Encoding Techniques in Genetic Algorithms

GA Operators

Following are the GA operators in Genetic Algorithms.

1. Encoding
2. Convergence test
3. Mating pool
4. Fitness Evaluation
5. Crossover
6. Mutation
7. Inversion

Encoding Operation

1. **Encoding**
2. Convergence test
3. Mating pool
4. Fitness Evaluation
5. Crossover
6. Mutation
7. Inversion

Different Encoding Schemes

- **Different GAs**
  - Simple Genetic Algorithm (SGA)
  - Steady State Genetic Algorithm (SSGA)
  - Messy Genetic Algorithm (MGA)
- **Encoding Schemes**
  - Binary encoding
  - Real value encoding
  - Order encoding
  - Tree encoding

Different Encoding Schemes

Often, GAs are specified according to the encoding scheme it follows.

For example:

- Encoding Scheme
- Binary encoding –> Binary Coded GA or simply **Binary GA**
- Real value encoding –> Real Coded GA or simply **Real GA**
- Order encoding –> **Order GA** (also called as **Permuted GA**)
- Tree encoding

Encoding Schemes in GA

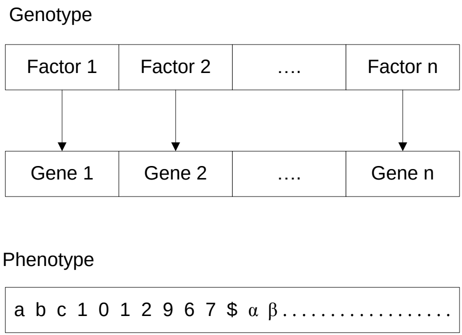
Genetic Algorithm uses metaphor consisting of two distinct elements :

1. Individual
2. Population

An individual is a single solution while a population is a set of individuals at an instant of searching process.

Individual Representation :Phenotype and Genotype

- An individual is defined by a chromosome. A chromosome stores genetic information (called phenotype) for an individual.
- Here, a chromosome is expressed in terms of factors defining a problem.



Individual Representation :Phenotype and Genotype

Note :

- A gene is the GA's representation of a single factor (i.e. a design parameter), which has a domain of values (continuous, discontinuous, discrete etc.) symbol, numbering etc.
- In GA, there is a mapping from genotype to phenotype. This eventually decideds the performance (namely speed and accuracy) of the problem solving.

Encoding techniques

There are many ways of encoding:

1. **Binary encoding:** Representing a gene in terms of bits (0s and 1s).
2. **Real value encoding:** Representing a gene in terms of values or symbols or string.
3. **Permutation (or Order) encoding:** Representing a sequence of elements)
4. **Tree encoding:** Representing in the form of a tree of objects.

Binary Encoding

In this encoding scheme, a gene or chromosome is represented by a string (fixed or variable length) of binary bits (0's and 1's)

A :	0 1 1 0 0 1 0 1 0 1 0 1 0 1 1 1 1 0	Individual 1
B :	0 0 1 0 1 0 1 1 1 0 1 0 1 0 1 0 0 0	Individual 2

Example: 0-1 Knapsack problem

- There are n items, each item has its own cost (c_i) and weight (w_i).
- There is a knapsack of total capacity w.
- The problem is to take as much items as possible but not exceeding the capacity of the knapsack.

This is an optimization problem and can be better described as follows.

Maximize :

$$\sum_i c_i * w_i * x_i$$

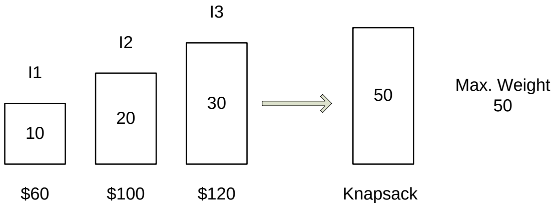
Subject to

$$\sum x_i * w_i \leq W$$

where $x_i \in [0 \cdots 1]$

Example: 0-1 Knapsack problem

Consider the fallowing, an instance of the 0-1 Knapsack problem.



Brute force approach to solve the above can be stated as follows:

- Select at least one item

  $[10], [20], [30], [10, 20], [10, 30], [, 20, 30], [10, 20, 30]$

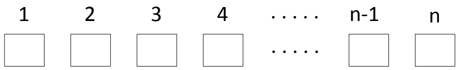
- So, for n-items, are there are $2^n - 1$ trials.
- 0 - means item not included and 1 - means item included

$$[100], [010], [011], [110], [101], [011], [111]$$

Example: 0-1 Knapsack problem

The encoding for the 0-1 Knapsack, problem, in general, for *n* items set would look as follows.

Genotype :



Phenotype :

0 1 0 1 1 0 1 0 1 0 1 0 1. . . . . . .1 0 1

A binary string of n-bits

Few more examples

- **Example 1 :**

  Minimize :

$$f(x) = \frac{x^2}{2} + \frac{125}{x}$$

where $0 \leq x \leq 15$ and x is any discrete integer value.

Genotype :

x

Phenotype :

0 1 1 0 1

A binary string of 5-bits

Few more examples

- **Example 2 :**

  Maximize :

$$f(x, y) = x^3 - x^2y + xy^2 + y^3$$

Subject to :

$$x + y \leq 10$$

and

$$1 \leq x \leq 10$$
$$-10 \leq y \leq 10$$

Genotype :

x	**y**

Phenotype :

0 1 1 0 1	**1 1 0 0 1**

Two binary string of 5-bits each

Pros and cons of Binary encoding scheme

- **Limitations:**
  1. Needs an effort to convert into binary from
  2. Accuarcy depends on the binary reprresentation

- **Advantages:**
  1. Since operations with binary represntation is faster, it provide a faster implementations of all GA operators and hence the execution of GAs.
  2. Any optimization problem has it binary-coded GA implementation

Real value encoding

- The real-coded GA is most suitable for optimization in a continuous search space.
- Uses the direct representations of the design paparmeters.
- Thus, avoids any intermediate encoding and decoding steps.

Genotype :

x	y

Phenotype :

5.28	-475.36

Real-value representation

Real value encoding with binary codes

Methodology: Step 1 [Deciding the precision]

For any continuous design variable x such that $X_L \leq x \leq X_U$, and if ε is the precision required, then string length n should be equal to

$$n = \log_2 \left(\frac{X_U - X_L}{\varepsilon} \right)$$

where $X_L \leq x \leq X_U$

Equivalently,

$$\varepsilon = \left(\frac{X_U - X_L}{2^n} \right)$$

In general, $\varepsilon = [0 \cdots 1]$. It is also called, *Obtaianable accuracy*

Note: If $\varepsilon = 0.5$, then 4.05 or 4.49 $\equiv$ 4 and 4.50 or 4.99 $\equiv$ 4.5 and so on.

Real value encoding: Illustration 1

1. **Example 1:**

   $1 \leq x \leq 16$, $n = 6$. What is the accuracy?

   $\varepsilon = \frac{16-1}{2^6} = \frac{15}{64} = 0.249 \approx 0.25$

2. **Example 2:**

   What is the obtainable accuracy, for the binary representation for a variable X in the range range $20.1 \leq X \leq 45.6$ with 8-bits?

3. **Example 3:**

   In the above case, what is the binary representation of $X = 34.35$?

Real value encoding with binary codes

Methodology: Step 2[Obtaining the binary representation]

Once, we know the length of binary string for an obtainable accuracy (i.e precision), then we can have the following mapping relation from a real value X to its binary equivalent decoded value X_B,which is given by

$$X = X_L + \frac{X_U - X_L}{2^n - 1} \times X_B$$

where X_B = Decoded value of a binary string,
n is the number of bits in the representation,
$X_L \rightarrow 0\,0\,0\,0 \cdots 0$ and
$X_U \rightarrow 1\,1\,1\,1 \cdots 1$
are the decoded values of the binary representation of the lower and upper values of X.

Real value encoding: Illustration 2

Example:

Suppose, $X_L = 2$ and $X_U = 17$ are the two extreme decoded values of a variable x.

$n = 4$ is the number of binary bits in the representation for x.

$X_B = 10 (= 1\ 0\ 1\ 0)$ is a decoded value for a given x.

What is the value of $x = ?$ and its binary representation??

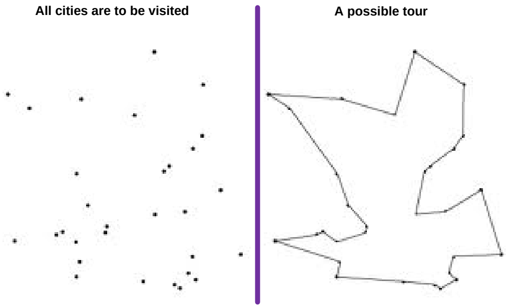
Here, $x = 2 + \frac{17-2}{2^4-1} \times 10 = 12$

Binary representation of $x = 1\ 1\ 0\ 0$

Order Encoding

Let us have a look into the following instance of the Traveling Salesman Problem (TSP).

TSP
- Visit all the cities
- One city once only
- Starting and ending city is the same



How we can formally define the TSP?

Order Encoding for TSP

Understanding the TSP:

There is a cost of visiting a city from another city and hence the total cost of visiting all the cities but exactly once (except the starting city).

Objective function: To find a tour (i.e. a simple cycle covering all the cities) with a minimum cost involved.

Constraints:

- All cities must be visited.
- There will be only one occurrence of each city (except the starting city).

Design parameters:

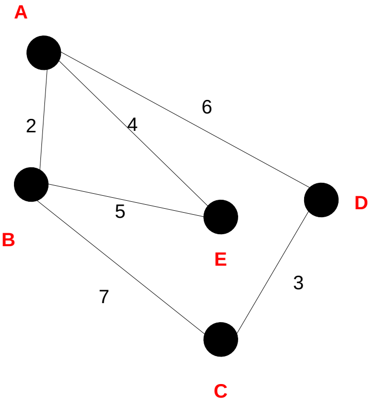
- Euclidean distance may be taken as the measurement of the cost, otherwise, if it is specified explicitly.
- The above stated information are the design variables in this case.

We are to search for the best path out of $n!$ possible paths.

A small instance of the TSP

d	A	B	C	D	E
A	0	2	∝	6	4
B	2	0	7	∝	5
C	∝	7	0	3	1
D	6	∝	3	0	∝
E	4	5	1	∝	0

d= Distance matrix



Connectivity among cities

Defining the TSP

Minimizing

$$\text{cost} = \sum_{i=0}^{n-2} d(c_i, c_{i+1}) + d(c_{n-1}, c_0)$$

Subject to

$P = [c_0, c_1, c_2, \cdots, c_{n-1}, c_0]$
where $c_i \in X$;
Here, P is an ordered collection of cities and $c_i \neq c_j$ such that $\forall i, j = 0, 1, \cdots, n-1$

Note: P represents a possible tour with the starting cities as c_0.

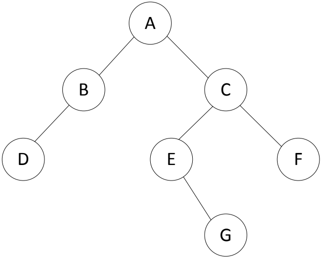
and

$X = x_1, x_2, \cdots, x_n$, set of n number of cities,

$d(x_i, x_j)$ is the distance between any two cities x_i and x_j.

Tree encoding

In this encoding scheme, a solution is encoded in the form of a binary tree.



A binary tree

D A B E G C F **(In-order)**

(T_L R T_R)

A B D C E G F **(Pre-order)**

(R T_L T_R)

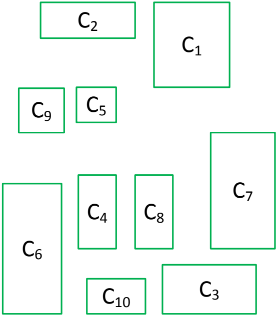
D B G E E C A **(Post-order)**

(T_L T_R R)

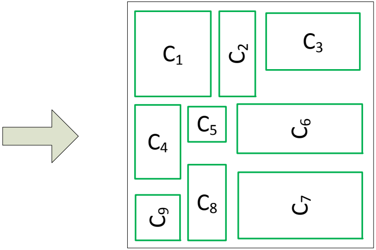
Three compact representation

Floor Planning : An example of tree encoding

Floor planning is a standard problem in VLSI design. Here, given *n* circuits of different area requirements, we are to arrange them into a floor of chip layout, so that all circuits are placed in a minimum layout possible.



10 circuits



A floor plan

Formulation of floor planning problem

A possible formulation of Floor planning problem of VLSI circuit is as follows.

Given :

1. A set of n rectangular blocks $B = b_1, b_2, \cdots, b_i, \cdots, b_n$
2. For each $b_i \in B$, we have the following specification:
   - the width w_i and height h_i (which are constant for rigid blocks and variable for flexible blocks)
   - ρ_i, the desirable aspect ratio about where $\frac{1}{\rho_i} \leq \frac{h_i}{w_i} \leq \rho_i$, where $\rho_i = 1$, if the block b_i is rigid.
   - $a_i = w_i \times h_i$, the area of each block b_i.

Formulation of floor planning problem

3. A set of nets $N = \{n_1, n_2, \cdots, n_k\}$ describing the connectivity information.

$$Wire = f_1(B, N)$$

4. Desirable floor plan aspect ratio ρ such that $\frac{1}{\rho} \leq \frac{H}{W} \leq \rho$, where H and W are the height and width of the floor plan, respectively.

$$Area = f_2(B, N, \rho)$$

5. Timing information.

$$Delay = f_3(B, N, \rho)$$

Formulation of Floor planning problem

A legal floor plan is a floor plan that satisfies the following constraints.

Constraints :

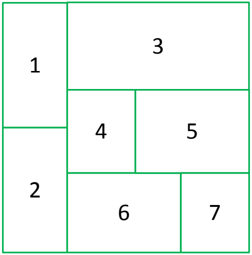
3. Each block b_i is assigned to a location say (x_i, y_i).
4. No two blocks overlap
5. For each flexible block say b_i, $a_i = w_i \times h_i$ and should meet aspect ratio constraint.

Objectives :

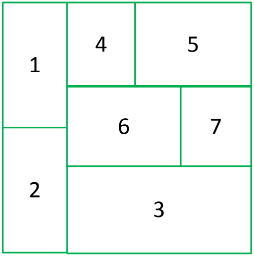
We are to find a floor plan, which would

1. Minimize floor plan area.
2. Minimize wire length.
3. Minimize circuit delay.

Tree encoding for Floor planning problem



Floor Plan I



Floor Plan II

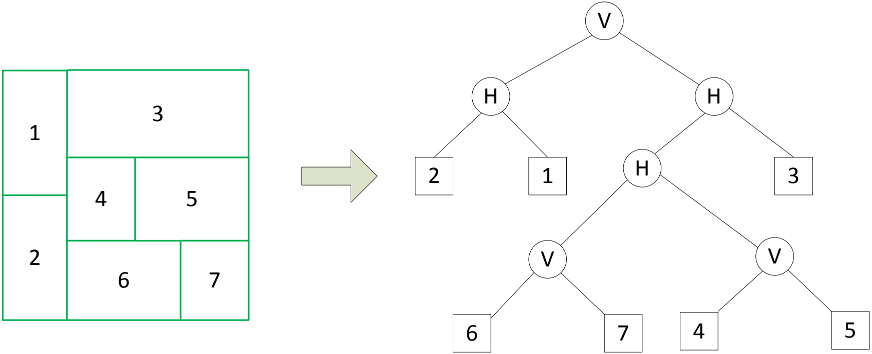
1. How many floor plans are possible?
2. Can we find a binary tree representation of a floor plan??

Binary tree representation for floor planning

A floor plan can be modeled by a binary tree with n leaves and $n-1$ nodes where

- each node represents a vertical cut-line or horizontal cut-line, and Letter V and H refer to vertical and horizontal cut-operators.
- each leaf node represents a rectangle blocks.

Example : Floor plane I



Floor Plan I

Binary tree representation of the floor plan I

Example : Floor plane I

Note 1:

The operators H and V expressed in polish notation carry the following meanings:

$ijH \rightarrow$ Block b_j is on top of the block b_i.

$ijV \rightarrow$ Block b_i is on the left of block b_j.

Note 2: A tree can be represented in a compact form using Polish notation

Note 3: Polish notation

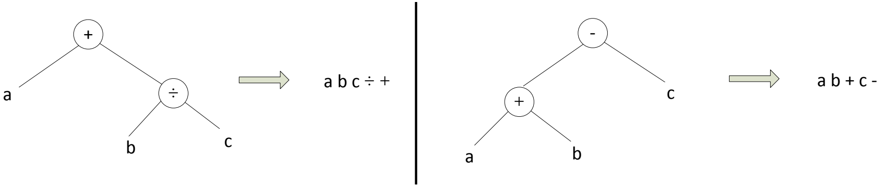
$a + b \div c = a + (b \div c) = abc \div +$

$a + b - c = ab + c-$

Example : Floor plane I

Note 4:

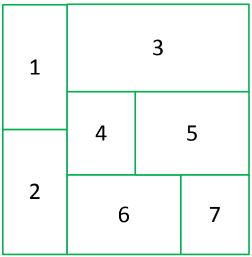
Post order traversal of a binary tree is equivalent to polish notation



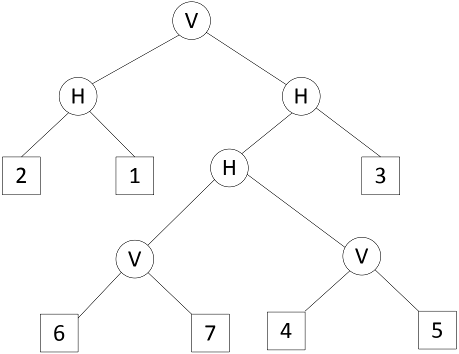
Note 5:

There is only one way to performing a post order traversal of a binary tree.

Example : Floor Plane I (with Polish notation



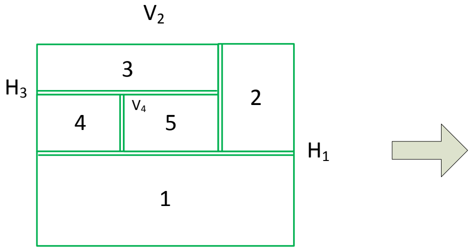
Floor Plan I



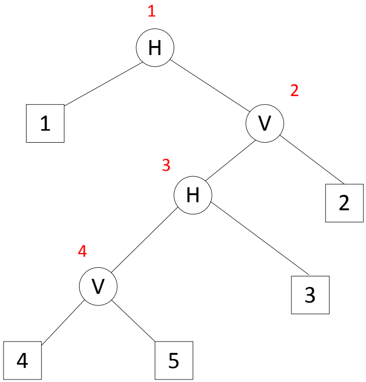
Binary tree representation of the floor plan I

Polish notation : 2 1 H 6 7 V 4 5 V H 3 H V

Example : H and V operators



Floor Plan



Binary tree

Polish notation : 4 5 V 3 H 2 V 1 H