

Bayesian Optimization of a Wearable Assistive Device Using an Estimator Stopping Process

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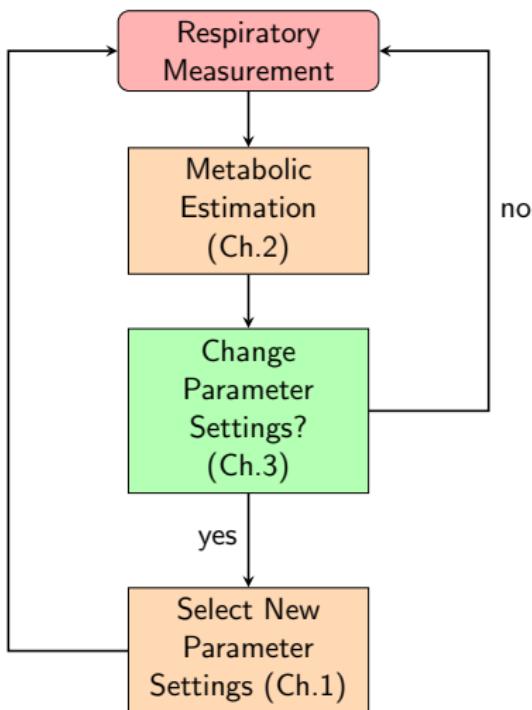
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Soft Exosuit

Process Overview



Standard Gaussian Process Model

Observations represent underlying process with some Gaussian noise

$$y \sim f + \mathcal{N}(0, \sigma_n^2)$$

$$f \sim \mathbb{G}(0, \kappa)$$

$$\kappa(x_i, x_j | \sigma_\theta^2, I) = \sigma_\theta^2 \exp\left(-\frac{1}{2} d^2\left(\frac{x_i}{I}, \frac{x_j}{I}\right)\right)$$

Gaussian Process Regression

Given some training data, closed form posterior distribution at any point x

$$\bar{\mu}(x) = K(X, x)^T [K(X, X) + \sigma_n^2 I]^{-1} Y$$

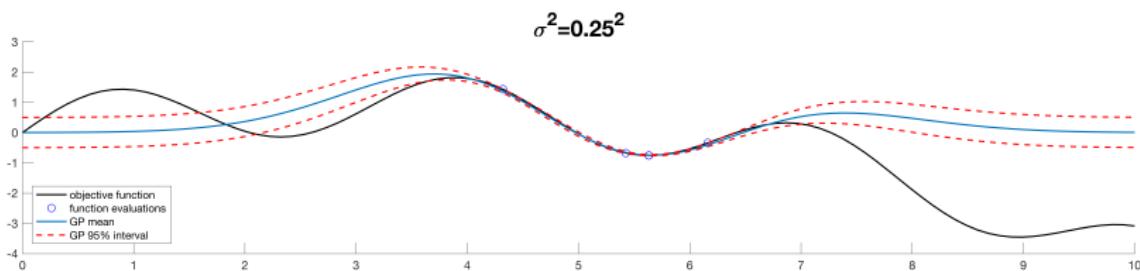
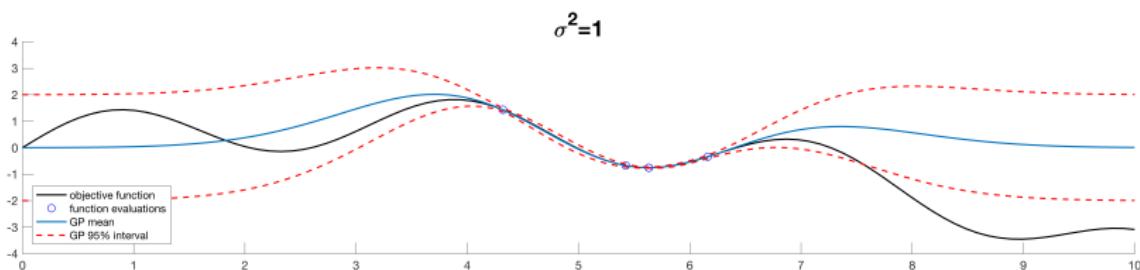
$$\bar{\sigma}^2(x) = \kappa(x, x | \theta) - K(X, x)^T [K(X, X) + \sigma_n^2 I]^{-1} K(X, x)$$

$$K(X, x)_i = \kappa(x_i, x | \theta)$$

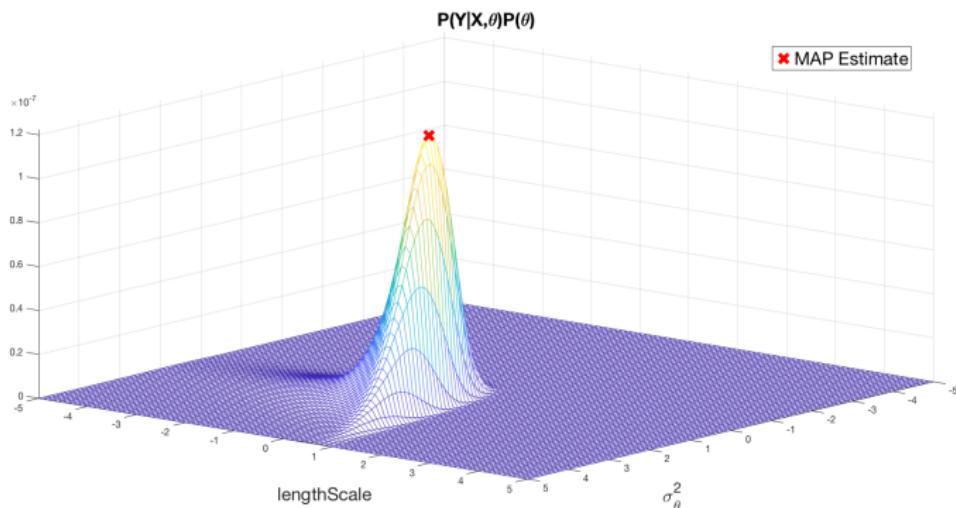
$$K(X, X)_{ij} = \kappa(x_i, x_j | \theta),$$

where $X = \{x_i\}_{i=1}^n$ and $Y = \{y_i\}_{i=1}^n$.

σ_θ^2 Effect



Hyperparameter Estimation



Expected Improvement

$$\begin{aligned} EI(x|\mathbb{S}) &= \int_{-\infty}^{\infty} \max(0, y^* - y) p(y|x) dy \\ &= z\bar{\sigma}(x)\Phi(z) + \bar{\sigma}(x)\phi(z) \\ z &= \frac{y^* - \bar{\mu}(x) + \xi}{\bar{\sigma}(x)}, \end{aligned}$$

where y^* is the best value observed so far, $\Phi(z)$ and $\phi(z)$ are the standard normal CDF and PDF functions, and ξ is a scaling parameter to adjust the tradeoff between exploration-exploitation

Bayesian Optimization

Objective Function $F(x)$

Acquisition Function $g(\mu, \sigma^2)$

Specify Exploration Points $\mathbb{E} = \{e_1, e_2, \dots, e_n\}$

Training Samples \mathbb{S}

for $i = 1$ **to** n

$\mathbb{S} = \mathbb{S} \cup \{e_i, F(e_i)\}$

end

while $t < T$

Update GP Hyperparameters θ

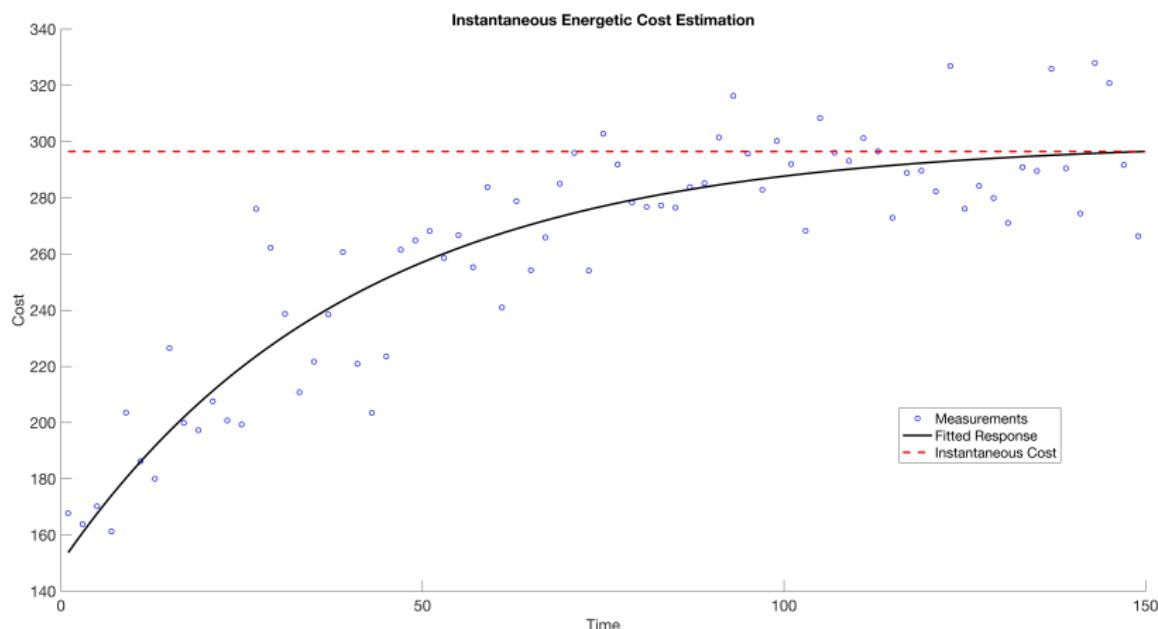
Given $f(x|\theta, \mathbb{S}) \sim \mathcal{N}(\mu_x, \sigma_x^2)$,

$x^* = \arg \max_{x \in \mathbb{X}} g(\mu_x, \sigma_x^2)$

$\mathbb{S} = \mathbb{S} \cup \{x^*, F(x^*)\}$

end

Instantaneous Energetic Cost



Kalman Filter

System Dynamics

$$\begin{aligned}x(t+1) &= F(x(t), v(t), t) \\z(t) &= H(x(t), w(t), t)\end{aligned}$$

Update Equations

$$\begin{aligned}\hat{x}(t) &= \hat{x}(t|t-1) + Ky \\P_x(t) &= P_x(t|t-1) - KP_y(t|t-1)K^T \\K &= P_{xy}(t|t-1)P_y^{-1}(t|t-1) \\y &= z(t) - H(\hat{x}(t|t-1), w(t), t)\end{aligned}$$

Unscented Transform

$$\chi_0 = \bar{x}$$

$$\chi_i = \bar{x} + (\sqrt{(N + \lambda)P_x})_i \quad i = 1, \dots, N$$

$$\chi_i = \bar{x} - (\sqrt{(N + \lambda)P_x})_{i-N} \quad i = N + 1, \dots, 2N$$

$$W_0^{(m)} = \lambda / (N + \lambda)$$

$$W_0^{(c)} = \lambda / (N + \lambda) + (1 - \alpha^2 + \beta)$$

$$W_i^m = W_i^c = 1 / \{(2(N + \lambda))\} \quad i = 1, \dots, 2N,$$

where $\lambda = \alpha^2(N + \kappa) - N$, and α , κ , and β are scaling parameters. Given $y = g(x)$,

$$\bar{y} \approx \sum_{i=0}^{2N} W_i^{(m)} g(\chi_i)$$

$$P_y \approx \sum_{i=0}^{2N} W_i^{(c)} (g(\chi_i) - \bar{y})(g(\chi_i) - \bar{y})^T$$

UKF Model

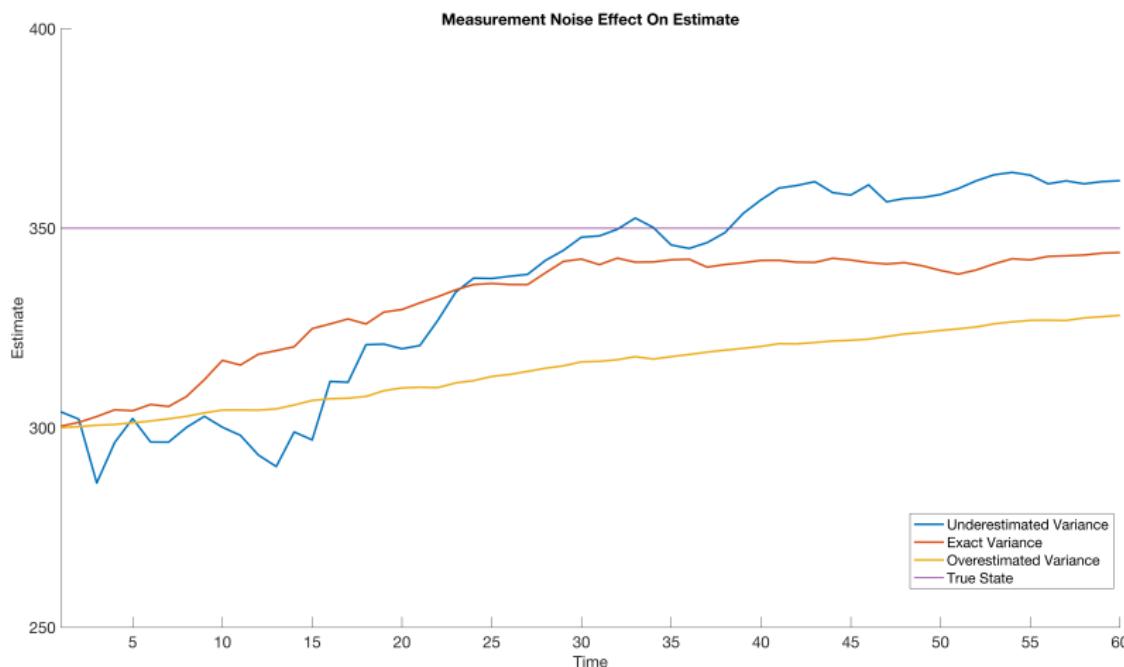
Time constants τ_0, τ characterizing rate of change of cost constants c_0, c

$$x = [c_0 \ c \ \tau_0 \ \tau]$$

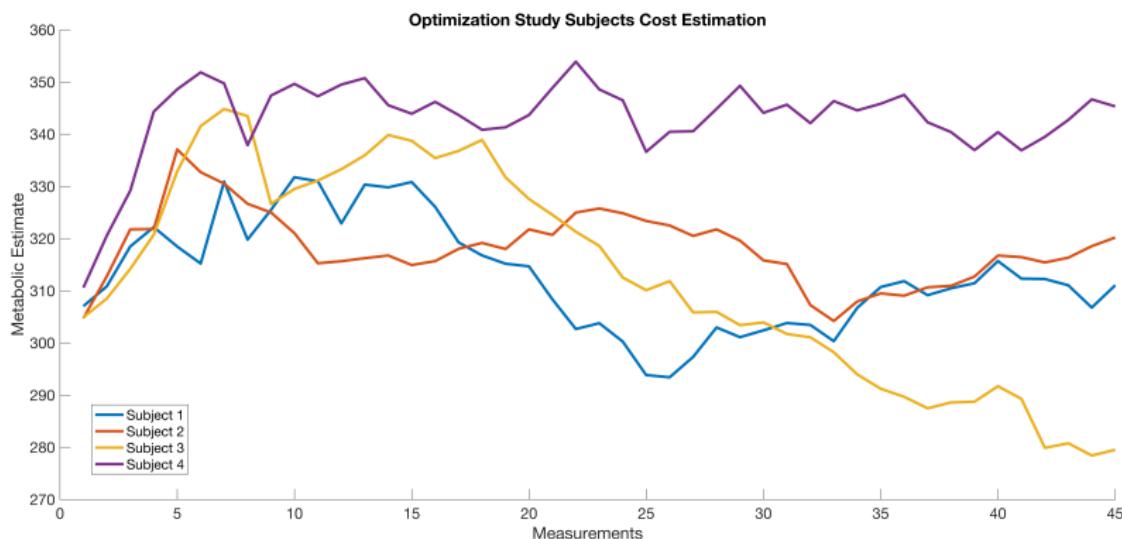
$$F(x(t), v(t), t) = x(t) + v(t)$$

$$H(x(t), w(t), t) = c(1 - e^{\frac{-t}{\tau}}) + c_0 e^{\frac{-t}{\tau_0}} + w(t)$$

Measurement Noise



Estimator in Subject Trials



Stopping Problem Formulation

 N

Finite horizon

 X_t State at time t $P(X_t|X_{t-1})$

State transitions, typically Markovian

 λ Discount Factor $\in (0, 1]$ $r(X)$ Bounded reward function for continuing at state X $g(X)$ Bounded reward function for stopping at state X

Optimal Stopping Point

Via backward induction

$$J_N(x) = g(x)$$

$$J_n(x) = \max\{g(x), r(x) + \lambda \mathbb{E}_{P(y|x)}[J_{n+1}(y)]\}$$

Optimal stopping point

$$\tau = \min_t J_t(X_t) = g(X_t)$$

Distribution of Interest

Given

$$\text{Current Estimate: } \hat{x}_t \sim \mathcal{N}(\mu_{x_t}, \sigma_{x_t}^2)$$

$$\text{Best Estimate: } \hat{x}^* \sim \mathcal{N}(\mu_{x^*}, \sigma_{x^*}^2)$$

We are concerned with

$$\hat{x}_t - \hat{x}^* \sim \mathcal{N}(\mu, \sigma^2)$$

$$\mu = \mu_{x_t} - \mu_{x^*}$$

$$\sigma^2 = \sigma_{x_t}^2 + \sigma_{x^*}^2$$

σ -Offset Model

$$X_t = \mu$$

$$P(X_t | X_{t-1}) = P(X_t) \sim \mathcal{N}(\mu, \sigma^2)$$

$$r(x) = K\sigma - x$$

$$g(x) = 0$$

Stopping condition $X_t > K\sigma$

Success/Failure Model

Define success as

$$\Phi\left(\frac{\mu}{\sigma}\right) > K,$$

for risk tolerance K . Define model as

$$X_t = (\alpha_t + \alpha_0, \beta_t + \beta_0)$$

$$P(X_{t+1}) = \begin{cases} (\alpha_t + \alpha_0 + 1, \beta_t + \beta_0) \\ \text{w.p. } \frac{\alpha_t + \alpha_0}{\alpha_t + \alpha_0 + \beta_t + \beta_0} \\ (\alpha_t + \alpha_0, \beta_t + \beta_0 + 1) \\ \text{w.p. } \frac{\beta_t + \beta_0}{\alpha_t + \alpha_0 + \beta_t + \beta_0} \end{cases}$$

$$r(X_t) = \frac{\alpha_t + \alpha_0}{\alpha_t + \alpha_0 + \beta_t + \beta_0},$$

with smoothing priors α_0, β_0 and success/failures α_t, β_t

Multi-Armed Bandit Problem

Which is the better option?

- ▶ Option 1: 20 successes, 20 failures
- ▶ Option 2: 3 successes, 4 failures

Assume $g(x) = g$ is a constant, define Gittins Index as

$$\nu(X_t) = (1 - \lambda) \min_g \{ J_t(X_t) = g \}$$

$$\nu((30, 30)) = 0.5133$$

$$\nu((4, 6)) = 0.5289$$

In the case of single option, must set another threshold $\nu(x) < V$

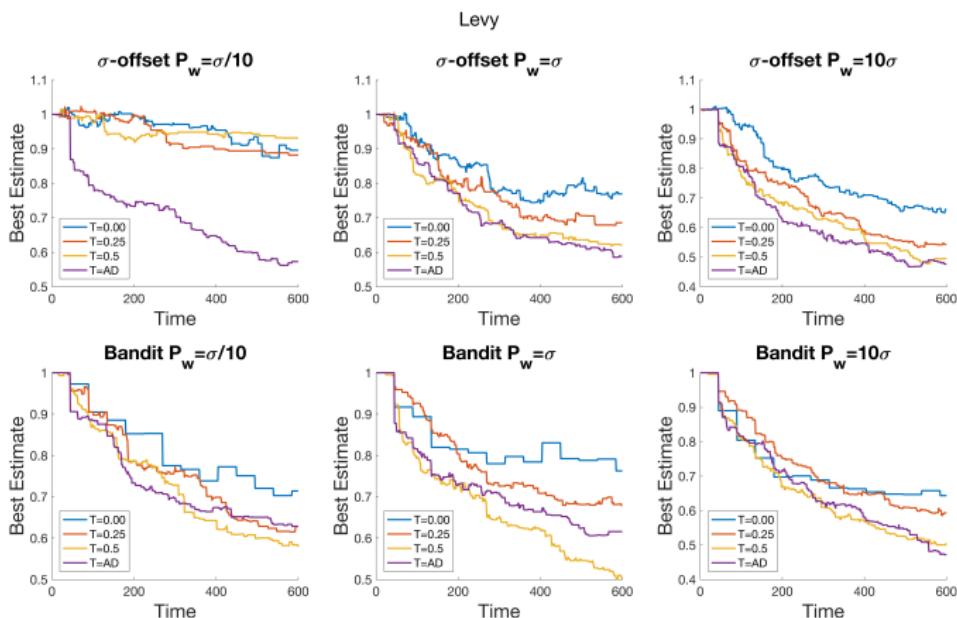
Adaptive Threshold

Use the exploration phase of Bayesian optimization to tune thresholds in either model, with final distribution for each point as "best" value. Consider a window of values at time t

e_{00}	e_{01}	e_{02}	e_{03}	e_{04}	e_{05}	e_{06}	e_{07}	e_{08}	e_{09}	
e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}	e_{18}	e_{19}	
e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	...
e_{30}	e_{31}	e_{32}	e_{33}	e_{34}	e_{35}	e_{36}	e_{37}	e_{38}	e_{39}	
⋮										

Use most risk averse threshold in window: max σ -offset, min Gittins

Simulations



Domain	Minimum	Mean	Std
$[-10, 10]^4$	0	42.544	27.939

Hip-Only Protocol

Onset	Peak	Offset	Force	Curv1	Curv2
0-25	15-40	30-55	100-300	20-70	20-70

Gittins Method $V = 0.35$

8 Exploration Points

5 Minute Break

40 Minute Optimization

5 Minute Break

5 Minute Slack/FIX/OPT Conditions

Hip-Only Protocol

Subj	OPT	FIX	OPT Peak F	FIX Peak F
1	35%	29%	223N	247N
2	7%	-4%	174N	230N

Multi-Joint Protocol

Hip Peak	Hip Offset	Ankle Onset	Ankle Peak
5-30	20-45	30-50	35-55

Fixed Hip Force = 30% BW, Ankle Force = 60% BW

Gittins Method $V = AD$

6 Exploration Points

5 Minute Break

30 Minute Optimization

5 Minute Break

5 Minute Slack/OPT Conditions

Multi-Joint Protocol

Subj	HP	HO	AO	AP	% ↓
1	11	33	42	53	22.0
2	16	37	40	52	40.7
3	12	33	44.5	55	17

Contribution

- ▶ Two stopping models based on a metabolic estimator
- ▶ Demonstrated comparable results to previous study
- ▶ Initial results with multi-joint optimization

Future Work

- ▶ Continue to tune Gaussian process in exploration/exploitation tradeoff
- ▶ Optimize metabolic estimator covariance parameters
- ▶ Conduct more subject trials, including using σ -offset method

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