Fuzzy Logic : Introduction

What is Fuzzy logic?

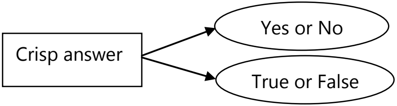
- Fuzzy logic is a mathematical language to express something.

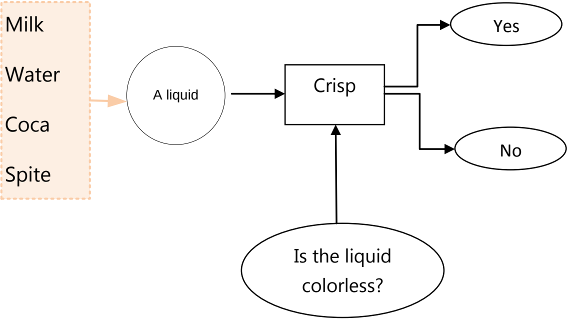
  This means it has grammar, syntax, semantic like a language for communication.

- There are some other mathematical languages also known
  - **Relational algebra** (operations on sets)
  - **Boolean algebra** (operations on Boolean variables)
  - **Predicate logic** (operations on well formed formulae (wff), also called predicate propositions)

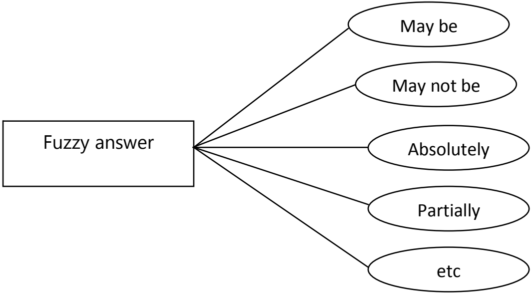
- **Fuzzy logic deals with Fuzzy set.**

Example : Fuzzy logic vs. Crisp logic

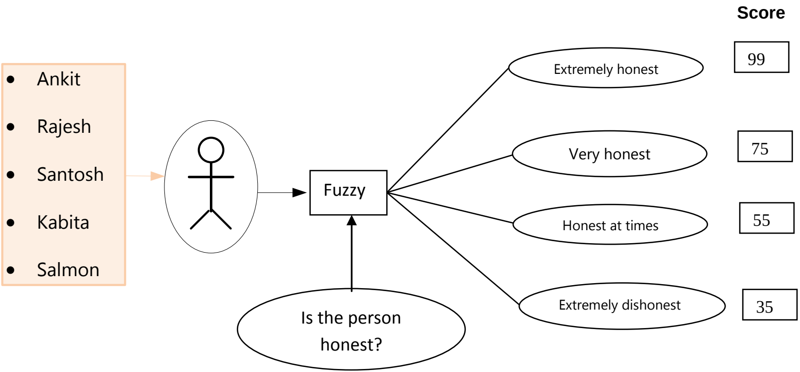




Example : Fuzzy logic vs. Crisp logic



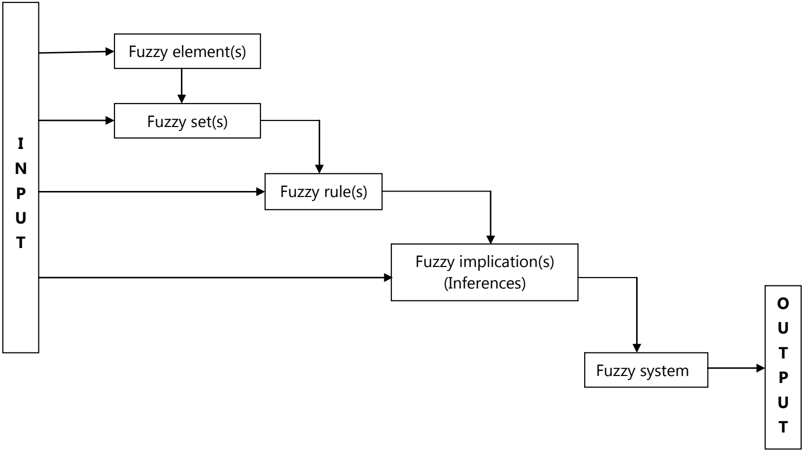
Example : Fuzzy logic vs. Crisp logic



World is fuzzy!

Our world is better described with fuzzily!

Concept of fuzzy system



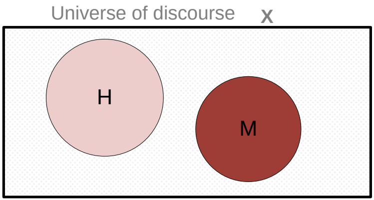
Concept of fuzzy set

To understand the concept of **fuzzy set** it is better, if we first clear our idea of **crisp set**.

X = The entire population of India.

H = All Hindu population = $\{ h_1, h_2, h_3, \ldots , h_L \}$

M = All Muslim population = $\{ m_1, m_2, m_3, \ldots , m_N \}$



Here, All are the sets of finite numbers of individuals.

Such a set is called crisp set.

Example of fuzzy set

Let us discuss about fuzzy set.

X = All students in class

S = All **Good students.**

S = $\{$ (s, g) $|$ s $\in$ X $\}$ and g(s) is a measurement of goodness of the student *s*.

Example:

S = $\{$ (Rajat, 0.8), (Kabita, 0.7), (Salman, 0.1), (Ankit, 0.9) $\}$ etc.

Fuzzy set vs. Crisp set

Crisp Set	Fuzzy Set
1. S = $\{$ s $\mid$ s $\in$ X $\}$	1. F = (s, μ) $\mid$ s $\in$ X and μ(s) is the degree of s.
2. It is a collection of elements.	2. It is collection of ordered pairs.
3. Inclusion of an element s $\in$ X into S is crisp, that is, has strict boundary **yes** or **no**.	3. Inclusion of an element s $\in$ X into F is fuzzy, that is, if present, then with a degree of **membership**.

Fuzzy set vs. Crisp set

Note: A crisp set is a fuzzy set, but, a fuzzy set is not necessarily a crisp set.

Example:

H = $\{ (h_1, 1), (h_2, 1), \ldots , (h_L, 1) \}$

Person = $\{ (p_1, 1), (p_2, 0), \ldots , (p_N, 1) \}$

In case of a crisp set, the elements are with extreme values of degree of membership namely either 1 or 0.

How to decide the degree of memberships of elements in a fuzzy set?

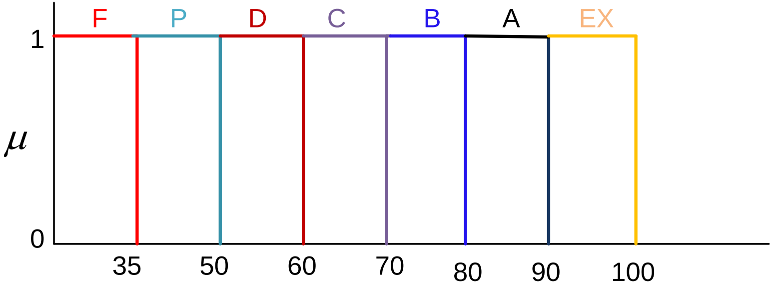
City	Bangalore	Bombay	Hyderabad	Kharagpur	Madras	Delhi
DoM	0.95	0.90	0.80	0.01	0.65	0.75

How the cities of comfort can be judged?

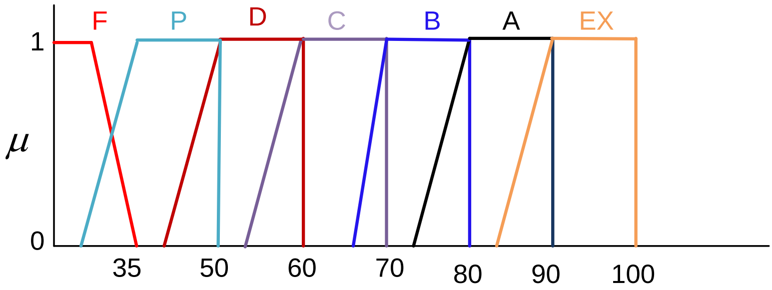
Example: Course evaluation in a crisp way

1. EX = Marks $\geq$ 90
2. A = 80 $\leq$ Marks $<$ 90
3. B = 70 $\leq$ Marks $<$ 80
4. C = 60 $\leq$ Marks $<$ 70
5. D = 50 $\leq$ Marks $<$ 60
6. P = 35 $\leq$ Marks $<$ 50
7. F = Marks $<$ 35

Example: Course evaluation in a crisp way



Example: Course evaluation in a fuzzy way



Few examples of fuzzy set

- High Temperature
- Low Pressure
- Color of Apple
- Sweetness of Orange
- Weight of Mango

Note: Degree of membership values lie in the range [0...1].

Some basic terminologies and notations

Definition 1: Membership function (and Fuzzy set)

If X is a universe of discourse and x $\in$ X, then a fuzzy set A in X is defined as a set of ordered pairs, that is

$A = \{(x, \mu_A(x)) | x \in X\}$ where μ_A(x) is called the membership function for the fuzzy set A.

Note:
μ_A(x) map each element of X onto a membership grade (or membership value) between 0 and 1 (both inclusive).

Question:
How (and who) decides μ_A(x) for a Fuzzy set A in X?

Some basic terminologies and notations

Example:

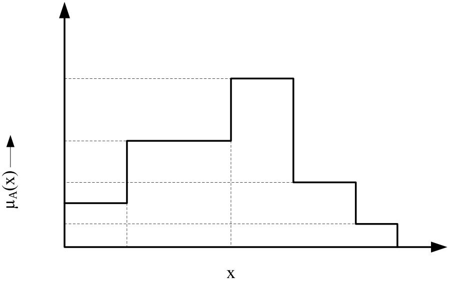
X = All cities in India

A = City of comfort

A=$\{$(New Delhi, 0.7), (Bangalore, 0.9), (Chennai, 0.8), (Hyderabad, 0.6), (Kolkata, 0.3), (Kharagpur, 0)$\}$

Membership function with discrete membership values

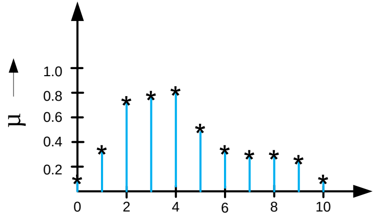
The membership values may be of discrete values.



A fuzzy set with discrete values of μ

Membership function with discrete membership values

Either elements or their membership values (or both) also may be of discrete values.



Number of children (X) ⟶

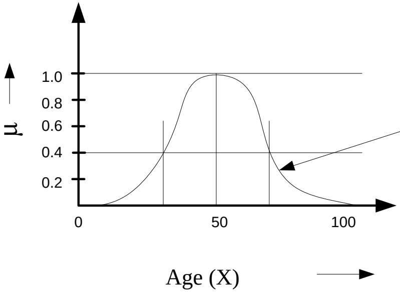
A = “Happy family”

A ={(0,0.1),(1,0.30),(2,0.78)……(10,0.1)}

Note : X = discrete value

How you measure happiness ??

Membership function with continuous membership values



$$\mu_B(x) = \frac{1}{1+\left(\frac{x-50}{10}\right)^4}$$

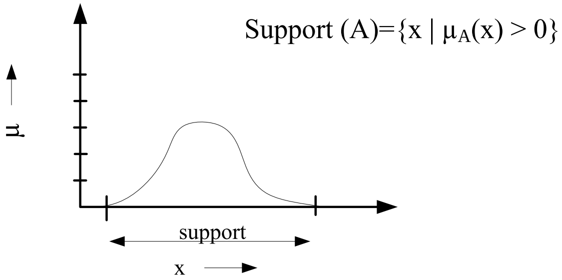
$$B=\{(x,\mu_B(x))\}$$

B = “Middle aged”

Note : x = real value $= R^+$

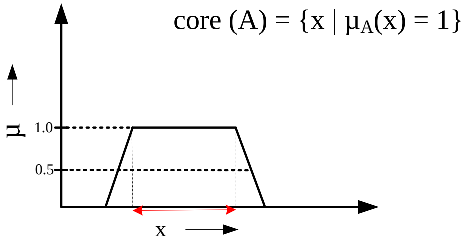
Fuzzy terminologies: Support

Support: The support of a fuzzy set A is the set of all points $x \in X$ such that $\mu_A(x) > 0$



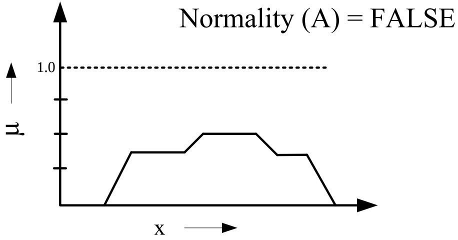
Fuzzy terminologies: Core

Core: The core of a fuzzy set A is the set of all points x in X such that $\mu_A(x) = 1$



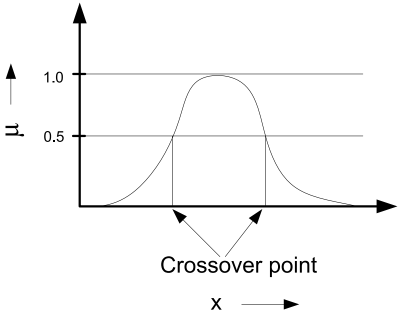
Fuzzy terminologies: Normality

Normality : A fuzzy set A is a normal if its core is non-empty. In other words, we can always find a point $x \in X$ such that $\mu_A(x) = 1$.



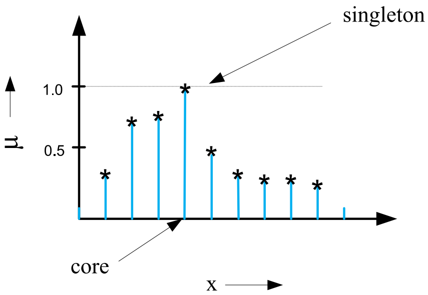
Fuzzy terminologies: Crossover points

Crossover point : A crossover point of a fuzzy set A is a point $x \in X$ at which $\mu_A(x) = 0.5$. That is
Crossover $(A) = \{x | \mu_A(x) = 0.5\}$.



Fuzzy terminologies: Fuzzy Singleton

Fuzzy Singleton : A fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called a fuzzy singleton. That is
|A| = { x | $\mu_A(x) = 1$}.



Fuzzy terminologies: α-cut and strong α-cut

α-cut and strong α-cut :

The α-cut of a fuzzy set A is a crisp set defined by

$$A_\alpha = \{x \mid \mu_A(x) \geq \alpha \}$$

Strong α-cut is defined similarly :

$$A_\alpha' = \{x \mid \mu_A(x) > \alpha \}$$

Note : Support(A) = A_0' and Core(A) = A_1.

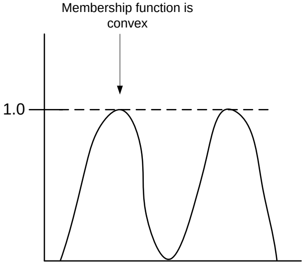
Fuzzy terminologies: Convexity

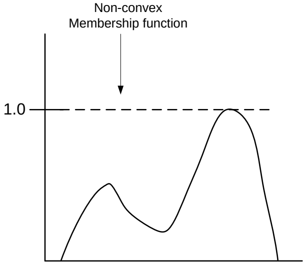
Convexity : A fuzzy set A is convex if and only if for any x_1 and $x_2 \in X$ and any $\lambda \in [0, 1]$

$$\mu_A\ (\lambda x_1 + (1 - \lambda) x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

Note :

- A is convex if all its α- level sets are convex.
- Convexity $(A_\alpha) \implies A_\alpha$ is composed of a single line segment only.





Fuzzy terminologies: Bandwidth

Bandwidth :

For a normal and convex fuzzy set, the bandwidth (or width) is defined as the distance the two unique crossover points:

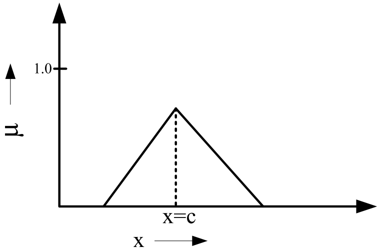
Bandwidth(A) = $\mid x_1 - x_2 \mid$

where $\mu_A(x_1) = \mu_A(x_2) = 0.5$

Fuzzy terminologies: Symmetry

Symmetry :

A fuzzy set *A* is symmetric if its membership function around a certain point $x = c$, namely μ_A(x + c) = μ_A(x - c) for all x $\in$ X.



Fuzzy terminologies: Open and Closed

A fuzzy set A is

Open left

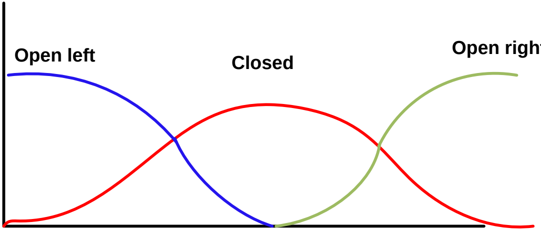
If $\lim_{x\to-\infty} \mu_A(x) = 1$ and $\lim_{x\to+\infty} \mu_A(x) = 0$

Open right:

If $\lim_{x\to-\infty}\mu_A(x) = 0$ and $\lim_{x\to+\infty} \mu_A(x) = 1$

Closed

If : $\lim_{x\to-\infty} \mu_A(x) = \lim_{x\to+\infty} \mu_A(x) = 0$



Fuzzy vs. Probability

Fuzzy : When we say about certainty of a thing

Example: A patient come to the doctor and he has to diagnose so that medicine can be prescribed.
Doctor prescribed a medicine with certainty 60% that the patient is suffering from flue. So, the disease will be cured with certainty of 60% and uncertainty 40%. Here, in stead of flue, other diseases with some other certainties may be.

Probability: When we say about the chance of an event to occur

Example: India will win the T20 tournament with a chance 60% means that out of 100 matches, India own 60 matches.

Prediction vs. Forecasting

The Fuzzy vs. Probability is analogical to Prediction vs. Forecasting

Prediction : When you start guessing about things.

Forecasting : When you take the information from the past job and apply it to new job.

The main difference:

Prediction is based on the best guess from experiences.
Forecasting is based on data you have actually recorded and packed from previous job.