7.5 Solved Problems

1. Find the power set and cardinality of the given set $X = \{2, 4, 6\}$. Also find cardinality of power set.

Solution: Since set X contains three elements, so its cardinal number is

$$n_X = 3$$

The power set of X is given by

$$P(X) = \{\phi, \{2\}, \{4\}, \{6\}, \{2,4\}, \{4,6\}, \{2,6\}, \{2,4,6\}\}$$

The cardinality of power set $P(X)$, denoted by $n_{P(X)}$, is found as

$$n_{P(X)} = 2^{n_X} = 2^3 = 8$$

2. Consider two given fuzzy sets

$$\underset{\sim}{A} = \left\{\frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8}\right\}$$

$$\underset{\sim}{B} = \left\{\frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8}\right\}$$

Perform union, intersection, difference and complement over fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

Solution: For the given fuzzy sets we have the following

(a) Union

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\} = \left\{\frac{1}{2} + \frac{0.4}{4} + \frac{0.5}{6} + \frac{1}{8}\right\}$$

(b) Intersection

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\} = \left\{\frac{0.5}{2} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.2}{8}\right\}$$

(c) Complement

$$\overline{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x) = \left\{\frac{0}{2} + \frac{0.7}{4} + \frac{0.5}{6} + \frac{0.8}{8}\right\}$$

$$\overline{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x) = \left\{\frac{0.5}{2} + \frac{0.6}{4} + \frac{0.9}{6} + \frac{0}{8}\right\}$$

(d) Difference

$$\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \left\{\frac{0.5}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0}{8}\right\}$$

$$\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}} = \left\{\frac{0}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{0.8}{8}\right\}$$

3. Given the two fuzzy sets

$$\underset{\sim}{B}_1 = \left\{\frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\right\}$$

$$\underset{\sim}{B}_2 = \left\{\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}\right\}$$

find the following:

(a) $\underset{\sim}{B}_1 \cup \underset{\sim}{B}_2$; (b) $\underset{\sim}{B}_1 \cap \underset{\sim}{B}_2$; (c) $\overline{\underset{\sim}{B}_1}$;
(d) $\overline{\underset{\sim}{B}_2}$; (e) $\underset{\sim}{B}_1 | \underset{\sim}{B}_2$; (f) $\overline{\underset{\sim}{B}_1 \cup \underset{\sim}{B}_2}$;

(g) $\overline{\underset{\sim}{B}_1 \cap \underset{\sim}{B}_2}$; (h) $\underset{\sim}{B}_1 \cap \overline{\underset{\sim}{B}_1}$; (i) $\underset{\sim}{B}_1 \cup \overline{\underset{\sim}{B}_1}$;

(j) $\underset{\sim}{B}_2 \cap \overline{\underset{\sim}{B}_2}$; (k) $\underset{\sim}{B}_2 \cup \overline{\underset{\sim}{B}_2}$

Solution: For the given fuzzy sets, we have the following:

(a) $$\underset{\sim}{B}_1 \cup \underset{\sim}{B}_2 = \left\{\frac{1}{1.0}+\frac{0.75}{1.5}+\frac{0.3}{2.0}+\frac{0.15}{2.5}+\frac{0}{3.0}\right\}$$

(b) $$\underset{\sim}{B}_1 \cap \underset{\sim}{B}_2 = \left\{\frac{1}{1.0}+\frac{0.6}{1.5}+\frac{0.2}{2.0}+\frac{0.1}{2.5}+\frac{0}{3.0}\right\}$$

(c) $$\overline{\underset{\sim}{B}_1} = \left\{\frac{0}{1.0}+\frac{0.25}{1.5}+\frac{0.7}{2.0}+\frac{0.85}{2.5}+\frac{1}{3.0}\right\}$$

(d) $$\overline{\underset{\sim}{B}_2} = \left\{\frac{0}{1.0}+\frac{0.4}{1.5}+\frac{0.8}{2.0}+\frac{0.9}{2.5}+\frac{1}{3.0}\right\}$$

(e) $$\underset{\sim}{B}_1 | \underset{\sim}{B}_2 = \underset{\sim}{B}_1 \cap \overline{\underset{\sim}{B}_2} = \left\{\frac{0}{1.0}+\frac{0.4}{1.5}+\frac{0.3}{2.0}+\frac{0.15}{2.5}+\frac{0}{3.0}\right\}$$

(f) $$\overline{\underset{\sim}{B}_1 \cup \underset{\sim}{B}_2} = \left\{\frac{0}{1.0}+\frac{0.25}{1.5}+\frac{0.7}{2.0}+\frac{0.85}{2.5}+\frac{1}{3.0}\right\}$$

(g) $$\overline{\underset{\sim}{B}_1 \cap \underset{\sim}{B}_2} = \left\{\frac{0}{1.0}+\frac{0.4}{1.5}+\frac{0.8}{2.0}+\frac{0.9}{2.5}+\frac{1}{3.0}\right\}$$

(h) $$\underset{\sim}{B}_1 \cap \overline{\underset{\sim}{B}_1} = \left\{\frac{0}{1.0}+\frac{0.25}{1.5}+\frac{0.3}{2.0}+\frac{0.15}{2.5}+\frac{0}{3.0}\right\}$$

(i) $$\underset{\sim}{B}_1 \cup \overline{\underset{\sim}{B}_1} = \left\{\frac{1}{1.0}+\frac{0.75}{1.5}+\frac{0.7}{2.0}+\frac{0.85}{2.5}+\frac{1}{3.0}\right\}$$

(j) $$\underset{\sim}{B}_2 \cap \overline{\underset{\sim}{B}_2} = \left\{\frac{0}{1.0}+\frac{0.4}{1.5}+\frac{0.2}{2.0}+\frac{0.1}{2.5}+\frac{0}{3.0}\right\}$$

(k) $$\underset{\sim}{B}_2 \cup \overline{\underset{\sim}{B}_2} = \left\{\frac{1}{1.0}+\frac{0.6}{1.5}+\frac{0.8}{2.0}+\frac{0.9}{2.5}+\frac{1}{3.0}\right\}$$

4. It is necessary to compare two sensors based upon their detection levels and gain settings. The table of gain settings and sensor detection levels with a standard item being monitored providing typical membership values to represent the detection levels for each sensor is given in Table 1.

Table 1

Gain setting	Detection level of sensor 1	Detection level of sensor 2
0	0	0
10	0.2	0.35
20	0.35	0.25
30	0.65	0.8
40	0.85	0.95
50	1	1

Now given the universe of discourse $X = \{0, 10, 20, 30, 40, 50\}$ and the membership functions for the two sensors in discrete form as

$$\underset{\sim}{D}_1 = \left\{\frac{0}{0}+\frac{0.2}{10}+\frac{0.35}{20}+\frac{0.65}{30}+\frac{0.85}{40}+\frac{1}{50}\right\}$$

$$\underset{\sim}{D}_2 = \left\{\frac{0}{0}+\frac{0.35}{10}+\frac{0.25}{20}+\frac{0.8}{30}+\frac{0.95}{40}+\frac{1}{50}\right\}$$

find the following membership functions:

(a) $\mu_{\underset{\sim}{D}_1 \cup \underset{\sim}{D}_2}(x)$; (b) $\mu_{\underset{\sim}{D}_1 \cap \underset{\sim}{D}_2}(x)$; (c) $\mu_{\overline{\underset{\sim}{D}_1}}(x)$;

(d) $\mu_{\overline{\underset{\sim}{D}_2}}(x)$; (e) $\mu_{\underset{\sim}{D}_1 \cup \overline{\underset{\sim}{D}_1}}(x)$; (f) $\mu_{\underset{\sim}{D}_1 \cap \overline{\underset{\sim}{D}_1}}(x)$;

(g) $\mu_{\underset{\sim}{D}_2 \cup \overline{\underset{\sim}{D}_2}}$; (h) $\mu_{\underset{\sim}{D}_2 \cap \overline{\underset{\sim}{D}_2}}(x)$; (i) $\mu_{\underset{\sim}{D}_1 | \underset{\sim}{D}_2}(x)$;

(j) $\mu_{\underset{\sim}{D}_2 | \underset{\sim}{D}_1}(x)$

Solution: For the given fuzzy sets we have

(a) $\mu_{\underset{\sim}{D}_1 \cup \underset{\sim}{D}_2}(x)$

$$= \max\{\mu_{\underset{\sim}{D}_1}(x), \mu_{\underset{\sim}{D}_2}(x)\}$$

$$= \left\{\frac{0}{0}+\frac{0.35}{10}+\frac{0.35}{20}+\frac{0.8}{30}+\frac{0.95}{40}+\frac{1}{50}\right\}$$

(b) $\mu_{\underset{\sim}{D}_1 \cap \underset{\sim}{D}_2}(x)$

$$= \min\{\mu_{\underset{\sim}{D}_1}(x), \mu_{\underset{\sim}{D}_2}(x)\}$$

$$= \left\{\frac{0}{0}+\frac{0.2}{10}+\frac{0.25}{20}+\frac{0.65}{30}+\frac{0.85}{40}+\frac{1}{50}\right\}$$

(c) $\mu_{\overline{\underset{\sim}{D}_1}}(x)$

$$= 1 - \mu_{\underset{\sim}{D}_1}(x)$$

$$= \left\{\frac{1}{0}+\frac{0.8}{10}+\frac{0.65}{20}+\frac{0.35}{30}+\frac{0.15}{40}+\frac{0}{50}\right\}$$

(d) $\mu_{\overline{D_2}}(x)$

$$= 1-\mu_{D_2}(x)$$
$$= \left\{\frac{1}{0}+\frac{0.65}{10}+\frac{0.75}{20}+\frac{0.2}{30}+\frac{0.05}{40}+\frac{0}{50}\right\}$$

(e) $\mu_{D_1\cup\overline{D_1}}(x)$

$$= \max\{\mu_{D_1}(x), \mu_{\overline{D_1}}(x)\}$$
$$= \left\{\frac{1}{0}+\frac{0.8}{10}+\frac{0.65}{20}+\frac{0.65}{30}+\frac{0.85}{40}+\frac{1}{50}\right\}$$

(f) $\mu_{D_1\cap\overline{D_1}}(x)$

$$= \min\{\mu_{D_1}(x), \mu_{\overline{D_1}}(x)\}$$
$$= \left\{\frac{0}{0}+\frac{0.2}{10}+\frac{0.35}{20}+\frac{0.35}{30}+\frac{0.15}{40}+\frac{0}{50}\right\}$$

(g) $\mu_{D_2\cup\overline{D_2}}(x)$

$$= \max\{\mu_{D_2}(x), \mu_{\overline{D_2}}(x)\}$$
$$= \left\{\frac{1}{0}+\frac{0.65}{10}+\frac{0.75}{20}+\frac{0.8}{30}+\frac{0.95}{40}+\frac{1}{50}\right\}$$

(h) $\mu_{D_2\cap\overline{D_2}}(x)$

$$= \min\{\mu_{D_2}(x), \mu_{\overline{D_2}}(x)\}$$
$$= \left\{\frac{0}{0}+\frac{0.35}{10}+\frac{0.25}{20}+\frac{0.2}{30}+\frac{0.05}{40}+\frac{0}{50}\right\}$$

(i) $\mu_{D_1|D_2}(x)$

$$= \mu_{D_1\cap\overline{D_2}}(x) = \min\{\mu_{D_1}(x), \mu_{\overline{D_2}}(x)\}$$
$$= \left\{\frac{0}{0}+\frac{0.2}{10}+\frac{0.35}{20}+\frac{0.2}{30}+\frac{0.05}{40}+\frac{0}{50}\right\}$$

(j) $\mu_{D_2|D_1}(x)$

$$= \mu_{D_2\cap\overline{D_1}}(x) = \min\{\mu_{D_2}(x), \mu_{\overline{D_1}}(x)\}$$
$$= \left\{\frac{0}{0}+\frac{0.35}{10}+\frac{0.25}{20}+\frac{0.35}{30}+\frac{0.15}{40}+\frac{0}{50}\right\}$$

5. Design a computer software to perform image processing to locate objects within a scene. The two fuzzy sets representing a plane and a train image are:

$$\text{Plane} = \left\{\frac{0.2}{\text{train}}+\frac{0.5}{\text{bike}}+\frac{0.3}{\text{boat}}+\frac{0.8}{\text{plane}}+\frac{0.1}{\text{house}}\right\}$$

$$\text{Train} = \left\{\frac{1}{\text{train}}+\frac{0.2}{\text{bike}}+\frac{0.4}{\text{boat}}+\frac{0.5}{\text{plane}}+\frac{0.2}{\text{house}}\right\}$$

Find the following:

(a) Plane ∪ Train; (b) Plane ∩ Train;
(c) $\overline{\text{Plane}}$; (d) $\overline{\text{Train}}$;
(e) Plane|Train; (f) $\overline{\text{Plane} \cup \text{Train}}$;
(g) $\overline{\text{Plane} \cap \text{Train}}$; (h) Plane ∪ $\overline{\text{Plane}}$;
(i) Plane ∩ $\overline{\text{Plane}}$; (j) Train ∪ $\overline{\text{Train}}$;
(k) Train ∪ $\overline{\text{Train}}$

Solution: For the given fuzzy sets we have the following:

(a) Plane ∪ Train

$$= \max\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$
$$= \left\{\frac{1.0}{\text{train}}+\frac{0.5}{\text{bike}}+\frac{0.4}{\text{boat}}+\frac{0.8}{\text{plane}}+\frac{0.2}{\text{house}}\right\}$$

(b) Plane ∩ Train

$$= \min\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$
$$= \left\{\frac{0.2}{\text{train}}+\frac{0.2}{\text{bike}}+\frac{0.3}{\text{boat}}+\frac{0.5}{\text{plane}}+\frac{0.1}{\text{house}}\right\}$$

(c) $\overline{\text{Plane}} = 1-\mu_{\text{Plane}}(x)$

$$= \left\{\frac{0.8}{\text{train}}+\frac{0.5}{\text{bike}}+\frac{0.7}{\text{boat}}+\frac{0.2}{\text{plane}}+\frac{0.9}{\text{house}}\right\}$$

(d) $\overline{\text{Train}} = 1-\mu_{\text{Train}}(x)$

$$= \left\{\frac{0}{\text{train}}+\frac{0.8}{\text{bike}}+\frac{0.6}{\text{boat}}+\frac{0.5}{\text{plane}}+\frac{0.8}{\text{house}}\right\}$$

(e) Plane|Train

$$= \text{Plane} \cap \overline{\text{Train}}$$
$$= \min\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Train}}}(x)\}$$
$$= \left\{\frac{0}{\text{train}}+\frac{0.5}{\text{bike}}+\frac{0.3}{\text{boat}}+\frac{0.5}{\text{plane}}+\frac{0.1}{\text{house}}\right\}$$

(f) $\overline{\text{Plane} \cup \text{Train}}$

$$= 1 - \max\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$
$$= \left\{\frac{0}{\text{train}}+\frac{0.5}{\text{bike}}+\frac{0.6}{\text{boat}}+\frac{0.2}{\text{plane}}+\frac{0.8}{\text{house}}\right\}$$

(g) $\overline{\underset{\sim}{\text{Plane}} \cap \underset{\sim}{\text{Train}}}$

$$= 1 - \min\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$

$$= \left\{\frac{0.8}{\text{train}} + \frac{0.8}{\text{bike}} + \frac{0.7}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.9}{\text{house}}\right\}$$

(h) $\underset{\sim}{\text{Plane}} \cup \overline{\underset{\sim}{\text{Plane}}}$

$$= \max\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Plane}}}(x)\}$$

$$= \left\{\frac{0.8}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.7}{\text{boat}} + \frac{0.8}{\text{plane}} + \frac{0.9}{\text{house}}\right\}$$

(i) $\underset{\sim}{\text{Plane}} \cap \overline{\underset{\sim}{\text{Plane}}}$

$$= \min\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Plane}}}(x)\}$$

$$= \left\{\frac{0.2}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.2}{\text{plane}} + \frac{0.1}{\text{house}}\right\}$$

(j) $\underset{\sim}{\text{Train}} \cup \overline{\underset{\sim}{\text{Train}}}$

$$= \max\{\mu_{\text{Train}}(x), \mu_{\overline{\text{Train}}}(x)\}$$

$$= \left\{\frac{1.0}{\text{train}} + \frac{0.8}{\text{bike}} + \frac{0.6}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.8}{\text{house}}\right\}$$

(k) $\underset{\sim}{\text{Train}} \cap \overline{\underset{\sim}{\text{Train}}}$

$$= \min\{\mu_{\text{Train}}(x), \mu_{\overline{\text{Train}}}(x)\}$$

$$= \left\{\frac{0}{\text{train}} + \frac{0.2}{\text{bike}} + \frac{0.4}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.2}{\text{house}}\right\}.$$

6. For aircraft simulator data the determination of certain changes in its operating conditions is made on the basis of hard break points in the mach region. We define two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ representing the condition of "near" a mach number of 0.65 and "in the region" of a mach number of 0.65, respectively, as follows

$\underset{\sim}{A}$ = near mach 0.65

$$= \left\{\frac{0}{0.64} + \frac{0.75}{0.645} + \frac{1}{0.65} + \frac{0.5}{0.655} + \frac{0}{0.66}\right\}$$

$\underset{\sim}{B}$ = in the region of mach 0.65

$$= \left\{\frac{0}{0.64} + \frac{0.25}{0.645} + \frac{0.75}{0.65} + \frac{1}{0.655} + \frac{0.5}{0.66}\right\}$$

For these two sets find the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$; (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$; (c) $\overline{\underset{\sim}{A}}$;

(d) $\overline{\underset{\sim}{B}}$; (e) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$; (f) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$

Solution: For the two given fuzzy sets we have the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$

$$= \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$$

$$= \left\{\frac{0}{0.64} + \frac{0.75}{0.645} + \frac{1}{0.65} + \frac{1}{0.655} + \frac{0.5}{0.66}\right\}$$

(b) $\underset{\sim}{A} \cap \underset{\sim}{B}$

$$= \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$$

$$= \left\{\frac{0}{0.64} + \frac{0.25}{0.645} + \frac{0.75}{0.65} + \frac{0.5}{0.655} + \frac{0}{0.66}\right\}$$

(c) $\overline{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x)$

$$= \left\{\frac{1}{0.64} + \frac{0.25}{0.645} + \frac{0}{0.65} + \frac{0.5}{0.655} + \frac{1}{0.66}\right\}$$

(d) $\overline{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x)$

$$= \left\{\frac{1}{0.64} + \frac{0.75}{0.645} + \frac{0.25}{0.65} + \frac{0}{0.655} + \frac{0.5}{0.66}\right\}$$

(e) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$

$$= 1 - \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$$

$$= \left\{\frac{1}{0.64} + \frac{0.25}{0.645} + \frac{0}{0.65} + \frac{0}{0.655} + \frac{0.5}{0.66}\right\}$$

(f) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$

$$= 1 - \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$$

$$= \left\{\frac{1}{0.64} + \frac{0.75}{0.645} + \frac{0.25}{0.65} + \frac{0.5}{0.655} + \frac{1}{0.66}\right\}$$

7. For the two given fuzzy sets

$$\underset{\sim}{A} = \left\{\frac{0.1}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{1}{4}\right\}$$

$$\underset{\sim}{B} = \left\{\frac{1}{0} + \frac{0.5}{1} + \frac{0.7}{2} + \frac{0.3}{3} + \frac{0}{4}\right\}$$

find the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$; (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$; (c) $\overline{\underset{\sim}{A}}$;

(d) $\overline{\underset{\sim}{B}}$; (e) $\underset{\sim}{A} \cup \overline{\underset{\sim}{A}}$; (f) $\underset{\sim}{A} \cap \overline{\underset{\sim}{A}}$;

(g) $\underset{\sim}{B} \cup \overline{\underset{\sim}{B}}$; (h) $\underset{\sim}{B} \cap \overline{\underset{\sim}{B}}$; (i) $\underset{\sim}{A} \cap \overline{\underset{\sim}{B}}$;

(j) $\underset{\sim}{A} \cup \overline{\underset{\sim}{B}}$; (k) $\underset{\sim}{B} \cap \overline{\underset{\sim}{A}}$; (l) $\underset{\sim}{B} \cup \overline{\underset{\sim}{A}}$;

(m) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$; (n) $\overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}}$

Solution: For the given sets we have:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B} = \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{1}{0} + \frac{0.5}{1} + \frac{0.7}{2} + \frac{0.6}{3} + \frac{1}{4}\right\}$$

(b) $\underset{\sim}{A} \cap \underset{\sim}{B} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.1}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.3}{3} + \frac{0}{4}\right\}$$

(c) $\overline{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x)$

$$= \left\{\frac{0.9}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.4}{3} + \frac{0}{4}\right\}$$

(d) $\overline{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x)$

$$= \left\{\frac{0}{0} + \frac{0.5}{1} + \frac{0.3}{2} + \frac{0.7}{3} + \frac{1}{4}\right\}$$

(e) $\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} = \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\overline{\underset{\sim}{A}}}(x)\}$

$$= \left\{\frac{0.9}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.6}{3} + \frac{1}{4}\right\}$$

(f) $\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\overline{\underset{\sim}{A}}}(x)\}$

$$= \left\{\frac{0.1}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.4}{3} + \frac{0}{4}\right\}$$

(g) $\underset{\sim}{B} \cup \overline{\underset{\sim}{B}} = \max\{\mu_{\underset{\sim}{B}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{1}{0} + \frac{0.5}{1} + \frac{0.7}{2} + \frac{0.7}{3} + \frac{1}{4}\right\}$$

(h) $\underset{\sim}{B} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{\underset{\sim}{B}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0}{0} + \frac{0.5}{1} + \frac{0.3}{2} + \frac{0.3}{3} + \frac{0}{4}\right\}$$

(i) $\underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0}{0} + \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.6}{3} + \frac{1}{4}\right\}$$

(j) $\underset{\sim}{A} \cup \overline{\underset{\sim}{B}} = \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0.1}{0} + \frac{0.5}{1} + \frac{0.4}{2} + \frac{0.7}{3} + \frac{1}{4}\right\}$$

(k) $\underset{\sim}{B} \cap \overline{\underset{\sim}{A}} = \min\{\mu_{\underset{\sim}{B}}(x), \mu_{\overline{\underset{\sim}{A}}}(x)\}$

$$= \left\{\frac{0.9}{0} + \frac{0.5}{1} + \frac{0.6}{2} + \frac{0.3}{3} + \frac{0}{4}\right\}$$

(l) $\underset{\sim}{B} \cup \overline{\underset{\sim}{A}} = \max\{\mu_{\underset{\sim}{B}}(x), \mu_{\overline{\underset{\sim}{A}}}(x)\}$

$$= \left\{\frac{1}{0} + \frac{0.8}{1} + \frac{0.7}{2} + \frac{0.4}{3} + \frac{0}{4}\right\}$$

(m) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = 1 - \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0}{0} + \frac{0.5}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0}{4}\right\}$$

(n) $\overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{\overline{\underset{\sim}{A}}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0}{0} + \frac{0.5}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0}{4}\right\}$$

8. Let U be the universe of military aircraft of interest' as defined below:

$$U = \{a10,\ b52,\ c130,\ f2,\ f9\}$$

Let $\underset{\sim}{A}$ be the fuzzy set of bomber class aircraft:

$$\underset{\sim}{A} = \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{1}{f9}\right\}$$

Let $\underset{\sim}{B}$ be the fuzzy set of fighter class aircraft:

$$\underset{\sim}{B} = \left\{\frac{0.1}{a10} + \frac{0.2}{b52} + \frac{0.8}{c130} + \frac{0.7}{f2} + \frac{0}{f9}\right\}$$

Find the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$; (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$; (c) $\overline{\underset{\sim}{A}}$; (d) $\overline{\underset{\sim}{B}}$;

(e) $\underset{\sim}{A} | \underset{\sim}{B}$; (f) $\underset{\sim}{B} | \underset{\sim}{A}$; (g) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$;

(h) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$; (i) $\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}}$; (j) $\overline{\underset{\sim}{B}} \cup \underset{\sim}{A}$

Solution: We have

(a) $\underset{\sim}{A} \cup \underset{\sim}{B} = \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.8}{c130} + \frac{0.7}{f2} + \frac{1}{f9}\right\}$$

(b) $\underset{\sim}{A} \cap \underset{\sim}{B} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.1}{a10} + \frac{0.2}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{0}{f9}\right\}$$

(c) $\overline{\underset{\sim}{A}} = 1-\mu_{\underset{\sim}{A}}(x)$

$$= \left\{\frac{0.7}{a10} + \frac{0.6}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{0}{f9}\right\}$$

(d) $\overline{\underset{\sim}{B}} = 1-\mu_{\underset{\sim}{B}}(x)$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{1}{f9}\right\}$$

(e) $\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{1}{f9}\right\}$$

(f) $\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}} = \min\{\mu_{\underset{\sim}{B}}(x), \mu_{\overline{\underset{\sim}{A}}}(x)\}$

$$= \left\{\frac{0.1}{a10} + \frac{0.2}{b52} + \frac{0.8}{c130} + \frac{0.7}{f2} + \frac{0}{f9}\right\}$$

(g) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = 1 - \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.7}{a10} + \frac{0.6}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{0}{f9}\right\}$$

(h) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = 1 - \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{1}{f9}\right\}$$

(i) $\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}} = \max\{\mu_{\overline{\underset{\sim}{A}}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{1}{f9}\right\}$$

(j) $\overline{\underset{\sim}{B}} \cup \underset{\sim}{A} = \max\{\mu_{\overline{\underset{\sim}{B}}}(x), \mu_{\underset{\sim}{A}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{1}{f9}\right\}$$

9. Consider two fuzzy sets

$$\underset{\sim}{A} = \left\{\frac{0.2}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.5}{4}\right\}$$

$$\underset{\sim}{B} = \left\{\frac{0.1}{1} + \frac{0.2}{2} + \frac{0.2}{3} + \frac{1}{4}\right\}$$

Find the algebraic sum, algebraic product, bounded sum and bounded difference of the given fuzzy sets.

Solution: We have

(a) Algebraic sum

$$\mu_{\underset{\sim}{A}+\underset{\sim}{B}}(x) = [\mu_{\underset{\sim}{A}}(x)+\mu_{\underset{\sim}{B}}(x)] - [\mu_{\underset{\sim}{A}}(x)\cdot\mu_{\underset{\sim}{B}}(x)]$$

$$= \left\{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4}\right\}$$

$$- \left\{\frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4}\right\}$$

$$= \left\{\frac{0.28}{1} + \frac{0.44}{2} + \frac{0.52}{3} + \frac{0}{4}\right\}$$

(b) Algebraic product

$$\mu_{\underset{\sim}{A}\cdot\underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x)\cdot\mu_{\underset{\sim}{B}}(x)$$

$$= \left\{\frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4}\right\}$$

(c) Bounded sum

$$\mu_{\underset{\sim}{A}\oplus\underset{\sim}{B}}(x)$$

$$= \min[1, \mu_{\underset{\sim}{A}}(x)+\mu_{\underset{\sim}{B}}(x)]$$

$$= \min\left\{1, \left\{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4}\right\}\right\}$$

$$= \left\{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4}\right\}$$

(d) Bounded difference

$$\mu_{\underset{\sim}{A}\odot\underset{\sim}{B}}(x)$$

$$= \max[0, \mu_{\underset{\sim}{A}}(x)-\mu_{\underset{\sim}{B}}(x)]$$

$$= \max\left\{0, \left\{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4}\right\}\right\}$$

$$= \left\{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4}\right\}$$

10. The discretized membership functions for a transistor and a resistor are given below:

$$\mu_{\underset{\sim}{T}} = \left\{\frac{0}{0} + \frac{0.2}{1} + \frac{0.7}{2} + \frac{0.8}{3} + \frac{0.9}{4} + \frac{1}{5}\right\}$$

$$\mu_{\underset{\sim}{R}} = \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.2}{3} + \frac{0.4}{4} + \frac{0.5}{5}\right\}$$

Find the following: (a) Algebraic sum; (b) algebraic product; (c) bounded sum; (d) bounded difference.

Solution: We have

(a) Algebraic sum

$$\begin{aligned}
&\mu_{\underset{\sim}{T}+\underset{\sim}{R}}(x) \\
&= [\mu_{\underset{\sim}{T}}(x)+\mu_{\underset{\sim}{R}}(x)] - [\mu_{\underset{\sim}{T}}(x)\cdot\mu_{\underset{\sim}{R}}(x)] \\
&= \left\{\frac{0}{0} + \frac{0.3}{1} + \frac{1.0}{2} + \frac{1.0}{3} + \frac{1.3}{4} + \frac{1.5}{5}\right\} \\
&\quad - \left\{\frac{0}{0} + \frac{0.02}{1} + \frac{0.21}{2} + \frac{0.16}{3} + \frac{0.36}{4} + \frac{0.5}{5}\right\} \\
&= \left\{\frac{0}{0} + \frac{0.28}{1} + \frac{0.79}{2} + \frac{0.84}{3} + \frac{0.94}{4} + \frac{1}{5}\right\}
\end{aligned}$$

(b) Algebraic product

$$\begin{aligned}
&\mu_{\underset{\sim}{T}\cdot\underset{\sim}{R}}(x) \\
&= \mu_{\underset{\sim}{T}}(x)\cdot\mu_{\underset{\sim}{R}}(x) \\
&= \left\{\frac{0}{0} + \frac{0.02}{1} + \frac{0.21}{2} + \frac{0.16}{3} + \frac{0.36}{4} + \frac{0.5}{5}\right\}
\end{aligned}$$

(c) Bounded sum

$$\begin{aligned}
&\mu_{\underset{\sim}{T}\oplus\underset{\sim}{R}}(x) \\
&= \min\{1, \mu_{\underset{\sim}{T}}(x)+\mu_{\underset{\sim}{R}}(x)\} \\
&= \min\left\{1, \left\{\frac{0}{0} + \frac{0.3}{1} + \frac{1.0}{2} + \frac{1.0}{3} + \frac{1.3}{4} + \frac{1.5}{5}\right\}\right\} \\
&= \left\{\frac{0}{0} + \frac{0.3}{1} + \frac{1.0}{2} + \frac{1.0}{3} + \frac{1.0}{4} + \frac{1.0}{5}\right\}
\end{aligned}$$

(d) Bounded difference

$$\begin{aligned}
&\mu_{\underset{\sim}{T}\odot\underset{\sim}{R}}(x) \\
&= \max\{0, \mu_{\underset{\sim}{T}}(x)-\mu_{\underset{\sim}{R}}(x)\} \\
&= \max\left\{0, \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.5}{4} + \frac{0.5}{5}\right\}\right\} \\
&= \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.5}{4} + \frac{0.5}{5}\right\}
\end{aligned}$$