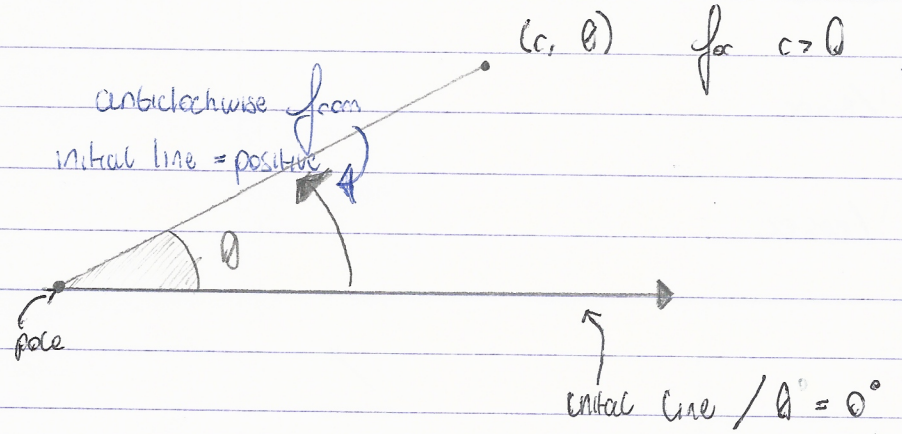
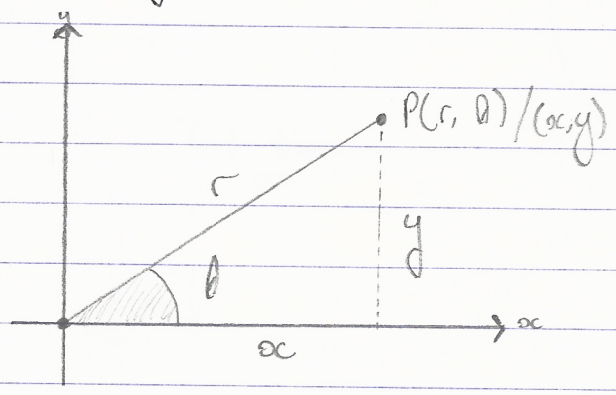
Polar Coordinates

Fundamentals



$r = \sqrt{x^2 + y^2}$ & $\theta = \tan^{-1}(y/x)$



$\Rightarrow \sin\theta = y/r$ & $\cos\theta = x/r$

Converting Forms

E.g., $r^2 = \cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \frac{x^2}{r^2} - \frac{y^2}{r^2} \Rightarrow r^4 = x^2 - y^2$

$(\sqrt{x^2+y^2})^4 = x^2 - y^2$

$(x^2+y^2)^2 = x^2 - y^2$

E.g., $y^2 = 8x$ & $\because y = \sin\theta\, r$, $x = r\cos\theta$

$y^2 = \sin^2\theta\, r^2$

$r^2\sin^2\theta = 8r\cos\theta \Rightarrow r\sin^2\theta = 8\cos\theta$

$r = \frac{8\cos\theta}{\sin^2\theta} = \frac{8\cos\theta}{\sin\theta}\,\frac{1}{\sin\theta}$

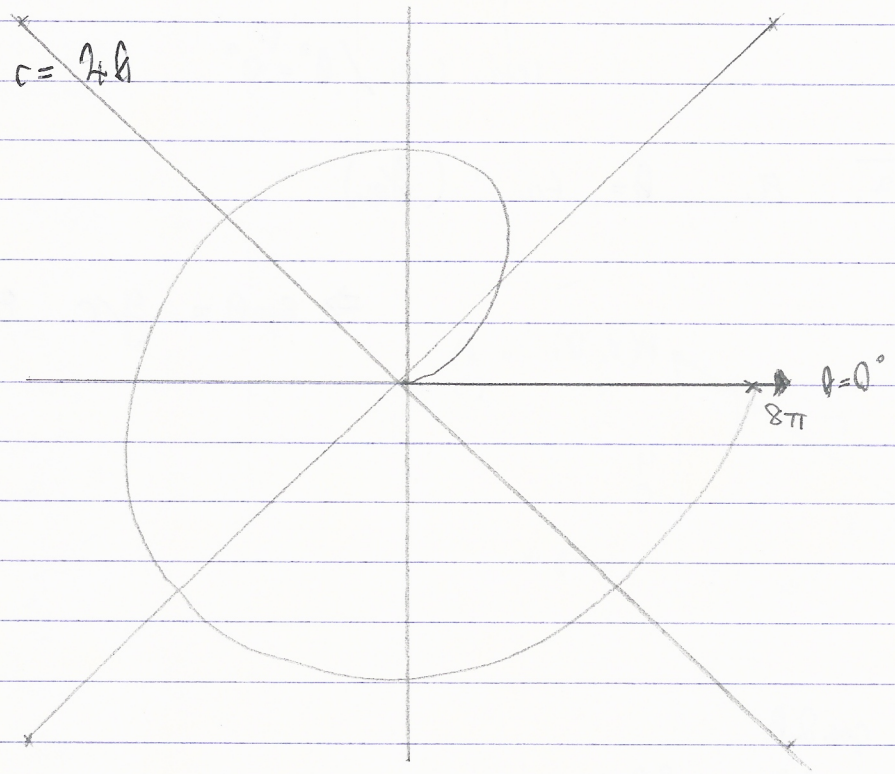
$= 8\cos\theta\csc\theta$

New Identities

- $\cot x = 1/\tan x$ Third letter!
- $\operatorname{cosec} x = 1/\sin x$
- $\sec x = 1/\cos x$

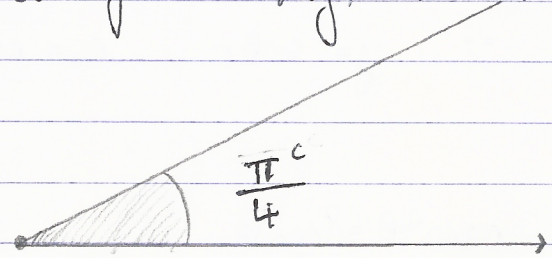
Spiral

$r = 2\theta$



θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r	0	π	2π	3π	4π	5π	6π	7π	8π

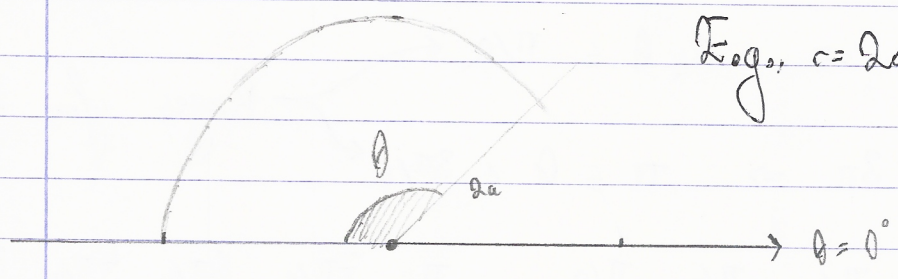
Half-lines $\theta = \alpha$ form. Eg., $\theta = \pi/4$



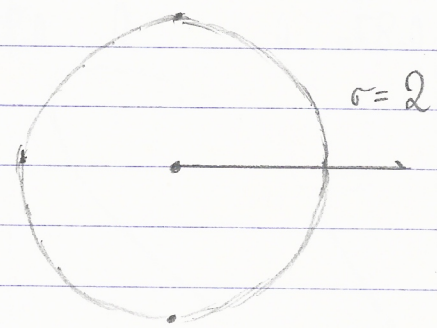
Circles & Arcs

- Format is $r = a$, circle centre O & radius a

E.g., $r = 2a$ $\left\{ \frac{\pi}{4} \le \theta \le \pi \right\}$



OR $r = 2$

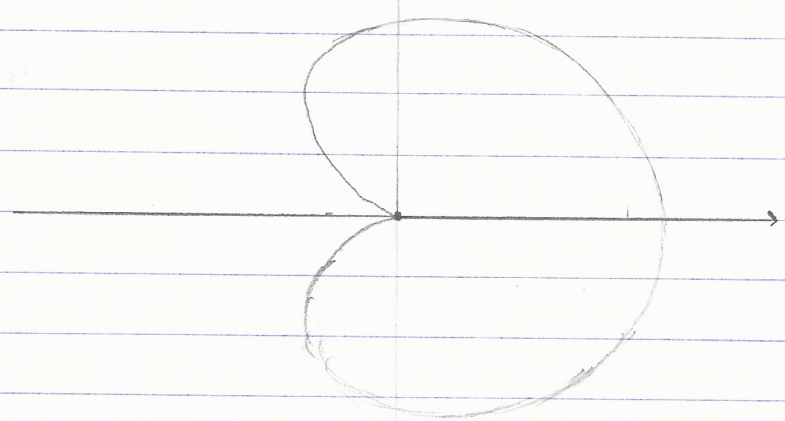


Cardiods

$r = a(1 + \cos\theta)$ or $r = a(1 + \sin\theta)$

E.g., $r = a(1 + \cos\theta)$

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r	$2a$	$\sim 1.7a$	a	$\sim 0.3a$	0	$\sim 0.3a$	a	$\sim 1.7a$	$2a$



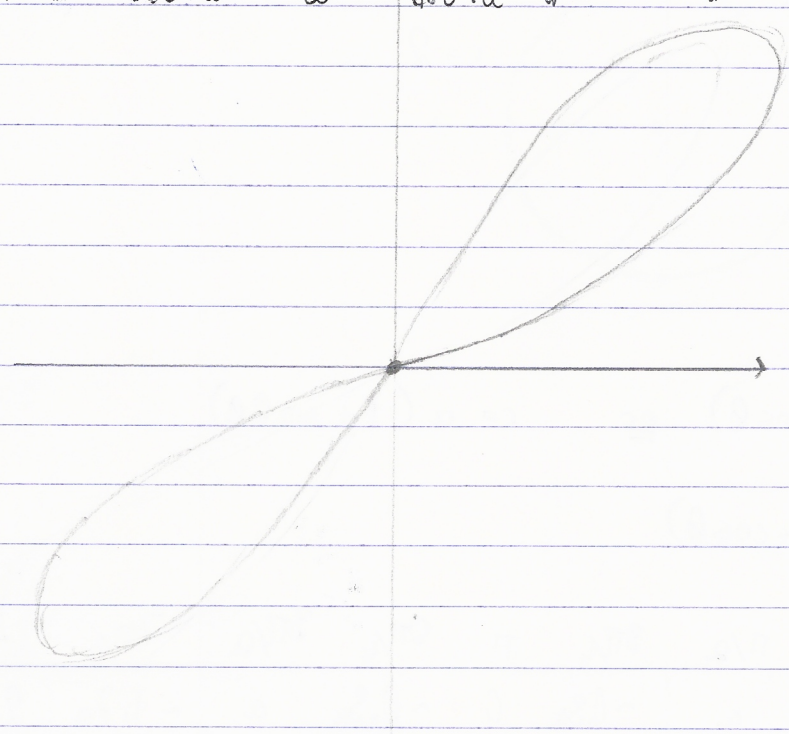
Loops

$r = a \sin 2\theta$

$0 \le 2\theta \le \pi \Rightarrow 0 \le \theta \le \pi/2$

&

$2\pi \le 2\theta \le 3\pi \Rightarrow \pi \le \theta \le 3\pi/2$

Ranges for $r > 0$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	...	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$
r	0	$0.87a$	a	$0.87a$	0		0	$0.87a$	a	$0.87a$	0