$$Li(x) = \int_{2}^{x} \frac{dt}{ln(t)}$$

$$\int_{2}^{x} \frac{1}{ln(t)} dt$$

Integración por partes

Derivar u

$$u = \frac{1}{ln(t)}$$

$$u = ln^{-1}(t)$$

$$\frac{du}{dt} = -ln^{-2}(t) \cdot \frac{1}{t}$$

$$du = -\frac{1}{t \cdot ln^{2}(t)} dt$$

Integrar dv

$$dv = dt$$

$$\int dv = \int dt$$

$$v = t$$

$$u \cdot v - \int v \cdot du$$

$$\frac{1}{ln(t)} \cdot t - \int t \cdot - \frac{1}{t \cdot ln^{2}(t)} dt$$

$$\left[\frac{t}{ln(t)} - \int - \frac{1}{ln^{2}(t)} dt \right]_{2}^{x}$$

$$\frac{t}{ln(t)} - \left(\int - \frac{1}{ln^2(t)} dt\right)$$

$$\boxed{1 = \frac{1}{t} \cdot e^{ln(t)}}$$

$$\frac{t}{ln(t)} - \left(\int - \frac{\frac{1}{t} \cdot e^{ln(t)}}{ln^2(t)} dt\right)$$

$$\frac{t}{ln(t)} - \left(\int - \frac{e^{ln(t)}}{ln^2(t)} \frac{1}{t} dt\right)$$

<u>Sustitución "u"</u>

$$u = ln(t)$$

$$\frac{du}{dt} = \frac{1}{t}$$

$$du = \frac{1}{t} dt$$

$$\frac{t}{ln(t)} - \left(\int - \frac{e^u}{u^2} du\right)$$

$$\frac{t}{ln(t)} - \left(-\int \frac{e^u}{u^2} du\right)$$

$$\frac{t}{ln(t)} - \left(-\int e^u \cdot \frac{1}{u^2} du\right)$$

Integración por partes

Derivar a

$$a = e^{u}$$

$$\frac{da}{du} = e^{u}$$

$$da = e^{u} du$$

Integrar db

$$db = \frac{1}{u^{2}}$$

$$\int db = \int \frac{1}{u^{2}} du$$

$$b = -\frac{1}{u}$$

$$\frac{t}{ln(t)} - (-(a \cdot b - \int b \cdot da))$$

$$\frac{t}{ln(t)} - (-(e^{u} \cdot - \frac{1}{u} - \int - \frac{1}{u} \cdot e^{u} du))$$

$$\frac{t}{ln(t)} - (-(-\frac{e^{u}}{u} - \int - \frac{e^{u}}{u} du))$$

$$\frac{t}{ln(t)} - (-(-\frac{e^{u}}{u} - (-\int \frac{e^{u}}{u} du)))$$

$$\int \frac{e^{x}}{x} = Ei(x) \Rightarrow \int \frac{e^{u}}{u} = Ei(u)$$

$$\frac{t}{ln(t)} - (-(-\frac{e^{u}}{u} - (-Ei(u))))$$

$$\frac{t}{ln(t)} - (-(- \frac{e^{u}}{u} + Ei(u)))$$

$$\frac{t}{ln(t)} - (-(- \frac{e^{ln(t)}}{ln(t)} + Ei(ln(t))))$$

$$\boxed{a^{log_a(b)} = b \Rightarrow e^{ln(t)} = t}$$

$$\frac{t}{ln(t)} - (-(- \frac{t}{ln(t)} + Ei(ln(t))))$$

$$\frac{t}{ln(t)} - (-(- \frac{t}{ln(t)}) - (Ei(ln(t)\ )\ )\ )$$

$$\frac{t}{ln(t)} - (-(- \frac{t}{ln(t)}) - (Ei(ln(t)\ )\ )\ )$$

$$\frac{t}{ln(t)} - (\frac{t}{ln(t)} - Ei(ln(t)\ )\ )$$

$$\frac{t}{ln(t)} - \frac{t}{ln(t)} + Ei(ln(t))$$

$$Ei(ln(t))$$

$$\left[\frac{t}{ln(t)} - \frac{t}{ln(t)} + Ei(ln(t))\right]_{2}^{x}$$

$$Li(x) = Ei(ln(x)) - Ei(ln(2))$$

Estimación de Ei(x)

$$Ei(x) = \int_{\infty}^{x} \frac{e^{t}}{t} dt \quad \text{o} \quad Ei(x) \approx \gamma + ln(x) \sum_{k=1}^{\infty} \frac{x^{k}}{k \cdot k!}$$

Tabla de valores(cantidad de primos hasta x)

$Li(10^2) = 30$

$Li(10^3) = 178$

$Li(10^4) = 1246$

$Li(10^5) = 9630$

$Li(10^6) = 78628$

$Li(10^7) = 664918$

$Li(10^8) = 5762209$

$Li(10^9) = 50849235$

$Li(10^{10}) = 455055614$

$Li(10^{11}) = 4118066401$

$Li(10^{12}) = 37607950281$

$Li(10^{13}) = 346065645810$

$Li(10^{14}) = 3204942065692$

$Li(10^{15}) = 29844571475288$

$Li(10^{16}) = 279238344248557$

$Li(10^{17}) = 2623557165610822$

$Li(10^{18}) = 24739954309690415$

$Li(10^{19}) = 234057667376222382$

$Li(10^{20}) = 2220819602783663484$