W Time Value of Money

Time value of money refers to the value of one unit of money is different in different time periods. In other words, the value of the money which is changed with the change in time, is called Time Value of Money. If an individual is offered an alternative either to accept ₹ 100 at present or one year later, he would prefer ₹ 100 at present. The basic reason of his choice of receiving ₹ 100 at present is the time value of money which means the value of the money is changed with the change in time. For this change of value, one rupee of today is more valuable than one rupee of tomorrow, because of the following reasons :

(i) **Investment opportunities :** An individual likes to take money at present than the same money in future due to investment opportunities. For example, if ₹ 100 is invested in a bank, ₹ 110 is received after one year. In such a situation, if an individual is asked to accept ₹ 100 at present or ₹ 100 after one year, he would accept ₹ 100 at present because this ₹ 100 will be ₹ 110 after one year if the money so received at present is invested in a bank.

(ii) **Uncertainty :** Future is uncertain. Due to such uncertainty, people prefer present money to future money. Because, there is no certainty whether the consumption of money utility which will be received in future is possible or not. So, most people prefer present consumption to that of future because future consumption may not be possible due to physical illness or death.

(iii) **Inflation :** The purchasing power of money reduces due to inflation *i.e.*, if inflation exists, lesser amount of goods or services are obtained in future than the amount at present against same amount of money. Thus, it is clear that the purchasing power of money at present is more than the purchasing power in future. So, people prefer to get money at present to future.

On the basis of above discussion, it can be concluded that the present value of certain sum of money is more valuable than future value of the same amount. In other words, the future value of certain sum of money is less than the present value of the same amount.

X Significance of Time Value of Money

Time value of money refers to the value of one unit of money is different in different time periods. In other words, the value of the money which is changed with the changes in time, is called **Time value of money**. The significance of time value of money is explained below :

(i) **Existence of interest :** If the present and future value of a certain amount of money have been equal, the system of paying interest did not exist, *i.e.*, the rate of interest would zero. But in reality, the rate of interest can never be zero. This is because of the existence of time value of money. That is to say, as money has time value, the question of paying interest arises. Thus, the rate of interest is more than zero due to existence of time value of money.

(ii) **Decision-making :** If investment is made into a project at present, return is obtained from that project in future. So, in order to take decision whether a project should be accepted or not, the future returns of the project have to be compared with present investment. But the money value of the future returns can never be equal to the money value of the present investment due to existence of time value of money. So, in order to take correct decision, the present value of the investment has to be compared with the present value of the future returns.

(iii) **Invention of compounding and discounting techniques :** The concepts of compounding and discounting have been invented for the existence of time value of money. The future value of present money and the present value of future money can be known with the help of compounding and discounting techniques respectively. Thus, if there is no existence of time value of money, there would be no importance of compounding and discounting techniques.

(iv) **Making the capital productive :** Interest is given on capital due to existence of time value of money *i.e.*, capital earns income. Thus, it can be said that time value of money makes the capital productive.

(v) **Time preference for money :** People prefers present money to future money. For example, if an individual is offered an alternative either to accept ₹100 at present or one year later, he would prefer ₹100 at present. The basic reason of his choice of receiving ₹100 at present is the time value of money which means that the value of the money is changed with the changes in time.

Y Compounding Technique

We know that the value of the money gradually decreases with the passage of time *i.e.*, the amount of goods or services which is received in future against certain sum of money is less than the amount of goods or services received at present with the same amount of money. In other words, the amount of goods or services so received at present against a certain sum of money, more amount of money will be required in order to get the same amount of the goods or services in future. Thus it can be said that, the value of a certain amount of money at present is equivalent to more amount of money in future. For this, additional amount is compounded with the present money in order to get future money of the equivalent amount of the present money. The technique of determining the future money of the equivalent amount of present money is called **Compounding Technique.**

The future value of a certain sum of money is less than its present value. So, if an individual accepts money in future instead of at present, he suffers loss. Interest is given for this loss, because interest is the compensation of money paid in future instead of making payment at present.

So, if interest is added to the present money, the future money of equal value of the present money is obtained. Such interest is added to the present money with the help of the compounding technique. Thus it can be said briefly, **the technique by which the future money of equal value of the present money is determined by adding necessary interest to the present money, is called Compounding Technique.** How the future money is determined by compounding interest with the present money is shown below :

Suppose, interest is paid @ 10% p.a. on the present amount of ₹ 10,000. Let us see how much amount of money can be obtained after 3 years for this amount.

Particulars	*1st Year* ₹	*2nd Year* ₹	*3rd Year* ₹
Opening Money (*At the beginning of the year*)	**10,000**	**11,000**	**12,100**
Add : Interest (*compensation*) (*During the year*)	$10{,}000 \times \frac{10}{100} = 1{,}000$	$11{,}000 \times \frac{10}{100} = 1{,}100$	$12{,}100 \times \frac{10}{100} = 1{,}210$
Closing Money (*At the end of the year*)	**11,000**	**12,100**	**13,310**

It is found from the above table that ₹ 10,000 of the first year becomes ₹ 13,310 at the end of 3rd year. In this case, ₹ (13,310 – 10,000) or, ₹ 3,310 is the interest or compensation of ₹ 10,000 for 3 years.

Z Technique of Determining the Compound Value

The process of investing money as well as re-investing the interest earned thereon is called Compounding. As a result of such compounding, the amount so received after a certain period is called **Compound Value** of the initial investment.

For example, if 'P' be the amount of initial investment and the rate of interest is r% per annum, the compound value of 'P' at the end of first year will be = $(P + Pr)$ or, $P(1 + r)$. Thus the amount of investment will be $P(1 + r)$ at the beginning of the second year. The compounded sum at the end of second year will be

$$= P(1 + r) + P(1 + r) \times r$$
$$= P(1 + r)(1 + r)$$
$$= P(1 + r)^2$$

Thus, the amount of investment will be $P(1 + r)^2$ at the beginning of third year. The compounded sum at the end of third year will be

$$= P(1 + r)^2 + P(1 + r)^2 \times r$$
$$= P(1 + r)^2(1 + r)$$
$$= P(1 + r)^3$$

Similarly, the compound value of 'P' at the end of nth year will be $P(1 + r)^n$. Thus, it can be said that if A be the compounded value of the initial investment of P after nth year at a interest @ r% p.a., then—

$$A = P(1 + r)^n$$

Here 'P' is the Principal or initial investment and $(1 + r)^n$ is the compound value of ₹ 1 at 'r' rate of interest per annum for 'n' years. So, the total compound value $(A) = P \times (1 + r)^n$. For example, if 'A' be the compound value of ₹ 10,000 after 5 years at interest @ 8% p.a.; then —

$$A = P \times (1 + r)^n$$

or, $A = 10{,}000 \times (1 + 0{\cdot}08)^5$

or, $A = 10{,}000 \times (1{\cdot}08)^5$

or, $A = 10{,}000 \times 1{\cdot}469$

$\therefore$ $A = ₹\ 14{,}690.$

Again the value of $(1 + r)^n$ can easily be determined with the help of compound value table which contains the number of years in the first column and the rate of interest in its first row. The value which is available at the intersection point of a certain number of year and certain rate of interest is the compound value of ₹ 1 at that certain year and certain rate of interest. For example, if the number of year (n) is 5 and the rate of interest (r) is 8% p.a., wer get 1.469 at the intersection point of 5 years and 8% rate of interest from the Compound Value Factor (CVE) table which is given at the end of this book. In this case, 1.469 is called Compound Value Factor of ₹ 1 for 5 years at 8% rate of interest ($CVF_{8,5}$) or Future Value Interest Factor of ₹ 1 for 5 years at 8% rate of interest ($FVIF_{8,5}$). That is to say, —

$$(1 + 0{\cdot}08)^5 = CVF_{8,5} = FVIF_{8,5} = 1.469$$

Similarly, $(1 + r)^n = CVF_{r,n} = FVIF_{r,n}$

Thus, $A = P \times (1 + r)^n = P \times CVF_{r,n} = P \times FVIF_{r,n}$

In this case, interest is compounded annually i.e. only one time in a year. But if interest is compounded 'm' times in a year, then —

$$A = P \times \left(1+\frac{r}{m}\right)^{mn}$$

or; $A = P \times CVF_{i,y} = P \times FVIF_{i,y}$;

where — $\dfrac{r}{m} = i$ and $mn = y$.

For example, if 'A' be the Compound Value of ₹ 20,000 after 5 years at interest @ 12% p.a. where interest is compounded quarterly; then —

$$A = P \times \left(1+\frac{r}{m}\right)^{mn}$$

Here, interest is compounded quarterly *i.e.*; 4 times in a year.

So, $m = 4$. Again, $P = ₹\ 20{,}000$, $r = 12\%$ or, $0{\cdot}12$ and $n = 5$.

Thus, —

$$A = 20{,}000 \times \left(1+\frac{0{\cdot}12}{4}\right)^{4\times5}$$

or, $A = 20{,}000 \times (1 + 0{\cdot}03)^{20}$

or, $A = 20{,}000 \times CVF_{3,\ 20}$

or, $A = 20{,}000 \times 1{\cdot}806$ [From Table A-1, $CVF_{3,\ 20} = 1{\cdot}806$]

or, $A = 36{,}120$

Hence, the required compound value is ₹ 36,120.

■ Z.1 Application of Compounding Technique

1. ***Annual Compounding*** **:** When interest is compounded only at the end of each year, it is known as Annual Compounding.

❑ **Example :** Mr. X invested ₹ 20,000 in a Savings Bank Account. The bank pays interest annually at 12% p.a. Find the compound value of the investment after 8 years.

● **Solution ⇒** Here, P = ₹ 20,000, $r = 12\%$ or, $0{\cdot}12$, $n = 8$.

Let 'A' be the compound value. Thus—

$$A = P(1+r)^n$$

or, $A = 20{,}000\,(1 + 0{\cdot}12)^8$

or, $A = 20{,}000 \times 2{\cdot}476$

or, $A = 49{,}520.$

Hence, required compound value is ₹ 49,520.

2. ***Semi-annual Compounding*** **:** When interest is compounded at the end of every six months *i.e.*, two times in a year, it is known as Semi-annual Compounding.

❑ **Example :** Compute the compound value when ₹ 20,000 is invested for 10 years and the interest on it is compounded at 12% p.a. semi-annual.

● **Solution ⇒** Here, semi-annual interest is compounded. This means, interest is paid 2 times in a year. Thus — $m = 2$, $P = 20{,}000$, $r = 12\%$ or, $0{\cdot}12$. Now, Let 'A' be the compound value. Thus—

$$A = P\left(1+\frac{r}{m}\right)^{mn}$$

or, $A = 20{,}000\left(1 + \dfrac{0.12}{2}\right)^{10 \times 2}$

or, $A = 20{,}000\,(1 + 0{\cdot}06)^{20}$

> *Note :* Here $(1 + 0{\cdot}06)^{20}$ is the compound value of rupee one at a interest @ 6% p.a. for 20 years.

∴ $A = 20{,}000 \times CVF_{6,\,20}$

or, $A = 20{,}000 \times 3{\cdot}207$ [From Table A–1, $CVF_{6,\,20} = 3.207$]

or, $A = 64{,}140.$

Hence, required compound value is ₹ 64,140.

3. ***Quarterly Compounding*** **:** When interest is compounded at the end of every three months *i.e.*, four times in a year, it is known as Quarterly Compounding.

■ **Example :** Compute the compound value when ₹ 5,000 is invested for 3 years and the interest on it is compounded at 12% p.a. quarterly.

● **Solution ⇒** Here, $m = 4$ times, $r = 12\%$ or, $0{\cdot}12$, $P = 5{,}000$.

Let 'A' be the compound value. Thus,—

$$A = P\left(1+\frac{r}{m}\right)^{mn}$$

or, $A = 5{,}000\left(1+\dfrac{0{\cdot}12}{4}\right)^{4 \times 3}$

or, $A = 5{,}000\,(1 + 0{\cdot}03)^{12}$

∴ $A = 5000 \times CVF_{3,\,12}$

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or, $A = 5,000 \times 1{\cdot}426$ [From Table A–1, $CVF_{3,\,12} = 1.426$]

or, $A = 7,345$

Hence, required compound value is ₹ 7,130.

4. ***Compound value of a series of payments :*** When different amounts of money are paid in different times, we have to find out the compound value of each payment and the compound values so found out are to be added in order to determine the compound value of the entire payments on a certain day in future.

■ **Example :** Mr. X deposits ₹ 4,000, ₹ 6,000, ₹ 3,000, ₹ 2,000 and ₹ 5,000 at the end of each year in his Savings Bank Account. The bank allows interest @ 8% p.a. He wants to determine the future value of his deposits at the end of 5th year. You are required to find out the compound value.

● **Solution** ⇒ Here deposits are made at the end of each year. So, the first year's deposit will earn interest for four years. Similarly, interests are to be received for 3, 2, 1 year on the second, third and fourth year's deposit respectively. No interest is to be received in fifth year on the deposit which was made at the end of that year. In this case, the value of total deposit will be after 5 years = compound value of ₹ 4,000 for 4 years + compound value of ₹ 6,000 for 3 years + compound value of ₹ 3,000 for 2 years + compound value of ₹ 2,000 for 1 year + compound value of ₹ 5,000 for zero year *i.e.,*—

$$A = 4{,}000\,(1 + 0{\cdot}08)^4 + 6{,}000\,(1 + 0{\cdot}08)^3 + 3{,}000\,(1 + 0{\cdot}08)^2 + 2{,}000\,(1 + 0{\cdot}08)^1 + 5{,}000\,(1 + 0{\cdot}08)^0, \text{ where}$$

A is the compounded value of the entire deposits.

Now, it is seen from the compound value table that —

$$(1 + 0{\cdot}08)^4 = CVF_{8,\,4} = 1{\cdot}360$$

$$(1 + 0{\cdot}08)^3 = CVF_{8,\,3} = 1{\cdot}260$$

$$(1 + 0{\cdot}08)^2 = CVF_{8.2} = 1{\cdot}166$$

$$(1 + 0{\cdot}08)^1 = CVF_{8,\,1} = 1{\cdot}080$$

Thus, $A = (4{,}000 \times 1{\cdot}360) + (6{,}000 \times 1{\cdot}260) + (3{,}000 \times 1{\cdot}166) + (2{,}000 \times 1{\cdot}080) + 5{,}000 \times 1 \quad [\because (1 + 0{\cdot}08)^0 = 1]$

or, $A = 5{,}440 + 7{,}560 + 3{,}498 + 2{,}160 + 5{,}000$

or, $A = 23{,}658.$

Hence, required compound value is ₹ 23,658.

How this type of problem can easily be solved is shown below :

No. of Year (n)	*No. of Years Compounded at 8%*	*Annual Deposit*	*Compound Value of ₹ 1 or, ($CVF_{8,\,n}$)*	*Product*
1	4	4,000	1·360	5,440
2	3	6,000	1·260	7,560
3	2	3,000	1·166	3,498
4	1	2,000	1·080	2,160
5	0	5,000	1·000	5,000
			Compound Value	**23,658**

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ZA Discounting Technique

The concept of discounting is the exact opposite of that of compounding. In case of compounding technique the future value of present cash flows is determined. On the other hand, the discounting technique is applied for determining the present value of the future cash flows. We know that the future amount of money to the equivalent sum of present money is more than the present amount of money. In other words, the present amount of money to the equivalent amount of future money is less than the future amount of money. For example, if ₹ 100 is deposited at a interest @ 10% p.a., ₹ 110 will be obtained at the end of first year. So, we can say, ₹ 100 is to be deposited at present in order to get ₹ 110 after one year at a interest @10% p.a. Alternatively, we can say the money whose value is ₹ 110 at the end of first year, its present value at the beginning of the first years is ₹ 100 if it is discounted @ 10% p.a. Thus, it is clear that the present amount of money is less than the future amount. This can be explained in different ways as given below :

The present value of money is more than the future value. So, the amount of goods or services which will be received in future against a certain sum of money, less amount of money is required in order to get the same amount of goods or services at present. So, the present amount of money is obtained if the future money is discounted. Since the present value is obtained by discounting the future cash flows, the concept of discounting is also known as Present Value Concept. In brief, ***the technique by which the present value of the certain sum of future money is determined, is called Discounting Technique.*** Let us see how the present value can be determined with the help of discounting process. Suppose, a sum of ₹ 14,641 will be required after 4 years. If the rate of interest is 10% p.a. then let us see how much money needs to be deposited at present —

At the beginning of 1st year	At the end of 1st year	At the end of 2nd year	At the end of 3rd year	At the end of 4th year
				₹ 14,641
			₹ 14,641 × $\frac{100}{110}$ = ₹ 13,310	
		₹ 13,310 × $\frac{100}{110}$ = ₹ 12,100		
	₹ 12,100 × $\frac{100}{110}$ = ₹ 11,000			
₹ 11,000 × $\frac{100}{110}$ = ₹ 10,000				

The above table shows that ₹ 10,000 of the present value is equivalent to ₹ 14,641 at the end of 4th year.

ZB Technique of Determining the Discounted (Present) Value

If future money is discounted in order to determine the present money, the amount so received after discounting will be reduced. Thus, it can be said that if the interest is deducted from the future money, present money is obtained. We know that if the present amount of ₹ '*P*' be ₹ '*A*' after *n* years at a interest @ *r*% p.a., then —

$$A = P(1+r)^n$$

$$\text{or,} \quad P = \frac{A}{(1+r)^n}$$

$$\therefore \quad P = A.\frac{1}{(1+r)^n}$$

Thus, it can be said, if an amount of ₹ '*A*' which will be obtained after '*n*' years is discounted at the rate of *r*%, the discounted value (present value) will be $A.\frac{1}{(1+r)^n}$.

Now, suppose, we have to determine the present value of ₹ 14,641 at a discount @ 10% p.a. for 4 years. In this case, *r* = 10%, *n* = 4 and *A* = 14,641. Now, if *P* be the present value, then—

$$P = 14{,}641 \times \frac{1}{(1+0{\cdot}10)^4}$$

$$\text{or,} \quad P = 14{,}641 \times \frac{1}{1{\cdot}10} \times \frac{1}{1{\cdot}10} \times \frac{1}{1{\cdot}10} \times \frac{1}{1{\cdot}10}$$

$$\text{or,} \quad P = 14{,}641 \times \frac{100}{110} \times \frac{100}{110} \times \frac{100}{110} \times \frac{100}{110}$$

$$\text{or,} \quad P = 14{,}641 \times \frac{10}{11} \times \frac{10}{11} \times \frac{10}{11} \times \frac{10}{11}$$

$$\text{or,} \quad P = 14{,}641 \times \frac{10{,}000}{14{,}641}$$

$$\text{or,} \quad P = 10{,}000.$$

$\frac{1}{(1+r)^n}$ is the Present Value Factor (PVF) or Present Value Interst Factor (**PVIF**) of ₹ 1 received after 'n' year at 'r' rate of discount (**$PVF_{r,n}$ or $PVIF_{r,n}$**). The value of $\frac{1}{(1+r)^n}$ can easily be determined with the help of present value Table (i.e., Table A-3) which has been given at the end of this book. The present value table contains number of year in its first column and rate of discount (or, interest) in its first row. The value which is available at the intersection point of a certain year and a certain rate of interest is the present value of ₹ 1 or Present Value Factor at that certain year and certain rate of interest. For example, the intersection point of 4th year and 10% rate of discount contains 0·683. This means that ₹ 0·683 is the present value of ₹ 1 received after 4th year at a discount @10% p.a. Thus, the present value of ₹ 14,641 received after 4 year at a discount @ 10% = 14,641 × 0·683 = 9999·803 or; ₹ 10,000 (Aprrox). Thus, we can say that —

$$P = A \times \frac{1}{(1+r)^n} = A \times PVF_{r,n} = A \times PVIF_{r,n}.$$

■ ZB.1. Application of Discounting Concept

1. Present Value of a Lump-sum Cash :

❑ **Example :** Mr. X expects to get ₹ 40,000 after 5 years. If the rate of interest is 8% p.a., calculate the amount which is required to be invested by Mr. X at present.

● **Solution ⇒** Here, A = ₹ 40,000, $r = 8\%$ or, 0·08, $n = 5$.

Let P be the present value. Thus—

$$P = A \times \frac{1}{(1+r)^n}$$

$$P = 40{,}000 \times \frac{1}{(1+0.08)^5}$$

$$\text{or, } P = 40{,}000 \times PVF_{8,5}$$

$$\text{or, } P = 40{,}000 \times 0{\cdot}681 \text{ [From Table A-3, } PVF_{8,5} = 0.681]$$

$$\text{or, } P = 27{,}240.$$

Thus, the required present value is ₹ 27,240.

2. Present Value of a Series of Cash Flow : When different amounts of cash flows take place in different times, the sum total of present value of each cash flow will be the total present value of the entire cash flows. Let $A_1, A_2, A_3 \cdots\cdots A_n$ are the cash flows of a project at the end of 1st, 2nd, 3rd .... nth year respectively. Now, if r be the rate of discounting, the present value of the entire cash flows will be—

$$P = \frac{A_1}{(1+r)^1} + \frac{A_2}{(1+r)^2} + \frac{A_3}{(1+r)^3} + \ldots\ldots\ldots\ldots + \frac{A_n}{(1+r)^n}$$

❑ **Example :** Mr. Joydeep Mukherjee considering an investment opportunity which will give him cash flows of ₹ 3,000, ₹ 4,000, ₹ 2,000, ₹ 6,000 and ₹ 5,000 respectively at the end of each of the next 5 years. You are required to calculate the present value of these cash inflows if his time preference rate is 10%.

Solution:

We know,

Present value of a stream of cash flows (P)

$$= \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \frac{A_3}{(1+r)^3} + \cdots + \frac{A_n}{(1+r)^n}$$

Here, $A_1 = 3000$, $A_2 = 4000$, $A_3 = 2000$, $A_4 = 6000$, $A_5 = 5000$, $r = 10\%$, $= 0.1$,

$n = 5$

Substituting the given values ~~we~~ we get,

$$P = \frac{3000}{1.1} + \frac{4000}{(1.1)^2} + \frac{2000}{(1.1)^3} + \frac{6000}{(1.1)^4} + \frac{5000}{(1.1)^5}$$

$= 2,727.27 + 3,305.79 + 1,502.63 + 4098.08 + 3,104.60$

$= 14,738.37$ Ans.