Bayesian Optimization of a Wearable Assistive Device Using an Estimator Stopping Process

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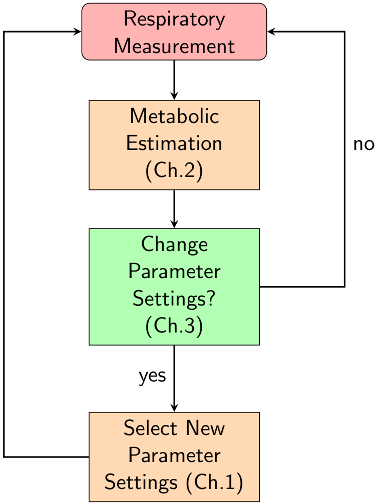
Soft Exosuit

Process Overview



Standard Gaussian Process Model

Observations represent underlying process with some Gaussian noise

$$y \sim f + \mathbb{N}(0, \sigma_n^2)$$
$$f \sim \mathbb{G}(0, \kappa)$$
$$\kappa(x_i, x_j | \sigma_\theta^2, l) = \sigma_\theta^2 \exp(-\frac{1}{2} d^2(\frac{x_i}{l}, \frac{x_j}{l}))$$

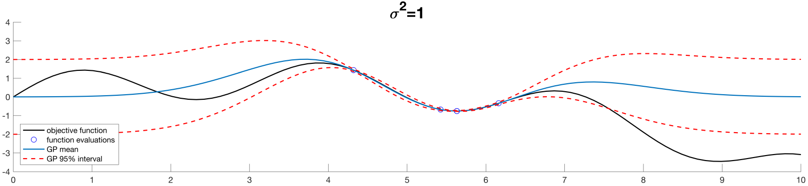
Gaussian Process Regression

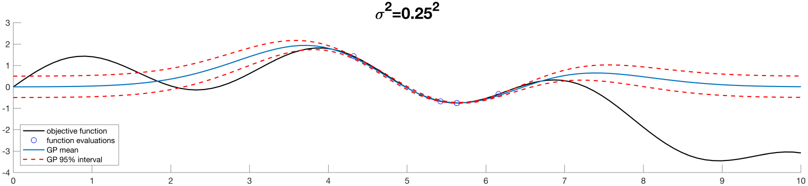
Given some training data, closed form posterior distribution at any point x

$$\bar{\mu}(x) = K(X,x)^T[K(X,X)+\sigma_n^2 I]^{-1}Y$$
$$\bar{\sigma}^2(x) = \kappa(x,x|\theta) - K(X,x)^T[K(X,X)+\sigma_n^2 I]^{-1}K(X,x)$$
$$K(X,x)_i = \kappa(x_i,x|\theta)$$
$$K(X,X)_{ij} = \kappa(x_i,x_j|\theta),$$

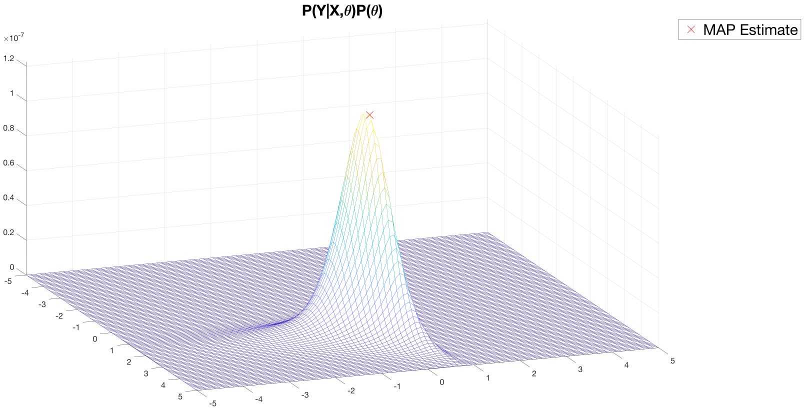
where $X = \{x_i\}_{i=1}^n$ and $Y = \{y_i\}_{i=1}^n$.

σ_θ^2 Effect





Hyperparameter Estimation



Expected Improvement

$$
\begin{aligned}
EI(x|\mathbb{S}) &= \int_{\infty}^{\infty} max(0, y^* - y)p(y|x)dy \\
&= z\bar{\sigma}(x)\Phi(z) + \bar{\sigma}(x)\phi(z) \\
z &= \frac{y^* - \bar{\mu}(x) + \xi}{\bar{\sigma}(x)},
\end{aligned}
$$

where y^* is the best value observed so far, $\Phi(z)$ and $\phi(z)$ are the standard normal CDF and PDF functions, and ξ is a scaling parameter to adjust the tradeoff between exploration-exploitation

Bayesian Optimization

Objective Function $F(x)$
Acquisition Function $g(\mu, \sigma^2)$
Specify Exploration Points $\mathbb{E} = \{e_1, e_2, ..., e_n\}$
Training Samples $\mathbb{S}$
for $i = 1$ **to** n
    $\mathbb{S} = \mathbb{S} \cup \{e_i, F(e_i)\}$
end
while $t < T$
    Update GP Hyperparameters θ
    Given $f(x|\theta, \mathbb{S}) \sim \mathcal{N}(\mu_x, \sigma_x^2)$,
    $x^* = \arg\max_{x \in \mathbb{X}} g(\mu_x, \sigma_x^2)$
    $\mathbb{S} = \mathbb{S} \cup \{x^*, F(x^*)\}$
end

Instantaneous Energetic Cost

Kalman Filter

Unscented Transform

UKF Model

Covariance Parameterization

Subject Trials

σ-Offset Model

Gittins Model

Hip-Only Trials

Acknowledgements: