A2 Trigonometry

Compound Angles Extended.

- $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\cos(\alpha-\beta) = \cos(\alpha)\cos(-\beta) - \sin(\alpha)\sin(-\beta)$

$= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

- $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\sin(\alpha-\beta) = \sin(\alpha)\cos(-\beta) + \cos(\alpha)\sin(-\beta)$

$= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

- Draw a triangle & say when an angle is acute.

Writing sine & cosine in terms of sine or cosine

$\sqrt{3}\sin\theta - \cos\theta$ can be written as $R\sin(\theta-\alpha)$

$\Rightarrow R\sin(\theta-\alpha) = R\left[\sin\theta\cos\alpha - \cos\theta\sin\alpha\right]$

$= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$\therefore \quad R\sin\theta\cos\alpha = \sqrt{3}\sin\theta \Rightarrow R\cos\alpha = \sqrt{3}$

$-R\cos\theta\sin\alpha = -\cos\theta \Rightarrow +R\sin\alpha = 1$

$$\frac{+R\sin\alpha}{R\cos\alpha} = \frac{1}{\sqrt{3}} \Rightarrow \tan\alpha = \frac{1}{\sqrt{3}}, \quad \alpha = \pi/6$$

$$R = 1/\sin\alpha = 1/\sin\left(\tfrac{\pi}{6}\right) = 2$$

$$\therefore \sqrt{3}\sin\theta - \cos\theta = 2\sin\left(\theta - \frac{\pi}{6}\right)$$

You can equate to:

$R\sin(\theta+\alpha)$

$R\sin(\theta-\alpha)$

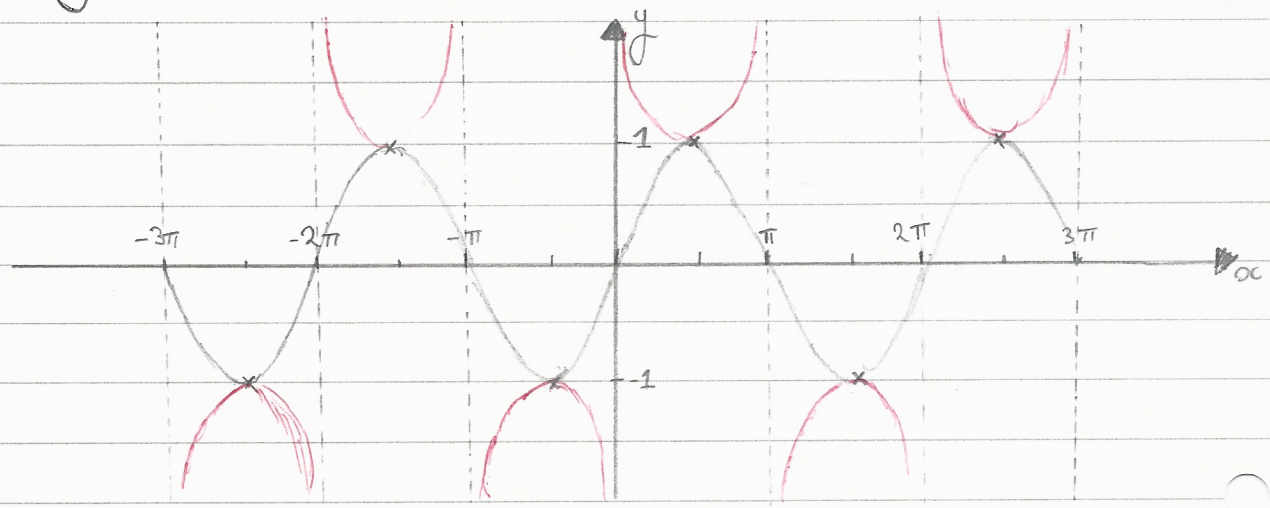
$R\cos(\theta+\alpha)$

$R\cos(\theta-\alpha)$

(ignore those which give you α negative $\because$ it makes it difficult to equate)

Reciprocal trigonometric functions

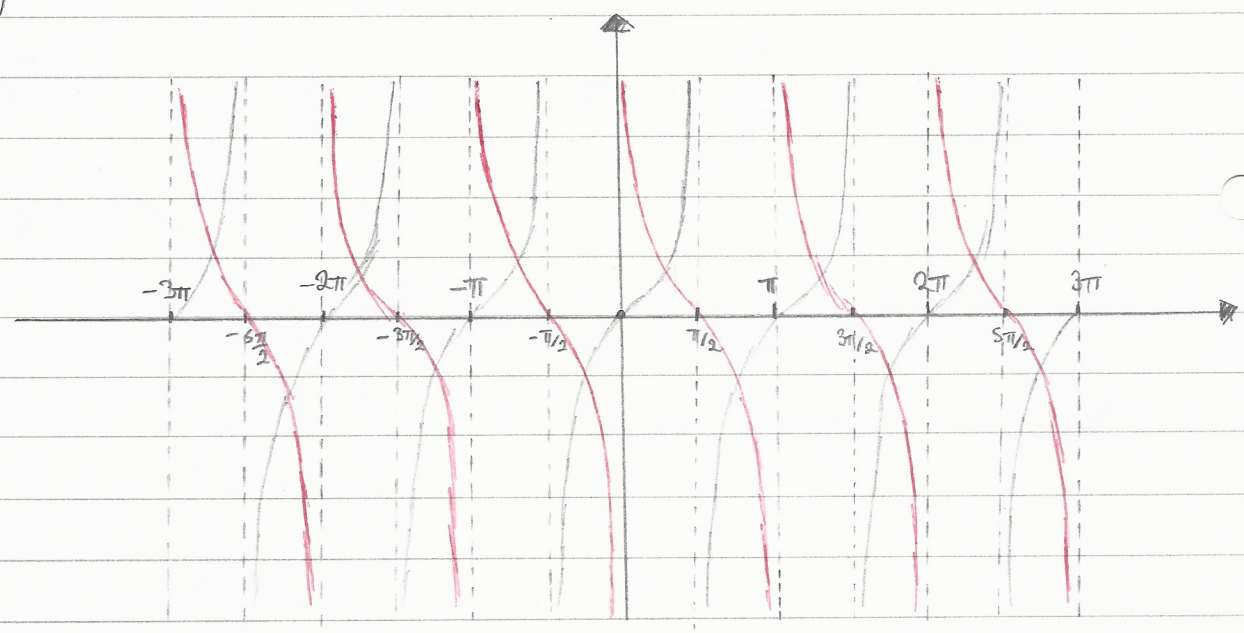
$\therefore \; y = \operatorname{cosec} x = 1/\sin x$



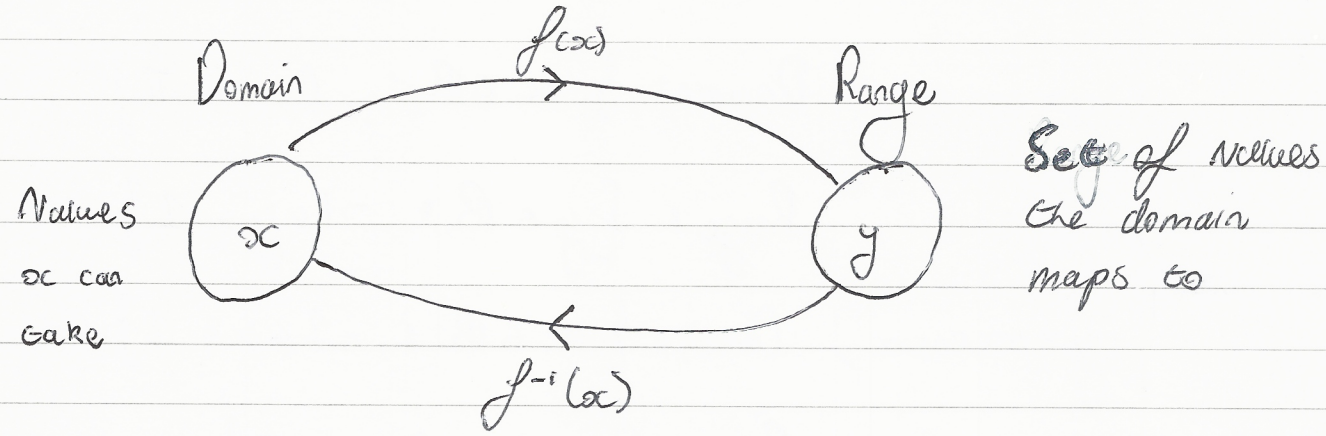
since $y = \cos(90^\circ - x) = \sin x \Rightarrow y = \cos x = \sin(90^\circ - x)$

$\therefore$ flip $y = \sin x$ in y-axis & move 90° right for both this & the inverse

$y = \underline{\cot x} = \underline{1/\tan x}$



Inverse functions & inverse trigonometric functions



You can have a many-to-one mapping e.g. $f(x) = x^2 + 4$

$\therefore f(2) = 8$

$f(-2) = 8$ <u>VET</u> $f^{-1}(8) = \pm 2$

& you cannot have both

$\therefore$ you need to restrict the domain e.g., $x > 0$

<u>SO</u> for $y = \sin x$, use $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ as the domain

$\therefore$ range $\Rightarrow -1 \le y \le 1$

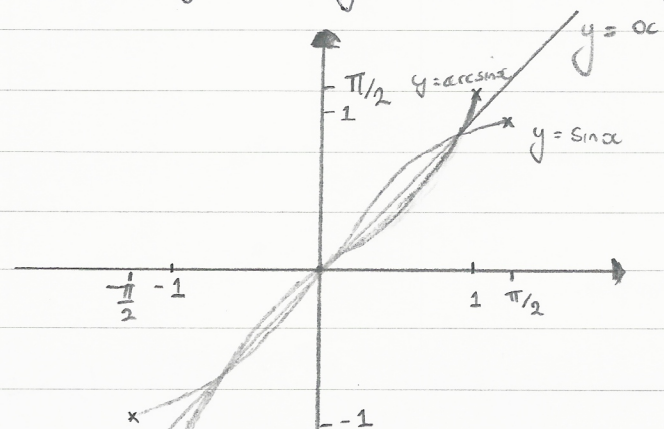
Graphs

$y = \sin x$, Domain: $\left\{ x \in \mathbb{R} : -\frac{\pi}{2} \le x \le \frac{\pi}{2} \right\}$

Range: $\{ y \in \mathbb{R} : -1 \le y \le 1 \}$

$y = \arcsin x$, Domain: $\{ x \in \mathbb{R} : -1 \le x \le 1 \}$

Range: $\left\{ y \in \mathbb{R} : -\frac{\pi}{2} \le y \le \frac{\pi}{2} \right\}$

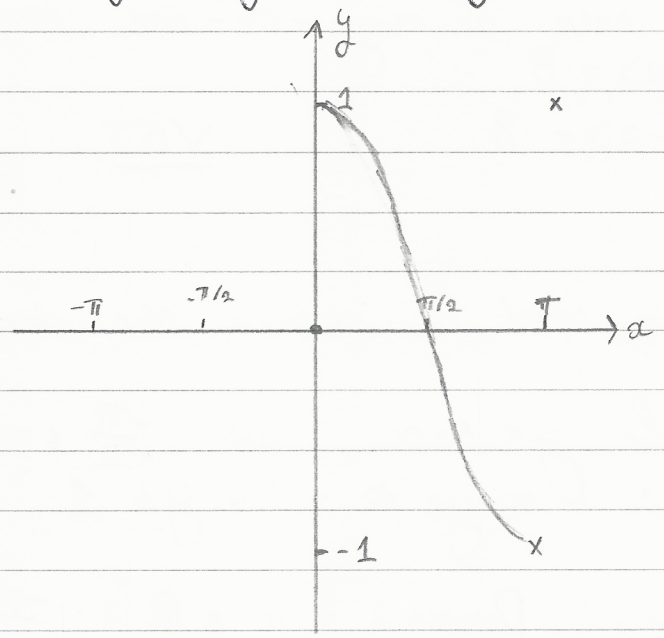


$y = \cos x$, Domain: $\{x \in \mathbb{R} : 0 \le x \le \pi\}$

Range: $\{y \in \mathbb{R} : -1 \le y \le 1\}$

$y = \arccos x$, Domain: $\{x \in \mathbb{R} : -1 \le x \le 1\}$

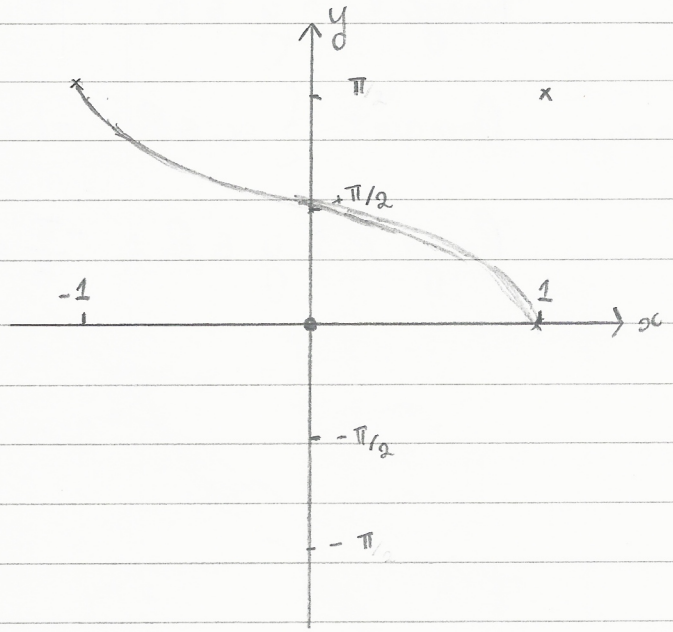
Range: $\{y \in \mathbb{R} : 0 \le y \le \pi\}$



$y = \cos x$

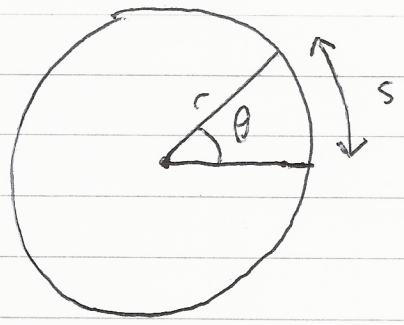
$\Downarrow$

$y = \arccos x$



Very similar approach for $y = \tan x$

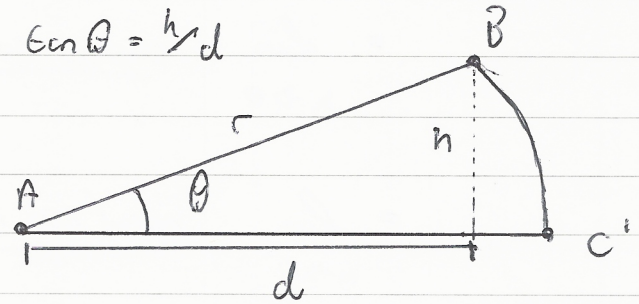
Radians & Small Angle ~~Aproxim~~ Approximation



$$s = \frac{\theta}{2\pi} \times 2\pi r$$
$$= \theta r \quad \text{(radians)}$$

$$\text{Area} = \pi r^2 \times \frac{\theta}{2\pi}$$
$$= \frac{\theta r^2}{2} \quad \text{(radians)}$$

$\sin\theta = h/r$, $\cos\theta = d/r$, $\tan\theta = h/d$



$\Rightarrow h = \sin\theta r$, $d = r\cos\theta$
$h = d\tan\theta$

$\therefore \quad h = \sin\theta r = d\tan\theta$

as $\theta \to 0$, $h \to r\theta$ & $d \to r$

$\therefore \quad h \approx \cancel{r}\theta \approx \cancel{r}\sin\theta \times \cancel{r}\tan\theta$
$\Rightarrow \quad \theta \approx \sin\theta \times \tan\theta$ } small angle for sin, tan

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$
$$\cos\theta = 1 - 2\sin^2\theta/2$$
$$= 1 - 2\left(\theta/2\right)^2$$
$$= 1 - \frac{\theta^2}{2}$$

Differentiating Using from first principles

We know that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \lim_{\theta\to 0}\frac{\theta}{\theta} = \lim_{\theta\to 0} 1 = 1$

& $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} = \lim_{\theta\to 0}\frac{1/2\,\theta^2}{\theta} = \lim_{\theta\to 0}\frac{1}{2}\theta = 0$

$$f(x) = \cos x$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$

$$= \lim_{h \to 0} \frac{\cos x(\cos h - 1)}{h} - \lim_{h \to 0} \frac{\sin x \sin h}{h}$$

$$= \cancel{-\cos x \lim_{h \to 0} \frac{1 - \cos h}{h} - \lim_{h \to 0}}$$

$$= -\cos x \lim_{h \to 0} \frac{1 - \cos h}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= -\cos x\,(0) - \sin x\,(1)$$

$$= -\sin x$$