Time-Varying Formation Control for Unmanned Aerial Vehicles: Theories and Applications

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***Abstract*— Formation control analysis and design problems for unmanned aerial vehicle (UAV) swarm systems to achieve time-varying formations are investigated. To achieve predefined time-varying formations, formation protocols are presented for UAV swarm systems first, where the velocities of UAVs can be different when achieving formations. Then, consensus-based approaches are applied to deal with the time-varying formation control problems for UAV swarm systems. Necessary and sufficient conditions for UAV swarm systems to achieve time-varying formations are proposed. An explicit expression of the time-varying formation center function is derived. In addition, a procedure to design the protocol for UAV swarm systems to achieve time-varying formations is given. Finally, a quadrotor formation platform, which consists of five quadrotors is introduced. Theoretical results obtained in this brief are validated on the quardrotor formation platform, and outdoor experimental results are presented.**

***Index Terms*— Formation control, swarm system, time-varying formation, unmanned aerial vehicle (UAV).**

I. Introduction

FORMATION control of unmanned aerial vehicle (UAV) swarm systems has received considerable interests in recent years due to its broad potential applications in civilian and military areas, such as drag reduction [1], surveillance and reconnaissance [2], [3], radiation detection and contour mapping [4], target search and localization [5], telecommunication relay [6], and so on. In the past decades, many formation control approaches have been proposed in robotics community, such as leader–follower [7], [8], behavior [9], virtual structure-based [10] approaches, and so on. With the development of UAV technology and the increasing demand for formation control of UAV swarm systems, more and more researchers try to deal with the formation problems for UAV swarm systems using these approaches.

Leader–follower-based approaches were applied to formation control of UAV swarm systems in [11]–[13]. Behavior-based formation control problems for UAV swarm systems were studied in [14]–[16]. Formation control results of UAV swarm systems based on virtual structure approaches can be found in [17]–[19]. However, as pointed out in [20], leader–follower, behavior, and virtual structure-based formation control approaches have their own strengths and weaknesses. For example, leader–follower approaches are easy to implement but have no explicit feedback on formation and lack of robustness due to the existence of the leader, behavior approaches can ensure collision avoidance but are difficult to analyze mathematically, and so on.

Recently, consensus problems of linear time-invariant swarm systems or multiagent systems have been studied extensively [21]–[29]. Consensus means that all agents reach an agreement on certain variables of interest. It has been revealed in [30] that consensus approaches can be used to deal with formation control problems, and leader–follower, behavior, and virtual structure-based formation control approaches can be treated as special cases of consensus-based ones. In addition, the weakness of the previous approaches can be overcome. Therefore, it is significant to study consensus-based formation control problems for UAV swarm systems. Based on consensus strategy, Abdessameud and Tayebi [31] proposed formation controllers for UAV swarm systems with undirected interaction topologies to achieve time invariant formations in the presence of communication delays. Seo *et al.* [32] applied a consensus protocol together with an output feedback linearization method to deal with formation control problems of UAV swarm systems, where the formation can be partially time-varying. Turpin *et al.* [33] studied formation control problems for UAV swarm systems using consensus approaches and presented indoor formation flight experiments for quadrotor swarm systems. Although many results have been obtained for consensus-based formation control problems of UAV swarm systems, necessary and sufficient conditions for UAV swarm systems to achieve time-varying formation have not been obtained, and distributed outdoor time-varying formation flight experiments using multiple UAVs have not been reported.

In this brief, consensus-based time-varying formation problems for UAV swarm systems are investigated and applications of the formation theories to quadrotor swarm systems are presented. On the formation control level, a UAV is regarded as a point-mass system, and the dynamics of each UAV is modeled by a double integrator. A consensus-based formation protocol for UAV swarm systems to achieve time-varying formations is proposed. Then, formation problems are transformed into consensus problems. Necessary and sufficient conditions for UAV swarm systems to achieve time-varying formation are presented, and an explicit expression of the

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time-varying formation center function is given. Furthermore, a procedure to design the protocol for UAV swarm systems to achieve time-varying formations is proposed. In order to demonstrate the theoretical results, a quadrotor formation platform is introduced. Finally, outdoor experimental applications for the quadrotor swarm system to achieve time-varying formations are presented.

Compared with the existing works on consensus-based formation control problems of UAV swarm systems, the new features of this brief are threefold. First, the formation can be time-varying. In [31] and [33], the formation is time invariant. In [32], the formation is partially time-varying; that is, when achieving formation the velocity components of all UAVs must be identical. However, in practical applications, there exist formations that require the velocity components of UAVs to be different, such as the rotation formation. In this case, only the time-varying formation can be used. Second, both formation analysis and design problems for UAV swarm systems to achieve time-varying formations are dealt with, where necessary and sufficient conditions for UAV swarm systems to achieve time-varying formations are proposed. Besides, an explicit expression of the formation center function is given. In previous literatures, such as [31]–[33], only sufficient conditions are derived and it can be shown that some of the previous results on formation control of swarm systems are special cases of this brief. Third, distributed outdoor flight experiments for quadrotor swarm systems to achieve time-varying formation are presented. In [31] and [32], the theoretical results were demonstrated by means of simulations. The time invariant formation experiments in [33] were carried out in lab, which are highly dependent on the indoor VICON motion capture system [34].

The rest of this brief is organized as follows. In Section II, basic concepts and useful results on graph theory are introduced and the problem formulation is presented. Main theoretical results are provided in Section III. In Section IV, a quadrotor formation platform is introduced. Simulation and experimental applications are shown in Section V. Finally, conclusions are drawn in Section VI.

Throughout this brief, for simplicity of notation, 0 will denote zero matrices of appropriate size with zero vectors and zero number as special cases. Let $\mathbf{1}_N$ be a column vector of size N with 1 as its elements, and I_n represent an identity matrix with dimension n. Let $\otimes$ denote Kronecker product. For a given $\Psi \in \mathbb{C}$, $\mathrm{Re}(\Psi)$ and $\mathrm{Im}(\Psi)$ denote the real part and the imaginary part of Ψ, respectively. The superscript H stands for the Hermitian adjoint of matrices.

II. Preliminaries and Problem Description

In this section, basic concepts and results on graph theory are introduced, and the problem description is presented.

A. Basic Concepts and Results on Graph Theory

A directed graph $G = \{Q, \varepsilon, W\}$ consists of a set of nodes $Q = \{q_1, q_2, \ldots, q_N\}$, a set of edges $\varepsilon \subseteq \{(q_i, q_j) : q_i, q_j \in Q\}$, and a weighted adjacency matrix $W = [w_{ij}] \in \mathbb{R}^{N\times N}$ with nonnegative elements w_{ij}. An edge of G is denoted by $e_{ij} = (q_i, q_j)$. In addition, $w_{ji} > 0$ if and only if $e_{ij} \in \varepsilon$, and $w_{ii} = 0$ for all $i \in \{1, 2, \ldots, N\}$. The set of neighbors of node q_i is denoted by $N_i = \{q_j \in Q : (q_j, q_i) \in \varepsilon\}$. The in-degree of node q_i is defined as $\deg_{\mathrm{in}}(q_i) = \sum_{j=1}^{N} w_{ij}$. The degree matrix of G is denoted by $D = \mathrm{diag}\{\deg_{\mathrm{in}}(q_i), i = 1, 2, \ldots, N\}$. The Laplacian matrix of G is defined as $L = D - W$. A directed graph is said to have a spanning tree if there exists at least one node having a directed path to all the other nodes. More details on graph theory can be found in [35]. The following lemma is useful in analyzing time-varying formation problems of UAV swarm systems.

Lemma 1 [23]: Let $L \in \mathbb{R}^{N\times N}$ be the Laplacian matrix of a directed graph G, then:

1) L has at least one zero eigenvalue, and $\mathbf{1}_N$ is the associated eigenvector; that is, $L\mathbf{1}_N = 0$;
2) if G has a spanning tree, then 0 is a simple eigenvalue of L, and all the other $N - 1$ eigenvalues have positive real parts.

B. Problem Description

Consider a UAV swarm system with N UAVs. The interaction topology of the UAV swarm system can be described by a directed graph G. For $i, j \in \{1, 2, \ldots, N\}$, UAV i can be denoted by node q_i in G and the interaction channel from UAV i to UAV j can be denoted by e_{ij}. It is assumed that G has a spanning tree.

For each of these UAVs, since the trajectory dynamics has much larger time constants than the attitude dynamics, if the formation is only concerned with positions and velocities, then the formation control can be implemented with an inner/outer-loop structure [18], [36]. In this configuration, the outer-loop can be used to drive the UAV toward the desired position with desired velocity while the inner-loop can be used to track the attitude. This brief mainly focuses on designing the outer-loop. Therefore, on the formation control level, a UAV can be regarded as a point-mass system, and the dynamics of each UAV can be approximately described by the following double integrator [2], [11], [16], [32]:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, N$, $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ denote the position and velocity vectors of UAV i, respectively, and $u_i(t) \in \mathbb{R}^n$ are the control inputs. In the following, for simplicity of description, it is assumed that $n = 1$, if not otherwise specified. However, it should be pointed out that similar analysis can also be done for the higher dimensional case by using Kronecker product, and all the results hereafter remain valid for $n > 1$.

Let $\theta_i(t) = [x_i(t), v_i(t)]^T$, $B_1 = [1, 0]^T$, and $B_2 = [0, 1]^T$. Then, UAV swarm system (1) can be rewritten as

$$\dot{\theta}_i(t) = B_1 B_2^T \theta_i(t) + B_2 u_i(t). \tag{2}$$

Let $h_i(t) = [h_{ix}(t), h_{iv}(t)]^T$ $(i = 1, 2, \ldots, N)$ be piecewise continuously differentiable vectors and $h(t) = [h_1^T(t), h_2^T(t), \ldots, h_N^T(t)]^T \in \mathbb{R}^{2N}$.

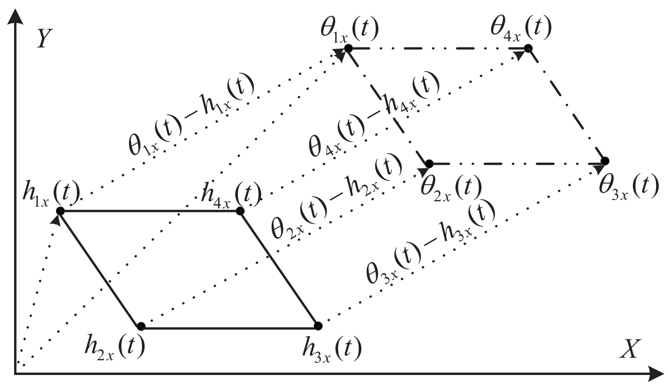


Fig. 1. Parallelogram formation in XY plane with $N = 4$.

Definition 1: A time-varying formation is specified by a vector $h(t)$. UAV swarm system (2) is said to achieve time-varying formation $h(t)$ if for appropriate initial states, there exists a vector-valued function $c(t) \in \mathbb{R}^2$ such that $\lim_{t\to\infty}(\theta_i(t) - h_i(t) - c(t)) = 0$ $(i = 1, 2, \ldots, N)$, where $c(t)$ is called a formation center function.

To illustrate Definition 1, consider a parallelogram formation for swarm system with four agents moving in the XY plane $(n = 2)$. Let $x_{iX}(t) \in \mathbb{R}$, $h_{ixX}(t) \in \mathbb{R}$, $c_{xX}(t) \in \mathbb{R}$ and $x_{iY}(t) \in \mathbb{R}$, $h_{ixY}(t) \in \mathbb{R}$, $c_{xY}(t) \in \mathbb{R}$ denote the position, formation, formation center function of UAV i along x-axis and y-axis, respectively. Let $\theta_{ix}(t) = [x_{iX}(t), x_{iY}(t)]^T$, $h_{ix}(t) = [h_{ixX}(t), h_{ixY}(t)]^T$, and $c_x(t) = [c_{xX}(t), c_{xY}(t)]^T$. From Fig. 1, one sees that if $\theta_{ix}(t) - h_{ix}(t) - c_x(t) \to 0$ as $t \to \infty$ for all $i = 1, 2, 3, 4$, then the two parallelograms formed by $h_{ix}(t)$ and $\theta_{ix}(t)$ $(i = 1, 2, 3, 4)$ are congruent; that is, the parallelogram formation is achieved.

Definition 2: UAV swarm system (2) is said to achieve consensus if for any given bounded initial states, there exists a vector-valued function $s(t) \in \mathbb{R}^2$ such that $\lim_{t\to\infty}(\theta_i(t) - s(t)) = 0$ $(i = 1, 2, \ldots, N)$, where $s(t)$ is called a consensus function.

Remark 1: From Definitions 1 and 2, one sees that when $h(t) \equiv 0$, UAV swarm system (2) achieves consensus if it achieves formation. In this case, the time-varying formation center function is equivalent to the consensus function. Therefore, for UAV swarm system described by (2), consensus problem is just a special case of formation problem.

Consider the following time-varying formation protocol:

$$u_i(t) = K_1(\theta_i(t) - h_i(t)) + K_2 \sum_{j\in N_i} w_{ij}((\theta_j(t) - h_j(t)) - (\theta_i(t) - h_i(t))) + \dot{h}_{iv}(t) \quad (3)$$

where $i = 1, 2, \ldots, N$, $K_1 = [k_{11}, k_{12}]$ and $K_2 = [k_{21}, k_{22}]$.

Remark 2: In protocol (3), K_1 can be used to design the motion modes of the time-varying formation center, while K_2 can be designed to drive all the UAVs to achieve the desired formation. It should be noted that protocol (3) presents a general framework for consensus-based formation control protocols. Many existing protocols, such as those shown in [30], [37], and [38], and so on can be regarded as special cases of protocol (3). However, it should be pointed out that collision avoidance cannot be ensured by protocol (3) for any initial conditions.

Let $\theta(t) = [\theta_1^T(t), \theta_2^T(t), \ldots, \theta_N^T(t)]^T$, $h_x(t) = [h_{1x}(t), h_{2x}(t), \ldots, h_{Nx}(t)]^T$, and $h_v(t) = [h_{1v}(t), h_{2v}(t), \ldots, h_{Nv}(t)]^T$. Under protocol (3), UAV swarm system (2) can be written in a compact form as follows:

$$\begin{aligned}\dot{\theta}(t) = &\left(I_N \otimes \left(B_2K_1 + B_1B_2^T\right) - L \otimes (B_2K_2)\right)\theta(t)\\ &-(I_N \otimes (B_2K_1) - L\otimes(B_2K_2))\,h(t) + (I_N \otimes B_2)\,\dot{h}_v(t).\end{aligned} \quad (4)$$

This brief mainly investigates the following three problems for UAV swarm system (4): 1) under what conditions the time-varying formation $h(t)$ can be achieved; 2) how to design protocol (3) to achieve the time-varying formation $h(t)$; and 3) how to demonstrate the theoretical results on practical quadrotor formation platform.

III. Time-Varying Formation Analysis and Protocol Design

In this section, first, time-varying formation problems for UAV swarm system (4) are transformed into consensus problems. Then, necessary and sufficient conditions to achieve time-varying formation $h(t)$ are presented, and an explicit expression of the time-varying formation center function is given. Finally, a procedure to determine the gain matrices in protocol (3) is proposed.

A. Time-Varying Formation Analysis

Let λ_i $(i = 1, 2, \ldots, N)$ be the eigenvalues of L corresponding to G, where $\lambda_1 = 0$ with the associated eigenvector $\bar{u}_1 = \mathbf{1}_N$ and $0 < \mathrm{Re}(\lambda_2) \le \cdots \le \mathrm{Re}(\lambda_N)$. Let $U^{-1}LU = J$, where $U = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_N]$, $U^{-1} = [\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_N]^H$ and J is the Jordan canonical form of L with $\bar{u}_i \in \mathbb{C}^N$ and $\tilde{u}_i \in \mathbb{C}^N$ $(i = 1, 2, \ldots, N)$. Let $\tilde{\theta}_i(t) = \theta_i(t) - h_i(t)$ $(i = 1, 2, \ldots, N)$ and $\tilde{\theta}(t) = [\tilde{\theta}_1^T(t), \tilde{\theta}_2^T(t), \ldots, \tilde{\theta}_N^T(t)]^T$. Then, swarm system (4) can be transformed into

$$\begin{aligned}\dot{\tilde{\theta}}(t) = &\left(I_N \otimes \left(B_2K_1 + B_1B_2^T\right) - L \otimes (B_2K_2)\right)\tilde{\theta}(t)\\ &+I_N \otimes \left(B_1B_2^T\right)h(t) + (I_N \otimes B_2)\dot{h}_v(t) - \dot{h}(t).\end{aligned} \quad (5)$$

Due to that $B_1 = [1, 0]^T$, $B_2 = [0, 1]^T$, and $\dot{h}(t) = (I_N \otimes I_2)\dot{h}(t)$, swarm system (5) can be rewritten as

$$\begin{aligned}\dot{\tilde{\theta}}(t) = &\left(I_N \otimes \left(B_2K_1 + B_1B_2^T\right) - L \otimes (B_2K_2)\right)\tilde{\theta}(t)\\ &+ (I_N \otimes B_1)\,(h_v(t) - \dot{h}_x(t)).\end{aligned} \quad (6)$$

The following lemma holds directly.

Lemma 2: UAV swarm system (4) achieves time-varying formation $h(t)$ if and only if swarm system (6) achieves consensus.

Let $c_1 \in \mathbb{R}^2$ and $c_2 \in \mathbb{R}^2$ be linearly independent vectors and $p_j = \bar{u}_i \otimes c_q$ $(j = 2(i-1)+q;\ i = 1, 2, \ldots, N;\ q = 1, 2)$. A consensus subspace (CS) is defined as the subspace $\mathbb{C}(U)$ spanned by $p_1 = \bar{u}_1 \otimes c_1 = \mathbf{1}_N \otimes c_1$ and $p_2 = \bar{u}_1 \otimes c_2 = \mathbf{1}_N \otimes c_2$, and a complement CS is defined as the subspace $\bar{\mathbb{C}}(U)$ spanned by $p_3, p_4, \ldots, p_{2N}$. Since p_j $(j = 1, 2, \ldots, 2N)$ are linearly independent, the following lemma can be obtained.

Lemma 3: $\mathbb{C}(U) \oplus \bar{\mathbb{C}}(U) = \mathbb{C}^{2N}$.

By Lemma 1, one gets $J = \mathrm{diag}\{0, \bar{J}\}$, where $\bar{J}$ consists of Jordan blocks corresponding to λ_i $(i = 2, 3, \ldots, N)$. Let $\tilde{U} = [\tilde{u}_2, \tilde{u}_3, \ldots, \tilde{u}_N]^H$, $\xi(t) = (\tilde{u}_1^H \otimes I_2)\tilde{\theta}(t)$, and $\varsigma(t) = (\tilde{U} \otimes I_2)\tilde{\theta}(t)$, then swarm system (6) can be transformed into

$$\dot{\xi}(t) = \left(B_2K_1 + B_1B_2^T\right)\xi(t) + \left(\tilde{u}_1^H \otimes B_1\right)(h_v(t) - \dot{h}_x(t)) \quad (7)$$

$$\dot{\varsigma}(t) = \left(I_{N-1} \otimes \left(B_2K_1 + B_1B_2^T\right) - \bar{J} \otimes (B_2K_2)\right)\varsigma(t) + \left(\tilde{U} \otimes B_1\right)(h_v(t) - \dot{h}_x(t)). \quad (8)$$

Lemma 4 [39]: The system $\dot{\varphi}(t) = M\varphi(t)$, where M is a 2×2 complex matrix with characteristic polynomial $f(s) = s^2 + a_1 s + a_2$, is asymptotically stable if and only if $\mathrm{Re}(a_1) > 0$ and $\mathrm{Re}(a_1)\mathrm{Re}(a_1\bar{a}_2) - \mathrm{Im}(a_2)^2 > 0$.

The following theorem presents a necessary and sufficient condition for UAV swarm system (4) to achieve time-varying formation $h(t)$.

Theorem 1: UAV swarm system (4) achieves time-varying formation $h(t)$ if and only if the following conditions hold simultaneously:

1) for any $i \in \{1, 2, \ldots, N\}$

$$\lim_{t\to\infty}\left(\left(h_{iv}(t) - h_{jv}(t)\right) - \left(\dot{h}_{ix}(t) - \dot{h}_{jx}(t)\right)\right) = 0, \quad j \in N_i; \quad (9)$$

2) for any $i \in \{2, 3, \ldots, N\}$

$$-k_{12} + \mathrm{Re}(\lambda_i)k_{22} > 0 \quad (10)$$

$$(-k_{12} + \mathrm{Re}(\lambda_i)k_{22})\,\psi_i - \mathrm{Im}(\lambda_i)^2 k_{21}^2 > 0 \quad (11)$$

where

$$\psi_i = k_{12}k_{11} - \mathrm{Re}(\lambda_i)\,(k_{12}k_{21} + k_{11}k_{22}) + \left(\mathrm{Re}(\lambda_i)^2 + \mathrm{Im}(\lambda_i)^2\right)k_{21}k_{22}.$$

Proof: Let $\tilde{\theta}_C(t) = (U \otimes I_2)[\xi^H(t), 0]^H$, $\tilde{\theta}_{\bar{C}}(t) = (U \otimes I_2)[0, \varsigma^H(t)]^H$, and $e_i \in \mathbb{R}^N$ be a vector that has 1 as its ith component and 0 elsewhere. Because c_1 and c_2 are linearly independent vectors, there exist $\alpha_1(t)$, $\alpha_2(t)$, and $\alpha_{2k+j}(t)$ $(k = 1, 2, \ldots, N-1; j = 1, 2)$ such that $\xi(t) = \alpha_1(t)c_1 + \alpha_2(t)c_2$ and $\varsigma(t) = [\alpha_3(t)c_1^H + \alpha_4(t)c_2^H, \ldots, \alpha_{2N-1}(t)c_1^H + \alpha_{2N}(t)c_2^H]^H$. Due to $[\xi^H(t), 0]^H = e_1 \otimes \xi(t)$, one has

$$\begin{aligned}\tilde{\theta}_C(t) &= (U \otimes I_2)\,(e_1 \otimes \xi(t))\\ &= \bar{u}_1 \otimes \xi(t)\\ &= \alpha_1(t)p_1 + \alpha_2(t)p_2 \in \mathbb{C}(U).\end{aligned} \quad (12)$$

By the structures of p_j $(j = 3, 4, \ldots, 2N)$, it can be shown

$$\begin{aligned}\tilde{\theta}_{\bar{C}}(t) &= \sum_{i=2}^{N}\left(\alpha_{2i-1}(t)\,(\bar{u}_i \otimes c_1) + \alpha_{2i}(t)\,(\bar{u}_i \otimes c_2)\right)\\ &= \sum_{j=3}^{2N}\alpha_j(t)p_j \in \bar{\mathbb{C}}(U).\end{aligned} \quad (13)$$

Note that $[\xi^H(t), \varsigma^H(t)]^H = (U^{-1} \otimes I_2)\tilde{\theta}(t)$, one has $\tilde{\theta}(t) = \tilde{\theta}_C(t) + \tilde{\theta}_{\bar{C}}(t)$. From Lemmas 2 and 3, UAV swarm system (4) achieves time-varying formation $h(t)$ if and only if $\lim_{t\to\infty}\tilde{\theta}_{\bar{C}}(t) = 0$; that is

$$\lim_{t\to\infty}\varsigma(t) = 0. \quad (14)$$

Let

$$\dot{\bar{\varsigma}}(t) = \left(I_{N-1} \otimes \left(B_2K_1 + B_1B_2^T\right) - \bar{J} \otimes (B_2K_2)\right)\bar{\varsigma}(t). \quad (15)$$

From (8) and (14), one knows that UAV swarm system (4) achieves formation $h(t)$ if and only if the system described by (15) is asymptotically stable and

$$\lim_{t\to\infty}\left(\tilde{U} \otimes B_1\right)\left(h_v(t) - \dot{h}_x(t)\right) = 0. \quad (16)$$

Next, it will be shown that conditions (1) and (2) in Theorem 1 are equivalent to the conditions that (16) holds and system (15) is asymptotically stable, respectively. If (9) holds, one has

$$\lim_{t\to\infty}(L \otimes B_1)\left(h_v(t) - \dot{h}_x(t)\right) = 0. \quad (17)$$

Substituting $L = UJU^{-1}$ into (17) and premultiplying both sides of (17) by $U^{-1} \otimes I$, one has

$$\lim_{t\to\infty}(\bar{J}\tilde{U} \otimes B_1)(h_v(t) - \dot{h}_x(t)) = 0. \quad (18)$$

Since G has a spanning tree, by Lemma 1 and the structure of J, $\bar{J}$ is nonsingular. Premultiplying both sides of (18) by $\bar{J}^{-1} \otimes I_2$ results in (16); that is, condition (1) is sufficient for (16).

If (16) holds, let $\tilde{U} = [\hat{U}, \hat{u}]$, where $\hat{U} \in \mathbb{C}^{(N-1)\times(N-1)}$ and $\hat{u} \in \mathbb{C}^{(N-1)\times 1}$ with $\hat{u}$ being the last column vector of $\tilde{U}$. Since $\mathrm{rank}(\tilde{U}) = N - 1$, without loss of generality, it is assumed that $\mathrm{rank}(\hat{U}) = N - 1$. From (16), one has

$$\lim_{t\to\infty}\left([\hat{U}, \hat{u}] \otimes B_1\right)\left(h_v(t) - \dot{h}_x(t)\right) = 0. \quad (19)$$

Since $\tilde{U}\mathbf{1}_N = 0$, it follows that:

$$\hat{u} = -\hat{U}\mathbf{1}_{N-1}. \quad (20)$$

Let $\bar{h}_x(t) = [h_{1x}(t), h_{2x}(t), \ldots, h_{(N-1)x}(t)]^T$ and $\bar{h}_v(t) = [\dot{h}_{1v}(t), \dot{h}_{2v}(t), \ldots, \dot{h}_{(N-1)v}(t)]^T$. From (19) and (20), it can be verified that

$$\lim_{t\to\infty}\left(\hat{U} \otimes I_2\right)\left((I_{N-1} \otimes B_1)\left(\bar{h}_v(t) - \dot{\bar{h}}_x(t)\right) - (\mathbf{1}_{N-1} \otimes B_1) \times \left(h_{Nv}(t) - \dot{h}_{Nx}(t)\right)\right) = 0. \quad (21)$$

Premultiplying the both sides of (21) by $\hat{U}^{-1} \otimes I_2$, one has for $\forall i \in \{1, 2, \ldots, N-1\}$

$$\lim_{t\to\infty}\left((h_{iv}(t) - h_{Nv}(t)) - \left(\dot{h}_{ix}(t) - \dot{h}_{Nx}(t)\right)\right) = 0. \quad (22)$$

By (22), it can be shown that condition (1) holds. Therefore, condition (1) in Theorem 1 is equivalent to condition (16).

From the structure of $\bar{J}$, one knows that the stability of system (15) is equivalent to these of the following $N - 1$ subsystems

$$\dot{\bar{\varsigma}}_i(t) = \left(B_2\,(K_1 - \lambda_i K_2) + B_1B_2^T\right)\bar{\varsigma}_i(t) \quad (i = 2, 3, \ldots, N). \quad (23)$$

Note that

$$\begin{aligned} &B_2(K_1 - \lambda_i K_2) + B_1 B_2^T \\ &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_{11} - \lambda_i k_{21} & k_{12} - \lambda_i k_{22} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ k_{11} - \lambda_i k_{21} & k_{12} - \lambda_i k_{22} \end{bmatrix}. \end{aligned}$$

One can obtain the characteristic polynomial of the state matrix of subsystems (23) as $f_i(s) = s^2 - (k_{12} - \lambda_i k_{22})\, s - (k_{11} - \lambda_i k_{21})$ $(i = 2, 3, \ldots, N)$, where s is a complex variable. From Lemma 4, it can be obtained that system (14) is asymptotically stable if and only if condition (2) holds. This completes the proof. ∎

Remark 3: Equation (9) indicates that there exists constraint on the formation which can be achieved. In fact, a group of UAVs cannot achieve any formations due to their dynamic limitation. Conditions (10) and (11) ensure the internal stability of subsystem (8), which ensures that the formation error converges to zero.

Remark 4: Formation problems for swarm system (2) have been investigated in [30] and [38], and sufficient conditions have been obtained to achieve time invariant formations. By Theorem 1, necessary and sufficient conditions can be obtained for swarm system (2) to achieve time-varying formations. In addition, by Theorem 1, it can be verified that the conditions in [38] are not only sufficient but also necessary.

If UAV swarm system (4) achieves $h(t)$, then from the proof of Theorem 1, one knows that $\tilde{\theta}_i(t) - \xi(t) \to 0$ as $t \to \infty$. Therefore, subsystem (7) determines the formation center function and an explicit expression of the time-varying formation center function can be obtained directly as follows.

Lemma 5: If UAV swarm system (4) achieves time-varying formation $h(t)$ with the formation center function $c(t)$, then $\lim_{t\to\infty} (c(t) - c_0(t) - c_h(t)) = 0$ where

$$c_0(t) = e^{(B_2K_1 + B_1B_2^T)t}\left(\tilde{u}_1^H \otimes I_2\right)\theta(0)$$

$$\begin{aligned} c_h(t) = &\int_0^t e^{(B_2K_1 + B_1B_2^T)(t-\tau)}\left(\tilde{u}_1^H \otimes B_2\right) \\ &\times\left(\dot{h}_v(\tau) - k_{12}h_v(\tau) - k_{11}h_x(\tau)\right)d\tau - \left(\tilde{u}_1^H \otimes I_2\right)h(t). \end{aligned}$$

Remark 5: In Lemma 5, $c_0(t)$ is said to be the consensus function, which also is the time-varying formation center function of the swarm system (4) with formation $h(t) \equiv 0$. $c_h(t)$ describes the effect of $h(t)$ on the formation center. If $h(t) \equiv 0$, $c(t)$ becomes the explicit expression of the consensus function for swarm system (4). From Theorem 2, one can see that K_1 can be used to design the motion modes [40] of the time-varying formation center by assigning the eigenvalues of $B_2K_1 + B_1B_2^T$.

B. Time-Varying Formation Protocol Design

In this section, a procedure to determine the gain matrices in protocol (3) for UAV swarm system (2) to achieve time-varying formations is proposed.

Theorem 2: If condition (1) in Theorem 1 holds, UAV swarm system (2) achieves time-varying formation by protocol (3) with $K_2 = [\mathrm{Re}(\lambda_2)]^{-1} B_2^T \bar{P}$, where $\bar{P}$ is the positive definite solution to the algebraic Riccati equation

$$\bar{P}\left(B_2K_1 + B_1B_2^T\right) + \left(B_2K_1 + B_1B_2^T\right)^T \bar{P} - \bar{P}B_2B_2^T\bar{P} + I = 0. \tag{24}$$

Proof: Consider the $N-1$ subsystems described by (24) and construct the following Lyapunov function candidate:

$$V_i(t) = \bar{\varsigma}_i^H(t)\bar{P}\bar{\varsigma}_i(t) \quad (i = 2, 3, \ldots, N). \tag{25}$$

Let $K_2 = [\mathrm{Re}(\lambda_2)]^{-1} B_2^T \bar{P}$. Taking the derivative of $V_i(t)$ with respect to t along the solution of subsystem (23), one has

$$\begin{aligned} \dot{V}_i(t) = &-\bar{\varsigma}_i^H(t)\bar{\varsigma}_i(t) + \left(1 - 2\mathrm{Re}(\lambda_i)[\mathrm{Re}(\lambda_2)]^{-1}\right) \\ &\times\bar{\varsigma}_i^H(t)\bar{P}B_2B_2^T\bar{P}\bar{\varsigma}_i(t). \end{aligned} \tag{26}$$

Since $0 < \mathrm{Re}(\lambda_2) \leq \cdots \leq \mathrm{Re}(\lambda_N)$, from (26), it holds that $\dot{V}_i(t) \leqslant -\bar{\varsigma}_i^H(t)\bar{\varsigma}_i(t)$ $(i = 2, 3, \ldots, N)$. According to Lyapunov's second method for stability, the $N-1$ subsystems described by (23) are asymptotically stable. From the proof of Theorem 1, one knows that UAV swarm system (2) achieves time-varying formation by protocol (3). The proof for Theorem 2 is completed. ∎

Based on the above results, a design procedure of protocol (3) can be summarized as follows. First, choose K_1 to design the motion modes of the formation center by assigning the eigenvalues of $B_2K_1 + B_1B_2^T$ at the desired locations in the complex plane. Since $(B_1B_2^T, B_2)$ is controllable, there always exists such K_1. Then, design K_2 to satisfy condition (11) using the conclusion of Theorem 2.

Remark 6: Compared with [41], this brief has the following three main theoretical contributions. First, only circle formation is investigated in [41]. In this brief, the formations can be any time-varying vectors satisfying condition (1). Second, in [41], each UAV is required to obtain both the position information of the formation center and the angle information of two immediate neighbors. In addition, the interaction topology in [41] must have a ring. In this brief, each UAV does not need to know the information mentioned previously. In addition, the interaction topology only needs to have a spanning tree. Third, as claimed by Kingston and Beard [41] themselves that the global stability of the approach cannot be conclusively shown and the results are demonstrated only by simulations. In this brief, necessary and sufficient conditions for UAV swarm systems to achieve time-varying formations are proven and experimental results are shown.

IV. Quadrotor Formation Platform

In this section, a quadrotor formation platform, which is used to demonstrate the theoretical results obtained in Section III is introduced.

The quadrotor formation platform consists of five quadrotors with flight control system (FCS) and one ground control station (GCS), which is shown in Fig. 2. The quadrotor frames together with accompanying brushless motors, propellers, and electrical speed controllers are provided by Xaircraft [42]. Each quadrotor has a tip-to-tip wingspan of 65 cm, a weight of about 1600 g, a battery life of 12 min, and a maximum take-off weight of 1800 g.

Fig. 2. Hardware structure of the quadrotor system.

To estimate the attitude and acceleration of the quadrotor, the FCS is equipped with three one-axis gyroscopes, a three-axis accelerometer, and a three-axis magnetometer. A global positioning system module with accuracy of 1.2-m circular error probable is used to measure the position and velocity of each quadrotor at a rate of 10 Hz. An ultrasonic range finder is adopted to get a more accurate height measurement. In addition, a micro-SD card is mounted onboard for data logging. Zigbee modules are used for wireless communication among quadrotors and the GCS. An RC receiver is also mounted on the quadrotor in case of emergency. All the computations are performed on the onboard TMS320F28335 DSP running at 135 MHz. Through the Zigbee network, control commands can be sent to a specified quadrotor or broadcast to all quadrotors, and the states of all quadrotors can be sent to the GCS. In addition, the states of all quadrotors can be monitored by the real-time display module on the GCS.

V. Simulation and Experimental Applications on Quadrotors

In this section, two applications on five quadrotors are shown to demonstrate the effectiveness of theoretical results in Section III. The interaction topology of the quadrotor swarm system in the applications is shown in Fig. 3. For simplicity, it is assumed that the interaction topology has 0–1 weights, and all quadrotors move in the XY plane; that is, $n = 2$.

The outer-loops for trajectory dynamics along x-axis and y-axis are controlled by the formation controller (3), and the inner-loops for attitudes rotating around x-axis and y-axis are controlled by PD controllers shown in [43]. The height and the yaw angle of each quadrotor are specified to be constants. The attitude controller runs at 500 Hz, while the formation controller runs at 10 Hz. It should be pointed out that when quadrotors are achieving formation, the GCS do not send commands to quadrotors.

Let $\theta_i(t) = [x_{iX}(t), v_{iX}(t), x_{iY}(t), v_{iY}(t)]^T$ and $h_i(t) = [h_{ixX}(t), h_{ivX}(t), h_{ixY}(t), h_{ivY}(t)]^T$. Using Kronecker product, the dynamics of each quadrotor in the case that $n = 2$ can be written as

$$\dot{\theta}_i(t) = \left(I_2 \otimes B_1 B_2^T\right)\theta_i(t) + (I_2 \otimes B_2)\, u_i(t).$$

Application 1: Consider a time-varying formation $h_i(t)$ ($i = 1, 2, \ldots, 5$) with $h_{ixX} = r\cos((\omega t + 2\pi(i-1)/5) - 1)g_i(t)$, $h_{ivX} = -\omega r \sin(\omega t + 2\pi(i-1)/5)g_i(t)$,

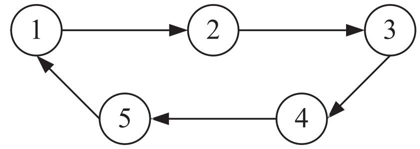


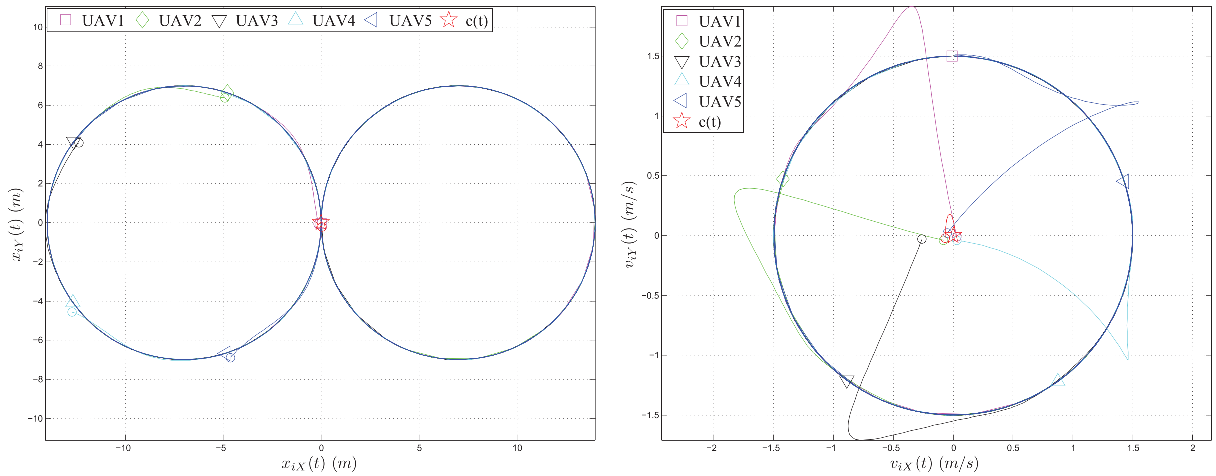
Fig. 3. Directed interaction topology G.

$h_{ixY} = r\sin(\omega t + 2\pi(i-1)/5)$, and $h_{ivY} = \omega r\cos(\omega t + 2\pi(i-1)/5)$, where $r = 7$ m, $\omega = 0.214$ rad$/s$, and $g_i(t) = \mathrm{sign}(\sin(\omega t/2 + \pi(i-1)/5))$. From $h_i(t)$ ($i = 1, 2, \ldots, 5$), one sees that if the quadrotor swarm system achieves the desired time-varying formation, then the five quadrotors will follow a figure eight pattern while keeping a phase separation of $2\pi/5$ rad. It should be pointed out that it is different from the results in [31]–[33] that when the time-varying formation is achieved, the velocities of quadrotors are not identical.

Due to the limitation of flight space and the requirement of performing flight experiments within a visual range, the motion modes of the formation center are placed at $-0.6 + 1.28j$ and $-0.6 - 1.28j$ with $j^2 = -1$ by $K_1 = I_2 \otimes [-2, -1.2]$, which means that when the quadrotor swarm system achieves time-varying formation, the formation center is stationary. In order to avoid collisions, the initial states of quadrotors will be chosen to be near the trajectories of formations. It can be verified that condition (1) in Theorem 1 is satisfied. From Theorem 2, one can obtain a K_2 as $K_2 = I_2 \otimes [0.3416, 0.7330]$. Choose the initial states of the quardrotors as $\theta_1(0) = [-0.16, 0.03, -0.07, -0.01]^T$, $\theta_2(0) = [-4.92, -0.08, 6.38, -0.04]^T$, $\theta_3(0) = [-12.37, -0.26, 4.08, -0.03]^T$, $\theta_4(0) = [-12.73, 0.03, -4.56, -0.04]$, and $\theta_5(0) = [-4.63, -0.05, -6.9, 0.02]^T$.

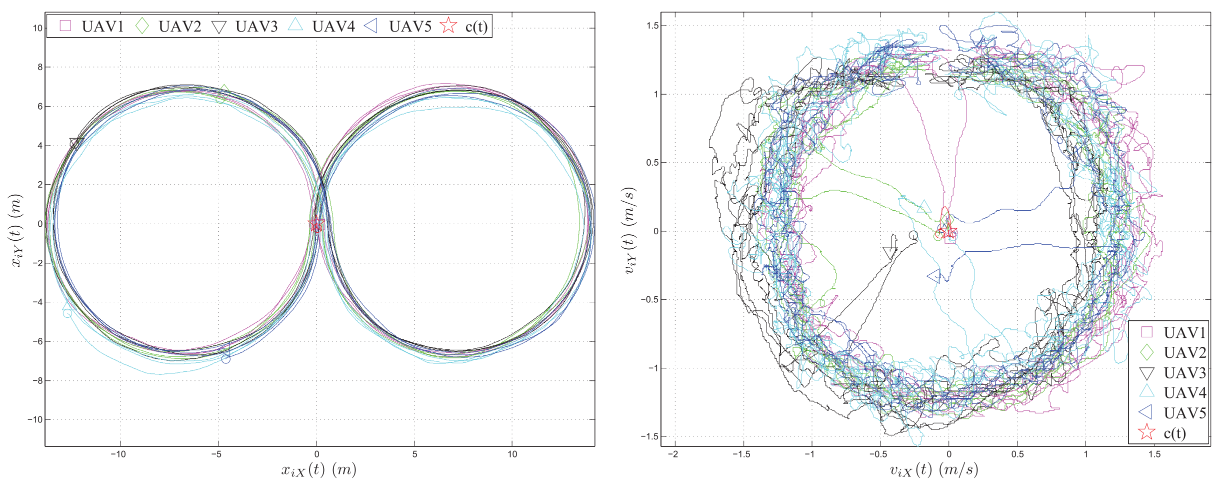
Figs. 4 and 5 show the state trajectories of the quadrotor swarm system in simulation and experiment, and the predefined formation center function $c(t)$ within 180 s, respectively, where the initial states of quadrotors and $c(0)$ are denoted by circles, and the final states of five quadrotors and $c(t)$ are denoted by square, diamond, down triangle, up triangle, left triangle, and pentagram, respectively. Fig. 6 shows the snapshots of positions and velocities of the five quadrotors in the experiment, and $c(t)$ from 16 to 21 s. From Figs. 4–6, one sees that by choosing appropriate initial states, the quadrotor swarm system achieves the predefined time-varying formation with collision avoidance in both simulation and experiment. The videos of the experiment in Application 1 and other two time-varying formation experiments can be found at https://youtu.be/wpRJMQvCQQQ or http://v.youku.com/v_show/id_XNjIwNjUyNDg4.html.

Application 2: In this application, the motion modes of the formation center function are assigned to move periodically. Due to the limitation of flight space and for the safety of the UAVs, only simulation results are given. Consider a time-varying formation $h(t)$ with $h_{ixX}(t) = r\sin(\omega t + 2(i-1)\pi/5)$, $h_{ivX}(t) = \omega r\cos(\omega t + 2(i-1)\pi/5)$, $h_{ixY}(t) = r\cos(\omega t + 2(i-1)\pi/5)$, and $h_{ivY}(t) = -\omega r\sin(\omega t + 2(i-1)\pi/5)$ ($i = 1, 2, \ldots, 5$) where $r = 10$ m and $\omega = 0.1$ rad$/s$. Choose $K_1 = I_2 \otimes [-0.25, 0]$ to assign the motion modes of the formation center at



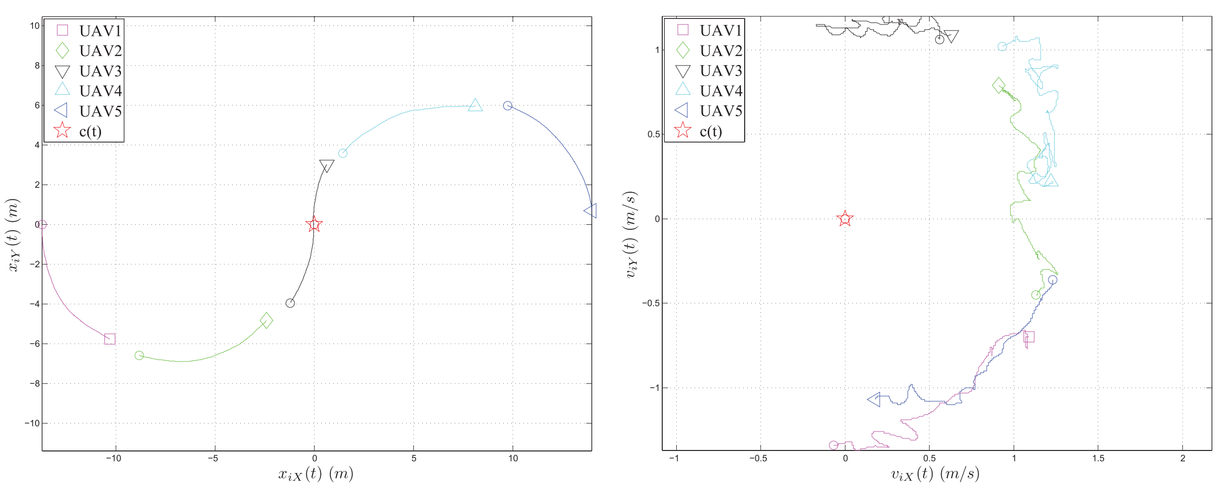
(a) (b)

Fig. 4. State trajectories of five quadrotors in simulation and $c(t)$. (a) Position trajectories in simulation. (b) Velocity trajectories in simulation.



(a) (b)

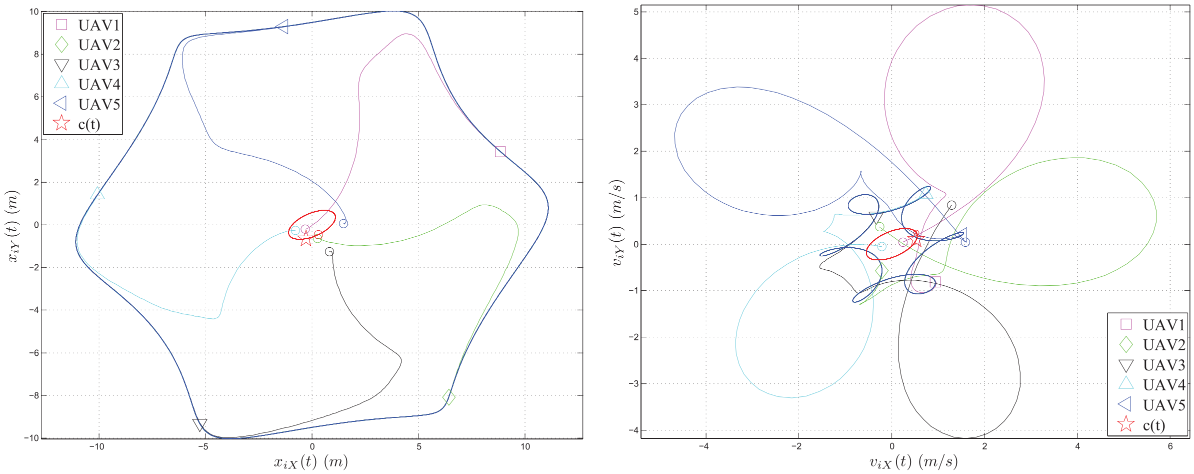
Fig. 5. State trajectories of five quadrotors in experiment and $c(t)$. (a) Position trajectories in experiment. (b) Velocity trajectories in experiment.



(a) (b)

Fig. 6. Snapshots of $\theta(t)$ in the experiment and $c(t)$ for $t \in [4\,\mathrm{s}, 49\,\mathrm{s}]$. (a) Position snapshots in experiment. (b) Velocity snapshots in experiment.

$0.5j$ and $-0.5j$ with $j^2 = -1$. Therefore, if the quadrotor swarm system achieves the desired hboxformation, then the five quadrotors will keep a rotation pentagram around the formation center while the formation center moves periodically. From Theorem 2, one can obtain a K_2 as $K_2 = I_2 \otimes [1.1299, 2.3162]$. Choose the initial states of the quardrotors



(a) (b)

Fig. 7. State trajectories of five quadrotors in simulation and $c(t)$. (a) Position trajectories in simulation. (b) Velocity trajectories in simulation.

as $\theta_i(0) = [i\Theta_{i1}, i\Theta_{i2}, i\Theta_{i3}, i\Theta_{i4}]^T$ $(i = 1, 2, \ldots, 5)$, where Θ_{i1}, Θ_{i2}, Θ_{i3}, and Θ_{i4} are pseudorandom values with a uniform distribution on the interval $(0, 1)$. Fig. 7 shows the position and velocity trajectories of the five quadrotors and $c(t)$ in simulation within 200 s. From Fig. 7, one sees that the quadrotor swarm system achieves the predefined time-varying formation.

VI. Conclusion

Formation control problems for UAV swarm systems to achieve time-varying formations were studied. Formation protocols were presented for UAV swarm systems to achieve predefined time-varying formations. Consensus approaches were used to deal with the formation control problems for UAV swarm systems. Necessary and sufficient conditions for UAV swarm systems to achieve time-varying formations were presented. An explicit expression of the formation center function was derived and an procedure to determine gain matrices in the protocol was proposed. Distributed outdoor time-varying formation flight experiments were carried out on a quadrotor formation platform with five quadrotors to show the effectiveness of theoretical results. As interesting and important research topics, one can aim at collision avoidance and the robustness problems of the protocols.

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