## 8.8 Solved Problems

1. The elements in two sets $A$ and $B$ are given as

$$A = \{2, 4\} \quad \text{and} \quad B = \{a, b, c\}$$

Find the various Cartesian products of these two sets.

**Solution:** The various Cartesian products of these two given sets are

$$A \times B = \{(2, a), (2, b), (2, c), (4, a), (4, b), (4, c)\}$$
$$B \times A = \{(a, 2), (a, 4), (b, 2), (b, 4), (c, 2), (c, 4)\}$$
$$A \times A = A^2 = \{(2, 2), (2, 4), (4, 2), (4, 4)\}$$

$B \times B = B^2 = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$

2. Consider the following two fuzzy sets:

$$\underset{\sim}{A} = \left\{ \frac{0.3}{x_1} + \frac{0.7}{x_2} + \frac{1}{x_3} \right\}$$

$$\text{and } \underset{\sim}{B} = \left\{ \frac{0.4}{y_1} + \frac{0.9}{y_2} \right\}$$

Perform the Cartesian product over these given fuzzy sets.

**Solution:** The fuzzy Cartesian product performed over fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ results in fuzzy relation $\underset{\sim}{R}$ given by $\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B}$. Hence

$$\underset{\sim}{R} = \begin{bmatrix} 0.3 & 0.3 \\ 0.4 & 0.7 \\ 0.4 & 0.9 \end{bmatrix}$$

The calculation for $\underset{\sim}{R}$ is as follows:

$$\begin{aligned}
\mu_{\underset{\sim}{R}}(x_1, y_1) &= \min[\mu_{\underset{\sim}{A}}(x_1), \mu_{\underset{\sim}{B}}(y_1)] \\
&= \min(0.3, 0.4) = 0.3 \\
\mu_{\underset{\sim}{R}}(x_1, y_2) &= \min[\mu_{\underset{\sim}{A}}(x_1), \mu_{\underset{\sim}{B}}(y_2)] \\
&= \min(0.3, 0.9) = 0.3 \\
\mu_{\underset{\sim}{R}}(x_2, y_1) &= \min[\mu_{\underset{\sim}{A}}(x_2), \mu_{\underset{\sim}{B}}(y_2)] \\
&= \min(0.7, 0.4) = 0.4 \\
\mu_{\underset{\sim}{R}}(x_2, y_2) &= \min[\mu_{\underset{\sim}{A}}(x_2), \mu_{\underset{\sim}{B}}(y_2)] \\
&= \min(0.7, 0.9) = 0.7 \\
\mu_{\underset{\sim}{R}}(x_3, y_1) &= \min[\mu_{\underset{\sim}{A}}(x_3), \mu_{\underset{\sim}{B}}(y_1)] \\
&= \min(1, 0.4) = 0.4 \\
\mu_{\underset{\sim}{R}}(x_3, y_2) &= \min[\mu_{\underset{\sim}{A}}(x_3), \mu_{\underset{\sim}{B}}(y_2)] \\
&= \min(1, 0.9) = 0.9
\end{aligned}$$

Thus, the Cartesian product between fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are obtained.

3. Two fuzzy relations are given by

$$\underset{\sim}{R} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

$$\text{and } \underset{\sim}{S} = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain fuzzy relation $\underset{\sim}{T}$ as a composition between the fuzzy relations.

**Solution:** The composition between two given fuzzy relations is performed in two ways as

(a) Max-min composition
(b) Max-product composition

(a) Max-min composition

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

The calculations for obtaining $\underset{\sim}{T}$ are as follows:

$$\begin{aligned}
\mu_{\underset{\sim}{T}}(x_1, z_1) &= \max\{\min[\mu_{\underset{\sim}{R}}(x_1, y_1), \mu_{\underset{\sim}{S}}(y_1, z_1)], \\
&\qquad \min[\mu_{\underset{\sim}{R}}(x_1, y_2), \mu_{\underset{\sim}{S}}(y_2, z_1)]\} \\
&= \max[\min(0.6, 1), \min(0.3, 0.8)] \\
&= \max(0.6, 0.3) = 0.6 \\
\mu_{\underset{\sim}{T}}(x_1, z_2) &= \max[\min(0.6, 0.5), \min(0.3, 0.4)] \\
&= \max(0.5, 0.3) = 0.5 \\
\mu_{\underset{\sim}{T}}(x_1, z_3) &= \max[\min(0.6, 0.3), \min(0.3, 0.7)] \\
&= \max(0.3, 0.3) = 0.3 \\
\mu_{\underset{\sim}{T}}(x_2, z_1) &= \max[\min(0.2, 1), \min(0.9, 0.8)] \\
&= \max(0.2, 0.8) = 0.8 \\
\mu_{\underset{\sim}{T}}(x_2, z_2) &= \max[\min(0.2, 0.5), \min(0.9, 0.4)] \\
&= \max(0.2, 0.4) = 0.4 \\
\mu_{\underset{\sim}{T}}(x_2, z_3) &= \max[\min(0.2, 0.3), \min(0.9, 0.7)] \\
&= \max(0.2, 0.7) = 0.7
\end{aligned}$$

(b) Max-product composition

$$\underset{\sim}{T} = \underset{\sim}{R} \bullet \underset{\sim}{S}$$

Calculations for $\underset{\sim}{T}$ are as follows:

$$\begin{aligned}
\mu_{\underset{\sim}{T}}(x_1, z_1) &= \max\{[\mu_{\underset{\sim}{R}}(x_1, y_1) \bullet \mu_{\underset{\sim}{S}}(y_1, z_1)], \\
&\qquad [\mu_{\underset{\sim}{R}}(x_1, y_2) \bullet \mu_{\underset{\sim}{S}}(y_2, z_1)]\} \\
&= \max(0.6, 0.24) = 0.6
\end{aligned}$$

$$\mu_{\underset{\sim}{T}}(x_1, z_2) = \max[(0.6 \times 0.5), (0.3 \times 0.4)]$$
$$= \max(0.3, 0.12) = 0.3$$
$$\mu_{\underset{\sim}{T}}(x_1, z_3) = \max[(0.6 \times 0.3), (0.3 \times 0.7)]$$
$$= \max(0.18, 0.21) = 0.21$$
$$\mu_{\underset{\sim}{T}}(x_2, z_1) = \max[(0.2 \times 1), (0.9 \times 0.8)]$$
$$= \max(0.2, 0.72) = 0.72$$
$$\mu_{\underset{\sim}{T}}(x_2, z_2) = \max[(0.2 \times 0.5), (0.9 \times 0.4)]$$
$$= \max(0.1, 0.36) = 0.36$$
$$\mu_{\underset{\sim}{T}}(x_2, z_3) = \max[(0.2 \times 0.3), (0.9 \times 0.7)]$$
$$= \max(0.06, 0.63) = 0.63$$

The fuzzy relation $\underset{\sim}{T}$ by max-product composition is given as

$$\underset{\sim}{T} = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.3 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix} \end{array}$$

4. For a speed control of DC motor, the membership functions of series resistance, armature current and speed are given as follows:

$$\underset{\sim}{R_{se}} = \left\{ \frac{0.4}{30} + \frac{0.6}{60} + \frac{1.0}{100} + \frac{0.1}{120} \right\}$$

$$\underset{\sim}{I_a} = \left\{ \frac{0.2}{20} + \frac{0.3}{40} + \frac{0.6}{60} + \frac{0.8}{80} + \frac{1.0}{100} + \frac{0.2}{120} \right\}$$

$$\underset{\sim}{N} = \left\{ \frac{0.35}{500} + \frac{0.67}{1000} + \frac{0.97}{1500} + \frac{0.25}{1800} \right\}$$

Compute relation $\underset{\sim}{T}$ for relating series resistance to motor speed, i.e., $\underset{\sim}{R_{se}}$ to $\underset{\sim}{N}$. Perform max-min composition only.

**Solution:** For relating series resistance to motor-speed, i.e., $R_{se}$ to $\underset{\sim}{N}$, we have to perform the following operations – two fuzzy cross-products and one fuzzy composition (max-min):

$$\underset{\sim}{R} = \underset{\sim}{R_{se}} \times \underset{\sim}{I_a}$$
$$\underset{\sim}{S} = \underset{\sim}{I_a} \times \underset{\sim}{N}$$
$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S}$$

Relation $\underset{\sim}{R}$ is obtained as the Cartesian product of $\underset{\sim}{R_{se}}$ and $\underset{\sim}{I_a}$, i.e.,

$$\underset{\sim}{R} = \underset{\sim}{R_{se}} \times \underset{\sim}{I_a}$$

$$= \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array}\begin{array}{c} \begin{array}{cccccc} 20 & 40 & 60 & 80 & 100 & 120 \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0.4 & 0.4 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.6 & 0.6 & 0.6 & 0.2 \\ 0.2 & 0.3 & 0.6 & 0.8 & 1.0 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

Relation $\underset{\sim}{S}$ is obtained as the Cartesian product of $\underset{\sim}{I_a}$ and $\underset{\sim}{N}$, i.e.,

$$\underset{\sim}{S} = \underset{\sim}{I_a} \times \underset{\sim}{N} = \begin{array}{c} \\ 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \end{array}\begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 & 0.25 \\ 0.35 & 0.6 & 0.6 & 0.25 \\ 0.35 & 0.67 & 0.8 & 0.25 \\ 0.35 & 0.67 & 0.97 & 0.25 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

Relation $\underset{\sim}{T}$ is obtained as the composition between relations $\underset{\sim}{R}$ and $\underset{\sim}{S}$, i.e.,

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array}\begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} 0.35 & 0.4 & 0.4 & 0.25 \\ 0.35 & 0.6 & 0.6 & 0.25 \\ 0.35 & 0.67 & 0.97 & 0.25 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

5. Consider two fuzzy sets given by

$$\underset{\sim}{A} = \left\{ \frac{1}{\text{low}} + \frac{0.2}{\text{medium}} + \frac{0.5}{\text{high}} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0.9}{\text{positive}} + \frac{0.4}{\text{zero}} + \frac{0.9}{\text{negative}} \right\}$$

(a) Find the fuzzy relation for the Cartesian product of $\underset{\sim}{A}$ and $\underset{\sim}{B}$, i.e., $\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B}$.

(b) Introduce a fuzzy set $\underset{\sim}{C}$ given by

$$\underset{\sim}{C} = \left\{ \frac{0.1}{\text{low}} + \frac{0.2}{\text{medium}} + \frac{0.7}{\text{high}} \right\}$$

(b) Find the relation between $\underset{\sim}{C}$ and $\underset{\sim}{B}$ using Cartesian product, i.e., find $\underset{\sim}{S} = \underset{\sim}{C} \times \underset{\sim}{B}$.

(c) Find $\underset{\sim}{C} \circ \underset{\sim}{R}$ using max-min composition.

(d) Find $\underset{\sim}{C} \circ \underset{\sim}{S}$ using max-min composition.

**Solution:**

(a) The Cartesian product between $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is obtained as

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)]$$

$$= \begin{array}{c} \\ \text{low} \\ \text{medium} \\ \text{high} \end{array} \begin{array}{c} \begin{array}{ccc} \text{positive} & \text{zero} & \text{negative} \end{array} \\ \begin{bmatrix} 0.9 & 0.4 & 0.9 \\ 0.2 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.5 \end{bmatrix} \end{array}$$

(b) The new fuzzy set is

$$\underset{\sim}{C} = \left\{ \frac{0.1}{\text{low}} + \frac{0.2}{\text{medium}} + \frac{0.7}{\text{high}} \right\}$$

The Cartesian product between $\underset{\sim}{C}$ and $\underset{\sim}{B}$ is obtained as

$$\underset{\sim}{S} = \underset{\sim}{C} \times \underset{\sim}{B} = \min[\mu_{\underset{\sim}{C}}(x), \mu_{\underset{\sim}{B}}(y)]$$

$$= \begin{array}{c} \\ \text{low} \\ \text{medium} \\ \text{high} \end{array} \begin{array}{c} \begin{array}{ccc} \text{positive} & \text{zero} & \text{negative} \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.7 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

(c)

$$\underset{\sim}{C} \circ \underset{\sim}{R} = [0.1 \;\; 0.2 \;\; 0.7]_{1\times 3} \begin{bmatrix} 0.9 & 0.4 & 0.9 \\ 0.2 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.5 \end{bmatrix}_{3\times 3}$$

$$= [0.5 \;\; 0.4 \;\; 0.5]$$

For instance,

$$\begin{aligned} \mu_{\underset{\sim}{C} \circ \underset{\sim}{R}}(x_1, y_1) &= \max[\min(0.1, 0.9), \min(0.2, 0.2), \\ &\qquad \min(0.7, 0.5)] \\ &= \max(0.1, 0.2, 0.5) = 0.5 \end{aligned}$$

(d)

$$\underset{\sim}{C} \circ \underset{\sim}{S} = [0.1 \;\; 0.2 \;\; 0.7]_{1\times 3} \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.7 & 0.4 & 0.7 \end{bmatrix}_{3\times 3}$$

$$= [0.7 \;\; 0.4 \;\; 0.7]$$

Hence max-min composition was used to find the relations.

6. Consider a universe of aircraft speed near the speed of sound as $X = \{0.72, 0.725, 0.75, 0.775, 0.78\}$ and a fuzzy set on this universe for the speed "near mach 0.75" $= \underset{\sim}{M}$ where

$$\underset{\sim}{M} = \left\{ \frac{0}{0.72} + \frac{0.8}{0.725} + \frac{1}{0.75} + \frac{0.8}{0.775} + \frac{0}{0.78} \right\}$$

Define a universe of altitudes as $Y = \{21, 22, 23, 24, 25, 26, 27\}$ in $k$-feet and a fuzzy set on this universe for the altitude fuzzy set "approximately 24,000 feet" $= \underset{\sim}{N}$ where

$$\underset{\sim}{N} = \left\{ \frac{0}{21k} + \frac{0.2}{22k} + \frac{0.7}{23k} + \frac{1}{24k} + \frac{0.7}{25k} + \frac{0.2}{26k} + \frac{0}{27k} \right\}$$

(a) Construct a relation $\underset{\sim}{R} = \underset{\sim}{M} \times \underset{\sim}{N}$.

(b) For another aircraft speed, say $\underset{\sim}{M_1}$, in the region of mach 0.75 where

$$\underset{\sim}{M_1} = \left\{ \frac{0}{0.72} + \frac{0.8}{0.725} + \frac{1}{0.75} + \frac{0.6}{0.775} + \frac{0}{0.78} \right\}$$

find relation $\underset{\sim}{S} = \underset{\sim}{M_1} \circ \underset{\sim}{R}$ using max-min composition.

**Solution:** The two given fuzzy sets are

$$\underset{\sim}{M} = \left\{ \frac{0}{0.72} + \frac{0.8}{0.725} + \frac{1}{0.75} + \frac{0.8}{0.775} + \frac{0}{0.78} \right\}$$

$$\underset{\sim}{N} = \left\{ \frac{0}{21k} + \frac{0.2}{22k} + \frac{0.7}{23k} + \frac{1}{24k} + \frac{0.7}{25k} + \frac{0.2}{26k} + \frac{0}{27k} \right\}$$

(a) Relation $\underset{\sim}{R} = \underset{\sim}{M} \times \underset{\sim}{N}$ is obtained by using Cartesian product

$$\underset{\sim}{R} = \min[\mu_{\underset{\sim}{M}}(x), \mu_{\underset{\sim}{N}}(y)]$$

$$= \begin{array}{c} \\ 0.72 \\ 0.725 \\ 0.75 \\ 0.775 \\ 0.78 \end{array} \begin{array}{c} \begin{array}{ccccccc} 21k & 22k & 23k & 24k & 25k & 26k & 27k \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.7 & 0.8 & 0.7 & 0.2 & 0 \\ 0 & 0.2 & 0.7 & 1 & 0.7 & 0.2 & 0 \\ 0 & 0.2 & 0.7 & 0.8 & 0.7 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b) Relation $\underset{\sim}{S} = \underset{\sim}{M}_1 \circ \underset{\sim}{R}$ is found by using max-min composition

$$\underset{\sim}{S} = \max\{\min[\mu_{\underset{\sim}{M}}(x), \mu_{\underset{\sim}{R}}(x,y)]\}$$

$$= \begin{bmatrix} 0 & 0.8 & 1 & 0.6 & 0 \end{bmatrix}_{1\times 5}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.7 & 0.8 & 0.7 & 0.2 & 0 \\ 0 & 0.2 & 0.7 & 1 & 0.7 & 0.2 & 0 \\ 0 & 0.2 & 0.7 & 0.8 & 0.7 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5\times 7}$$

$$\underset{\sim}{S} = \begin{bmatrix} 0 & 0.2 & 0.7 & 1 & 0.7 & 0.2 & 0 \end{bmatrix}_{1\times 7}$$

7. Consider two relations

$$\underset{\sim}{R} = \begin{array}{c} \\ 9 \\ 18 \\ 27 \\ 36 \end{array} \begin{array}{c} \begin{array}{ccccc} -100 & -50 & 0 & 50 & 100 \end{array} \\ \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1 & 0.8 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

and

$$\underset{\sim}{S} = \begin{array}{c} \\ -100 \\ -50 \\ 0 \\ 50 \\ 100 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 4 & 8 & 16 & 20 \end{array} \\ \begin{bmatrix} 1 & 0.8 & 0.6 & 0.3 & 0.1 \\ 0.7 & 1 & 0.7 & 0.5 & 0.4 \\ 0.5 & 0.6 & 1 & 0.8 & 0.8 \\ 0.3 & 0.4 & 0.6 & 1 & 0.9 \\ 0.9 & 0.3 & 0.5 & 0.7 & 1 \end{bmatrix} \end{array}$$

If $\underset{\sim}{R}$ is a relationship between frequency and temperature and $\underset{\sim}{S}$ represents a relation between temperature and reliability index of a circuit, obtain the relation between frequency and reliability index using (a) max-min composition and (b) max-product composition.

**Solution:**

(a) Max-min composition is performed as follows.

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \max\{\min[\mu_{\underset{\sim}{R}}(x,y), \mu_{\underset{\sim}{S}}(x,y)]\}$$

$$= \begin{array}{c} \\ 9 \\ 18 \\ 27 \\ 36 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 4 & 8 & 16 & 20 \end{array} \\ \begin{bmatrix} 0.9 & 0.6 & 0.7 & 1 & 0.9 \\ 0.8 & 0.6 & 0.7 & 1 & 0.9 \\ 0.6 & 0.6 & 0.8 & 0.9 & 0.9 \\ 0.9 & 1 & 0.8 & 0.8 & 0.8 \end{bmatrix} \end{array}$$

(b) Max-product composition is performed as follows.

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \max\{\min[\mu_{\underset{\sim}{R}}(x,y) \times \mu_{\underset{\sim}{S}}(x,y)]\}$$

$$= \begin{array}{c} \\ 9 \\ 18 \\ 27 \\ 36 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 4 & 8 & 16 & 20 \end{array} \\ \begin{bmatrix} 0.81 & 0.5 & 0.7 & 1.0 & 0.9 \\ 0.72 & 0.5 & 0.7 & 1.0 & 0.9 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.81 \\ 0.9 & 1.0 & 0.8 & 0.64 & 0.64 \end{bmatrix} \end{array}$$

Thus the relation between frequency and reliability index has been found using composition techniques.

8. Three fuzzy sets are given as follows:

$$\underset{\sim}{P} = \left\{\frac{0.1}{2} + \frac{0.3}{4} + \frac{0.7}{6} + \frac{0.4}{8} + \frac{0.2}{10}\right\}$$

$$\underset{\sim}{Q} = \left\{\frac{0.1}{0.1} + \frac{0.3}{0.2} + \frac{0.3}{0.3} + \frac{0.4}{0.4} + \frac{0.5}{0.5} + \frac{0.2}{0.6}\right\}$$

$$\underset{\sim}{T} = \left\{\frac{0.1}{0} + \frac{0.7}{0.5} + \frac{0.3}{1}\right\}$$

The following operations are performed over the fuzzy sets:

(a) $\underset{\sim}{R} = \underset{\sim}{P} \times \underset{\sim}{Q} = \min[\mu_{\underset{\sim}{P}}(x), \mu_{\underset{\sim}{Q}}(y)]$

$$= \begin{array}{cc} & \begin{array}{cccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 \end{array} \\ \begin{array}{c} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{array} & \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.4 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.4 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

(b) $\underset{\sim}{S} = \underset{\sim}{Q} \times \underset{\sim}{T} = \min[\mu_{\underset{\sim}{Q}}(x), \mu_{\underset{\sim}{T}}(y)]$

$$= \begin{array}{cc} & \begin{array}{ccc} 0 & 0.5 & 1 \end{array} \\ \begin{array}{c} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \end{array} & \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.4 & 0.3 \\ 0.1 & 0.5 & 0.3 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

(c) $\underset{\sim}{M} = \underset{\sim}{R} \circ \underset{\sim}{S} = \max\{\min[\mu_{\underset{\sim}{R}}(x,y), \mu_{\underset{\sim}{S}}(x,y)]\}$

$$= \begin{array}{cc} & \begin{array}{ccc} 0 & 0.5 & 1 \end{array} \\ \begin{array}{c} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{array} & \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.5 & 0.3 \\ 0.1 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

(d) $\underset{\sim}{M} = \underset{\sim}{R} \circ \underset{\sim}{S} = \max[\mu_{\underset{\sim}{R}}(x,y) \times \mu_{\underset{\sim}{S}}(x,y)]$

$$= \begin{array}{cc} & \begin{array}{ccc} 0 & 0.5 & 1 \end{array} \\ \begin{array}{c} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{array} & \begin{bmatrix} 0.01 & 0.05 & 0.03 \\ 0.03 & 0.05 & 0.09 \\ 0.05 & 0.25 & 0.15 \\ 0.04 & 0.20 & 0.12 \\ 0.02 & 0.0 & 0.06 \end{bmatrix} \end{array}$$

Thus the operations were performed over the given fuzzy sets.

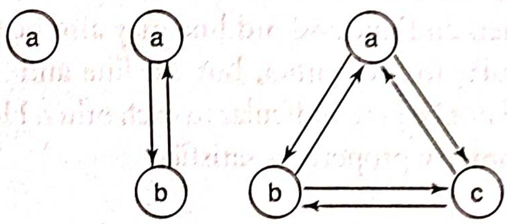
9. Which of the following are equivalence relations?

| No. | Set | Relation on the set |
|---|---|---|
| (i) | People | is the brother of |
| (ii) | People | has the same parents as |
| (iii) | Points on a map | is connected by a road to |
| (iv) | Lines in plane geometry | is perpendicular to |
| (v) | Positive integers | for some integer $k$, equals $10^k$ times |

Draw graphs of the equivalence relations.

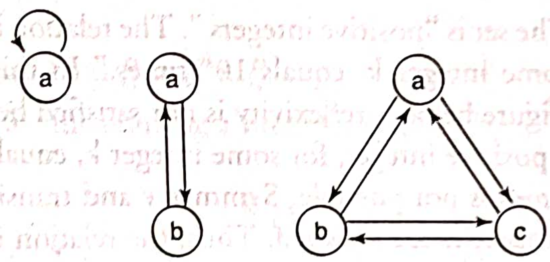
**Solution:**

(a) The set is people. The relation of the set "is the brother of." The relation (figure below) is not equivalence relation because people considered cannot be brothers to themselves. So, reflexive property is not satisfied. But symmetry and transitive properties are satisfied.




The figure illustrates that the relation is not an equivalence relation.

(b) The set is people. The relation is "has the same parents as." In this case (figure below), all the three properties are satisfied, hence it is an equivalence relation.




Thus the relation is an equivalence relation.

(c) The set is "points on a map." The relation is "is connected by a road to." This relation (figure on next page) is not an equivalence relation because the transitive property is not satisfied. The road may connect 1st point and 2nd point; 2nd point