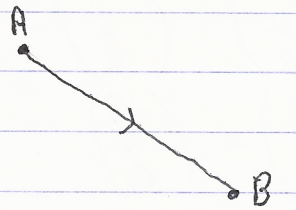
Vectors

Vectors and Scalars

- Vectors have both direction & magnitude.
- Scalars only have magnitude.

Displacement Vectors



$\overrightarrow{AB}$ represents this vector however it may just be represented by a letter, $\underline{a}$ or $\underset{\sim}{a}$

It represents the position of one point relative to another.

Column Vectors

- Written as $\begin{pmatrix} x \\ y \end{pmatrix}$ and there exist the column vectors:

$$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

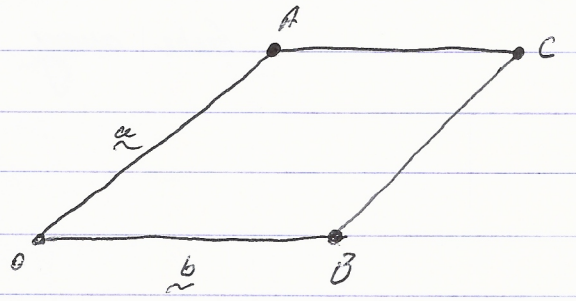
x-direction, y-direction, z-direction

These are the base unit vectors

Position Vectors

- These always start at the origin. Such as position vector $B(3, 2) = 3i + 2j$ $\underline{OC}$ as $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Parallel



$\overline{BC} = \underset{\sim}{a}$ $\because$ $\overline{BC} = \overline{OC}$ as same magnitude and direction (relative to their respective parts)

Eg $AC = \underset{\sim}{b}$

If two vectors are <u>parallel</u> & have a point in common, they must be on the same straight line $\Rightarrow$ colinear.

E.g., $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} -1 \\ 8 \\ 7 \end{pmatrix}$.

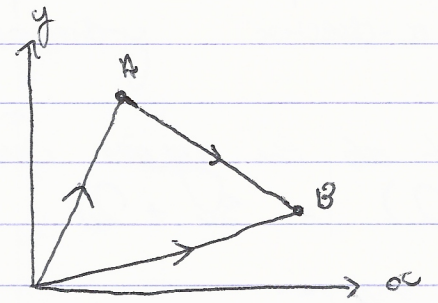
$$\Rightarrow \overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

$$\& \ AC = -\overrightarrow{OA} + \overrightarrow{OC} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 8 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \\ 6 \end{pmatrix}$$

$$\Rightarrow \overrightarrow{AC} = 3\overrightarrow{AB} \ \because \ \overrightarrow{AC} = 3\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \\ 6 \end{pmatrix}$$

$\Rightarrow \because$ they are parallel & both pass through the point A, they are colinear

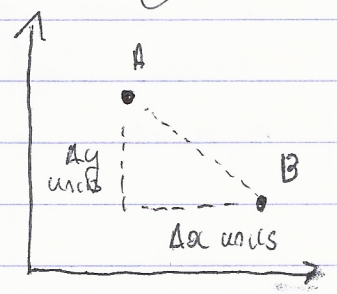
Essentially... • For position vectors, you have to go back to the origin.



$$\Rightarrow \overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$-\overrightarrow{OA}$ so flip arrow direction & take away

<u>OB</u> based directly...



Units Vectors

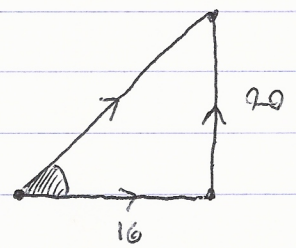
- All have a magnitude of one unit. E.g.

$$\hat{q} = \frac{q}{|q|} = \frac{3i + 4j}{5} = \frac{3}{5}i + \frac{4}{5}j$$

"the modulus of q" / the magnitude

- Shrinks vectors down to a unit of 1.

Example ① 16 units east & 20 units north
(arrows follow on)



$$\text{magnitude} = \sqrt{16^2 + 20^2}$$
$$= 4\sqrt{41} \text{ units}$$

$$\text{bearing} = \tan^{-1}\left(\frac{20}{16}\right) = \underline{038.7^\circ}$$