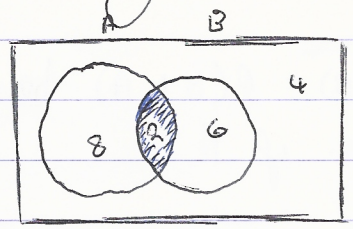
Probability

Conditional (Dependent) Events

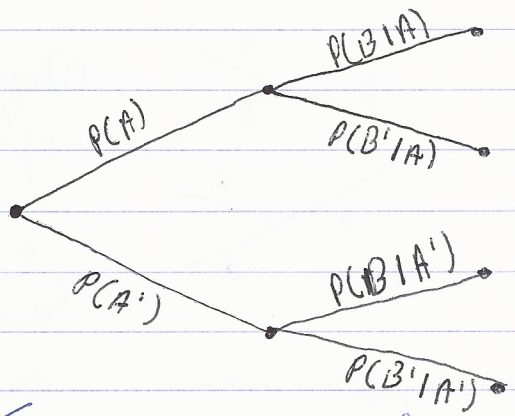


The probability of A given that your in B / B_2 is true $= P(A|B) = \frac{2}{8}$

Essentially you did $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

OR $P\left(\begin{matrix}\text{one event} \\ \text{occurs}\end{matrix} \middle/ \begin{matrix}\text{another} \\ \text{occurred}\end{matrix}\right) = \frac{P(\text{both occur})}{P(\text{given one occured})}$

IS W/O REPLACEMENT



In terms of sweets, $P(B|A)$ is dependent on A as fewer sweets remain

$\Rightarrow$ probability of taking A & B, $P(A \cap B) = P(A) \times P(B|A)$ (go along branches)

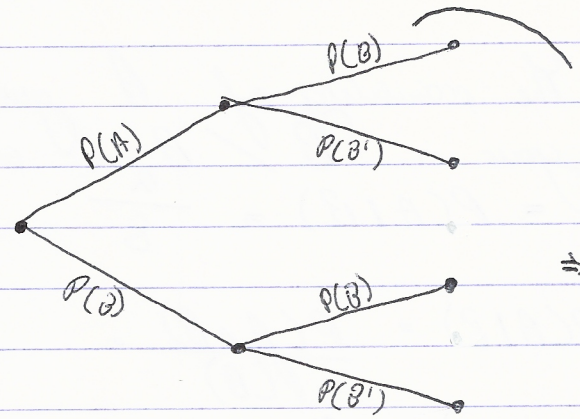
(Tree diagrams are useful)

However, if $P(A|B) = P(A) \Rightarrow$ A doesn't depend on B as the probability is the same regardless whether B has occured or not. However, if $P(A|B) \neq P(A)$ the $\Rightarrow$ dependent. E.g., P(delayed) $\neq$ P(delayed / snow) as more likely to be delayed if snowed.

Independent Events

$P(A \cap B) = P(A) \times P(B)$ $\because$ if $P(A) = P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A \cap B) = P(A) \times P(B)$

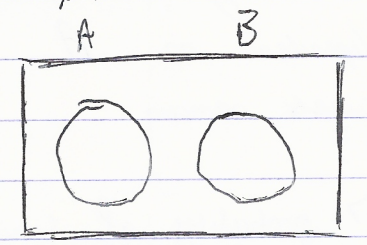


$P(B) = P(B \mid A)$ because it doesn't matter if A has

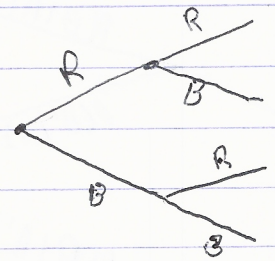
$\Rightarrow P(A \cap B) = P(A) \times P(B)$ as going along branches.

Mutually Exclusive Events

- Events which cannot happen at the same time (opposite is non-mutually exclusive).
- Examples:



or



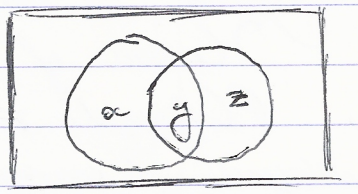
P(having two of the same colour) = P(2R) + P(2B)
(which cannot occur at the same time

For non-mutually exclusive events, you have...

$$P(A \cup B) = P(A) + P(B) - \underline{P(A \cap B)}$$

(however for mutually exclusive, this is 0)

Proof of Formula



$$P(A \cup B)$$
$$= P(A) + P(B) - P(A \cap B)$$
$$= x + y + y + z - y$$
$$= x + y + z$$

More Examples

- Flipping a coin twice $\Rightarrow$ HH, HT, TH, TT
  each is mutually exclusive as you cannot have both HH & TT.

Example

$P(A) = 0.6, \quad P(A|B) = 0.7$

$P(B) = 0.5, \quad P(B|A) = ?$

$$\underset{0.35}{P(A \cap B)} = \underset{0.7}{P(A|B)} \times \underset{0.5}{P(B)} = P(B|A) \times \underset{0.6}{P(A)}$$

$\because$ of arrangement $\Rightarrow 0.6x = 0.35$

$\Rightarrow x = 0.58...$

Advice

- Use the appropriate diagram for each question (e.g., Table, Venn Diagram, Tree).
- A question should say "given that", otherwise assume "and".