Engineering Statistics

Descriptive Statistics

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Descriptive statistics

Describing your data

Spread	Central tendency
Variability	Selecting right plot

Variables

Categorical	Numeric

Important

N
Mean
Median
Mode
SD
IQR
Skewness
Kurtosis

統計名詞

Some important terms:

- **Data** (**數據**)- collections of facts
- **Population (母體)** - a well-defined collection of objects
- **Census** (**普查**) – collecting desired information for all objects in the population
- **Sample** (**樣本**) - a subset of the population
- **Variable** (**變數**) - any characteristic whose value may change from one object to another in the population
- **Univariate data (單變量)** - observations on a single variable
- **Bivariate data** (**雙變量**) - observations on each of two variables

籃球選手身高、體重

- **Multivariate data** (**多變量**) - observations on more than two variables

舒張壓、收縮壓與血脂

Example 1.1: Charity Business in the US (慈善機構)籌款活動佔總費用支出的比例

- A sample of 5500 charitable organizations
- For some efficiently-operated charities, only a small percentage of total expenses are spent on fund-raising and administrative activities
- Others spend a high percentage of what they take in to perform the same activities
- Data on fund-raising expenses as a percentage of total expenditures for a random sample of 60 charities:

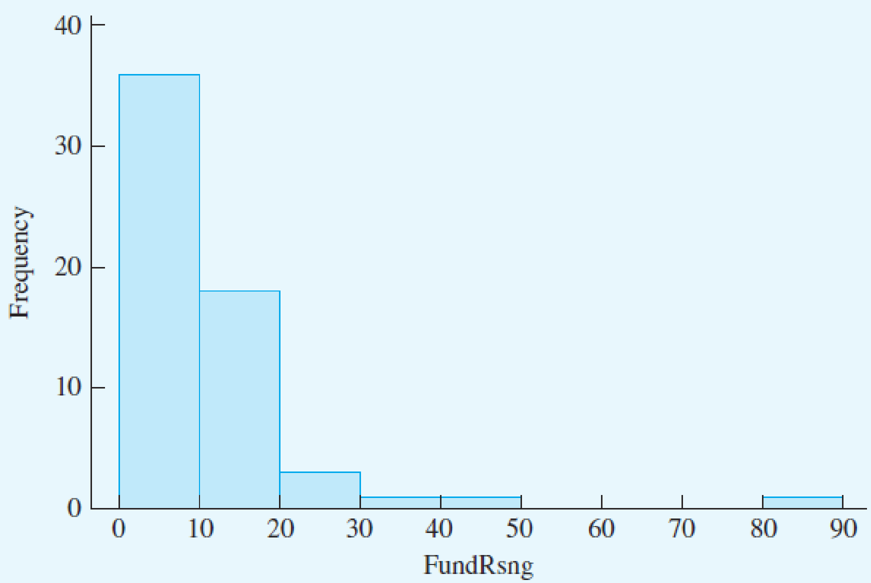
Data:

6.1	12.6	34.7	1.6	18.8	2.2	3.0	2.2	5.6	3.8
2.2	3.1	1.3	1.1	14.1	4.0	21.0	6.1	1.3	20.4
7.5	3.9	10.1	8.1	19.5	5.2	12.0	15.8	10.4	5.2
6.4	10.8	83.1	3.6	6.2	6.3	16.3	12.7	1.3	0.8
8.8	5.1	3.7	26.3	6.0	48.0	8.2	11.7	7.2	3.9
15.3	16.6	8.8	12.0	4.7	14.7	6.4	17.0	2.5	16.2

Descriptive statistics

Display the data to find the anwers:

```
Stem-and-leaf of FundRsng N = 60
Leaf Unit = 1.0
   0 | 01111122223333333344
   0 | 5555666666778888
   1 | 0001222244
   1 | 55666789
   2 | 01
   2 | 6
   3 | 4
   3 |
   4 |
   4 | 8
   5 |
   5 |
   6 |
   6 |
   7 |
   7 |
   8 | 3
```



- Observe how the percentages are distributed over the range of possible values from 0 to 100.
- A substantial majority of the charities in the sample obviously spend less than 20% on fund-raising.
- Only a few percentages might be viewed as beyond the bounds of sensible practice.

Displays for univariate data

Stem-and-leaf (莖葉圖)
Histograms
…

Stem-and-Leaf Displays (莖葉圖)

- Separate each observation into two parts:
  - A **stem**: consists of one or more leading digits
  - A **leaf**: consists of the remaining or trailing digit(s)
- Stem values are listed in a single column
- Leaf of each observation are then placed on the row of the corresponding stem

Example: Use of alcohol by college students (各所大學學生喝酒習慣比例)

0	4
1	1345678889
2	1223456666777889999
3	0112233344555666677777888899999
4	11122222334444556666666777888888999
5	0011122222334556666667777888899
6	01111244455666778

Stem: tens digit
Leaf: ones digit

Stem-and-Leaf Displays (莖葉圖)

- A stem-and-leaf display conveys
  - Identification of a **typical** or representative value
  - **Extent of spread** about the typical value
  - Presence of any **gaps** in the data
  - Extent of **symmetry** in the distribution of values
  - Number and location of **peaks**
  - Presence of any **outlying** values

Stem-and-Leaf Displays (莖葉圖)

A display of the binge-drinking data with repeated stems

```
Stem-and-leaf of pct binge N = 140
Leaf Unit = 1.0

   1   0   4
   1   0
   4   1   134
  11   1   5678889
  16   2   12234
  30   2   56666777889999
  40   3   0112233344
  61   3   555666677777888899999
 (14)  4   11122222334444
  65   4   5566666677788888999
  46   5   00111222334
  34   5   55666667777788899
  17   6   011112444
   8   6   55666778
```

median

Stem-and-Leaf Displays (莖葉圖)

- A comparative stem-and-leaf display

	9	658618
9447	8	13754380
2208965655	7	5312267
2432875	6	45104
5882	5	9

Displays for univariate data: most appropriate for smaller datasets

Package: aplpack

Displays for univariate data: most appropriate for smaller datasets

stem.leaf(
data, unit, m
).

TRY it in R

R: Descriptive statistics

R_descriptive_a.R

Histograms

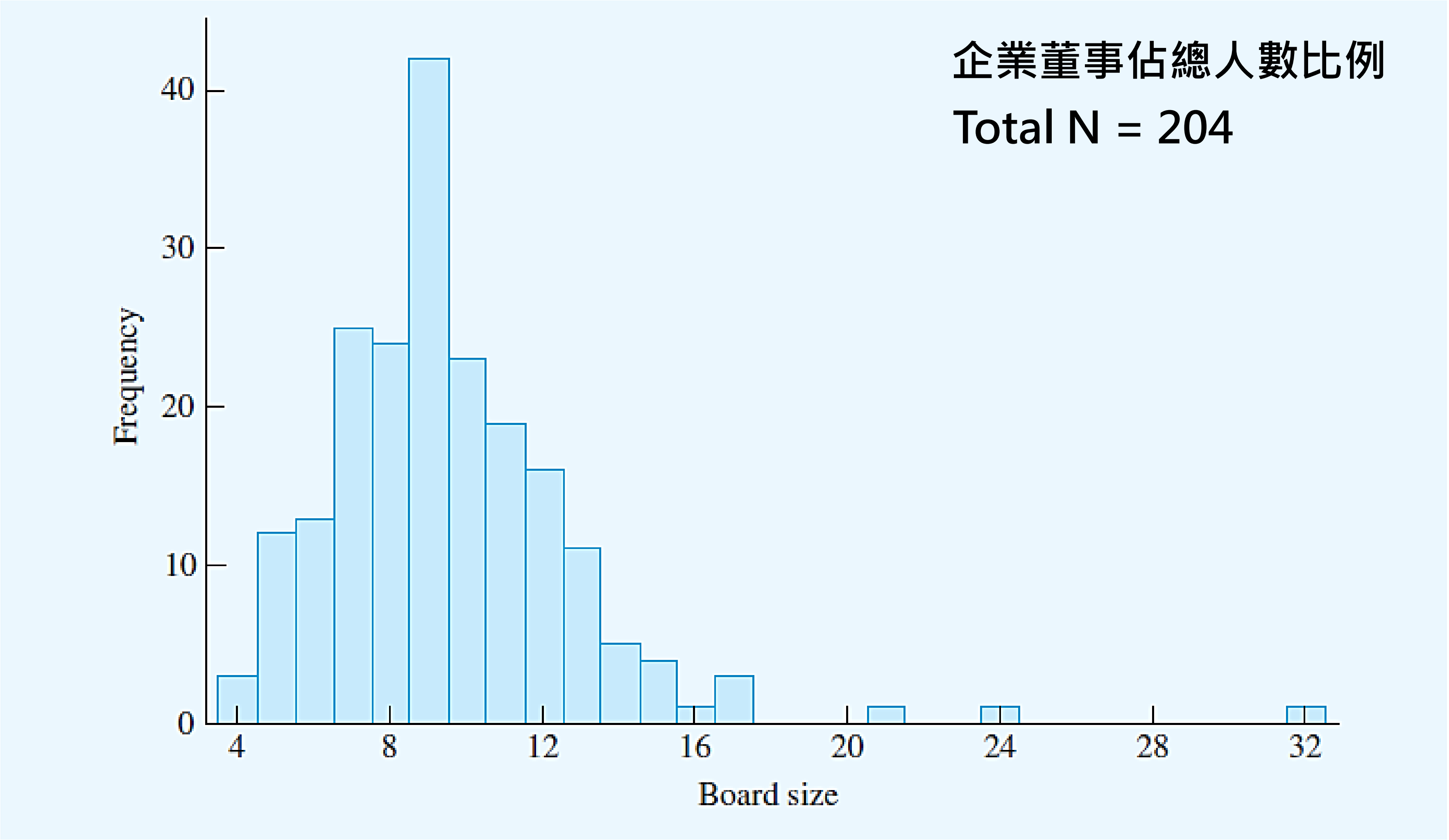
Definitions:

- **Discrete variable** - possible values either is finite or else can be listed in an infinite sequence

(可以一一列舉)

- **Continuous variable** - possible values consist of an entire interval on the number line
- **Frequency** - the number of times a particular value occurs in the data set
- **Relative frequency** - the fraction or proportion of time the value occurs

Positive Skew (正偏)



Histograms- class interval

- 選定特定區間來計數，但若數值落在邊界上，只能重新調整區間。
- 區間數量約可以用總數據個數開根號來決定。

$$\textbf{number of classes} \;\square\; \sqrt{\textbf{number of observations}}$$

Histograms- class interval

能源消耗數據

90 gas-heated homes

Power companies need information about customer usage to obtain accurate forecasts of demand. Investigators from Wisconsin Power and Light determined energy consumption (BTUs) during a particular period for a sample of 90 gas-heated homes. An adjusted consumption value was calculated as follows:

$$\text{adjusted consumption} = \frac{\text{consumption}}{(\text{weather, in degree days})(\text{house area})}$$

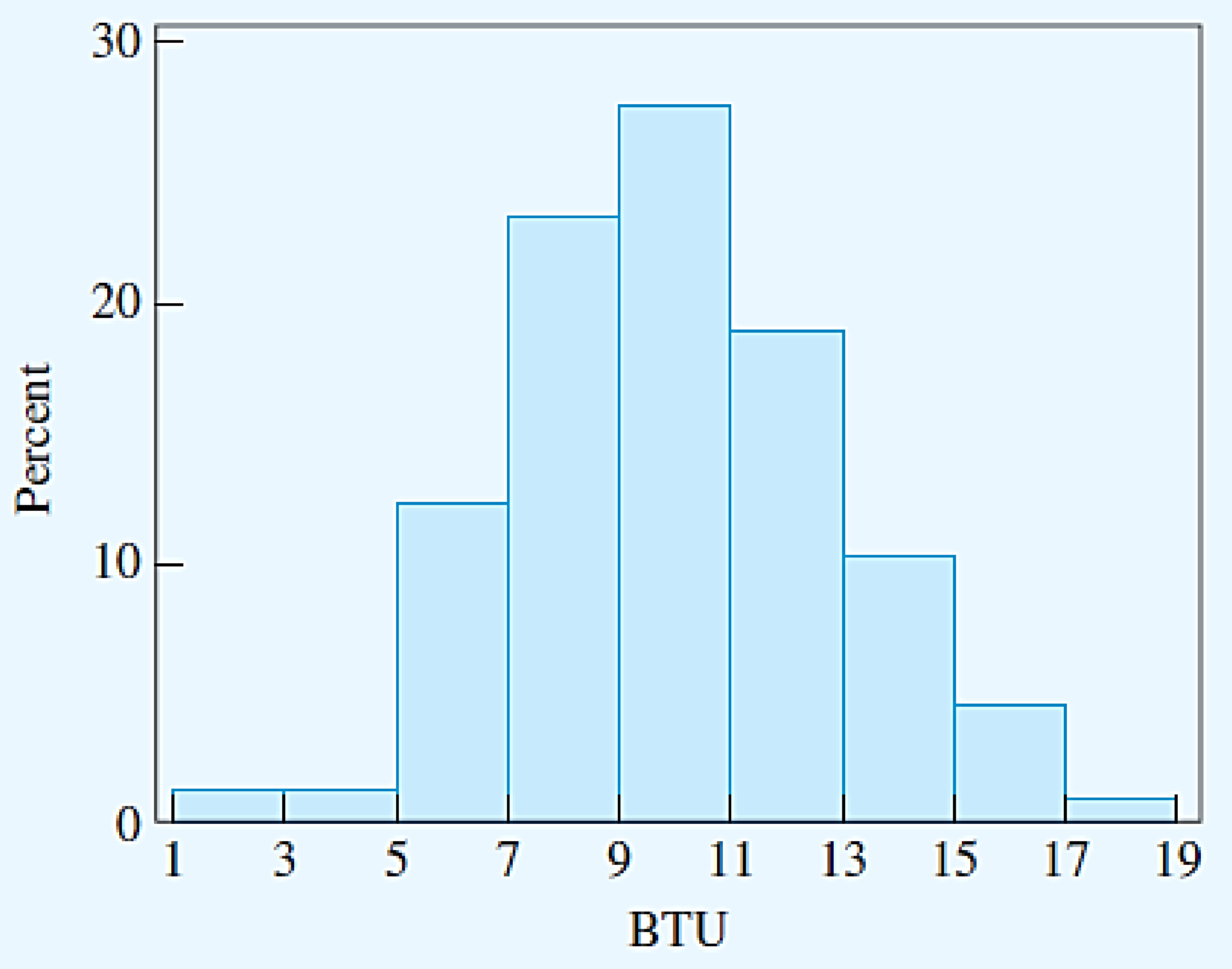
normalization

This resulted in the accompanying data (part of the stored data set FURNACE.MTW available in Minitab, which we have ordered from smallest to largest):

2.97	4.00	5.20	5.56	5.94	5.98	6.35	6.62	6.72	6.78
6.80	6.85	6.94	7.15	7.16	7.23	7.29	7.62	7.62	7.69
7.73	7.87	7.93	8.00	8.26	8.29	8.37	8.47	8.54	8.58
8.61	8.67	8.69	8.81	9.07	9.27	9.37	9.43	9.52	9.58
9.60	9.76	9.82	9.83	9.83	9.84	9.96	10.04	10.21	10.28
10.28	10.30	10.35	10.36	10.40	10.49	10.50	10.64	10.95	11.09
11.12	11.21	11.29	11.43	11.62	11.70	11.70	12.16	12.19	12.28
12.31	12.62	12.69	12.71	12.91	12.92	13.11	13.38	13.42	13.43
13.47	13.60	13.96	14.24	14.35	15.12	15.24	16.06	16.90	18.26

Sorting data

Histograms- class interval



90筆資料

9 classes

Histograms- class interval

Question: 若依照此分類統計結果，將如何計算BTU小於10的比例為何?

Class:	1–<3	3–<5	5–<7	7–<9	9–<11	11–<13	13–<15	15–<17	17–<19
Frequency:	1	1	11	21	25	17	9	4	1
Relative frequency:	.011	.011	.122	.233	.278	.189	.100	.044	.011

From the histogram,

$$\text{proportion of observations less than 9} \approx .01 + .01 + .12 + .23 = .37$$

(exact value = 34/90 = .378)

The relative frequency for the 9 – <11 class is about .27, so roughly half of this, or .135, should be between 9 and 10. Thus

$$\text{proportion of observations less than 10} \approx .37 + .135 = .505 \quad \text{(slightly more than 50\%)}$$

The exact value of this proportion is 47/90 = .522.

Density Histograms 密度直方圖

- For **Continuous Data** (**Unequal Class Widths**):
  - After determining the frequencies and relative frequencies, calculate height of each rectangle:

$$\text{rectangle height} = \frac{\text{relative frequency of the class}}{\textbf{class width}}$$

  - Resulting rectangle heights are called **densities**; the vertical scale is the **density scale**
  - Will also work for equal class widths

Density Histograms

Example: Corrosion of reinforcing steel in concrete structures (n = 48)

鋼筋混凝土建築的腐蝕問題，透過glass-fiber-reinforced plastic來包覆混凝土外圍的處理

11.5	12.1	9.9	9.3	7.8	6.2	6.6	7.0	13.4	17.1	9.3	5.6
5.7	5.4	5.2	5.1	4.9	10.7	15.2	8.5	4.2	4.0	3.9	3.8
3.6	3.4	20.6	25.5	13.8	12.6	13.1	8.9	8.2	10.7	14.2	7.6
5.2	5.5	5.1	5.0	5.2	4.8	4.1	3.8	3.7	3.6	3.6	3.6

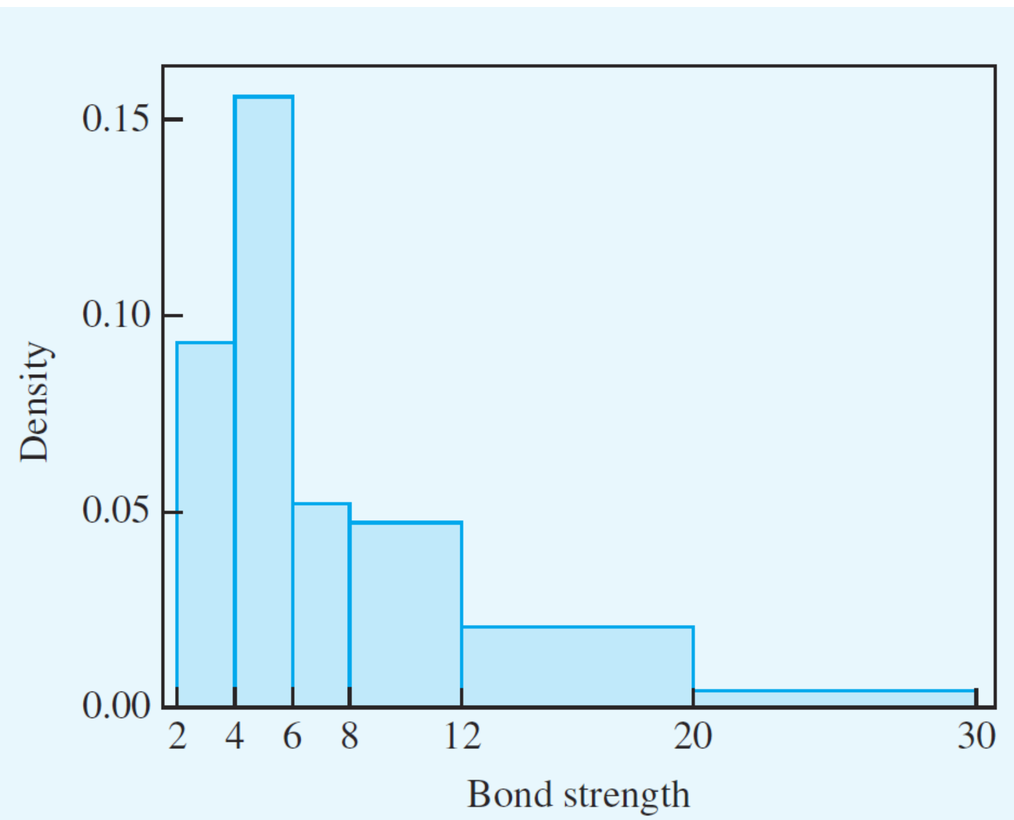
Class:	2–<4	4–<6	6–<8	8–<12	12–<20	20–<30
Frequency:	9	15	5	9	8	2
Relative frequency:	.1875	.3125	.1042	.1875	.1667	.0417
Density:	.094	.156	.052	.047	.021	.004

Density Histograms

relative frequency = (class width) (density)

= (rectangle width) (rectangle height)

= **rectangle area**



底下面積總=1

Histograms Shapes

- **Unimodal (單峰)** – rises to a single peak and then declines
  - **positively skewed:** if the right or upper tail is stretched out compared with the left or lower tail
  - **negatively skewed:** if the longer tail extends to the left
- **Bimodal (雙峰)** – has two different peaks
- **Multimodal (多峰)** – more that two peaks
- **Symmetric** – if the left half is a mirror image of the right half

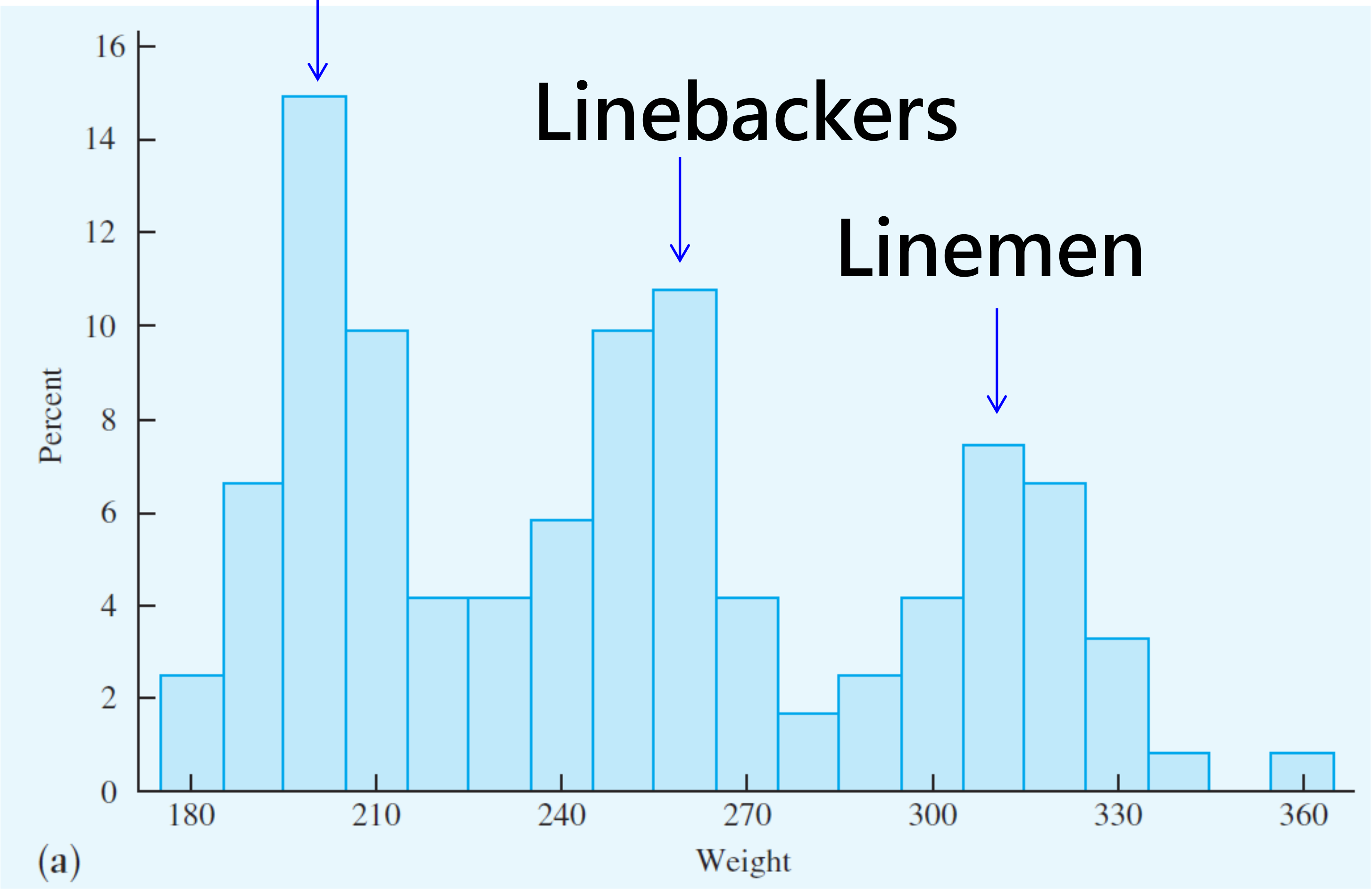
Histograms Shapes

Example: Histogram and smoothed histogram

- the weights (lbs) of 121 NFL players

others

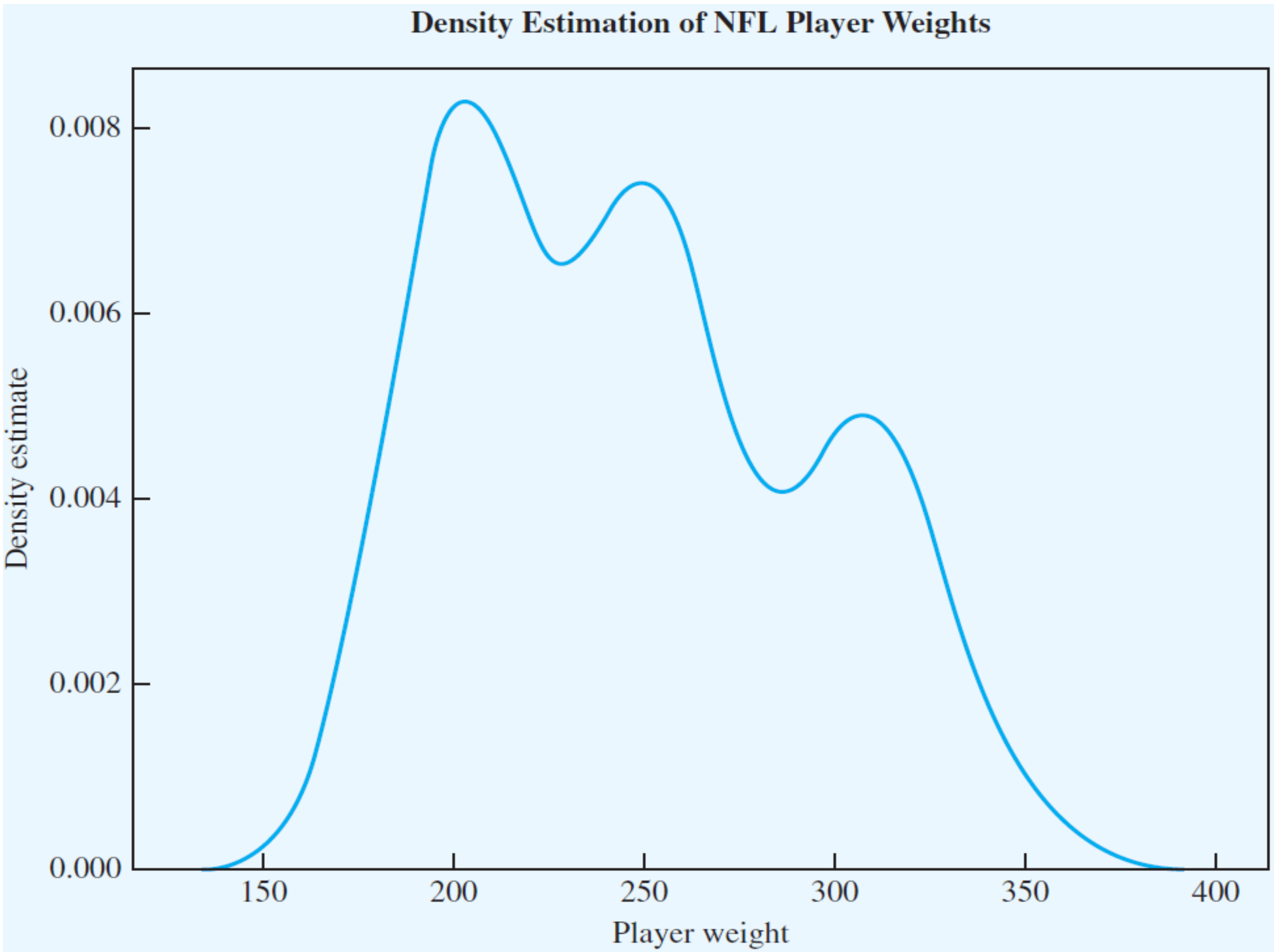
Multimodal



Histograms Smoothed-density

Example: Histogram and smoothed histogram

- the weights (lbs) of 121 NFL players



Histograms Smoothed-density

平滑化直方圖優勢: 可以更清楚判釋直方圖的形貌特徵，易於定義說明資料特徵。

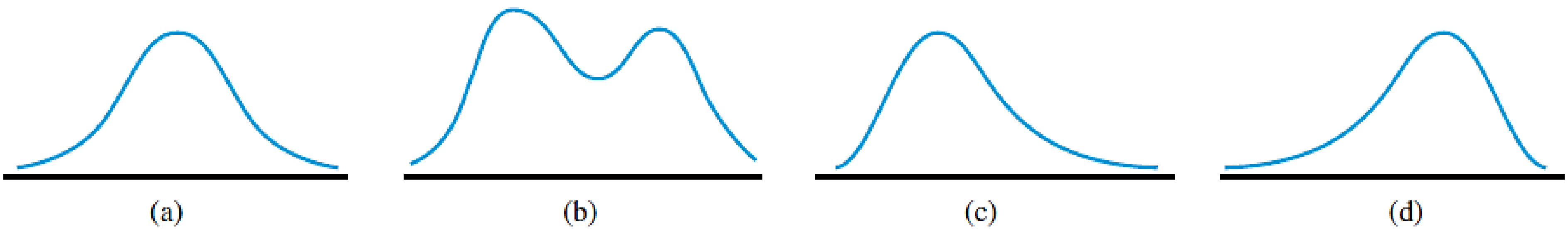


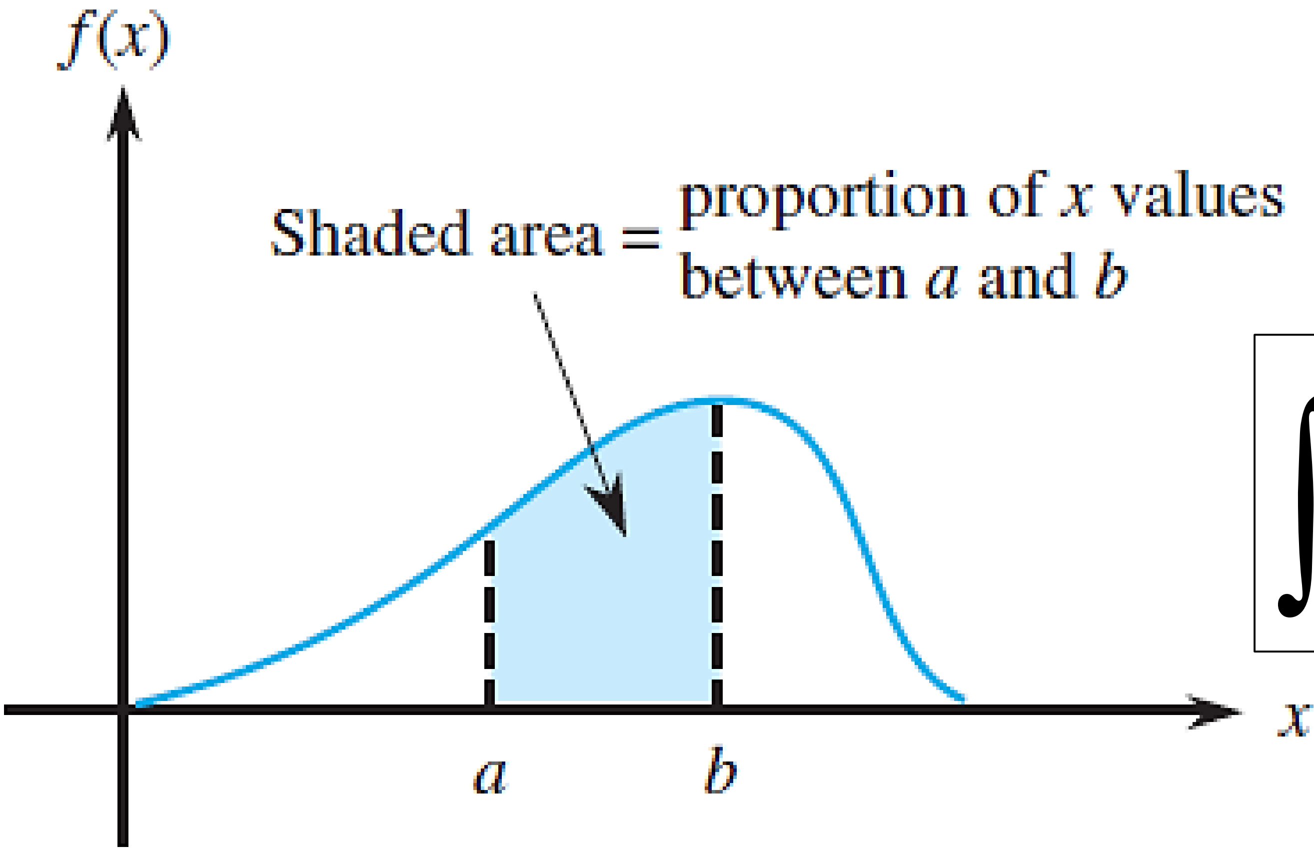
圖 1.11 平滑直方圖：(a) 對稱單峰；(b) 雙峰；(c) 正偏；(d) 負偏。

Histograms Smoothed-density

- **Density function** $f(x)$: used to describe population or process distribution of a continuous variable x
- **Density curve:** graph of $f(x)$
- The following properties must be satisfied:
  1. $f(x) \geq 0$
  2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$
  3. For any two numbers a and b with $a < b$, proportion of x values between a and b =

$$\int_{a}^{b} f(x)\,dx$$

Histograms Smoothed-density



$$\int_a^b f(x)\,dx$$

圖 1.14　密度曲線下方的面積等於區間內數值在整個數線上的比例。

Displays for univariate data: most appropriate for smaller datasets

geom_histogram(bins, binwidth).

Frequency

ggplot(data = , aes(x))

Density

ggplot(data = , aes(x,..density..))

TRY it in R

R: Descriptive statistics

R_descriptive_b.R

R語言常用描述統計函數

length()	資料長度
mean()	平均數
median()	中位數
range()	全距
quantile()	四分位數
IQR()	四分位差
summary()	描述統計摘要
sd()	標準差
var()	變異數
skewness()	偏度
kurtosis()	峰度

R語言數學運算符號

加: +
減: -
乘: *
除: /
整除: %/%
餘數: %%
冪次: ^

R語言數學函式

log()
log10()
exp()
sin()
cos()
asin()
acos()

Measures of Center

- Measures of Center for Data
  - *The Sample Mean*
  - *The Sample Median*
  - *Trimmed Means*

- Measures of Center for Distributions
  - *Discrete Distributions*
  - *Continuous Distributions*
  - μ *and* $\overline{x}$
  - *The Median of a Distribution*

Measures of Center for Data: Mean

- Suppose that the sample consists of observations on a numerical variable *x*
- *n* represents the sample size
- The individual observations: $x_1, x_2, \ldots, x_n$

The Sample Mean

$$\bar{x} = \frac{x_1 + x_2 + \ldots}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Measures of Center for Data: Example

✓ stem-and-leaf display of the data

```
0H | 6
1L | 024
1H | 556777889
2L |
2H |
3L | 0
```

Observation:

- Sample mean: $\bar{x} = \frac{229.0}{14} = 16.36$
- $x = 30.5$ is an **outlier**

Without this outlier: $\bar{x} = 16.36$ □ $\bar{x} = 15.27$

✓ **<u>樣本平均值容易受Outlier影響!!!</u>**

The Sample Median:

- 相對於樣本平均值，樣本中位數較不受Outlier影響

$$\tilde{x} = \begin{cases} \left(\dfrac{n+1}{2}\right)\text{th value on ordered list} & n \text{ odd} \\ \text{average of } \dfrac{n}{2}\text{th and } \left(\dfrac{n}{2}+1\right)\text{th values} & n \text{ even} \end{cases}$$

Measures of Center for Data: Example

Music composer’ s instructions
(12位演奏家呈現特定的曲目其演奏時間)
✓Durations (min) are listed in increasing order:

62.3	62.8	63.6	65.2	65.7	66.4	67.4	68.4	68.8	70.8	75.7	79.0

- The sample median: $\tilde{x} = (66.4 + 67.4)/2 = 66.90$

✓If the largest observation 79.0 had not been included:

$$\tilde{x} = 66.9 \;\square\; \tilde{x} = 66.4$$

Measures of Center for Data: Trimmed

Trimmed Means

- A trimmed mean is a compromise between $\bar{x}$ and $\tilde{x}$.
- **Less sensitive** to outliers than the mean but more sensitive than the median.
- The observations are **first ordered** from smallest to largest.
- A trimming percentage 100*r*% is chosen, where *r* is a number between 0 - 0.5.
- The sample mean is a 0% trimmed mean.
- The median is a trimmed mean corresponding to the largest possible trimming percentage.

Measures of Center for Data: Example

Lifetime (hr) of a certain type of incandescent lamp (燈泡壽命)

✓Consider 20 observations, ordered from smallest to largest:

612	623	666	744	883	898	964	970	983	1003
1016	1022	1029	1058	1085	1088	1122	1135	1197	1201

$$\bar{x} = 19{,}299 / 20 = 965.0 \text{ and } \tilde{x} = (1003 + 1016) / 2 = 1009.5$$

- <u>The 10% trimmed mean (20x0.1=2):</u>

$$\bar{x}_{\text{tr}(10)} = \frac{19{,}299 - 612 - 623 - 1197 - 1201}{16} = 979.1$$

Measures of Center for Distribution

The primary measure of center for a discrete distribution is the mean value; for continuous distributions, both the mean value and the median are frequently used.

Measures of Center for Distribution: Discrete

Definitions:

- The **mean** value of a **discrete variable x**

$$\mu_x = \sum x \cdot p(x)$$

描述隨機變數水準的統計量稱為期望值(expected value)。

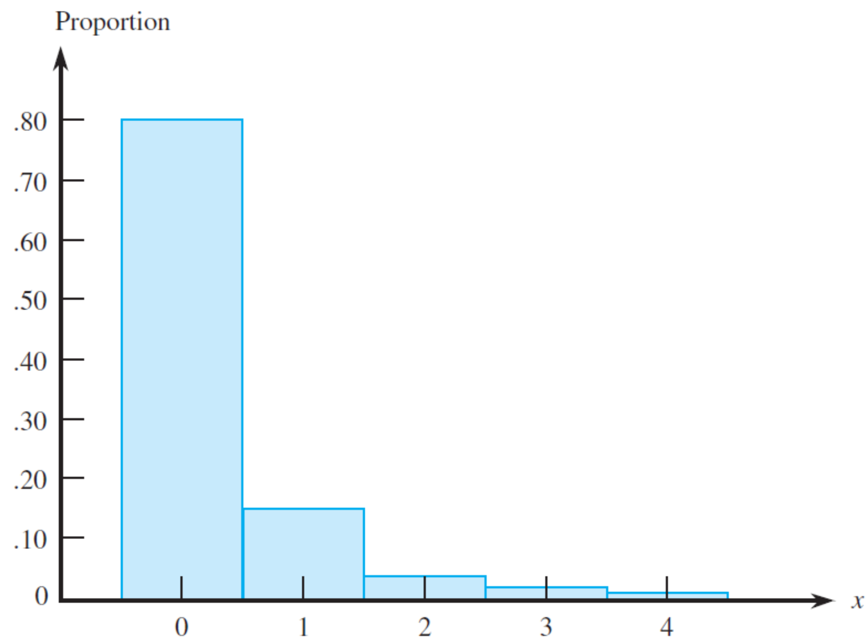
Discrete Distributions

Example: Manufacturing plastic parts

- Let x represent the number of defects on a single part

x:	0	1	2	3	4
$p(x)$:	.80	.14	.03	.02	.01

- Where is this distribution centered?
- What is the mean or long-run average value of x?



Measures of Center for Distribution: Discrete Example

Example 2.4

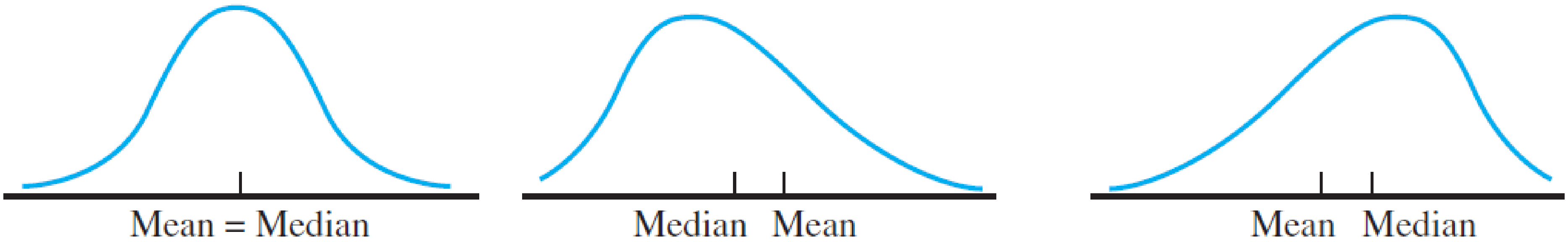
We return now to the plastic part scenario introduced at the outset of this subsection. The mean value of x, the number of defects on a part, is

$$
\begin{aligned}
\mu_x &= \sum_{x=0}^{4} x \cdot p(x) \\
&= 0(p(0)) + 1(p(1)) + 2(p(2)) + 3(p(3)) + 4(p(4)) \\
&= (0)(.80) + (1)(.14) + (2)(.03) + (3)(.02) + (4)(.01) \\
&= .30
\end{aligned}
$$

When we consider the population of all such parts, the population mean value of x is .30. Alternatively, .30 is the long-run average value of x when part after part is monitored. It can also be shown that the histogram of the distribution of Figure 2.4

產品平均缺陷數量=0.3 is not a possible value of x

Measures of Center for Distribution: Mean & Median



偏度(skewness)是指資料分配的不對稱性。測度資料分配不對稱性的統計量稱為偏度係數(coefficient of skewness, SK)。

$$k_3 = \square \frac{(x-\overline{x})^3}{n}$$

$$k_2 = \square \frac{(x-\overline{x})^2}{n}$$

$$SK = \frac{k_3}{(k_2)^{3/2}}$$

- ✓ 負值表示左偏分配
- ✓ 正值表示右偏分配
- ✓ SK等於0: 對稱分配
- ✓ 若SK大於1或小於-1，視為嚴重偏斜分配

Measures of Variability

- Measures of Variability for Sample Data
- The Variance and Standard Deviation of a Discrete Distribution
- The Variance and Standard Deviation of a Continuous Distribution
  - *The Case of a Normal Distribution*
  - *Other Continuous Distributions*
  - σ^2 *and* s^2

Measures of Variability for Sample Data

- **Range:** the difference between the largest and smallest sample values
- **Deviations from the mean:** $x_1 - \bar{x}, x_2 - \bar{x}, \ldots$
- **Sample variance: (樣本變異數)**

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{S_{xx}}{n-1} \qquad S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2$$

- **Sample standard deviation: (樣本標準差)**

$$s = \sqrt{s^2}$$

Measures of Variability: Discrete

- The **variance of a discrete distribution** for a variable *x*, mass function *p*(*x*), is

$$\sigma^2 = \sum (x-\mu)^2 \cdot p(x)$$

- The **standard deviation** is σ, the positive square root of the variance

Measures of Variability: Continuous

- **The variance of a continuous distribution** specified by density function $f(x)$ is

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\, dx$$

- The **standard deviation** is σ, the positive square root of the variance

Measures of Variability: Kurtosis

峰度(kurtosis)是指資料分佈峰值高低。通常是與標準常態分佈比較而言。由於標準常態分配的峰度係數為3，當K大於3時為尖峰分配，資料的分佈相對集中；當K小於3時為扁平分佈，資料的分佈相對分散。

$$m_4 = \square \frac{(x-\overline{x})^4}{n}$$

$$m_2 = \square \frac{(x-\overline{x})^2}{n}$$

$$KT = \frac{m_4}{(m_2)^2}$$

R: Descriptive data: Normal Distribution

Function:

rnorm()

dnorm()

R: Descriptive data: Data shape

Package: moments

R: Descriptive data: Data shape

skewness().

kurtosis().

TRY it in R

R: Descriptive data: Data shape

R_descriptive_c.R

More Detailed Summary Quantities

描述樣本資料離散程度的統計量，除了變異數和標準差，主要還有**四分位距**
(quartile deviation or inter-quartile range)。

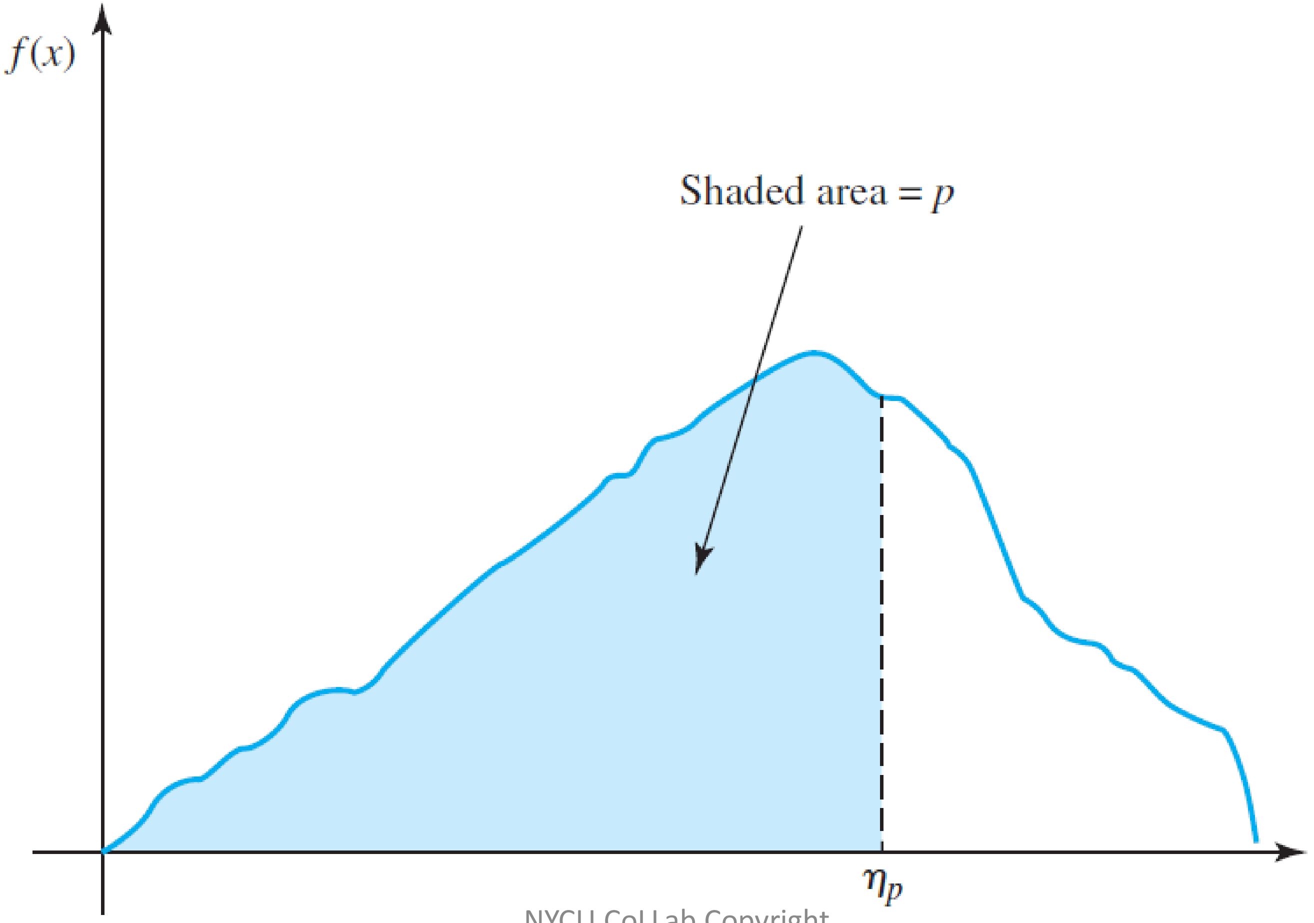
$$IQR = Q_{75\%} - Q_{25\%}$$

More Detailed Summary Quantities

$$\int_{-\infty}^{\eta_p} f(x)\,dx = p$$

Percentiles (百分位數)

p is the area under the density curve to the left of η_p.



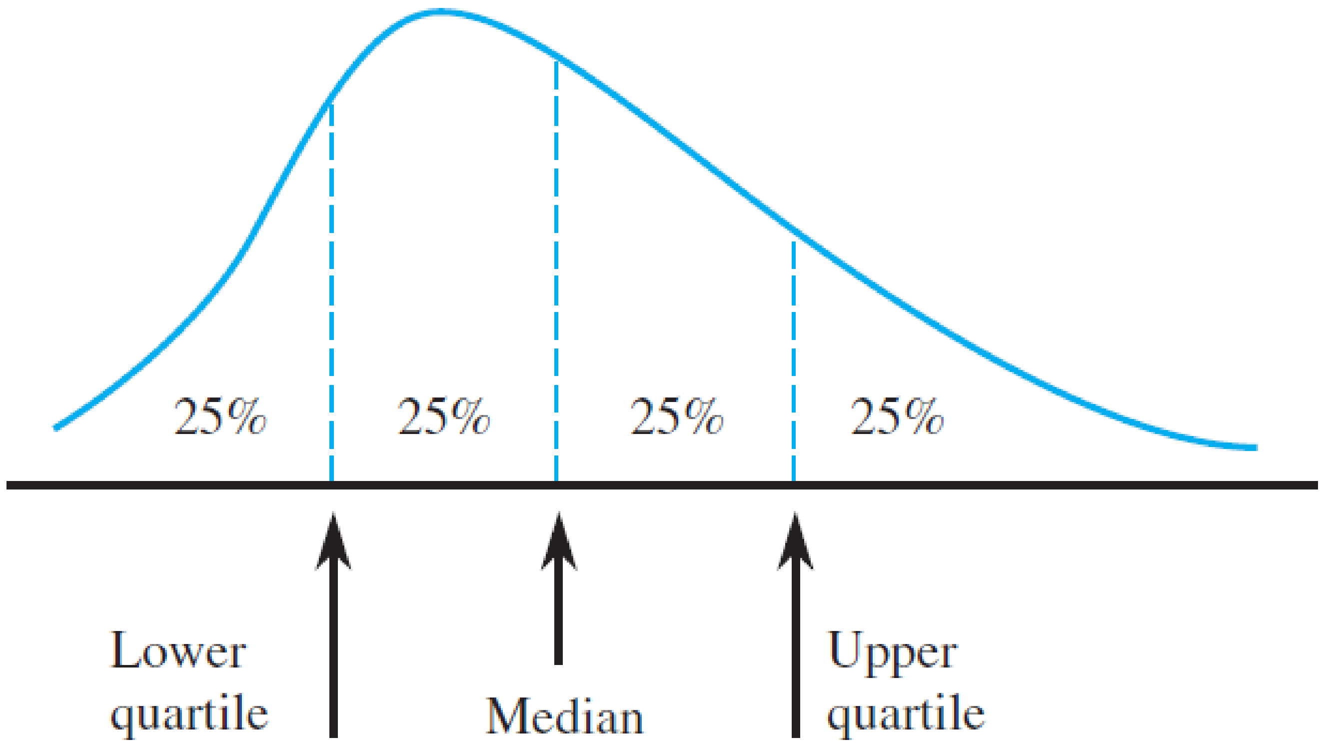
More Detailed Summary Quantities

The **median** and **IQR** can be used together to give a concise yet informative visual summary of sample data called a

✔ boxplot (盒鬚圖)

More Detailed Summary Quantities

- Quartiles and the Interquartile Range (IQR)
- Boxplots
- Boxplots That Show Outliers
- Percentiles



Quartiles & the IQR: Example

- A stem-and-leaf display of the 27 observations follows:

5	9
6	3 3 5 8 8
7	0 0 2 3 4 6 7 7 8 8 9
8	1 2 7
9	0 7 7
10	7
11	3 6 7

Stem: ones digit
Leaf: tenths digit

- Because $n = 27$ is odd, the median $\tilde{x} = 7.7$ is included in each half of the data:

Lower half:	5.9	6.3	6.3	6.5	6.8	6.8	7.0	7.0	7.2	7.3	7.4	7.6	7.7	7.7
Upper half:	7.7	7.8	7.8	7.9	8.1	8.2	8.7	9.0	9.7	9.7	10.7	11.3	11.6	11.8

$$\text{lower quartile} = \frac{7.0 + 7.0}{2} = 7.0 \qquad \text{upper quartile} = \frac{8.7 + 9.0}{2} = 8.85$$

$$\text{IQR} = 8.85 - 7.0 = 1.85$$

✔數列最大及最小值若改變將不影響IQR，但卻會影響變異數及標準差

Quartiles & the IQR: normal distribution

Definition:

- A continuous variable *x* is said to have a **normal distribution with parameters** μ **and** σ, where $-\infty < \mu < \infty$ and $\sigma > 0$, if the density function of *x* is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\square < \mathrm{x} < \square$$

- Many population and process variables have distributions **that can be closely fit** by a normal curve

Quartiles & the IQR: normal distribution

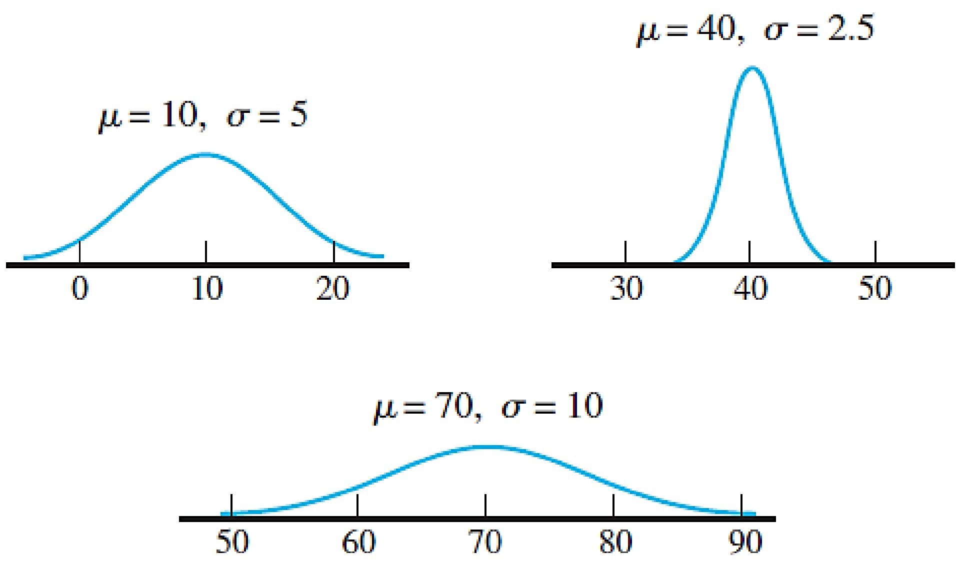


圖 1.19　數種常態密度曲線。

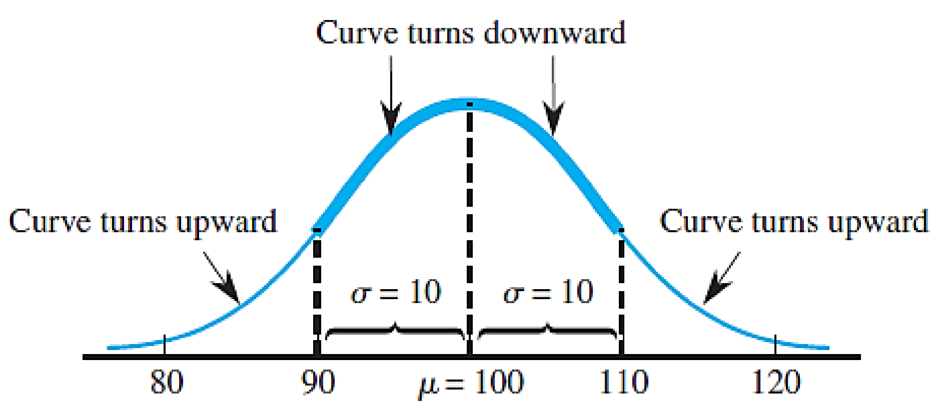


圖 1.20　μ 跟 σ 的視覺化識別。

Quartiles & the IQR: Example

常態分佈尋找下四分位數(25%)及上四分位數(75%)

Example 2.12 The quartiles of a normal distribution are easily expressed in terms of μ and σ. First, consider a variable z having the standard normal distribution. Symmetry of the standard normal curve about 0 implies that $\tilde{\mu} = 0$. Looking for .2500 inside Appendix Table I, we obtain the following information:

$$\text{area to the left of } -.67{:}\quad .2514$$
$$\text{area to the left of } -.68{:}\quad .2483$$

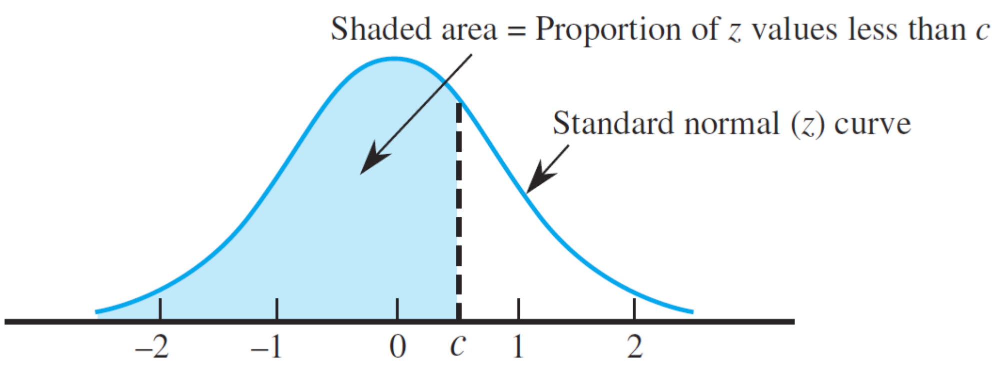
Since .25 is roughly halfway between these two tabled areas, we take $-.675$ as the lower quartile. By symmetry, .675 is the upper quartile.

It is then easily verified that if x has a normal distribution with mean value μ and standard deviation σ,

$$\text{upper quartile} = \mu + .675\sigma \qquad \text{lower quartile} = \mu - .675\sigma$$

我們可以發現常態分佈的IQR會等於1.35倍的標準差。
換句話說，可檢驗樣本資料的IQR值是否等於1.35倍樣本標準差來看資料是否符合常態分佈。

Quartiles & the IQR: Example



pnorm(c, mean =, sd =)
Here:
mean = 算數平均數
sd = 標準差

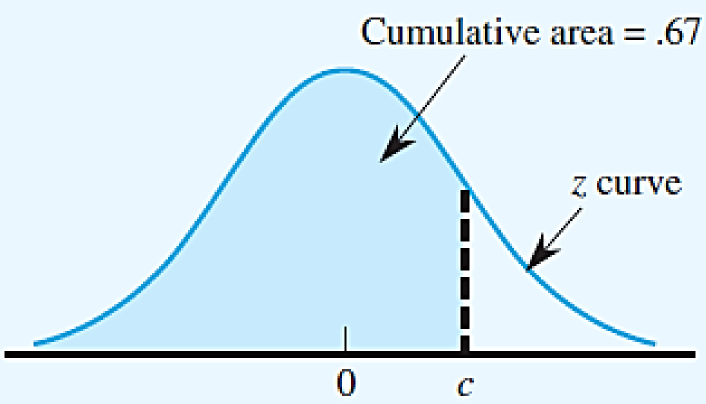


圖 1.24 決定某一項特定累加面積的 c。

qnorm(p, mean =, sd =)
Here:
p = 0.67
mean = 算數平均數
sd = 標準差

R: Descriptive data: IQR in normal distribution

pnorm().

qnorm().

TRY it in R

R: Descriptive data: IQR in normal distribution

R_descriptive_d.R

Boxplot (盒鬚圖)

- A **boxplot** is a visual display of data based on the following **five-number** summary:
  - smallest x_i
  - lower quartile
  - median
  - upper quartile
  - largest x_i

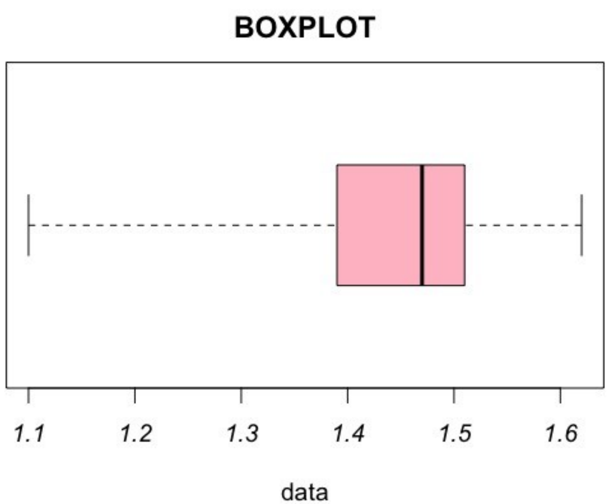
Boxplot: Example

材料樣品之測量比重值

1.10	1.29	1.38	1.39	1.40	1.45	1.46
1.48	1.49	1.50	1.51	1.51	1.56	1.62

The five-number summary

Smallest $x_i = 1.10$ lower quartile $= 1.39$ $\tilde{x} = 1.47$ upper quartile $= 1.51$
Largest $x_i = 1.62$



Boxplot: Outlier

- A boxplot can be embellished to indicate explicitly the presence of outliers.
- Any observation farther than 1.5 IQR from the closest quartile is an **outlier.**
- An outlier is **extreme** if it is more than 3 IQR from the nearest quartile.

Boxplot: Outlier

Thus, any observation smaller than $48 - 34.5 = 13.5$ or larger than $71 + 34.5 = 105.5$ is an outlier. There is one outlier at the lower end of the sample and two at the upper end. Because $71 + 69 = 140$, the largest observation of 144 is an extreme outlier; the other outlier is mild. The whiskers extend out to 32 and 76, the most extreme observations that are not outliers. The resulting boxplot is in Figure 2.11.

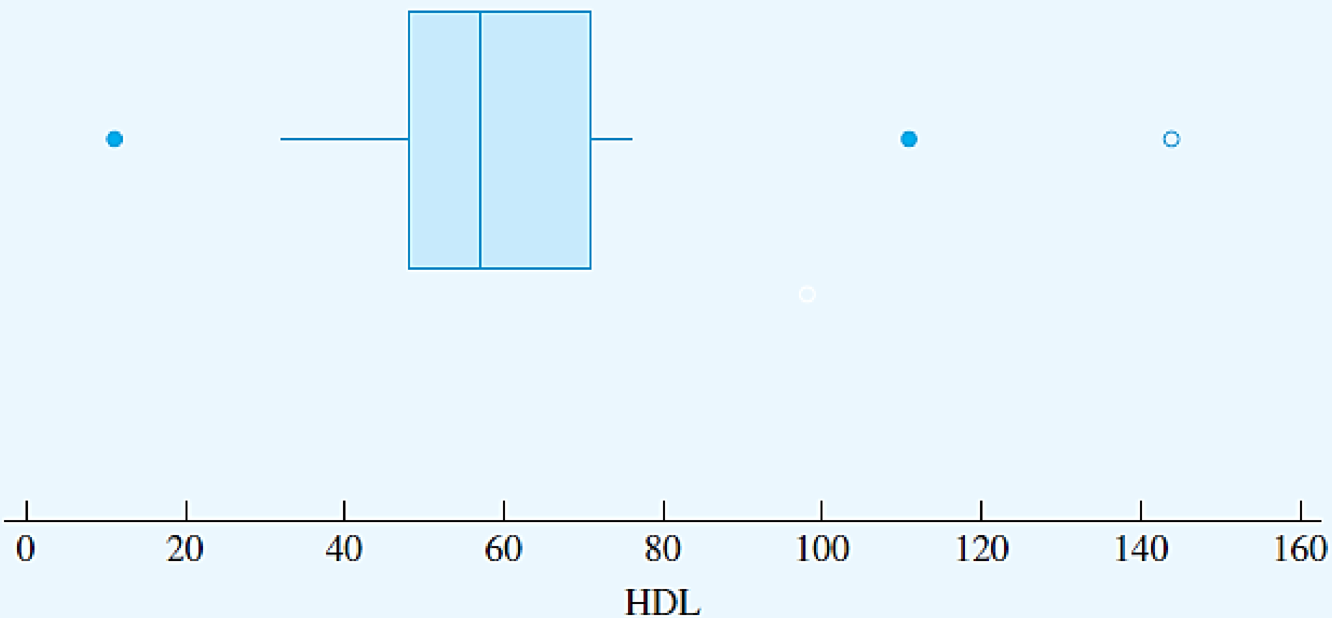


圖 2.11　HDL 膽固醇數據的盒型圖顯示有中度與極度離群值。

R: Descriptive data: display data distribution

geom_boxplot(
coef =,
outlier.shape=,
outlier.size=).

TRY

it

in

R

R: Descriptive data: display data distribution

R_descriptive_e.R

Categorical Data

- **Bar chart (長條圖):** often used to describe histogram for categorical data
- **Pareto (柏拉圖) diagram:** a bar chart where categories appear in order of decreasing frequency; if a miscellaneous category is required, it is placed last

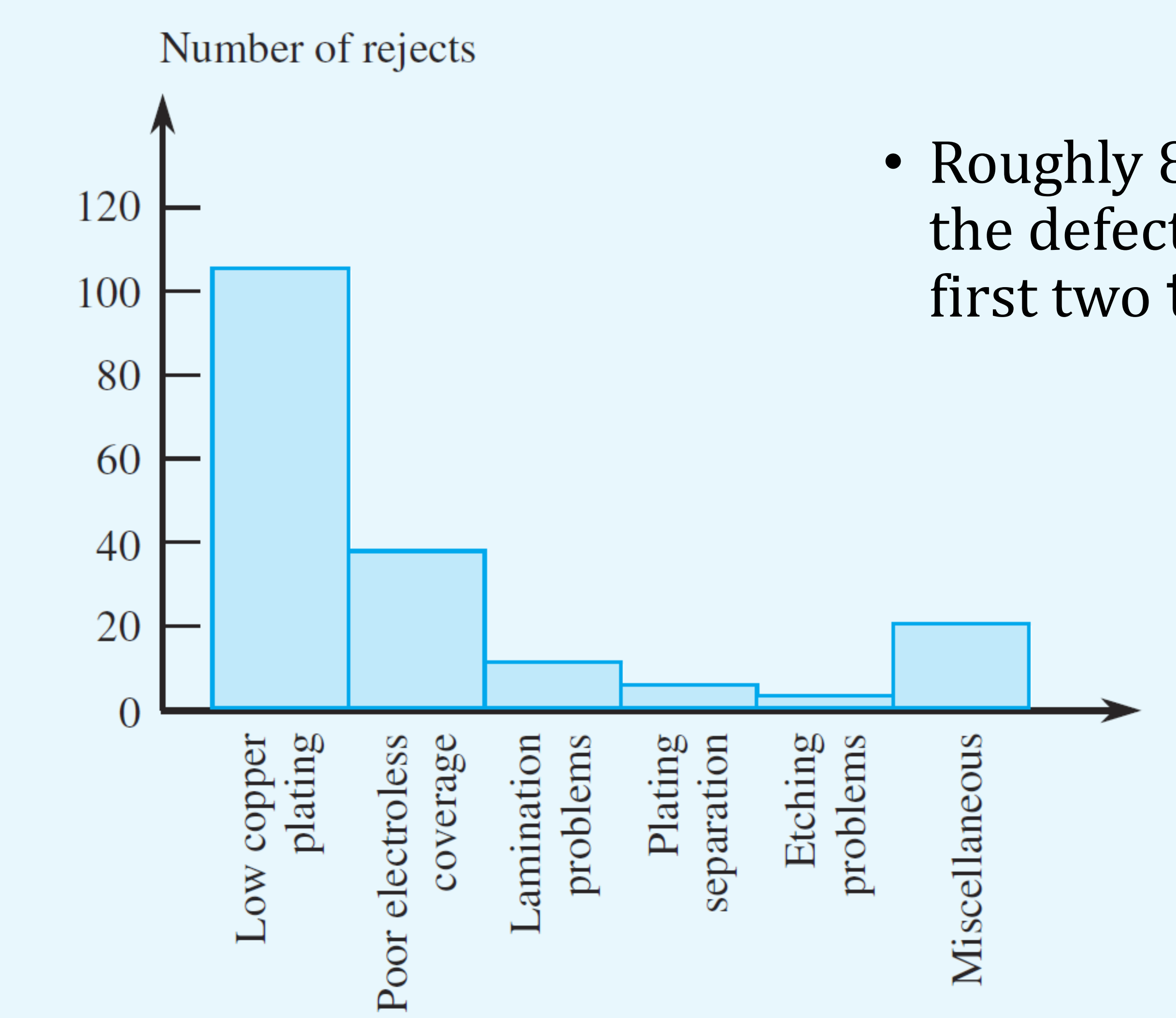
Categorical Data: An Example

- **Example: Manufacture of printed circuit boards**
- Type of defect for each board rejected:

(產品的製成缺陷分類項目，包覆塗料的缺陷、電鍍不完整、層狀問題、塗料剝落、侵蝕剝落)

Type of defect	Frequency	Relative frequency
Low copper plating	112	.615
Poor electroless coverage	35	.192
Lamination problems	10	.055
Plating separation	8	.044
Etching problems	5	.027
Miscellaneous	12	.066

Categorical Data: An Example



- Roughly 80% (.615 1 + .192) of the defects were of one of the first two types

R: Descriptive data: quick to get more details

Package: psych

R: Descriptive data: quick to get more details describe().

課堂練習: 學號-姓名-ch5-Descriptive.R

30個圓柱狀試體的彈性模數大小數據:

20.0 25.0 30.0 37.0 37.5 38.1 40.0 40.2 40.8 41.0 42.0 43.1 43.9 44.1 44.6 45.0 46.1 47.0 50.2 55.0 56.0 57.0 58.0 62.0 64.3 68.8 70.1 74.5 90.0 100.0

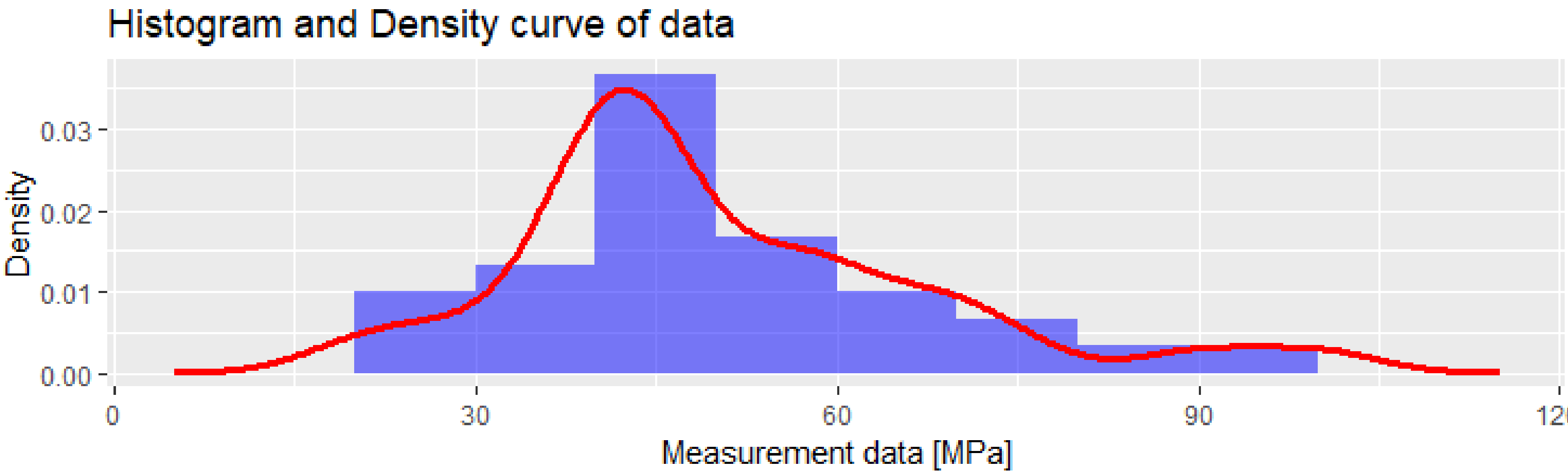
單位: MPa

試著回答以下問題:

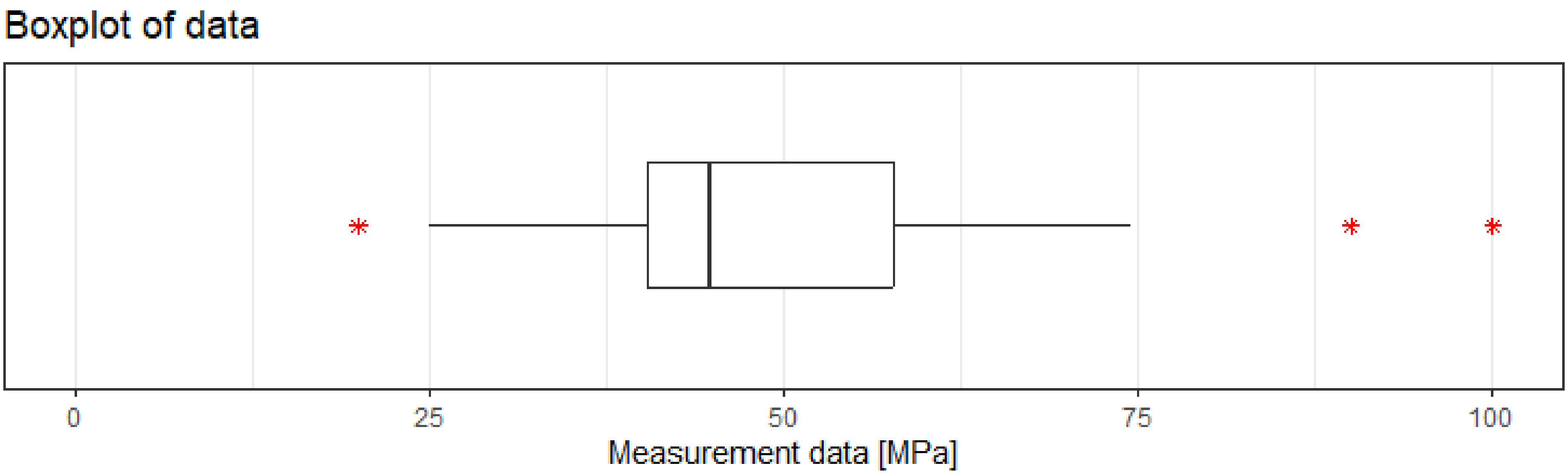
(1) Construct a histogram with the density curve.
(hist(data, breaks = 6, plot = FALSE); density(x,width = 20))

(2) What are the skewness and kurtosis values? Please make a short conclusion.

(3) Construct a boxplot display of the data. Does there appear to be any outlying values? (coef = 1.0)
(去除y軸座標指令: scale_y_discrete())



Histogram and Density curve of data
Density
0.00
0.01
0.02
0.03
0
30
60
90
120
Measurement data [MPa]



Boxplot of data
0
25
50
75
100
Measurement data [MPa]