Advanced Macro 2 - Assignment 1

Unterweger L., Oberbrinkmann S.

2023-04-17

Preliminary

We hereby declare that the answers to the given assignment are entirely our own, resulting from our own work effort only. Our team members contributed to the answers of the assignment in the following proportions:\

- Unterweger Lucas: 50
- Oberbrinkmann Sophia: 50

Question 1: Business cycles stylized facts (4 Points)

We'll start by setting up our coding environment by importing necessary packages. (Code not shown.)

Now, we can import and clean our data set.

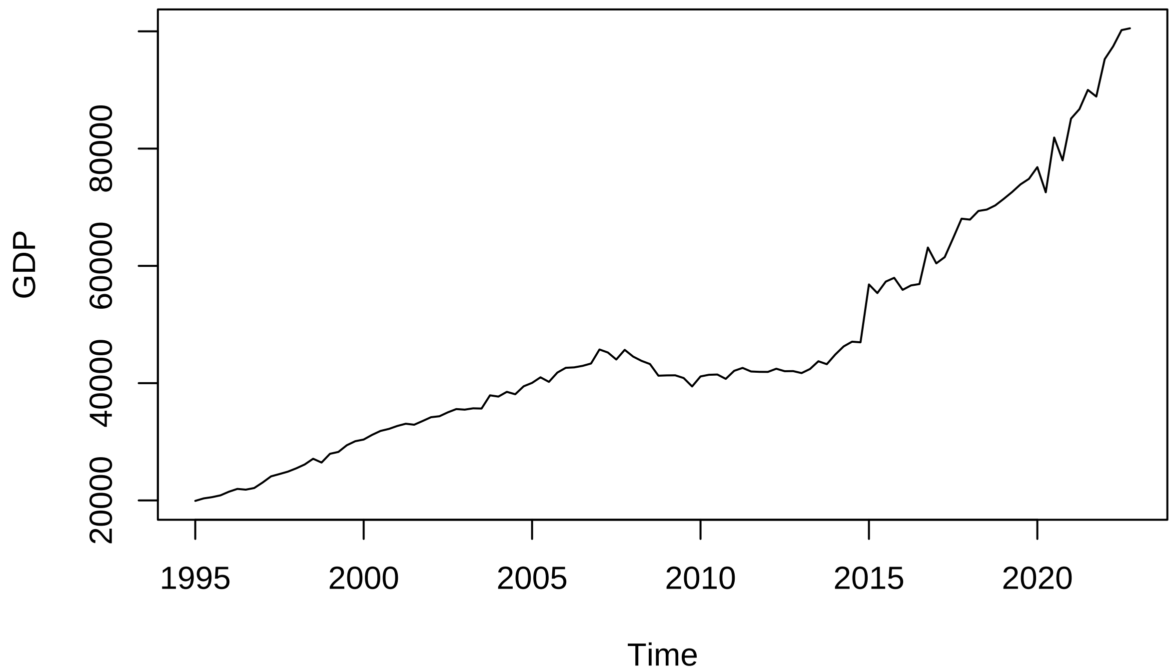
```
ireland <- read_excel("data/Ireland_GDPData.xlsx", sheet = 4)
```

```
## New names:
## * `` -> `...1`
```

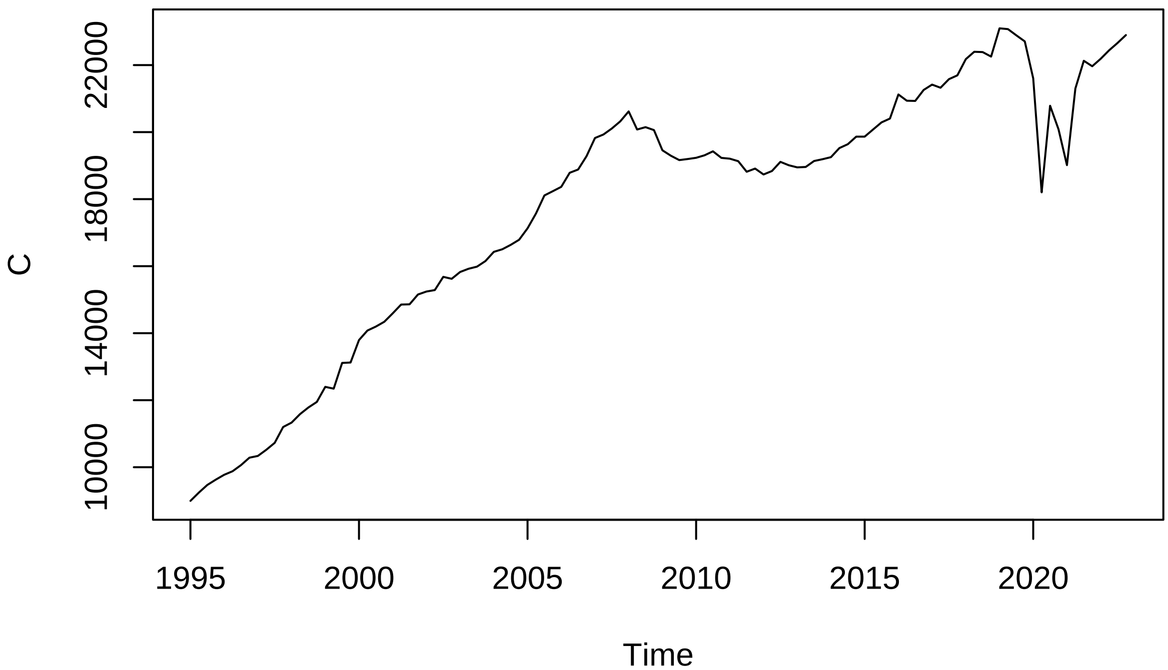
```
colnames(ireland) <- c("t","Y","G","C","I")
head(ireland)
```

```
## # A tibble: 6 x 5
##   t             Y     G     C     I
##   <chr>     <dbl> <dbl> <dbl> <dbl>
## 1 1995-Q1 19924. 4407. 8996. 3573.
## 2 1995-Q2 20344. 4281. 9243. 3754.
## 3 1995-Q3 20559. 4288. 9470. 4123.
## 4 1995-Q4 20866  4138. 9628. 4168.
## 5 1996-Q1 21485. 4561. 9773  4158.
## 6 1996-Q2 21964. 4419. 9879. 4636.
```

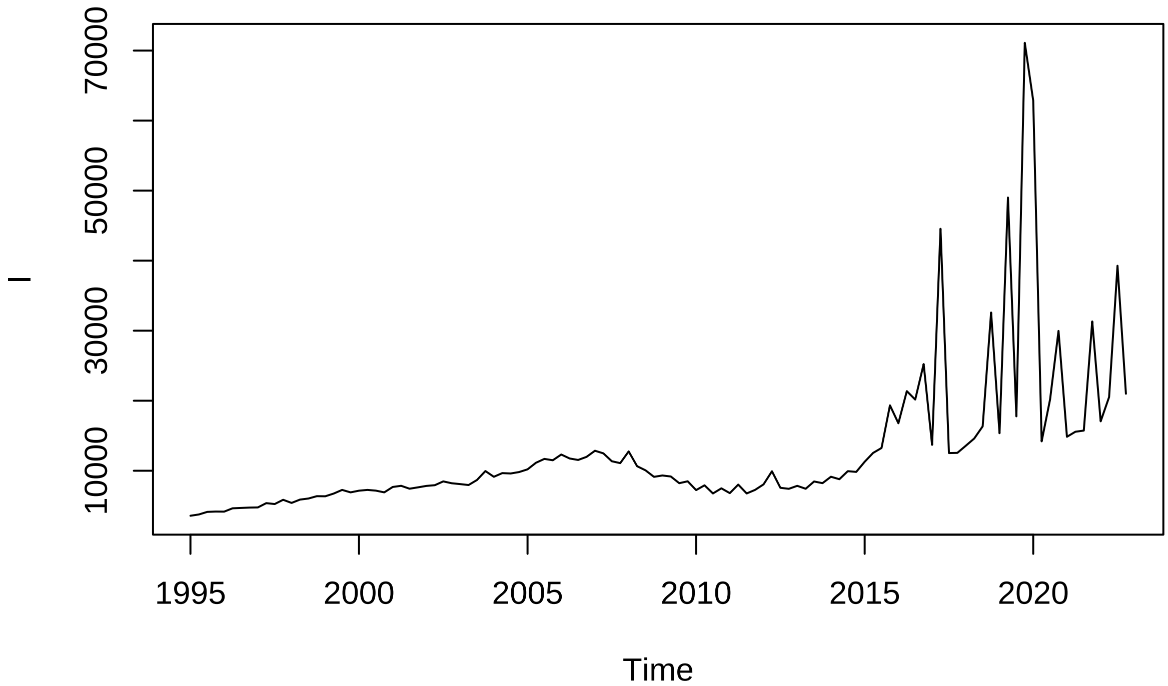
```
GDP <- ts(ireland$Y, start = 1995.0, frequency = 4)
C <- ts(ireland$C, start = 1995.0, frequency = 4)
I <- ts(ireland$I, start = 1995.0, frequency = 4)
G <- ts(ireland$G, start = 1995.0, frequency = 4)
plot(GDP)
```


GDP
20000
40000
60000
80000
1995
2000
2005
2010
2015
2020
Time

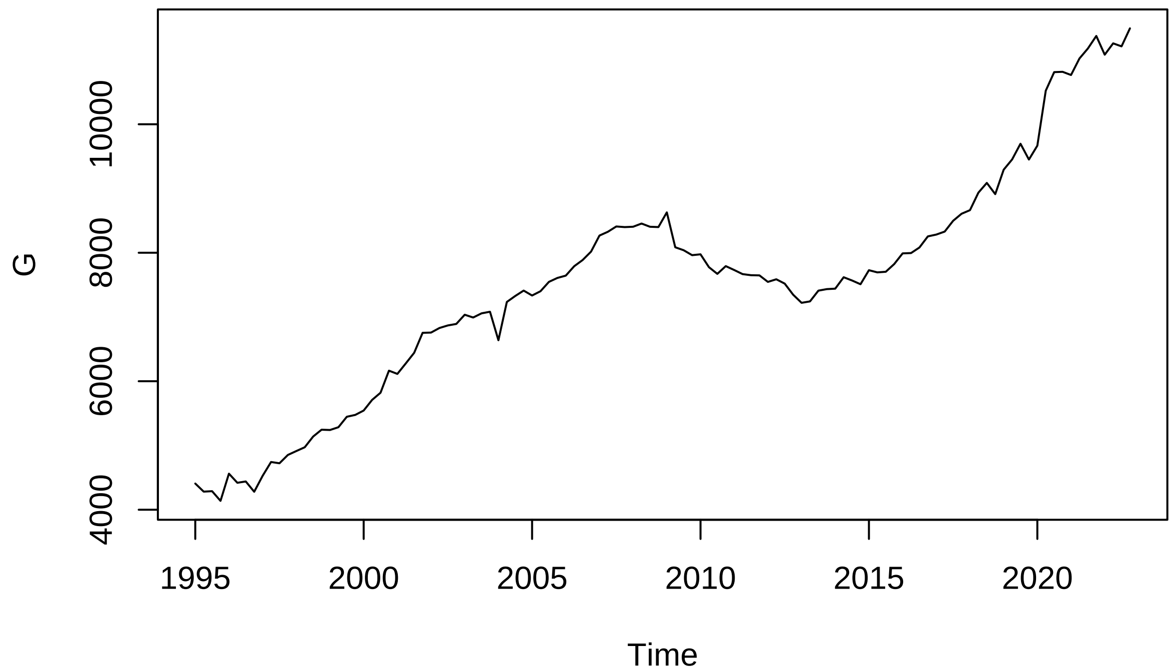
```
plot(C)
```




```
plot(I)
```




```
plot(G)
```



Sample Statistics Let's compute the sample means:

```
CdivGDP <- C/GDP
IdivGDP <- I/GDP
GdivGDP <- G/GDP

mean(CdivGDP)
```

```
## [1] 0.4063485
```

```
mean(IdivGDP)
```

```
## [1] 0.2530171
```

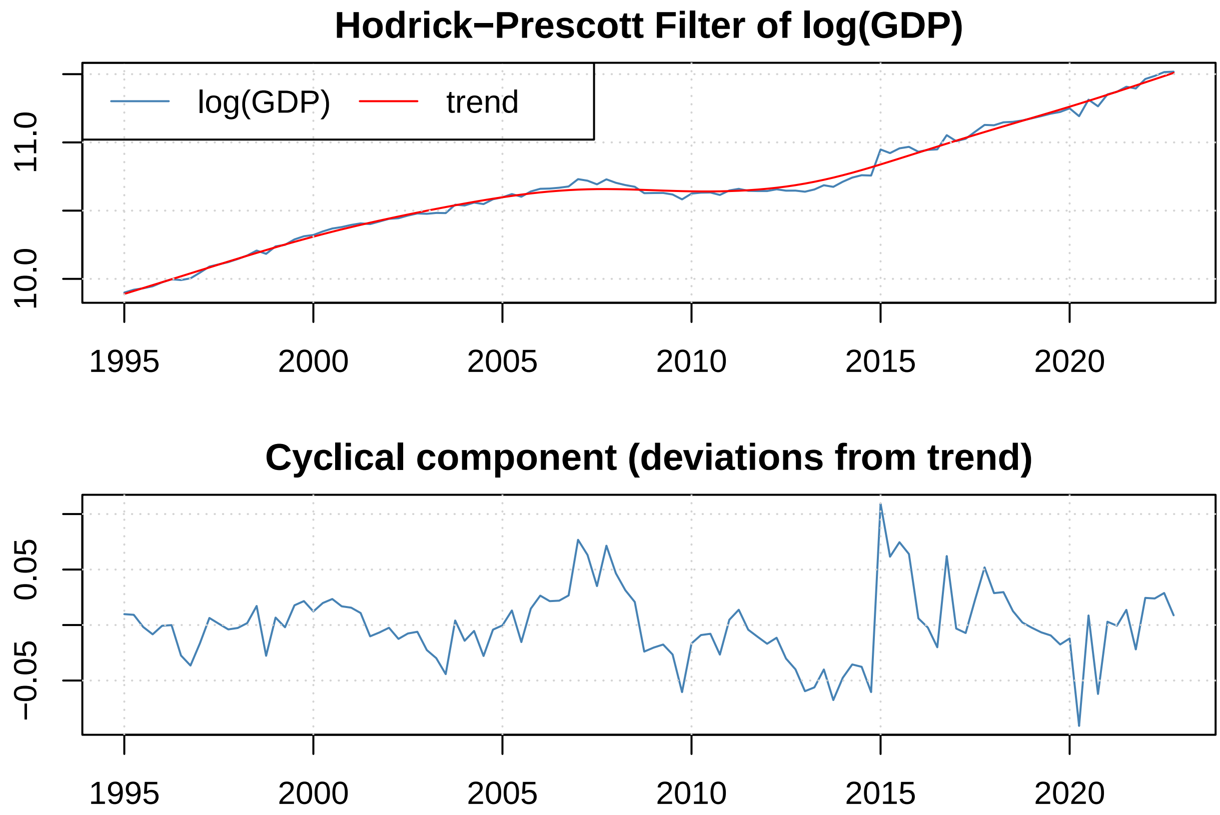
```
mean(GdivGDP)
```

```
## [1] 0.1719054
```

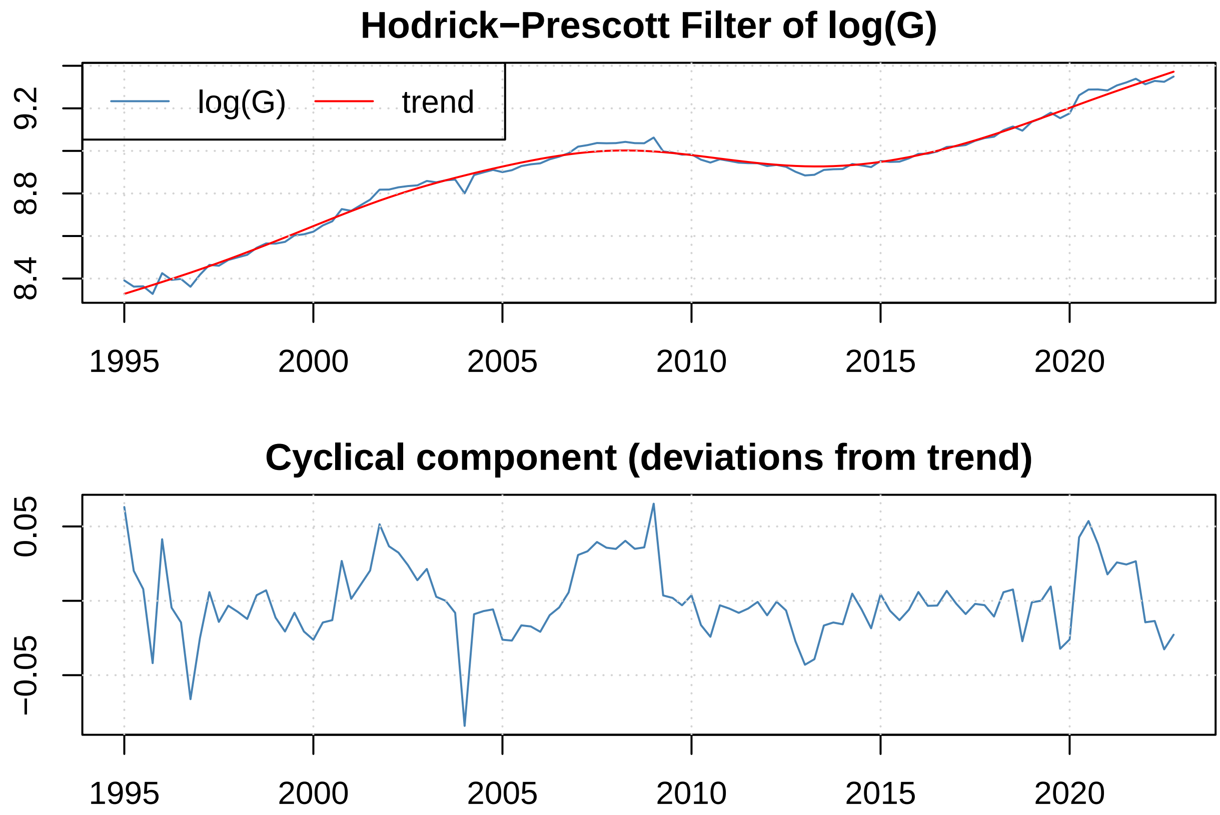
Detrending

Let's apply the HP filter:

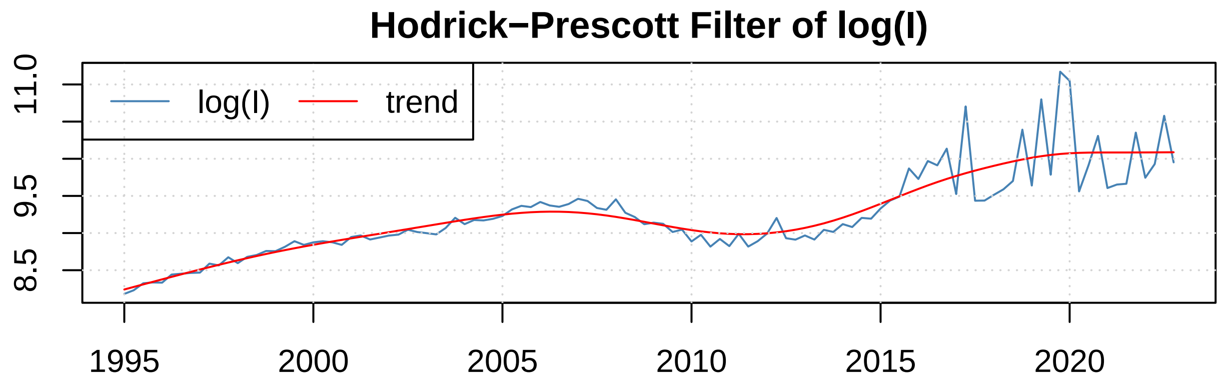
```
plot(hpfilter(log(GDP)))
```

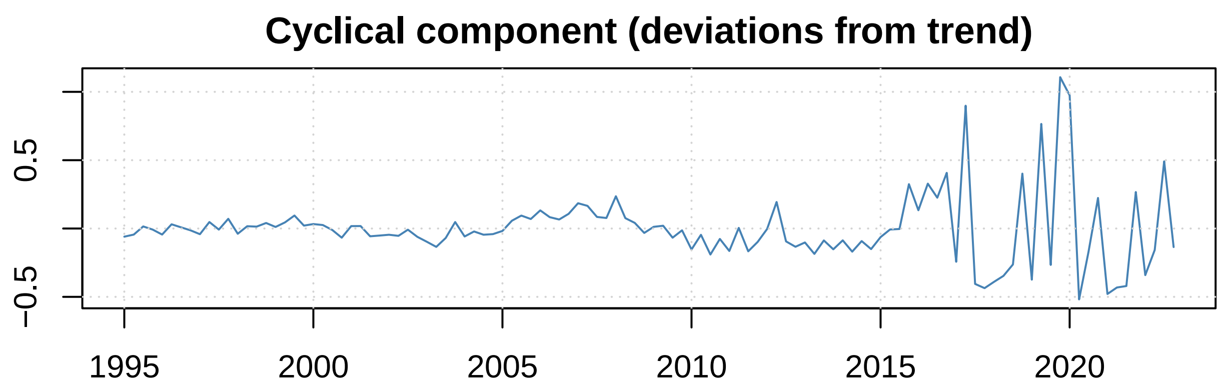



```
plot(hpfilter(log(G)))
```

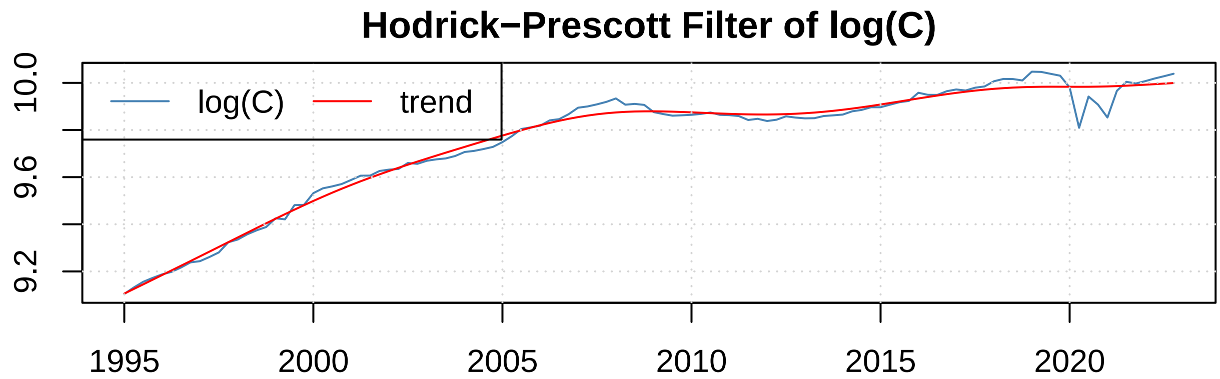


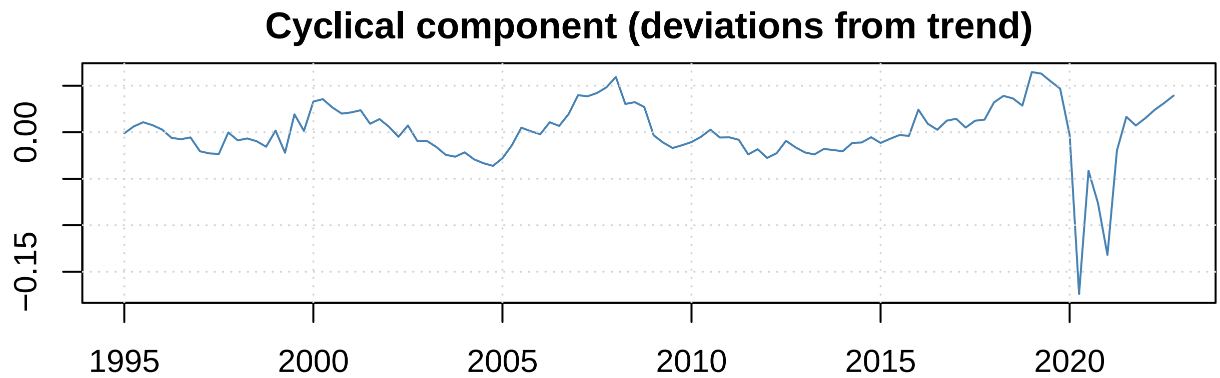

```
plot(hpfilter(log(I)))
```






```
plot(hpfilter(log(C)))
```





Business Cycle Stylized Facts

```
trend_G <- hpfilter(log(G))[1]$cycle
trend_C <- hpfilter(log(C))[1]$cycle
trend_I <- hpfilter(log(I))[1]$cycle
trend_GDP <- hpfilter(log(GDP))[1]$cycle

output_table <- data.frame(
  names = c("GDP","I","C","G"),
  standard_deviation = c(sd(trend_GDP),sd(trend_I),sd(trend_C),sd(trend_G)),
  relative_sd = c(sd(trend_GDP)/sd(trend_GDP),sd(trend_I)/sd(trend_GDP), sd(trend_C)/sd(trend_GDP),sd(t
  cont_output_corr = c(cor(trend_GDP,trend_GDP), cor(trend_I,trend_GDP), cor(trend_C,trend_GDP), cor(tr
    )
output_table
```

```
##   names standard_deviation relative_sd cont_output_corr
## 1   GDP         0.03230920   1.0000000        1.0000000
## 2     I         0.25468238   7.8826591        0.1112197
## 3     C         0.03193810   0.9885142        0.4768935
## 4     G         0.02470573   0.7646655        0.1549818
```

Window 1

```
GDP_W1 <- window(GDP, end=2007.75)
I_W1 <- window(I, end=2007.75)
```

```r
C_W1 <- window(C, end=2007.75)
G_W1 <- window(G, end=2007.75)

trend_G <- hpfilter(log(G_W1))[1]$cycle
trend_C <- hpfilter(log(C_W1))[1]$cycle
trend_I <- hpfilter(log(I_W1))[1]$cycle
trend_GDP <- hpfilter(log(GDP_W1))[1]$cycle

output_table_W1 <- data.frame(
  names = c("GDP","I","C","G"),
  standard_deviation = c(sd(trend_GDP),sd(trend_I),sd(trend_C),sd(trend_G)),
  relative_sd = c(sd(trend_GDP)/sd(trend_GDP),sd(trend_I)/sd(trend_GDP), sd(trend_C)/sd(trend_GDP),sd(t
  cont_output_corr = c(cor(trend_GDP,trend_GDP), cor(trend_I,trend_GDP), cor(trend_C,trend_GDP), cor(tr
  )
```

Window 2

```r
GDP_W2 <- window(GDP, start=2008.0)
I_W2 <- window(I, start=2008.0)
C_W2 <- window(C, start=2008.0)
G_W2 <- window(G, start=2008.0)

trend_G <- hpfilter(log(G_W2))[1]$cycle
trend_C <- hpfilter(log(C_W2))[1]$cycle
trend_I <- hpfilter(log(I_W2))[1]$cycle
trend_GDP <- hpfilter(log(GDP_W2))[1]$cycle

output_table_W2 <- data.frame(
  names = c("GDP","I","C","G"),
  standard_deviation = c(sd(trend_GDP),sd(trend_I),sd(trend_C),sd(trend_G)),
  relative_sd = c(sd(trend_GDP)/sd(trend_GDP),sd(trend_I)/sd(trend_GDP), sd(trend_C)/sd(trend_GDP),sd(t
  cont_output_corr = c(cor(trend_GDP,trend_GDP), cor(trend_I,trend_GDP), cor(trend_C,trend_GDP), cor(tr
    )
```

Output

```r
output_table
```

```
##   names standard_deviation relative_sd cont_output_corr
## 1   GDP         0.03230920   1.0000000        1.0000000
## 2     I         0.25468238   7.8826591        0.1112197
## 3     C         0.03193810   0.9885142        0.4768935
## 4     G         0.02470573   0.7646655        0.1549818
```

```r
output_table_W1
```

```
##   names standard_deviation relative_sd cont_output_corr
## 1   GDP         0.01582877    1.000000        1.0000000
## 2     I         0.05050510    3.190716        0.4005525
## 3     C         0.01591679    1.005561        0.4630622
## 4     G         0.02558119    1.616120        0.1368501
```

```r
output_table_W2
```

```
##   names standard_deviation relative_sd cont_output_corr
```

```
## 1   GDP          0.03803500   1.0000000      1.000000000
## 2     I          0.34357331   9.0330832      0.054996823
## 3     C          0.03783447   0.9947279      0.392198387
## 4     G          0.01911173   0.5024774      0.006569063
```

Question 2: A real business cycle model with energy price shocks (5 points)

Expected discounted utility of the representative household:

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta_t \frac{C_t^{1-\sigma}}{1-\sigma} + \theta \log(1 - N_t) \right] \tag{1}$$

subject to

$$C_t + K_{t+1} + P_t EN_t \leq A_t K_t^{\alpha} N_t^{\gamma} EN_t^{1-\alpha-\gamma} + (1-\delta) K_t \tag{2}$$

(a) Social Planner's Intertemporal Optimization Problem

We start by setting up our Lagrangian system where we maximize (1) which the constraint (2). The Lagrangian is then given by

$$L_t = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \theta \log(1 - N_t) + \lambda_t \left(A_t K_t^{\alpha} N_t^{\gamma} EN_t^{1-\alpha-\gamma} + (1-\delta) K_t - C_t - K_{t+1} - P_t EN_t \right) \right\} \right]$$

We can now use *first order optimality conditions* and optimize w.r.t the *current consumption* C_t,*tomorrows capital stock* K_{t+1},*hours worked* N_t,*energy consumption* EN_t and the *Lagrange multiplier* λ_t. This results in the following five equations:

$$\frac{\partial L}{\partial C_t} = E_0 \beta^t \left\{ (1-\sigma) \frac{C_t^{-\sigma}}{1-\sigma} - \lambda_t \right\} \stackrel{!}{=} 0 \tag{3}$$

$$\iff C_t^{-\sigma} = \lambda_t \tag{4}$$

Question 3: Understanding impulse responses and model simulation (total of 8 points)

(a) Steady State

The following parameters are given:\ $\beta = 0.99$, $\sigma = 1$, $\gamma = 0.65$,$\theta = 3.48$, $\alpha = 0.3$, $\delta = 0.025$, $\bar{A} = 1$, $\bar{P} = 1$, $\rho_A = 0.95$, $\sigma_A = 0.007$, $\rho_P = 0.5$, $\sigma_P = 0.00001$