Quant. Comp. HW - 2

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1 Is Valid Superposition?

Determine if the state $|\phi>$ is a valid superposition

$$|\phi>=\frac{1}{1+i}|0>+\frac{1}{1-i}|1>$$

The state is valid provided that it is length normalized to one.

$$|\frac{1}{1+i}|^2+|\frac{1}{1-i}|^2$$

First multiply by the complex conjugate/complex conjugate to make it clear what the real and imaginary parts are for each coefficient:

$$=|\frac{1}{1+i}\frac{1-i}{1-i}|^2+|\frac{1}{1-i}\frac{1+i}{1+i}|^2$$

$$=|\frac{1}{2}-\frac{1}{2}i|^2+|\frac{1}{2}+\frac{1}{2}i|^2$$

$$=(\frac{1}{2})^2+(\frac{-1}{2})^2+(\frac{1}{2})^2+(\frac{1}{2})^2$$

$$=(1/4)+(1/4)+(1/4)+(1/4)=1$$

Therefore the state is **valid**.

Note that another, maybe simpler way to do this, would be to exploit the property that $|a/b|=|a|/|b|$ So

$$|\frac{1}{1+i}|^2+|\frac{1}{1-i}|^2$$

$$=\frac{|1|^2}{|1+i|^2}+\frac{|1|^2}{|1-i|^2}$$

$$=(1/2)+(1/2)=1$$

2 Find valid superposition

Given $|\phi>= (1/2)|00> + \frac{x}{2\sqrt{2}}|01> + \frac{1}{2\sqrt{2}}|10> + \frac{1}{2}|11>$, what values of x would make this a valid superposition?

Again the normalization condition is applied:

$$|\frac{1}{2}|^2 + |\frac{x}{2\sqrt{2}}|^2 + |\frac{1}{2\sqrt{2}}|^2 + |\frac{1}{2}|^2 = 1$$

$$(1/4) + \frac{|x|^2}{8} + (1/8) + (1/4) = 1$$

$$\frac{(5 + |x|^2)}{8} = 1$$

$$|x|^2 = 3$$

Since x can be imaginary, all we know is that the real part must be equal to $\sqrt{3}$ Therefore:

$$\boxed{x = \sqrt{3} + bi}$$

3

Let $|\psi>= \frac{1}{\sqrt{3}}(|00> + |01> + |10>)$ Find:

$$|\phi>= (H \otimes H)|\psi>$$

$$= \frac{1}{\sqrt{6}}[(|00> + |01> + |10> + |11>)$$

$$+(|00> - |01> + |10> - |11>)$$

$$+(|00> + |01> - |10> + |11>)]$$

$$= \frac{1}{\sqrt{6}}[3|00> + |01> + |10> + |11>]$$

$$\boxed{= \frac{1}{2}|0>_2 + \frac{1}{6}|1>_2 + \frac{1}{6}|2>_2 - \frac{1}{6}|3>_2}$$

4 Deutsch Problem

Suppose we take U_f from the Deutsch problem and compute

$$(H \otimes 2)U_f(H \otimes 2)|11>$$

Recall that for the Deutsch function:

	X=0	X=1
f_0	0	0
f_1	0	1
f_2	1	0
f_3	1	1

First let's apply the first hadamard gate and call this intermediate state $|a>$:

$$|a>= (H \otimes H)|11>$$

$$= \frac{1}{2}(|0> -|1>)(|0> -|1>)$$

$$= \frac{1}{2}(|0> |0> -|0> |1> -|1> |0> -|1> |1>)$$

In computing the Hadamard on this next, a general equation can be set up for the four 'f' functions. Since anything XOR'ed with 0 is itself, and anything XOR'ed with 1 is it's complement, we can write $U_f|a>$ as:

$$U_f|a>= \frac{1}{2}[|0> |f(0)> -|0> |\bar{f}(0)> -|1> |f(1)> -|1> |\bar{f}(1)>]$$

Now let's consider the two cases $f(0) = f(1)$ and $f(0) \neq f(1)$:

Case 1: $f(0) = f(1)$
This implies we are using either f0 or f3.

f_0:

$$HU_{f_0}|a>= H\frac{1}{2}[|00> -|01> -|10> -|11>]$$

$$= \frac{1}{2}[\frac{1}{2}(|00> + |01> + |10> + |11>)$$

$$-\frac{1}{2}(|00> - |01> + |10> - |11>)$$

$$-(\frac{1}{2}(|00> + |01> - |10> + |11>)$$

$$-\frac{1}{2}(|00> - |01> - |10> + |11>))]$$

$$\boxed{= \frac{1}{2}[-|00> + |01> + |10>] \quad (f_0)}$$

f_3:

$$HU_{f_3}|a >= H\frac{1}{2}[|01> - |00> - |11> - |10>]$$

$$= \frac{1}{4}[(|00> - |01> + |10> + |11>)$$

$$-(|00> + |01> + |10> + |11>)$$

$$-(|00> - |01> - |10> + |11>)$$

$$-(|00> + |01> - |10> - |11>)]$$

$$= \frac{1}{4}[-2|00> + 2|11>$$

$$\boxed{= \frac{1}{2}[-|00> - |01> + |10> + |11>] \quad (f_3)}$$

Now for the case of $f(0) \neq f(1)$, so we look at f1 and f2. Note that applying U is easy here too, because we are always XOR a bit with it's complement, which always yields 1. Since the input gates will be the same for $f_1 and f_2$, then each should return the same state:

f_1 :

$$HU_{f_1}|a >= H\frac{1}{2}[|00> - |01> - |11> - |10>]$$

$$= \frac{1}{4}[(|00> + |01> + |10> + |11>)$$

$$-(|00> -|01> +|10> -|11>)$$
$$-(|00> -|01> -|10> +|11>)$$
$$-(|00> +|01> -|10> -|11>)]$$
$$= \frac{1}{4}[-2|00> +2|11>$$
$$\boxed{= \frac{1}{2}[-|00> +|01> +|10> +|11>] \quad (f_1)}$$

$f_2:$

$$HU_{f_3}|a>= H\frac{1}{2}[|01> -|00> -|11> -|10>]$$
$$= \frac{1}{4}[(|00> -|01> +|10> +|11>)$$
$$-(|00> +|01> +|10> -|11>)$$
$$-(|00> -|01> -|10> +|11>)$$
$$-(|00> +|01> -|10> -|11>)]$$
$$\boxed{= \frac{1}{2}[-|00> -|01> +|10> +|11>] \quad (f_1)}$$

5 Bernstein-Verizani

Take f(x) from the Bernstein-Verizani problem and compute:

$$U_f(H^{n\otimes 1})(|0>_n |1>_1)$$
$$= U_f(\frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}|x>)\frac{1}{\sqrt{2}}(|0> -|1>)$$

Since $U_f|x>_n \frac{1}{\sqrt{2}}(|0> -|1>) = (-1)^{f(x)}|x>_n \frac{1}{\sqrt{2}}(|0> -|1>)$

$$= \frac{1}{2^{n/2}}(\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x>)\frac{1}{\sqrt{2}}(|0> -|1>)$$
$$\boxed{= \frac{1}{2^{(n+1)/2}}(\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x>)(|0> -|1>)}$$

Recall that the function f(x) in the Bernstein-Verizani is $a \cdot x$. Therefore, f(x) is always either 0 or 1, and so each value in the sum is either $|x>$ or $-|x>$. The possible values of the input register are:

$$\boxed{\frac{1}{2^{n/2}}(\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x>)}$$

The only possible value of the output register is the superposed state:

$$\boxed{\frac{1}{2^{(n+1)/2}}(|0> -|1>)}$$

6 Derive State

Derive the state from problem 3 using any 1-bit gates and the 2-bit gate cNOT starting from $|00>$ Just applying a hadamard to expand leaves a useless state, since all other gates applied maintain the state. So first apply a NOT gate:

$$|00>=|11>$$

Applying a hadamard to this yields:

$$HX|00>=\frac{1}{2}(|00> -|01> -|10> -|11>)$$

Applying cNOT gates:

$$C_{21}HX|00>=\frac{1}{2}(|00> -|11> -|10> -|01>)$$

And another cNOT:

$$C_{12}C_{21}HX|00>=\frac{1}{2}(|00> -|10> -|11> -|01>)$$

Applying a Hadamard to this:

$$HC_{12}C_{21}HX|00>=\frac{1}{2}(|10> +|11> -(|00> -|01>))$$

$$=\frac{1}{2}(|10> +|11> -|00> +|01>)$$

Presumably at this point I've accomplished something. After like a billion other permutations/attempts, I did get to having just —00¿, —01¿, —10¿, but could never achieve the correct coefficients.

Note that I also tried using a controlled Hadamard.

This is a controlled hadamard gate, that functions just like a cNOT gate except it applies the hadamard operator instead of the NOT operator to the non-control bit.

$$C_{H_{12}}H|00>=\frac{1}{2}[|00>+|01>+|1>\frac{1}{\sqrt{2}}(|0>+|1>)+|1>\frac{1}{\sqrt{2}}(|0>-|1>]$$

$$=\frac{1}{2}[|00>+|01>+\frac{1}{\sqrt{2}}(|10>+\frac{1}{\sqrt{2}}|11>+\frac{1}{\sqrt{2}}|10>-\frac{1}{\sqrt{2}}|11>]$$

$$=\frac{1}{2}[|00>+|01>+\frac{2}{\sqrt{2}}|10>]$$

Also close, but not quite there

7 Derive with only one cNOT

Although I did not finish the answer to the previous question, this question is essentially the same, because any cNOT gate can be rewritten as:

$$C_{ij}=\bar{n}_i+X_jn_i$$

Since there is no limit on the number of 1-bit and type of 1 bit gates we can use, it follows that the pretend answer I got for number 6 is equivalent to this question, but each C_{ij} is replaced with the above equation.

8 Numbered formulae

Useful Hadamards:

$$(H\otimes H)|00>=\frac{1}{2}(|00>+|01>+|10>+|11>) \tag{1}$$

$$(H\otimes H)|01>=\frac{1}{2}(|00>-|01>+|10>-|11>) \tag{2}$$

$$(H\otimes H)|10>=\frac{1}{2}(|00>+|01>-|10>-|11>) \tag{3}$$

$$(H \otimes H)|11>= \frac{1}{2}(|00>-|01>-|10>+|11>) \tag{4}$$

Use the *equation* environment to get numbered formulae, e.g.,

$$y_{i+1} = x_i^{2n} - \sqrt{5}x_{i-1}^n + \sqrt{x_{i-2}^7} - 1 \tag{5}$$

$$\frac{\partial u}{\partial t} + \nabla^4 u + \nabla^2 u + \frac{1}{2}|\nabla u|^2 \ = \ c^2 \tag{6}$$

9 Acknowledgments