Quant. Comp. HW - 2

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1 Is Valid Superposition?

Determine if the state $|\phi>$ is a valid superposition

$$|\phi> = \frac{1}{1+i}|0> + \frac{1}{1-i}|1>$$

The state is valid provided that it is length normalized to one.

$$|\frac{1}{1+i}|^2 + |\frac{1}{1-i}|^2$$

First multiply by the complex conjugate/complex conjugate to make it clear what the real and imaginary parts are for each coefficient:

$$= |\frac{1}{1+i}\frac{1-i}{1-i}|^2 + |\frac{1}{1-i}\frac{1+i}{1+i}|^2$$

$$= |\frac{1}{2} - \frac{1}{2}i|^2 + |\frac{1}{2} + \frac{1}{2}i|^2$$

$$= (\frac{1}{2})^2 + (\frac{-1}{2})^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2$$

$$= (1/4) + (1/4) + (1/4) + (1/4) = 1$$

Therefore the state is **valid**.
Note that another, maybe simpler way to do this, would be to exploit the property that $|a/b| = |a|/|b|$ So

$$|\frac{1}{1+i}|^2 + |\frac{1}{1-i}|^2$$

$$= \frac{|1|^2}{|1+i|^2} + \frac{|1|^2}{|1-i|^2}$$

$$= (1/2) + (1/2) = 1$$

2 Find valid superposition

Given $|\phi >= (1/2)|00 > +\frac{x}{2\sqrt{2}}|01 > +\frac{1}{2\sqrt{2}}|10 > +\frac{1}{2}|11 >$, what values of x would make this a valid superposition?

Again the normalization condition is applied:

$$|\frac{1}{2}|^2 + |\frac{x}{2\sqrt{2}}|^2 + |\frac{1}{2\sqrt{2}}|^2 + |\frac{1}{2}|^2 = 1$$

$$(1/4) + \frac{|x|^2}{8} + (1/8) + (1/4) = 1$$

$$\frac{(5 + |x|^2)}{8} = 1$$

$$|x|^2 = 3$$

Since x can be imaginary, all we know is that the real part must be equal to $\sqrt{3}$ Therefore:

$$\boxed{x = \sqrt{3} + bi}$$

3

Let $|\psi >= \frac{1}{\sqrt{3}}(|00 > +|01 > +|10 >)$ Find:

$$|\phi >= (H \otimes H)|\psi >$$

$$= \frac{1}{\sqrt{6}}[(|00 > +|01 > +|10 > +|11 >)$$

$$+(|00 > -|01 > +|10 > -|11 >)$$

$$+(|00 > +|01 > -|10 > +|11 >)]$$

$$= \frac{1}{\sqrt{6}}[3|00 > +|01 > +|10 > +|11 >]$$

$$\boxed{= \frac{1}{2}|0 >_2 + \frac{1}{6}|1 >_2 + \frac{1}{6}|2 >_2 - \frac{1}{6}|3 >_2}$$

4 Deutsch Problem

Suppose we take U_f from the Deutsch problem and compute

$$(H \otimes 2)U_f(H \otimes 2)|11>$$

Recall that for the Deutsch function:

	X=0	X=1
f_0	0	0
f_1	0	1
f_2	1	0
f_3	1	1

First let's apply the first hadamard gate and calls this intermediate state $|a>$:

$$|a>= (H \otimes H)|11>$$

$$= \frac{1}{2}(|0> -|1>)(|0> -|1>)$$

$$= \frac{1}{2}(|0> |0> -|0> |1> -|1> |0> -|1> |1>)$$

In computing the Hadamard on this next, a general equation can be set up for the four 'f' functions. Since anything XOR'ed with itself is 0, and anything XOR'ed with 1 is it's complement, we can write $U_f|a>$ as:

$$U_f|a>= \frac{1}{2}[|0> |f(0)> -|0> |\bar{f}(0)> -|1> |f(1)> -|1> |\bar{f}(1)>]$$

Now let's consider the two cases $f(0) = f(1)$ and $f(0) \neq f(1)$:

Case 1: $f(0) = f(1)$
This implies we are using either f0 or f3.

f_0:

$$HU_{f_0}|a>= H\frac{1}{2}[|00> -|01> -|10> -|11>]$$

$$= \frac{1}{2}[-|00> +|11> +|11> +|01> +|10>]$$

$$\boxed{= \frac{1}{2}[-|00> +2|11> +|01> +|10>] \quad (f_0)}$$

f_3:

$$HU_{f_3}|a>= H\frac{1}{2}[|01> -|00> -|11> -|10>]$$

$$= \frac{1}{4}[(|00> +|01> -|10> +|11>) - (|00> +|01> +|10> +|11>)$$
$$-(|00> -|01> -|10> -|11>) - (|00> +|01> -|10> -|11>)]$$

$$= \frac{1}{4}[-2|00> +2|11>$$

$$\boxed{= \frac{1}{2}[|00> +|11> \quad (f_3)}$$

Now for the case of $f(0) \neq f(1)$, so we look at f1 and f2. Note that applying U is easy here too, because we are always XOR a bit with it's complement, which always yields 1. Since the input gates will be the same for $f_1 and f_2$, then each should return the same state:

f_1 :

$$HU_{f_1}|a>= H\frac{1}{2}[|00> -|01> -|11> -|10>]$$

$$= \frac{1}{4}[2|01> +2|11> -2|00> +2|10> +2|11>]$$

$$\boxed{= \frac{1}{2}[-|00> +|01> +|10> +|11>] \quad (f_1)}$$

f_2 :

$$HU_{f_3}|a>= H\frac{1}{2}[|01> -|00> -|11> -|10>]$$

$$= \frac{1}{4}[-2|01> -2|11> -2|00> +2|10> +2|11>$$

$$\boxed{= \frac{1}{2}[-|00> +|01> +|10>] \quad (f_2)}$$

5 Bernstein-Verizani

Take f(x) from the Bernstein-Verizani problem and compute:

$$U_f(H^{n\otimes 1})(|0>_n|1>_1)$$

$$= U_f(\frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}|x>)\frac{1}{\sqrt{2}}(|0>-|1>)$$

Since $U_f|x>_n \frac{1}{\sqrt{2}}(|0>-|1>) = (-1)^{f(x)}|x>_n \frac{1}{\sqrt{2}}(|0>-|1>)$

$$= \frac{1}{2^{n/2}}(\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x>)\frac{1}{\sqrt{2}}(|0>-|1>)$$

$$\boxed{= \frac{1}{2^{(n+1)/2}}(\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x>)(|0>-|1>)}$$

Recall that the function f(x) in the Bernstein-Verizani is $a \cdot x$. Therefore, f(x) is always either 0 or 1, and so each value in the sum is either $|x>$ or $-|x>$. The possible values of the input register are:

$$\boxed{\frac{1}{2^{n/2}}(\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x>)}$$

The only possible value of the output register is the superposed state:

$$\boxed{\frac{1}{2^{(n+1)/2}}(|0>-|1>)}$$

6 Derive State

Derive the state from problem 3 using any 1-bit gates and the 2-bit gate cNOT starting from $|00>$ Just applying a hadamard to expand leaves a useless state, since all other gates applied maintain the state. So first apply a NOT gate:

$$|00>=|11>$$

Applying a hadamard to this yields:

$$HX|00>=\frac{1}{2}(|00>-|01>-|10>-|11>)$$

Applying cNOT gates:

$$C_{21}HX|00>=\frac{1}{2}(|00>-|11>-|10>-|01>)$$

And another cNOT:

$$C_{12}C_{21}HX|00>=\frac{1}{2}(|00>-|10>-|11>-|01>)$$

Applying a Hadamard to this:

$$HC_{12}C_{21}HX|00>=\frac{1}{2}(|10>+|11>-(|00>-|01>-|11>))$$

$$=\frac{1}{2}(|10>+2|11>-|00>+|01>)$$

Now I can just NOT each bit

$$(X\otimes 1)(X\otimes 2)H(X\otimes 2)C_{12}C_{21}H(X\otimes 1)|00>$$

$$\boxed{=\frac{1}{\sqrt{3}}[|00>+|01>+|10>]}$$

7 Numbered formulae

Useful Hadamards:

$$(H\otimes H)|00>=\frac{1}{2}(|00>+|01>+|10>+|11>) \tag{1}$$

$$(H\otimes H)|01>=\frac{1}{2}(|00>-|01>+|10>-|11>) \tag{2}$$

$$(H\otimes H)|10>=\frac{1}{2}(|00>+|01>-|10>-|11>) \tag{3}$$

$$(H\otimes H)|11>=\frac{1}{2}(|00>-|01>-|10>-|11>) \tag{4}$$

Use the *equation* environment to get numbered formulae, e.g.,

$$y_{i+1}=x_i^{2n}-\sqrt{5}x_{i-1}^n+\sqrt{x_{i-2}^7}-1 \tag{5}$$

$$\frac{\partial u}{\partial t}+\nabla^4 u+\nabla^2 u+\frac{1}{2}|\nabla u|^2 \ = \ c^2 \tag{6}$$

8 Acknowledgments

I'm now going to assume I have a C_H, which is a controlled hadamard gate, that functions just like a cNOT gate except it applies the hadamard operator instead of the NOT operator to the non-control bit.

$$C_{H_{12}} H|00> = \frac{1}{2}[|00> + |01> + |1> \frac{1}{\sqrt{2}}(|0> + |1>) + |1> \frac{1}{\sqrt{2}}(|0> - |1>]$$

$$= \frac{1}{2}[|00> + |01> + \frac{1}{\sqrt{2}}(|10> + \frac{1}{\sqrt{2}}|11> + \frac{1}{\sqrt{2}}|10> - \frac{1}{\sqrt{2}}|11>]$$

$$= \frac{1}{2}[|00> + |01> + \frac{2}{\sqrt{2}}|10>]$$