Quant. Comp. HW - 2

Steven MacCoun

Oct. 18, 2005

1 Is Valid Superposition?

Determine if the state $|\phi >$ is a valid superposition

$$|\phi >= \frac{1}{1+i}|0> + \frac{1}{1-i}|1>$$

The state is valid provided that it is length normalized to one.

$$|\frac{1}{1+i}|^2 + |\frac{1}{1-i}|^2$$

$$= \frac{1}{(1+i)(1-i)} + \frac{1}{(1-i)(1+i)}$$

$$= (1/2) + (1/2) = 1$$

Therefore the state is **valid**.

2 Find valid superposition

3

4 Deutsch Problem

Suppose we take U_f from the Deutsch problem and compute

$$(H \otimes 2)U_f(H \otimes 2)|11>$$

	X=0	X=1
f_0	0	0
f_1	0	1
f_2	1	0
f_3	1	1

Recall that for the Deutsch function:

First let's apply the first hadamard gate and calls this intermediate state $|a>$:

$$|a>= (H \otimes H)|11>$$
$$= \frac{1}{2}(|0> -|1>)(|0> -|1>)$$
$$= \frac{1}{2}(|0> |0> -|0> |1> -|1> |0> -|1> |1>)$$

In computing the Hadamard on this next, a general equation can be set up for the four 'f' functions. Since anything XOR'ed with itself is 0, and anything XOR'ed with 1 is it's complement, we can write $U_f|a>$ as:

$$U_f|a>= \frac{1}{2}[|0> |f(0)> -|0> |\bar{f}(0)> -|1> |f(1)> -|1> |\bar{f}(1)>]$$

Now let's consider the two cases $f(0) = f(1)$ and $f(0) \neq f(1)$:

Case 1: $f(0) = f(1)$
This implies we are using either f0 or f3.

f_0:

$$HU_{f_0}|a>= H\frac{1}{2}[|00> -|01> -|10> -|11>]$$
$$= \frac{1}{2}[-|00> +|11> +|11> +|01> +|10>]$$
$$\boxed{= \frac{1}{2}[-|00> +2|11> +|01> +|10>] \quad (f_0)}$$

f_3:

$$HU_{f_3}|a>= H\frac{1}{2}[|01> -|00> -|11> -|10>]$$

$$= \frac{1}{4}[(|00> +|01> -|10> +|11>) - (|00> +|01> +|10> +|11>)$$
$$-(|00> -|01> -|10> -|11>) - (|00> +|01> -|10> -|11>)]$$
$$= \frac{1}{4}[-2|00> +2|11>$$
$$\boxed{= \frac{1}{2}[|00> +|11> \qquad (f_3)}$$

Now for the case of $f(0) \neq f(1)$, so we look at f1 and f2. Note that applying U is easy here too, because we are always XOR a bit with it's complement, which always yields 1. Since the input gates will be the same for $f_1 and f_2$, then each should return the same state:

f_1 :
$$HU_{f_1}|a >= H\frac{1}{2}[|00> -|01> -|11> -|10>]$$
$$= \frac{1}{4}[2|01> +2|11> -2|00> +2|10> +2|11>]$$
$$\boxed{= \frac{1}{2}[-|00> +|01> +|10> +|11>] \quad (f_1)}$$

f_2 :
$$HU_{f_3}|a >= H\frac{1}{2}[|01> -|00> -|11> -|10>]$$
$$= \frac{1}{4}[-2|01> -2|11> -2|00> +2|10> +2|11>$$
$$\boxed{= \frac{1}{2}[-|00> +|01> +|10>] \quad (f_2)}$$

4.1 Numbered formulae

Useful Hadamards:

$$(H \otimes H)|00>= \frac{1}{2}(|00> +|01> +|10> +|11>) \tag{1}$$

$$(H \otimes H)|01>= \frac{1}{2}(|00> -|01> +|10> -|11>) \tag{2}$$

$$(H \otimes H)|10>= \frac{1}{2}(|00> +|01> -|10> -|11>) \tag{3}$$

$$(H \otimes H)|11>= \frac{1}{2}(|00> -|01> -|10> -|11>) \tag{4}$$

Use the *equation* environment to get numbered formulae, e.g.,

$$y_{i+1} = x_i^{2n} - \sqrt{5}x_{i-1}^n + \sqrt{x_{i-2}^7} - 1 \tag{5}$$

$$\frac{\partial u}{\partial t} + \nabla^4 u + \nabla^2 u + \frac{1}{2}|\nabla u|^2 \; = \; c^2 \tag{6}$$

5 Acknowledgments

Thanks to my buddies Æschyulus and Chloë, who helped me define the macro `\piRsquare` which is πr^2. The end.