3Blue1Brown's Videos

Basics

- Transformations are functions with vector inputs & outputs.
- A whole grid moves under a transformation, imagine every point on the plane moving to a new point.

- Linear : • all lines remain lines & the origin is fixed under the transformations
  • grid lines remain parallel & evenly spaced too

- Transformations are based of $\hat{i}$ & $\hat{j}$. If you know where they go after a transformation, you know where every point goes.

If $\hat{i} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ & $\hat{j} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ then for any vector:

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow x \begin{bmatrix} 1 \\ -2 \end{bmatrix} + y \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1x + 3y \\ -2x + 0y \end{bmatrix}$$

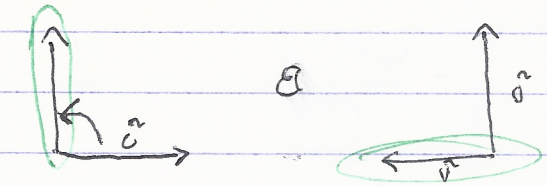
the transformation can be described as...

$$\begin{array}{cc} \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \\ \hat{i} & \hat{j} \end{array}$$

why matrix multiplication is this way

- In the general case : $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\begin{bmatrix} a \\ c \end{bmatrix}$ is where the first base vector lands & $\begin{bmatrix} b \\ d \end{bmatrix}$ is where the last base vector lands.

Rotations - A $90°$ anticlockwise rotation



$\hat{i}$ lands on $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ & $\hat{j}$ lands on $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Shear - A shear moves $\hat{j}$ to $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\Rightarrow$ Shearing everyone.

- $\hat{i}$ remains at $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\Rightarrow$ the transformation is $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Compositions - They are one single action <u>rather</u> than 2 successive ones.

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Shear}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{rotation}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{composition}}$$

$\longleftarrow$

read right to left $\Rightarrow$ rotation followed by shear :: $f(g(x))$ is read right to left (inner most)

- You break it down in multiplication:

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \text{for column 1}$$

& $$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \text{for column 2}$$

- This isn't commutative ∴ 90° ac → Shear ≠ Shear → 90° cc
- Yet, is associative ∴
- $(AB)C = A(BC)$, you're still doing $C \to B \to A$

Determinant
- Under the transformation, $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, a 1×1 block's area has increased $6\times$.
- The determinant is by how much the area of a shape is changed under a transformations.
- If $\det(A) = 0$, its been squished onto a line.
- If $\det(A) < 0$, $\hat{i}$ & $\hat{j}$ flip.

Inverses
- $A\underset{\sim}{x} = \underset{\sim}{v}$ where A = transformation causing $\underset{\sim}{x}$ to land on $\underset{\sim}{v}$.

$$A^{-1}A = I \implies I\underset{\sim}{x} = A^{-1}\underset{\sim}{v}$$
$$\times A^{-1}$$

which transformation $\underset{\sim}{v}$ back onto $\underset{\sim}{x}$