
MAX School on Advanced Materials and Molecular Modelling
with QUANTUM ESPRESSO

QE-2021: Hands-on session – Day-4

Advanced functionals

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Outline

Exercise 1: DFT+*U* study of FeO

Exercise 2: DFT with hybrid functionals: study of Si

Exercise 3: DFT with Van der Waals functionals: study of graphite

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DFT+*U*

The DFT+*U* total energy:

$$E_{\text{DFT+}U} = E_{\text{DFT}} + E_U$$

The Hubbard correction energy:

$$E_U = \frac{1}{2} \sum_{I,m,m',\sigma} \underbrace{(U^I - J^I)}_{U_{\text{eff}}^I} (\delta_{mm'} - n_{mm'}^{I\sigma}) n_{m'm}^{I\sigma}$$

effective Hubbard parameter

The occupation matrix:

$$n_{mm'}^{I\sigma} = \sum_{v,\mathbf{k}} f_{v\mathbf{k}}^\sigma \langle \psi_{v\mathbf{k}}^\sigma | \varphi_{m'}^I \rangle \langle \varphi_m^I | \psi_{v\mathbf{k}}^\sigma \rangle$$

The total occupation of localized states (*d* or *f*) at site *I*:

$$n^I = \sum_{m,\sigma} n_{mm}^{I\sigma}$$

The Kohn-Sham equation:

$$\left[-\frac{1}{2} \nabla^2 + \hat{V}_{\text{KS}}^\sigma + \hat{V}_U^\sigma \right] \psi_{v\mathbf{k}}^\sigma(\mathbf{r}) = \varepsilon_{v\mathbf{k}}^\sigma \psi_{v\mathbf{k}}^\sigma(\mathbf{r})$$

Localized-states manifold

How do we know to what states of a Hubbard atom the U_{eff} correction will be applied?



Currently, this is hard-coded (in Quantum ESPRESSO 6.7)



Check two subroutines in **quantum-espresso/Modules** :



set_hubbard_n.f90

principal quantum number “ n ”



set_hubbard_l.f90

orbital quantum number “ l ”

For Fe : $n = 3$



For Fe : $l = 2$



Apply the Hubbard U_{eff} correction to the $3d$ electrons of Fe

Input file pw.FeO.scf.in

```
&control
  calculation='scf'          ← SCF calculation
  restart_mode='from_scratch',
  prefix='FeO'
  pseudo_dir = '.../.../pseudo'
 outdir='./tmp/'
  verbosity='high'
/
&system
  ibrav = 0,
  celldm(1) = 8.19,           ← Experimental lattice parameter
  nat = 4,
  ntyp = 3,
  ecutwfc = 30.0,
  ecutrho = 240.0,
  occupations = 'smearing',
  smearing = 'mv',           ← 'MV' smearing for metals (FeO is metallic @PBEsol)
  degauss = 0.02,
  nspin = 2,
  starting_magnetization(1) = 0.5,
  starting_magnetization(2) = -0.5
  lda_plus_u = .true.,
  lda_plus_u_kind = 0,
  U_projection_type = 'atomic',
  Hubbard_U(1) = 1.d-8
  Hubbard_U(2) = 1.d-8       ← Spin-polarized (collinear) calculation
                                (antiferromagnetic ordering)
/
&electrons
  conv_thr = 1.d-9
  mixing_beta = 0.3
/
ATOMIC_SPECIES
Fe1 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
Fe2 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
O   16.0   O.pbesol-n-kjpaw_psl.0.1.UPF
CELL_PARAMETERS {alat}
  0.50  0.50  1.00
  0.50  1.00  0.50
  1.00  0.50  0.50
ATOMIC_POSITIONS {crystal}
Fe1  0.00  0.00  0.00
Fe2  0.50  0.50  0.50
O   0.25  0.25  0.25
O   0.75  0.75  0.75
K_POINTS {automatic}
  3 3 3 0 0 0
```

SCF calculation

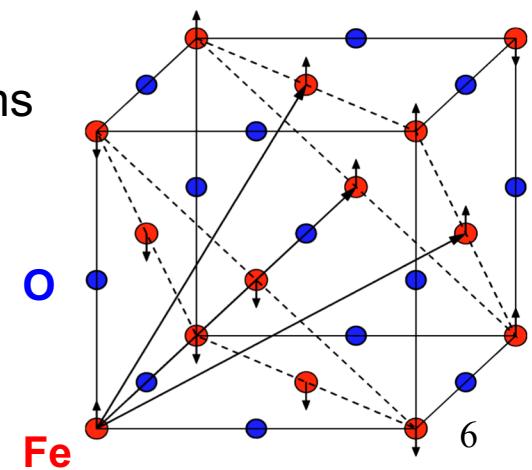
Experimental lattice parameter

'MV' smearing for metals (FeO is metallic @PBEsol)

Spin-polarized (collinear) calculation
(antiferromagnetic ordering)

Initialize Hubbard parameters for two types of Fe

Two sublattices of Fe atoms



Input file pw.FeO.nscf.in

```
&control
  calculation='nscf' ←
  restart_mode='from_scratch',
  prefix='FeO'
  pseudo_dir = '.../.../pseudo'
 outdir='./tmp/'
/
&system
  ibrav = 0,
  celldm(1) = 8.19,
  nat = 4,
  ntyp = 3,
  ecutwfc = 30.0,
  ecutrho = 240.0,
  occupations = 'smearing',
  smearing = 'mv',
  degauss = 0.02,
  nspin = 2,
  starting_magnetization(1) = 0.5,
  starting_magnetization(2) = -0.5
  lda_plus_u = .true.,
  lda_plus_u_kind = 0,
  U_projection_type = 'atomic',
  Hubbard_U(1) = 1.d-8
  Hubbard_U(2) = 1.d-8
  nbnd = 35 ←
/
&electrons
  conv_thr = 1.d-9
  mixing_beta = 0.3
/
ATOMIC_SPECIES
Fe1 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
Fe2 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
O 16.0 O.pbesol-n-kjpaw_psl.0.1.UPF
CELL_PARAMETERS {alat}
  0.50 0.50 1.00
  0.50 1.00 0.50
  1.00 0.50 0.50
ATOMIC_POSITIONS {crystal}
  Fe1 0.00 0.00 0.00
  Fe2 0.50 0.50 0.50
  O 0.25 0.25 0.25
  O 0.75 0.75 0.75
K_POINTS {automatic}
  6 6 6 0 0 0 ←
```

NSCF calculation

NSCF input is very similar to the SCF input,
but there are a few differences which are
highlighted here

Number of electronic bands

Projected DOS must be converged with respect to
the number of **k** points. Also, number of bands must
be large enough, to have converged projected DOS
in a specific energy range. All this is controlled here.

k points mesh which is denser than in SCF

Input file projwfc.FeO.in

```
&projwfc
prefix='FeO'
outdir='./tmp/'
ngauss = 0,
degauss = 0.005,
Emin = -15.0,
Emax = 30.0,
DeltaE = 0.01
/
```

Simple Gaussian broadening of PDOS

Value of Gaussian broadening, in Ry (not eV!)

Minimum and maximum value of energy for the plot, in eV

Energy grid step

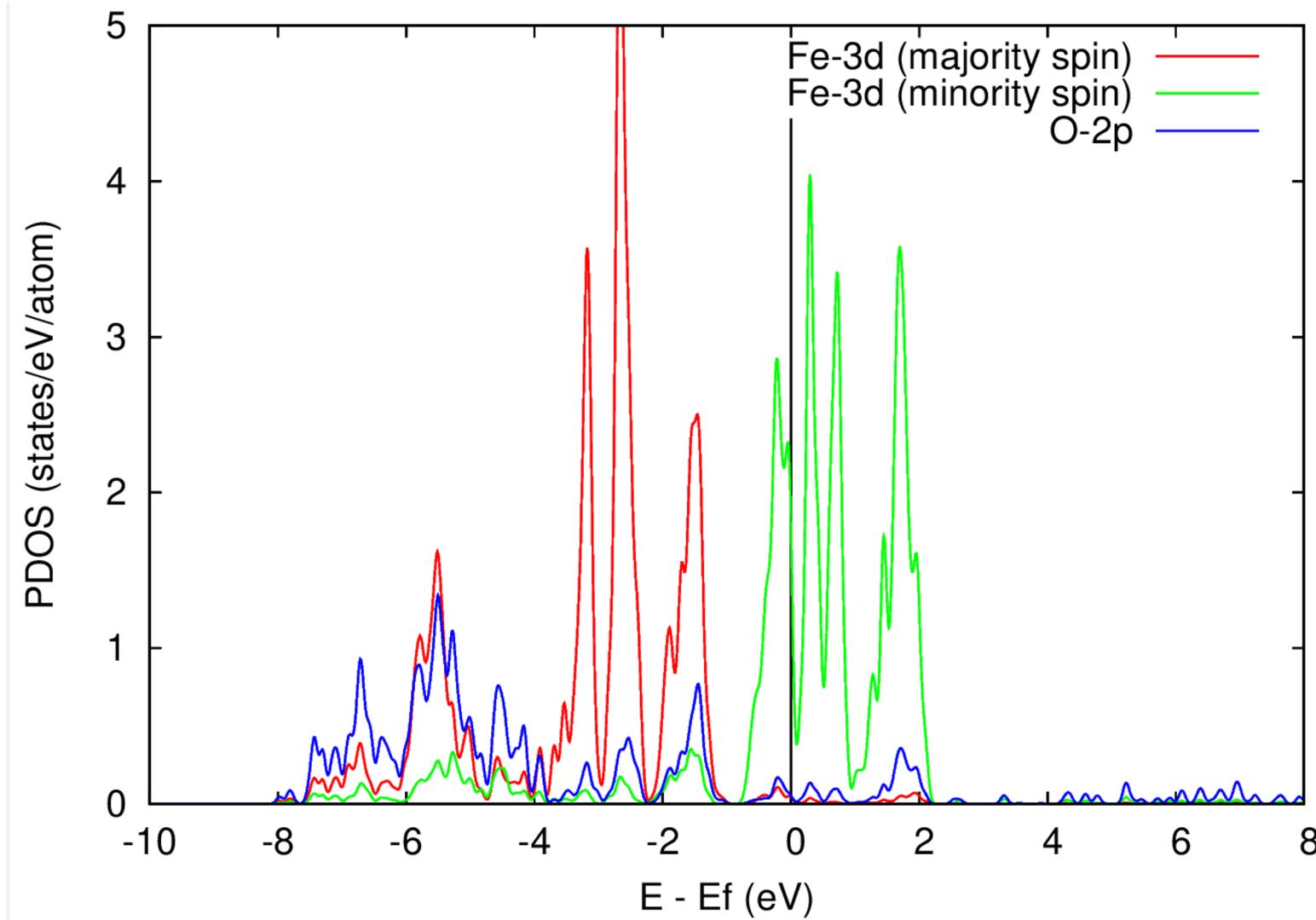
Gnuplot script: plot_pdos.gp

Inspect the script: it aims at plotting Fe-3d state (majority spin and minority spin) and O-2p states.

PDOS is shifted such that the Fermi energy corresponds to the zero of energy.

Visualize the file “**FeO_PDOS.eps**”

PDOS of FeO from standard DFT calculation



DFT predicts FeO to be metallic (**but this is wrong!**)



Let's try **DFT+U**

Experimentally FeO is known to be insulating

Input file pw.FeO.scf.in for DFT+U

```
&control
  calculation='scf'
  restart_mode='from_scratch',
  prefix='FeO'
  pseudo_dir = '../pseudo'
 outdir='./tmp/'
  verbosity='high'
/
&system
  ibrav = 0,
  celldm(1) = 8.19,
  nat = 4,
  ntyp = 3,
  ecutwfc = 30.0,
  ecutrho = 240.0,
  occupations = 'smearing',
  smearing = 'mv',
  degauss = 0.02,
  nspin = 2,
  starting_magnetization(1) = 0.5,
  starting_magnetization(2) = -0.5
  lda_plus_u = .true.,
  lda_plus_u_kind = 0,
  U_projection_type = 'atomic',
  Hubbard_U(1) = 4.6
  Hubbard_U(2) = 4.6
/
&electrons
  conv_thr = 1.d-9
  mixing_beta = 0.3
/
ATOMIC_SPECIES
Fe1 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
Fe2 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
O    16.0   O.pbesol-n-kjpaw_psl.0.1.UPF
CELL_PARAMETERS {alat}
  0.50  0.50  1.00
  0.50  1.00  0.50
  1.00  0.50  0.50
ATOMIC_POSITIONS {crystal}
  Fe1  0.00  0.00  0.00
  Fe2  0.50  0.50  0.50
  O   0.25  0.25  0.25
  O   0.75  0.75  0.75
K_POINTS {automatic}
  3 3 3 0 0 0
```

Here we want to put a Hubbard U correction for 3d electrons of Fe atoms

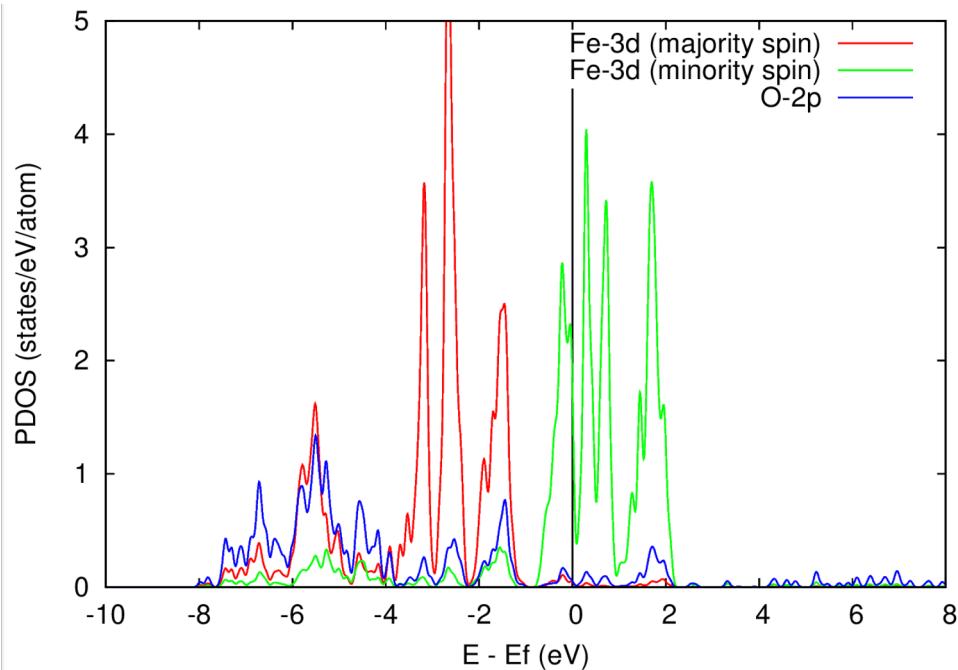
Which value of U to use?

For every material Hubbard U is different; here we use $U = 4.6$ eV for Fe-3d states (see next slides)

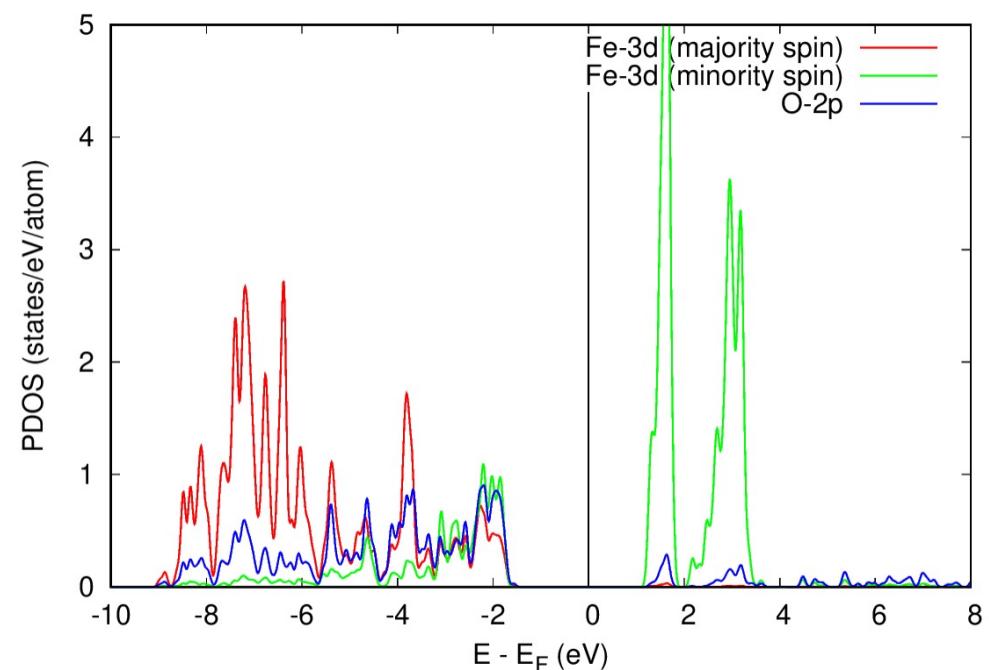
Hubbard U values (in eV) for atomic type 1 & type 2

DFT vs DFT+ U

DFT



DFT+ U



The solution with DFT+ U is metallic

The solution with DFT+ U is **insulating**



Agreement with the experiment!

How to compute the Hubbard U parameter?

Linear response theory



Density-functional perturbation theory (DFPT)

$$U^I = (\chi_0^{-1} - \chi^{-1})_{II}$$



$$\chi_{IJ} = \sum_{\sigma,m} \frac{dn_{mm}^{I\sigma}}{d\lambda^J}$$



$$\frac{dn_{m_1 m_2}^{sl\sigma}}{d\lambda^{s'l'}} = \frac{1}{N_{\mathbf{q}}} \sum_{\mathbf{q}}^{N_{\mathbf{q}}} e^{i\mathbf{q}\cdot(\mathbf{R}_l - \mathbf{R}_{l'})} \Delta_{\mathbf{q}}^{s'} \bar{n}_{m_1 m_2}^{s\sigma}$$



Sum over all \mathbf{q} points in a \mathbf{q} mesh

How to compute the Hubbard U parameter?

Using the **HP** (Hubbard Parameters) code of **Quantum ESPRESSO**

Input file “hp.FeO.in”:

```
&inpuhp
prefix = 'FeO'
outdir = './tmp/'
nq1 = 1, nq2 = 1, nq3 = 1
/
```

Prefix and outdir must be exactly the same as in
the input file LiCoO₂.pw.in

Size of the q point mesh: 1x1x1 in this case



$$U = 4.6 \text{ eV}$$

Warning: This U value is not converged with respect to the k and q meshes!!!

Documentation: [quantum-espresso/HP/Doc/INPUT_HP.txt](#)

Theory: I. Timrov, N. Marzari, M. Cococcioni, PRB **98**, 085127 (2018)

I. Timrov, N. Marzari, M. Cococcioni, PRB **103**, 045141 (2021)

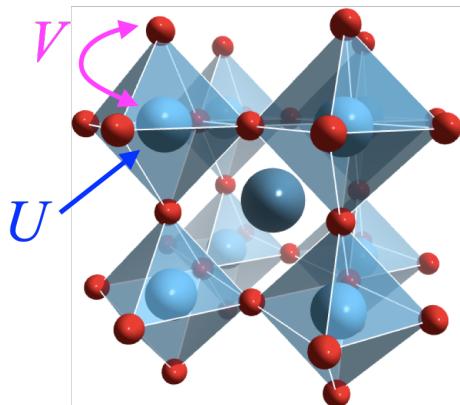
DFT+ U + V

DFT+ U + V is an **extended formulation** that takes into account **inter-site Hubbard interactions**:

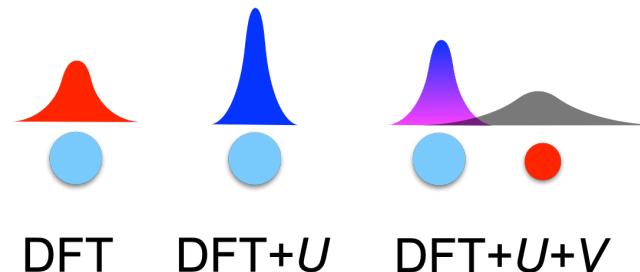
$$E = E_{\text{DFT}} + \frac{1}{2} \sum_{I,\sigma} U^I \text{Tr} [(\mathbf{1} - \mathbf{n}^{I\sigma}) \mathbf{n}^{I\sigma}] - \frac{1}{2} \sum_{I,J,\sigma} V^{IJ} \text{Tr} [\mathbf{n}^{IJ\sigma} \mathbf{n}^{JI\sigma}]$$

effective on-site Hubbard interaction

inter-site Hubbard interactions



atomic localisation & inter-atomic hybridisations



Both U and V can be computed using the HP code

[1] V.L. Campo Jr and M. Cococcioni, JPCM 22, 055602 (2010).

Theory:

V.L. Campo Jr and M. Cococcioni, JPCM 22, 055602 (2010)

I. Timrov, N. Marzari, M. Cococcioni, PRB 103, 045141 (2021)

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The $\mathbf{q} + \mathbf{G} = 0$ divergence

In periodic systems, the exact exchange contains a divergence when $\mathbf{q} + \mathbf{G} = 0$:

$$E_x = -\frac{4\pi}{2\Omega} \times \frac{\Omega}{(2\pi)^3} \int d\mathbf{q} \sum_{\mathbf{G}} \frac{A(\mathbf{q} + \mathbf{G})}{|\mathbf{q} + \mathbf{G}|^2}$$

where

We need to setup the \mathbf{q} mesh

$$A(\mathbf{q} + \mathbf{G}) = \frac{\Omega}{(2\pi)^3} \int d\mathbf{k} |\rho_{\mathbf{k},v}^{\mathbf{k}-\mathbf{q},v'}(\mathbf{q} + \mathbf{G})|^2 \equiv \frac{1}{N_k} \sum_{\mathbf{k}} |\rho_{\mathbf{k},v}^{\mathbf{k}-\mathbf{q},v'}(\mathbf{q} + \mathbf{G})|^2$$

(the finite sum over N_k \mathbf{k} -points is what we actually compute) and

$$\rho_{\mathbf{k},v}^{\mathbf{k}-\mathbf{q},v'}(\mathbf{r}) = \psi_{\mathbf{k}-\mathbf{q},v'}^*(\mathbf{r}) \psi_{\mathbf{k},v}(\mathbf{r}).$$

This divergence is integrable, see: F.Gygi and A.Baldereschi, PRB 34, 4405 (1986)

Input file pw.Si.scf.in

Hybrid functionals are very computationally expensive!

```
&control
    calculation='scf'           ←
    restart_mode='from_scratch',
    prefix='Si'
    pseudo_dir = '.../.../pseudo'
   outdir='./tmp/'
    verbosity='high'
/
&system
    ibrav = 2,
    celldm(1) = 10.20,
    nat = 2,
    ntyp = 1,
    ecutwfc = 20.0,
    input_dft = 'pbe0',          ←
    nqx1 = 1, nqx2 = 1, nqx3 = 1, ←
    x_gamma_extrapolation = .true., ←
    exxdiv_treatment = 'gygi-baldereschi' ←
/
&electrons
    conv_thr = 1.d-9
    mixing_beta = 0.3
/
ATOMIC_SPECIES
    Si 28.086 Si.pbe-rrkj.UPF
ATOMIC_POSITIONS {alat}
    Si 0.00 0.00 0.00
    Si 0.25 0.25 0.25
K_POINTS {automatic}
    8 8 8 1 1 1
```

SCF calculation

Override the functional written in the pseudo-potential (PP) file.

Important: use a PP for the closest GGA (PBE in this case), there are no PP's for hybrids.

Mesh for the \mathbf{q} points 1x1x1 is the minimal grid ($\mathbf{q}=0$), it is fast but not very accurate

If .true. use extrapolation for the $\mathbf{q} \rightarrow 0$ limit

Specifies that the singularity at $\mathbf{q} \rightarrow 0$ is analytically integrated

Popular hybrid functionals

input_dft = "pbe0"

J.P.Perdew, M. Ernzerhof, K.Burke, JCP 105, 9982 (1996)

C. Adamo, V. Barone, JCP 110, 6158 (1999)

input_dft = "b3lyp"

P.J. Stephens,F.J. Devlin,C.F. Chabalowski,M.J. Frisch, J.Phys.Chem 98, 11623 (1994)

input_dft = "hse"

Heyd, Scuseria, Ernzerhof, J. Chem. Phys. 118, 8207 (2003)

Heyd, Scuseria, Ernzerhof, J. Chem. Phys. 124, 219906 (2006)

Types of treatment of divergences at q->0

exxdiv_treatment	CHARACTER
-------------------------	-----------

Default: 'gygi-baldereschi'

Specific for EXX. It selects the kind of approach to be used for treating the Coulomb potential divergencies at small q vectors.

'gygi-baldereschi' :
appropriate for cubic and quasi-cubic supercells

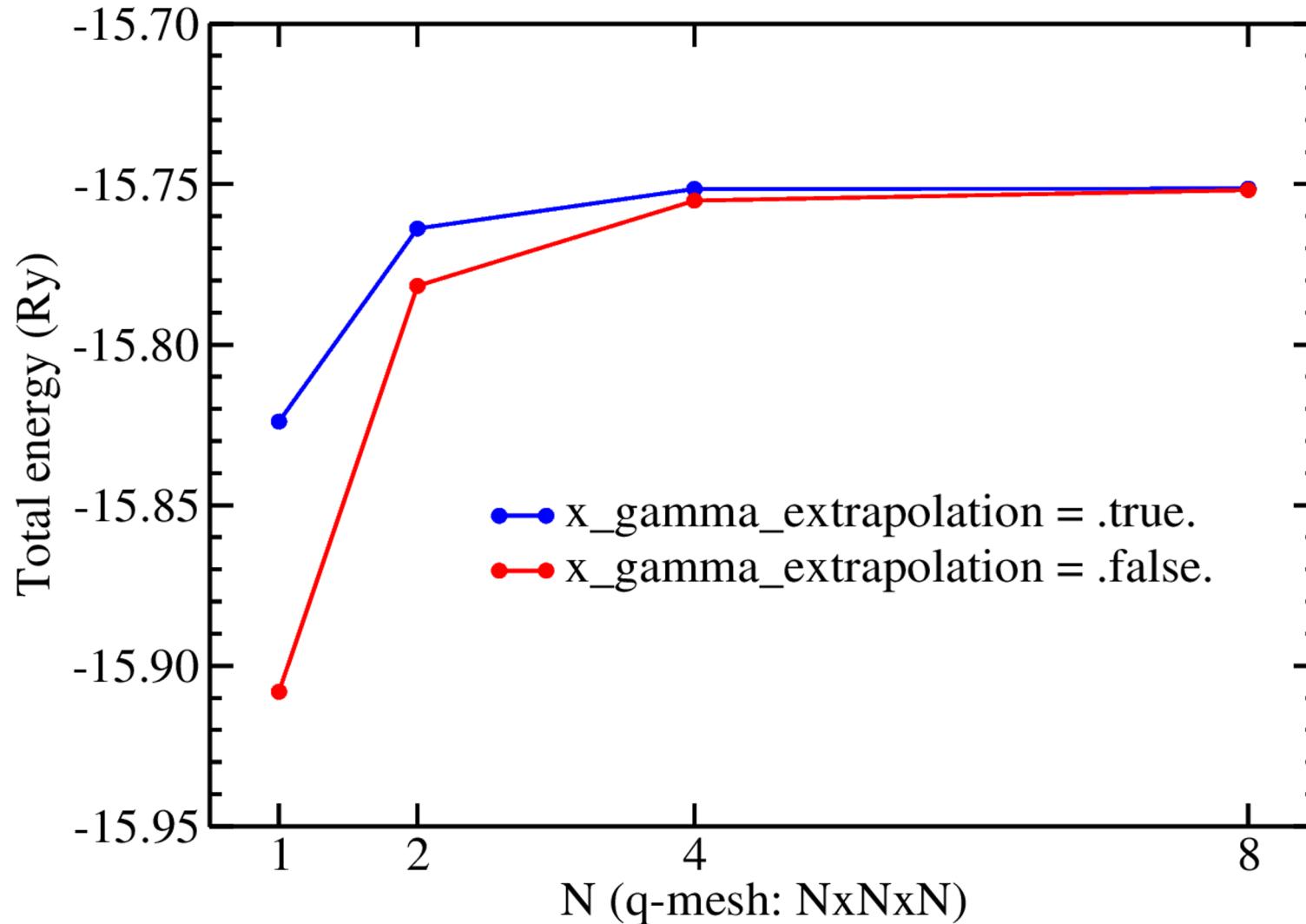
'vcut_spherical' :
appropriate for cubic and quasi-cubic supercells

'vcut_ws' :
appropriate for strongly anisotropic supercells, see also [ecutvcut](#).

'none' :
sets Coulomb potential at G,q=0 to 0.0 (required for GAU-PBE)

Convergence of the total energy wrt q point grid

Slow calculations → do for scarce q meshes and check the “reference” folder



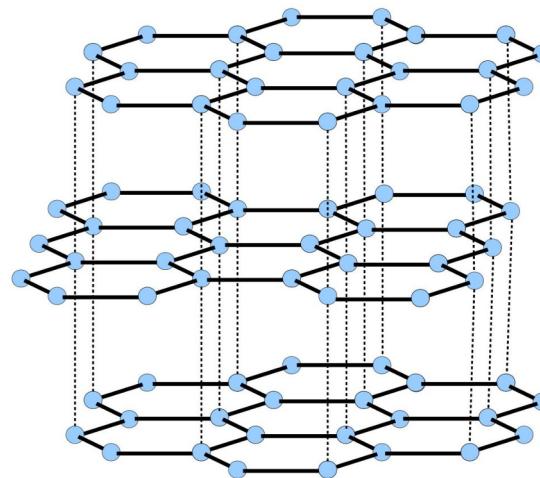
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Graphite



Graphite has Van der Waals (dispersive) interactions between layers.

The equilibrium inter-layer distance is too small with LDA, and too large with GGA with respect to the experimental value (3.336 Å).

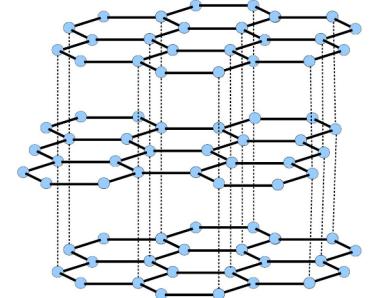


Van der Waals interactions must be taken into account.

Input file pw.graphite.vc-relax.in

```
&control
    calculation='vc-relax' ←
    restart_mode='from_scratch',
    prefix='graphite'
    pseudo_dir = '.../.../pseudo'
   outdir='./tmp/'
    verbosity='high'
    etot_conv_thr = 1.0D-5
    forc_conv_thr = 1.0D-4
/
&system
    ibrav = 4,
    a = 2.466,
    c = 6.411,
    nat = 4,
    ntyp = 1,
    ecutwfc = 30.0,
    ecutrho = 240.0,
    occupations = 'smearing',
    smearing = 'mv',
    degauss = 0.02,
    input_dft = 'vdw-DF' ←
/
&electrons
    conv_thr = 1.d-9
    mixing_beta = 0.3
/
&ions
/
&cell
/
ATOMIC_SPECIES
C 12.011 C.pbe-rrkjus.UPF
ATOMIC_POSITIONS {crystal}
C 0.000000 1.000000 0.75000
C 0.666667 0.333333 0.75000
C 0.000000 1.000000 0.25000
C 0.333333 0.666667 0.25000
K_POINTS {automatic}
4 4 2 1 1 1 ←
```

Variable-cell optimization



Hexagonal lattice, a and c are in Angstrom!

Override the functional written in the PP file.
Important: use PP for the closest GGA (PBE in this case), because there are no PP for non-local functionals

If we do not specify anything inside these namelists, then the default values will be used for various control parameters

The \mathbf{k} point grid is denser along xy than along z, reflecting shorter periodicity in the xy plane

Types of the van der Waals correction

vdw_corr	CHARACTER
<i>Default:</i>	'none'
<i>See:</i>	london_s6 , london_rcut , london_c6 , london_rvdw , dftd3_version , dftd3_threebody , ts_vdw_econv_thr , ts_vdw_isolated , xdm_a1 , xdm_a2
Type of Van der Waals correction. Allowed values:	
'grimme-d2' , ' Grimme-D2 ', ' DFT-D ', ' dft-d ' :	
Semiempirical Grimme's DFT-D2. Optional variables: london_s6 , london_rcut , london_c6 , london_rvdw	
S. Grimme, J. Comp. Chem. 27, 1787 (2006), doi:10.1002/jcc.20495	
V. Barone et al., J. Comp. Chem. 30, 934 (2009), doi:10.1002/jcc.21112	
'grimme-d3' , ' Grimme-D3 ', ' DFT-D3 ', ' dft-d3 ' :	
Semiempirical Grimme's DFT-D3. Optional variables: dftd3_version , dftd3_threebody	
S. Grimme et al, J. Chem. Phys 132, 154104 (2010), doi:10.1002/jcc.20495	
'TS' , ' ts ', ' ts-vdw ', ' ts-vdW ', ' tkatchenko-scheffler ' :	
Tkatchenko-Scheffler dispersion corrections with first-principle derived C6 coefficients. Optional variables: ts_vdw_econv_thr , ts_vdw_isolated	
See A. Tkatchenko and M. Scheffler, PRL 102, 073005 (2009) .	
'XDM' , ' xdm ' :	
Exchange-hole dipole-moment model. Optional variables: xdm_a1 , xdm_a2	
A. D. Becke et al., J. Chem. Phys. 127, 154108 (2007), doi:10.1063/1.2795701	
A. Otero de la Roza et al., J. Chem. Phys. 136, 174109 (2012), doi:10.1063/1.4705760	

Note that non-local functionals (eg vdw-DF) are NOT specified here but in [input_dft](#)

Structural optimization in graphite

```
Perform a variable-cell optimization : `pw.x < pw.graphite.vc-relax.in > pw.graphite.vc-relax.out`
```

Study different cases:

1. `input_dft = 'vdw-DF'` @PBE pseudo (non-local)
2. `input_dft = 'vdw-DF2'` @PBE pseudo (non-local)
3. `input_dft = 'rVV10'` @PBE pseudo (non-local)
4. `vdw_corr = 'DFT-D'` @PBE pseudo (semi-empirical)
5. `vdw_corr = 'DFT-D3'` @PBE pseudo (semi-empirical)
6. Normal PBE calculation @PBE pseudo
7. Normal LDA calculation @LDA pseudo

Compare the optimized inter-layer distances with the experimental value of 3.336 Å.