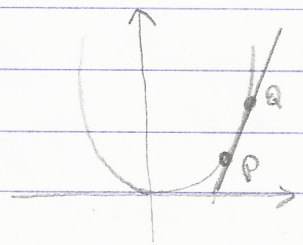
AS

Differentiation and Integration

25-10-17 (Introduction)



The chord join P & Q has a gradient that gets closer and closer to the tangent at P as Q grows closer to P.

Let $P = (1, 1)$ & $Q = (1 + dx, (1+dx)^2)$ for $y = x^2$

$$\Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{(1+dx)^2 - 1}{1 + dx - 1} = \frac{\cancel{1} + 2dx + (dx)^2 \cancel{-1}}{\cancel{1} + \cancel{dx} \cancel{-1}} = 2 + dx$$

so as $2 + dx$ is the m & $\lim_{dx \to 0} 2 + dx = 2$

(this can be done for a general case by using x instead of 1)

$y = x^3 \Rightarrow P(x, \overset{x^3}{y})$ to $Q(x + h, (x+h)^3)$

$$m = \frac{\Delta y}{\Delta x} = \frac{(x+h)^3 - x}{x + h - x} = \frac{\cancel{x^3} + 3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}3} \cancel{-x^3}}{\cancel{h}}$$

$$= 3x^2 + 3xh + h^2$$

$\lim_{h \to 0} (3x^2 + 3xh + h^2) = 3x^2$ as h approaches zero, the gradient at the point approaches $3x^2$

Format

$$\lim_{dx \to 0} \frac{dy}{dx} = \frac{dy}{dx} \quad \text{OR} \quad f'(x) = \frac{dy}{dx}$$

General Case

$y = ax^n \Rightarrow \frac{dy}{dx} = anx^{n-1}$ for each term in a polynomial

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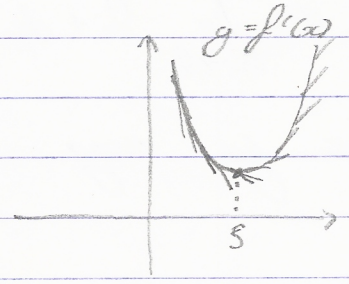
Stationary Points

- The idea is to find $f'(x)$ for a polynomial <u>AND</u> as the gradient of any stationary point $= 0$, solve $f'(x) = 0$

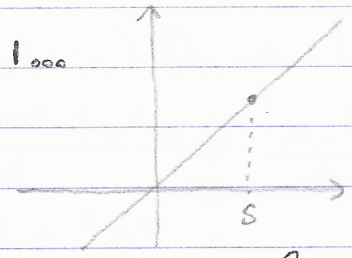
Second Derivative Test

- Use to find if a point is a maximum or minimum.
- If $\frac{d^2y}{dx^2} < 0$, maximum point <u>or</u> if $\frac{d^2y}{dx^2} > 0$, minimum point.
- This works as shown below:



As you approach the minimum point, the gradient increases from negative to positive. Thus $y = f''(x)$ can be plotted as:

$-3 \to -2 \to -1 \to 0 \to 1 \ldots$



And as any point here has the same positive gradient of a ~~maximum~~ point.

The converse argument is used for maximum points (gradient goes positive to negative $\Rightarrow \frac{d^2y}{dx^2}$ will have a negative slope with same gradient at all points)

It doesn't work if it $= 0$. It could be a no or point of inflection. You don't know what's happening!

Gradient Either Side Test

- Always test both sides to be sure!
- You can devise a method by logic.

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Integration Introduction

also called anti-derivatives

Indefinite integrals can be found using $\frac{dy}{dx} = ax^m \Rightarrow y = \frac{a}{m+1}x^{m+1} + C$

Increase power by 1 & divide coefficient by power.

• E.g. $\int x^3 \, dx = \frac{1}{4}x^4 + C$

Read as "the integral of x^3 with respect to x"

important as you don't know the final value removed under differentiation

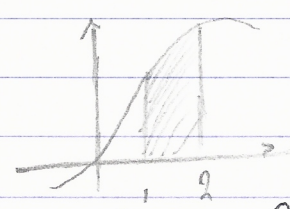
Definite integrals $\left(\int_a^b f(x)\,dx\right)$ find the area under the curve between a & b.

Fundamental Theorem of Calculus

$$\int_1^2 8x^3 + 6x + 2 \, dx = \Big[8x^4 + 3x^2 + 2x + C\Big]_1^2$$

the +C doesn't matter here as it will be cancelled anyway.

$= 48 - 7 = 41$ units^2



OR more generalised $\int_a^b f'(x)\,dx = f(b) - f(a)$

& $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$