CHAPTER 7

Testing Hypothesis
One Sample Tests

In Chapter 6, we have estimated the population parameters, such as mean and standard deviation using central limit theorem. In this chapter, we use central limit theorem to seek answers for questions such as:

(a) whether quality has improved with the introduction of a new process in comparison with the old process
(b) the turnover of an industrial sector has improved over the last year
(c) the returns from large cement companies are larger than the returns from small cement companies.

These questions reduce to the following general questions:

1. whether a given sample comes from some known population
2. whether two given samples came from the same population.

We answer Question 1 by a test of significance applied on a large sample when σ is known or not known. We do the same thing for a small population using t-distributions. We answer Question 2 in Chapter 8.

Interval estimation and testing hypothesis have several things in common. How an answer to interval estimation leads to an answer to test hypothesis is also discussed in this chapter. The chapter ends with a discussion on the probabilities of two types of errors we commit while testing a hypothesis.

7.1 WHAT IS A HYPOTHESIS?

We come across statements:

(a) in day-to-day life (in arguments with friends, political platforms, TV panel discussions, etc.)

(b) made by scientists
(c) by business analysts, management consultants in business research that are useful as a guide to managerial decisions.

In day-to-day life or political platforms, statements are made and established by convincing arguments or by raising the pitch of one's voice. We can call these statements *propositions*.

In general, **propositions** are statements over observable phenomena and judged as true or false by others.

The scientists use *deduction* (a process of proving a general statement using facts and valid arguments—a valid argument is one which follows rules of logic) or *induction* (proving a general statement from specific situations). They also use both the processes—going from *general to the specific* or from *specific to general*. These are also propositions.

The process of deduction and induction can be illustrated in our context using an example. An MBA student learns various theories and also discuss case studies. When he applies a theoretical concept to a business problem, he uses 'deduction'. He goes from a general principle applicable to any industry to a particular company. When he studies a case he studies the salient features of a particular company (discussed in the case) and applies it to the industry in general or some other company later. In this case, he moves from a specific company to an industry. This is similar to 'induction'.

In business research, the business analysts and management consultants make propositions related to a problem a company is facing or an opportunity the company can exploit to its advantage. They do not stop with propositions; they test it and present it to the company. In this case, the proposition is called a *hypothesis* (see Box 7.1).

Box 7.1

A **hypothesis** is a proposition formulated for empirical testing.

By **empirical testing** we mean relying on practical experience rather than theories.

The business analysts use a recent sample for testing a hypothesis. We are interested only in a hypothesis formulated by business analysts.

7.1.1 Types of Hypotheses

There are three types of hypotheses. They are: research hypothesis, statistical hypothesis and substantive hypothesis.

Research hypothesis

A research hypothesis is any hypothesis framed by a business researcher in order to study relationship between variables, effectiveness of a process, etc. It can be *descriptive*—stating the distribution of some variable. For example, listing causes for discontent among workers is a descriptive hypothesis. It can be relational—describing the relationship between two variables. For example, study of relationship between budget on R&D and sales is relational. It can be *correlational*—one variable being cause of another. For example, study of impact of spending an advertisement on sales is correlational.

Statistical hypothesis

A statistical hypothesis is a hypothesis regarding a population. We can attach a level of precision for the test of a statistical hypothesis. All hypotheses we are going to test in this chapter and later chapters are statistical.

Substantive hypothesis

The outcome of a substantive hypothesis gives a result that is useful to the decision-maker. For example, a rise of the market share from 25% to 25.3% may be indicated by the testing of a statistical hypothesis. When a mere 0.3% rise in market share is not useful to the decision-maker, the statistical hypothesis is not substantive. When the market shares of the company and its nearest competitor are very close, even a 0.3% rise in market share is useful to the decision-maker. In this case the hypothesis is substantive.

7.2 TESTING HYPOTHESIS

In this section, we give the procedure for testing a statistical hypothesis.

In hypothesis testing, we test a hypothesis involving population parameter using sample information. Broadly, hypotheses are tested on three occasions. We test:

1. research hypothesis (Example 7.2 illustrates this)
2. a claim made by an advertisement (Example 7.3 illustrates this)
3. under decision-making situations (Example 7.1 illustrates this)

7.2.1 Brief Sketch of Hypothesis Testing Procedure

We formulate the hypothesis to be tested (actually, two hypotheses are formulated) and fix the level of significance (level of precision associated with the test). A suitable statistic is chosen. It is calculated for the sample data and the calculated value is used to make a decision regarding the hypothesis with the help of a decision rule.

Before proceeding further, we give Examples 7.1–7.3.

EXAMPLE 7.1 A readymade shop selling men's garments outsources stitching of shirts to a local supplier. The supplier is known to supply shirts of size 42 and the distribution of the sizes of shirts with lable 42 has a standard deviation of 1.5 inches. The readymade shop is very particular about correct size of shirts and accepts/rejects consignments after examining 30 shirts from the consignment.

EXAMPLE 7.2 The Speciality Retail Industry had a very good year to year EPS net growth of 36% in $Q3$ (third quarter) of 2013. There is a feeling that the growth in $Q4$ (fourth quarter) of 2013 will not be that much good. In order to test this, an analyst examines the year to year EPS growth of 30 companies in $Q4$ of 2013.

EXAMPLE 7.3 Centuro is a model introduced by Mahindra & Mahindra claiming a mileage of 85.4 km/litre. A college student wants to test the company's claim, borrows a Centuro from his friends, makes 10 rides and calculates the average mileage. (He has to use weighted average.)

We will come back to these examples while going through the steps of the testing procedure. We will deal with tests on population mean (μ) and then tests on other parameters. As these tests are for testing hypotheses about population parameters, these tests are called *parametric tests*. In Chapter 8, we deal with parametric tests involving two populations. The entire discussion in this section is regarding tests on population mean.

7.2.2 Framing the Hypothesis

Life is not as simple as answering a question with YES/NO options. Even when we say YES, more often we say it with some reservations (advertisements add 'conditions apply'; under 'conditions apply' you will be denied the promised gift). When we say NO, we say it with a reluctance. In testing hypothesis also we have a similar situation. We formulate two hypotheses while testing. They are: *null hypothesis* (H_0) and *alternate hypothesis* (H_1).

Null hypothesis (H_0) and alternate hypothesis (H_1)

Usually, a null hypothesis is formulated in such a way that it is one we want to reject it.

We take some corrective action on the basis of rejection of null hypothesis. Usually, a null hypothesis denotes the continuation of status quo or old method or current practice. So, a corrective action is taken on the rejection of null hypothesis. The negation of null hypothesis is called *alternate hypothesis*. We act on the acceptance of alternate hypothesis. We take H_1 as the hypothesis to be established by the test.

What should we do when we are not rejecting a full hypothesis? The obvious choice is to accept the null hypothesis. But statisticians do not accept the null hypothesis when they are unable to reject it. (Recall, what we said in the beginning.) The reason is historical. Mathematicians accept a proof by deduction as a complete proof, but do not give such weightage to a proof by induction. So, proving a statement is difficult in mathematics, whereas disproving is easier; you simply given an example where the statement fails. (It is similar to the following: Passing MBA examinations and getting a degree is not easy but failing MBA examinations is easier; fail in only one subject.) In the same way, statisticians never accept that null hypothesis can be proved on the basis of one sample (method of induction). That is why they call it null hypothesis. We have a similar situation in our legal system. We will discuss it under Type I and Type II errors. Earlier statisticians never said they accepted H_0. Instead, they said they did not reject H_0. They had no reservations in accepting H_1 while they rejected H_0.

Nowadays, the term Accept H_0 is treated as synonymous with 'Do not reject H_0'. However, no action is initiated on the acceptance of H_0 (except in decision-making situations).

7.2.3 Significance Level

The significance level is used to define the level of precision we expect from the test.

We resort to a hypothesis test when we see a noticeable deviation of the sample mean from the population mean. In this case, we say that the difference $\overline{x} - \mu$ has statistical significance if we have sufficient reasons to believe that the difference is not due to random sample fluctuations alone. As we associate a significance level with a test, a test of hypothesis is also called a *test of significance* (we have confidence level in interval estimation and level of significance in a test of significance).

Type I and Type II errors

In a test, we have two types of errors: *Type I error* and *Type II error*.

When we reject H_0 when H_0 is true, we commit a Type I error.

When we accept H_0 when H_0 is false, we commit a Type II error. Let us represent these two types of errors in Table 7.1.

Table 7.1 Type I and Type II Errors

Conclusion	Population condition	
	H_0 true	H_0 false
Accept H_0	Correct decision	Type II error
Reject H_0	Type I error	Correct decision

The significance level α is the probability of committing a Type I error.
β is the probability of committing a type II error. We record these in Box 7.2.

Box 7.2

Level of significance: $\alpha = P(H_0 \text{ is rejected} | H_0 \text{ is true})$
$\beta = P(H_0 \text{ is accepted} | H_0 \text{ is false})$.

When the significance level is α, the level of precision is $1-\alpha$. (This corresponds to confidence level in estimation.)

Ideally, we would like to reduce both types of errors. But there are some problems in reducing both simultaneously. We will come back to this problem after going through Example 7.4.

EXAMPLE 7.4 Let us illustrate our legal process in punishing a person charged with a murder as a test of significance. We formulate H_0 and H_1 as follows:

H_0 : The accused in innocent H_1 : The accused is not innocent (guilty)

We can draw Table 7.2 which is similar to Table 7.1.

Table 7.2 Errors in Judgement of a Case

Action	States of nature	
	Innocent	Guilty
Acquit	Correct decision	Type II error
Punish	Type I error	Correct decision

The Indian legal system acts on the following principle: It does not matter if hundred criminals are acquitted; one innocent person should not be punished wrongly. In other words, courts strive to avoid Type I error; they release many accused persons giving them the benefit of doubt. This is beneficial to the accused. But this leads to an increase in Type II errors which seem to be detrimental to the society at large. The justification made by the legal system is as follows. If an innocent is punished, two errors occur—the error of punishing the innocent and acquitting the real guilty. On the other hand, if a culprit is acquitted wrongly, only one error occurs—that of releasing the culprit.

Let us go back to our discussion on types of errors. Example 7.4 deals with the difficulty in controlling both errors simultaneously. The same problem arises in a test of significance. If the probability of Type I error is reduced then the probability of Type II error increases. In the case of legal system both types of errors can be controlled by increasing the number of witnesses, witnesses allowed to tell the truth without fear and with clarity so that the defence lawyers are unable to break the witnesses of the prosecution, etc. In the case of tests of significance a larger sample will solve the problem. We will discuss this once again in Section 7.8.

Choice of significance level

Let us consider the problem of deciding the significance level α. We naturally feel that a lower value for α is good since it reduces the probability of Type I error. But we need to tolerate a higher value for β if we choose a lower value for α and vice versa. Because of the trade off between these two errors, we have to choose α dpending on the situation and the cost or penalty attached to both types of errors. By increasing the sample size, we can control both types of errors, but increase in the cost of data collection creeps in.

Let us consider the effect of Type I and Type II errors in the case of manufacturing. Let us define H_0 by

H_0 : The item is good

Then, Type I error is committed when a good item is rejected by the consumer and Type II error is committed when a bad item is accepted by the consumer. The significance level α is called a *producer's risk* since a good item is 'wasted' and the producer suffers. The probability of Type II error β is called a *consumer's risk* since he is accepting a bad item and this is a loss to him.

Also, a manufacturer may prefer a lower α in same situations and a lower β in some other situations. A pharmaceutical company will prefer a lower α for life-saving drugs and a lower β for OTC drugs (over the counter drugs). A TV company may prefer a lower β since it can satisfy the customers with a warranty for a longer period; this is possible when the cost of warranty is lower than cost of higher quality control.

Usually, a significance level of 0.05 is chosen. In case where type I error involves more cost or penalty we can take $\alpha = 0.01$.

Before proceeding further let us apply these concepts of H_0, H_1 and α to Examples 7.1–7.3.

EXAMPLE 7.5 Formulate the null hypothesis, alternate hypothesis and choose α for (a) Example 7.1 (b) Example 7.2 (c) Example 7.3.

Solution: The readymade shop requires shirts of size 42 and wants to test whether there is significant difference between the exact size required and sizes of shirts received from the local supplier.

Let X denote the size of shirts supplied by the local supplier. As the exact size is 42 inches, we can take the population mean to be 42. We define the null hypothesis by:

$$H_0 : \mu = 42.$$

As the readymade shop rejects shirts which are oversized or undersized, the hypothesis to be tested is $\mu \neq 42$. So, we define the alternate hypothesis by:

$$H_1 : \mu \neq 42.$$

We can take α to be 0.05.

(b) The Speciality Retail Industry has a year to year EPS net growth of 36% in $Q3$. We want to test whether the growth has decreased in $Q4$. As the average growth of all companies in Speciality Retail Industry in $Q3$ is 36, we can define the null hypothesis by:

$$H_0 : \mu = 36.$$

As we are testing for decrease in growth. The alternate hypothesis can be taken as:

$$H_1 : \mu < 36.$$

We take α to be 0.05

(c) The mileage given by Centuro is 85.4 km/litre as per the claim of Mahindra & Mahindra Co. The student wants to test the claim of the company regarding mileage. As a consumer he doubts the claim of the company. So, we can define H_0 and H_1 by:

$$H_0 : \mu = 85.4 \quad H_1 : \mu < 85.4$$

H_1 is chosen with the understanding that the student will not buy the vehicle if H_0 is rejected; he may not buy or buy after some more rides if he does not reject the hypothesis.

7.2.4 Two-tailed and One-tailed Tests and Region of Rejection

In Example 7.4, we saw two types of alternate hypotheses:

$$H_1 : \mu \neq 42 \text{ (for Example 7.1)}$$

$H_1 : \mu < 36$ (for Example 7.2) and $H_1 : \mu < 85.4$ (for Example 7.3)

We do not tolerate significant difference on either side of $\mu = 42$ when the alternate hypothesis is $\mu \neq 42$, but ignore significant difference on the right of $\mu = 36$ and $\mu = 85.4$ in the other two cases. (Increase in Year to Year EPS growth is welcome for a company and a better mileage is welcome for the student.) So, we reject H_0 when there is significant difference on either side of $\mu = 42$, i.e., when H_1 is $\mu \neq 42$; we reject H_0 when there is significant difference on the left of $\mu = 36$ and $\mu = 85.4$ alone.

For testing a hypothesis we construct a test statistic t which is obtained by transforming the sampling distribution of $\bar{x}$ into standard normal distribution. Then we divide the distribution of t into a region of rejection and region of acceptance (hardcore statisticians will object to this terminology but the terminology is accepted now).

For alternate hypothesis of the form $\mu \neq \mu_0$, μ_0 being some specified value (μ_0 is called a *hypothesised value*) for population mean, the region of rejection consists of two symmetric tails since significant difference on either side is not tolerated. The test is called a *two-tailed test*.

For an alternate hypothesis of the form $\mu < \mu_0$, the region of rejection is the left tail. The test is called a *one-tailed test* or *left-tailed test* to be more precise. When, H_1 is $\mu > \mu_0$, the region of rejection is the right tail and the test is a right-tailed test.

We define the regions of rejection in the case of two-tailed and one-tailed test as follows:

In the case of a two-tailed test with significance level α, we find a value to satisfy the condition $P(|t| > t_0) = \alpha$ and the region of rejection consists of two tails defined by $t > t_0$ and $t < -t_0$ (see Figure 7.1).

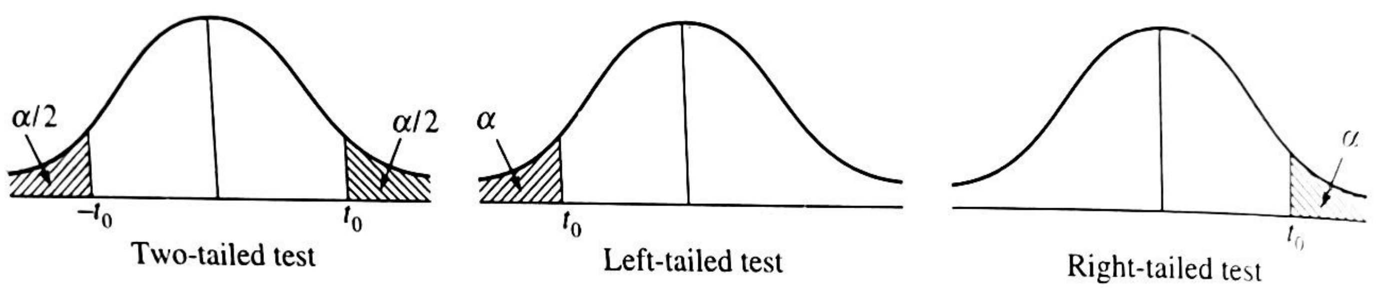

Figure 7.1 Regions of rejection.

Note, that $P(t < -t_0) = P(t > t_0) = \alpha/2$ since the two tails are symmetric. In other words, the area under each tail is $\alpha/2$.

In the case of a left-tailed test, we find to such that $P(t < t_0) = \alpha$; the region of rejection is the left tail defined by $t < t_0$. The area under the left tail is α.

In the case of a right-tailed test, the region of rejection is the right tail defined by $t > t_0$. The area under the tail is α. In the case of a two-tailed test, t_0 and $-t_0$ define the two tails of rejection. In the case of a left-tailed test t_0 (a negative value) defines the left tail and in the case of a right-tailed test t_0 (a positive value) defines the right tail.

t_0 is called the **critical value** in the case of one-tailed tests. $\pm t_0$ are called *critical values* in the case of a two-tailed test.

We summarise our discussion on region of rejection and critical value in Table 7.3.

Table 7.3 Two-tailed and One-tailed Tests

Alternate hypothesis	Nature of test	Calculation of t_0	Region of rejection and critical value
$H_1 : \mu \neq \mu_0$	Two-tailed	$P(\|t\| > t_0) = \alpha$ or $P(t > t_0) = \alpha/2$	Two tails defined by $t < -t_0$, $t > t_0$. $\pm t_0$ are critical values
$H_1 : \mu < \mu_0$	Left-tailed	$P(t < t_0) = \alpha$	Left tail defined by $t < t_0$. t_0 is the critical value
$H_1 : \mu > \mu_0$	Right-tailed	$P(t > t_0) = \alpha$	Right tail defined by $t > t_0$. t_0 is the critical value

7.2.5 Choice of Test Statistic and Critical Value

In Table 7.3, we have seen that the region of rejection consists of two tails in the case of two-tailed test and a single tail in one-tailed test. The tails are defined by the critical value t_0. The calculation of a critical value t_0 depends on the sample statistic and significance level. We will see how this is done.

For testing the population mean, the null hypothesis is taken in the form $H_0 : \mu = \mu_0$. We assume the truth of the null hypothesis for conducting the test. So, the population mean is μ_0.

If $\bar{x}$ denotes the mean of a large sample of size n (i.e., $n \geq 30$) then

$$\boxed{t = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}} \tag{7.1}$$

follows the standard normal distribution [by Box (5.4)]

Let us calculate the value of t_0 for a two-tailed test. If $\alpha = 0.05$, then t_0 is given by $P(t > t_0) = 0.025$ or $P(0 < t \leq t_0) = 0.475$. The z value corresponding to an area of 0.475 is 1.96. So, $\pm$ 1.96 are the critical values and the region of rejection consists of two tails $t < -1.96$ and $t > 1.96$ [see Figure 7.2(a)].

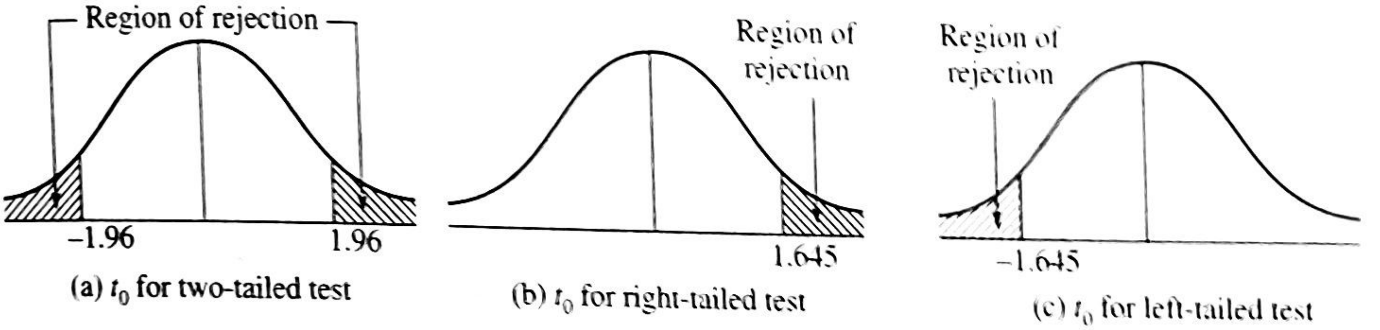

Figure 7.2 Critical values for tests when $\alpha = 0.05$.

In the case of a right-tailed test, t_0 is given by:

$$P(t > t_0) = 0.05 \quad \text{or} \quad P(0 < t \leq t_0) = 0.45$$

The z value corresponding to an area of 0.45 is 1.645. So, the critical value is 1.645 and the region of rejection is $t > 1.645$ [see Figure 7.2(b)].

In the same way, we see that the critical value is $-$ 1.645 for a left-tailed test and the region of rejection is $t < -1.645$ [see Figure 7.2(c)].

We can tabulate the critical values and the regions of rejection for two-tailed and one-tailed tests for different significance levels.

Table 7.4 Critical Values for Two-tailed and One-tailed Tests

Significance level (α)	Two-tailed test	Left-tailed test	Right-tailed test
0.10	$\pm$ 1.645	−1.28	1.28
0.05	$\pm$ 1.96	−1.645	1.645
0.02	$\pm$ 2.33	−2.05	2.05
0.01	$\pm$ 2.575	−2.33	2.33

Rejection rule

We calculate the value of test statistic [given by Eq. (7.1)] for the sample data. This value is called the *calculated sample statistic*.

The rejection rules are:

Reject H_0

(a) if the calculated test statistic is greater than t_0 or less than $-t_0$ (two-tailed test) (see Figure 7.3)

(b) the calculated test statistic is greater than t_0 (right-tailed test) (see Figure 7.3)

(c) the calculated test statistic is less than $-t_0$ (left-tailed test) (see Figure 7.3)

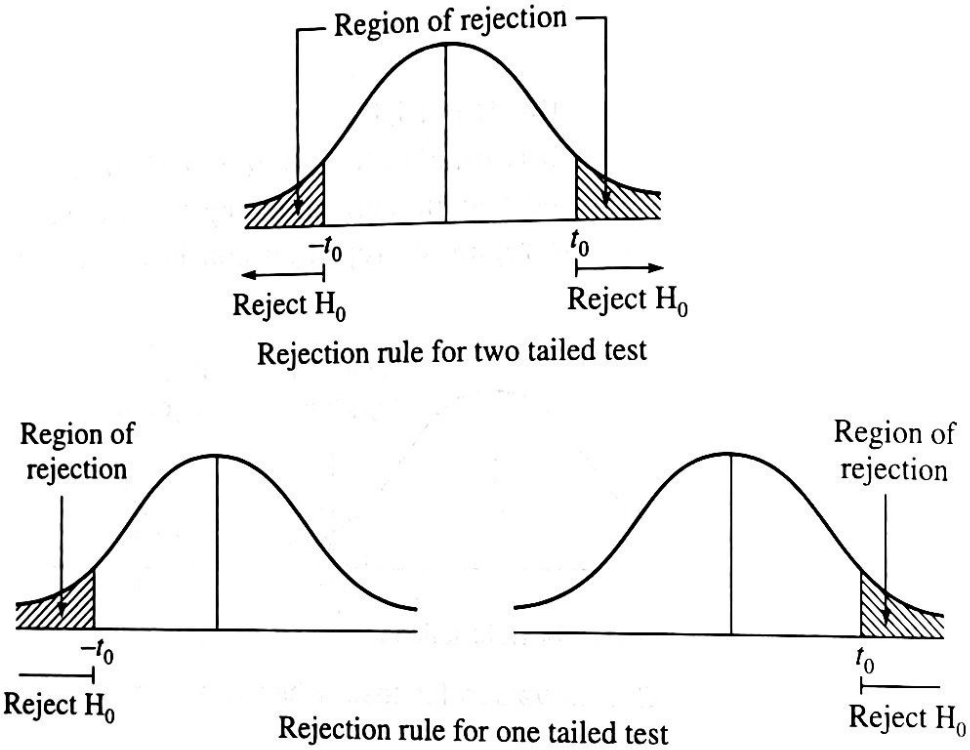

Figure 7.3 Rejection rule for two-tailed and one-tailed tests.

7.2.6 Hypothesis Testing Procedure

Step 1 Develop null hypothesis H_0 and alternative hypothesis H_1. H_1 states what we want to establish by the test.

Step 2 Specify the level of significance α (usually, α is taken as 0.05).

Step 3 Select the test statistic to be used for testing the hypothesis.

Step 4 Use level of significance and H_1 to calculate critical value (s). Find the region of rejection and state the rejection rule (see Figure 7.4).

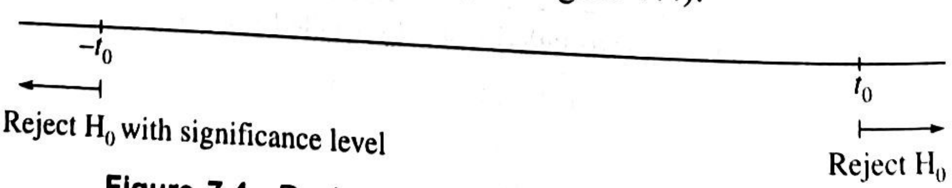

Figure 7.4 Region of rejection for a two-tailed test.

Step 5 Collect sample data and calculate test statistic chosen in Step 3.

Step 6 Use the computed test statistic (got in Step 5) and the rejection rule (Step 4) to determine whether to reject H_0 or not.

Step 7 Use statistical decision (got in Step 6) to make a business decision.

7.2.7 Interpretation of Hypothesis Test

1. The significance level is the probability of Type I error. It also means that it is the proportion of sample means lying in the region of rejection.
2. If $\alpha_1 < \alpha_2$, then the region of rejection with significance level α_2 is a subset of the region of rejection with significance level α_1. In simpler terms, the region of rejection becomes smaller if the significance level is increased (see Figure 7.5).

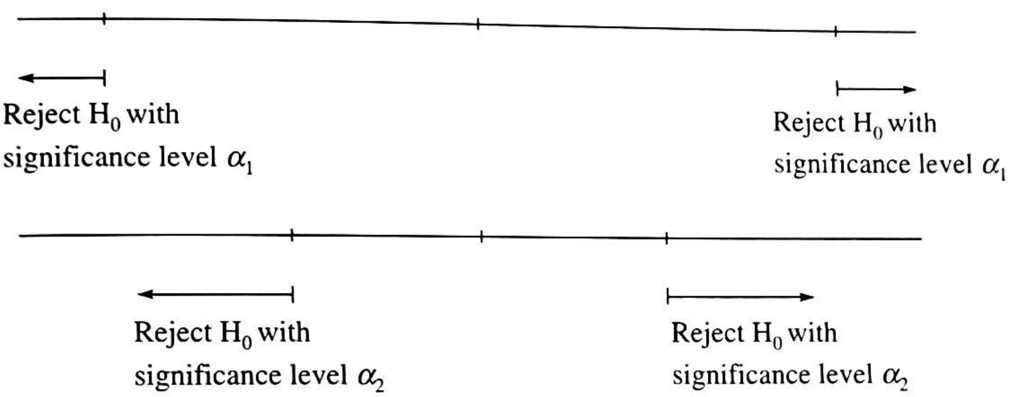

Figure 7.5 Regions of rejection with α_1 and α_2 ($\alpha_1 < \alpha_2$).

3. When we do not reject H_0, it only means that there is no statistical evidence to disprove H_0 and we do not take any action. However, we have to take action even when H_0 is not rejected in tests conducted in decision-making situations. [For example, in exercising quality control, either we reject the lot when H_0 is rejected or we accept the lot when H_0 is not rejected. However, we have to keep an eye on the probability of type II error in the second case (in accepting the lot).]
4. Type I and Type II errors do not occur simultaneously; they are inversely related. (We will see more about this in Section 7.8.)
5. The finite population multiplier should be introduced while calculating the standard error $\left(\frac{\sigma}{\sqrt{n}}\right)$ when $\frac{n}{N} \geq 0.05$ (n and N are the sizes of the sample and population.)

7.3 HYPOTHESIS TESTING OF MEAN WHEN σ IS KNOWN

In this section, we discuss hypothesis testing using large samples when σ is known. According to Eq. 7.1 the test statistic is

$$t = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

This test statistic can be used for small samples also when the population is normal. If σ is not known we can use the estimate s in place of σ for large samples and the same test statistic is used with σ replaced by s. (Some statisticians object to this and insist on using t-distribution in this case irrespective of the sample size; however, the difference between using normal distribution and t-distribution when $n \geq 30$ is small.)

If σ is not known and the sample is small we use the sample statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

which follows t-distribution with n–1 degrees of freedom (see Section 7.5).

EXAMPLE 7.6 According to the data available in relevant websites, the average earnings of an industrial worker in Ambala was ₹ 270 per day in 2013. A sample of 50 workers in Ambala was taken in March 2014. The sample had a mean of ₹ 274.50 with a standard deviation of ₹ 18.20.

(a) Test whether the daily earnings per day has changed in the year 2014 at a significance level of 0.05

(b) Test whether the daily earnings has increased in the year 2014 at a significance level of 0.05

(c) Is there any change in your conclusions [in (a) and (b)]

If so, explain.

Solution: Let X denote the earnings of a worker in Ambala. We are given the following data:

$$\mu_0 = 270 \quad \bar{x} = 274.50 \quad s = 18.20 \quad n = 50$$

(a) We want to test whether the daily earnings have changed in the year 2014. We frame the null hypothesis and alternative hypothesis as follows:

Step 1 $H_0 : \mu = 270 \quad H_1 : \mu \neq 270$

Step 2 $\alpha = 0.05$

Step 3 As $n \geq 30$, the sample statistic is:

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Step 4 As H_1 is $\mu \neq 270$, we perform a two-tailed test. So, the region of rejection consists of two tails [see Figure 7.6].

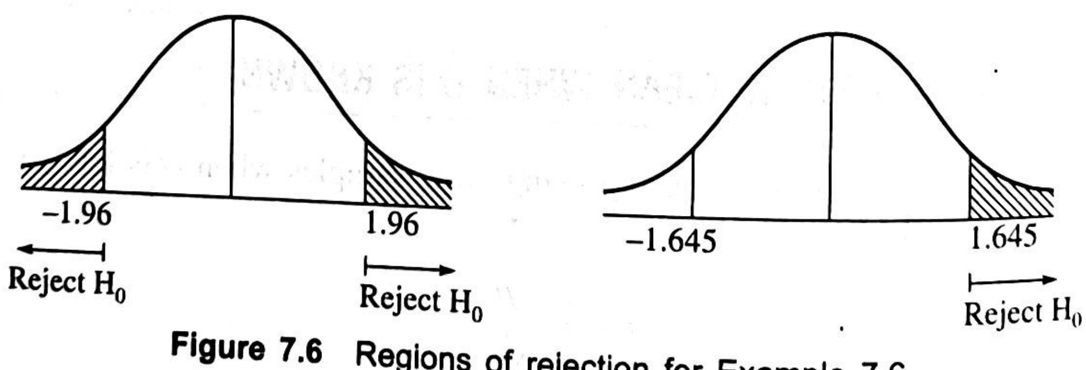

Figure 7.6 Regions of rejection for Example 7.6.

By Table 7.4, the critical values are $\pm$ 1.96

Rejection rule: Reject H_0 if the calculated test statistic is greater than 1.96 or less than -1.96

Step 5 As σ is not given we take the estimate s for σ.

$$\text{Calculated test statistic} = \frac{274.50 - 270}{18.20/\sqrt{50}} = \frac{(4.50)(7.07)}{18.50}$$

$$= 1.719$$

Step 6 As $1.719 < 1.96$, we do not reject H_0. So, there is not enough statistical evidence to conclude that the daily earnings has changed.

Step 7 We cannot take any business decision in this case.

(b) We want to test whether the daily earnings has increased.

So,

Step 1 $H_0 : \mu = 270 \quad H_1 : \mu > 270$

Step 2 $\alpha = 0.05$

Step 3 $t = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}}$ [as in (a)]

Step 4 As H_1 is $\mu > 270$, we perform a right-tailed test and the region of rejection is a right-tail. [See Figure 7.6] and $t_0 = 1.645$.

Rejection rule: Reject H_0 if $t > 1.645$

Step 5 The calculated test statistic = 1.719 [as in (a)]

Step 6 As $1.719 > 1.645$, we reject H_0

Step 7 So, there is enough statistical evidence to conclude that the daily earnings has increased.

(c) As the region of rejection is spread over two tails in a two-tailed test [subdivision (c)], the critical point on the right side is farther than the critical point for a right-tailed test [subdivision (b)].

[t_0 for (a) = $1.96 > t_0$; for (b) = 1.645.] This is the reason for a change in conclusion of (a) and (b). In a two-tailed test an allowance of 2.5% error is given for deviation on either side, whereas the entire allowance of 5% error can be used for deviation on the right side alone in a right-tailed test and the deviation on the left side is not a matter of concern for us. So, we reject H_0 in (b) at the cost of making more error on the right side.

EXAMPLE 7.7 An automatic filling machine has to fill eyedrops with a mean of 10 ml per bottle. Both overfilling and underfilling are not desirable. A quality control inspector takes a sample of 30 bottles in every half an hour in order to decide whether he has to stop operation in the case of overfilling and underfilling beyond a certain level for adjusting the machine. If the sample has a mean of 9.8 ml with a standard deviation of 0.5 ml, give the decision rule for the quality control inspector, taking 0.05 as the significance level.

Solution: We are given the following data:

$$\mu_0 = 10 \quad \bar{x} = 9.8 \quad s = 0.5 \quad n = 30$$

As we are not tolerating deviation on either side of $\mu_0 = 10$, we frame H_0 and H_1 as follows:

Step 1 $H_0 : \mu = 10 \quad H_1 : \mu \neq 10$

Step 2 $\alpha = 0.05$

Step 3 As $n \geq 30$, the sample statistic is

$$t = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \text{ [as in Solution 7.6(a)]}$$

Step 4 As $H_1 : \mu \neq 10$, we perform a two-tailed test. So, the region of rejection consists of two tails (see Figure 7.7).

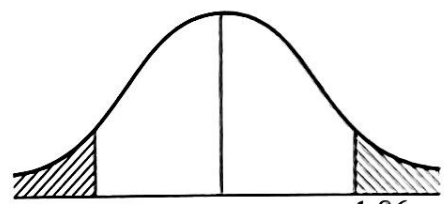

Figure 7.7 Region of rejection for Example 7.7.

By Table 7.4, the critical values are ± 1.96. As we have a decision-making situation we frame rejection rule as follows:

Rejection rule: Reject H_0 if the calculated test statistic is greater than 1.96 or less than −1.96. Otherwise, accept H_0.

Step 5 As σ is not given we take the estimate s for σ.

$$\text{Calculated test statistic} = \frac{9.8 - 10}{0.5/\sqrt{30}} = \frac{(-0.2)(5.48)}{0.5}$$

$$= -2.192$$

Step 6 As − 2.192 < −1.96, we reject H_0.

Step 7 The quality control inspector has to stop the operation and correct the machine.

Note: If the calculated test statistic were 1.2 or −1.2 then we have to accept H_0 and act on it. That is, we do not interrupt the operation until the next test is conducted.

EXAMPLE 7.8 An ancillary unit in Trichy receives order from BHEL and many other industries in and around Trichy. Usually, it receives an average of 9.1 orders per week. At present the ancillary unit feels that the number of orders will decline due to recession. It considers the average number of orders per week during 32 weeks and finds that the average number of orders has declined to 7.8 per week with a standard deviation of 1.15 orders per week. Test whether the average number of orders per week has declined significantly at a significance level of 0.05.

Solution: We are given the following data:

$$\mu_0 = 9.1 \quad \bar{x} = 7.8 \quad s = 1.15 \quad n = 32$$

We want to test whether the number of orders per week has declined now. So, we frame the null hypothesis and alternate hypothesis as follows:

Step 1 $H_0 : \mu = 9.1 \quad H_1 : \mu < 9.1$

Step 2 $\alpha = 0.05$

Step 3 As $n \geq 30$, the sample statistics is $t = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Step 4 As $H_1 : \mu < 9.1$, we perform a left-tailed test. The region of rejection is a single left tail (see Figure 7.8). By Table 7.4, the critical value is –1.645.

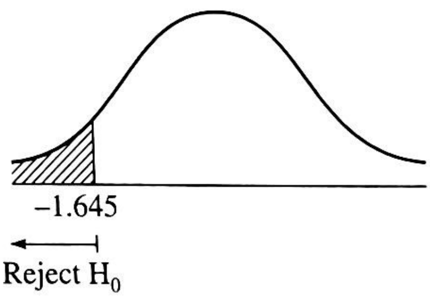

Figure 7.8 Region of rejection for Example 7.8.

Rejection rule: Reject H_0 if the calculated test statistic is less than –1.645.

Step 5 As σ is not given we take the estimate s for σ.

$$\text{Calculated test statistic } = \frac{7.8 - 9.1}{1.15/\sqrt{32}} = \frac{-(1.3)\left(\sqrt{32}\right)}{1.15}$$

$$= -6.395$$

Step 6 As – 6.395 < –1.645, we reject H_0.

Step 7 So, we conclude that there is a decrease in the average number of orders per week at a significance level of 0.05. The ancillary unit has to take extra efforts to increase the number of orders.

7.4 HYPOTHESIS TESTING OF PROPORTIONS

Suppose, 65% of the students in a class pass an internal test. The professor gives some coaching/extra assignment to the students to improve their performance. In order to test whether the coaching/extra assignment is effective, he conducts another test and observes the percentage of students passing the second test. If the percentage is 67% (say) there seems to be a change in the percentage of students passing the test. To test whether the coaching has improved their performance we conduct hypothesis testing of proportions.

7.5 HYPOTHESIS TESTING OF MEAN USING A SMALL SAMPLE WHEN σ IS NOT KNOWN

We have seen that t-distribution is used for small samples ($n < 30$) from a normal population whose standard deviation is not known (in interval estimation). The same thing happens in the case of hypothesis testing also.

The choice of null hypothesis and alternate hypothesis are similar to the earlier choices.

$H_0 : \mu = \mu_0$
H_1 is $\mu < \mu_0$ in the case of left-tailed test.
H_1 is $\mu > \mu_0$ in the case of right-tailed test.
H_1 is $\mu \neq \mu_0$ in the case of two-tailed distribution.

The sample statistic is:

$$\boxed{t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}} \tag{7.3}$$

and it follows a t-distribution with n–1 degrees of freedom. The other steps of hypothesis testing are as in earlier cases. We have to be a bit careful while using the t-distribution table.

In the t-distribution table (A3) we are given the area of both tails. So, we take the table value directly for a two-tailed test. In the case of one-tailed tests, we have to take the area under the column corresponding to twice the significance level. For example, the t-value for a significance level of 0.10 for a t-distribution with 19 degrees of freedom is used when we perform a one-tailed test with a significance level of 0.05.

Table 7.5 will be helpful while we are performing small sample tests.

Table 7.5 Table Values for a t-distribution with 19 Degrees of Freedom

Significance level	Two-tailed test	One-tailed test
0.05	2.093	1.729
0.025	–	2.093
0.02	2.539	2.861
0.01	2.861	–

Note: In general, the t value for one-tailed test with a significance level of α is equal to $t_{n,2\alpha}$ (see, Section 6.9)

EXAMPLE 7.11 A teller in a bank estimates that 10 persons on an average come to the bank in the last 10 minutes of working hours. In order to test his estimate he counts the number of customers coming in the last 10 minutes on 12 days and arrives at the following data:

15 15 13 12 8 9 7 10 16 13 17 13

Using the sample data, test whether the teller's estimate of number of persons coming in the last 10 minutes is wrong at a significance level of 0.05 (assume the population to be normal).

Solution: Let μ denote the average number of persons coming in the last 10 minutes. First of all let us find $\bar{x}$ and s^2.

$$\bar{x} = \frac{1}{12}[15 + 15 + 13 + 12 + 8 + 9 + 7 + 10 + 16 + 13 + 17 + 13] = 12.333$$

$$s^2 = \frac{1}{11}[(15^2 + 15^2 + 13^2 + 12^2 + 8^2 + 9^2 + 7^2 + 10^2 + 16^2 + 13^2 + 17^2 + 13^2] - 12(12.333)^2]$$

$= 10.424$

So, $s = 3.229$

The given data are:

$$n = 12 \quad \bar{x} = 12.333 \quad s = 3.229 \quad \mu_0 = 10$$

As we want to test whether teller's estimate ($\mu = 12$) is wrong, we frame H_0 and H_1 as follows:

Step 1 $H_0 : \mu = 10 \quad H_1 : \mu \neq 10$

Step 2 $\alpha = 0.05$

Step 3 The test statistic is:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

It follows a t-distribution with 11 degrees of freedom.

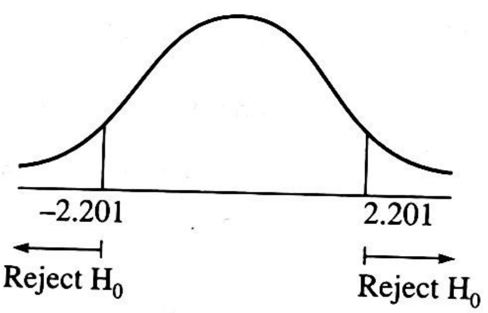

Figure 7.11 Region of rejection for Example 7.11.

Step 4 As we have a two-tailed test, we have to take the value under the column corresponding to 0.05 (i.e., $t_{11,\,0.05}$).

The critical values are $\pm$ 2.201

Rejection rule: Reject H_0 if calculated test statistic is greater than 2.201 or less than −2.201.

Step 5 Calculated test statistic $= \dfrac{12.333 - 12}{3.229/\sqrt{12}} = \dfrac{(0.333)(3.464)}{3.229}$

$= 0.357$

Step 6 As $0.357 < 2.201$, we cannot reject H_0. So, there is no statistical evidence to question the estimate of the teller.

EXAMPLE 7.12 Test the hypothesis

$$H_0 : \mu = 75 \quad H_1 = \mu < 75$$

Using the following sample data

$$n = 8 \quad \bar{x} = 73.42 \quad s^2 = 25.68 \quad s = 5.068 \quad \alpha = 0.01$$

(Assume the normality of population)

Solution: As the alternate hypothesis is $\mu < 75$, we have to apply a left-tailed test. The test statistic is:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

It follows a t-distribution with 7 degrees of freedom.

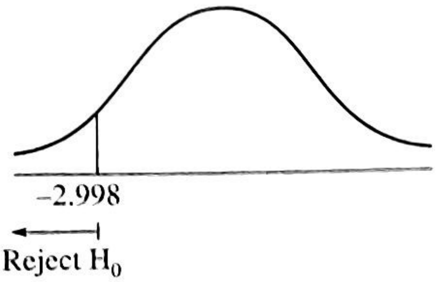

Figure 7.12 Region of rejection for Example 7.12.

As we have a left-tailed test, we have to take the value under the column corresponding to 0.02 (i.e. $t_{7,\ 0.02}$). So, the critical value is –2.998.

Rejection rule: Reject H_0 if calculated test statistic < –2.998.

$$\text{Calculated test statistic} = \frac{73.42 - 75}{5.068/\sqrt{8}} = \frac{-(1.58)\,(2.828)}{5.068}$$

$$= -\,0.88$$

As –0.88 is not greater than –2.998, we cannot reject H_0.

EXAMPLE 7.13 It is claimed that the average age of managers in a public sector bank is 40. A sample of 20 managers was taken and found to have a mean of 46 years and a standard deviation of 9.96 years. Do you have enough statistical evidence to claim that the average of age of managers is more than 40 at a significance level of 0.05?

Solution: We are given the following data:

$$n = 20 \quad \bar{x} = 46 \quad s = 9.96 \quad \mu_0 = 40$$

As we want to test whether the average age is more than 40, we frame H_0 and H_1 as follows:

Step 1 $H_0 : \mu = 40 \quad H_1 : \mu > 40$

Step 2 $\alpha = 0.05$

Step 3 The test statistic is:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

It follows a t-distribution with 19 degrees of freedom.

Step 4 As we have a right-tailed test, we have to take the value under the column corresponding to 0.10 (i.e., $t_{19,\,0.1}$). So, the critical value is 1.729.

Rejection rule: If calculated test statistic > 1.729 reject H_0.

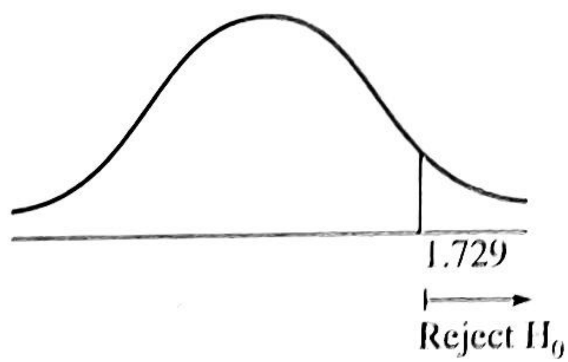

Figure 7.13 Region of rejection for Example 7.13.

Step 5 Calculated test statistic $= \dfrac{46 - 40}{9.96/\sqrt{20}} = \dfrac{(6)(4.472)}{9.96}$

$$= 2.694$$

Step 6 As 2.694 > 1.729, we reject H_0. So, the average age of managers is more than 40.

7.7 PROBABILITY VALUE METHOD (*P*-VALUE METHOD)

Let us consider hypothesis testing using a right-tailed test. We reject H_0 if the calculated test statistic, denoted by z_0 is greater than the critical value, denoted by c_1, corresponding to a given significance level α. Equivalently, we reject H_0 if the area under the right tail defined by $Z > z_1$ is less than the area under the right tail defined by $Z > c_1$ (see Figure 7.14).

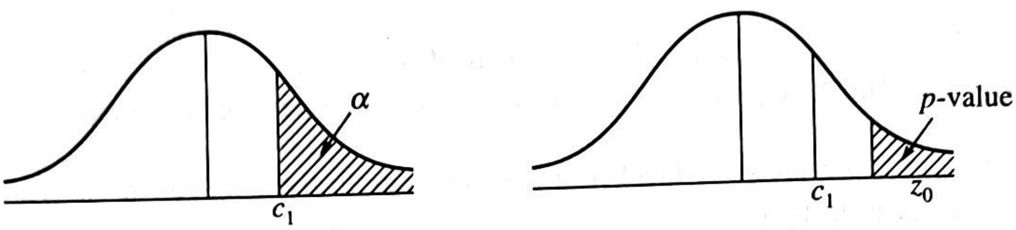

Figure 7.14 Rejection rule in terms of *p*-value.

We define *p*-value as given in Box 7.4:

Box 7.4
The area under the right-tail given by $Z > z_0$ is called the *p*-value corresponding to the calculated test statistic z_0.

So, the rejection rule in terms of *p*-value is given in Box 7.5

Box 7.5
Reject H_0, if the *p*-value corresponding to z_0 is less than the significance level α.

The *p*-value means something more and we shall see what *p*-value actually means with the help of an example.

Consider the problem of testing.

$$H_0: \mu = 15 \quad \text{and} \quad H_1: \mu > 15$$

With the following data:

$$\bar{x} = 16 \quad n = 36 \quad \sigma = 3$$

$$z_0 = \frac{16-15}{3/\sqrt{36}} = 2$$

The *p*-value corresponding to $z_0 = 2$ is the area of the right tail $Z > 2$. So, the *p*-value is $0.50 - 0.4772 = 0.228$ [see Figure 7.15(a)].

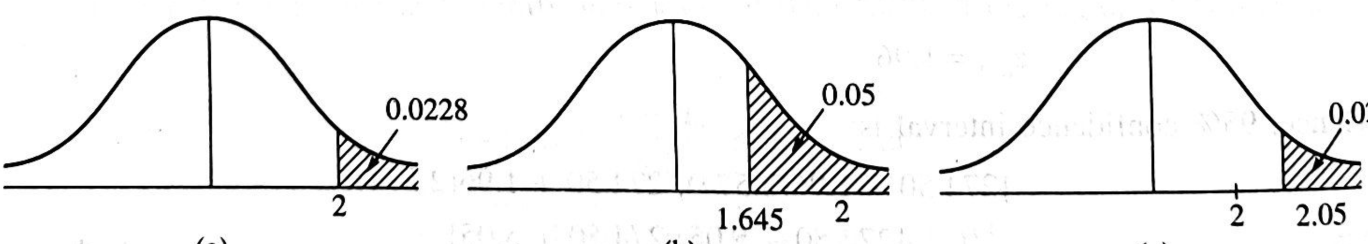

Figure 7.15 Regions of rejections at $\alpha = 0.05$ and $\alpha = 0.02$.

As the p-value is less than 0.05, we reject H_0 at a significance level of 0.05 [see Figure 7.15(b)].

As the p-value is not less than 0.02, we accept H_0 at a significance level of 0.02 [see Figure 7.15(c)].

Also, we accept H_0 at a significance level of 0.0228 or less and reject H_0 at any level larger than 0.0228.

Box 7.6

The p-value is the largest significance level at which we accept H_0.

Figure 7.16 illustrates Box 7.6.

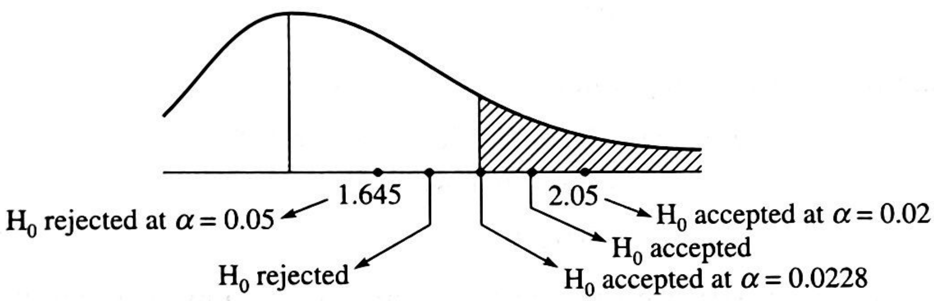

Figure 7.16 Rejection|acceptance of H_0 at various levels.

So far, our discussion on p-values was regarding a right-tailed test. The p-value corresponding to a left-tailed test is the area under the left tail. As the normal curve is symmetric about the y-axis, it is equal to the area under the right tail corresponding to the numerical value of z_0 (that is, we take 1.56 if $z = -1.56$). **In the case of a two-tailed test, we take the p-value to be twice the area of the right tail.** Thus, in all cases, the rejection rule is given by Box 7.5.

Advantages of using p-value method for hypothesis testing:

1. Once the p-value is known for a given sample information, we can test H_0 at any given level of significance.
2. The rejection rule is the same for all cases. Reject H_0 if the p-value is less than α.

7.7.1 How to Calculate the *p*-value?

1. The p-value corresponding to a calculated test statistic z_0 is the area of the right tail defined by $Z > z_0$. For large samples, we can calculate the p-values using standard normal table (A2).
2. For small sample tests, we can use t-distribution table only to say that the p-value lies between 0.10 and 0.05 or 0.05 and 0.02 or 0.02 and 0.01.
3. Computer statistics packages give the exact p-values for small sample tests using t-distributions.

EXAMPLE 7.16 A tyre company claims that the life of a tyre manufactured by them is at least 60,000 km. A sample of 64 tyres yielded the mean of 57,900 km and a standard deviation of 8500 km. Assume the life of tyres to be normally distributed.

Find the largest significance level at which we can accept company's claim.

(a) Find the largest significance level at which we can accept company's claim.
(b) Is the company's claim justified at a significance level of 0.05?
(c) Use p-value method to test

$$H_0 : \mu = 60{,}000,\ H_1 : \mu \neq 60{,}000, \text{ at a significance level of } 0.05$$

Solution: The calculated test statistic is:

$$z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{57{,}900 - 60{,}000}{8500/\sqrt{64}} = \frac{-(2100)\,(8)}{8500}$$

$$= -1.976$$

(a) We are required to find p-value

$$p\text{-value} = 0.5 - \text{Table value for } 1.98$$
$$= 0.5 - 0.4761$$
$$= 0.0239$$

(b) As the p-value < 0.05, we can reject the company's claim regarding the life of their tyres.

(c) As the test is two-tailed, p-value is twice the size of the tail.
So, p-value $= 2(0.0239) = 0.0478$
As $0.0478 < 0.05$, we reject H_0.

SUPPLEMENTARY EXAMPLES

EXAMPLE 7.20 Formulate H_0 and H_1 for the following situations:

(a) An automatic filling machine should fill 200 ml of a cough syrup in bottles. The machine has to be stopped in case of overfilling or underfilling.

(b) A label on a pack of soap powder says that the net weight of soap powder is 200 gm. A consumer tests the claim on the label.

(c) The students of a coaching centre for CAT examination score 70% in a preliminary test. After coaching them for one month the centre conducts another test for testing the efficiency of their coaching method.

(d) A patient has to test whether brisk walking for a month has reduced his weight. His weight was 63 kg before he started the exercise of walking.

Solution:

(a) $H_0 : \mu = 200 \quad H_1 : \mu \neq 200$

(b) $H_0 : \mu = 200 \quad H_1 : \mu < 200$

(c) $H_0 : p = 70 \quad H_1 : p > 70$

(d) $H_0 : \mu = 63 \quad H_1 : \mu < 63$

EXAMPLE 7.21 Give the rejection rules for the following data:

$H_1 : \mu \neq 45 \quad \alpha = 0.05 \quad n = 30$

$H_1 : \mu < 45 \quad \alpha = 0.05 \quad n = 30$

$H_1 : \mu > 45 \quad \alpha = 0.05 \quad n = 30$

$H_1 : \mu \neq 45 \quad \alpha = 0.02 \quad n = 50$

$H_1 : \mu > 45 \quad \alpha = 0.01 \quad n = 40$

Solution: All samples are large. So, we can use normal approximation to the sampling distributions.

(a) The test is two-tailed and $\alpha = 0.05$ by Table 7.4, the critical values are ± 1.96

Rejection rule: Reject H_0 if calculated test statistic is greater than 1.96 or less than -1.96

(b) The test is left-tailed and $\alpha = 0.05$ by Table 7.4, the critical values is -1.645

Rejection rule: Reject H_0 if the calculated test statistic is less than -1.645

(c) The test is right-tailed and $\alpha = 0.05$ by Table 7.4, the critical value is 1.645

Rejection rule: Reject H_0 if the calculated test statistic is greater than 1.645

(d) The test is two-tailed and $\alpha = 0.02$ by Table 7.4, the critical values are ± 2.33

Rejection rule: Reject H_0 if the calculated test statistic is greater than 2.33 or less than -2.33

(e) The test is right-tailed and $\alpha = 0.01$ by Table 7.4, the critical value is 2.578

Rejection rule: Reject H_0 if the calculated test statistic is greater than 2.578

EXAMPLE 7.22 The station master of Chennai Central Station claims that at least 80% of the trains arrive at the Central Station within 30 minutes of the schedule arrival time. Frame H_0 and H_1 so that his claim may be rejected after a sample study.

Solution: Let p denote the proportion of trains arriving within 30 minutes of the scheduled arrival time. Then:

$$H_0 : p = 0.80 \quad H_1 : p < 0.80$$

EXAMPLE 7.23 Can you use t-distribution for testing $H_0 : \mu = 45$ $H_1 : \mu \neq 45$ using a sample of size 12 from a left-skewed population? We are given that $\bar{x} = 55$ and $s = 15$. Answer the same question when the sample is of size 30.

Solution: As the population is left-skewed, it is not symmetrical and hence not normal. So, we cannot apply t-distribution. When the sample is of size 30, it becomes a large sample and so we can use central limit theorem for getting a normal test statistic.

EXAMPLE 7.24 An insurance agent dealing with products of LIC and some other private insurance companies, claims that the average amount for which all policies are taken by workers of BHEL, Trichy is 3 lakhs. An investment consultant takes a sample of 36 workers (there are about 10,000 workers in BHEL, Trichy) and finds that the average amount for which insurance policy was taken is ₹ 2,95,000 with a standard deviation of ₹ 12,500.

(a) Does the investment consultant have enough evidence to reject the claim of the insurance agent at a significance level of 0.05.
(b) Rework (a) if the average amount for which policies were taken is ₹ 3,20,000.

Solution: Let μ denote the average amount for which insurance policies were taken by BHEL workers.

(a) The given data are:

$$\mu_0 = 3{,}00{,}000 \quad \bar{x} = 2{,}95{,}000 \quad s = 12{,}500 \quad n = 36$$

The null and alternate hypotheses are:

$$H_0 : \mu = 3{,}00{,}000 \quad H_1 : \mu \neq 3{,}00{,}000$$
$$\alpha = 0.05$$

As the sample is large, the sample statistic is:

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

As H_1 is $\mu \neq 3{,}00{,}000$, we use a two-tailed test. As $\alpha = 0.05$, critical values are ± 1.96 (by Table 7.4)

Rejection rule: Reject H_0 if the calculated test statistic is greater than 1.96 or less than -1.96

As σ is not given, we take the estimate s in place of σ.

$$\text{Calculated test statistic} = \frac{2,95,000 - 30,000}{12,500/\sqrt{36}} = \frac{-(5000)(6)}{12,500}$$

$$= -2.4$$

As $-2.4 < -1.96$, H_0 is rejected.

So, the investment consultant has statistical evidence to reject the claim of the insurance agent.

EXAMPLE 7.25 According to a survey, the MBA students of top ranking B-schools used the net with an average of 16 hours per week (for academic purposes). A sample of 60 students was taken from other B-schools and it was found that the average usage of net was 14 hours/week by the students of the sample with a standard deviation of 7.5 hours. Do you feel that the use of net by students of other B-schools is less than the use of net by top ranking B-schools. Take a significance level of 0.05.

Solution: We are given the following data:

$$\mu_0 = 16 \quad \bar{x} = 14 \quad s = 7.5 \quad n = 60 \quad \alpha = 0.05$$

As we want to test whether the usage of net by students of other B-schools is less than the usage of students of top ranking B-schools, we frame H_0 and H_1 as follows:

$$H_0 : \mu = 16 \quad H_1 : \mu < 16$$

$$\alpha = 0.05$$

As the sample is large we take the test statistic

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

As H_1 is $\mu < 16$, we use a left-tailed test.

As $\alpha = 0.05$, the critical value is -1.645 (by Table 7.4). So,

Rejection rule: Reject H_0 if calculated test statistic is less than -1.645

As σ is not known, we take the estimate s in place of σ.

$$\text{Calculated test statistic} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$$= \frac{14 - 16}{7.5/\sqrt{60}} = \frac{-(2)(7.746)}{7.5}$$

$$= -2.066$$

As $-2.066 < -1.645$, we reject H_0.

So, we can conclude that usage of net by students of other B-schools is less than usage by students of top ranking B-schools.

EXAMPLE 7.26 The crimes recorded in the area under a town police station in the past revealed that the average number of crimes per day was 28 and the standard deviation was 8. During the last 32 days, the average number of crimes per day rose to 30.3 per day. Should the police be concerned about the spurt in crimes in the last 32 days? You can choose a significance level of 0.05.

Solution: We are given the following data:

$$\mu_0 = 28 \quad \bar{x} = 30.3 \quad \sigma = 8 \quad n = 32 \quad \alpha = 0.05$$

As we want to test whether the daily crime rate has increased, we define the null and alternate hypotheses as follows:

$$H_0 : \mu = 28 \quad H_1 : \mu > 28$$
$$\alpha = 0.05$$

As the sample is large, the sample statistic is:

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

As H_1 is $\mu > 28$, we use a right-tailed test.
As $\alpha = 0.05$, the critical value is 1.645 (by Table 7.4). So,

Rejection rule: Reject H_0 if calculated test statistic is greater than 1.645

$$\text{Calculated test statistic} = \frac{30.3 - 28}{8/\sqrt{32}} = \frac{(2.3)(5.659)}{8}$$
$$= 1.63$$

As $1.63 < 1.645$, we do not reject H_0. So, the sample data does not suggest any immediate action, but the police should be watchful of the crime rate in the coming days.

EXAMPLE 7.29 A private insurance company estimates that policy documents are dispatched from the head office within a month. The company wants to test its claim. It gathers information from 10 investors regarding the number of days taken by the insurance company to disptach the policy documents and arrive at the following data:

32	26	27	35	31	23	27	26	30	31

Can the company justify its claim at a significance level of 0.05?

Solution: We calculate $\bar{x}$ and s for the sample data

$$\bar{x} = \frac{1}{10}[32 + 26 + 27 + 35 + 31 + 23 + 27 + 26 + 30 + 31]$$

$$= \frac{1}{10}(288)$$

$$= 28.8$$

$$s^2 = \frac{1}{9}[(32^2 + 26^2 + 27^2 + 35^2 + 31^2 + 23^2 + 27^2 + 26^2 + 30^2 + 31^2] - 10(28.8)^2]$$

$$= \frac{1}{9}(8410 - 8294.4)$$

$$= \frac{1}{9}(115.6) = 12.84$$

$\therefore$ $s = 3.583$

The given data are:

$$n = 10 \quad \bar{x} = 28.8 \quad s = 3.583 \quad \mu_0 = 30$$

As the company wants to claim that $\mu \leq 30$, H_0 and H_1 are taken as follows:

$$H_0 : \mu = 30 \quad H_1 : \mu < 30$$

$$\alpha = 0.05$$

The test statistic is:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

It follows a t-distribution with 9 degrees of freedom. As we have a right-tailed test, we have to take the value under 0.10 for a t-distribution with 9 degrees of freedom (i.e., $t_{9,\,0.1}$). So, the critical value is 1.833.

Rejection rule: Reject H_0 if $t < -1.833$

$$\text{Calculated test statistic} = \frac{28.8 - 30}{3.583/\sqrt{10}} = \frac{-(1.2)(3.162)}{3.583}$$

$$= -1.059$$

As the calculated test statistic is not less than -1.833, the company cannot reject H_0. So, the company can say that there is no statistical evidence at a significance level of 0.05 to refute its claim.

EXAMPLE 7.30 The average cost of a two bedroom flat in a developing residential area of Chennai is found to be 35 lakhs. However, a recent sample of 10 flats had a mean cost of 39 lakhs with a standard deviation of 5 lakhs. Test whether the average cost of two bedroom flat in a developing area of Chennai has increased recently at a significance level of 0.05.

Solution: Let us take the cost of flats in units of lakhs.

$$\bar{x} = 39 \quad s = 5 \quad n = 10 \quad \mu_0 = 35$$

As we want to test whether the cost has increased, we take H_0 and H_1 as follows:

$$H_0 : \mu = 35 \quad H_1 : \mu > 35$$

$$\alpha = 0.05$$

The test statistic is:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

It follows a t-distribution with 9 degrees of freedom. As we have a right-tailed test, we have to take the value under the column corresponding to 0.10 (i.e., $t_{9,\,0.1}$). It is 1.833. So, the critical value is 1.833.

Rejection rule: If the calculated test statistic is greater than 1.833 reject H_0.

$$\text{Calculated test statistic} = \frac{39 - 35}{5/\sqrt{10}} = \frac{(4)\,(3.162)}{5}$$

$$= 2.5296$$

As the calculated test statistic is greater than 1.833, we reject H_0.
So, the average price of two bedroom flats in Chennai has increased recently.

EXAMPLE 7.31 If a sample of 15 values has a mean of 32 and a variance of 4, test the hypothesis that the population mean is 34 against the hypothesis that the population mean is different from 34 at a significance level of 0.02.

Solution: The given data are:

$$\bar{x} = 32 \quad s = \sqrt{4} = 2 \quad n = 15 \quad \mu_0 = 34$$

We are given that

$$H_0 : \mu = 34 \quad H_1 : \mu \neq 34$$

$$\alpha = 0.02$$

The test statistic is:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

It follows a t-distribution with 14 degrees of freedom. As we have a two-tailed test, we have to take the value under the column corresponding to 0.02 (i.e., $t_{14,\,0.02}$). It is 2.624. So, the critical values are ± 2.624.

Rejection rule: If the calculated test statistic is greater than 2.624 or less than –2.624, reject H_0.