[0], &b[0], &c[0], &t[0], n);
  c.rsz(sa+sb-1); return c;
}
```

Poly.h

Description: Basic poly ops including division and interpolation.

"ModInt.h" d632b6, 59 lines

```
typedef mi T; using poly = vector<T>;
void remz(poly& p) { while (sz(p) && p.bk==0) p.pop_back(); }
poly rev(poly p) { reverse(all(p)); return p; }
poly shift(poly p, int x) { p.insert(begin(p),x,0); return p; }
poly RSZ(poly p, int x) { p.rsz(x); return p; }
T eval(const poly& p, T x) {
  T res = 0; ROF(i,sz(p)) res = x*res+p[i];
  return res; }
poly dif(const poly& p) { // differentiate
  poly res; FOR(i,1,sz(p)) res.pb(i*p[i]);
  return res; }
poly integ(const poly& p) { // integrate
  poly res(sz(p)+1); FOR(i,sz(p)) res[i+1] = p[i]/(i+1);
  return res; }

poly& operator+=(poly& l, const poly& r) {
  l.rsz(max(sz(l),sz(r))); FOR(i,sz(r)) l[i] += r[i];
  return l; }
poly& operator-=(poly& l, const poly& r) {
  l.rsz(max(sz(l),sz(r))); FOR(i,sz(r)) l[i] -= r[i];
  return l; }
poly& operator*=(poly& l, const T& r) { trav(t,l) t *= r;
  return l;  }
poly& operator/=(poly& l, const T& r) { trav(t,l) t /= r;
  return l; }
poly operator+(poly l, const poly& r) { return l += r; }
poly operator-(poly l, const poly& r) { return l -= r; }
poly operator-(poly l) { trav(t,l) t *= -1; return l; }
poly operator*(poly l, const T& r) { return l *= r; }
poly operator*(const T& r, const poly& l) { return l*r; }
poly operator/(poly l, const T& r) { return l /= r;  }
poly operator*(const poly& l, const poly& r) {
  if (!min(sz(l),sz(r))) return {};
  poly x(sz(l)+sz(r)-1);
  FOR(i,sz(l)) FOR(j,sz(r)) x[i+j] += l[i]*r[j];
  return x;
}
poly& operator*=(poly& l, const poly& r) { return l = l*r; }

pair<poly,poly> quoRem(poly a, poly b) {
  assert(sz(b)); auto B = b.bk; assert(B != 0);
  B = 1/B; trav(t,b) t *= B;
  norm(a); poly q(max(sz(a)-sz(b)+1,0));
  while (sz(a) >= sz(b)) {
    q[sz(a)-sz(b)] = a.bk;
    FOR(i,sz(b)) a[i+sz(a)-sz(b)] -= a.bk*b[i];
    norm(a);
  }
  trav(t,q) t *= B;
  return {q,a};
}
poly interpolate(vector<pair<T,T>> v) {
```

```
  poly ret, prod = {1}; trav(t,v) prod *= poly({-t.f,1});
  FOR(i,sz(v)) {
    T fac = 1; FOR(j,sz(v)) if (i != j) fac *= v[i].f-v[j].f;
    ret += v[i].s/fac*quoRem(prod,{-v[i].f,1}).f;
  }
  return ret;
}
```

PolyRoots.h

Description: Finds the real roots of a polynomial.
Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(N^2 \log(1/\epsilon)\right)$

"Poly.h" 75b07e, 20 lines

```
typedef ld T;
poly polyRoots(poly p, T xmin, T xmax) {
  if (sz(p) == 2) { return {-p[0]/p[1]}; }
  auto dr = polyRoots(dif(p),xmin,xmax);
  dr.pb(xmin-1); dr.pb(xmax+1); sort(all(dr));
  poly ret;
  FOR(i,sz(dr)-1) {
    T l = dr[i], h = dr[i+1];
    bool sign = eval(p,l) > 0;
    if (sign^(eval(p,h) > 0)) {
      FOR(it,60) { // while (h-l > 1e-8)
        auto m = (l+h)/2, f = eval(p,m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.pb((l+h)/2);
    }
  }
  return ret;
}
```

FFT.h

Description: Multiply two polynomials. For xor convolution don't multiply v by `roots[ind]`.
Time: $\mathcal{O}(N \log N)$

"ModInt.h" d3ad97, 39 lines

```
typedef complex<db> cd; typedef vector<cd> vcd;
const int root = 3; // for NTT
// const int MOD = (119<<23)+1; // = 998244353
// For p < 2^30 there is also e.g. (5<<25, 3), (7<<26, 3),
// (479<<21, 3) and (483<<21, 5). Last two are > 10^9.

int size(int s) { return s > 1 ? 32-__builtin_clz(s-1) : 0; }
void genRoots(vcd& roots) { // primitive n-th roots of unity
  int n = sz(roots); db ang = 2*PI/n;
  FOR(i,n) roots[i] = cd(cos(ang*i),sin(ang*i));
}
void genRoots(vmi& roots) {
  int n = sz(roots); mi r = pow(mi(root),(MOD-1)/n);
  roots[0] = 1; FOR(i,1,n) roots[i] = roots[i-1]*r;
}

template<class T> void fft(vector<T>& a,
  const vector<T>& roots, bool inv = 0) {
  int n = sz(a); // sort #s from 0 to n-1 by reverse binary
  for (int i = 1, j = 0; i < n; i++) {
    int bit = n>>1; for (; j&bit; bit /= 2) j ^= bit;
    j ^= bit; if (i < j) swap(a[i],a[j]);
  }
  for (int len = 2; len <= n; len *= 2)
    for (int i = 0; i < n; i += len) FOR(j,len/2) {
      int ind = n/len*j; if (inv && ind) ind = n-ind;
      auto u = a[i+j], v = a[i+j+len/2]*roots[ind];
      a[i+j] = u+v, a[i+j+len/2] = u-v;
    }
  if (inv) { T i = T(1)/T(n); trav(x,a) x *= i; }
}
template<class T> vector<T> mul(vector<T> a, vector<T> b) {
  if (!min(sz(a),sz(b))) return {};
  int s = sz(a)+sz(b)-1, n = 1<<size(s);
  vector<T> roots(n); genRoots(roots);
  a.rsz(n), fft(a,roots); b.rsz(n), fft(b,roots);
  FOR(i,n) a[i] *= b[i];
  fft(a,roots,1); a.rsz(s); return a;
}
```

FFTmod.h

Description: Multiply two polynomials with arbitrary MOD. Ensures precision by splitting into halves.
Time: ~0.8s when `sz(a)=sz(b)=1<<19`

"FFT.h" 8a6e6d, 29 lines

```
vl mulMod(const vl& a, const vl& b) {
  if (!min(sz(a),sz(b))) return {};
  int s = sz(a)+sz(b)-1, n = 1<<size(s), cut = sqrt(MOD);
  vcd roots(n); genRoots(roots);
  vcd ax(n), bx(n);
  // ax(x)=a1(x)+i*a0(x)
  FOR(i,sz(a)) ax[i] = cd((int)a[i]/cut, (int)a[i]%cut);
  // bx(x)=b1(x)+i*b0(x)
  FOR(i,sz(b)) bx[i] = cd((int)b[i]/cut, (int)b[i]%cut);
  fft(ax,roots), fft(bx,roots);
  vcd v1(n), v0(n);
  FOR(i,n) {
    int j = (i ? (n-i) : i);
    // v1 = a1*(b1+b0*cd(0,1));
    v1[i] = (ax[i]+conj(ax[j]))*cd(0.5,0)*bx[i];
    // v0 = a0*(b1+b0*cd(0,1));
    v0[i] = (ax[i]-conj(ax[j]))*cd(0,-0.5)*bx[i];
  }
  fft(v1,roots,1), fft(v0,roots,1);
  vl ret(n);
  FOR(i,n) {
    ll V2 = (ll)round(v1[i].real()); // a1*b1
    ll V1 = (ll)round(v1[i].imag())+(ll)round(v0[i].real());
    // a0*b1+a1*b0
    ll V0 = (ll)round(v0[i].imag()); // a0*b0
    ret[i] = ((V2%MOD*cut+V1)%MOD*cut+V0)%MOD;
  }
  ret.rsz(s); return ret;
}
```

PolyConv.h

Description: multiply two polynomials directly if small, otherwise use FFT

"Poly.h", "FFT.h" 15ccb2, 10 lines

```
bool small(const poly& a, const poly& b) { // multiply directly
  return (ll)sz(a)*sz(b) <= 500000; }
vmi smart(const vmi& a, const vmi& b) { return mul(a,b); }
vl smart(const vl& a, const vl& b) {
  auto X = mul(vcd(all(a)),vcd(all(b)));
  vl x(sz(X)); FOR(i,sz(X)) x[i] = round(X[i].real());
  return x;
}
poly conv(const poly& a, const poly& b) {
  return small(a,b) ? a*b : smart(a,b); }
```

PolyInv.h

Description: computes A^{-1} such that $AA^{-1} \equiv 1 \pmod{x^n}$. Newton's method: If you want $F(x) = 0$ and $F(Q_k) \equiv 0 \pmod{x^a}$ then $Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2a}}$ satisfies $F(Q_{k+1}) \equiv 0 \pmod{x^{2a}}$.
Usage: `vmi v={1,5,2,3,4}; ps(exp(2*log(v,9),9)); // squares v`
Time: $\mathcal{O}(N \log N)$

"PolyConv.h" a1f1c2, 32 lines

```
poly inv(poly A, int n) { // Q-(1/Q-A)/(-Q^{-2})
  poly B = {1/A[0]};
  while (sz(B) < n) {
    int x = 2*sz(B);
    B = RSZ(2*B-conv(RSZ(A,x),conv(B,B)),x);
  }
  return RSZ(B,n);
}
poly sqrt(const poly& A, int n) {  // Q-(Q^2-A)/(2Q)
  assert(A[0] == 1); poly B = {1};
  while (sz(B) < n) {
    int x = 2*sz(B);
    B = T(1)/T(2)*RSZ(B+mul(RSZ(A,x),inv(B,x)),x);
  }
  return RSZ(B,n);
}
pair<poly,poly> divi(const poly& f, const poly& g) {
  if (sz(f) < sz(g)) return {{},f};
  auto q = mul(inv(rev(g),sz(f)-sz(g)+1),rev(f));
  q = rev(RSZ(q,sz(f)-sz(g)+1));
  auto r = RSZ(f-mul(q,g),sz(g)-1); return {q,r};
}
poly log(poly A, int n) { assert(A[0] == 1); // (ln A)' = A'/A
  return RSZ(integ(conv(dif(A),inv(A,n))),n); }
poly exp(poly A, int n) { // Q-(lnQ-A)/(1/Q)
  assert(A[0] == 0); poly B = {1};
  while (sz(B) < n) {
    int x = 2*sz(B);
    B = RSZ(B+conv(B,RSZ(A,x)-log(B,x)),x);
  }
  return RSZ(B,n);
}
```

6.3 Misc

LinRec.h

Description: Berlekamp-Massey, computes linear recurrence of order N for sequence of $2N$ terms
Time: $\mathcal{O}\left(N^2\right)$

"Poly.h", "ModInt.h" a7ade8, 31 lines

```
struct LinRec {
  vmi x, C, rC;
  void init(const vmi& _x) { // original sequence
    x = _x; int n = sz(x), m = 0;
    vmi B; B = C = {1}; // B is fail vector
    mi b = 1; // B gives 0,0,0,...,b
    FOR(i,n) {
      m ++;
      mi d = x[i]; FOR(j,1,sz(C)) d += C[j]*x[i-j];
      if (d == 0) continue; // rec still works
      auto _B = C; C.rsz(max(sz(C),m+sz(B)));
      // subtract rec that gives 0,0,0,...,d
      mi coef = d/b; FOR(j,m,m+sz(B)) C[j] -= coef*B[j-m];
      if (sz(_B) < m+sz(B)) { B = _B; b = d; m = 0; }
    }
    rC = C; reverse(all(rC)); // poly for getPo
    C.erase(begin(C)); trav(t,C) t *= -1;
    // x[i]=sum_{j=0}^{sz(C)-1}C[j]*x[i-j-1]
  }
  vmi getPo(int n) {
    if (n == 0) return {1};
    vmi x = getPo(n/2); x = rem(x*x,rC);
    if (n&1) { vmi v = {0,1}; x = rem(x*v,rC); }
    return x;
  }
```

```
  mi eval(int n) { // evaluate n-th term
    vmi t = getPo(n);
    mi ans = 0; FOR(i,sz(t)) ans += t[i]*x[i];
    return ans;
  }
};
```

Integrate.h

Description: Integration of a function over an interval using Simpson's rule. The error should be proportional to dif^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

693e87, 7 lines

```
// db f(db x) { return x*x+3*x+1; }
db quad(db (*f)(db), db a, db b) {
  const int n = 1000;
  db dif = (b-a)/2/n, tot = f(a)+f(b);
  FOR(i,1,2*n) tot += f(a+i*dif)*(i&1?4:2);
  return tot*dif/3;
}
```

IntegrateAdaptive.h

Description: Unused. Fast integration using adaptive Simpson's rule.
Usage:
```
db z, y;
db h(db x) { return x*x + y*y + z*z <= 1; }
db g(db y) { ::y = y; return quad(h, -1, 1); }
db f(db z) { ::z = z; return quad(g, -1, 1); }
db sphereVol = quad(f,-1,1), pi = sphereVol*3/4;
```

b15f55, 10 lines

```
db simpson(db (*f)(db), db a, db b) {
  db c = (a+b)/2; return (f(a)+4*f(c)+f(b))*(b-a)/6; }
db rec(db (*f)(db), db a, db b, db eps, db S) {
  db c = (a+b)/2;
  db S1 = simpson(f,a,c), S2 = simpson(f,c,b), T = S1+S2;
  if (abs(T-S)<=15*eps || b-a<1e-10) return T+(T-S)/15;
  return rec(f,a,c,eps/2,S1)+rec(f,c,b,eps/2,S2);
}
db quad(db (*f)(db), db a, db b, db eps = 1e-8) {
  return rec(f,a,b,eps,simpson(f,a,b)); }
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage:
```
vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
```
Time: $\mathcal{O}(NM \cdot \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^N\right)$ in the general case.

db1e87, 74 lines

```
typedef db T; typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1/.0;

#define ltj(X) if (s==-1 || mp(X[j],N[j])<mp(X[s],N[s])) s=j
struct LPSolver {
  int m, n; // # contraints, # variables
  vi N, B; vvd D;
  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
    FOR(i,m) FOR(j,n) D[i][j] = A[i][j];
    FOR(i,m) {
      B[i] = n+i, D[i][n] = -1, D[i][n+1] = b[i];
      // B[i]: add basic variable for each constraint,
      // convert ineqs to eqs
      // D[i][n]: artificial variable for testing feasibility
    }
    FOR(j,n) {
      N[j] = j; // non-basic variables, all zero
      D[m][j] = -c[j]; // minimize -c^T x
    }
    N[n] = -1; D[m+1][n] = 1;
  }
  void pivot(int r, int s) { // r = row, c = column
    T *a = D[r].data(), inv = 1/a[s];
    FOR(i,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), binv = b[s]*inv;
      FOR(j,n+2) b[j] -= a[j]*binv;
      // make column corresponding to s all 0s
      b[s] = a[s]*binv; // swap N[s] with B[r]
    }
    // equation for r scaled so x_r coefficient equals 1
    FOR(j,n+2) if (j != s) D[r][j] *= inv;
    FOR(i,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv; swap(B[r], N[s]); // swap basic w/ non-basic
  }
  bool simplex(int phase) {
    int x = m+phase-1;
    while (1) {
      int s = -1; FOR(j,n+1) if (N[j] != -phase) ltj(D[x]);
      // find most negative col for nonbasic (nb) variable
      if (D[x][s] >= -eps) return 1;
      // can't get better sol by increasing nb variable
      int r = -1;
      FOR(i,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || mp(D[i][n+1] / D[i][s], B[i])
              < mp(D[r][n+1] / D[r][s], B[r])) r = i;
        // find smallest positive ratio
        // -> max increase in nonbasic variable
      }
      if (r == -1) return 0; // unbounded
      pivot(r,s);
    }
  }
  T solve(vd& x) {
    int r = 0; FOR(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) { // run simplex, find feasible x!=0
      pivot(r, n); // N[n] = -1 is artificial variable
      // initially set to smth large
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      // D[m+1][n+1] is max possible value of the negation of
      // artificial variable, optimal value should be zero
      // if exists feasible solution
      FOR(i,m) if (B[i] == -1) { // ?
        int s = 0; FOR(j,1,n+1) ltj(D[i]);
        pivot(i,s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    FOR(i,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

Graphs (7)

Erdos-Gallai: $d_1 \ge \cdots \ge d_n$ can be degree sequence of simple graph on n vertices iff their sum is even and $\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k), \forall 1 \le k \le n$.

7.1 Cycles

DirectedCycle.h

Description: use stack

1ddbcc, 24 lines

```
template<int SZ> struct DirCyc {
  vi adj[SZ], st;
  bool inSt[SZ], vis[SZ];
  vi dfs(int x) {
    st.pb(x); inSt[x] = vis[x] = 1;
    trav(i,adj[x]) {
      if (inSt[i]) {
        int ind = sz(st)-1; while (st[ind] != i) ind --;
        return vi(begin(st)+ind,end(st));
      } else if (!vis[i]) {
        vi v = dfs(i); if (sz(v)) return v;
      }
    }
    st.pop_back(); inSt[x] = 0;
  }
  vi init(int n) {
    st.clear(); FOR(i,n) inSt[i] = vis[i] = 0;
    FOR(i,n) if (!vis[i]) {
      vi v = dfs(i);
      if (sz(v)) return v;
    }
    return {};
  }
};
```

NegativeCycle.h

Description: use bellman ford

03972f, 11 lines

```
vi negCyc(int n, vector<pair<pi,int>> ed) {
  vl dist(n);  vi pre(n);
  FOR(i,n) trav(t,ed) if (ckmin(dist[t.f.s],dist[t.f.f]+t.s))
    pre[t.f.s] = t.f.f;
  trav(t,ed) if (ckmin(dist[t.f.s],dist[t.f.f]+t.s)) {
    int x = t.f.s; FOR(i,n) x = pre[x];
    vi cyc; for (int v=x;v!=x||sz(cyc)>1;v=pre[v])cyc.pb(v);
    reverse(all(cyc)); return cyc;
  }
  return {};
}
```

7.2 DSU

DSU.h

Description: Disjoint Set Union with path compression. Add edges and test connectivity. Use for Kruskal's minimum spanning tree.
Time: $\mathcal{O}(\alpha(N))$

cc5aa3, 11 lines

```
struct DSU {
  vi e; void init(int n) { e = vi(n,-1); }
  int get(int x) { return e[x] < 0 ? x : e[x] = get(e[x]); }
  bool sameSet(int a, int b) { return get(a) == get(b); }
  int size(int x) { return -e[get(x)]; }
  bool unite(int x, int y) { // union-by-rank
    x = get(x), y = get(y); if (x == y) return 0;
    if (e[x] > e[y]) swap(x,y);
    e[x] += e[y]; e[y] = x; return 1;
  }
};
```

ManhattanMST.h

Description: Given N points, returns up to $4N$ edges which are guaranteed to contain a minimum spanning tree for the graph with edge weights $w(p,q) = |p.x - q.x| + |p.y - q.y|$. Edges are in the form {distance, {src, dst}}. Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"DSU.h" 3aa99a, 23 lines

```
vector<pair<int,pi>> manhattanMst(vpi v) {
  vi id(sz(v)); iota(all(id),0);
  vector<pair<int,pi>> ed;
  FOR(k,4) {
    sort(all(id),[&](int i, int j) {
      return v[i].f+v[i].s < v[j].f+v[j].s; });
    map<int,int> sweep;
    trav(i,id) { // find neighbors for first octant
      for (auto it = sweep.lb(-v[i].s);
        it != end(sweep); sweep.erase(it++)) {
        int j = it->s;
        pi d={v[i].f-v[j].f,v[i].s-v[j].s};if (d.s>d.f)break;
        ed.pb({d.f+d.s,{i,j}});
      }
      sweep[-v[i].s] = i;
    }
    trav(p,v) {
      if (k&1) p.f *= -1;
      else swap(p.f,p.s);
    }
  }
  return ed;
}
```

7.3 Trees

LCAjump.h

Description: Calculates least common ancestor in tree with root R using binary jumping.
Time: $\mathcal{O}(N \log N)$ build, $\mathcal{O}(\log N)$ query

c28cba, 27 lines

```
template<int SZ> struct LCA {
  static const int BITS = 32-__builtin_clz(SZ);
  int N, R = 1, par[BITS][SZ], depth[SZ];
  vi adj[SZ];
  void ae(int u, int v) { adj[u].pb(v), adj[v].pb(u); }
  void dfs(int u, int prev){
    par[0][u] = prev; depth[u] = depth[prev]+1;
    trav(v,adj[u]) if (v != prev) dfs(v, u);
  }
  void init(int _N) {
    N = _N; dfs(R,0);
    FOR(k,1,BITS) FOR(i,1,N+1)
      par[k][i] = par[k-1][par[k-1][i]];
  }
  int getPar(int a, int b) {
    R0F(k,BITS) if (b&(1<<k)) a = par[k][a];
    return a; }
  int lca(int u, int v){
    if (depth[u] < depth[v]) swap(u,v);
    u = getPar(u,depth[u]-depth[v]);
    R0F(k,BITS) if (par[k][u] != par[k][v])
      u = par[k][u], v = par[k][v];
    return u == v ? u : par[0][u];
  }
  int dist(int u, int v) {
    return depth[u]+depth[v]-2*depth[lca(u,v)]; }
};
```

Centroid.h

Description: The centroid of a tree of size N is a vertex such that after removing it, all resulting subtrees have size at most $\frac{N}{2}$. Can support tree path queries and updates.
Time: $\mathcal{O}(N \log N)$

305933, 34 lines

```
template<int SZ> struct Centroid {
  vi adj[SZ];
  bool done[SZ]; // processed as centroid yet
  int sub[SZ], par[SZ]; // subtree size, current par
  pi cen[SZ]; // immediate centroid anc
  vi dist[SZ]; // dists to all centroid ancs
  void ae(int a, int b) { adj[a].pb(b), adj[b].pb(a); }
  void dfs(int x) {
    sub[x] = 1;
    trav(y,adj[x]) if (!done[y] && y != par[x]) {
      par[y] = x; dfs(y); sub[x] += sub[y]; }
  }
  int centroid(int x) {
    par[x] = -1; dfs(x);
    for (int sz = sub[x];;) {
      pi mx = {0,0};
      trav(y,adj[x]) if (!done[y] && y != par[x])
        ckmax(mx,{sub[y],y});
      if (mx.f*2 <= sz) return x;
      x = mx.s;
    }
  }
  void genDist(int x, int p) {
    dist[x].pb(dist[p].bk+1);
    trav(y,adj[x]) if (!done[y] && y != p) genDist(y,x);
  } // CEN = {centroid above x, label of centroid subtree}
  void gen(pi CEN, int x) {
    done[x = centroid(x)] = 1; cen[x] = CEN;
    dist[x].pb(0); int co = 0;
    trav(y,adj[x]) if (!done[y]) genDist(y,x);
    trav(y,adj[x]) if (!done[y]) gen({x,co++},y);
  }
  void init() { gen({-1,0},1); }
};
```

HLD.h

Description: Heavy-Light Decomposition, add val to verts and query sum in path/subtree
Time: any tree path is split into $\mathcal{O}(\log N)$ parts

"LazySeg.h" 790bb5, 43 lines

```
template<int SZ, bool VALS_IN_EDGES> struct HLD {
  int N; vi adj[SZ];
  int par[SZ], sz[SZ], depth[SZ];
  int root[SZ], pos[SZ];
  void ae(int a, int b) { adj[a].pb(b), adj[b].pb(a); }
  void dfsSz(int v = 1) {
    if (par[v]) adj[v].erase(find(all(adj[v]),par[v]));
    sz[v] = 1;
    trav(u,adj[v]) {
      par[u] = v; depth[u] = depth[v]+1;
      dfsSz(u); sz[v] += sz[u];
      if (sz[u] > sz[adj[v][0]]) swap(u, adj[v][0]);
    }
  }
  void dfsHld(int v = 1) {
    static int t = 0; pos[v] = t++;
    trav(u,adj[v]) {
      root[u] = (u == adj[v][0] ? root[v] : u);
      dfsHld(u); }
  }
  void init(int _N) {
    N = _N; par[1] = depth[1] = 0; root[1] = 1;
    dfsSz(); dfsHld();
  }
  LazySeg<ll,SZ> tree;
  template <class BinaryOp>
  void processPath(int u, int v, BinaryOp op) {
    for (; root[u] != root[v]; v = par[root[v]]) {
      if (depth[root[u]] > depth[root[v]]) swap(u, v);
      op(pos[root[v]], pos[v]); }
    if (depth[u] > depth[v]) swap(u, v);
    op(pos[u]+VALS_IN_EDGES, pos[v]);
  }
  void modifyPath(int u, int v, int val) {
    processPath(u,v,[this, &val](int l,int r) {
      tree.upd(l,r,val); }); }
  void modifySubtree(int v, int val) {
    tree.upd(pos[v]+VALS_IN_EDGES,pos[v]+sz[v]-1,val); }
  ll queryPath(int u, int v) {
    ll res = 0; processPath(u,v,[this,&res](int l,int r) {
      res += tree.qsum(l,r); });
    return res; }
};
```

7.3.1 SqrtDecompton

HLD generally suffices. If not, here are some common strategies:

- Rebuild the tree after every $\sqrt{N}$ queries.
- Consider vertices with $>$ or $<$ $\sqrt{N}$ degree separately.
- For subtree updates, note that there are $O(\sqrt{N})$ distinct sizes among child subtrees of any node.

Block Tree: Use a DFS to split edges into contiguous groups of size $\sqrt{N}$ to $2\sqrt{N}$.

Mo's Algorithm for Tree Paths: Maintain an array of vertices where each one appears twice, once when a DFS enters the vertex (st) and one when the DFS exists (en). For a tree path $u \leftrightarrow v$ such that st[u]<st[v],

- If u is an ancestor of v, query [st[u],st[v]].
- Otherwise, query [en[u],st[v]] and consider $LCA(u,v)$ separately.

7.4 DFS Algorithms

SCC.h

Description: Kosaraju's Algorithm, DFS two times to generate strongly connected components in topological order. a,b in same component if both $a \to b$ and $b \to a$ exist.
Time: $\mathcal{O}(N+M)$

6f2369, 18 lines

```
template<int SZ> struct SCC {
  int N, comp[SZ];
  vi adj[SZ], radj[SZ], todo, allComp;
  bitset<SZ> visit;
  void ae(int a, int b) { adj[a].pb(b), radj[b].pb(a); }
  void dfs(int v) {
    visit[v] = 1; trav(w,adj[v]) if (!visit[w]) dfs(w);
```

```
    todo.pb(v); }
  void dfs2(int v, int val) {
    comp[v] = val;
    trav(w,radj[v]) if (comp[w] == -1) dfs2(w,val); }
  void init(int _N) { // fills allComp
    N = _N; FOR(i,N) comp[i] = -1, visit[i] = 0;
    FOR(i,N) if (!visit[i]) dfs(i);
    reverse(all(todo));
    trav(i,todo) if (comp[i] == -1) dfs2(i,i), allComp.pb(i);
  }
};
```

SCC2.h

Description: Tarjan's Algorithm, generate SCCs in topological order.
Time: $\mathcal{O}(N+M)$

01e4ea, 19 lines

```
template<int SZ> struct SCC {
  vi adj[SZ], allComp, st;
  int N, val[SZ], comp[SZ];
  void ae(int u, int v) { adj[u].pb(v); }
  int dfs(int u) {
    static int ti = 0; int low = val[u] = ++ti; st.pb(u);
    trav(i,adj[u]) if (comp[i] < 0) ckmin(low,val[i]?:dfs(i));
    if (low == val[u]) {
      allComp.pb(u); int x;
      do { comp[x=st.bk] = u; st.pop_back(); } while (x!=u);
    }
    return val[u] = low;
  }
  void init(int _N) {
    N = _N; FOR(i,N) val[i] = 0, comp[i] = -1;
    FOR(i,N) if (comp[i] < 0) dfs(i);
    reverse(all(allComp));
  }
};
```

TwoSAT.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a\|\|b)\&\&(!a\|\|c)\&\&(d\|\|!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).
Usage: TwoSat ts;

```
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.setVal(2); // Var 2 is true
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(N); // Returns true iff it is solvable
ts.ans[0..N-1] holds the assigned values to the vars
```

"SCC.h" b2dd9d, 34 lines

```
template<int SZ> struct TwoSat {
  SCC<2*SZ> S;
  bitset<SZ> ans;
  int N = 0;
  int addVar() { return N++; }
  void either(int x, int y) {
    x = max(2*x,-1-2*x), y = max(2*y,-1-2*y);
    S.ae(x^1,y); S.ae(y^1,x);
  }
  void implies(int x, int y) { either(~x,y); }
  void setVal(int x) { either(x,x); }
  void atMostOne(const vi& li) {
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    FOR(i,2,sz(li)) {
      int next = addVar();
      either(cur,~li[i]);either(cur,next);either(~li[i],next);
      cur = ~next;
    }
    either(cur,~li[1]);
  }
  bool solve(int _N) {
    if (_N != -1) N = _N;
    S.init(2*N);
    for (int i = 0; i < 2*N; i += 2)
      if (S.comp[i] == S.comp[i^1]) return 0;
    reverse(all(S.allComp));
    vi tmp(2*N);
    trav(i,S.allComp) if (tmp[i] == 0)
      tmp[i] = 1, tmp[S.comp[i^1]] = -1;
    FOR(i,N) if (tmp[S.comp[2*i]] == 1) ans[i] = 1;
    return 1;
  }
};
```

EulerPath.h

Description: Eulerian path for both directed and undirected graphs, if it exists.
Time: $\mathcal{O}(N+M)$

ec081d, 26 lines

```
template<int SZ, bool directed> struct Euler {
  int N, M = 0;
  vpi adj[SZ]; vpi::iterator its[SZ];
  vector<bool> used;
  void ae(int a, int b) {
    if (directed) adj[a].pb({b,M});
    else adj[a].pb({b,M}), adj[b].pb({a,M});
    used.pb(0); M ++;
  }
  vpi solve(int _N, int src = 1) {
    N = _N; FOR(i,1,N+1) its[i] = begin(adj[i]);
    vector<pair<pi,int>> ret, s = {{{src,-1},-1}};
    while (sz(s)) {
      int x = s.bk.f.f;
      auto& it = its[x], en = end(adj[x]);
      while (it != en && used[it->s]) it ++;
      if (it == en) {
        if (sz(ret) && ret.bk.f.s != s.bk.f.f) return {};
        ret.pb(s.bk), s.pop_back();
      } else { s.pb({{it->f,x},it->s}); used[it->s] = 1; }
    }
    if (sz(ret) != M+1) return {};
    vpi ans; trav(t,ret) ans.pb({t.f.f,t.s});
    reverse(all(ans)); return ans;
  }
};
```

BCC.h

Description: Biconnected components, removing any vertex in component doesn't disconnect it. To get block-cut tree, create a bipartite graph with the original vertices on the left and a vertex for each BCC on the right. Draw edge $u \leftrightarrow v$ if u is contained within the BCC for v.
Time: $\mathcal{O}(N+M)$

74680e, 29 lines

```
template<int SZ> struct BCC {
  vpi adj[SZ], ed;
  void ae(int u, int v) {
    adj[u].pb({v,sz(ed)}), adj[v].pb({u,sz(ed)});, ed.pb({u,v}); }
  int N, disc[SZ];
  vi st; vector<vi> bccs; // edges for each bcc
  int bcc(int u, int p = -1) { // return lowest disc
    static int ti = 0; int low = disc[u] = ++ti, child = 0;
    trav(i,adj[u]) if (i.s != p) {
      if (!disc[i.f]) {
        child ++; st.pb(i.s);
        int LOW = bcc(i.f,i.s); ckmin(low,LOW);
        // if (disc[u] < LOW) -> bridge
        if (disc[u] <= LOW) { // get edges in bcc
          // if (p != -1 || child > 1) -> u is articulation pt
          bccs.eb(); vi& tmp = bccs.bk; // new bcc
          for (bool done = 0; !done; tmp.pb(st.bk),
            st.pop_back()) done |= st.bk == i.s;
        }
      } else if (disc[i.f] < disc[u])
        ckmin(low,disc[i.f]), st.pb(i.s);
    }
    return low;
  }
  void init(int _N) {
    N = _N; FOR(i,N) disc[i] = 0;
    FOR(i,N) if (!disc[i]) bcc(i);
  }
};
```

MaximalCliques.h

Description: Used only once. Finds all maximal cliques.
Time: $\mathcal{O}\left(3^{N/3}\right)$

8f66d2, 16 lines

```
typedef bitset<128> B; B adj[128];
int N;
// possibly in clique, not in clique, in clique
void cliques(B P = ~B(), B X={}, B R={}) {
  if (!P.any()) {
    if (!X.any()) // do smth with R
    return;
  }
  int q = (P|X)._Find_first();
  // clique must contain q or non-neighbor of q
  B cands = P&~adj[q];
  FOR(i,N) if (cands[i]) {
    R[i] = 1; cliques(P&adj[i],X&adj[i],R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

MaxClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). To find maximum independent set consider complement.
Time: Runs in about 1s for $n = 155$ and worst case random graphs ($p = .90$). Faster for sparse graphs.

c30f68, 43 lines

```
struct MaxClique {
  db limit=0.025, pk=0; // # of steps
  struct Vertex { int i, d=0; Vertex(int _i):i(_i){} };
  typedef vector<Vertex> vv; vv V;
  vector<bitset<200>> e;
  vector<vi> C; // colors
  vi qmax,q,S,old; // max/current clique, sum # steps up to lev
  void init(vv& r) {
    trav(v,r) v.d = 0;
    trav(v,r) trav(j,r) v.d += e[v.i][j.i]; // degree
    sort(all(r),[](Vertex a,Vertex b) { return a.d > b.d; });
    int mxD = r[0].d; FOR(i,sz(r)) r[i].d = min(i,mxD)+1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev-1]-old[lev]; old[lev] = S[lev-1];
    while (sz(R)) {
      if (sz(q)+R.bk.d <= sz(qmax)) return; // no larger clique
      q.pb(R.bk.i); // insert node with max col into clique
      vv T; trav(v,R) if (e[R.bk.i][v.i]) T.pb({v.i});
      if (sz(T)) {
        if (S[lev]++/++pk < limit) init(T); // recalc degs
        int j = 0, mxk = 1, mnk = max(sz(qmax)-sz(q)+1,1);
        C[1].clear(), C[2].clear();
        trav(v,T) {
```

```
      int k = 1; auto f = [&](int i) { return e[v.i][i]; };
      while (any_of(all(C[k]),f)) k++;
      if (k > mxk) mxk = k, C[mxk+1].clear(); // new set
      if (k < mnk) T[j++].i = v.i;
      C[k].pb(v.i);
    }
    if (j > 0) T[j-1].d = 0; // >=1 vert >=j part of clique
    FOR(k,mnk,mxk+1) trav(i,C[k]) T[j].i = i, T[j++].d = k;
    expand(T,lev+1);
   } else if (sz(q) > sz(qmax)) qmax = q;
   q.pop_back(), R.pop_back(); // R.bk not in set
  }
 }
 vi solve(vector<bitset<200>> conn) {
  e = conn; C.rsz(sz(e)+1), S.rsz(sz(C)), old = S;
  FOR(i,sz(e)) V.pb({i});
  init(V), expand(V); return qmax;
 }
};
```

7.5 Flows

Konig's Theorem: In a bipartite graph, max matching = min vertex cover.

Dilworth's Theorem: For any partially ordered set, the sizes of the largest antichain and of the smallest chain decomposition are equal. Equivalent to Konig's theorem on the bipartite graph (U, V, E) where $U = V = S$ and (u, v) is an edge when $u < v$.

Dinic.h

Description: Fast flow. After computing flow, edges $\{u, v\}$ such that $level[u] \neq -1$, $level[v] = -1$ are part of min cut.
Time: $\mathcal{O}(N^2M)$ flow, $\mathcal{O}\left(M\sqrt{N}\right)$ bipartite matching

94d932, 43 lines

```
template<int SZ> struct Dinic {
  typedef ll F; // flow type
  struct Edge { int to, rev; F flow, cap; };
  int N,s,t;
  vector<Edge> adj[SZ];
  typename vector<Edge>::iterator cur[SZ];
  void ae(int u, int v, F cap) {
    assert(cap >= 0); // don't try smth dumb
    Edge a{v, sz(adj[v]), 0, cap}, b{u, sz(adj[u]), 0, 0};
    adj[u].pb(a), adj[v].pb(b);
  }
  int level[SZ];
  bool bfs() { // level = shortest distance from source
    FOR(i,N) level[i] = -1, cur[i] = begin(adj[i]);
    queue<int> q({s}); level[s] = 0;
    while (sz(q)) {
      int u = q.ft; q.pop();
      trav(e,adj[u]) if (level[e.to] < 0 && e.flow < e.cap)
        q.push(e.to), level[e.to] = level[u]+1;
    }
    return level[t] >= 0;
  }
  F sendFlow(int v, F flow) {
    if (v == t) return flow;
    for (; cur[v] != end(adj[v]); cur[v]++) {
      Edge& e = *cur[v];
      if (level[e.to]!=level[v]+1||e.flow==e.cap) continue;
      auto df = sendFlow(e.to,min(flow,e.cap-e.flow));
      if (df) { // saturated at least one edge
        e.flow += df; adj[e.to][e.rev].flow -= df;
        return df;
      }
    }
    return 0;
  }
  F maxFlow(int _N, int _s, int _t) {
    N = _N, s = _s, t = _t; if (s == t) return -1;
    F tot = 0;
    while (bfs()) while (auto df = sendFlow(s,
      numeric_limits<F>::max())) tot += df;
    return tot;
  }
};
```

MCMF.h

Description: Minimum-cost maximum flow, assumes no negative cycles. Edges may be negative only during first run of SPFA.
Time: $\mathcal{O}(FM \log M)$ if caps are integers and F is max flow

c48252, 51 lines

```
template<class T> using pqg = priority_queue<T,vector<T>,
  ↪greater<T>>;
template<class T> T poll(pqg<T>& x) {
  T y = x.top(); x.pop(); return y; }

template<int SZ> struct MCMF {
  typedef ll F; typedef ll C;
  struct Edge { int to, rev; F flow, cap; C cost; };
  vector<Edge> adj[SZ];
  void ae(int u, int v, F cap, C cost) {
    assert(cap >= 0);
    Edge a{v,sz(adj[v]),0,cap,cost}, b{u,sz(adj[u]),0,0,-cost};
    adj[u].pb(a), adj[v].pb(b);
  }
  int N, s, t;
  pi pre[SZ]; // previous vertex, edge label on path
  pair<C,F> cost[SZ]; // tot cost of path, amount of flow
  C totCost, curCost; F totFlow;
  bool spfa() { // find lowest cost path to send flow through
    FOR(i,N) cost[i] = {numeric_limits<C>::max(),0};
    cost[s] = {0,numeric_limits<F>::max()};
    pqg<pair<C,int>> todo; todo.push({0,s});
    while (sz(todo)) {
      auto x = poll(todo); if (x.f > cost[x.s].f) continue;
      trav(a,adj[x.s]) if (a.flow < a.cap
        && ckmin(cost[a.to].f,x.f+a.cost)) {
        // if costs are doubles, add some small constant so
        // you don't traverse some ~0-weight cycle repeatedly
        pre[a.to] = {x.s,a.rev};
        cost[a.to].s = min(a.cap-a.flow,cost[x.s].s);
        todo.push({cost[a.to].f,a.to});
      }
    }
    return cost[t].s;
  }
  void backtrack() {
    F df = cost[t].s; totFlow += df;
    curCost += cost[t].f; totCost += curCost*df;
    for (int x = t; x != s; x = pre[x].f) {
      adj[x][pre[x].s].flow -= df;
      adj[pre[x].f][adj[x][pre[x].s].rev].flow += df;
    }
    FOR(i,N) trav(p,adj[i]) p.cost += cost[i].f-cost[p.to].f;
    // all reduced costs non-negative
    // edges on shortest path become 0
  }
  pair<F,C> calc(int _N, int _s, int _t) {
    N = _N; s = _s, t = _t; totFlow = totCost = curCost = 0;
    while (spfa()) backtrack();
    return {totFlow,totCost};
  }
};
```

GlobalMinCut.h

Description: Used only once. Stoer-Wagner, find a global minimum cut in an undirected graph as represented by an adjacency matrix.
Time: $\mathcal{O}(N^3)$

8d671d, 25 lines

```
pair<int, vi> GlobalMinCut(vector<vi> wei) {
  int N = sz(wei);
  vi par(N); iota(all(par),0);
  pair<int,vi> bes = {INT_MAX,{}};
  R0F(phase,N) {
    vi w = wei[0]; int lst = 0;
    vector<bool> add(N,1); FOR(i,1,N) if (par[i]==i) add[i]=0;
    F0R(i,phase) {
      int k = -1;
      FOR(j,1,N) if (!add[j] && (k==-1 || w[j]>w[k])) k = j;
      if (i+1 == phase) {
        if (w[k] < bes.f) {
          bes = {w[k],{}};
          FOR(j,N) if (par[j] == k) bes.s.pb(j);
        }
        FOR(j,N)wei[lst][j]+=wei[k][j],wei[j][lst]=wei[lst][j];
        FOR(j,N) if (par[j] == k) par[j] = lst; // merge
      } else { // greedily add closest
        FOR(j,N) w[j] += wei[k][j];
        add[lst = k] = 1;
      }
    }
  }
  return bes;
}
```

GomoryHu.h

Description: Returns edges of Gomory-Hu tree. Max flow between pair of vertices of undirected graph is given by min edge weight along tree path. Uses the fact that for any i, j, k, $\lambda_{ik} \geq \min(\lambda_{ij}, \lambda_{jk})$, where λ_{ij} denotes the flow between i and j.
Time: $\mathcal{O}(N)$ calls to Dinic

"Dinic.h" 3ec20a, 17 lines

```
template<int SZ> struct GomoryHu {
  vector<pair<pi,int>> ed;
  void ae(int a, int b, int c) { ed.pb({{a,b},c}); }
  vector<pair<pi,int>> init(int N) {
    vpi ret(N+1,mp(1,0));
    FOR(i,2,N+1) {
      Dinic<SZ> D;
      trav(t,ed) D.ae(t.f.f,t.f.s,t.s),D.ae(t.f.s,t.f.f,t.s);
      ret[i].s = D.maxFlow(N+1,i,ret[i].f);
      FOR(j,i+1,N+1) if (ret[j].f == ret[i].f
        && D.level[j] != -1) ret[j].f = i;
    }
    vector<pair<pi,int>> res;
    FOR(i,2,N+1) res.pb({{i,ret[i].f},ret[i].s});
    return res;
  }
};
```

7.6 Matching

DFSmatch.h

Description: naive bipartite matching
Time: $\mathcal{O}(NM)$

37ad8b, 23 lines

```
template<int SZ> struct MaxMatch {
  int N, flow = 0, match[SZ], rmatch[SZ];
  bitset<SZ> vis; vi adj[SZ];
  MaxMatch() {
    memset(match,0,sizeof match);
    memset(rmatch,0,sizeof rmatch);
  }
  void connect(int a, int b, bool c = 1) {
    if (c) match[a] = b, rmatch[b] = a;
    else match[a] = rmatch[b] = 0;
  }
  bool dfs(int x) {
    if (!x) return 1;
    if (vis[x]) return 0;
    vis[x] = 1;
    trav(t,adj[x]) if (t != match[x] && dfs(rmatch[t]))
      return connect(x,t),1;
    return 0;
  }
  void tri(int x) { vis.reset(); flow += dfs(x); }
  void init(int _N) { N = _N;
    FOR(i,1,N+1) if (!match[i]) tri(i); }
};
```

Hungarian.h

Description: Given array of (possibly negative) costs to complete each of N jobs w/ each of M workers ($N \leq M$), finds min cost to complete all jobs such that each worker is assigned to at most one job. Basically just Dijkstra with potentials.
Time: $\mathcal{O}\left(N^2 M\right)$

d8824c, 33 lines

```
int hungarian(const vector<vi>& a) {
  int n = sz(a)-1, m = sz(a[0])-1; // jobs 1..n, workers 1..m
  vi u(n+1), v(m+1); // potentials
  vi p(m+1); // p[j] -> job picked by worker j
  FOR(i,1,n+1) { // find alternating path with job i
    p[0] = i; int j0 = 0; // add "dummy" worker 0
    vi dist(m+1,INT_MAX), pre(m+1,-1);
    vector<bool> done(m+1, false);
    do { // dijkstra
      done[j0] = true; // fix dist[j0], update dists from j0
      int i0 = p[j0], j1; int delta = INT_MAX;
      FOR(j,1,m+1) if (!done[j]) {
        auto cur = a[i0][j]-u[i0]-v[j];
        if (ckmin(dist[j],cur)) pre[j] = j0;
        if (ckmin(delta,dist[j])) j1 = j;
      }
      FOR(j,m+1) { // subtract constant from all edges going
        // from done -> not done vertices, lowers all
        // remaining dists by constant
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]); // Potentials adjusted so all edge weights
    // are non-negative. Perfect matching has zero weight and
    // costs of augmenting paths do not change.
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1]; j0 = j1;
    }
  }
  return -v[0]; // min cost
}
```

UnweightedMatch.h

Description: Edmond's Blossom Algorithm. General unweighted matching with 1-based indexing.
Time: $\mathcal{O}\left(N^2 M\right)$

151cc7, 63 lines

```
template<int SZ> struct UnweightedMatch {
  int match[SZ], N;
  vi adj[SZ];
  void ae(int u, int v) { adj[u].pb(v), adj[v].pb(u); }
  void init(int _N) {
    N = _N; FOR(i,1,N+1) adj[i].clear(), match[i] = 0; }
  queue<int> Q;
  int par[SZ], vis[SZ], orig[SZ], aux[SZ], t;
  void augment(int u, int v) { // toggle edges on u-v path
    int pv = v, nv;
    do {
      pv = par[v]; nv = match[pv];
      match[v] = pv; match[pv] = v;
      v = nv;
    } while (u != pv);
  }
  int lca(int v, int w) { // find LCA in O(dist)
    ++t;
    while (1) {
      if (v) {
        if (aux[v] == t) return v;
        aux[v] = t; v = orig[par[match[v]]];
      }
      swap(v,w);
    }
  }
  void blossom(int v, int w, int a) {
    while (orig[v] != a) {
      par[v] = w; w = match[v]; // go other way around cycle
      if (vis[w] == 1) Q.push(w), vis[w] = 0;
      orig[v] = orig[w] = a; // merge into supernode
      v = par[w];
    }
  }
  bool bfs(int u) {
    FOR(i,N+1) par[i] = aux[i] = 0, vis[i] = -1, orig[i] = i;
    Q = queue<int>(); Q.push(u); vis[u] = t = 0;
    while (sz(Q)) {
      int v = Q.ft; Q.pop();
      trav(x,adj[v]) {
        if (vis[x] == -1) {
          par[x] = v; vis[x] = 1;
          if (!match[x]) return augment(u, x), 1;
          Q.push(match[x]); vis[match[x]] = 0;
        } else if (vis[x] == 0 && orig[v] != orig[x]) {
          int a = lca(orig[v], orig[x]); // odd cycle
          blossom(x,v,a); blossom(v,x,a);
        }
      }
    }
    return 0;
  }
  int calc() {
    int ans = 0; // find random matching, constant improvement
    vi V(N-1); iota(all(V),1); shuffle(all(V),rng);
    trav(x,V) if (!match[x]) trav(y,adj[x]) if (!match[y]) {
      match[x] = y, match[y] = x;
      ++ans; break;
    }
    FOR(i,1,N+1) if (!match[i] && bfs(i)) ++ans;
    return ans;
  }
};
```

7.7 Advanced

LCT.h

Description: Link-Cut Tree. Given a function $f(1 \ldots N) \to 1 \ldots N$, evaluates $f^b(a)$ for any a, b. `x->access()` brings `x` to the top and propagates it; its left subtree will be the path from `x` to the root and its right subtree will be empty. `sz` is for path queries; `sub`, `vsub` are for subtree queries.
Time: $\mathcal{O}(\log N)$

a16b3a, 126 lines

```
typedef struct snode* sn;
struct snode {
  //////// VARIABLES
  sn p, c[2]; // parent, children
  sn extra; // extra cycle node for "The Applicant"
  bool flip = 0; // subtree flipped or not
  int val, sz; // value in node, # nodes in current splay tree
  int sub, vsub = 0; // vsub stores sum of virtual children
  snode(int _val) : val(_val) {
    p = c[0] = c[1] = extra = NULL;
    calc();
  }
  friend int getSz(sn x) { return x?x->sz:0; }
  friend int getSub(sn x) { return x?x->sub:0; }
  void prop() { // lazy prop
    if (!flip) return;
    swap(c[0],c[1]);
    FOR(i,2) if (c[i]) c[i]->flip ^= 1;
    flip = 0;
  }
  void calc() { // recalc vals
    FOR(i,2) if (c[i]) c[i]->prop();
    sz = 1+getSz(c[0])+getSz(c[1]);
    sub = 1+getSub(c[0])+getSub(c[1])+vsub;
  }
  //////// SPLAY TREE OPERATIONS
  int dir() {
    if (!p) return -2;
    FOR(i,2) if (p->c[i] == this) return i;
    return -1; // p is path-parent pointer
  }
  bool isRoot() { return dir() < 0; }
  friend void setLink(sn x, sn y, int d) {
    if (y) y->p = x;
    if (d >= 0) x->c[d] = y;
  }
  void rot() { // assume p and p->p propagated
    assert(!isRoot()); int x = dir(); sn pa = p;
    setLink(pa->p, this, pa->dir());
    setLink(pa, c[x^1], x);
    setLink(this, pa, x^1);
    pa->calc(); calc();
  }
  void splay() {
    while (!isRoot() && !p->isRoot()) {
      p->p->prop(), p->prop(), prop();
      dir() == p->dir() ? p->rot() : rot();
      rot();
    }
    if (!isRoot()) p->prop(), prop(), rot();
    prop();
  }
  //////// BASE OPERATIONS
  void access() { // bring this to top of tree, propagate
    for (sn v = this, pre = NULL; v; v = v->p) {
      v->splay(); // now switch virtual children
      if (pre) v->vsub -= pre->sub;
      if (v->c[1]) v->vsub += v->c[1]->sub;
      v->c[1] = pre; v->calc(); pre = v;
    }
```

```
    splay(); assert(!c[1]); // right subtree is empty
  }
  void makeRoot() {
    access(); flip ^= 1; access();
    assert(!c[0] && !c[1]);
  }
  //////// QUERIES
  friend sn lca(sn x, sn y) {
    if (x == y) return x;
    x->access(), y->access(); if (!x->p) return NULL;
    x->splay(); return x->p?:x;
  }
  friend bool connected(sn x, sn y) { return lca(x,y); }
  int distRoot() { access(); return getSz(c[0]); }
  sn getRoot() { // get root of LCT component
    access(); auto a = this;
    while (a->c[0]) a = a->c[0], a->prop();
    a->access(); return a;
  }
  sn getPar(int b) { // get b-th parent
    access(); b = getSz(c[0])-b; assert(b >= 0);
    auto a = this;
    while (1) {
      int z = getSz(a->c[0]);
      if (b == z) { a->access(); return a; }
      if (b < z) a = a->c[0];
      else a = a->c[1], b -= z+1;
      a->prop();
    }
  }
  //////// MODIFICATIONS
  friend void link(sn x, sn y, bool force = 0) {
    assert(!connected(x,y));
    if (force) y->makeRoot(); // make x par of y
    else { y->access(); assert(!y->c[0]); }
    x->access(); setLink(y,x,0); y->calc();
  }
  friend void cut(sn y) { // cut y from its parent
    y->access(); assert(y->c[0]);
    y->c[0]->p = NULL; y->c[0] = NULL; y->calc();
  }
  friend void cut(sn x, sn y) { // if x, y adj in tree
    x->makeRoot(); y->access();
    assert(y->c[0] == x && !x->c[0] && !x->c[1]);
    cut(y);
  }
};
sn LCT[MX];

//////// THE APPLICANT SOLUTION
void setNex(sn a, sn b) { // set f[a] = b
  if (connected(a,b)) a->extra = b;
  else link(b,a);
}
void delNex(sn a) { // set f[a] = NULL
  auto t = a->getRoot();
  if (t == a) { t->extra = NULL; return; }
  cut(a); assert(t->extra);
  if (!connected(t,t->extra))
    link(t->extra,t), t->extra = NULL;
}
sn getPar(sn a, int b) { // get f^b[a]
  int d = a->distRoot(); if (b <= d) return a->getPar(b);
  b -= d+1; auto r = a->getRoot()->extra; assert(r);
  d = r->distRoot()+1; return r->getPar(b%d);
}
```

DirectedMST.h

Description: Chu-Liu-Edmonds algorithm. Computes minimum weight directed spanning tree rooted at r, edge from $par[i] \to i$ for all $i \neq r$. Use DSU with rollback if need to return edges.
Time: $\mathcal{O}(M \log M)$

"DSUrb.h" 4a0958, 64 lines

```
struct Edge { int a, b; ll w; };
struct Node { // lazy skew heap node
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, a->r = merge(b, a->r));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll,vi> dmst(int n, int r, const vector<Edge>& g) {
  DSUrb dsu; dsu.init(n);
  vector<Node*> heap(n); // store edges entering each vertex
  // in increasing order of weight
  trav(e,g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0; vi seen(n,-1); seen[r] = r;
  vpi in(n,{-1,-1}); // edge entering each vertex in MST
  vector<pair<int,vector<Edge>>> cycs;
  F0R(s,n) {
    int u = s, w;
    vector<pair<int,Edge>> path;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      seen[u] = s;
      Edge e = heap[u]->top(); path.pb({u,e});
      heap[u]->delta -= e.w, pop(heap[u]);
      res += e.w, u = dsu.get(e.a);
      if (seen[u] == s) { // found cycle, contract
        Node* cyc = 0; cycs.eb();
        do {
          cyc = merge(cyc, heap[w = path.bk.f]);
          cycs.bk.s.pb(path.bk.s);
          path.pop_back();
        } while (dsu.unite(u,w));
        u = dsu.get(u); heap[u] = cyc, seen[u] = -1;
        cycs.bk.f = u;
      }
    }
    trav(t,path) in[dsu.get(t.s.b)] = {t.s.a,t.s.b};
    // found path from root to s, done
  }
  while (sz(cycs)) { // expand cycs to restore sol
    auto c = cycs.bk; cycs.pop_back();
    pi inEdge = in[c.f];
    trav(t,c.s) dsu.rollback();
    trav(t,c.s) in[dsu.get(t.b)] = {t.a,t.b};
    in[dsu.get(inEdge.s)] = inEdge;
  }
  vi par(n); F0R(i,n) par[i] = in[i].f;
  // i == r ? in[i].s == -1 : in[i].s == i
  return {res,par};
}
```

DominatorTree.h

Description: Used only a few times. Assuming that all nodes are reachable from *root*, *a* dominates *b* iff every path from *root* to *b* passes through *a*.
Time: $\mathcal{O}(M \log N)$

080316, 43 lines

```
template<int SZ> struct Dominator {
  vi adj[SZ], ans[SZ]; // input edges, edges of dominator tree
  vi radj[SZ], child[SZ], sdomChild[SZ];
  int label[SZ], rlabel[SZ], sdom[SZ], dom[SZ], co = 0;
  int par[SZ], bes[SZ];
  void ae(int a, int b) { adj[a].pb(b); }
  int get(int x) { // DSU with path compression
    // get vertex with smallest sdom on path to root
    if (par[x] != x) {
      int t = get(par[x]); par[x] = par[par[x]];
      if (sdom[t] < sdom[bes[x]]) bes[x] = t;
    }
    return bes[x];
  }
  void dfs(int x) { // create DFS tree
    label[x] = ++co; rlabel[co] = x;
    sdom[co] = par[co] = bes[co] = co;
    trav(y,adj[x]) {
      if (!label[y]) {
        dfs(y);
        child[label[x]].pb(label[y]);
      }
      radj[label[y]].pb(label[x]);
    }
  }
  void init(int root) {
    dfs(root);
    ROF(i,1,co+1) {
      trav(j,radj[i]) ckmin(sdom[i],sdom[get(j)]);
      if (i > 1) sdomChild[sdom[i]].pb(i);
      trav(j,sdomChild[i]) {
        int k = get(j);
        if (sdom[j] == sdom[k]) dom[j] = sdom[j];
        else dom[j] = k;
      }
      trav(j,child[i]) par[j] = i;
    }
    FOR(i,2,co+1) {
      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
      ans[rlabel[dom[i]]].pb(rlabel[i]);
    }
  }
};
```

EdgeColor.h

Description: Used only once. Naive implementation of Misra & Gries edge coloring. By Vizing's Theorem, a simple graph with max degree d can be edge colored with at most $d+1$ colors
Time: $\mathcal{O}(N^2 M)$

a3b607, 44 lines

```
template<int SZ> struct EdgeColor {
  int N = 0, maxDeg = 0, adj[SZ][SZ], deg[SZ];
  EdgeColor() {
    memset(adj,0,sizeof adj);
    memset(deg,0,sizeof deg);
  }
  void ae(int a, int b, int c) {
    adj[a][b] = adj[b][a] = c; }
  int delEdge(int a, int b) {
    int c = adj[a][b]; adj[a][b] = adj[b][a] = 0;
    return c; }
  vector<bool> genCol(int x) {
```

```
    vector<bool> col(N+1); F0R(i,N) col[adj[x][i]] = 1;
    return col; }
  int freeCol(int u) {
    auto col = genCol(u);
    int x = 1; while (col[x]) x ++; return x;
  }
  void invert(int x, int d, int c) {
    F0R(i,N) if (adj[x][i] == d)
      delEdge(x,i), invert(i,c,d), ae(x,i,c);
  }
  void ae(int u, int v) {
    // check if you can add edge w/o doing any work
    assert(N); ckmax(maxDeg,max(++deg[u],++deg[v]));
    auto a = genCol(u), b = genCol(v);
    FOR(i,1,maxDeg+2) if (!a[i] && !b[i])
      return ae(u,v,i);
    vector<bool> use(N); vi fan = {v}; use[v] = 1;
    while (1) {
      auto col = genCol(fan.bk);
      if (sz(fan) > 1) col[adj[fan.bk][u]] = 0;
      int i=0; while (i<N && (use[i] || col[adj[u][i]])) i++;
      if (i < N) fan.pb(i), use[i] = 1;
      else break;
    }
    int c = freeCol(u), d = freeCol(fan.bk); invert(u,d,c);
    int i = 0; while (i < sz(fan) && genCol(fan[i])[d]
      && adj[u][fan[i]] != d) i ++;
    assert (i != sz(fan));
    F0R(j,i) ae(u,fan[j],delEdge(u,fan[j+1]));
    ae(u,fan[i],d);
  }
};
```

Geometry (8)

8.1 Primitives

PointShort.h

Description: Use in place of complex<T>.

ca9652, 29 lines

```
typedef ld T;
int sgn(T a) { return (a>0)-(a<0); }
T sq(T a) { return a*a; }

typedef pair<T,T> P; typedef vector<P> vP;
T norm(P p) { return sq(p.f)+sq(p.s); }
T abs(P p) { return sqrt(norm(p)); }
T arg(P p) { return atan2(p.s,p.f); }
P conj(P p) { return P(p.f,-p.s); }
P perp(P p) { return P(-p.s,p.f); }
P operator+(P l, P r) { return P(l.f+r.f,l.s+r.s); }
P operator-(P l, P r) { return P(l.f-r.f,l.s-r.s); }
P operator*(P l, T r) { return P(l.f*r,l.s*r); }
P operator/(P l, T r) { return P(l.f/r,l.s/r); }
P operator*(P l, P r) { // complex # multiplication
  return P(l.f*r.f-l.s*r.s,l.s*r.f+l.f*r.s); }
P operator/(P l, P r) { return l*conj(r)/norm(r); }

P unit(P p) { return p/abs(p); }
T dot(P a, P b) { return a.f*b.f+a.s*b.s; }
T cross(P a, P b) { return a.f*b.s-a.s*b.f; }
T cross(P p, P a, P b) { return cross(a-p,b-p); }
P reflect(P p, P a, P b) {
  return a+conj((p-a)/(b-a))*(b-a); }
P foot(P p, P a, P b) { return (p+reflect(p,a,b))/(T)2; }
bool onSeg(P p, P a, P b) {
  return cross(a,b,p) == 0 && dot(p-a,p-b) <= 0; }
ostream& operator<<(ostream& os, P p) {
  return os << "(" << p.f << "," << p.s << ")"; }
```

AngleCmp.h

Description: Sorts points in ccw order about origin in the same way as atan2, which returns real in $(-\pi,\pi]$ so points on negative x-axis come last.

"Point.h" c792bf, 4 lines

```
bool half(P x) { return x.s == 0 ? x.f < 0 : x.s > 0; }
bool angleCmp(P a, P b) {
  bool A = half(a), B = half(b);
  return A == B ? cross(a,b) > 0 : A < B; }
```

SegDist.h

Description: computes distance between P and line (segment) AB

"Point.h" d105ae, 6 lines

```
T lineDist(P p, P a, P b) {
  return abs(cross(p,a,b))/abs(a-b); }
T segDist(P p, P a, P b) {
  if (dot(p-a,b-a) <= 0) return abs(p-a);
  if (dot(p-b,a-b) <= 0) return abs(p-b);
  return lineDist(p,a,b); }
```

LineIsect.h

Description: Computes the intersection point(s) of lines AB, CD. Returns $\{-1,\{0,0\}\}$ if infinitely many, $\{0,\{0,0\}\}$ if none, $\{1,x\}$ if x is the unique point.

"Point.h" 517078, 6 lines

```
P ext(P a, P b, P c, P d) { // extension in asymptote
  T x = cross(a,b,c), y = cross(a,b,d);
  return (d*x-c*y)/(x-y); }
pair<int,P> lineIsect(P a, P b, P c, P d) {
  return cross(b-a,d-c) == 0 ? mp(-(cross(a,c,d) == 0),P())
  : mp(1,ext(a,b,c,d)); }
```

SegIsect.h

Description: computes the intersection point(s) of line segments AB, CD

"Point.h" 5933ca, 10 lines

```
vP segIsect(P a, P b, P c, P d) {
  T x = cross(a,b,c), y = cross(a,b,d);
  T X = cross(c,d,a), Y = cross(c,d,b);
  if (sgn(x)*sgn(y) < 0 && sgn(X)*sgn(Y) < 0)
    return {(d*x-c*y)/(x-y)}; // interior
  set<P> s;
  #define i(a,b,c) if (onSeg(a,b,c)) s.insert(a)
  i(a,c,d); i(b,c,d); i(c,a,b); i(d,a,b);
  return {all(s)};
}
```

8.2 Polygons

Centroid.h

Description: centroid (center of mass) and signed area of a polygon with constant mass per unit area

Time: $\mathcal{O}(N)$

"Point.h" ab93f2, 8 lines

```
pair<P,T> cenArea(const vP& v) {
  P cen(0,0); T area = 0;
  F0R(i,sz(v)) {
    int j = (i+1)%sz(v); T a = cross(v[i],v[j]);
    cen += a*(v[i]+v[j]); area += a;
  }
  return {cen/area/(T)3,area/2};
}
```

InPoly.h

Description: tests whether a point is inside, on, or outside of the perimeter of a polygon

Time: $\mathcal{O}(N)$

"Point.h" 8f2d6a, 10 lines

```
string inPoly(const vP& p, P z) {
  int n = sz(p), ans = 0;
  F0R(i,n) {
    P x = p[i], y = p[(i+1)%n];
    if (onSeg(z,x,y)) return "on";
    if (x.s > y.s) swap(x,y);
    if (x.s <= z.s && y.s > z.s && cross(z,x,y) > 0) ans ^= 1;
  }
  return ans ? "in" : "out";
}
```

ConvexHull.h

Description: top-bottom convex hull

Time: $\mathcal{O}(N \log N)$

"Point.h" 1dcf49, 18 lines

```
pair<vi,vi> ulHull(const vP& P) {
  vi p(sz(P)), u, l; iota(all(p), 0);
  sort(all(p), [&P](int a, int b) { return P[a] < P[b]; });
  trav(i,p) {
    #define ADDP(C, cmp) while (sz(C) > 1 && cross(\
      P[C[sz(C)-2]],P[C.bk],P[i]) cmp 0) C.pop_back(); C.pb(i);
    ADDP(u, >=); ADDP(l, <=);
  }
  return {u,l};
}
vi hullInd(const vP& P) {
  vi u,l; tie(u,l) = ulHull(P); if (sz(l) <= 1) return l;
  if (P[l[0]] == P[l[1]]) return {0};
  l.insert(end(l),1+rall(u)-1); return l;
}
vP hull(const vP& P) {
  vi v = hullInd(P); vP res; trav(t,v) res.pb(P[t]);
  return res; }
```

Diameter.h

Description: rotating caliphers, gives greatest distance between two points in P

Time: $\mathcal{O}(N)$ given convex hull

"ConvexHull.h" 38208a, 9 lines

```
ld diameter(vP P) {
  P = hull(P);
  int n = sz(P), ind = 1; ld ans = 0;
  F0R(i,n) for (int j = (i+1)%n;;ind = (ind+1)%n) {
    ckmax(ans,abs(P[i]-P[ind]));
    if (cross(P[j]-P[i],P[(ind+1)%n]-P[ind]) <= 0) break;
  }
  return ans;
}
```

HullTangents.h

Description: Given convex polygon with no three points collinear and a point strictly outside of it, computes the lower and upper tangents.

Time: $\mathcal{O}(\log N)$

"Point.h" 85b807, 37 lines

```
bool lower;
bool better(P a, P b, P c) {
  T z = cross(a,b,c);
  return lower ? z < 0 : z > 0;
}
int tangent(const vP& a, P b) {
  if (sz(a) == 1) return 0;
```

```
  int lo, hi;
  if (better(b,a[0],a[1])) {
    lo = 0, hi = sz(a)-1;
    while (lo < hi) {
      int mid = (lo+hi+1)/2;
      if (better(b,a[0],a[mid])) lo = mid;
      else hi = mid-1;
    }
    lo = 0;
  } else {
    lo = 1, hi = sz(a);
    while (lo < hi) {
      int mid = (lo+hi)/2;
      if (!better(b,a[0],a[mid])) lo = mid+1;
      else hi = mid;
    }
    hi = sz(a);
  }
  while (lo < hi) {
    int mid = (lo+hi)/2;
    if (better(b,a[mid],a[(mid+1)%sz(a)])) lo = mid+1;
    else hi = mid;
  }
  return lo%sz(a);
}
pi tangents(const vP& a, P b) {
  lower = 1; int x = tangent(a,b);
  lower = 0; int y = tangent(a,b);
  return {x,y};
}
```

LineHull.h

Description: `lineHull` accepts line and ccw convex polygon. If all vertices in poly lie to one side of the line, returns a vector of closest vertices to line as well as orientation of poly with respect to line (± 1 for above/below). Otherwise, returns the range of vertices that lie on or below the line. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log N)$

"Point.h" 34d6ab, 41 lines

```
typedef array<P,2> Line;
#define cmp(i,j) sgn(-dot(dir,poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i+1,i) >= 0 && cmp(i,i-1+n) < 0
int extrVertex(const vP& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo+1 < hi) {
    int m = (lo+hi)/2;
    if (extr(m)) return m;
    int ls = cmp(lo+1,lo), ms = cmp(m+1,m);
    (ls < ms || (ls == ms && ls == cmp(lo,m)) ? hi : lo) = m;
  }
  return lo;
}
vi same(Line line, const vP& poly, int a) {
  // points on same parallel as a
  int n = sz(poly); P dir = perp(line[0]-line[1]);
  if (cmp(a+n-1,a) == 0) return {(a+n-1)%n,a};
  if (cmp(a,a+1) == 0) return {a,(a+1)%n};
  return {a};
}
#define cmpL(i) sgn(cross(line[0],line[1],poly[i]))
pair<int,vi> lineHull(Line line, const vP& poly) {
  int n = sz(poly); assert(n>1);
  int endA = extrVertex(poly,perp(line[0]-line[1])); // lowest
  if (cmpL(endA) >= 0) return {1,same(line,poly,endA)};
  int endB = extrVertex(poly,perp(line[1]-line[0])); // highest
  if (cmpL(endB) <= 0) return {-1,same(line,poly,endB)};
  array<int,2> res;
  FOR(i,2) {
    int lo = endA, hi = endB; if (hi < lo) hi += n;
    while (lo < hi) {
      int m = (lo+hi+1)/2;
      if (cmpL(m%n) == cmpL(endA)) lo = m;
      else hi = m-1;
    }
    res[i] = lo%n; swap(endA,endB);
  }
  if (cmpL((res[0]+1)%n) == 0) res[0] = (res[0]+1)%n;
  return {0,{(res[1]+1)%n,res[0]}};
}
```

PolyUnion.java

Description: Compute union of two polygons and compute the area of the resulting figure with `java.awt.geom`.
Time: Runs quite quickly for two convex polygons with 10^5 vertices each.

7e1053, 55 lines

```
import java.awt.geom.*;
import java.io.*;
import java.util.*;
public class AreaIntersect {
  static int nextI(StringTokenizer st) {
    return Integer.parseInt(st.nextToken()); }
  static double nextD(StringTokenizer st) {
    return Double.parseDouble(st.nextToken()); }
  public static void main(String[] args) throws IOException {
    BufferedReader br = new BufferedReader(new
      ↪InputStreamReader(System.in));
    PrintWriter pw = new PrintWriter(new BufferedWriter(new
      ↪OutputStreamWriter(System.out)));
    StringTokenizer st = new StringTokenizer(br.readLine());
    int n = nextI(st); int m = nextI(st);
    double[] first = loadPolygon(n, br);
    double[] second = loadPolygon(m, br);
    Area ret = makeArea(first); ret.add(makeArea(second));
    pw.printf("%.9f\n", computeArea(ret)); pw.close();
  }
  public static double[] loadPolygon(int n, BufferedReader br)
    ↪throws IOException {
    double[] ret = new double[2*n];
    for(int i = 0; i < n; i++) {
      StringTokenizer st = new StringTokenizer(br.readLine());
      ret[2*i] = nextD(st); ret[2*i+1] = nextD(st);
    }
    return ret;
  }
  public static Area makeArea(double[] pts) {
    Path2D.Double p = new Path2D.Double();
    p.moveTo(pts[0],pts[1]);
    for (int i=2;i<pts.length;i+=2) p.lineTo(pts[i],pts[i+1]);
    p.closePath(); return new Area(p);
  }
  public static double computeArea(Area a) {
    PathIterator iter = a.getPathIterator(null);
    double[] buffer = new double[6]; double ret = 0;
    ArrayList<Double> ps = new ArrayList<Double>();
    while (!iter.isDone()) {
      switch (iter.currentSegment(buffer)) {
        case PathIterator.SEG_MOVETO:
        case PathIterator.SEG_LINETO:
          ps.add(buffer[0]); ps.add(buffer[1]);
          break;
        case PathIterator.SEG_CLOSE:
          ps.add(ps.get(0)); ps.add(ps.get(1));
          Double[] qs = ps.toArray(new Double[0]);
          for (int i = 0; i+2 < ps.size(); i += 2)
            ret -= qs[i]*qs[i+3]-qs[i+1]*qs[i+2];
          ps.clear();
          break;
      }
      iter.next();
    }
    return ret/2;
  }
}
```

8.3 Circles

Circle.h

Description: represent circle as {center,radius}

"Point.h" 6cd5ca, 5 lines

```
typedef pair<P,T> circ;
bool on(circ x, P y) { return abs(y-x.f) == x.s; }
bool in(circ x, P y) {  return abs(y-x.f) <= x.s; }
T arcLen(circ x, P a, P b) {
  P d = (a-x.f)/(b-x.f); return x.s*acos(d.f); }
```

CircleIsect.h

Description: circle intersection points and intersection area

"Circle.h" 5dfd7c, 15 lines

```
vP isectPoint(circ x, circ y) {
  T d = abs(x.f-y.f), a = x.s, b = y.s;
  if (d == 0) { assert(a != b); return {}; }
  T C = (a*a+d*d-b*b)/(2*a*d); if (abs(C) > 1) return {};
  T S = sqrt(1-C*C); P tmp = (y.f-x.f)/d*x.s;
  return {x.f+tmp*P(C,S),x.f+tmp*P(C,-S)};
}
T isectArea(circ x, circ y) { // not thoroughly tested
  T d = abs(x.f-y.f), a = x.s, b = y.s; if (a < b) swap(a,b);
  if (d >= a+b) return 0;
  if (d <= a-b) return PI*b*b;
  auto ca = (a*a+d*d-b*b)/(2*a*d), cb = (b*b+d*d-a*a)/(2*b*d);
  auto s = (a+b+d)/2, h = 2*sqrt(s*(s-a)*(s-b)*(s-d))/d;
  return a*a*acos(ca)+b*b*acos(cb)-d*h;
}
```

CircleTangents.h

Description: internal and external tangents between two circles

"Circle.h" bb7166, 22 lines

```
P tangent(P x, circ y, int t = 0) {
  y.s = abs(y.s); // abs needed because internal calls y.s < 0
  if (y.s == 0) return y.f;
  T d = abs(x-y.f);
  P a = pow(y.s/d,2)*(x-y.f)+y.f;
  P b = sqrt(d*d-y.s*y.s)/d*y.s*unit(x-y.f)*dir(PI/2);
  return t == 0 ? a+b : a-b;
}
vector<pair<P,P>> external(circ x, circ y) {
  vector<pair<P,P>> v;
  if (x.s == y.s) {
    P tmp = unit(x.f-y.f)*x.s*dir(PI/2);
    v.pb(mp(x.f+tmp,y.f+tmp));
    v.pb(mp(x.f-tmp,y.f-tmp));
  } else {
    P p = (y.s*x.f-x.s*y.f)/(y.s-x.s);
    FOR(i,2) v.pb({tangent(p,x,i),tangent(p,y,i)});
  }
  return v;
}
vector<pair<P,P>> internal(circ x, circ y) {
  x.s *= -1; return external(x,y); }
```

Circumcenter.h

Description: returns {circumcenter,circumradius}

"Circle.h" cfb851, 5 lines

```
circ ccCenter(P a, P b, P c) {
  b -= a; c -= a;
  P res = b*c*(conj(c)-conj(b))/(b*conj(c)-conj(b)*c);
  return {a+res,abs(res)};
}
```

MinEnclosingCirc.h

Description: minimum enclosing circle

Time: expected $\mathcal{O}(N)$

"Circumcenter.h" 53963d, 13 lines

```
circ mec(vP ps) {
  shuffle(all(ps), rng);
  P o = ps[0]; T r = 0, EPS = 1+1e-8;
  FOR(i,sz(ps)) if (abs(o-ps[i]) > r*EPS) {
    o = ps[i], r = 0; // point is on MEC
    FOR(j,i) if (abs(o-ps[j]) > r*EPS) {
      o = (ps[i]+ps[j])/2, r = abs(o-ps[i]);
      FOR(k,j) if (abs(o-ps[k]) > r*EPS)
        tie(o,r) = ccCenter(ps[i],ps[j],ps[k]);
    }
  }
  return {o,r};
}
```

8.4 Misc

ClosestPair.h

Description: line sweep to find two closest points

Time: $\mathcal{O}(N \log N)$

"Point.h" 34bbb1, 17 lines

```
pair<P,P> solve(vP v) {
  pair<ld,pair<P,P>> bes; bes.f = INF;
  set<P> S; int ind = 0;
  sort(all(v));
  FOR(i,sz(v)) {
    if (i && v[i] == v[i-1]) return {v[i],v[i]};
    for (; v[i].f-v[ind].f >= bes.f; ++ind)
      S.erase({v[ind].s,v[ind].f});
    for (auto it = S.ub({v[i].s-bes.f,INF});
      it != end(S) && it->f < v[i].s+bes.f; ++it) {
      P t = {it->s,it->f};
      ckmin(bes,{abs(t-v[i]),{t,v[i]}});
    }
    S.insert({v[i].s,v[i].f});
  }
  return bes.s;
}
```

KDtree.h

Description: find nearest neighbor to point and squared dist

Time: supposedly $\mathcal{O}(\log N)$ on average for randomly distributed points

"Point.h" 3a542a, 39 lines

```
struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *f = 0, *s = 0;
  T distance(const P& p) { // min squared dist to point p
    T x = min(max(p.f,x0),x1), y = min(max(p.s,y0),y1);
    return norm(P(x,y)-p); }
  Node(vP&& vp) : pt(vp[0]) {
    for (P p : vp) { ckmin(x0,p.f), ckmax(x1,p.f);
      ckmin(y0,p.s), ckmax(y1,p.s); }
    if (sz(vp) > 1) { // split on x if the box is
      // wider than high (not best heuristic...)
      if (x1-x0 >= y1-y0) sort(all(vp));
      else sort(all(vp),[](P a,P b) { return a.s < b.s; });
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      f = new Node({begin(vp),begin(vp)+half});
      s = new Node({half+all(vp)});
    }
  }
};
struct KDtree {
  Node* root; KDtree(const vP& vp):root(new Node({all(vp)})){}
  pair<T,P> search(Node *node, const P& p) {
    if (!node->f) { // should not find the point itself
      if (p == node->pt) return {INF, P()};
      return mp(norm(p-node->pt), node->pt);
    }
    Node *f = node->f, *s = node->s;
    T bf = f->distance(p), bs = s->distance(p);
    if (bf > bs) swap(bs, bf), swap(f, s);
    // search closest side f, other side if needed
    auto best = search(f,p);
    if (bs < best.f) ckmin(best,search(s,p));
    return best;
  }
  pair<T,P> nearest(const P& p) { return search(root,p); }
};
```

DelaunayIncremental.h

Description: Bowyer-Watson where not all points collinear. Works for $|x|, |y| \leq 10^4$, assuming that all circumradii in final triangulation are $\ll 10^9$.

Time: $\mathcal{O}(N^2 \log N)$

"DelaunayFast.h" 9ab4a7, 21 lines

```
const T BIG = 1e9; // >>(10^4)^2
vector<array<int,3>> triIncrement(vP v) {
  v.pb({-BIG,-BIG}); v.pb({BIG,0}); v.pb({0,BIG});
  vector<array<int,3>> ret, tmp;
  ret.pb({sz(v)-3,sz(v)-2,sz(v)-1});
  FOR(i,sz(v)-3) {
    map<pi,int> m;
    trav(a,ret) {
      if (inCircle(v[i],v[a[0]],v[a[1]],v[a[2]]))
        m[{a[0],a[1]}]++, m[{a[1],a[2]}]++, m[{a[0],a[2]}]++;
      else tmp.pb(a);
    }
    trav(a,m) if (a.s == 1) {
      array<int,3> x = {a.f.f,a.f.s,i};
      sort(all(x)); tmp.pb(x);
    }
    swap(ret,tmp); tmp.clear();
  }
  trav(a,ret) if (a[2] < sz(v)-3) tmp.pb(a);
  return tmp;
}
```

DelaunayFast.h

Description: Fast Delaunay triangulation assuming no duplicates and not all points collinear (in latter case, result will be empty). Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in ccw order. Each circumcircle will contain none of the input points.

Time: $\mathcal{O}(N \log N)$

"Point.h" 165069, 81 lines

```
// typedef ll T;
typedef __int128 lll; // (can be ll if coords are < 2e4)
bool inCircle(P p, P a, P b, P c) {
  a -= p, b -= p, c -= p; // assert(cross(a,b,c)>0);
  lll x = (lll)norm(a)*cross(b,c)+(lll)norm(b)*cross(c,a)
    +(lll)norm(c)*cross(a,b);
  return x*(cross(a,b,c)>0?1:-1) > 0;
}

P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point
typedef struct Quad* Q;
struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
};
Q makeEdge(P orig, P dest) {
  Q q[] = {new Quad{0,0,0,orig}, new Quad{0,0,0,arb},
        new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
  FOR(i,4) q[i]->o = q[-i & 3], q[i]->rot = q[(i+1) & 3];
  return *q;
}
void splice(Q a, Q b) { swap(a->o->rot->o, b->o->rot->o); swap(
  ↪a->o, b->o); }
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next()); splice(q->r(), b);
  return q;
}
pair<Q,Q> rec(const vP& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.bk);
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = cross(s[0], s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (cross(e->F(),H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s)-half});
  tie(B, rb) = rec({sz(s)-half+all(s)});
  while ((cross(B->p,H(A)) < 0 && (A = A->next())) ||
      (cross(A->p,H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (inCircle(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  while (1) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && inCircle(H(RC), H(LC))))
      base = connect(RC, base->r());
    else base = connect(base->r(), LC->r());
  }
  return {ra, rb};
}
vector<array<P,3>> triangulate(vP pts) {
  sort(all(pts)); assert(unique(all(pts)) == end(pts));
  if (sz(pts) < 2) return {};
  Q e = rec(pts).f; vector<Q> q = {e};
```

```
  while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.pb(c->p); \
  q.pb(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  int qi = 0; while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  vector<array<P,3>> ret(sz(pts)/3);
  FOR(i,sz(pts)) ret[i/3][i%3] = pts[i];
  return ret;
}
```

8.5 3D

Point3D.h

Description: Basic 3D geometry.

"Point.h" 087260, 81 lines

```
typedef array<T,3> P3;
typedef array<P3,3> tri;
typedef vector<P3> vP3;
T norm(const P3& x) {
  T sum = 0; FOR(i,3) sum += sq(x[i]);
  return sum; }
T abs(const P3& x) { return sqrt(norm(x)); }

P3& operator+=(P3& l, const P3& r) { FOR(i,3) l[i] += r[i];
  return l; }
P3& operator-=(P3& l, const P3& r) { FOR(i,3) l[i] -= r[i];
  return l; }
P3& operator*=(P3& l, const T& r) { FOR(i,3) l[i] *= r;
  return l; }
P3& operator/=(P3& l, const T& r) { FOR(i,3) l[i] /= r;
  return l; }
P3 operator-(P3 l) { l *= -1; return l; }
P3 operator+(P3 l, const P3& r) { return l += r; }
P3 operator-(P3 l, const P3& r) { return l -= r; }
P3 operator*(P3 l, const T& r) { return l *= r; }
P3 operator*(const T& r, const P3& l) { return l*r; }
P3 operator/(P3 l, const T& r) { return l /= r; }

P3 unit(const P3& x) { return x/abs(x); }
T dot(const P3& a, const P3& b) {
  T sum = 0; FOR(i,3) sum += a[i]*b[i];
  return sum; }
P3 cross(const P3& a, const P3& b) {
  return {a[1]*b[2]-a[2]*b[1],a[2]*b[0]-a[0]*b[2],
      a[0]*b[1]-a[1]*b[0]}; }
P3 cross(const P3& a, const P3& b, const P3& c) {
  return cross(b-a,c-a); }
P3 perp(const P3& a, const P3& b, const P3& c) {
  return unit(cross(a,b,c)); }

bool isMult(const P3& a, const P3& b) { // for long longs
  P3 c = cross(a,b); FOR(i,sz(c)) if (c[i] != 0) return 0;
  return 1; }
bool collinear(const P3& a, const P3& b, const P3& c) {
  return isMult(b-a,c-a); }
bool coplanar(const P3&a,const P3&b,const P3&c,const P3&d) {
  return isMult(cross(b-a,c-a),cross(b-a,d-a)); }
bool op(const P3& a, const P3& b) {
  int ind = 0; // going in opposite directions?
  FOR(i,1,3) if (std::abs(a[i]*b[i])>std::abs(a[ind]*b[ind]))
    ind = i;
  return a[ind]*b[ind] < 0;
}
// coplanar points, b0 and b1 on opposite sides of a0-a1?
bool opSide(const P3&a,const P3&b,const P3&c,const P3&d) {
  return op(cross(a,b,c),cross(a,b,d)); }
// coplanar points, is a in triangle b
bool inTri(const P3& a, const tri& b) {
  FOR(i,3)if(opSide(b[i],b[(i+1)%3],b[(i+2)%3],a))return 0;
  return 1; }

// point-seg dist
T psDist(const P3&p,const P3&a,const P3&b) {
  if (dot(a-p,a-b) <= 0) return abs(a-p);
  if (dot(b-p,b-a) <= 0) return abs(b-p);
  return abs(cross(p,a,b))/abs(a-b);
}
// projection onto line
P3 foot(const P3& p, const P3& a, const P3& b) {
  P3 d = unit(b-a); return a+dot(p-a,d)*d; }
// rotate p about axis
P3 rotAxis(const P3& p, const P3& a, const P3& b, T theta) {
  P3 dz = unit(b-a), f = foot(p,a,b);
  P3 dx = p-f, dy = cross(dz,dx);
  return f+cos(theta)*dx+sin(theta)*dy;
}
// projection onto plane
P3 foot(const P3& a, const tri& b) {
  P3 c = perp(b[0],b[1],b[2]);
  return a-c*(dot(a,c)-dot(b[0],c)); }
// line-plane intersection
P3 lpIntersect(const P3&a0,const P3&a1,const tri&b) {
  P3 c = unit(cross(b[2]-b[0],b[1]-b[0]));
  T x = dot(a0,c)-dot(b[0],c), y = dot(a1,c)-dot(b[0],c);
  return (y*a0-x*a1)/(y-x);
}
```

Hull3D.h

Description: 3D convex hull where not all points are coplanar. Normals to returned faces point outwards.
Time: $\mathcal{O}\left(N^2\right)$

"Point3D.h" c6fa66, 31 lines

```
bool above(P3 a, P3 b, P3 c, P3 p) { // is p on or above plane
  return dot(cross(a,b,c),p-a) >= 0; }
typedef array<int,3> F; // face
vector<F> hull3d(vP3& p) { // make first four points form tetra
  int N = sz(p); FOR(i,1,N) if (p[0] != p[i]) swap(p[1],p[i]);
  FOR(i,2,N) if (!collinear(p[0],p[1],p[i])) swap(p[2],p[i]);
  FOR(i,3,N)if (!coplanar(p[0],p[1],p[2],p[i]))swap(p[3],p[i]);
  vector<F> hull;
  auto ad = [&](int a, int b, int c) { hull.pb({a,b,c}); };
  int a = 0, b = 1, c = 2, d = 3;
  if (above(p[a],p[b],p[c],p[d])) swap(c,d);
  ad(a,b,c); ad(b,a,d); ad(b,d,c), ad(d,a,c);
  vector<vector<bool>> in(N,vector<bool>(N));
  FOR(i,4,N) { // incremental construction
    vector<F> def, HULL; swap(hull,HULL);
    auto ins = [&](int a, int b, int c) {
      if (in[b][a]) in[b][a] = 0; // kill reverse face
      else in[a][b] = 1, ad(a,b,c);
    };
    trav(f,HULL) {
      int i0 = f[0], i1 = f[1], i2 = f[2];
      if (above(p[i0],p[i1],p[i2],p[i])) {
        ins(i0,i1,i), ins(i1,i2,i), ins(i2,i0,i);
      } else def.pb({i0,i1,i2});
    }
    trav(t,hull) if (in[t[0]][t[1]])
      in[t[0]][t[1]] = 0, def.pb(t);
    swap(hull,def);
  }
  return hull;
}
```

PolySaVol.h

Description: surface area and volume of polyhedron, normals to faces must point outwards

"Hull3D.h" 4a77f6, 8 lines

```
pair<T,T> SaVol(vP3 p, vector<F> faces) {
  T s = 0, v = 0;
  trav(i,faces) {
    s += abs(cross(p[i[0]],p[i[1]],p[i[2]]));
    v += dot(cross(p[i[0]],p[i[1]]),p[i[2]]);
  }
  return {s/2,v/6};
}
```

Delaunay3.h

Description: compute Delaunay triangulation with 3D hull
Time: $\mathcal{O}\left(N^2\right)$

"Point.h", "Hull3D.h" 6d37ff, 13 lines

```
vector<array<P,3>> triHull(vP p) {
  vector<array<P,3>> res;
  if (sz(p) == 3) {
    int d = (cross(p[0],p[1],p[2]) < 0);
    res.pb({p[0],p[1+d],p[2-d]}); return res;
  }
  vector<P3> p3; trav(x,p) p3.pb({x.f,x.s,norm(x)});
  #define nor(x) P(p3[x][0],p3[x][1])
  trav(t, hull3d(p3))
    if (dot(cross(p3[t[0]],p3[t[1]],p3[t[2]]),{0,0,1}) < 0)
      res.pb({nor(t[0]),nor(t[2]),nor(t[1])});
  return res;
}
```

Strings (9)

9.1 Light

KMP.h

Description: f[i] equals the length of the longest proper suffix of the i-th prefix of s that is a prefix of s
Time: $\mathcal{O}(N)$

a3579b, 15 lines

```
vi kmp(str s) {
  int N = sz(s); vi f(N+1); f[0] = -1;
  FOR(i,1,N+1) {
    f[i] = f[i-1];
    while (f[i] != -1 && s[f[i]] != s[i-1]) f[i] = f[f[i]];
    f[i] ++;
  }
  return f;
}
vi getOc(str a, str b) { // find occurrences of a in b
  vi f = kmp(a+"@"+b), ret;
  FOR(i,sz(a),sz(b)+1) if (f[i+sz(a)+1] == sz(a))
    ret.pb(i-sz(a));
  return ret;
}
```

Z.h

Description: for each index i, computes the the maximum len such that s.substr(0,len) == s.substr(i,len)
Time: $\mathcal{O}(N)$

75b3ce, 16 lines

```
vi z(str s) {
  int N = sz(s); s += '#';
  vi ans(N); ans[0] = N;
  int L = 1, R = 0;
  FOR(i,1,N) {
```

```
    if (i <= R) ans[i] = min(R-i+1,ans[i-L]);
    while (s[i+ans[i]] == s[ans[i]]) ans[i] ++;
    if (i+ans[i]-1 > R) L = i, R = i+ans[i]-1;
  }
  return ans;
}
vi getPrefix(str a, str b) { // find prefixes of a in b
  vi t = z(a+b), T(sz(b));
  FOR(i,sz(T)) T[i] = min(t[i+sz(a)],sz(a));
  return T;
}
```

Manacher.h

Description: length of largest palindrome centered at each character of string and between every consecutive pair
Time: $\mathcal{O}(N)$

503c5f, 13 lines

```
vi manacher(str s) {
  str s1 = "@"; trav(c,s) s1 += c, s1 += "#";
  s1.bk = '&';
  vi ans(sz(s1)-1); int lo = 0, hi = 0;
  FOR(i,1,sz(s1)-1) {
    if (i != 1) ans[i] = min(hi-i,ans[hi-i+lo]);
    while (s1[i-ans[i]-1] == s1[i+ans[i]+1]) ans[i] ++;
    if (i+ans[i] > hi) lo = i-ans[i], hi = i+ans[i];
  }
  ans.erase(begin(ans));
  FOR(i,sz(ans)) if ((i&1) == (ans[i]&1)) ans[i] ++;
  return ans;
}
```

MinRotation.h

Description: minimum cyclic shift
Time: $\mathcal{O}(N)$

57b7f2, 10 lines

```
int minRotation(str s) {
  int a = 0, N = sz(s); s += s;
  FOR(b,N) FOR(i,N) {
    // a is current best rotation found up to b-1
    if (a+i==b || s[a+i]<s[b+i]) { b += max(0,i-1); break; }
    // b to b+i-1 can't be better than a to a+i-1
    if (s[a+i] > s[b+i]) { a = b; break; } // new best found
  }
  return a;
}
```

LyndonFactor.h

Description: A string is "simple" if it is strictly smaller than any of its own nontrivial suffixes. The Lyndon factorization of the string s is a factorization $s = w_1 w_2 \dots w_k$ where all strings w_i are simple and $w_1 \geq w_2 \geq \dots \geq w_k$. Min rotation gets min index i such that cyclic shift of s starting at i is minimum.
Time: $\mathcal{O}(N)$

5af83e, 19 lines

```
vs duval(str s) {
  int n = sz(s); vs factors;
  for (int i = 0; i < n; ) {
    int j = i+1, k = i;
    for (; j < n && s[k] <= s[j]; j++) {
      if (s[k] < s[j]) k = i;
      else k ++;
    }
    for (; i <= k; i += j-k) factors.pb(s.substr(i,j-k));
  }
  return factors;
}
int minRotation(str s) {
  int n = sz(s); s += s;
  auto d = duval(s); int ind = 0, ans = 0;
  while (ans+sz(d[ind]) < n) ans += sz(d[ind++]);
  while (ind && d[ind] == d[ind-1]) ans -= sz(d[ind--]);
  return ans;
}
```

HashRange.h

Description: Polynomial hash for substrings with two bases.

1cfa42, 26 lines

```
uniform_int_distribution<int> MULT_DIST(0.1*MOD,0.9*MOD);
typedef array<int,2> T; // bases not too close to ends
const T base = {MULT_DIST(rng),MULT_DIST(rng)};
T operator+(const T& l, const T& r) {
  T x; FOR(i,2) x[i] = (l[i]+r[i])%MOD;
  return x; }
T operator-(const T& l, const T& r) {
  T x; FOR(i,2) x[i] = (l[i]-r[i]+MOD)%MOD;
  return x; }
T operator*(const T& l, const T& r) {
  T x; FOR(i,2) x[i] = (ll)l[i]*r[i]%MOD;
  return x; }

struct HashRange {
  str S;
  vector<T> pows, cum;
  void init(str _S) {
    S = _S; pows.rsz(sz(S)), cum.rsz(sz(S)+1);
    pows[0] = {1,1}; FOR(i,1,sz(S)) pows[i] = pows[i-1]*base;
    FOR(i,sz(S)) {
      int c = S[i]-'a'+1;
      cum[i+1] = base*cum[i]+T{c,c};
    }
  }
  T hash(int l, int r) { return cum[r+1]-pows[r+1-l]*cum[l]; }
};
```

ReverseBW.h

Description: Used only once. Burrows-Wheeler Transform appends # to a string, sorts the rotations of the string in increasing order, and constructs a new string that contains the last character of each rotation. This function reverses the transform.
Time: $\mathcal{O}(N \log N)$

339117, 8 lines

```
str reverseBW(str s) {
  vi nex(sz(s)); vi v(sz(s)); iota(all(v),0);
  stable_sort(all(v),[&s](int a,int b){return s[a]<s[b];});
  FOR(i,sz(v)) nex[i] = v[i];
  int cur = nex[0]; str ret;
  for (; cur; cur = nex[cur]) ret += s[v[cur]];
  return ret;
}
```

9.2 Heavy

ACfixed.h

Description: Aho-Corasick for fixed alphabet. For each prefix, stores link to max length suffix which is also a prefix.
Time: $\mathcal{O}(N \sum)$

fe2603, 28 lines

```
struct ACfixed { // fixed alphabet
  static const int ASZ = 26;
  struct node { array<int,ASZ> to; int link; };
  vector<node> d = {{}};
  int add(str s) { // add word
    int v = 0;
    trav(C,s) {
      int c = C-'a';
      if (!d[v].to[c]) d[v].to[c] = sz(d), d.eb();
      v = d[v].to[c];
    }
    return v;
  }
  void init() { // generate links
    d[0].link = -1;
    queue<int> q; q.push(0);
    while (sz(q)) {
      int v = q.ft; q.pop();
      FOR(c,ASZ) {
        int u = d[v].to[c]; if (!u) continue;
        d[u].link = d[v].link == -1 ? 0 : d[d[v].link].to[c];
        q.push(u);
      }
      if (v) FOR(c,ASZ) if (!d[v].to[c])
        d[v].to[c] = d[d[v].link].to[c];
    }
  }
};
```

PalTree.h

Description: Used infrequently. Palindromic tree computes number of occurrences of each palindrome within string. `ans[i][0]` stores min even x such that the prefix $s[1..i]$ can be split into exactly x palindromes, `ans[i][1]` does the same for odd x.
Time: $\mathcal{O}(N \sum)$ for addChar, $\mathcal{O}(N \log N)$ for updAns

4f5ea4, 42 lines

```
struct PalTree {
  static const int ASZ = 26;
  struct node {
    array<int,ASZ> to = array<int,ASZ>();
    int len, link, oc = 0; // # occurrences of pal
    int slink = 0, diff = 0;
    array<int,2> seriesAns;
    node(int _len, int _link) : len(_len), link(_link) {}
  };
  str s = "@"; vector<array<int,2>> ans = {{0,MOD}};
  vector<node> d = {{0,1},{-1,0}}; // dummy pals of len 0,-1
  int last = 1;
  int getLink(int v) {
    while (s[sz(s)-d[v].len-2] != s.bk) v = d[v].link;
    return v;
  }
  void updAns() { // serial path has O(log n) vertices
    ans.pb({MOD,MOD});
    for (int v = last; d[v].len > 0; v = d[v].slink) {
      d[v].seriesAns=ans[sz(s)-1-d[d[v].slink].len-d[v].diff];
      if (d[v].diff == d[d[v].link].diff)
        FOR(i,2) ckmin(d[v].seriesAns[i],
              d[d[v].link].seriesAns[i]);
      // start of previous oc of link[v]=start of last oc of v
      FOR(i,2) ckmin(ans.bk[i],d[v].seriesAns[i^1]+1);
    }
  }
  void addChar(char C) {
    s += C; int c = C-'a'; last = getLink(last);
    if (!d[last].to[c]) {
      d.eb(d[last].len+2,d[getLink(d[last].link)].to[c]);
      d[last].to[c] = sz(d)-1;
      auto& z = d.bk; z.diff = z.len-d[z.link].len;
      z.slink = z.diff == d[z.link].diff
        ? d[z.link].slink : z.link;
      // max suf with different dif
    }
    last = d[last].to[c]; d[last].oc ++;
    updAns();
  }
  void numOc() { ROF(i,2,sz(d)) d[d[i].link].oc += d[i].oc; }
};
```

SuffixArray.h

Description: Sort suffixes.
Time: $\mathcal{O}(N \log N)$

49d824, 35 lines

```
struct SuffixArray {
  str S; int N;
  void init(str _S) { S = _S, N = sz(S); genSa(), genLcp(); }
  vi sa, isa; // indices of suffixes in sorted order, inverses
  void genSa() {
    sa.rsz(N), isa.rsz(N); iota(all(sa),0);
    sort(all(sa),[&](int a, int b) { return S[a] < S[b]; });
    FOR(i,N) {
      bool same = i && S[sa[i]] == S[sa[i-1]];
      isa[sa[i]] = same ? isa[sa[i-1]] : i;
    }
    for (int len = 1; len < N; len *= 2) {
      // sufs currently sorted by first len chars
      vi is(isa), s(sa), nex(N); iota(all(nex),0);
      FOR(i,-1,N) { // rearrange sufs by 2*len
        int s1 = (i == -1 ? N : s[i])-len;
        if (s1 >= 0) sa[nex[isa[s1]]++] = s1;
      } // to make faster, break when all ints in sa distinct
      FOR(i,N) { // update isa for 2*len
        bool same = i && sa[i-1]+len < N
                && is[sa[i]] == is[sa[i-1]]
                && is[sa[i]+len] == is[sa[i-1]+len];
        isa[sa[i]] = same ? isa[sa[i-1]] : i;
      }
    }
  }
  vi lcp; // common prefix of every two indices in sa
  void genLcp() { // Kasai's Algo
    lcp = vi(N-1); int h = 0;
    FOR(i,N) if (isa[i]) {
      for (int j=sa[isa[i]-1]; j+h<N && S[i+h]==S[j+h]; h++);
      lcp[isa[i]-1] = h; if (h) h--;
    }
  }
};
```

SuffixAutomaton.h

Description: Used infrequently. Constructs minimal deterministic finite automaton (DFA) that recognizes all suffixes of a string
Time: $\mathcal{O}(N \log \sum)$

7658f9, 67 lines

```
struct SuffixAutomaton {
  struct state {
    int len = 0, firstPos = -1, link = -1;
    bool isClone = 0;
    map<char, int> next;
    vi invLink;
  };
  vector<state> st;
  int last = 0;
  void extend(char c) {
    int cur = sz(st); st.eb();
    st[cur].len=st[last].len+1, st[cur].firstPos=st[cur].len-1;
    int p = last;
    while (p != -1 && !st[p].next.count(c)) {
      st[p].next[c] = cur;
      p = st[p].link;
    }
    if (p == -1) st[cur].link = 0;
    else {
      int q = st[p].next[c];
      if (st[p].len+1 == st[q].len) {
        st[cur].link = q;
      } else {
        int clone = sz(st); st.pb(st[q]);
        st[clone].len = st[p].len+1, st[clone].isClone = 1;
        while (p != -1 && st[p].next[c] == q) {
          st[p].next[c] = clone;
          p = st[p].link;
        }
        st[q].link = st[cur].link = clone;
      }
    }
    last = cur;
  }
  void init(str s) {
    st.eb(); trav(x,s) extend(x);
    FOR(v,1,sz(st)) st[st[v].link].invLink.pb(v);
  }
  // APPLICATIONS
  void getAllOccur(vi& oc, int v) {
    if (!st[v].isClone) oc.pb(st[v].firstPos);
    trav(u,st[v].invLink) getAllOccur(oc,u);
  }
  vi allOccur(str s) {
    int cur = 0;
    trav(x,s) {
      if (!st[cur].next.count(x)) return {};
      cur = st[cur].next[x];
    }
    vi oc; getAllOccur(oc,cur); trav(t,oc) t += 1-sz(s);
    sort(all(oc)); return oc;
  }
  vl distinct;
  ll getDistinct(int x) {
    if (distinct[x]) return distinct[x];
    distinct[x] = 1;
    trav(y,st[x].next) distinct[x] += getDistinct(y.s);
    return distinct[x];
  }
  ll numDistinct() { // # distinct substrings including empty
    distinct.rsz(sz(st)); return getDistinct(0); }
  ll numDistinct2() { // another way to do above
    ll ans = 1;
    FOR(i,1,sz(st)) ans += st[i].len-st[st[i].link].len;
    return ans;
  }
};
```

SuffixTree.h

Description: Used infrequently. Ukkonen's algorithm for suffix tree.
Time: $\mathcal{O}(N \log \sum)$

b54cd3, 68 lines

```
struct SuffixTree {
  str s; int node, pos;
  struct state { // edge to state is s[fpos,fpos+len)
    int fpos, len, link = -1;
    map<char,int> to;
    state(int fpos, int len) : fpos(fpos), len(len) {}
  };
  vector<state> st;
  int makeNode(int pos, int len) {
    st.pb(state(pos,len)); return sz(st)-1;
  }
  void goEdge() {
    while (pos>1 && pos>st[st[node].to[s[sz(s)-pos]]].len) {
      node = st[node].to[s[sz(s)-pos]];
      pos -= st[node].len;
    }
  }
  void extend(char c) {
    s += c; pos ++; int last = 0;
    while (pos) {
      goEdge();
      char edge = s[sz(s)-pos];
      int& v = st[node].to[edge];
      char t = s[st[v].fpos+pos-1];
      if (v == 0) {
        v = makeNode(sz(s)-pos,MOD);
        st[last].link = node; last = 0;
      } else if (t == c) {
        st[last].link = node;
        return;
      } else {
        int u = makeNode(st[v].fpos,pos-1);
        st[u].to[c] = makeNode(sz(s)-1,MOD); st[u].to[t] = v;
        st[v].fpos += pos-1; st[v].len -= pos-1;
        v = u; st[last].link = u; last = u;
      }
      if (node == 0) pos --;
      else node = st[node].link;
    }
  }
  void init(str _s) {
    makeNode(-1,0); node = pos = 0;
    trav(c,_s) extend(c);
    extend('$'); s.pop_back(); // terminal char
  }
  int maxPre(str x) { // max prefix of x which is substring
    int node = 0, ind = 0;
    while (1) {
      if (ind==sz(x) || !st[node].to.count(x[ind])) return ind;
      node = st[node].to[x[ind]];
      FOR(i,st[node].len) {
        if (ind == sz(x) || x[ind] != s[st[node].fpos+i])
          return ind;
        ind ++;
      }
    }
  }
  vi sa; // generate suffix array
  void genSa(int x = 0, int len = 0) {
    if (!sz(st[x].to)) { // terminal node
      sa.pb(st[x].fpos-len);
      if (sa.bk >= sz(s)) sa.pop_back();
    } else {
      len += st[x].len;
      trav(t,st[x].to) genSa(t.s,len);
    }
  }
};
```

TandemRepeats.h

Description: Used only once. Main-Lorentz algorithm finds all (x, y) such that `s.substr(x,y-1) == s.substr(x+y,y-1)`.
Time: $\mathcal{O}(N \log N)$

"Z.h" fe5c66, 46 lines

```
struct TandemRepeats {
  str S;
  vector<array<int,3>> al;
  // (t[0],t[1],t[2]) -> exists repeating substr starting
  // at x with length t[0]/2 for all t[1] <= x <= t[2]
  vector<array<int,3>> solveLeft(str s, int m) {
    vector<array<int,3>> v;
    vi v2 = getPrefix(str(begin(s)+m+1,end(s)),
              str(begin(s),begin(s)+m+1));
    str V = str(begin(s),begin(s)+m+2); reverse(all(V));
    vi v1 = z(V); reverse(all(v1));
    FOR(i,m+1) if (v1[i]+v2[i] >= m+2-i) {
      int lo = max(1,m+2-i-v2[i]), hi = min(v1[i],m+1-i);
      lo = i-lo+1, hi = i-hi+1; swap(lo,hi);
      v.pb({2*(m+1-i),lo,hi});
```

```
    }
    return v;
  }
  void divi(int l, int r) {
    if (l == r) return;
    int m = (l+r)/2; divi(l,m); divi(m+1,r);
    str t(begin(S)+l,begin(S)+r+1);
    m = (sz(t)-1)/2;
    auto a = solveLeft(t,m);
    reverse(all(t));
    auto b = solveLeft(t,sz(t)-2-m);
    trav(x,a) al.pb({x[0],x[1]+l,x[2]+l});
    trav(x,b) {
      int ad = r-x[0]+1;
      al.pb({x[0],ad-x[2],ad-x[1]});
    }
  }
  void init(str _S) { S = _S; divi(0,sz(S)-1); }
  vi genLen() {
    // min length of repeating substr starting at each index
    priority_queue<pi,vpi,greater<pi>> m; m.push({MOD,MOD});
    vpi ins[sz(S)]; trav(a,al) ins[a[1]].pb({a[0],a[2]});
    vi len(sz(S));
    FOR(i,sz(S)) {
      trav(j,ins[i]) m.push(j);
      while (m.top().s < i) m.pop();
      len[i] = m.top().f;
    }
    return len;
  }
};
```

Various (10)

10.1 Dynamic programming

When doing DP on intervals:
$a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j,

- one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$.
- This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c)+f(b,d) \le f(a,d)+f(b,c)$ for all $a \le b \le c \le d$.
- Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

CircLCS.h
Description: Used only twice. For strs a, b calculates longest common subsequence of a with all rotations of b
Time: $\mathcal{O}(N^2)$
574233, 46 lines

```
pi dp[2001][4001];
str A,B; // both of len <= 2000
void init() {
  FOR(i,1,sz(A)+1) FOR(j,1,sz(B)+1) {
    // naive LCS, store where value came from
    pi& bes = dp[i][j]; bes = {-1,-1};
    ckmax(bes,{dp[i-1][j].f,0});
    ckmax(bes,{dp[i-1][j-1].f+(A[i-1] == B[j-1]),-1});
    ckmax(bes,{dp[i][j-1].f,-2});
    bes.s *= -1;
  }
}
void adjust(int col) { // remove col'th character of b, fix DP
  int x = 1; while (x <= sz(A) && dp[x][col].s == 0) x ++;
  if (x > sz(A)) return; // no adjustments to dp
  pi cur = {x,col}; dp[cur.f][cur.s].s = 0;
  while (cur.f <= sz(A) && cur.s <= sz(B)) {
    // every dp[cur.f][y >= cur.s].f decreased by 1
    if (cur.s < sz(B) && dp[cur.f][cur.s+1].s == 2) {
      cur.s ++;
      dp[cur.f][cur.s].s = 0;
    } else if (cur.f < sz(A) && cur.s < sz(B)
      && dp[cur.f+1][cur.s+1].s == 1) {
      cur.f ++, cur.s ++;
      dp[cur.f][cur.s].s = 0;
    } else cur.f ++;
  }
}
int getAns(pi x) {
  int lo = x.s-sz(B)/2, ret = 0;
  while (x.f && x.s > lo) {
    if (dp[x.f][x.s].s == 0) x.f --;
    else if (dp[x.f][x.s].s == 1) ret ++, x.f --, x.s --;
    else x.s --;
  }
  return ret;
}
int circLCS(str a, str b) {
  A = a, B = b+b; init();
  int ans = 0;
  FOR(i,sz(b)) {
    ckmax(ans,getAns({sz(a),i+sz(b)}));
    adjust(i+1);
  }
  return ans;
}
```

10.2 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

10.3 Optimization tricks

10.3.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `FOR(b,k) FOR(i,1<<K) if (i&1<<b) D[i] += D[i^(1<<b)];` computes all sums of subsets.

10.3.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

BumpAllocator.h
Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.
745db2, 7 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf; assert(s < i);
  return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

SmallPtr.h
Description: Unused. A 32-bit pointer that points into BumpAllocator memory.
"BumpAllocator.h" 2dd6c9, 9 lines

```
template<class T> struct ptr {
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
    assert(ind < sizeof buf); }
  T& operator*() const { return *(T*)(buf + ind); }
  T* operator->() const { return &**this; }
  T& operator[](int a) const { return (&**this)[a]; }
  explicit operator bool() const { return ind; }
};
```

BumpAllocatorSTL.h
Description: Unused. BumpAllocator for STL containers.
Usage: `vector<vector<int, small<int>>> ed(N);`
bb66d4, 13 lines

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;
template<class T> struct small {
  typedef T value_type;
  small() {}
  template<class U> small(const U&) {}
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);
  }
  void deallocate(T*, size_t) {}
};
```

FastIO.h

Description: Fast input and output.
Time: input is ~300ms faster for 10^6 long longs on CF

ef38ab, 37 lines

```cpp
namespace FastIO {
  const int BSZ = 1<<15; ////// INPUT
  char ibuf[BSZ]; int ipos, ilen;
  char nc() { // next char
    if (ipos == ilen) {
      ipos = 0; ilen = fread(ibuf,1,BSZ,stdin);
      if (!ilen) return EOF;
    }
    return ibuf[ipos++];
  }
  void rs(str& x) { // read str
    char ch; while (isspace(ch = nc()));
    do { x += ch; } while (!isspace(ch = nc()) && ch != EOF);
  }
  template<class T> void ri(T& x) { // read int or ll
    char ch; int sgn = 1;
    while (!isdigit(ch = nc())) if (ch == '-') sgn *= -1;
    x = ch-'0'; while (isdigit(ch = nc())) x = x*10+(ch-'0');
    x *= sgn;
  }
  template<class T, class... Ts> void ri(T& t, Ts&... ts) {
    ri(t); ri(ts...); } // read ints
  ////// OUTPUT (call initO() at start)
  char obuf[BSZ], numBuf[100]; int opos;
  void flushOut() { fwrite(obuf,1,opos,stdout); opos = 0; }
  void wc(char c) { // write char
    if (opos == BSZ) flushOut();
    obuf[opos++] = c; }
  void ws(str s) { trav(c,s) wc(c); } // write str
  template<class T> void wi(T x, char after = '\0') {
    if (x < 0) wc('-'), x *= -1;
    int len = 0; for (;x>=10;x/=10) numBuf[len++] = '0'+(x%10);
    wc('0'+x); R0F(i,len) wc(numBuf[i]);
    if (after) wc(after);
  }
  void initO() { assert(atexit(flushOut) == 0); }
}
```

10.4 Other languages

Main.java

Description: Basic template/info for Java

11488d, 14 lines

```java
import java.util.*;
import java.math.*;
import java.io.*;
public class Main {
  public static void main(String[] args) throws Exception {
    BufferedReader br = new BufferedReader(new
      ↪InputStreamReader(System.in));
    PrintStream out = System.out;
    StringTokenizer st = new StringTokenizer(br.readLine());
    assert st.hasMoreTokens(); // enable with java -ea main
    out.println("v=" + Integer.parseInt(st.nextToken()));
    ArrayList<Integer> a = new ArrayList<>();
    a.add(1234); a.get(0); a.remove(a.size()-1); a.clear();
  }
}
```

Python3.py

Description: not PyPy3, solves CF Factorisation Collaboration

40 lines

```python
from math import *
import sys, random
def nextInt():
  return int(input())
def nextStrs():
  return input().split()
def nextInts():
  return list(map(int,nextStrs()))

n = nextInt()
v = [n]
def process(x):
  global v
  x = abs(x)
  V = []
  for t in v: # print(type(t)) -> <class 'int'>
    g = gcd(t,x)
    if g != 1:
      V.append(g)
    if g != t:
      V.append(t//g)
  v = V
for i in range(50):
  x = random.randint(0,n-1)
  if gcd(x,n) != 1:
    process(x)
  else:
    sx = x*x%n # assert(gcd(sx,n) == 1)
    print(f"sqrt {sx}") # print value of var
    sys.stdout.flush()
    X = nextInt()
    process(x+X)
    process(x-X)
print(f'! {len(v)}',end='')
for i in v:
  print(f' {i}',end='')
print()
sys.stdout.flush() # sys.exit(0) -> exit
# sys.setrecursionlimit(int(1e9)) -> stack size
# print(f'{ans:=.6f}') -> print ans to 6 decimal places
```

Kotlin.kt

Description: Kotlin tips for dummies

e27a45, 87 lines

```kotlin
/* sorting
 * 1 (ok)
  val a = nextLongs().sorted() // a is mutable list
 * 2 (ok)
  val a = arrayListOf<Long>() // or ArrayList<Long>()
  a.addAll(nextLongs())
  a.sort()
 * 3 (ok)
  val A = nextLongs()
  val a = Array<Long>(n,{0})
  for (i in 0..n-1) a[i] = A[i]
  a.sort()
 * 4 (ok)
  val a = ArrayList(nextLongs())
  a.sort()
 * 5 (NOT ok, quicksort)
  val a = LongArray(N) // or nextLongs().toLongArray()
  Arrays.sort(a)
 */
/* 2D array
 * val ori = Array(n, {IntArray(n)})
 * val ori = arrayOf(
  intArrayOf(8, 9, 1, 13),
  intArrayOf(3, 12, 7, 5),
  intArrayOf(0, 2, 4, 11),
  intArrayOf(6, 10, 15, 14)
  )
 */
/* printing variables:
 * println("${l+1} and $r")
 * print d to 8 decimal places: String.format("%.8g%n", d)
 * try to print one stringbuilder instead of multiple prints
 */
/* comparing pairs
  val pq = PriorityQueue<Pair<Long,Int>>({x,y -> x.first.
     ↪compareTo(y.first)})
                 ~ (compareBy {it.first})
  val A = arrayListOf(Pair(1,3),Pair(3,2),Pair(2,3))
  val B = A.sortedWith(Comparator<Pair<Int,Int>>{x,y -> x.first
     ↪.compareTo(y.first)})
  sortBy
 */
/* hashmap
  val h = HashMap<String,Int>()
  for (i in 0..n-2) {
    val w = s.substring(i,i+2)
    val c = h.getOrElse(w){0}
    h.put(w,c+1)
  }
 */
/* basically switch, can be used as expression
  when (x) {
    0,1 -> print("x <= 1")
    2 -> print("x == 2")
    else -> { // Note the block
      print("x is neither 1 nor 2")
    }
  }
*/
// swap : a = b.also { b = a }
// arraylist remove element at index: removeAt, not remove ...
// lower bound: use .binarySearch()

import java.util.*

val MOD = 1000000007
val SZ = 1 shl 18
val INF = (1e18).toLong()

fun add(a: Int, b: Int) = (a+b) % MOD // from tourist :o
fun sub(a: Int, b: Int) = (a-b+MOD) % MOD
fun mul(a: Int, b: Int) = ((a.toLong() * b) % MOD).toInt()

fun next() = readLine()!!
fun nextInt() = next().toInt()
fun nextLong() = next().toLong()
fun nextInts() = next().split(" ").map { it.toInt() }
fun nextLongs() = next().split(" ").map { it.toLong() }

val out = StringBuilder()
fun YN(b: Boolean):String { return if (b) "YES" else "NO"  }

fun solve() {}
fun main(args: Array<String>) {
  val t = 1 // # of test cases
  for (i in 1..t) {
    solve()
  }
}
```