SUMMARY SHEET – DECISION MATHS

GRAPHICAL LINEAR PROGRAMMING

The main ideas are covered in

AQA	Edexcel	MEI	OCR
D1	D1	D1	D1

The main ideas in this chapter are

Formulating a problem as a linear programming problem, solving a Linear Programming Problem (maximisation and minimisation) and Integer Programming.

Before the exam you should:

- Practice formulating linear programming problems. This can often be the trickiest part of the problem. Remember to be consistent with units.
- Learn the terminology – the OBJECTIVE FUNCTION is what you have to maximise or minimise subject to a number of CONSTRAINTS.
- Make sure you are able to draw straight line graphs quickly from the constraints by considering where they cross the x and y axes. ⇒ Substitute zero
- You must be able to find the solution to problems from the graph. Make sure you can draw graphs accurately.
- Remember to shade OUT the unacceptable region to keep the feasible region clear and easy to identify.
- You must be able to find correct solutions to problems where the answer must be an integer.

logical

being consistent is key

terminology

eg

check shading if it satisfies inequalities

Formulating a problem as a Linear Programming Problem

First: identify the variables about which a decision is to be made. These are sometimes called the decision variables. For example if your problem is to decide how many chairs to make and how many tables to make to maximise profit, begin with a statement like – let x be the number of chairs and let y be the number of tables. If your problem is to work out how many grams of wheatgerm and how grams of oat flour there should be in a new food product to meet nutritional requirements and minimise cost then let x be the number of grams of wheatgerm and let y be the number of grams of oat flour.

Next: Decide what the objective function is (this is the value you are trying to maximise or minimise) and what the constraints are as inequalities involving x and y.

Be careful to use the same units consistently. For example it's possible that some distances appearing in a problem are given in metres and some are given in centimetres. Or some times they could be given in seconds with some given in minutes. Choose one type of units and convert everything into those units.

remember to check units

Example

Example:
A clothing retailer needs to order at least 200 jackets to satisfy demand over the next sales period. He stocks two types of jacket which cost him £10 and £30 to purchase. He sells them at 20 pounds and 50 pounds respectively. He has 2700 pounds to spend on jackets.

The cheaper jackets are bulky and each need 20cm of hanging space. The expensive jackets need only 10cm each. He has 40m of hanging space for jackets.

The retailer wishes to maximise profit. Assuming that all jackets will be sold, formulate a linear program, the solution of which will indicate how many jackets of each type should be ordered.

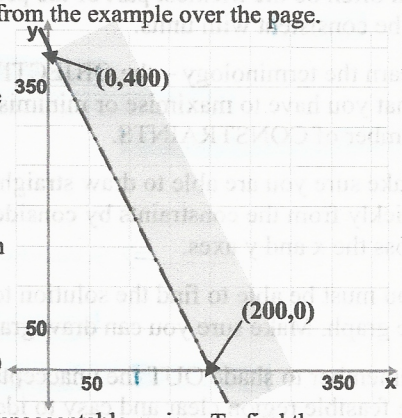
Formulation as a linear program
The decision is about how many of two types of jacket need to be ordered.
Let x = **number of cheaper jackets ordered**
Let y = **number of expensive jackets ordered**
The profit, P, given by selling all of these, is $P = 10x + 20y$, since the profit made on a cheaper jacket is 10 pounds and the profit made on an expensive one is 20 pounds.
The constraints are:
1. "needs to order at least 200" giving $x + y \geq 200$
2. "cost him 10 pounds and 30 pounds" and "has 2700 pounds to spend" giving $10x + 30y \leq 2700$
3. "20cm of hanging space" and "10cm" and "has 40m of hanging space" giving $0.2x + 0.1y \leq 40$

Disclaimer: Every effort has gone into ensuring the accuracy of this document. However, the FM Network can accept no responsibility for its content matching each specification exactly.

Solving a Linear Programming Problem

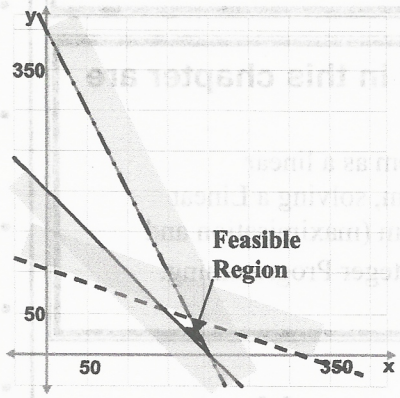
Draw a graph in which each constraint is represented by a line with shading. The unacceptable side of the line should be shaded. This leaves a "feasible region". The solution of the problem will be one of the vertices of the feasible region. These can be checked to find the best. We do this below for the example introduced over the page.

Drawing the line representing a constraint.
As an example, take the constraint $0.2x + 0.1y \leq 40$ from the example over the page.
The initial aim is to draw the line $0.2x + 0.1y = 40$. We know this is a straight line so it's enough to find two points on the line and join them. When $x = 0$, $y = 400$ and when $y = 0$, $x = 200$. So the points (0, 400) and (200, 0) are on the line.



Then shade out, the unacceptable region. To find the unacceptable region just test a point to see if it satisfies the constraint or not. For example, in this case (10, 10) clearly satisfies the constraint and so is in the acceptable region.

The feasible region.
Once you have drawn all the constraints, the feasible region is the intersection of the acceptable regions for all of them.

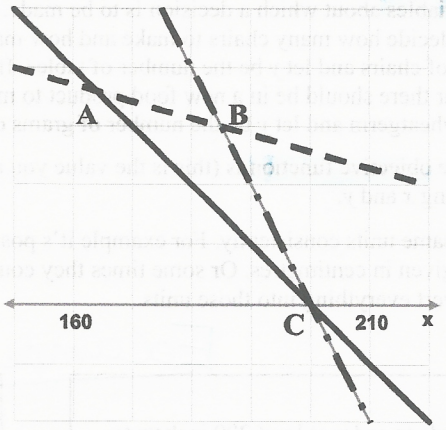


Constraint 1
$x + y \geq 200$

Constraint 2
$10x + 30y \leq 2700$

Constraint 3
$0.2x + 0.1y \leq 40$

Finding the solution
The solution of the problem will be at one of the vertices of the feasible region. You will need to solve simultaneous equations to find the co-ordinates of these vertices. Then each vertex must be checked to find the best. For example in the above we have a feasible region as in the diagram on the right. The coordinates of point A are found by solving $x + y = 200$ and $10x + 30y = 2700$ simultaneously. The solutions are $x = 165$ and $y = 35$. So the point is (165, 35) and the profit at that point is $P = 10x + 20y = 1650 + 700 = 2350$. Similarly it can be seen point B is (186, 28) giving a profit of 2420. Point C is (200, 0) giving a profit of 2000. So the best profit that can be made is by buying 165 cheap coats and 35 expensive coats.



Considering Gradients.
By calculating the gradients of each of the constraints and the gradient of the objective function, it's possible to predict in advance which vertex will give the optimal solution.

Minimisation problems are solved in exactly the same way. Just remember that this time you are looking for the vertex which makes the objective function the lowest.

Integer Programming

If the solution to the problem has to have integer values then points with integer value coordinates, close to the optimal point can be checked. This is likely to reveal the optimal solution but it is not guaranteed to. For example suppose the Objective Function is $2x + 3y$ and that this should be maximised. The optimal point may be (30.6, 40. 8) but do not assume that (30.40) will give the best solution; you must look at all the points with integer coordinates that are nearby: (31, 40), (30, 41), (30, 40) and (31, 41).

However (31, 41) and (31, 40) are not in the feasible region. You can check this by substituting in the values into the constraints. Of the two points nearby which are in the feasible region, namely (30, 41) and (30, 40), it can be seen that (30, 41) provides the best profit.

Disclaimer: Every effort has gone into ensuring the accuracy of this document. However, the FM Network can accept no responsibility for its content matching each specification exactly.