

Quantum...

Teleportation



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D You

Lets plug in some numbers



The system is a time machine!!!!!!!!!!!!!!!!!!!!!!!!!!!!

What is the probability of 12?

The order of measurement doesn't matter.

The only fact that matters is that the bits are entangled

I didn't get this



ChatGPT

The image seems to continue the discussion of a two-qubit quantum system and emphasizes that the system is entangled. Entanglement is a unique quantum phenomenon where the state of one qubit cannot be described independently of the state of the other, no matter the distance between them.

The statement about the "system is a time machine" is not accurate within the known laws of physics. It likely is an exaggerated way to express surprise or confusion about the counterintuitive nature of quantum entanglement, which can sometimes lead to misconceptions about causality and time.

It's also correct that the order of measurement does not matter in an entangled system. Whether you measure qubit 0 or qubit 1 first, the measurement outcomes will be correlated because of the entanglement.

If there's a specific part of the content or the quantum mechanics concepts involved that you'd like to discuss further, please let me know, and I can provide more detailed explanations.

More on entanglement

- How do you test if the qubits in the following state are *not* entangled?

$$|\psi\rangle = |q_0q_1\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

More on entanglement

- How do you test if the qubits in the following state are not entangled?

$$|\psi\rangle = |q_0q_1\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- The measurement probabilities for q_1 when q_0 is measured as 0 are in the ratio a/b
- The measurement probabilities for q_1 when q_0 is measured as 1 are in the ratio c/d
- These must be equal:

$$\frac{a}{b} = \frac{c}{d}$$

More on entanglement

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More on entanglement

- How do you test if the qubits in the following state not are entangled?

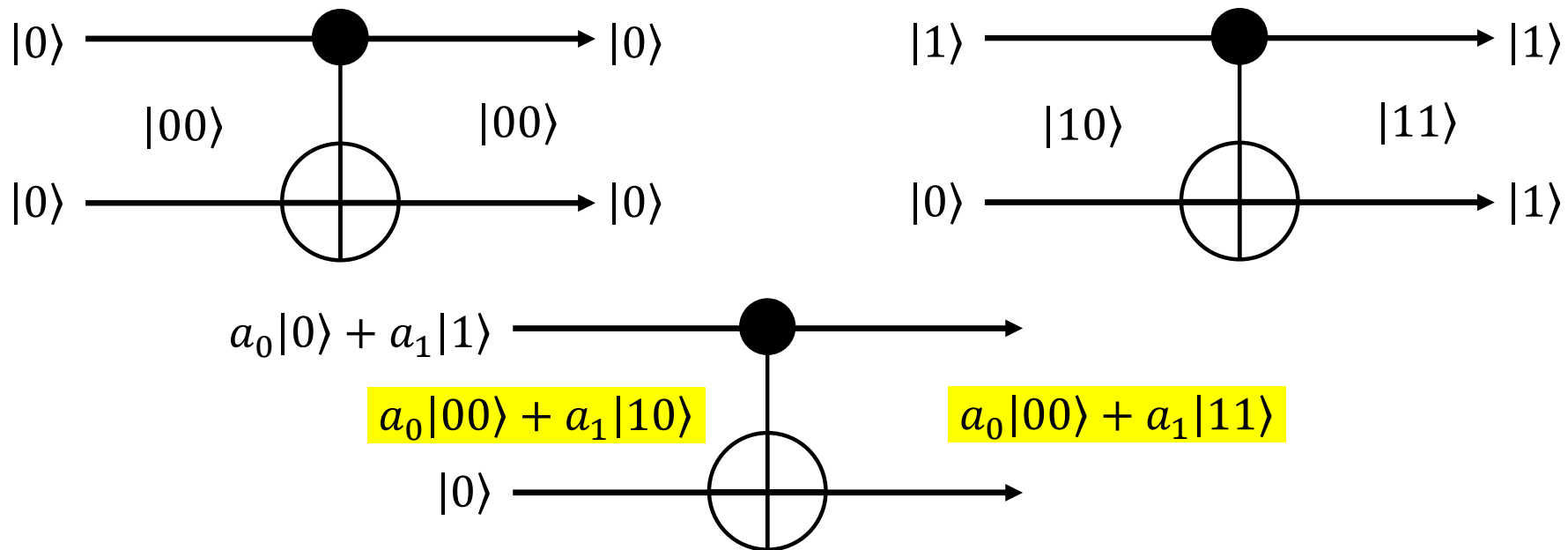
$$|\psi\rangle = |q_0q_1\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- These two tests of non-entanglement are actually identical:

$$\frac{a}{b} = \frac{c}{d}$$

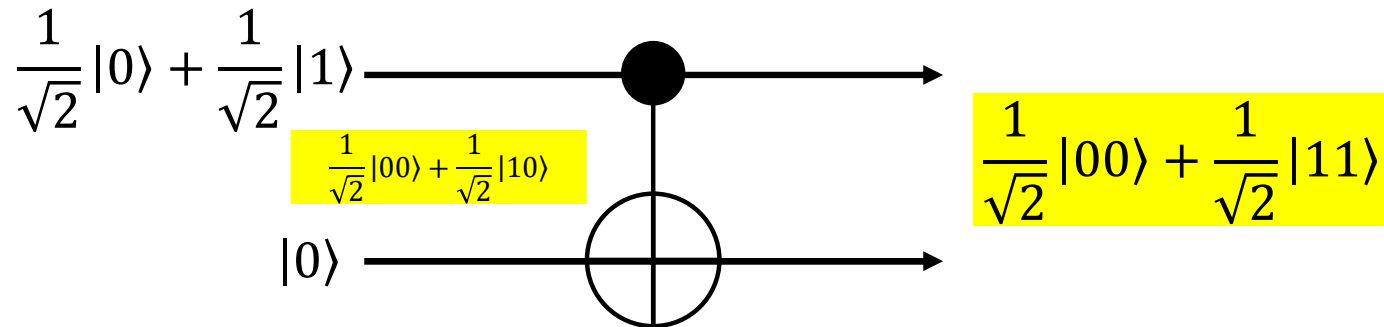
$$\frac{a}{c} = \frac{b}{d}$$

How to entangle bits

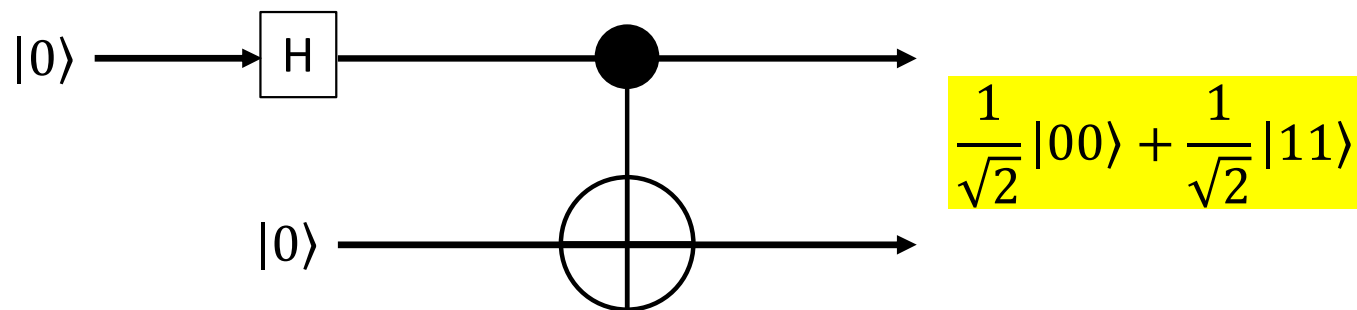


- The CNOT gate: The input entangles with the control (qu)bit
 - When the input is $|0\rangle$, the output is in the $a_0|00\rangle + a_1|11\rangle$ Bell state
 - If the input bit is $|0\rangle$, what is the output?
 - What about when the input too is in superposition

Entangling qubits with the CNOT



- Sending a uniform superposition and a 0 results in a standard Bell state
- This can also be achieved by inputting two 0s, with a Hadamard gate on the first qubit



Entangled bits are still individual bits

$$a_0|00\rangle + a_1|11\rangle = a_0 \left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \right) |0\rangle + a_1 \left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle \right) |1\rangle$$

- Each bit can be represented using a different basis
- Representing the first qubit of entangled qubits in Bell state in sign bases...

$$= \frac{1}{\sqrt{2}} (a_0|+0\rangle + a_0| -0\rangle + a_1|+1\rangle - a_1| -1\rangle)$$

- In spite of the changed representation, they remain entangled
 - What you measure on one qubit affects the phasor of the other

Entangled bits are still individual bits – the uniform Bell state

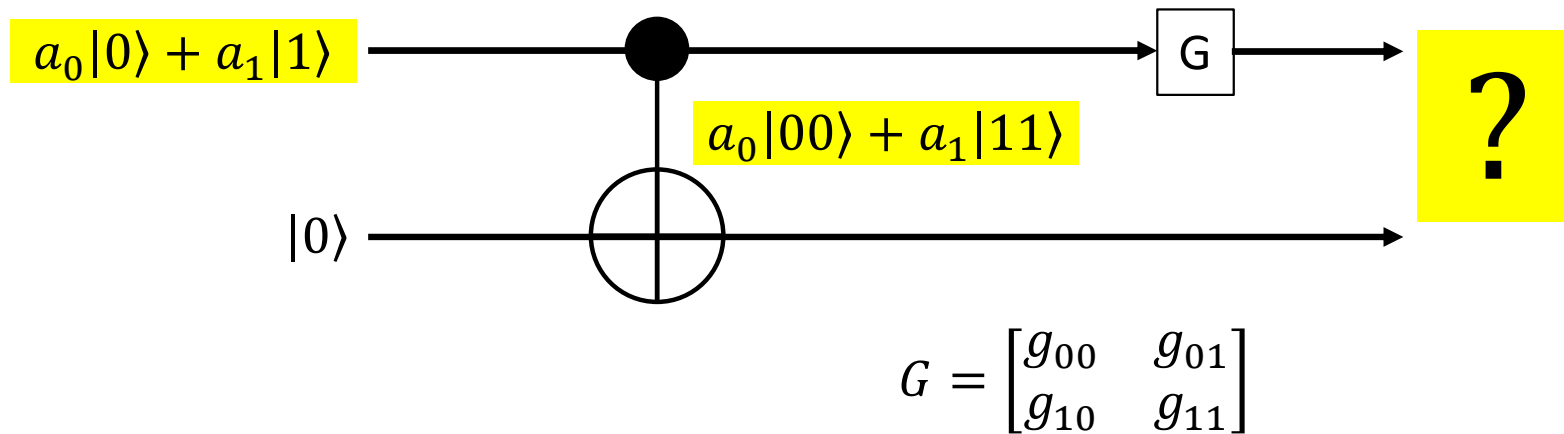
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right)|0\rangle + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right)|1\rangle$$

- Each bit can be represented using a different basis
- Representing the first qubit of entangled qubits in Bell state in sign bases...

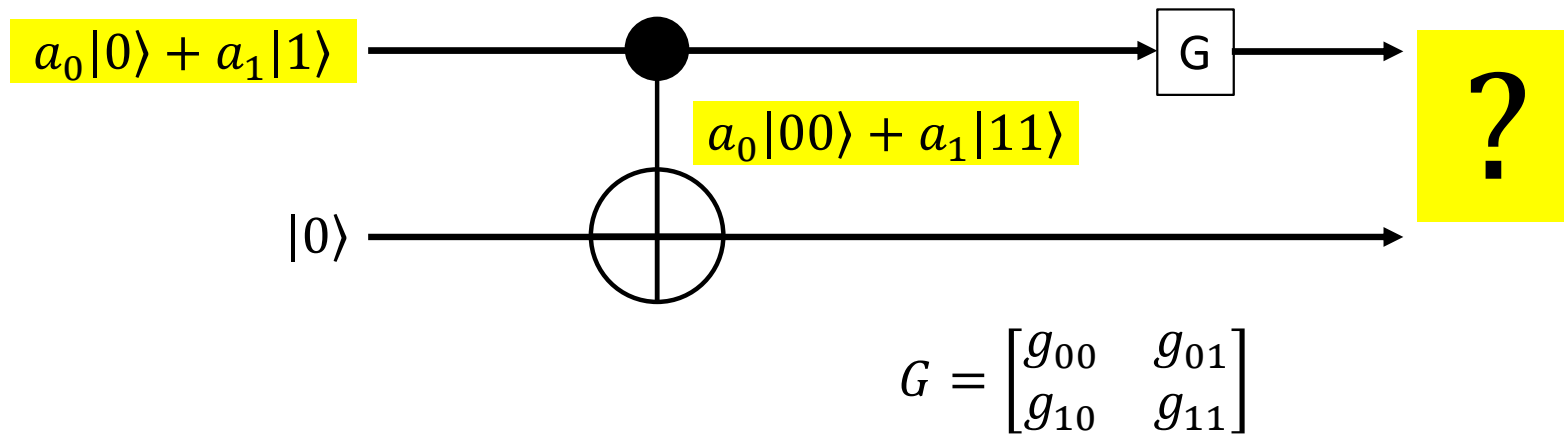
$$= \frac{1}{2}(|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle)$$

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 - What you measure on one qubit affects the phasor of the other

Entangled bits are still individual bits

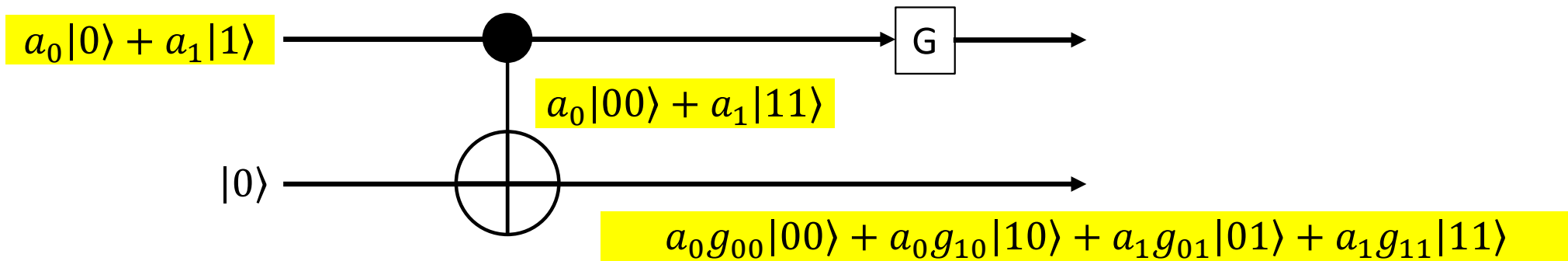


Entangled bits are still individual bits



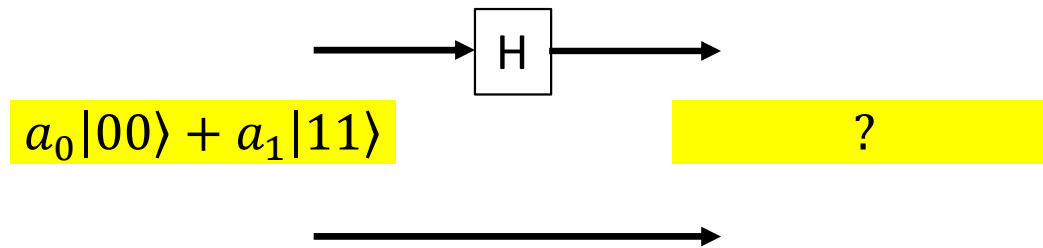
$$a_0(g_{00}|0\rangle + g_{10}|1\rangle)|0\rangle + a_1(g_{01}|0\rangle + g_{11}|1\rangle)|1\rangle$$

Entangled bits are still individual bits



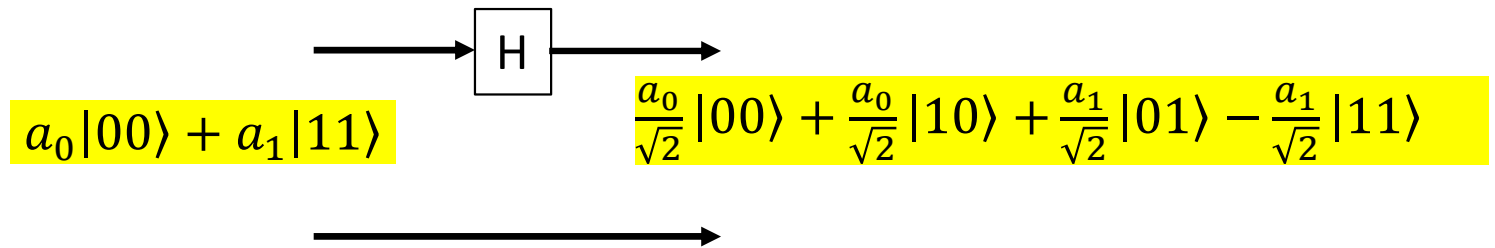
- Each bit has its own life and can be individually manipulated
- This, simply results in a different entangling of the bits

Entangled bits are still individual bits



- Rotating the first qubit of a 2-qubit system in the Bell State by a Hadamard transform

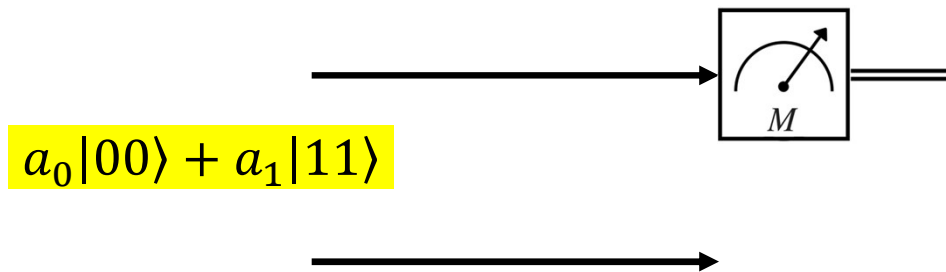
Entangled bits are still individual bits



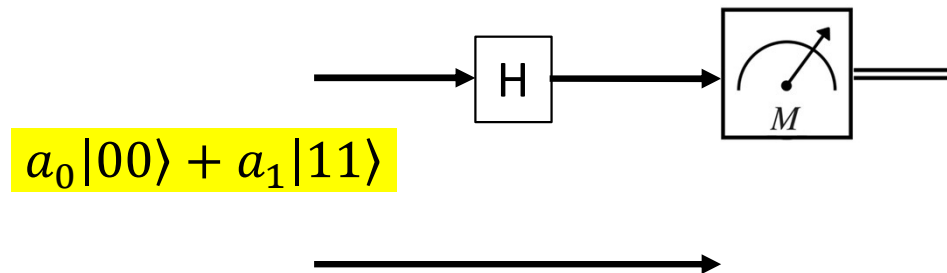
- Rotating the first qubit of a 2-qubit system in the Bell State by a Hadamard transform

This extends to measurement

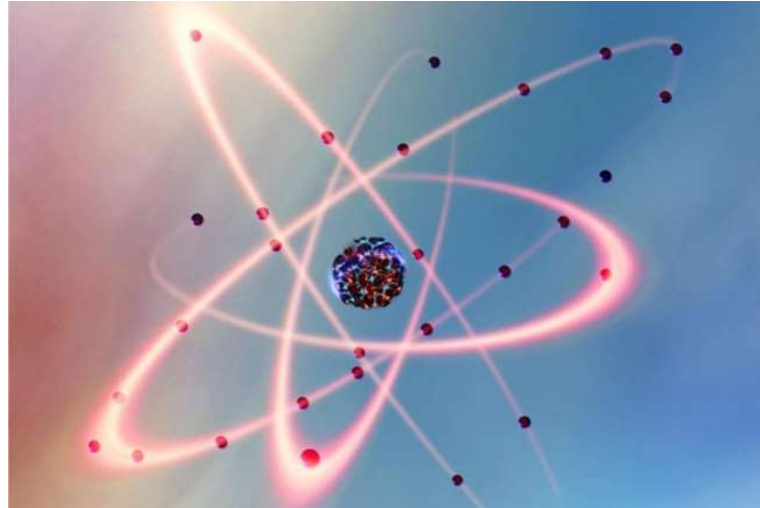
- Each basis can be measure using its own basis
- Measuring q0 of the Bell State in bit bases



- Measuring q0 of the Bell State in sign bases...



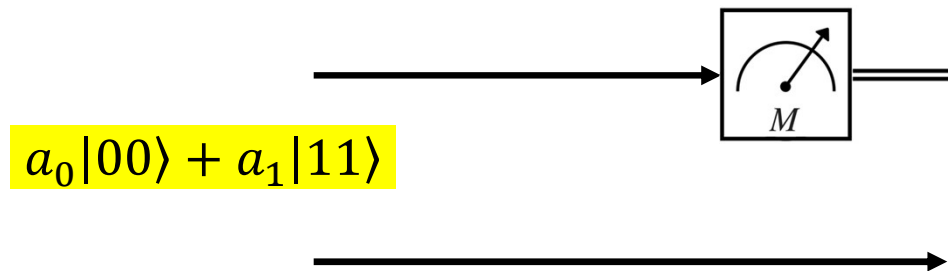
Axioms of quantum mechanics



1. Qubits always reside in a superposition of states
 2. Measurement collapses them to one of the states
 3. Transformations of a qubit comprise solely of unitary transforms
 - Rotations
- Axioms 1 and 3 fit our new math; 2 causes challenges

Measurement destroys entanglement

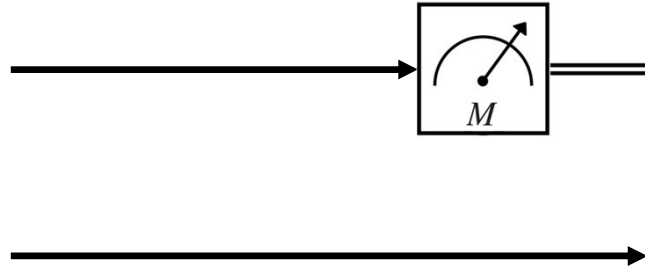
- Measurement of one qubit restricts the other



- The actual measurement of the first qubit results in a random value for the second qubit
 - Depending on the bases used by the first qubit
- At this point they get decoupled
 - Subsequent manipulations of either qubit do not influence the other

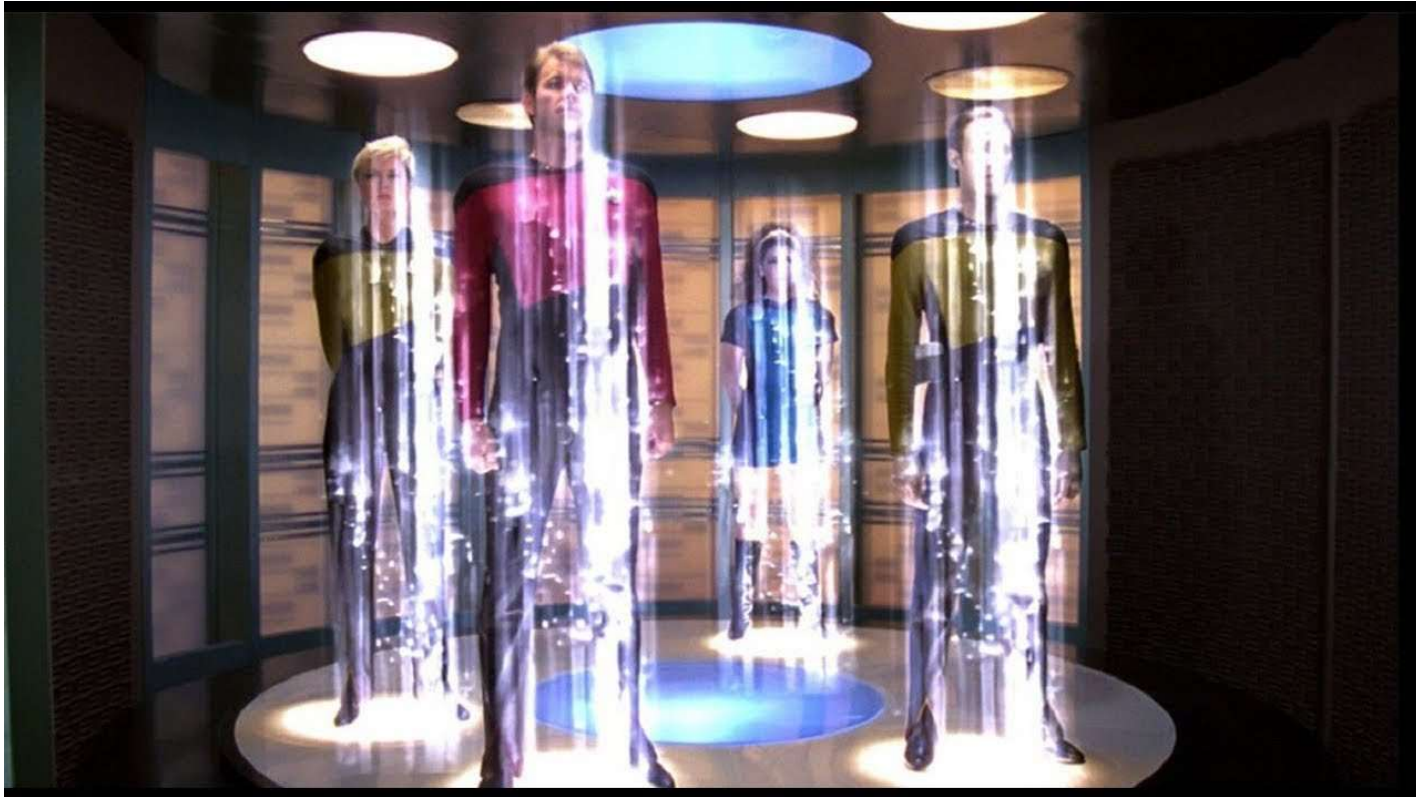
Moving on...

- There's nothing like a bit of randomness to destroy a good relationship...



- But moving on...

Today: How to use entanglement, part 2



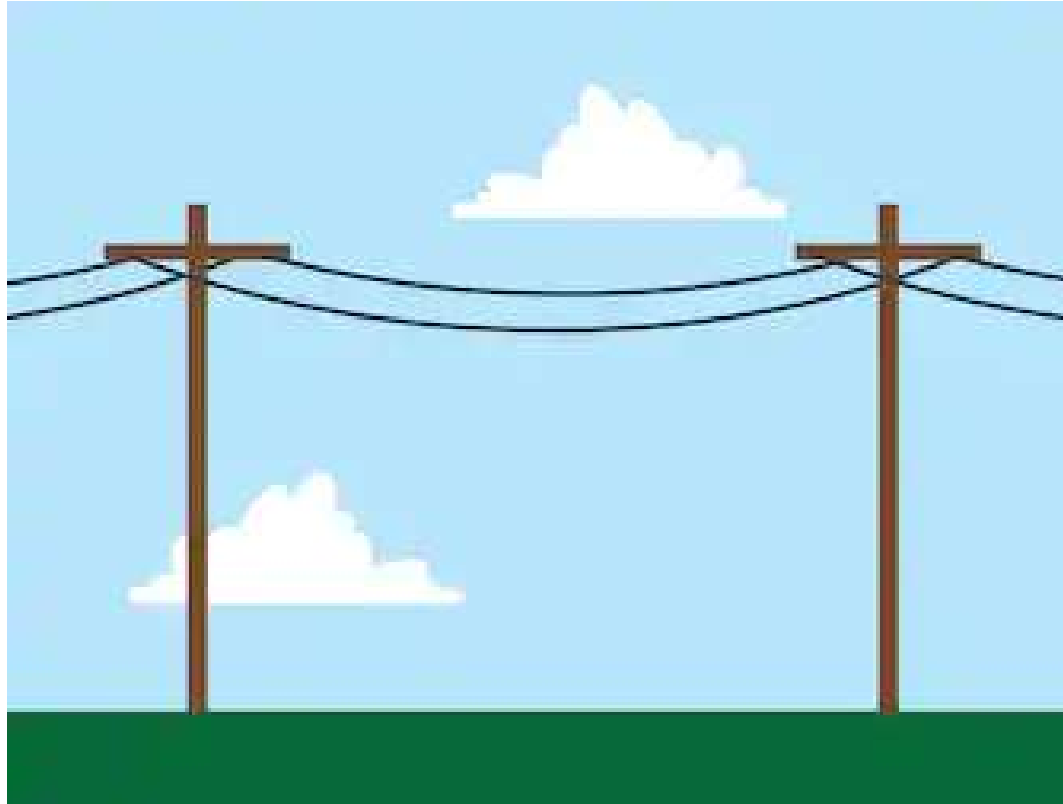
- To teleport ...

The history of communication



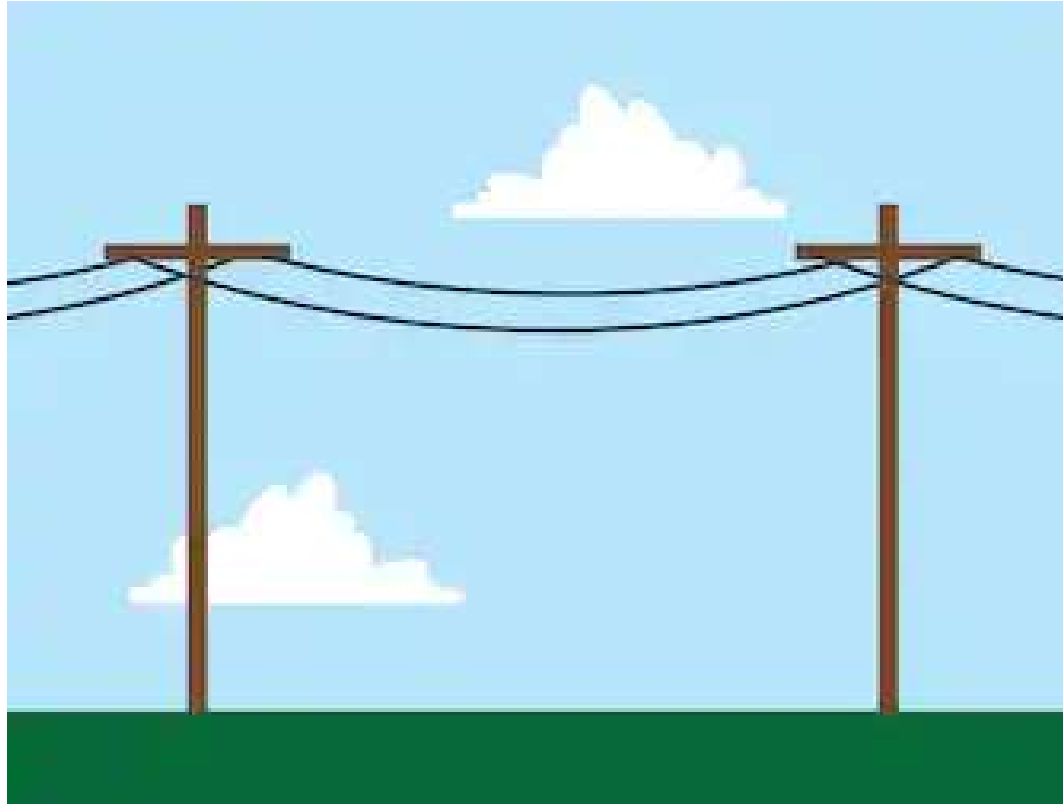
- For thousands of years messages were communicated from point A to point B via written messages on a physical medium
 - Papyrus, paper, bark, the human scalp

Then in 1816 something happened



- The first long-distance installation of telegraph by Francis Reynolds
 - What changed?

Then in 1816 something happened



- The first long-distance installation of telegraph by Francis Reynolds
 - What changed?
 - Messages could now be sent from A to B *with no transfer of medium*
 - *Not a single electron moved so much as a micrometer, yet the message got from A to B*

And now...

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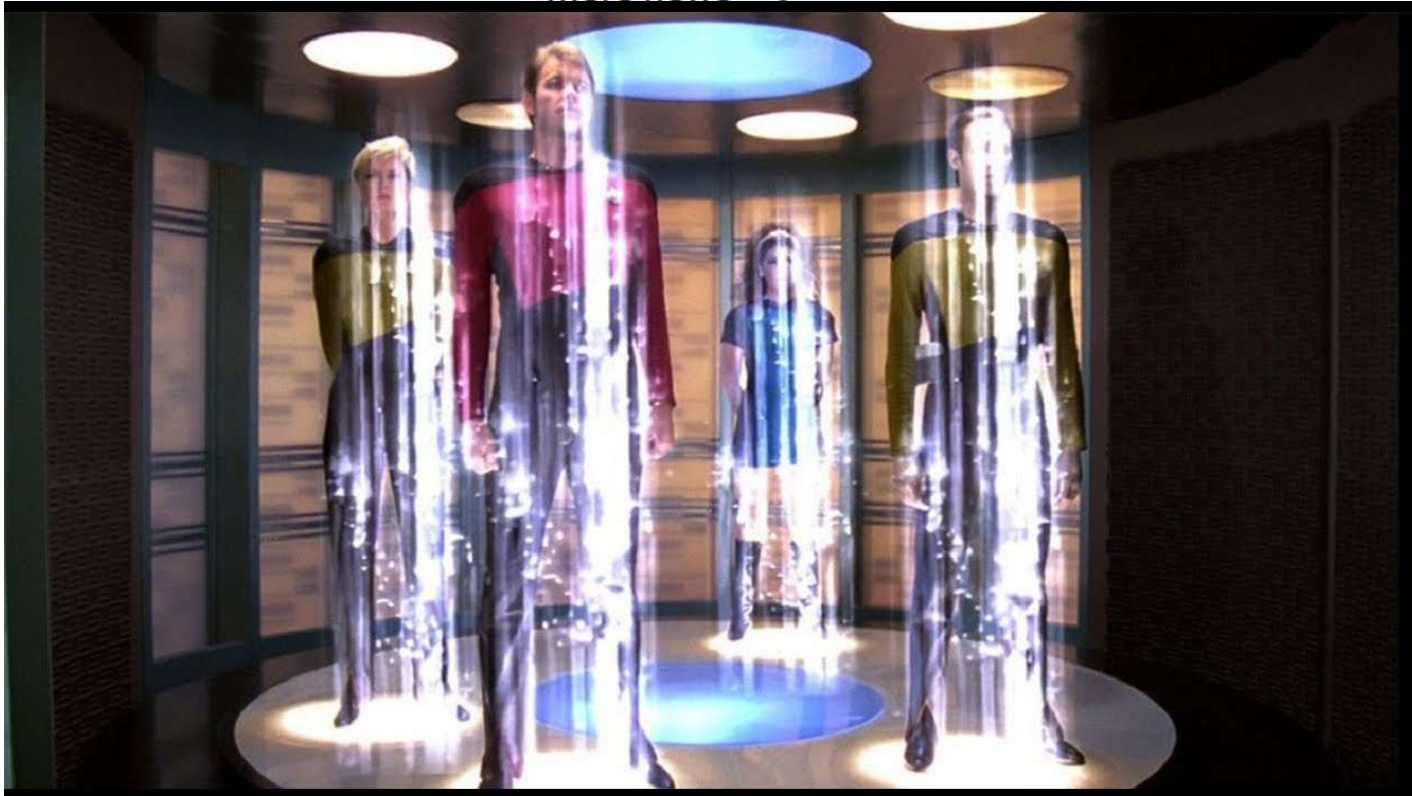
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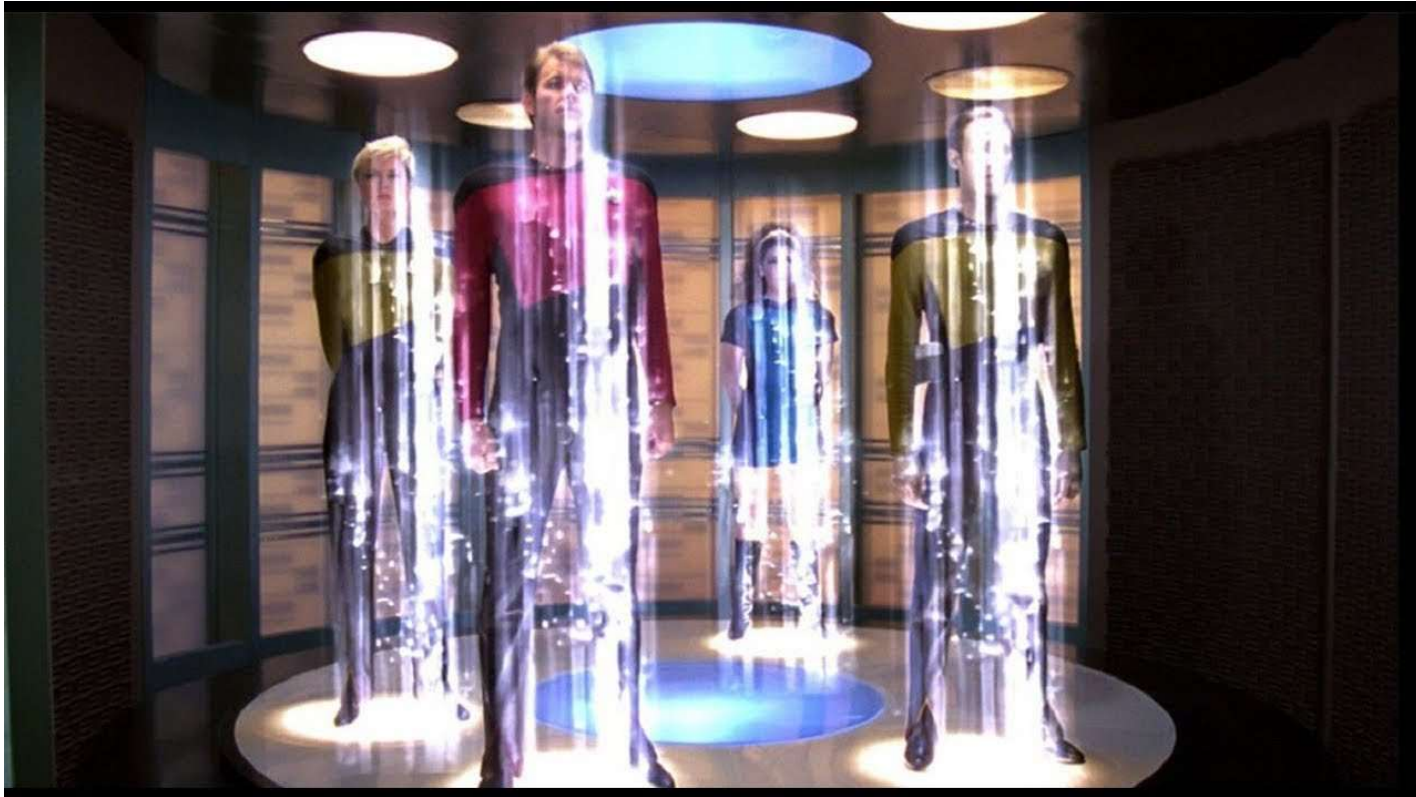
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Today: How to use entanglement, part 2



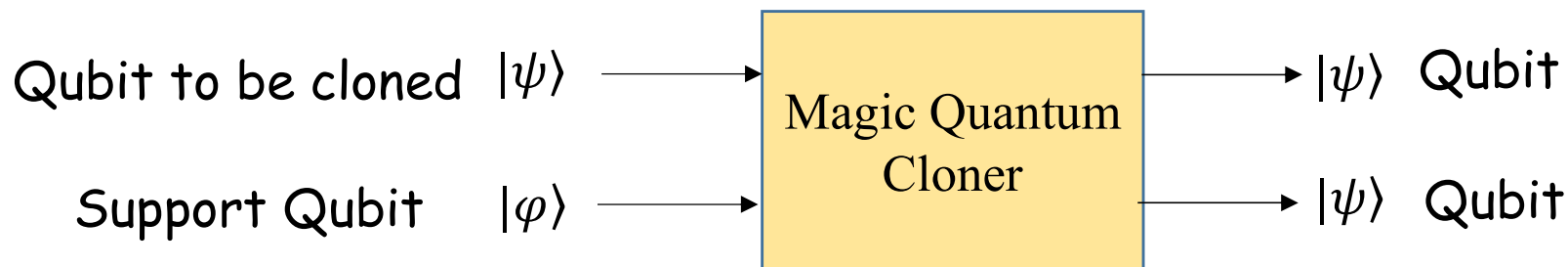
- To teleport a *qubit*

Quantum “teleportation”

- How do you transport a phasor? You don't.
 - You communicate the information about the state, so that it can be perfectly constructed on the receiving end
- How to transmit the state of a quantum system exactly to another location?
 - I.e how to transmit the coordinates of the phasor
 - They're complex, and cannot be measured!
- We will consider the simple case: how to transmit the phasor for *one* qubit exactly?

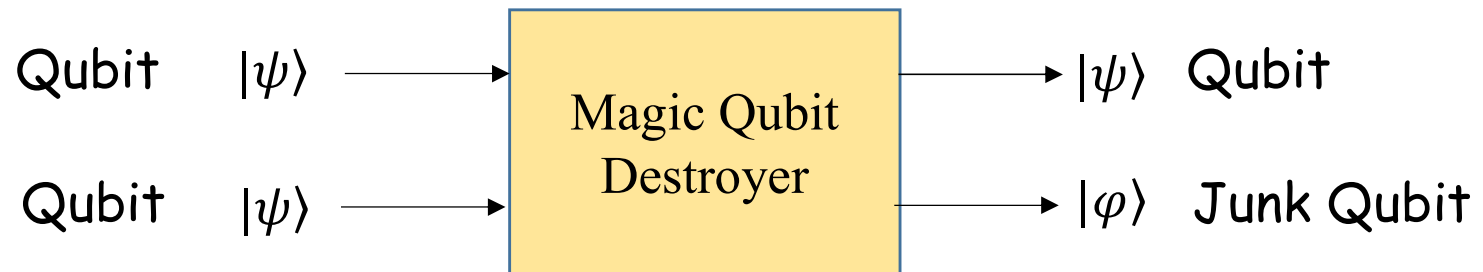
Recap: The no cloning theorem

- You cannot clone a qubit
 - Given an *unknown* qubit, you cannot simply make 2 independent copies of it



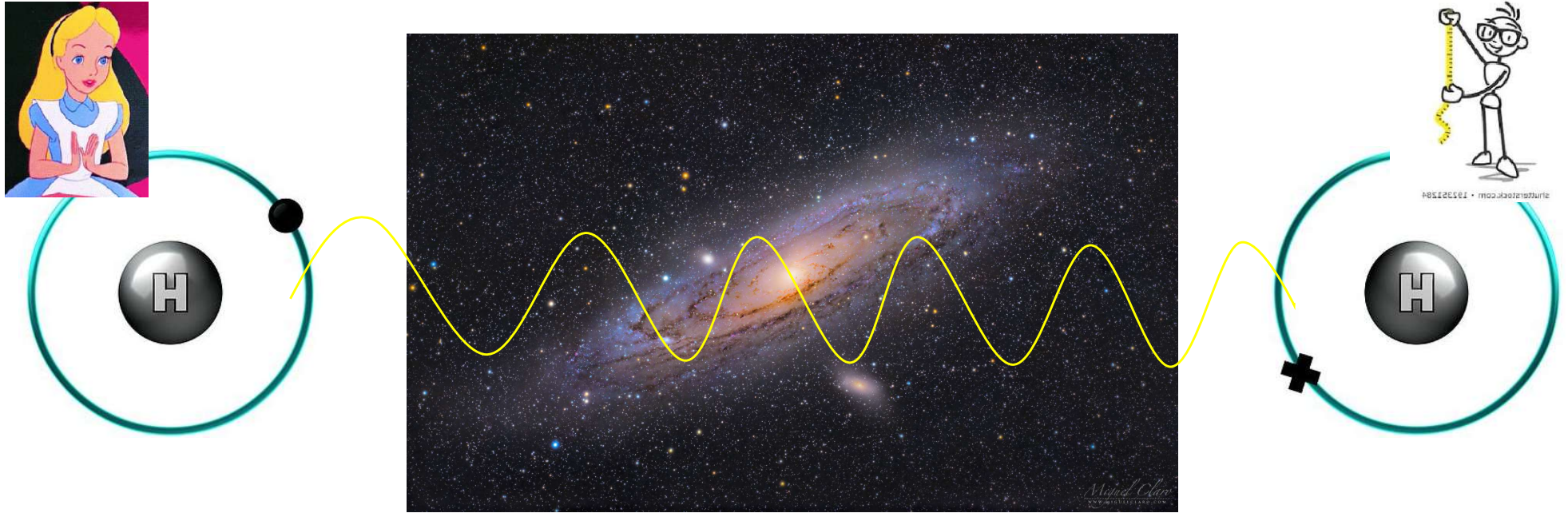
Consequence: No deleting theorem

- Given two identical qubits you cannot delete one of them
 - Deletion \rightarrow destroying the information in one of them
- A system that does this:



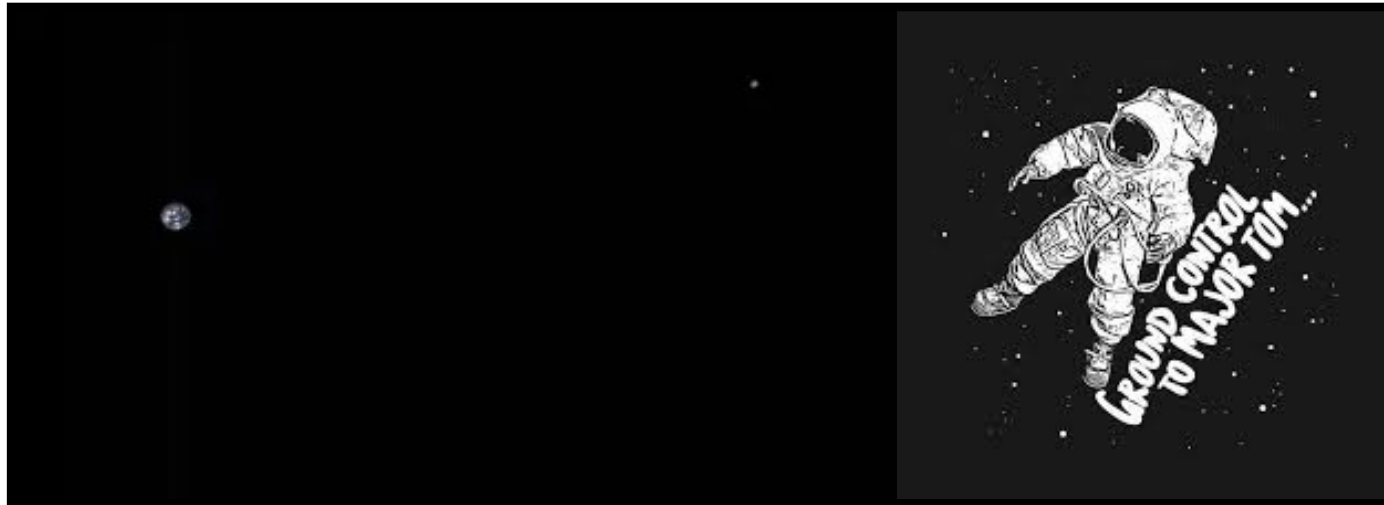
- This cannot exist

The no communication theorem



- Entangled qubits may collapse to the same state when measured, but this cannot be used to actively communicate a bit
 - The “No Communication Theorem” : You cannot use entanglement to instantly signal new information beyond the state of the qubit itself
- There is *no protocol* that enables Alice to communicate *any* information to Bob

So... Quantum teleportation



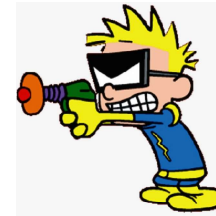
- How do you transport an exact phasor (state) from a sender to a receiver
 - Who is not connected to you by a physical quantum communication channel?

Setup

$$|x\rangle = a_0|0\rangle + a_1|1\rangle$$

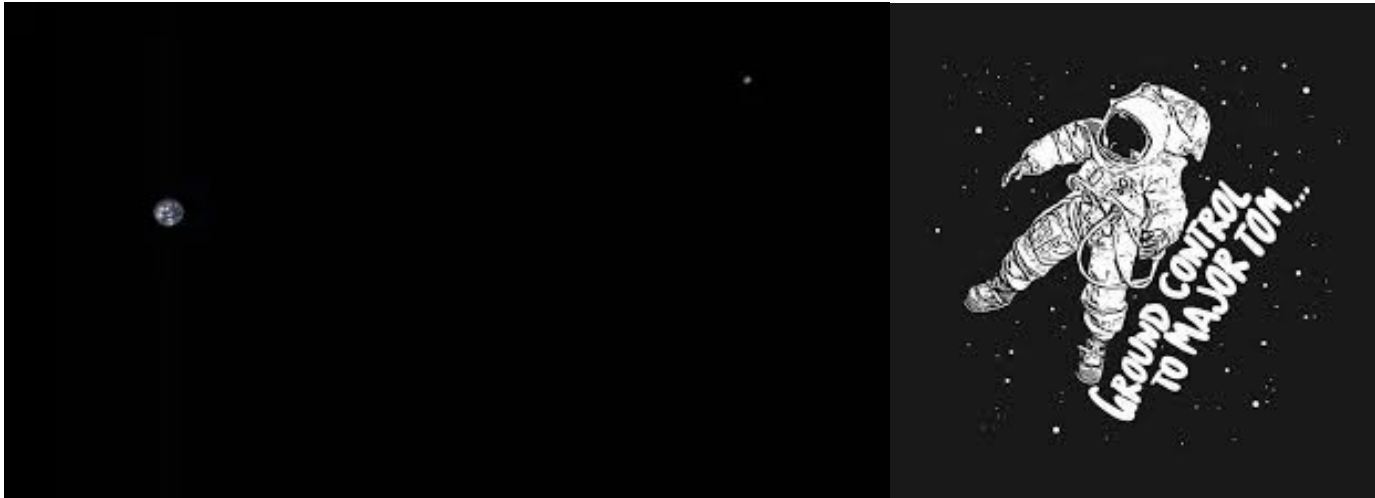


$$|x_{copy}\rangle = a_0|0\rangle + a_1|1\rangle$$



- Alice has a qubit
- She must convey it to Bob
- Alice and Bob share a bunch of entangled qubits that she could use
- Can she somehow teleport her qubit to Bob?

Transferring a Qubit...



- No communication: You *cannot* transfer your qubit to Major Tom without a physical channel!
- There's no such thing as 'teleportation'
- There's only 'transmission' – of information
- You cannot use entangled qubits to communicate information
 - See "no communication" theorem

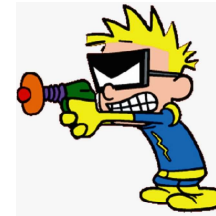
The communication setup

$$|x\rangle = a_0|0\rangle + a_1|1\rangle$$



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$$|x_{copy}\rangle = a_0|0\rangle + a_1|1\rangle$$



- Alice and Tom share entangled qubits
 - That *is* the quantum communication channel, there is no other line
- They *also* share a standard communication channel
 - You cannot work without it
- Alice has a qubit, and she must use these to communicate the qubit to Major Tom

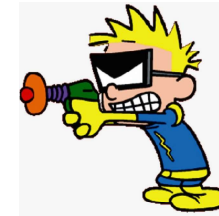
The problem with sending the exact state of a phasor

$$|x\rangle = a_0|0\rangle + a_1|1\rangle$$



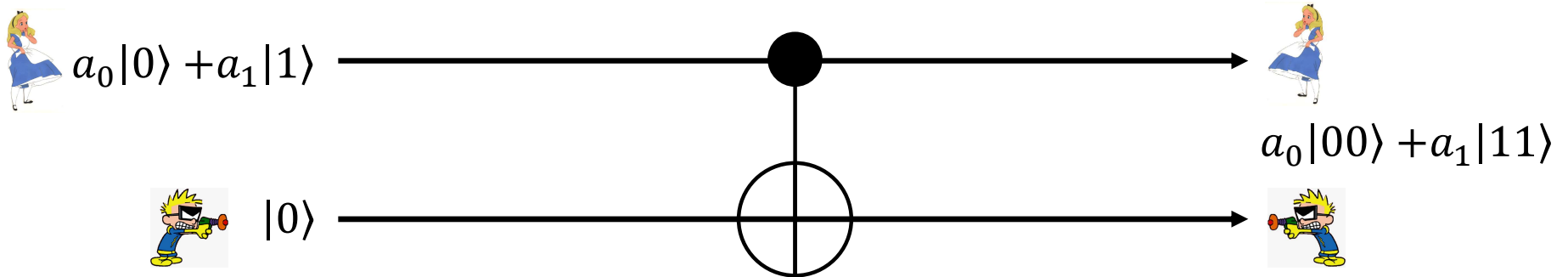
$$a_0 = 0.3, \text{ and} \\ a_1 = 0.995$$

$$|x_{copy}\rangle = a_0|0\rangle + a_1|1\rangle$$



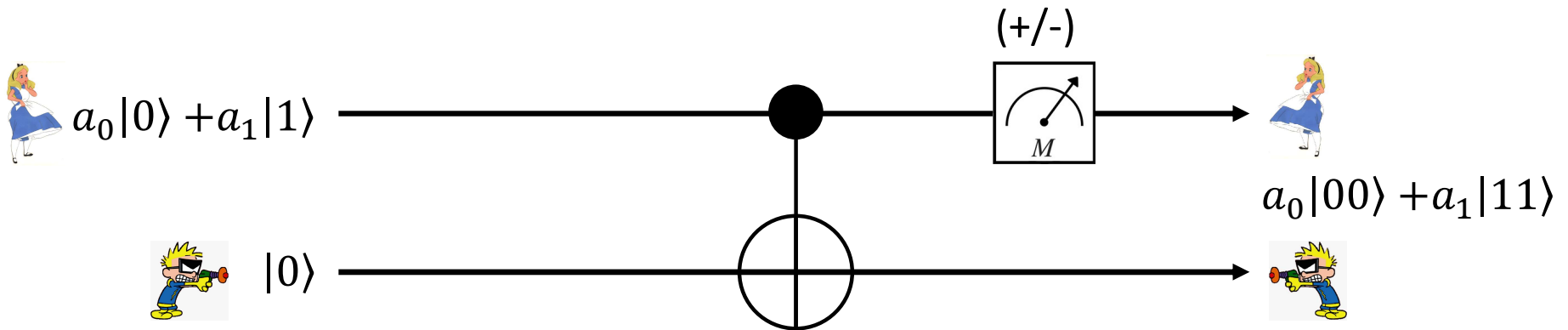
- You cannot measure it without collapsing it
 - Otherwise, you could just measure a_0 and a_1 , call up major Tom and send him the numbers
 - He could then construct a qubit for himself with those a values

Communicating a phasor with a CNOT



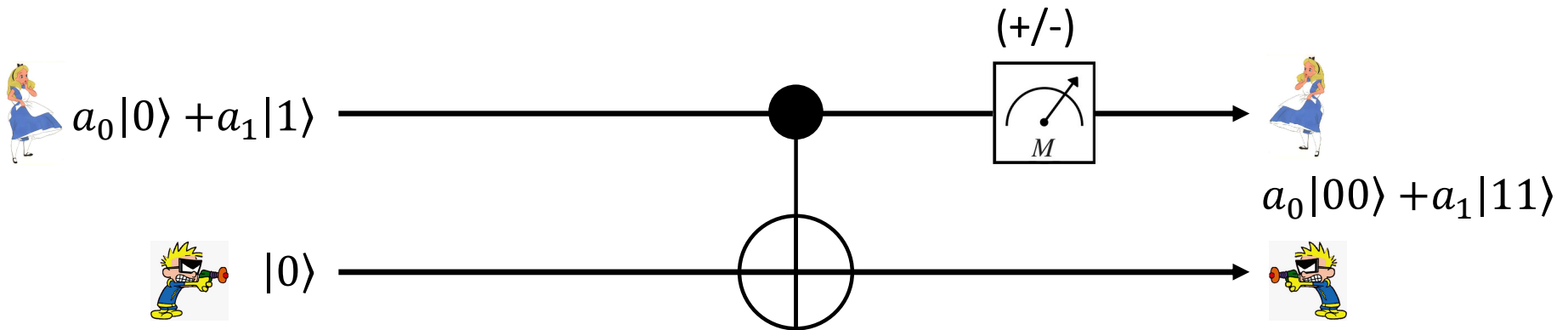
- Assume (hypothetically) that you have a CNOT gate stretched across space between you and Tom
 - *Tom* owns the target qubit
- You input your phasor $|x\rangle = a_0|0\rangle + a_1|1\rangle$
- You and Tom now share $a_0|00\rangle + a_1|11\rangle$
- Objective: Tom should somehow get exactly $|x_{copy}\rangle = a_0|0\rangle + a_1|1\rangle$ from this
 - You can eliminate your qubit from the entangled mess by measuring it

Communicating a phasor with a CNOT



- You and Tom now share $a_0|00\rangle + a_1|11\rangle$
- You now measure *your* qubit in the sign bases
 - How
 - What will you get?

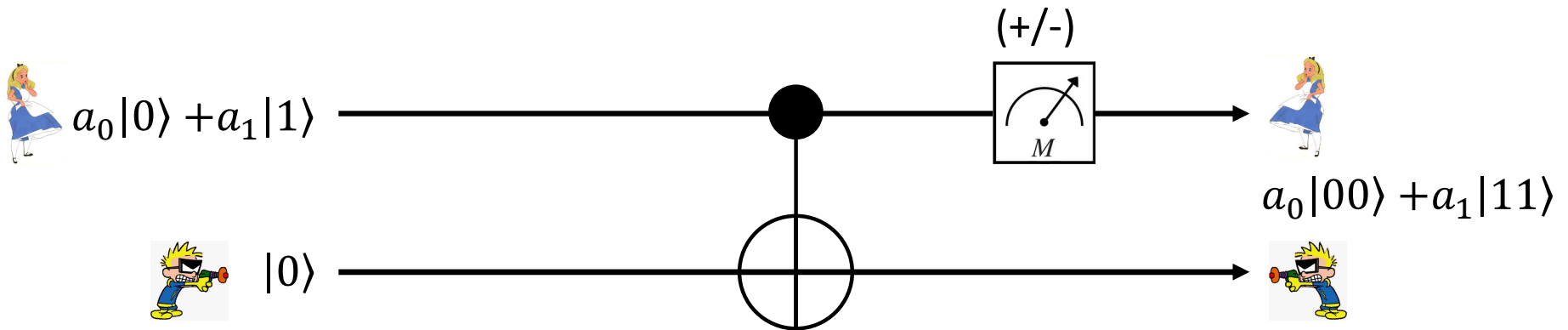
Communicating a phasor with a CNOT



- You and Tom now share $a_0|00\rangle + a_1|11\rangle$
- Note you can write *your* bit in sign bases and you will get

$$\begin{aligned}
 & a_0 \left(\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \right) \otimes |0\rangle \right) + a_1 \left(\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle \right) \otimes |1\rangle \right) \\
 &= \frac{1}{\sqrt{2}} (a_0|+0\rangle + a_0| -0\rangle + a_1|+1\rangle - a_1| -1\rangle)
 \end{aligned}$$

Communicating a phasor with a CNOT

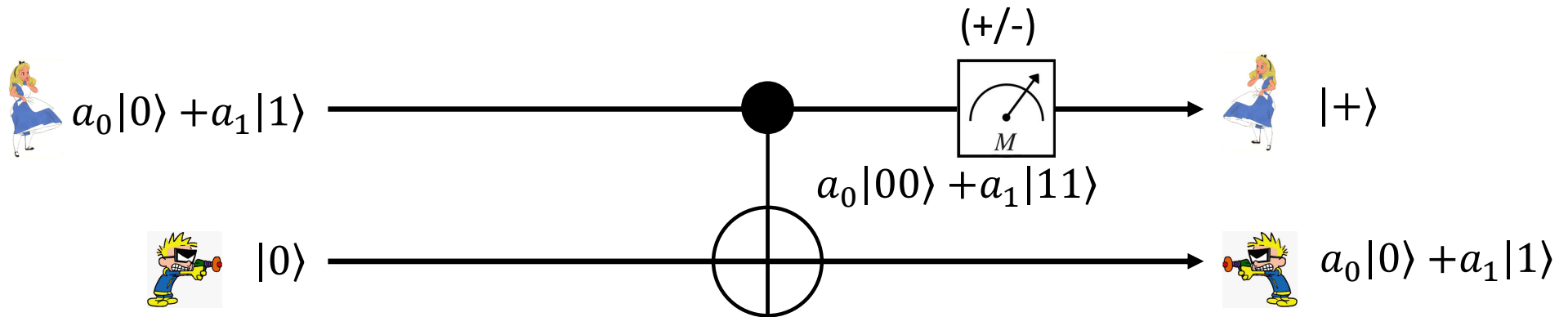


- You and Tom now share $a_0|00\rangle + a_1|11\rangle$
- Note you can write *your* bit in sign bases and you will get

$$\frac{1}{\sqrt{2}} (a_0|+0\rangle + a_0|-0\rangle + a_1|+1\rangle - a_1|-1\rangle)$$

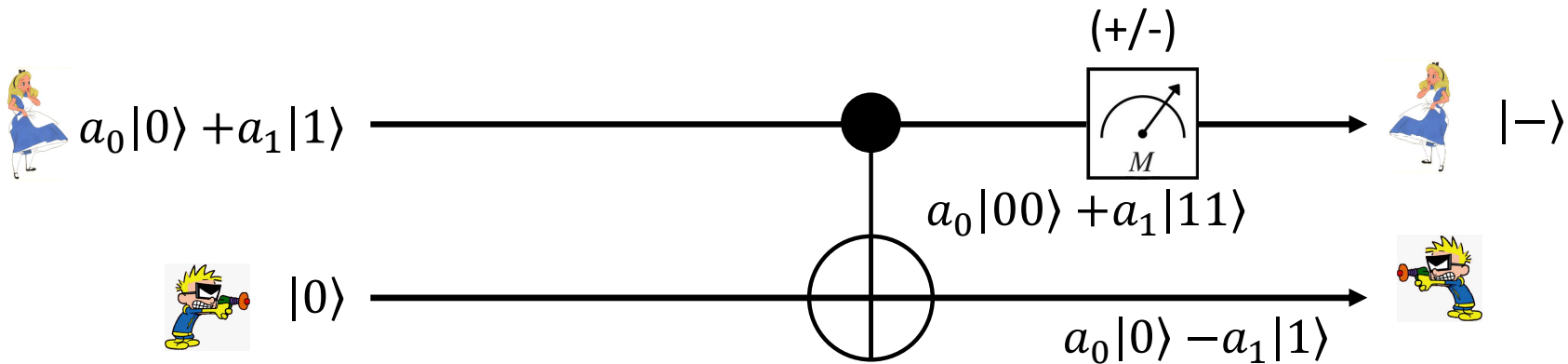
- Measuring your qubit in the sign bases will give you ??

Communicating a phasor with a CNOT



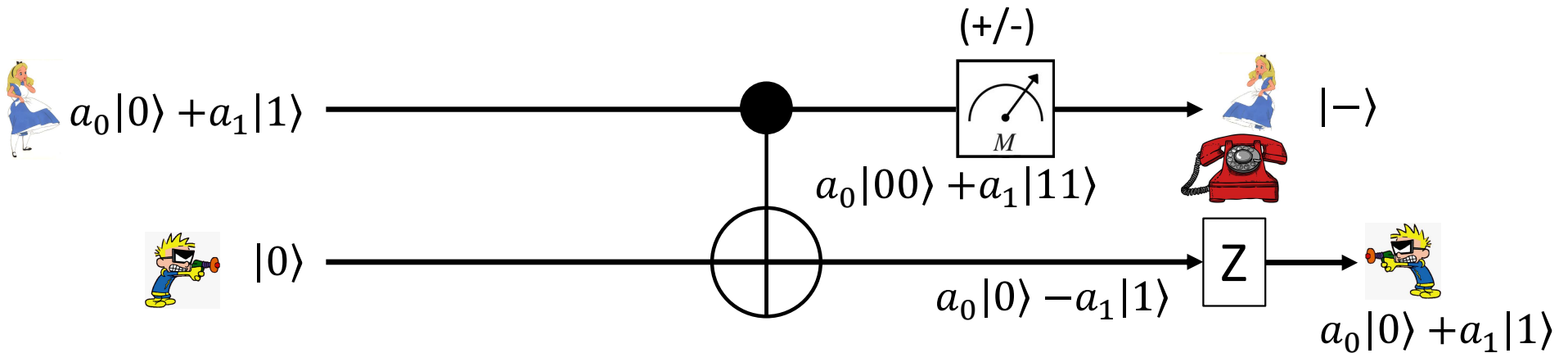
- You and Tom share $\frac{1}{\sqrt{2}} (a_0|+0\rangle + a_0| -0\rangle + a_1|+1\rangle - a_1| -1\rangle)$
- You measure your qubit in the sign bases
 - With $P = 0.5$, you get $|+\rangle$ and Tom gets $a_0|0\rangle + a_1|1\rangle$
 - Teleportation complete

Communicating a phasor with a CNOT



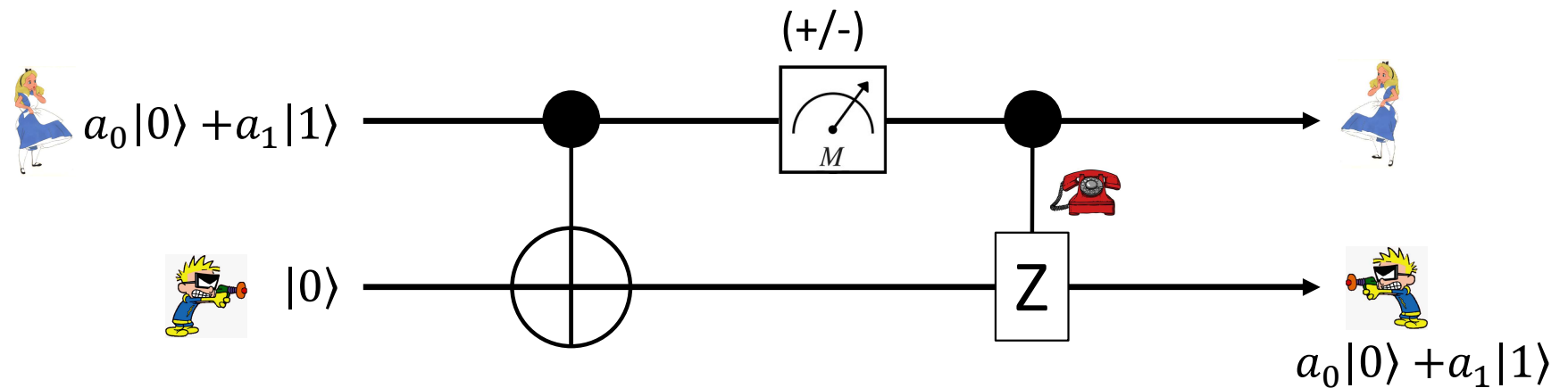
- You and Tom share $\frac{1}{\sqrt{2}} (a_0|+0\rangle + a_0| -0\rangle + a_1|+1\rangle - a_1| -1\rangle)$
- You measure your qubit in the sign bases
 - With $P = 0.5$, you get $|-\rangle$ and Tom gets $a_0|0\rangle - a_1|1\rangle$
 - Tom must phase flip his qubit to get the answer

Communicating a phasor with a CNOT



- You and Tom share $\frac{1}{\sqrt{2}} (a_0|+0\rangle + a_0| -0\rangle + a_1|+1\rangle - a_1| -1\rangle)$
- You measure your qubit in the sign bases
 - With $P = 0.5$, you get $|-\rangle$ and Tom gets $a_0|0\rangle - a_1|1\rangle$
 - Tom must phase flip his qubit to get the answer
 - You call up Tom and tell him to phase flip his qubit
 - Tom does so and gets $a_0|0\rangle + a_1|1\rangle$
 - Teleportation complete

Teleporting a phasor with a CNOT

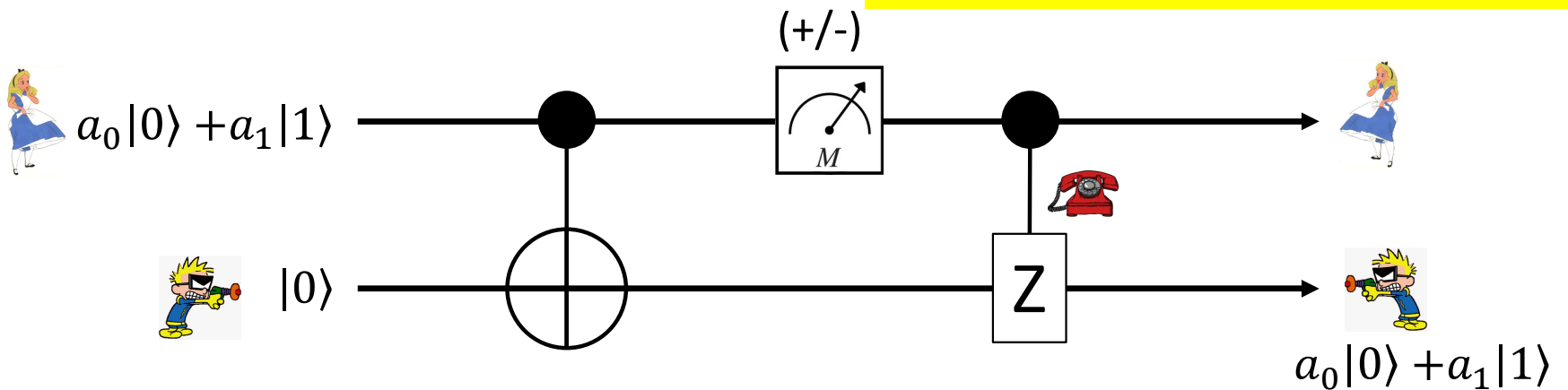


- The strange gate is a *controlled phase flip* gate
- The control, however, is through classical communication
 - Your telephone provides the control line

Teleporting a phasor with a CNOT

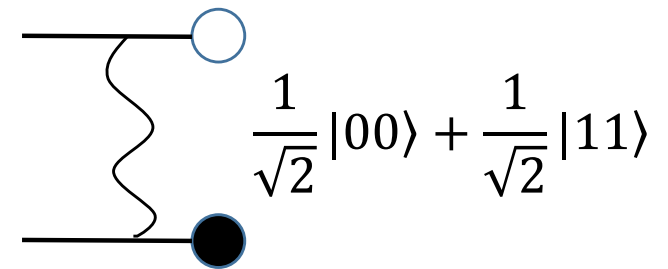
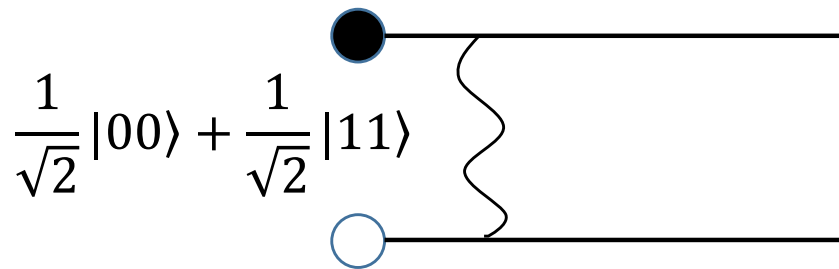
In reality we don't have a CNOT between you and Tom

And no channel either!



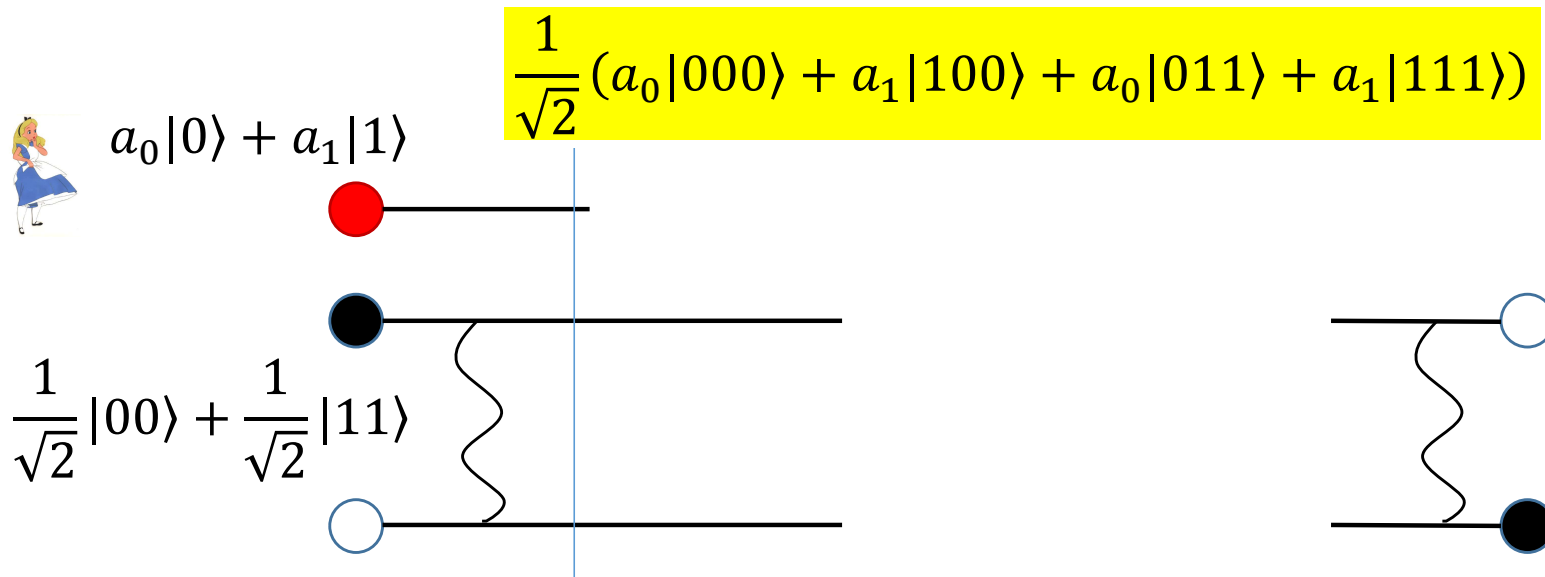
- The strange gate is a *controlled phase flip* gate
- The control, however, is through classical communication
 - Your telephone provides the control line

Teleporting a qubit using entanglement



- Assume before Tom left, he and you shared an entangled bit in a Bell state
- There is no physical connection now, but you each have one qubit of a bell-entangled pair

Teleporting a qubit using entanglement



- You add your qubit to the system

- The overall system is

$$\frac{1}{\sqrt{2}}(a_0|000\rangle + a_1|100\rangle + a_0|011\rangle + a_1|111\rangle)$$

- It is a 3-qubit system

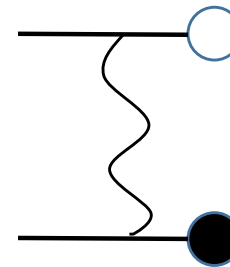
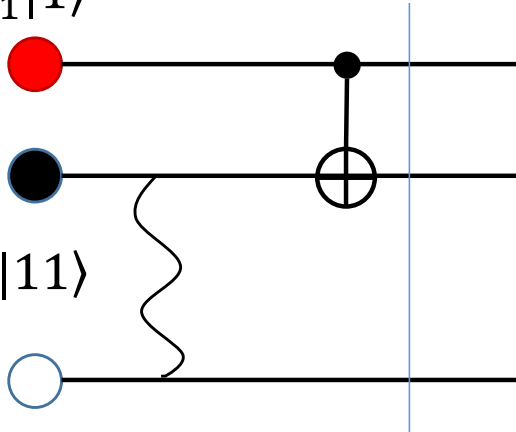
Teleporting a qubit using entanglement



$$a_0|0\rangle + a_1|1\rangle$$

$$\frac{1}{\sqrt{2}}(a_0|000\rangle + a_1|110\rangle + a_0|011\rangle + a_1|101\rangle)$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$



- You add your qubit to the system

- The overall system is

$$\frac{1}{\sqrt{2}}(a_0|000\rangle + a_1|100\rangle + a_0|011\rangle + a_1|111\rangle)$$

- It is a 3-qubit system

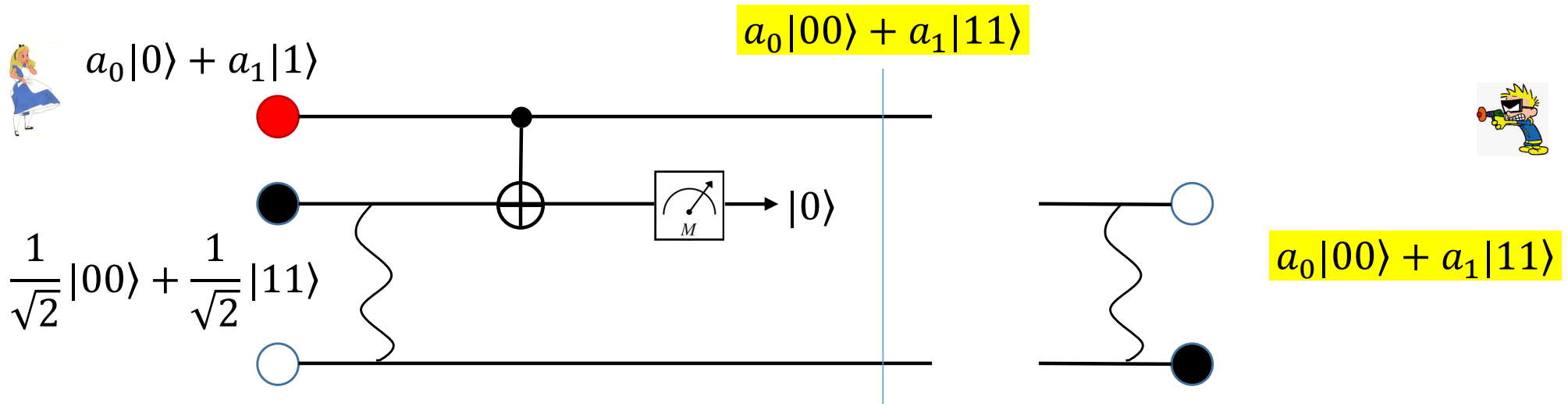
- You CNOT *your* bit with your phasor

- The outcome is:

$$\frac{1}{\sqrt{2}}(a_0|000\rangle + a_1|110\rangle + a_0|011\rangle + a_1|101\rangle)$$

- As before, you have to measure your bits to eliminate the extra bits

Teleporting a qubit using entanglement

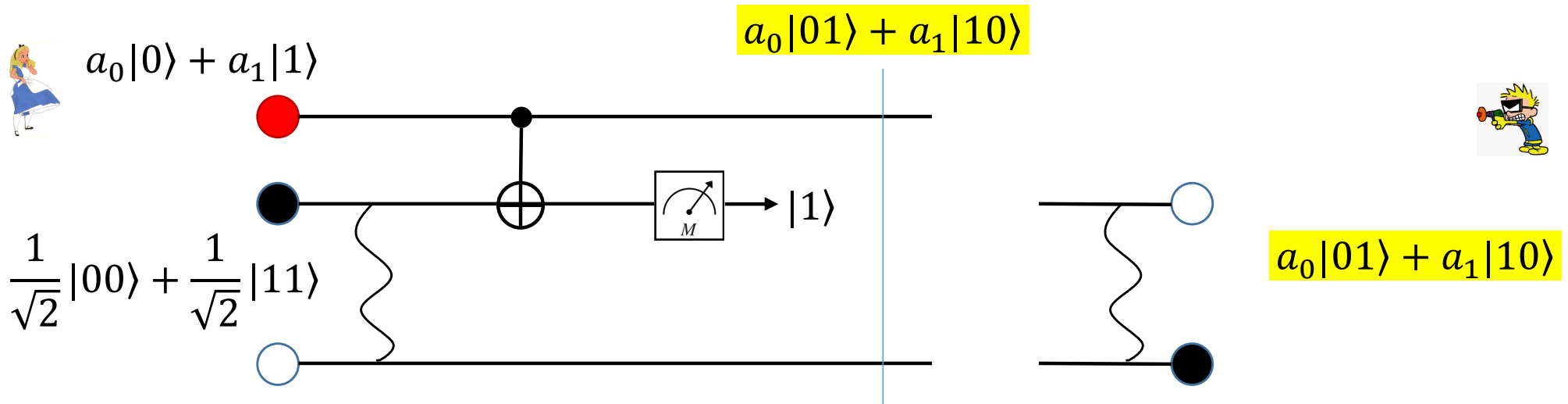


- The 3-qubit system is now at:

$$\frac{1}{\sqrt{2}} (a_0|000\rangle + a_1|110\rangle + a_0|011\rangle + a_1|101\rangle)$$

- You measure your share of the entangled qubit
- If you measure a $|0\rangle$, the system collapses to $a_0|00\rangle + a_1|11\rangle$
 - And Tom and you now share $a_0|00\rangle + a_1|11\rangle$

Teleporting a qubit using entanglement

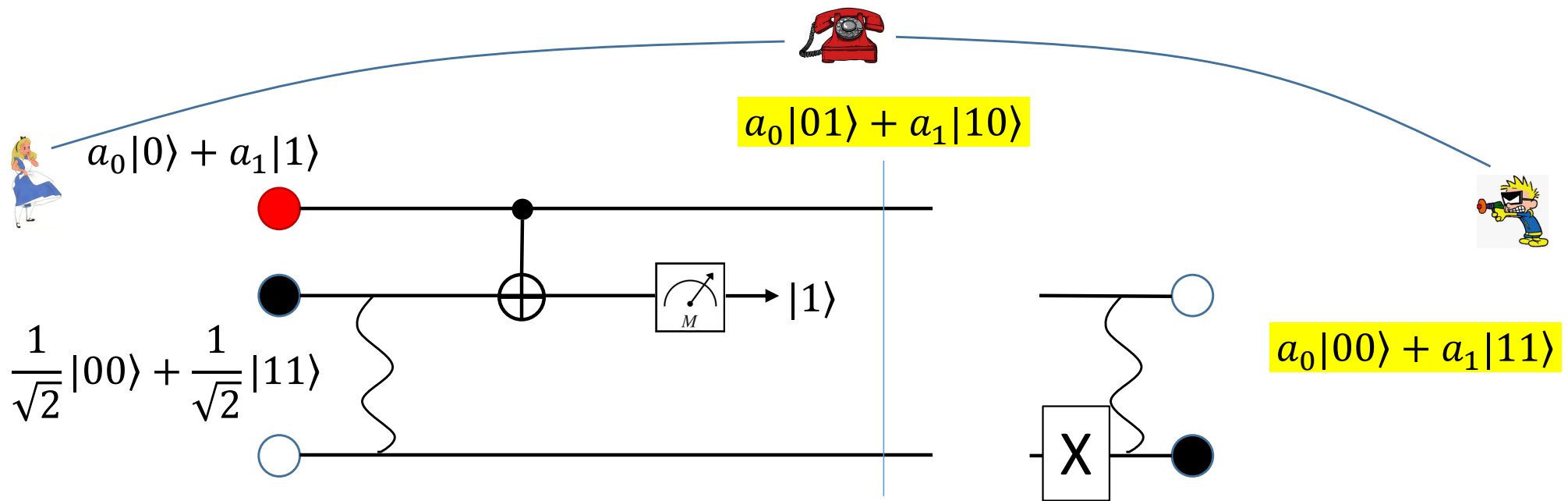


- The 3-qubit system is now at:

$$\frac{1}{\sqrt{2}} (a_0|000\rangle + a_1|110\rangle + a_0|011\rangle + a_1|101\rangle)$$

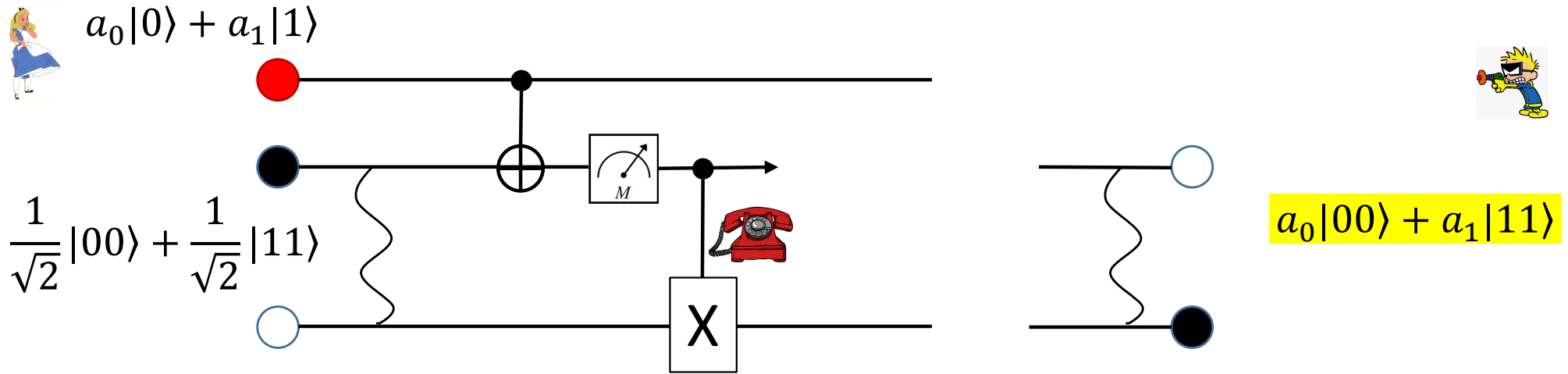
- You measure your share of the entangled qubit
- If you measure a $|0\rangle$, the system collapses to $a_0|00\rangle + a_1|11\rangle$
 - And Tom and you now share $a_0|00\rangle + a_1|11\rangle$
- If you measure a $|1\rangle$, the system collapses to $a_0|01\rangle + a_1|10\rangle$
 - And Tom and you now share $a_0|01\rangle + a_1|10\rangle$

Teleporting a qubit



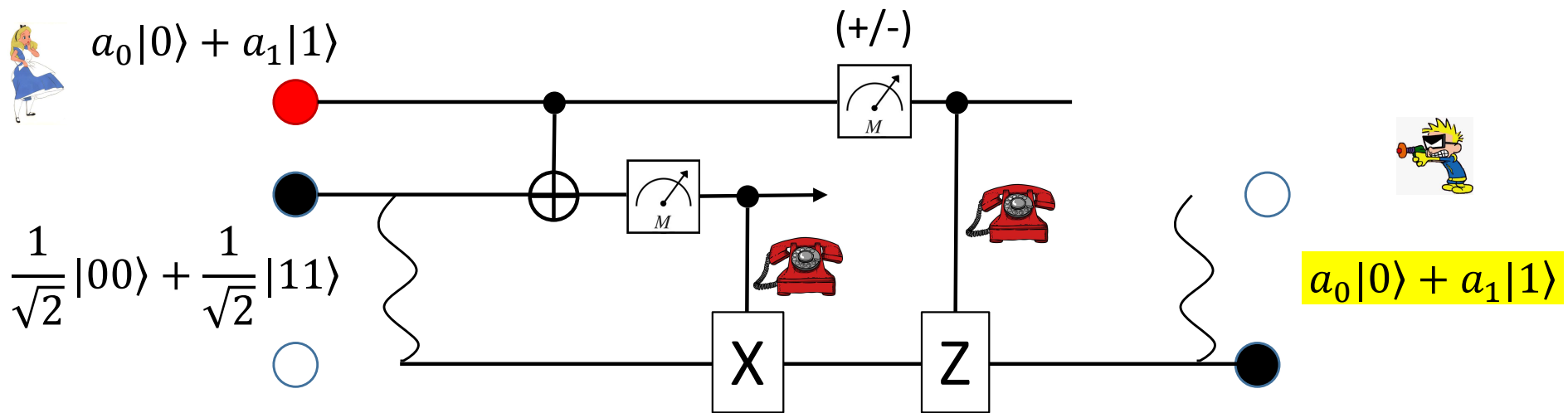
- If you measure a $|0\rangle$, the system collapses to $a_0|00\rangle + a_1|11\rangle$
 - And Tom and you now share $a_0|00\rangle + a_1|11\rangle$
- If you measure a $|1\rangle$, the system collapses to $a_0|01\rangle + a_1|10\rangle$
 - And Tom and you now share $a_0|01\rangle + a_1|10\rangle$
 - You call up Tom and ask him to do a bitflip on his qubit
 - Now, once again, Tom has $a_0|00\rangle + a_1|11\rangle$

Teleporting a qubit using entanglement



- The overall system so far
- Uses the *controlled sign-flip gate*
 - Control line is a classical communication link
- The output is: Tom now has $a_0|00\rangle + a_1|11\rangle$
 - This must somehow be collapsed to $a_0|0\rangle + a_1|1\rangle$
- You have to measure your other bit to eliminate it

Teleporting a qubit using entanglement



- Tom obtains $a_0|0\rangle + a_1|1\rangle$
 - Done using only a classical communication channel and an entangled qubit
 - No quantum communication
- We “teleported” a qubit to Tom!!
 - Who can now get back to earth...

So what on earth is going on here?

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