Question 5

Use the definition of θ in order to show the following:

a. $5n^3 + 2n^2 + 3n = \theta(n^3)$

There exists positive real numbers c_1 *and* c_2 *and a positive integer* n_0 *such that*

$$c_2 g(n) \leq f(n) \leq c_1 g(n)$$

$$5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3 = 10n^3$$

$$\boldsymbol{c_1 = 10}$$

$$5n^3 \leq 5n^3 + 2n^2 + 3n$$

$$\boldsymbol{c_2 = 5}$$

$$2n^2 + 3n \geq 0$$

$$2n + 3 \geq 0$$

$$n \geq -1.5$$

$$\boldsymbol{n_0 = -1}$$

b. $\sqrt{7n^2 + 2n - 8} = \theta(n)$

There exists positive real numbers c_1 *and* c_2 *and a positive integer* n_0 *such that*

$$c_2 g(n) \leq f(n) \leq c_1 g(n)$$

$$\sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n^2} \leq \sqrt{9n^2} \leq 3n$$

$$\boldsymbol{c_1 = 3}$$

$$\sqrt{7n^2} \leq \sqrt{7n^2 + 2n - 8}$$

$$\boldsymbol{c_2 = \sqrt{7} = 2.65}$$

$$2n - 8 \geq 0$$

$$2n \geq 8$$

$$n \geq 4$$

$$\boldsymbol{n_0 = 4}$$