Quant. Comp. HW - 2

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1 Is Valid Superposition?

Determine if the state $|\phi >$ is a valid superposition

$$|\phi >= \frac{1}{1+i}|0 > + \frac{1}{1-i}|1 >$$

The state is valid provided that it is length normalized to one.

$$|\frac{1}{1+i}|^2 + |\frac{1}{1-i}|^2$$

$$= \frac{1}{(1+i)(1-i)} + \frac{1}{(1-i)(1+i)}$$

$$= (1/2) + (1/2) = 1$$

Therefore the state is **valid**.

2 Find valid superposition

3

4 Deutsch Problem

Suppose we take U_f from the Deutsch problem and compute

$$(H \otimes 2)U_f(H \otimes 2)|11 >$$

	X=0	X=1
f_0	0	0
f_1	0	1
f_2	1	0
f_3	1	1

Recall that for the Deutsch function:

First let's apply the first hadamard gate and calls this intermediate state $|a>$:

$$|a>= (H \otimes H)|11>$$

$$= \frac{1}{2}(|0> -|1>)(|0> -|1>)$$

$$= \frac{1}{2}(|0> |0> -|0> |1> -|1> |0> -|1> |1>)$$

Now let's consider the two cases $f(0) = f(1)$ and $f(0) \neq f(1)$:

Case 1: $f(0) = f(1)$

This implies we are using either f0 or f3.

For f3:

$$U_{f_3}|a>= U_{f_3}[\frac{1}{2}(|0> |0> -|0> |1> -|1> |0> -|1> |1>)]$$

$$= (1/2)[|0> |0 \otimes 1> -|0> |1 \otimes 1> -|1> |0 \otimes 1> -|1> |1 \otimes 1>$$

$$= (1/2)[|0> |1> -|0> |0> -|1> |1> -|1> |0>]$$

Applying the final Hadamard to this:

$$(H \otimes H)\frac{1}{2}[|01> -|00> -|11> -|10>]$$

$$= \frac{1}{4}[(|00> +|01> -|10> +|11>) - (|00> +|01> +|10> +|11>)$$

$$-(|00> -|01> -|10> -|11>) - (|00> +|01> -|10> -|11>)]$$

$$= \frac{1}{4}[-2|00> +2|11>$$

$$= \frac{1}{2}[|00> +|11>$$

For f0:

$$U_{f_0}|a>= U_{f_0}[\frac{1}{2}(|0>|0>-|0>|1>-|1>|0>-|1>|1>)]$$

Since U_{f_0} always returns 0, the computation is straightforward since anything XOR'ed with 0 is just itself

$$=(1/2)[|0>|0>-|0>|1>-|1>|0>-|1>|1>$$

4.1 Numbered formulae

Useful Hadamards:

$$(H\otimes H)|00>=\frac{1}{2}(|00>+|01>+|10>+|11>) \tag{1}$$

$$(H\otimes H)|01>=\frac{1}{2}(|00>-|01>+|10>-|11>) \tag{2}$$

$$(H\otimes H)|10>=\frac{1}{2}(|00>+|01>-|10>-|11>) \tag{3}$$

$$(H\otimes H)|11>=\frac{1}{2}(|00>-|01>-|10>-|11>) \tag{4}$$

Use the *equation* environment to get numbered formulae, e.g.,

$$y_{i+1}=x_i^{2n}-\sqrt{5}x_{i-1}^n+\sqrt{x_{i-2}^7}-1 \tag{5}$$

$$\frac{\partial u}{\partial t}+\nabla^4 u+\nabla^2 u+\frac{1}{2}|\nabla u|^2 \;=\; c^2 \tag{6}$$

5 Acknowledgments

Thanks to my buddies Æschyulus and Chloë, who helped me define the macro `\piRsquare` which is πr^2. The end.