ENGS 26: Control Theory

Duck Car Final Project

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Part 1. System Modeling

1st approach: Current Probe

This method requires attaching a current probe to our Duck Car and feeding a square wave into it. On an oscilloscope, we can observe the motor transient response to find our time constant. Additionally, measuring the motor current transient allows us to calculate the torque, since current is proportional to torque. The absolute current corresponds to the absolute torque, which is determined by the gear ratio, wheel diameter, and wheel shaft. With this information, we can then determine the acceleration of the car.

2nd approach (the method we chose): Tracker app

The process involved filming the duck car running along a straight path, specifically under the overpass between Maclean and Cummings (Figure 1). The video captured the duck car from a stationary position until it reached a steady-state velocity.

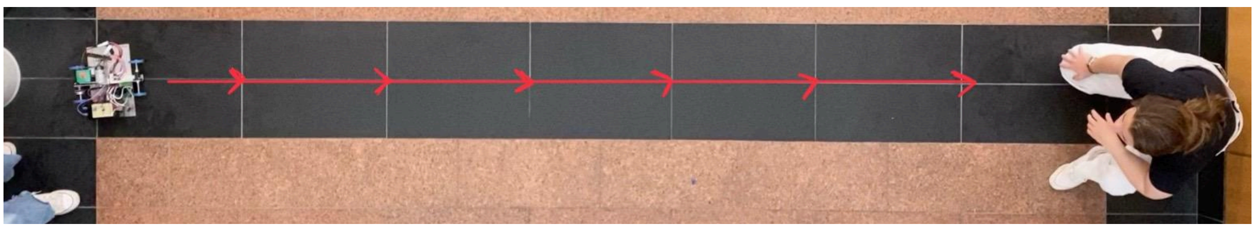

Figure 1. Overhead video of our duck car running along a straight path that got uploaded into the tracker software.

After filming, we uploaded the video into the Tracker app and tracked the car's position, time, and velocity. Transferring the data to MATLAB, we plotted the data to find relevant values such as the time constant (**τ**) and gain (K) of the motor (Figure 2).

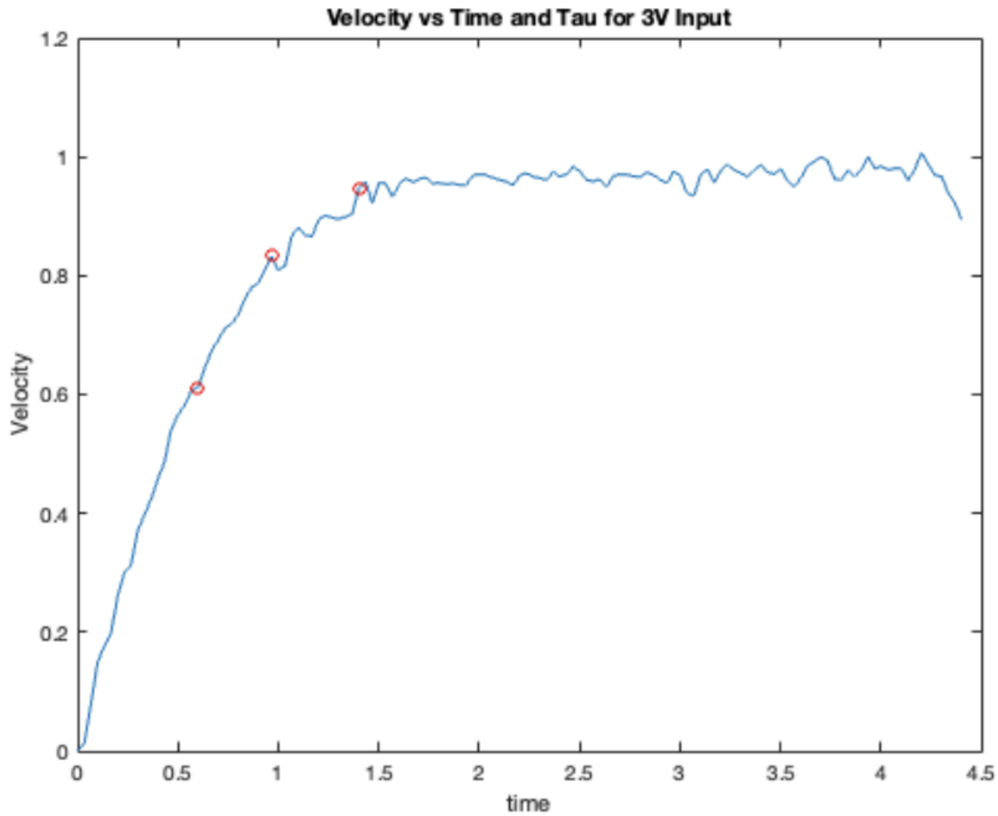

Figure 2: Plot of velocity vs time of the duck car using data from video capture out of Tracker App.

We analyzed this graph to find the steady state velocity and the time constant, **τ**, to define the motor of the system. First, the motor gain was determined by measuring the steady state velocity of our velocity vs time plot in Figure 3. Doing so the following motor gain value was determined:

$$K = 0.87481 \text{ m/s/V}$$

We determined **τ** by calculating the time it took the system to reach 63% (**τ**), 86% (2**τ**), and 95% (3**τ**) of the steady state. Our work was done in MATLAB and is shown in Figure 3. By averaging these values, we obtained the best approximation of the motor's time constant (Figure 4). After running the average code, the following time constant value was determined:

$$\boldsymbol{\tau} = 0.9894\text{s}$$

```
%Find Steady State and Tau Values
SteadyStategood1 = mean(velocitygood1(52:end))
ssz1 = 0.63*SteadyStategood1;
```

```
ssz2 = 0.86*SteadyStategood1;
ssz3 = 0.96*SteadyStategood1;

indexssgood1 = find(velocitygood > ssgood1+velocitygood(1), 1, 'first');
indexssgood2 = find(velocitygood > ssgood2+velocitygood(1), 1, 'first');
indexssgood3 = find(velocitygood > ssgood3+velocitygood(1), 1, 'first');

%Tau values at 0.63, 0.86, 0.95 percent of Steady State
tgood1 = timegood(indexssgood1);
tgood2 = timegood(indexssgood2);
tgood3 = timegood(indexssgood3);
```

*Figure 3. Matlab Code to find 1***τ***, 2***τ***, and 3***τ*

```
%Tau Value
taugood = mean([tgood1,tgood2,tgood3])
```

Figure 4: Matlab code to find final (mean) tau, by averaging the three different tau values

We know the motor transform transfer function comes in the following form: $\frac{K}{s(\boldsymbol{\tau} s+1)}$. We used the values we calculated above and plugged them into this form to find the motor transfer function. Plugging in our **τ** and K value above, we get the following transfer function:

$$G_m(s) = \frac{0.87481}{s(0.9894s+1)}$$

Next, to find the sensor gain, we connected the duck car to a voltmeter and tested it at different distances from a wall. We noted the sensor's voltage at each distance and then plotted the distances and its accompanying sensor voltages. The following relationship is displayed in Figure 5.

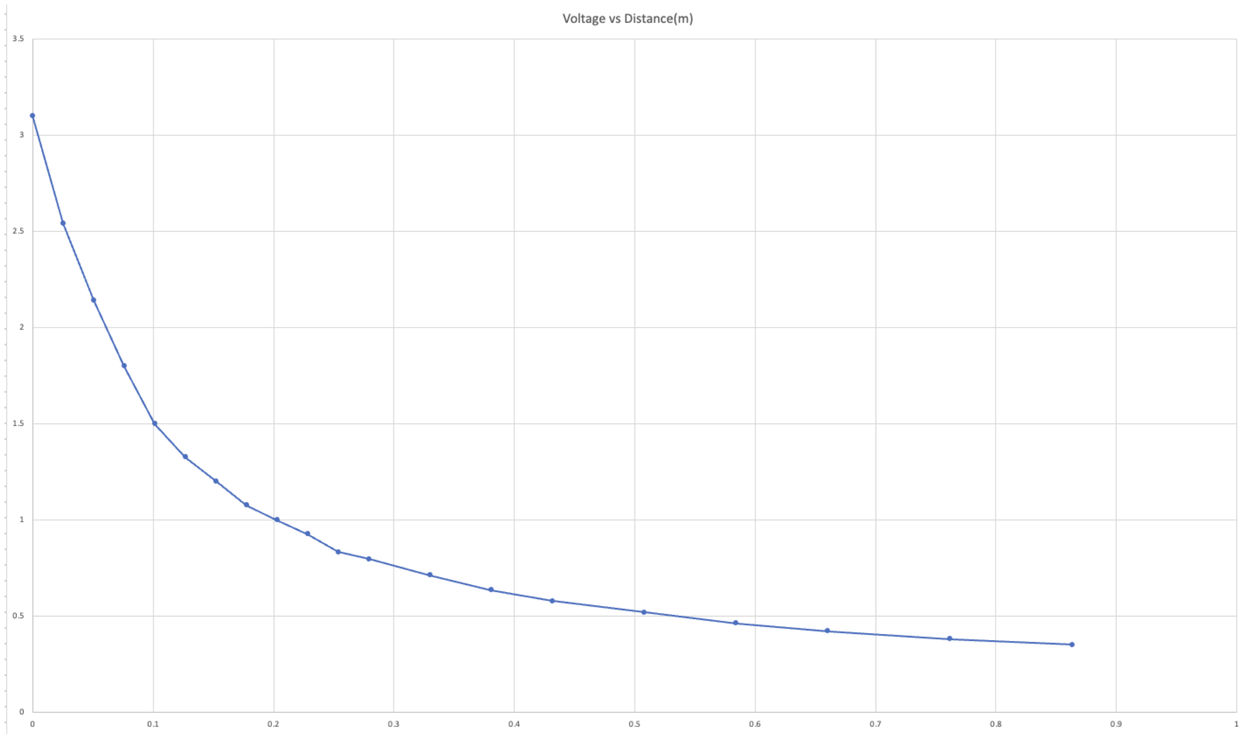

Figure 5: Sensor Voltage vs Distance from wall measured in meters

We decided to use a reference voltage of 2V which made our operating point approximately 2.5 inches from the wall. 2.5in is enough for a small buffer zone to account for some overshoot and ensures our car does not slam into the wall. However, it also allows for our car to be more reactive and faster. We linearize the system around our target voltage of 2V by finding the slope of the tangent line at that point. This tangent line is displayed in Figure 6.

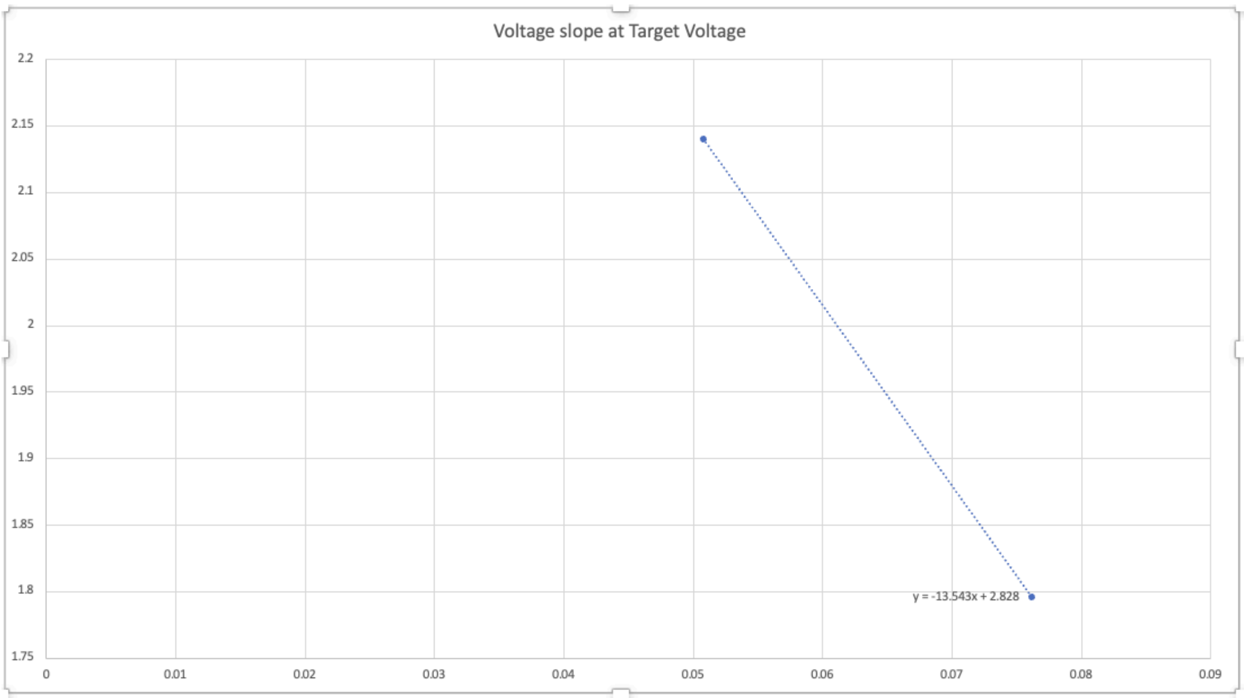

Figure 6. A graph showing the linearized slope at 2V from Figure 5

The linearized slope at target voltage 2 V = 13.543. This value is also our sensor gain value, therefore Ka = 13.543 V/m.

Block Diagram

Combining all the components to create an open loop transfer function, we refer to the following block diagram of our system (Figure 7):

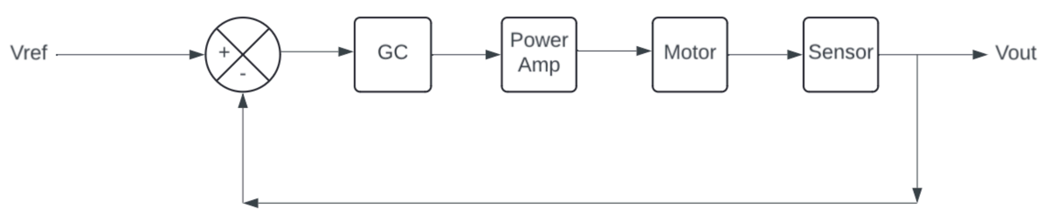

Figure 7: Block diagram of our transfer function with Compensator (G_c), Power Amp, Motor, and Sensor.

The equation of our open loop transfer function is of the following form:

$$G(s) = G_c \times Power\ Amp \times Motor \times Sensor$$

$$G(s) = 1 \times 1 \times \frac{0.87481}{s(0.9894s+1)} \times 13.543$$
$$= \frac{11.848}{s(2.263s + 1)}$$

We assumed that the power amp, K_a, is equal to 1. We set the compensator $G_c = 1$ because the system is uncompensated.

Part 2. Analysis of the Uncompensated System

The following figures show the analysis of our uncompensated system in MATLAB sisotools.

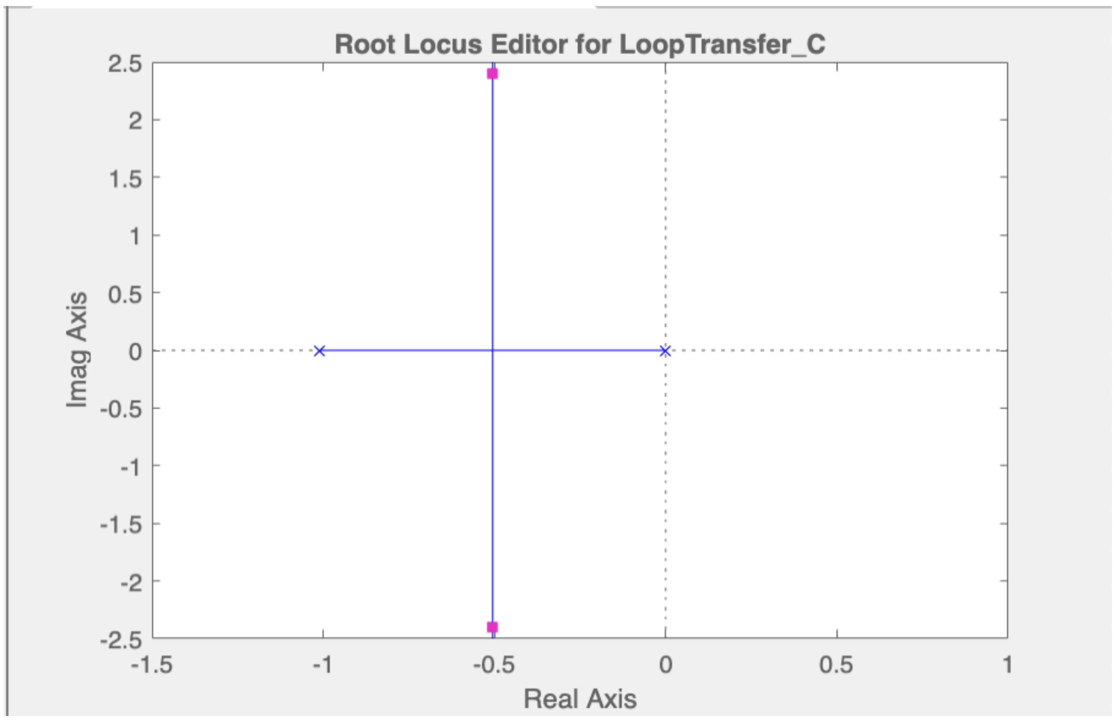

Figure 8. Root locus of our uncompensated system

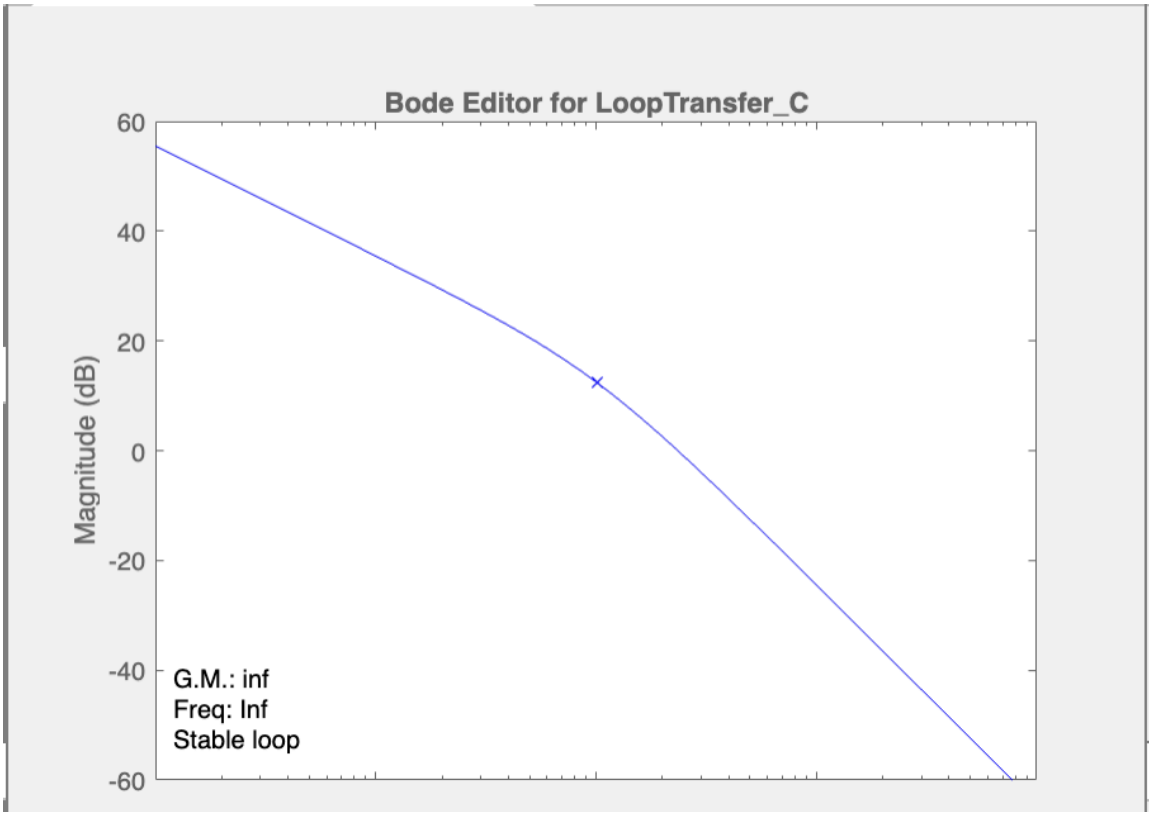

Figure 9. Gain bode plot of uncompensated system

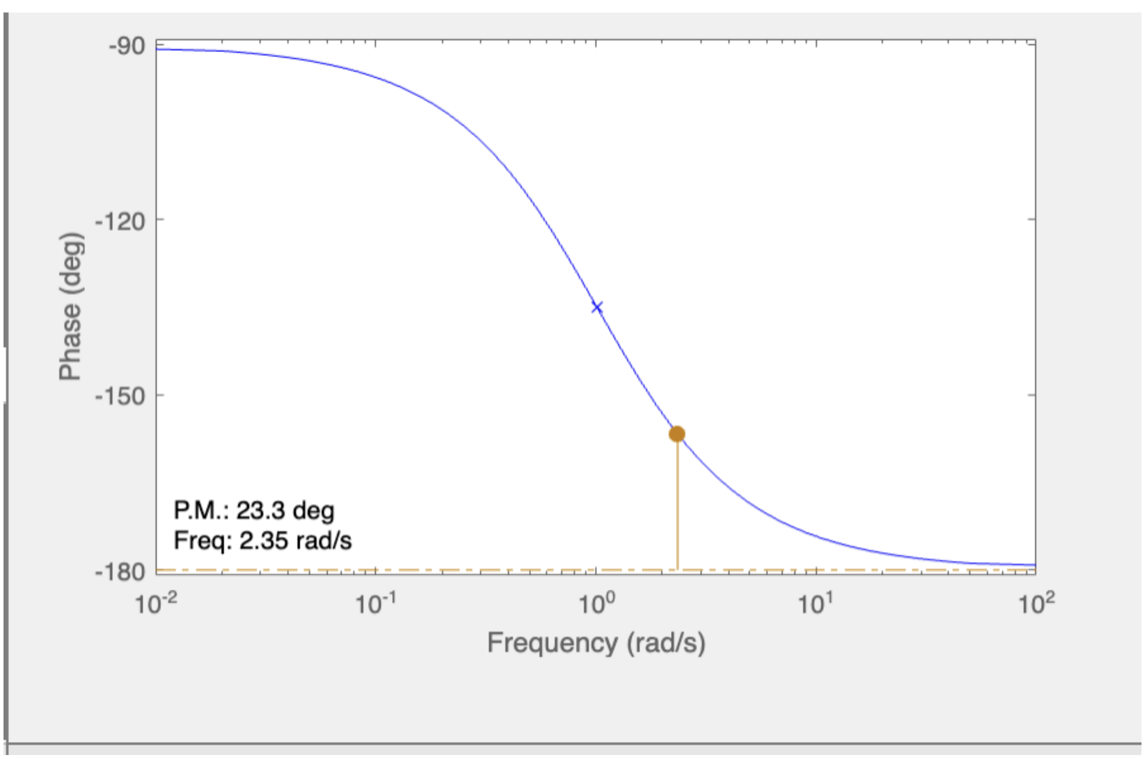

Figure 10. Phase Bode plot of uncompensated system

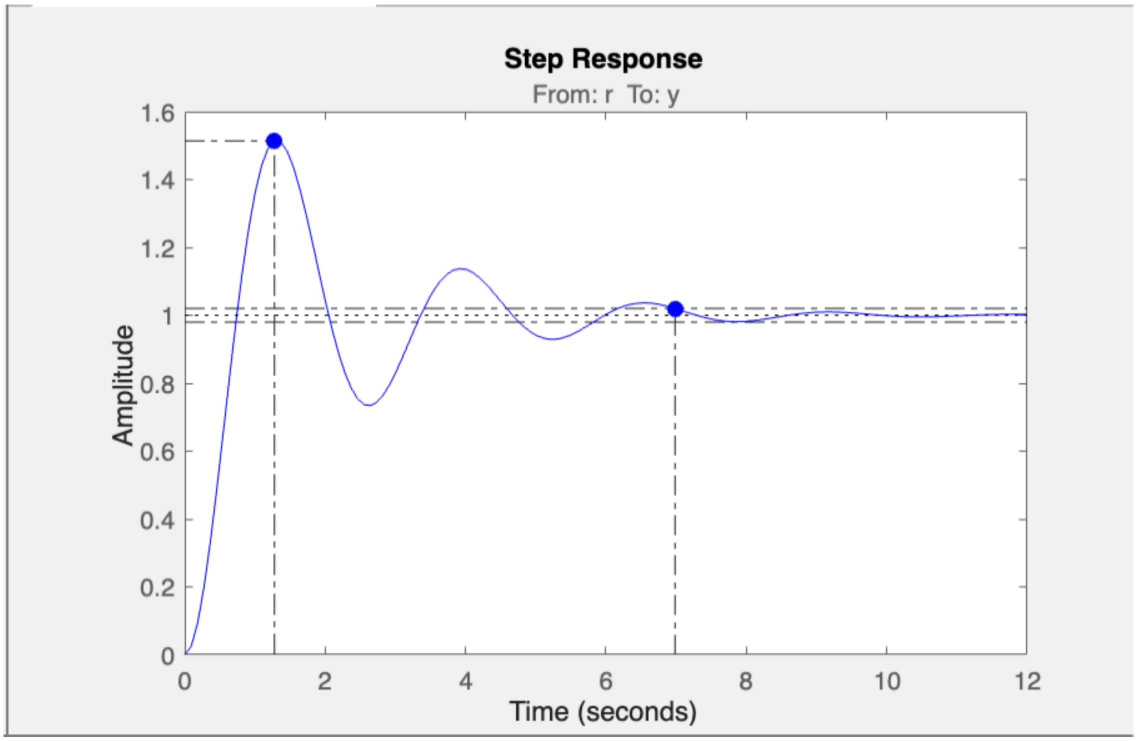

Figure 11. Step response of our uncompensated system

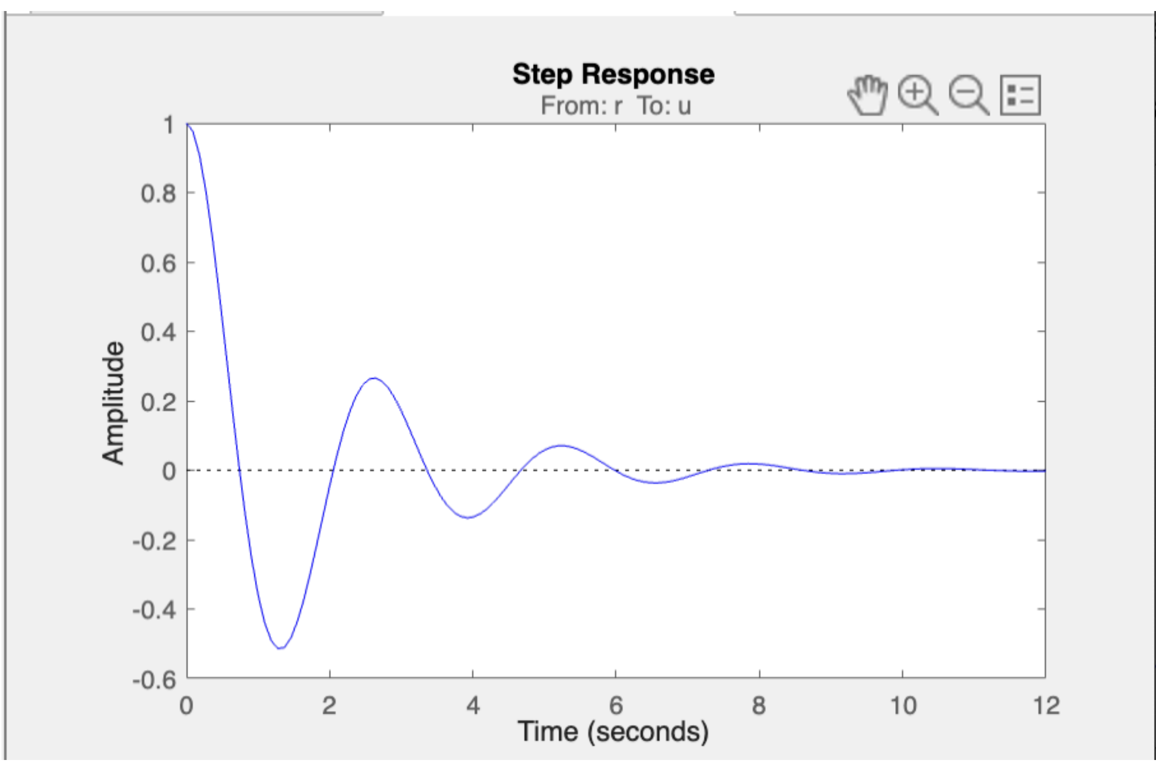

Figure 12. Control effort of our uncompensated system

Note: in the images above, our gain is equal to one.

Time domain analysis

The system is stable since all poles and zeros and the root locus are located in the left half plane of the complex plane. This indicates that the natural response of the system will decay over time, ensuring stability.

A settling time of 6.98 seconds is considered far too high, implying that the system has a slow transient response. This slow response can be undesirable in many applications where quick stabilization is critical.

A peak overshoot of 51.4% is quite high, indicating that the system experiences significant oscillations before settling to its final value. This level of overshoot suggests that the system has poor damping characteristics, potentially leading to oscillatory behavior that may be undesirable in precise control applications.

An amplitude of 1.5 is considered satisfactory, indicating that the system does not require excessive control input to achieve its response. This suggests that the system is stable and the control effort is within acceptable limits, making the control strategy effective.

Frequency domain analysis

A phase margin of 23.3º is generally satisfactory, indicating that the system has a reasonable buffer before becoming unstable. This suggests that the system is stable but might be close to the threshold where instability could occur if additional phase lag is introduced.

An infinite gain margin indicates that no finite increase in gain will make the system unstable. This is a very good gain margin, implying that the system is robustly stable with respect to gain variations.

Part 3. Time- and Frequency-Domain Design Specifications

1. The closed loop system must be stable. All poles and zeros must be on the left side of the root locus.

The poles and zeros of our uncompensated system are initially located on the left-hand side of the s-plane. To maintain the stability characteristics and initial positions of these poles and zeros, it is essential to place our compensator on the left side as well. This strategic placement ensures that the compensator enhances system performance without altering the fundamental stability properties of the uncompensated system. By doing so, we can achieve the desired improvements while preserving the original system dynamics.

2. Settling Time < 3s

The initial analysis of the uncompensated system revealed a settling time of 6.49 seconds. This duration is excessively long for our duck car demonstration, necessitating an improvement in system performance. To ensure a more responsive and practical demonstration, a revised settling time of under 2 seconds is deemed appropriate. While a settling time of less than 1 second could be ideal for some applications, it might impose overly stringent requirements on our compensator design, potentially leading to unnecessary complexity and instability. Therefore, the target settling time range between 1 and 2 seconds offers a balanced approach, providing a swift yet achievable performance enhancement.

3. Overshoot: <20%

The uncompensated system exhibited an overshoot value of 26.3%, which is considerably high. To improve system performance, our goal is to reduce this overshoot to less than 20%. Achieving an overshoot below 20% represents a significant improvement while remaining feasible. While aiming for an overshoot of less than 10% could be beneficial, it may impose excessively stringent requirements on our compensator design. Thus, targeting an overshoot reduction to less than 20% strikes a balance between improved performance and practical implementation.

4. Closed loop e_{ss} to a step input = 0

This specification is essential for our system, as it ensures that the output precisely follows any sudden change in the desired input without persistent deviation. Such precision is particularly important for applications like our duck car demonstration, where accurate tracking of the setpoint is crucial for performance.

5. Max output for power amp: 12V

The control effort of our uncompensated system has an amplitude of 0.5. The max output of the power amp is 12V making it a reasonable max spec. We still have a lot of wiggle room in terms of control effort when designing our compensator.

6. Gain margin of at least 6dB

The gain margin of our uncompensated system is effectively infinite, indicating a robust level of stability well beyond the required specification. It is crucial that the addition of a compensator does not compromise this high gain margin. Our design goal is to enhance system performance while maintaining, or minimally impacting, the existing gain margin to ensure continued stability and reliability.

7. Phase margin of at least 30 degrees

The phase margin of our uncompensated system is 42.2 degrees, which is approaching the critical threshold of 30 degrees. This proximity necessitates careful consideration when adding a compensator, as any changes must not compromise the phase margin specification. Ensuring that the compensator maintains or improves the phase margin is essential to preserving system stability and avoiding potential performance degradation.

8. The maximum magnitude of the closed-loop frequency response = 1 dB above the DC value.

The specification that the maximum magnitude of the closed-loop frequency response is 1 dB above the DC value ensures system stability and performance. This tight limit prevents excessive amplification of certain frequencies, which could lead to instability or oscillations. By keeping the frequency response within 1 dB of the DC gain, we maintain consistent and predictable system behavior.

9. Closed loop bandwidth: must be maximized and optimized

This specification can be optimized by placing pole(s) as far left as possible.

10. Voltage into motor is no more than 5V

To avoid saturation of the system ensure that the input voltage does not lead the system to process more than 12 volts. By supplying the motor with more than 5V the system is more likely to saturate.

Part 4. The Compensator design

The following figures show our compensated system design and analysis using MATLAB sisotools. We decided to implement a lead compensator with a stop resistor.

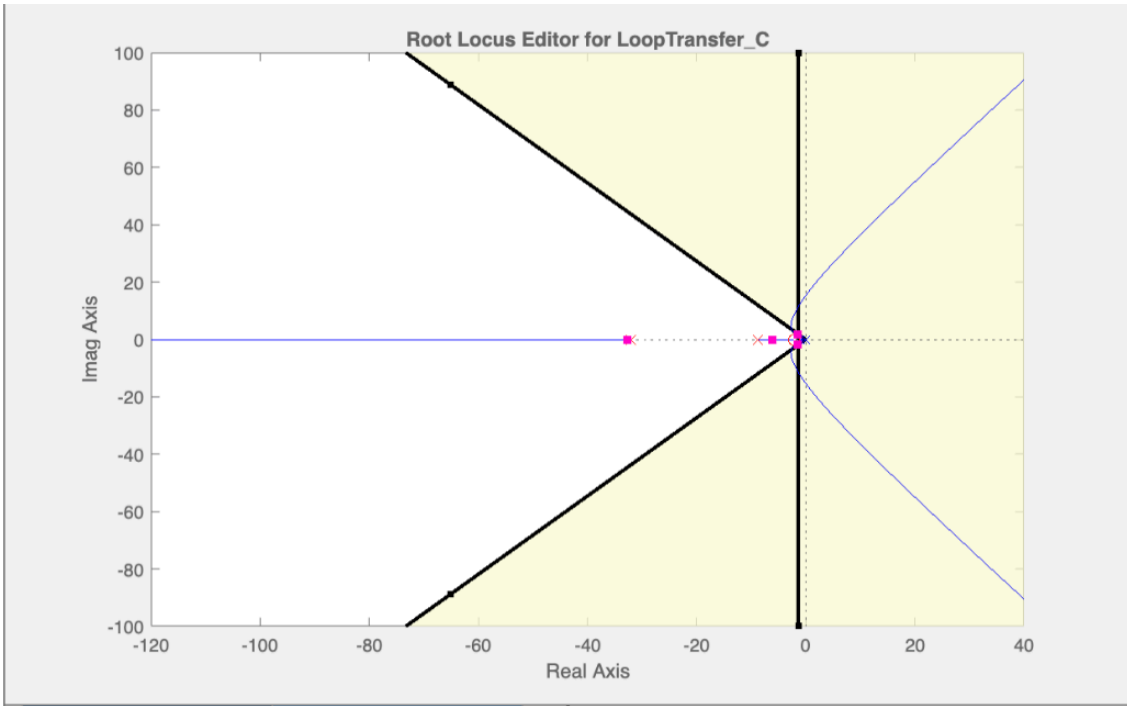

Figure 13. Root locus of compensated system

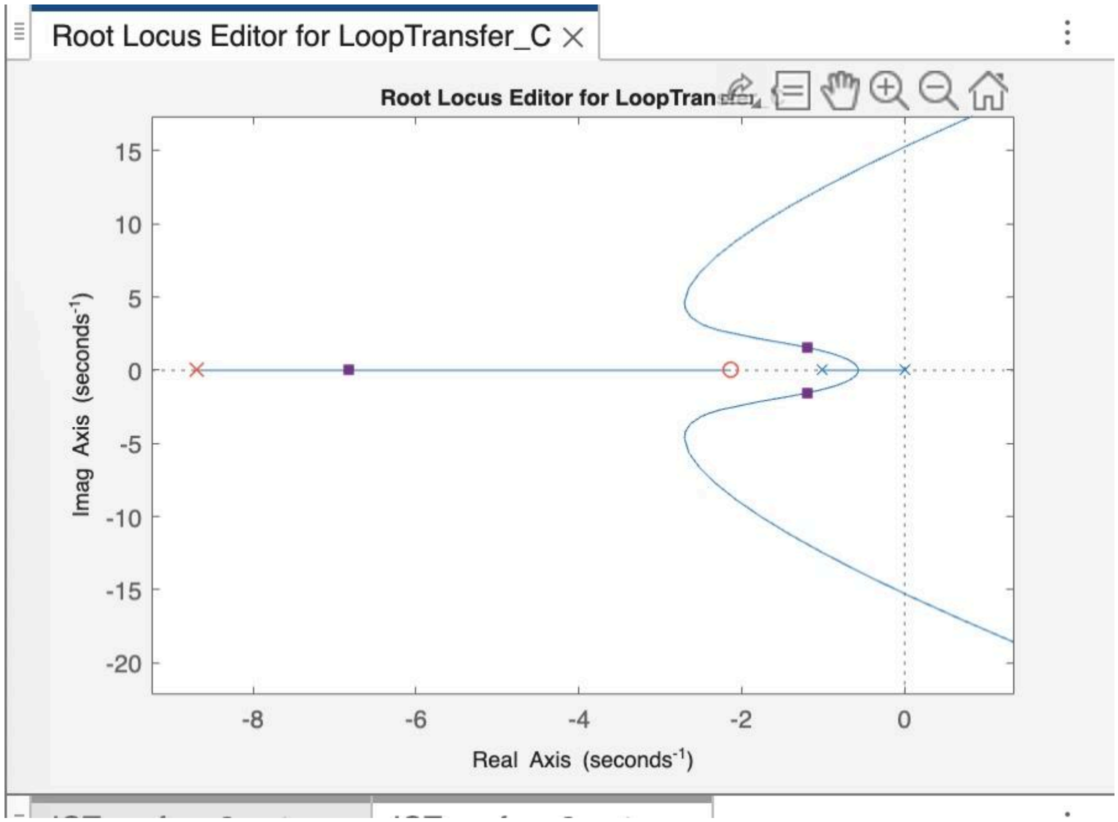

Figure 14. A zoomed in root locus of the compensated system, showing the effect of the lead compensator more clearly

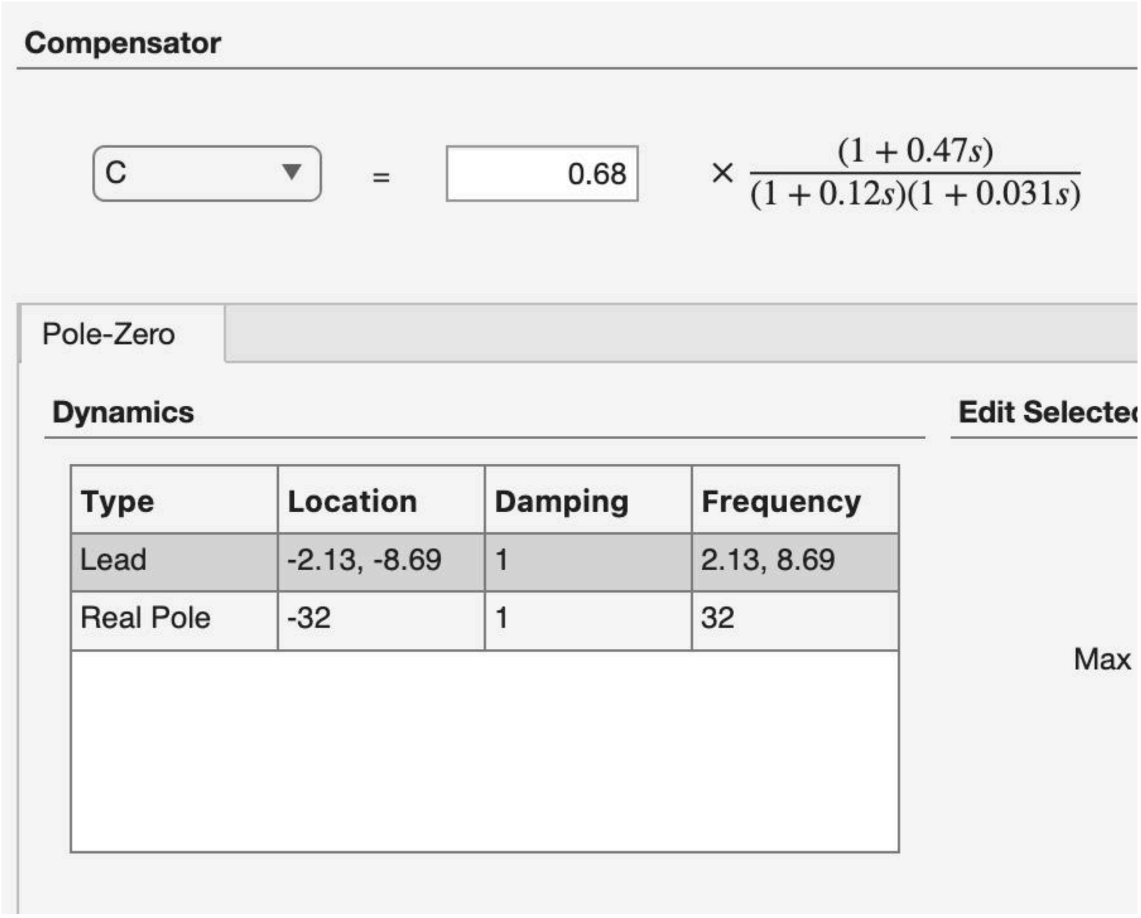

Figure 15. The designed compensator showing the location of the lead (pole and zero), other pole, and gain value of compensator

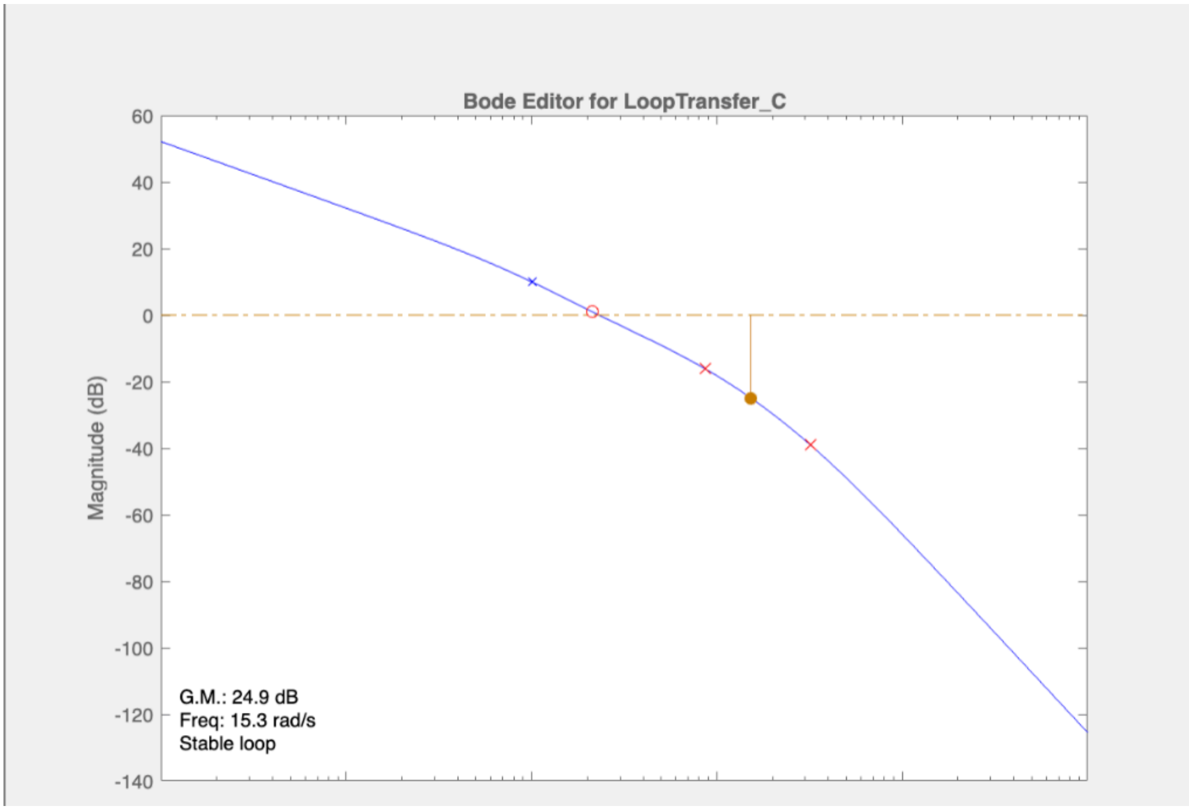

Figure 16. Gain bode plot of compensated system

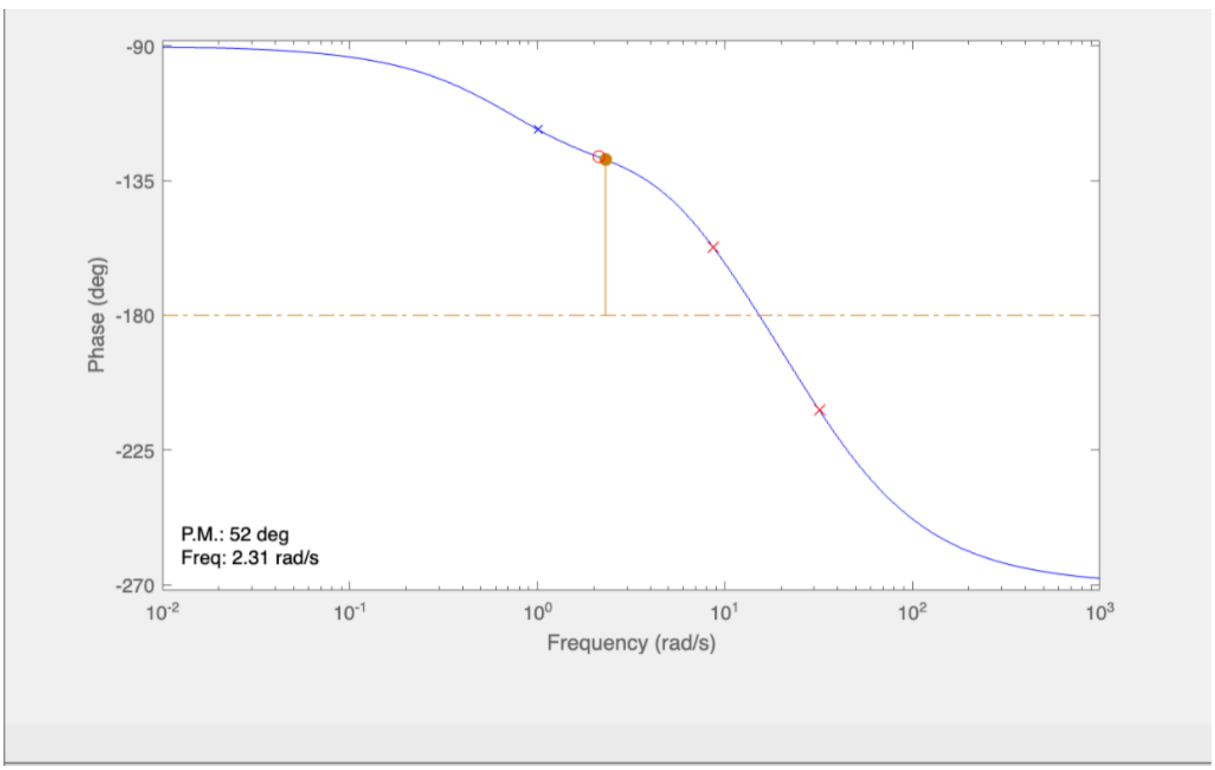

Figure 17. Phase bode plot of compensated system

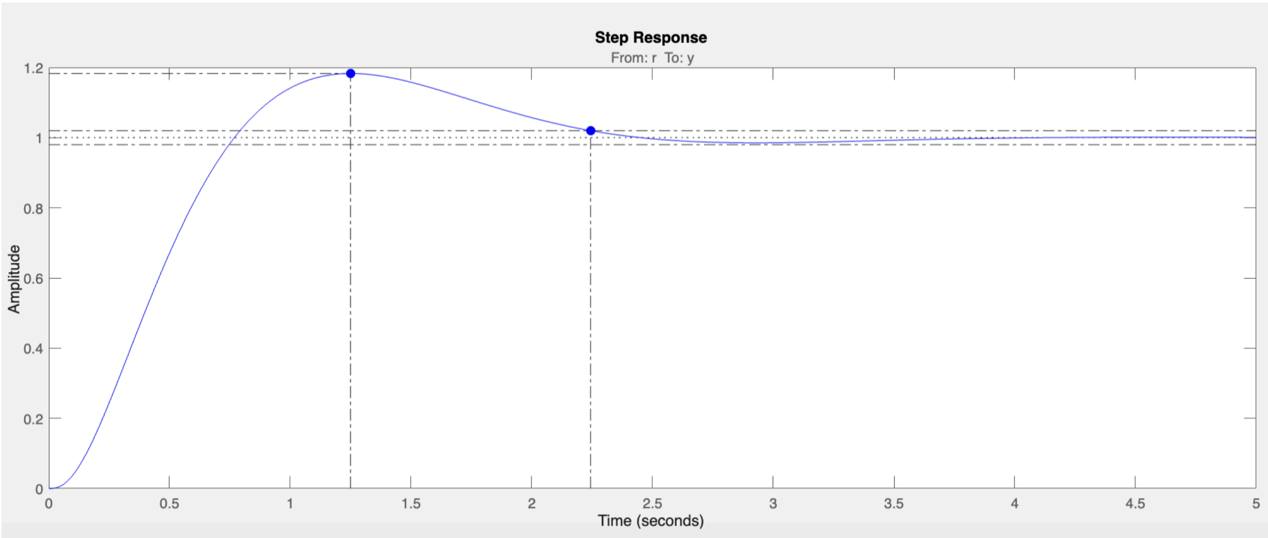

Figure 18. Step response of compensated system

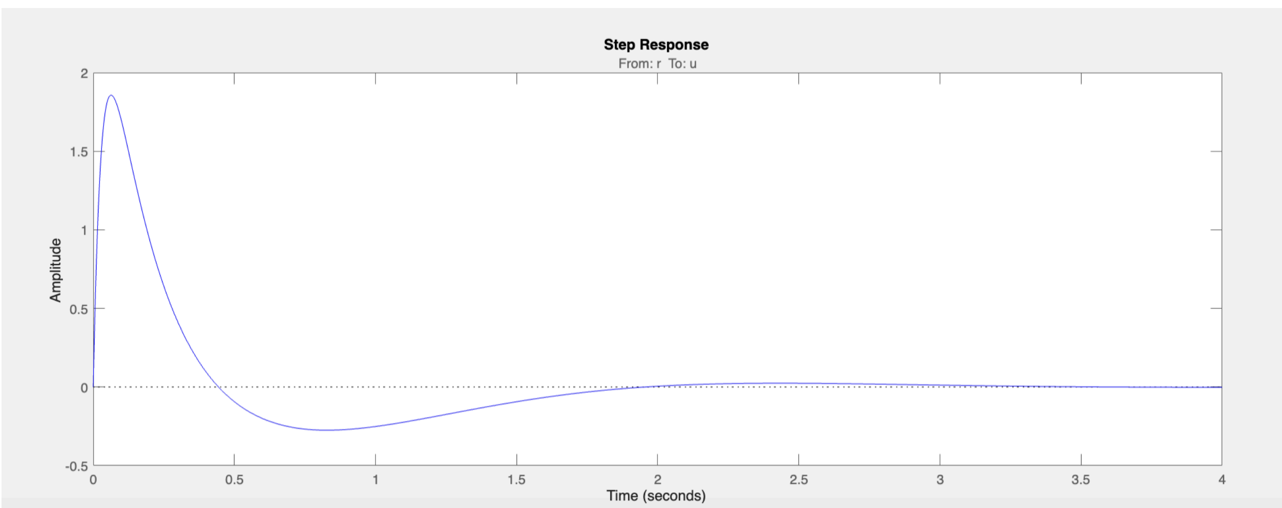

Figure 19. Control effort of compensated system

Verification that our system meets the specifications:

Design Specifications	Value	Our Compensator Value	Does it meet the spec?
Stable - Poles and Zeros on left hand plane	N/A	Stable - Poles and Zeros on left hand plane	YES
Peak Response	< 20%	18.3%	YES

Settling Time	< 3s	2.24s	YES
Control Effort	< 12V	2V	YES
Gain margin	≥ 6dB	24.9 dB	YES
Phase margin	≥ 30º	52º	YES
Input voltage	≥5V	2.2V	YES
Bandwidth	Maximized and optimazed	To meet this requirement, we extended our lead pole as much as possible without compromising the system's responsiveness. Pushing the bandwidth further would risk saturating our control efforts and delay the settling time.	YES
Voltage into motor	< 5V	2V * 2= 4	YES

Table 1. Demonstrating how our compensator meets the specifications we set in part 2 of this report.

By graphing our uncompensated system in sisotools we were able to see the effect different competitors had on our specifications. Once the root locus and compensator met our specs on the unit step response graph and the compensator was noted satisfactory, we built the compensator into our physical system. This process is noted in part 5. The compensator was then tested through visual analysis and deemed satisfactory or unsatisfactory. We ran through this process numerous times taking note of how the system responded to each different compensator.

We noted that a lead compensator reduced settling time and peak overshoot well, but at the expense of a higher control effort. The extra pole, stop resistor, was added as it significantly reduced control effort. The final compensator was thus determined as a lead with a stop resistor.

Part 5. Implementation of the compensator

To take our compensator and design a circuit, we had to compare our transfer function to the system model. A lead compensator with a stop resistor has the transfer function:

$$G_c(s) \;=\; -\frac{R_2}{R_1}\left(\frac{1+R_1C_1s}{1+R_2C_2s}\right)\frac{1}{1+R_3C_1}$$

The circuit diagram for this compensator and transfer function is shown in Figure 20 below.

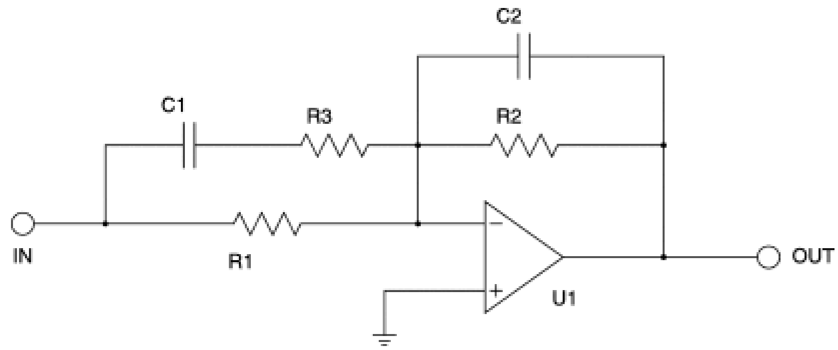

Figure 20. The schematic of a lead compensator with a stop resistor.

To find these specific values for the capacitors and resistors, we related the general equation (above) to our compensator equation:

$$G_c(s) \;=\; -\,0.68\left(\frac{1+0.9s}{1+0.069s}\right)\frac{1}{0.033s}$$

From this, we were able to make the following equations:

$$R_1C_1 \;=\; 0.09$$

$$R_2C_2 \;=\; 0.069$$

$$R_3C_1 \;=\; 0.033$$

$$\frac{R_2}{R_1} \;=\; 0.68$$

Solving this system of equations we were able to solve for the specific component values. We ended up implementing a lead compensator with a stop resistor with the following values:

$$R_1 \;=\; 10k\Omega$$

$$R_2 \;=\; 6.8k\Omega$$

$$R_3 \;=\; 680\Omega$$

$$C_1 \;=\; 47\mu F$$

$$C_2 \;=\; 0.22\mu F$$

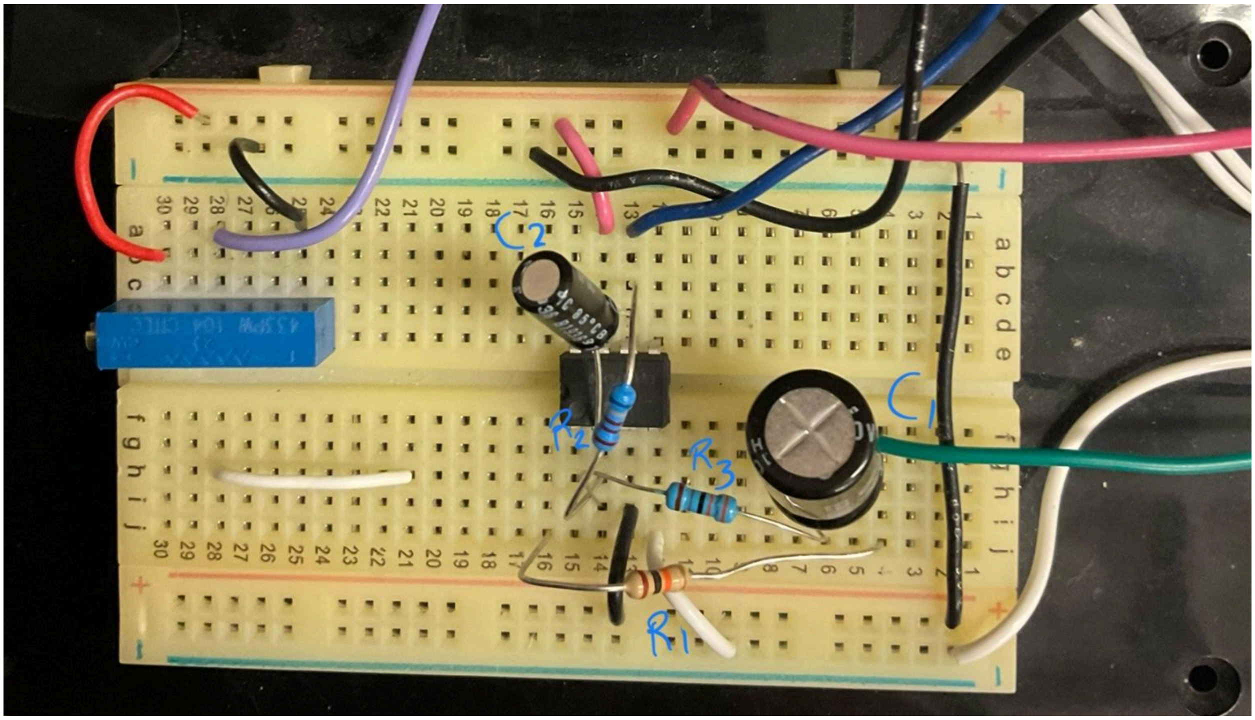

Figure 21. A photo of our breadboard circuit labeled with the component values.

Adding this compensator into to initial system, we get the following open loop transfer function:

$$G(s) = \frac{1.901s+4.045}{0.003681s^4+0.1531s^3+1.14s^2+s}$$

```
%Define Compensated System Response
gaincomp = 0.68
numcomp = [0.47 1]
dencomp = [0.00372 0.151 1]
comtf = gaincomp*tf(numcomp,dencomp)

gaincar = 6.8
numcar = [0.8748]
dencar = [0.9894 1 0]
cartf = gaincar*tf(numcar, dencar)

totaltf = comtf*cartf

%Plot Theoretical System Response
figure(7)
bode(totaltf)

%Find Theoretical Settling Time and Peak Overshoot
SteadyStatetf = 1
ts = 1.02*SteadyStatetf %Settling Time is 2.2 seconds
%The difference between our theoretical and experiemtnal settling time and
%overshoot can be atrributed to the linearizatrion of the sensor gain
%Scale
positioncutscaled = (positioncom(146:end)-positioncom(146))/SteadyStatecut;
figure(8)
step(feedback(totaltf,1))
hold on
plot(timecut, positioncutscaled)
xlabel('time')
ylabel('Position')
title('Theoretical and Exprimental Compensated System Step Response')
hold off
```

Figure 22: Closing the Loop and Graphing the Compensated System

In the above code, the transfer function is built by combining the compensator and system transfer function. The loop is then closed by adding a feedback to the transfer function, and the closed loop system step response is graphed.

Part 6. Demonstration that specifications are met

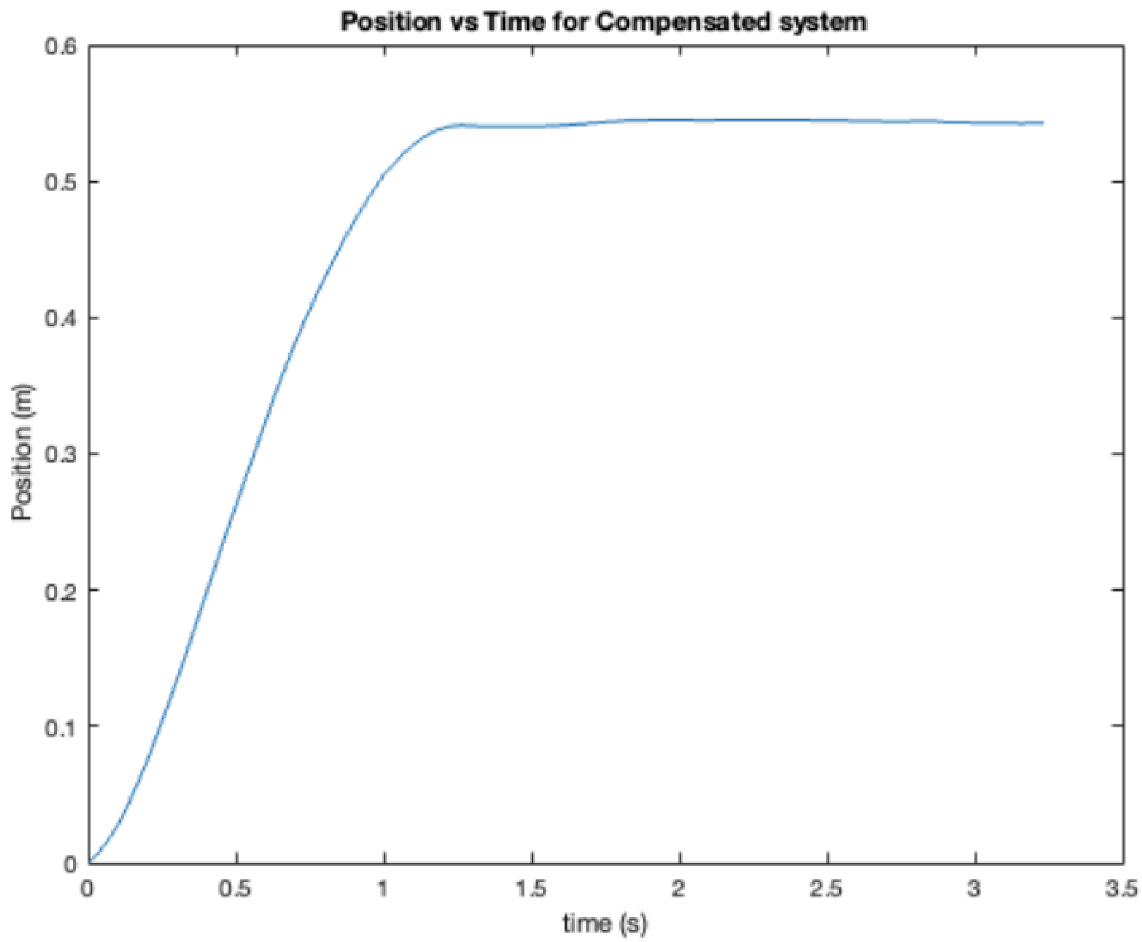

Figure 23. The position vs time graph for a step input on our compensated system

From this graph we were able to calculate the settling time and peak overshoot. Our settling time was t_s = 1.02 seconds and our peak overshoot was 0%. These values differ from our theoretical values, but are well within our specifications.

```
%Grab Time and Velocity Data
timecom = datacom(:,1);
timecut = timecom(146:end)-timecom(146);

%Cut Position
positioncom = datacom(:,2);
positioncut = (positioncom(146:end)-positioncom(146));

%Cut Velocity
velocitycom = datacom(:,3);
velocitycut = velocitycom(146:end)-velocitycom(146);

%Find Experimental Steady-State Value and Settling Time
SteadyStatecut = mean(positioncut(90:end))
ts = 0.98*SteadyStatecut;

%The index of position value for 98% of Steady State is 36, the corresponding
%index for time is equal to settling time
settlingtime = timecut(36)

%Percent Overshoot
overshoot = ((max(positioncut)-SteadyStatecut)/SteadyStatecut)*100

%Plot Position
figure(5)
plot(timecut, positioncut)
xlabel('time (s)')
ylabel('Position (m)')
title('Position vs Time for Compensated system')
```

Figure 24. The code for calculating the experimental settling time and peak overshoot.

The tracker data was imported into MATLAB, time and position values were formatted to begin at zero and plotted. The settling time is then found to be within 2% of steady state value.

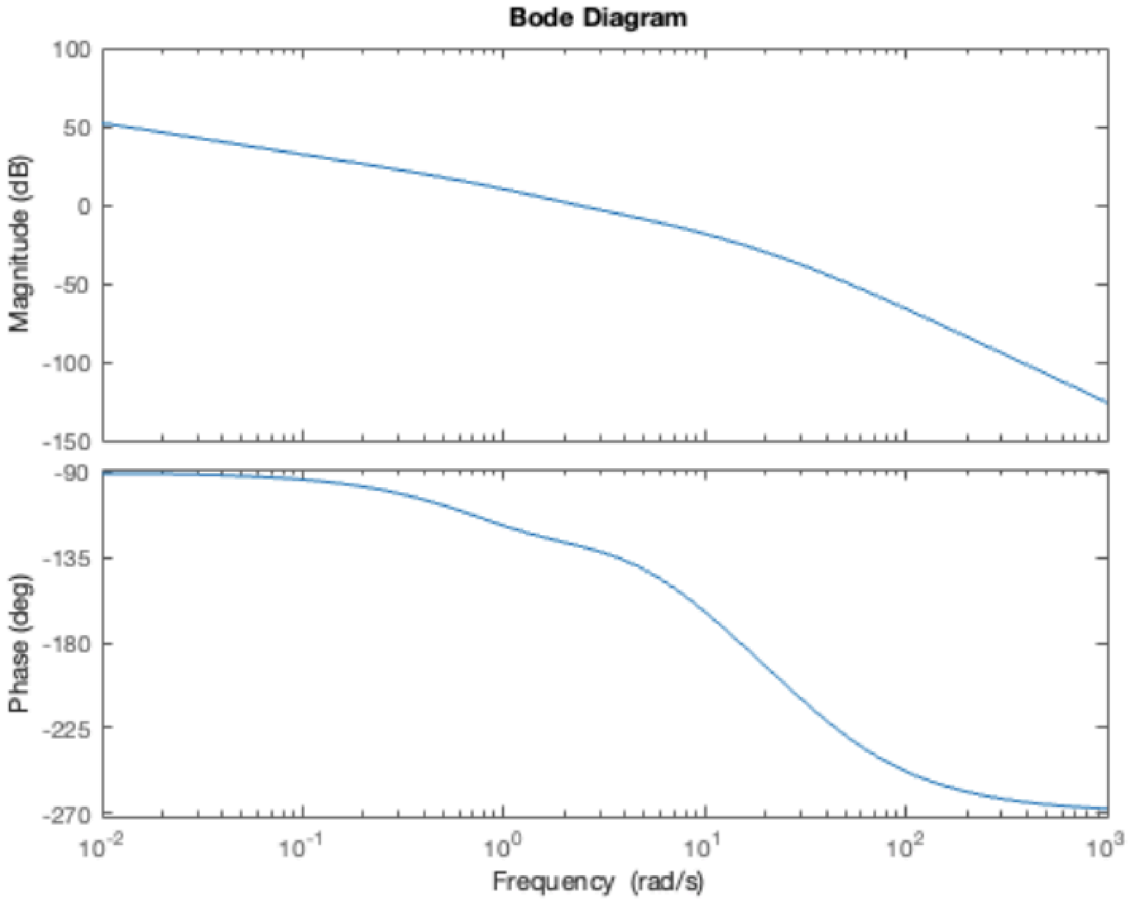

Figure 25. Bode plots for the compensated system

By analyzing the bode plots in MATLAB, we find:

$$\text{Gain margin, } g_m = 24.9\text{dB}$$
$$\text{Phase margin, } p_m = 51.4^{\circ}$$

The above values fall within the specifications we provided in part 3 and further demonstrate that the system is stable.

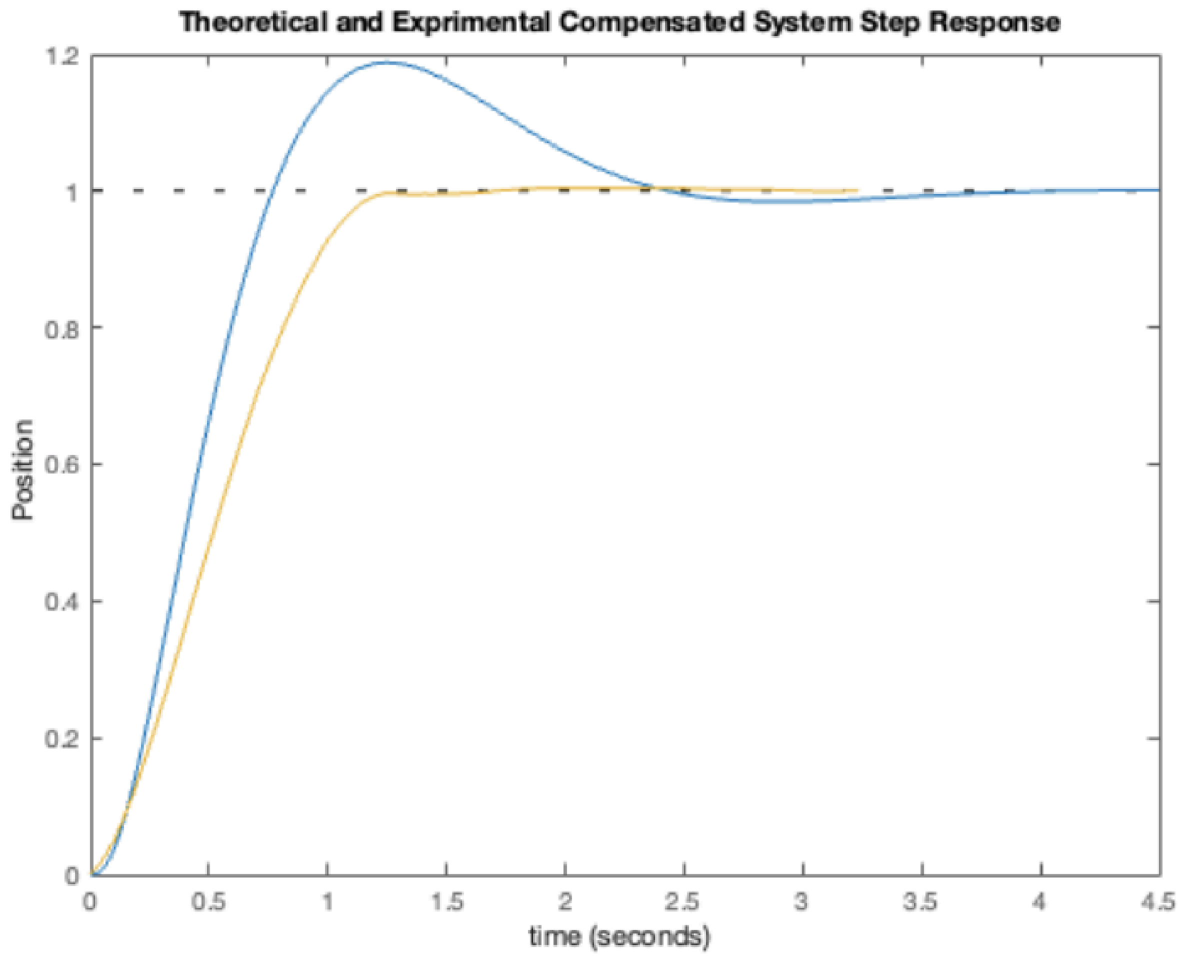

Figure 26. Visually overlaying our theoretical and experimental system step response

Our rise time was almost exactly the same, while the settling time and peak overshoot we found experimentally were significantly smaller compared to our theoretical values.

To check the control effort and the steady state error we hooked up the system to the oscilloscope and recorded the system response. We simulated the demonstration test by placing two boards in front of the car. The board closest to the car is subsequently removed and the car accelerates towards the second board stopping right before it.

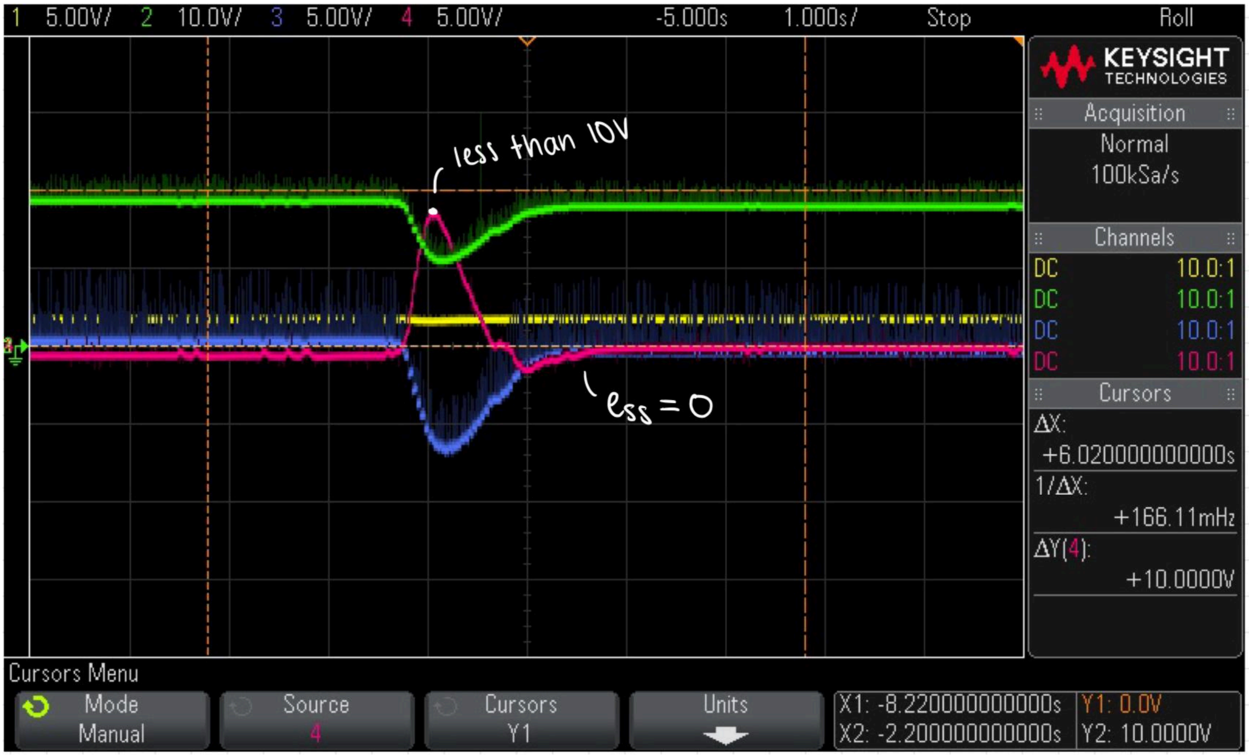

Figure 27. An oscilloscope printout showing the system as it responds to a step input (when a board is removed and the car drives towards and stops in front of another board).

The blue line shows the steady state error of the system, which is initially at zero. When the board is removed, e_{ss} increases as the car goes towards the farther wall, and ultimately returns to zero. This shows that our system has a closed-loop $e_{ss} = 0$, as the specifications define. The pink line shows the control effort. The control effort also spikes when the car moves forwards towards the further board before returning to zero. An important note here is that the effort doesn't exceed 10V, meeting another one of our specifications.