INTRODUCTION TO ROC CURVE ANALYSIS WITH APPLICATIONS IN FUNCTIONAL GENOMICS

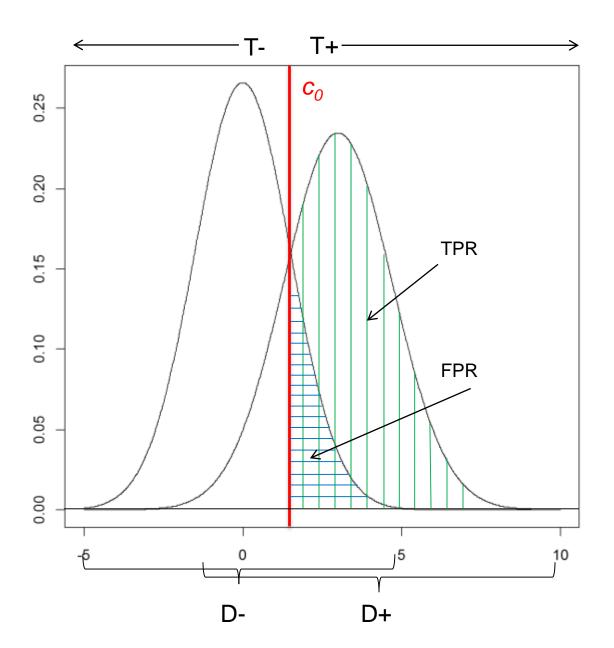
Presented by: Shana White March 31, 2015

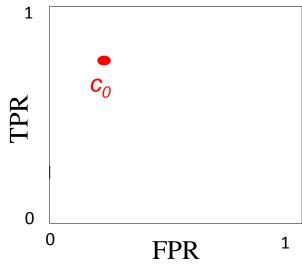
OUTLINE

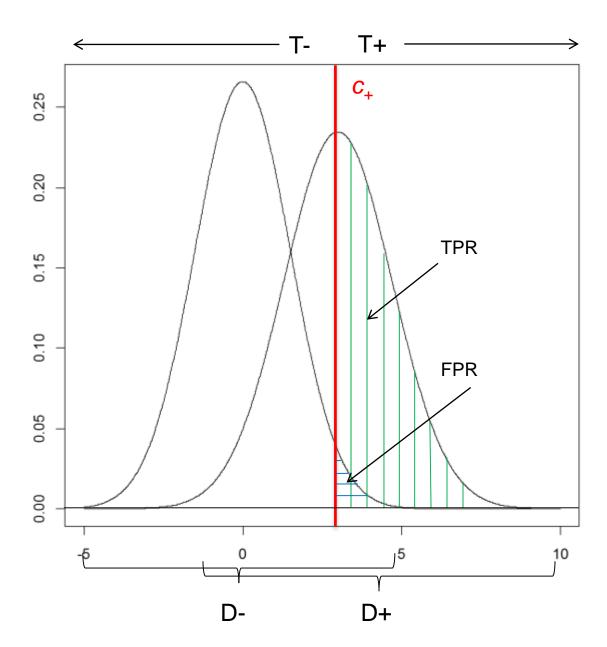
- Introduction
 - Parametric ROC Curve
 - Non-Parametric ROC Curve
- Examples
 - Single Cutoff-Point: Single mutation
 - Continuous outcome: Gene expression levels
 - Discreet outcome: Gene ranks
- Extensions
 - 'Non-Proper' ROC Curves
 - Multi-class ROC Analysis

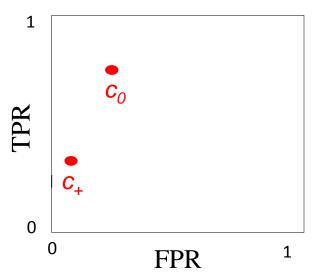
ROC CURVES

- ROC = Receiver-operator characteristic (signal detection)
- Purpose: evaluation of classification model that predicts binary group membership
 - Model can be based on supervised or unsupervised methods, but 'true' group membership (as determined by 'gold standard') is known
- Depict tradeoffs between sensitivity and specificity over a range of cutoff values. In terms of disease:
 - Sensitivity (True positive rate [TPR]): identification of those who truly have the disease (D+) as having the disease (T+)
 - Specificity (True negative rate [TNR]): identification of those that do not actually have the disease (D-) as not having the disease (T-)
 - The value [1-Specificity] (False positive rate [FPR]) is used in graphs
 - Ideal classification models have high TPR while maintaining low FPR
 - TPR and FPR increase/decrease together

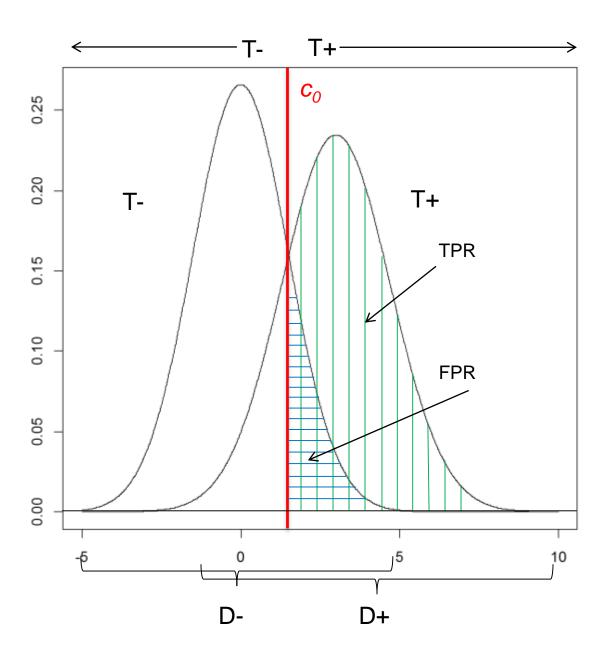


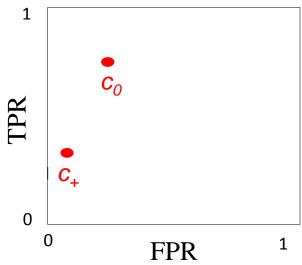


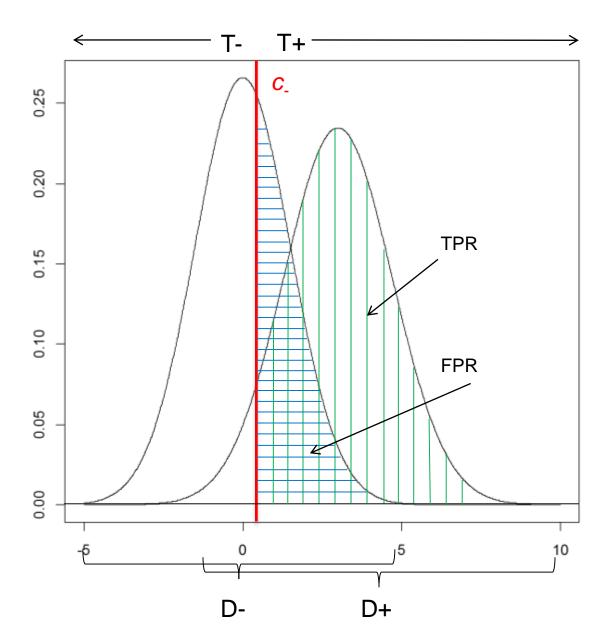


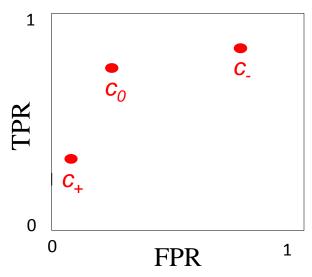


- Increasing the cutoff point (making the test more conservative) decreases TPR and FPR
 - Less false alarms, but also less true positives identified





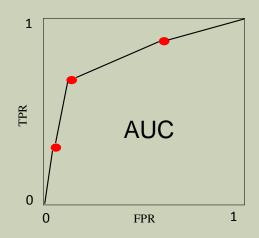




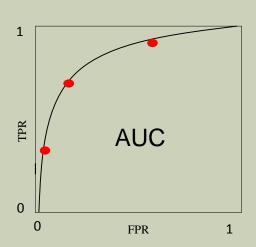
- Decreasing the cutoff point (making the test *less* conservative) increases both TPR and FPR
 - More false alarms, but also more true positives identified

AUC

Non-Parametric

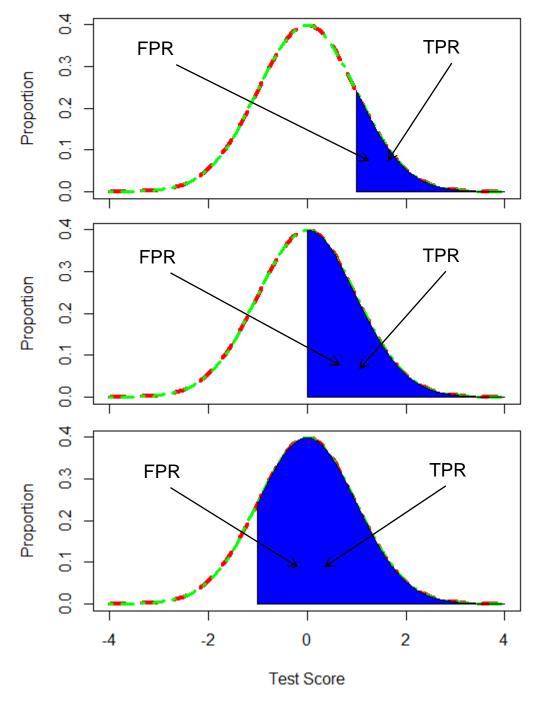


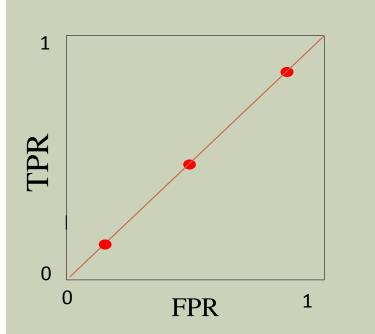
Parametric



The area under the ROC curve (AUC) is a statistical summary of overall accuracy

- Equivalent to the probability that a test (or classification model) will correctly classify a random pair of observations when one is from the D+ group and the other is from the D- group
 - → measure of how well the populations are separated by the classification model





- AUC Values range: 0.5 1.0
 - 0.5 test is no better than a random class assignment
 - 1.0 perfect class prediction

PARAMETRIC CALCULATION (NORMAL DISTRIBUTION)

Let X_0 and X_1 be test scores for 2 different populations of interest (such as no disease versus disease).

If
$$X_0 \sim N(\mu_0, \sigma_0)$$
 and $X_1 \sim N(\mu_1, \sigma_1)$, then
$$\text{AUC} = \Phi\left(\frac{a}{\sqrt{1+\,b^2}}\right)$$
 Where $a = \frac{\mu_1 - \mu_0}{\sigma_1}$, $b = \frac{\sigma_0}{\sigma_1}$, and Φ is the cumulative normal distribution function.

I think it helps to look at the equation a bit differently:

$$\left(\frac{a}{\sqrt{1+|b|^2}}\right) = \frac{\frac{\mu_1 - \mu_0}{\sigma_1}}{\sqrt{1+\left(\frac{\sigma_0}{\sigma_1}\right)^2}} = \frac{\frac{\mu_1 - \mu_0}{\sigma_1}}{\sqrt{\left(\frac{\sigma_1}{\sigma_1}\right)^2 + \left(\frac{\sigma_0}{\sigma_1}\right)^2}} = \frac{\frac{\mu_1 - \mu_0}{\sigma_1}}{\sqrt{\sigma_1^2 + \sigma_0^2}} = \frac{\mu_1 - \mu_0}{\sqrt{\sigma_1^2 + \sigma_0^2}}$$

It is now readily apparent that AUC:

Increases as distance between means increases.

Decreases when standard deviations increase.

What happens if distance between means is 0?

"DATA-BASED" NON-PARAMETRIC ROC CURVE

Sort data by score.

Above cutoff point: Predicted Group = 1
Below cutoff point: Predicted Group = 0

		Group	Score	FPR	TPR
	TP	1	10	0	0.1
Ī	FN	1	9.4		
	FN	1	9.2		
	FN	1	7.6		
	TN	0	7.4		
	FN	1	7.3		
	TN	0	7.1		
	FN	1	6.5		
	TN	0	6.3		
	TN	0	6.2		
	TN	0	5.6		
	FN	1	5.4		
	TN	0	5.3		
	FN	1	4.7		
	FN	1	4.6		
	TN	0	4.2		
	FN	1	4.1		
	TN	0	3.9		

1.8

1.5

TN

TN

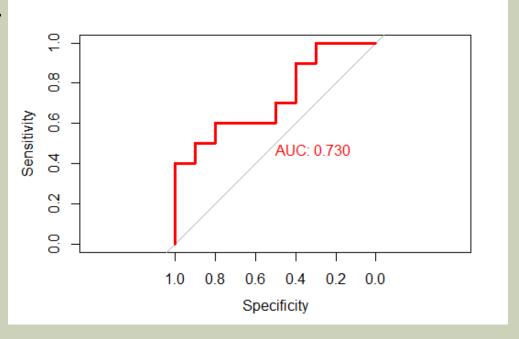
	Group	Score	
TP	1	10	
TP	1	9.4	
TP	1	9.2	
TP	1	7.6	
FP	0	7.4	
FN	1	7.3	
TN	0	7.1	
FN	1	6.5	
TN	0	6.3	
TN	0	6.2	
TN	0	5.6	
FN	1	5.4	
TN	0	5.3	
FN	1	4.7	
FN	1	4.6	
TN	0	4.2	
FN	1	4.1	
TN	0	3.9	
TN	0	1.8	
TN	0	1.5	

FPR	TPR	
0	0.1	
0	0.2	
0	0.3	
0	0.4	
0.1	0.4	

"DATA-BASED" NON-PARAMETRIC ROC CURVE

Above cutoff point: Predicted Group = 1
Below cutoff point: Predicted Group = 0

Gro	up	Score	FPR	TPR
	1	10	0	0.1
	1	9.4	0	0.2
	1	9.2	0	0.3
	1		0	0.4
	0		0.1	0.4
	1		0.1	0.5
	0		0.2	0.5
	1		0.2	0.6
	0		0.3	0.6
	0		0.4	0.6
	0		0.5	0.6
	1		0.5	0.7
	0		0.6	0.7
	1		0.6	0.8
	1		0.6	0.9
	0		0.7	0.9
	1		0.7	1
	0		0.8	1
	0		0.9	1
	0	1.5	1	1



Interpretation of AUC = If I randomly select one person from each group, what is the probability that a person from Group(1) has a higher score than the person from Group(0)?

USING RANKS TO BUILD AN ROC CURVE

Group	Score
0	1.5
0	6.2
0	5.3
0	4.2
0	3.9
0	7.1
0	5.6
0	7.4
0	1.8
0	6.3
1	9.2
1	10
1	4.1
1	6.5
1	5.4
1	9.4
1	4.6
1	7.3
1	4.7
1	7.6

Group	Score
1	10
1	9.4
1	9.2
1	7.6
0	7.4
1	7.3
0	7.1
1	6.5
0	6.3
0	6.2
0	5.6
1	5.4
0	5.3
1	4.7
1	4.6
0	4.2
1	4.1
0	3.9
0	1.8
0	1.5

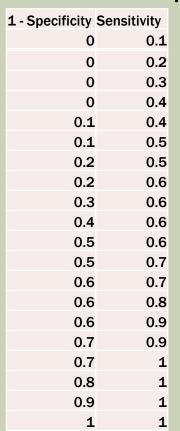
Group	Score	Rank	
1	10	1	
1	9.4	2	
1	9.2	3	
1	7.6	4	
0	7.4	5	
1	7.3	6	
0	7.1	7	
1	6.5	8	
0	6.3	9	1
0	6.2	10	
0	5.6	11	
1	5.4	12	
0	5.3	13	
1	4.7	14	
1	4.6	15	
0	4.2	16	
1	4.1	17	
0	3.9	18	
0	1.8	19	
0	1.5	20	

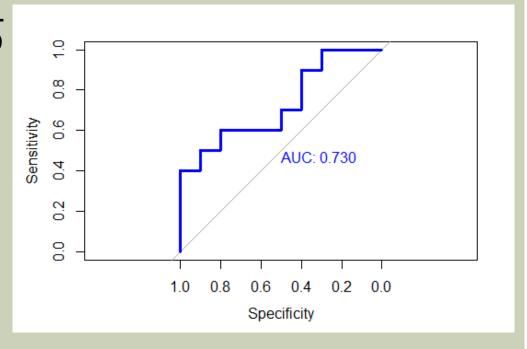
Group	Rank	
1	1	
1	2	
1		
1	4	
0	5	
1	6	
0	7	
1	8	
0	9	
0	10	
0	11	
1	12	
0	13	
1	14	
1	15	
0	16	
1	17	
0	18	
0	19	
0	20	

"RANK-BASED" NON-PARAMETRIC ROC CURVE

Above cutoff point: Predicted Group = 1
Below cutoff point: Predicted Group = 0

Group	Rank
1	1
1	2
1	3
1	4
0	5
1	6
0	7
1	8
0	9
0	10
0	11
1	12
0	13
1	14
1	15
0	16
1	17
0	18
0	19
0	20

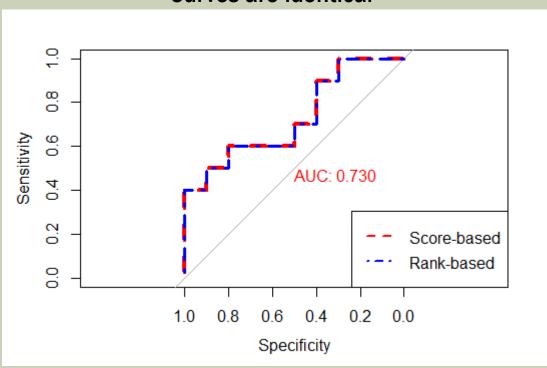




Interpretation of AUC = If I randomly select one person from each group, what is the probability that a person from Group(1) is ranked higher than the person from Group(0)?

RANK-BASED VS SCORE-BASED NON-PARAMETRIC ROC CURVE

Curves are identical



Non-parametric (or "data-based" or "Empirical") curves are inherently rank-based "Rankiness" → nice statistical properties without the need to assume a particular distribution

NON-PARAMETRIC CALCULATION

Let X_0 and X_1 be test scores OR ranks based on scores for 2 different populations of interest (such as no disease versus disease). If there is no distributional assumption for the values of X_0 and X_1 , then

$$\text{AUC} = \frac{1}{n_0 n_1} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} I(X_{i1}, X_{j0})$$
 where $I(X_{i1}, X_{j0}) = \begin{cases} 1 \text{ if } X_{i1} > X_{j0} \\ 1/2 \text{ if } X_{i1} = X_{j0} \\ 0 \text{ if } X_{i1} < X_{j0} \end{cases}$

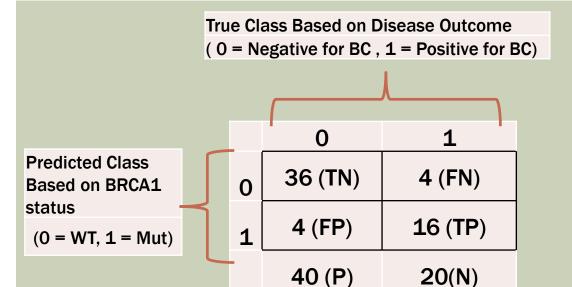
AUC is equal to the Mann-Whitney *U* statistic; it compares the sums of ranks.

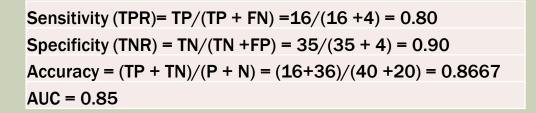
- Approximately normally distributed for large samples.
- Robust (not sensitive to outliers)

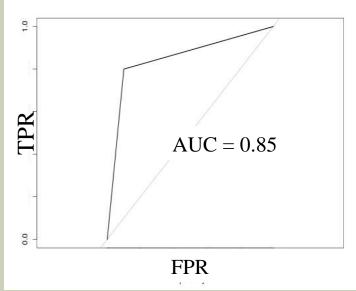
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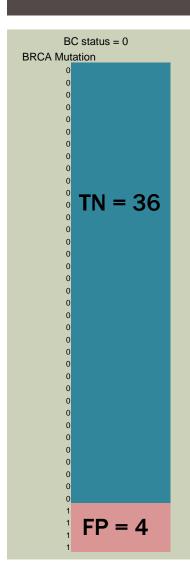
EXAMPLE: SINGLE CUT-POINT, SINGLE MUTATION

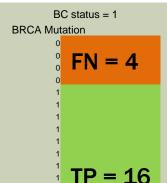






A CLOSER LOOK AT AUC





Total number of pairwise comparisons : (TN + FP)(FN + TP) = (40)(20) = 800

Number of times correct $(X_{i1} > X_{j0})$: TN*TP = 576

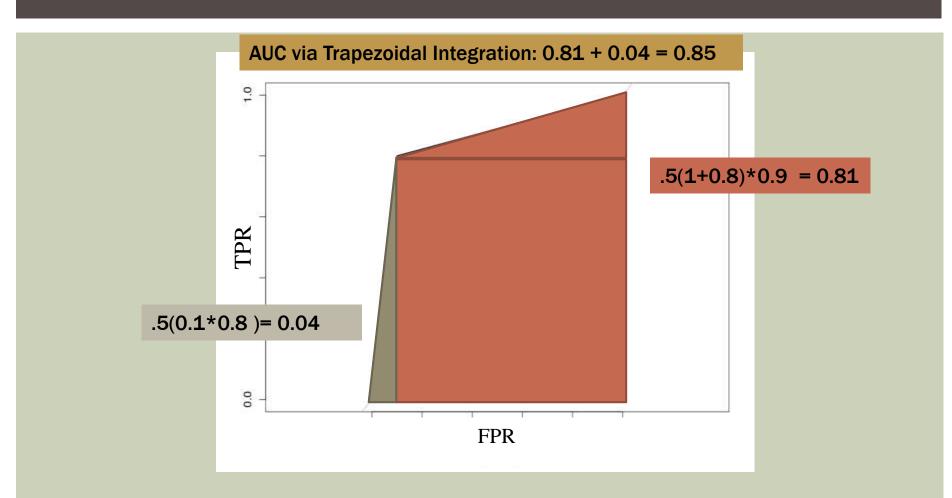
Number of ties $(X_{i1} = X_{j0})$: TP*FP + FN*TN = 64 + 144

$$AUC = \frac{576 + \frac{1}{2}(208)}{800} = \frac{680}{800} = 0.85$$

For one cutoff point:

$$AUC = \frac{2 (TP*TN) + (TP*FP) + (TP*FN)}{2 (TN+FP)(FN+TP)} = \frac{Sens. + Spec.}{2}$$

A GEOMETRIC LOOK AT AUC



*ROC analysis is not typically performed for single cutoff scenarios but hopefully this sheds light on the probabilistic/geometric nature of AUC

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- Hypothetical situation:
 - I read about an cell-line experiment that found certain genes upregulated in MCF-7 (breast cancer) cells. I want to see how the results translate to human BC tumors.
 - I choose two genes and go to the Genomics Portals website to obtain gene expression data

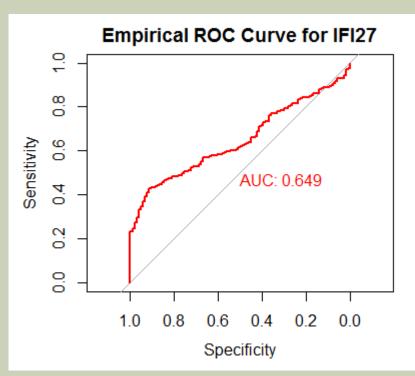
Dataset: Breast invasive carcinoma from TCGA

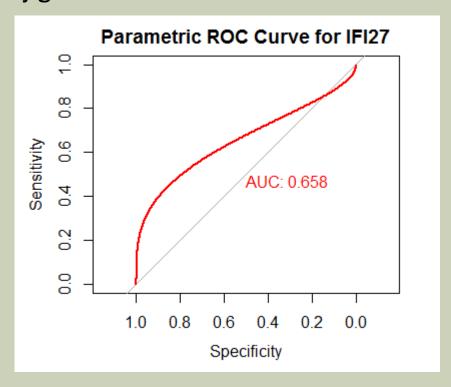
Pubmed source: TCGA_BRCA_exp_HiSeqV2

Genes:

- IFI27 (interferon, alpha-inducible protein 27) promotes cell death and is associated with healing
- FN1 (fibronectin 1) cell adhesion and migration processes
- How well do the corresponding gene expression levels discriminate tumor samples from normal tissue samples?

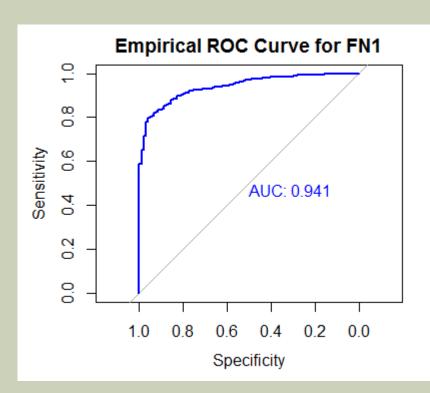
Potential discriminatory gene #1: IFI27

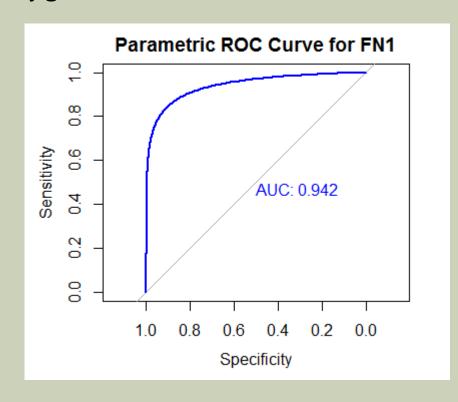




AUC = Probability that that if we randomly select one tumor and one normal sample gene expression for IFI27(tumor) > gene expression for IFI27(normal)

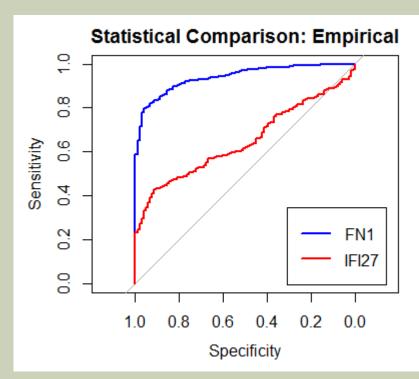
Potential discriminatory gene #2: FN1



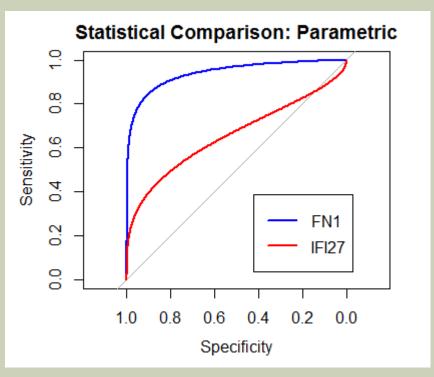


AUC = Probability that that if we randomly select one tumor and one normal sample gene expression for FN1(tumor) > gene expression for FN1(normal)

Is FN1 significantly better at discriminating amongst BC tumor samples than IFI27?



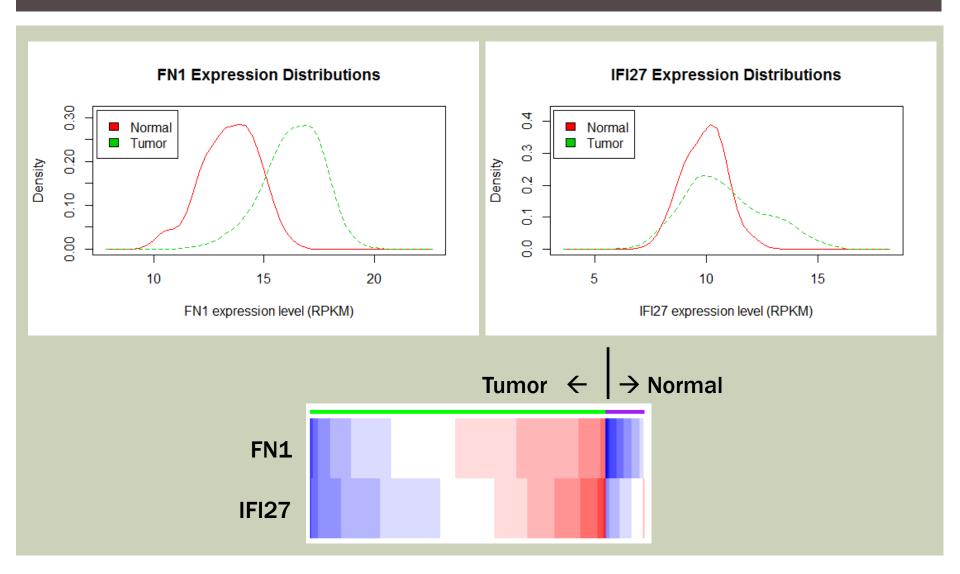
P-value = 1.06 e-46



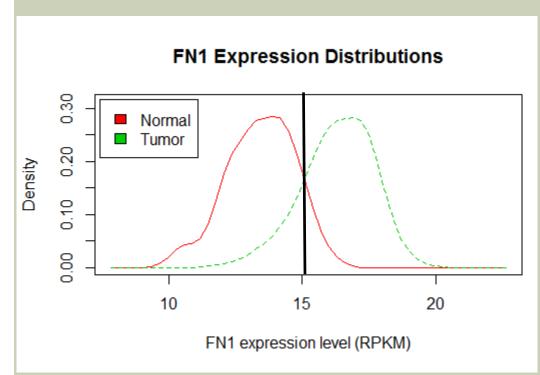
P-value = 4.69 e-42

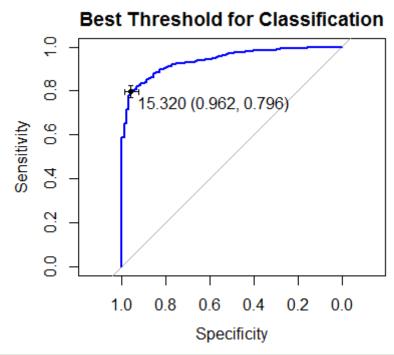
Note: pROC takes into account correlation between measurements when determining significance

A CLOSER LOOK: DATA DISTRIBUTIONS



I decide to go with FN1, now I want to know a little more.





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EXAMPLE: DISCREET OUTCOME - RANKING OF DE GENES

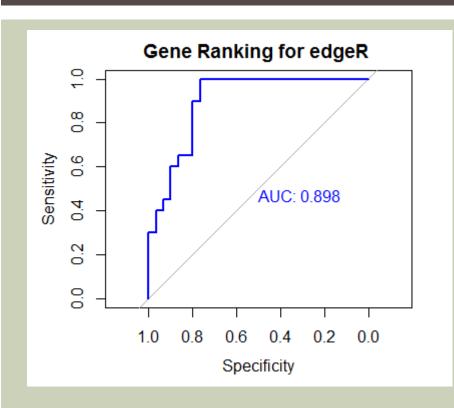
Hypothetical situation:

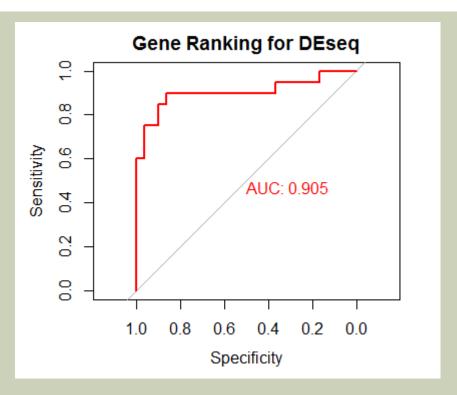
- I have been using DEseq for differential expression analysis. I want to start using edgeR, but I hear it is less conservative than DEseq.
 - Since edgeR is less conservative, I am concerned that it will return more false positives than DEseq.
 - On the other hand, I am concerned that DEseq will return less true positives than edgeR.
 - I construct a simulation data set to represent two groups and expression levels for 50 genes.
 - I choose at random 20 genes to be up- or down-regulated (DE) between groups when I simulate the data
 - (for more detail see Soneson & Delorenzi 2013)
 - I run DE on DEseq and edgeR and then rank the genes by p-value.

EXAMPLE: DISCREET OUTCOME DATA SET-UP

GeneID	DEstatus	rankedgeR	rankDEseq
gene1	1	1	1
gene2	1	2	2
gene3	1	3	3
gene4	1	4	
gene5	1	5	5
gene6	1	6	6
gene7	1	8	
gene8	1	9	8
gene9	1	11	9
gene10	1	13	10
gene 11	1	14	11
gene12	1	15	12
gene13	1	17	14
gene14	1	20	15
gene37	0	37	35
gene38	0	38	36
gene39	0	39	37
gene40	0	40	39
gene41	0	41	40
gene42	0	42	41
gene43	0	43	42
gene44	0	44	43
gene45	0	45	44
gene46	0	46	46
gene47	0	47	47
gene48	0	48	
gene49	0	49	49
gene50	0	50	50

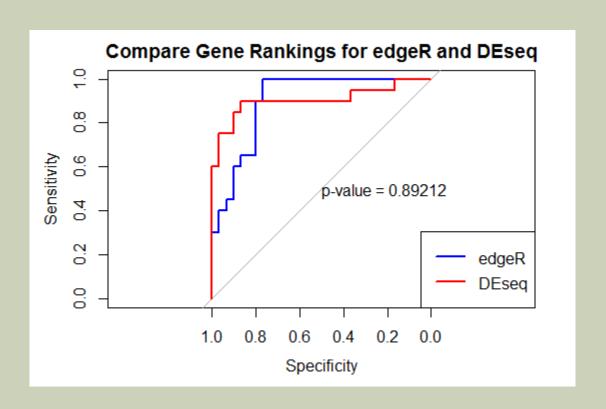
ROC CURVES FOR GENE RANKS





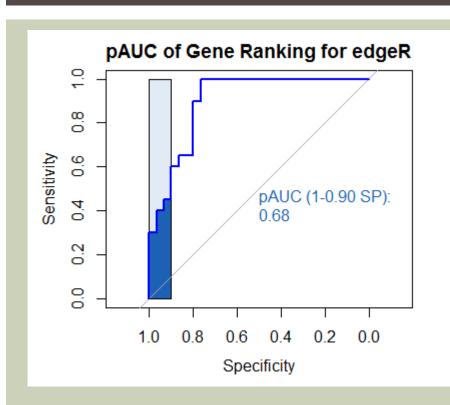
AUC = Probability that that if we randomly select one DE and one non-DE gene: gene rank(DE gene) > gene rank (non-DE gene)

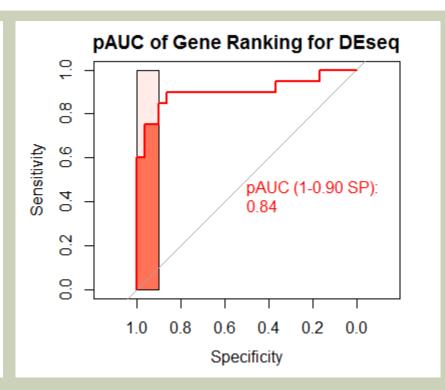
DIFFERENCES IN PERFORMANCE



Is there another way to identify differences in performance?

COMPARISON OVER SPECIFIC FPR

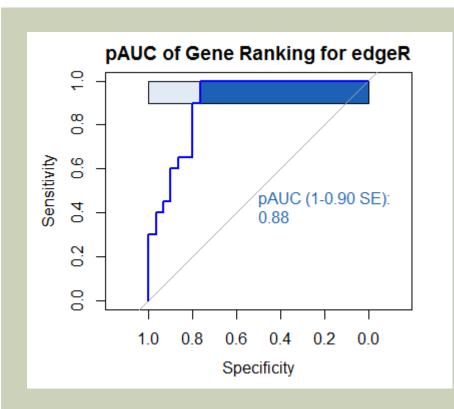


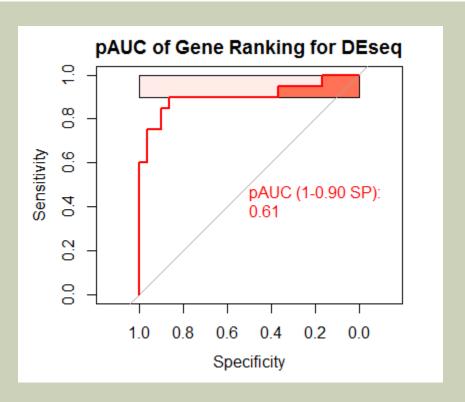


p-value for difference in pAUC= 0.001041

If we are mainly focused on maintaining a low FPR, DEseq outperforms edgeR

COMPARISON OVER SPECIFIC TPR





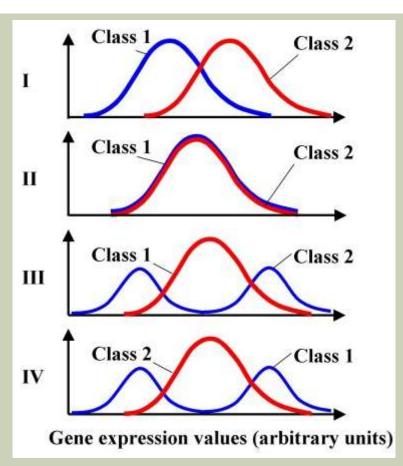
p-value for difference in pAUC= 0.03188

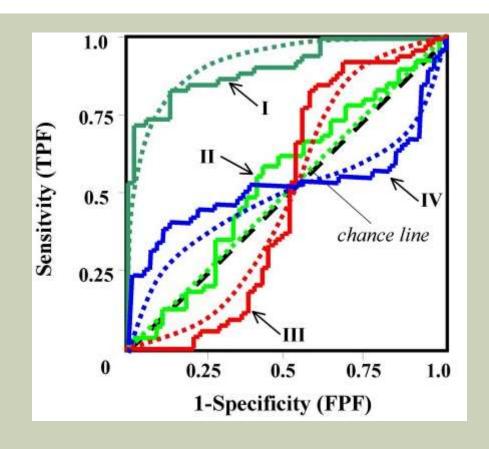
If we want to identify as many genes as possible, edgeR outperforms DEseq at a lower threshold

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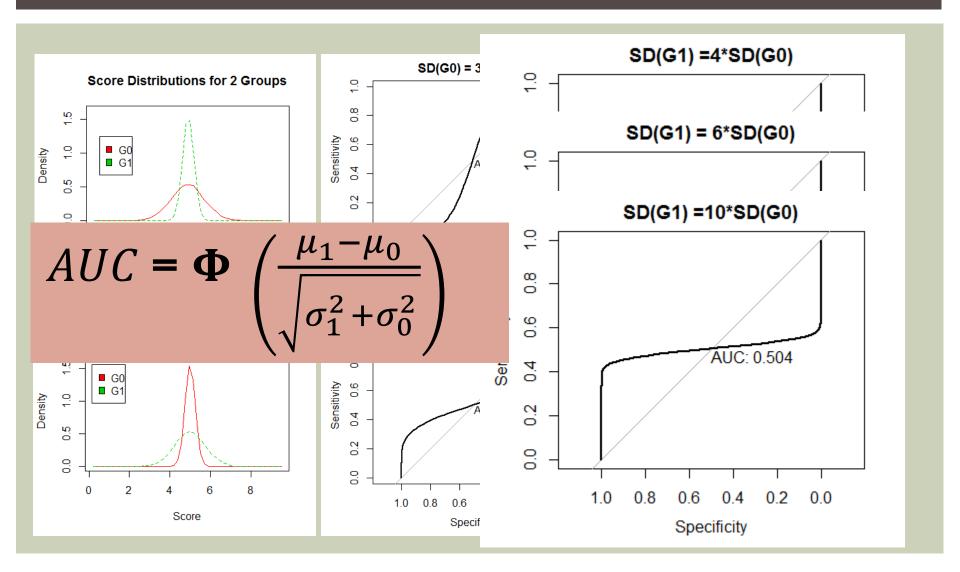
SCENARIOS FOR 'NOT-PROPER' ROC CURVES





Parodi et al. BMC Bioinformatics 2008 9:410 doi:10.1186/1471-2105-9-410

SIMPLIFIED SCENARIO: SAME MEAN, DIFFERENT SD'S



ANALYSIS WITH NOT-PROPER ROCS

- Rationale: If there is no difference in mean measurement between groups, we are 'constrained' to have AUC = 0.50
 - Useful if difference in measurements between groups is not reflected by an overall difference in means.
 - Can account for bimodal distributions, 'nested' distributions
 - May capture unique aspects of biological variability
- ABCR = area between the ROC curve and the rising diagonal
- *TNRC* = Test for Not-proper ROC Curves

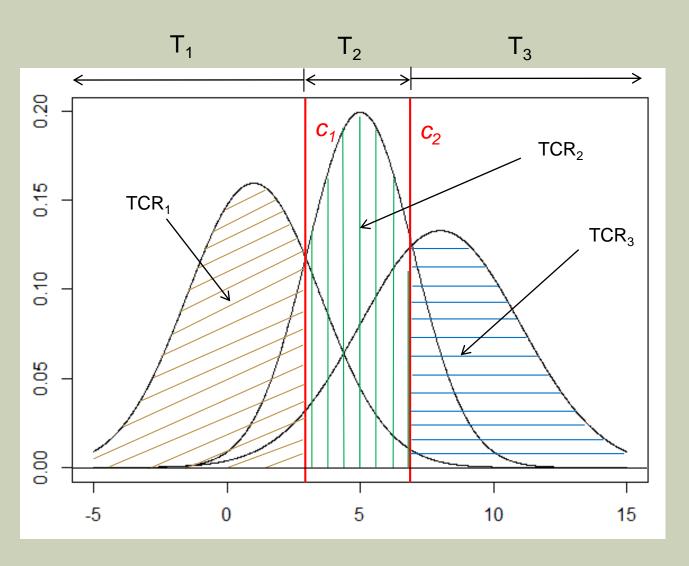
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MULTI-CLASS ROC

- Oftentimes, category labels are made binary for ease of interpretation and/or analysis
 - Ex) [cancer, no cancer], [normal, abnormal], [positive, negative]
- Traditional ROC methods adhere to this 'rule'
 - If there is more than one class we are interested in, we have to 'force dichotomization' by combining two [or more] classes
 - Ex) Types of breast cancer (LA, LB, HER2, Basal) would all be classified as cancer if compared to normal
- Multi-class ROC methods are natural extensions of traditional methods
 - Instead of area [under the curve], we are interested in volume under the surface (VUS, 3-classes) or hypervolume under the manifold (HUM, more than three classes)
 - Baseline value for comparison:

$$\frac{1}{k!}$$
 where k = number of classes

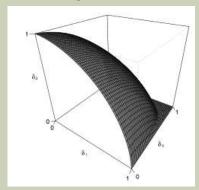


Instead of TPR and FPR, we are concerned with the **three** true classification rates (TCR) for any two cutoff points (c_{1}, c_{2}) , where $c_{1} < c_{2}$

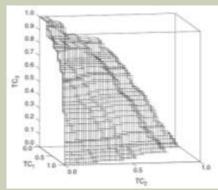
Class 1 Class 2 Class 3

VOLUME UNDER THE SURFACE

- Each possible pair of cutoff points yields a triplet of true classification rates
 - Triplet is coordinates for point in 3-D space
 - Points are plotted and connected to form an ROC surface

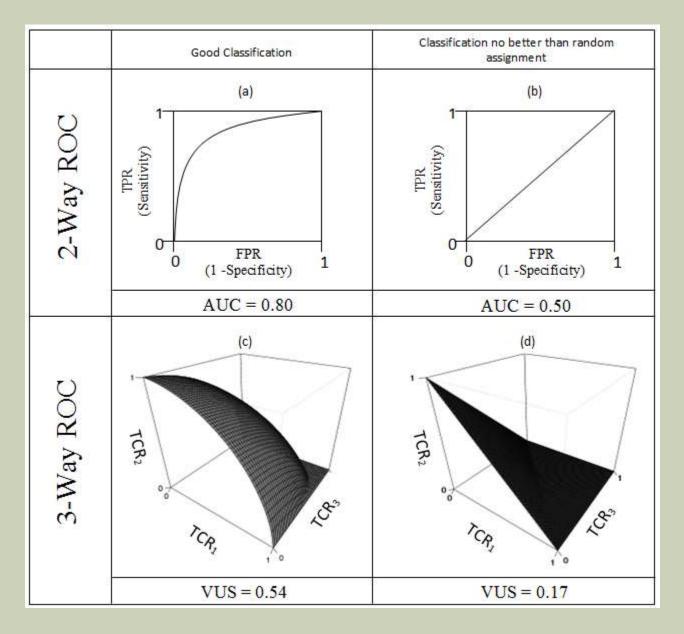


Parametric (Kang & Tian 2013)



Non-Parametric (Nakas & Yiannoutsos 2004)

- Volume under the ROC surface (VUS) is analogous to AUC
- VUS of $\frac{1}{6}$ (\approx 0.167) indicative of random class assignment



Note: (c) and (d) adapted from (Kang & Tian 2013)

EXAMPLE: MICROSATELLITE INSTABILITY IN COLON CANCER

- Microsatellite instability (MSI) has been associated with colon cancer. Results from a panel of genetic markers are used to determine MSI status on three levels:
 - 1. MSS: microsatellite stable, no markers detected
 - 2. MSI-L: MSI-Low; at least 1 but < 30% across panel
 - 3. MSI-H: MSI-High; > 30% mutation rate across panel
 - I read a paper (Molecular Cancer 2007, 6:54) that says they found a gene that is upregulated in MSI-H vs. MSS-type tumors (they dichotomized their data)
 - I want to see if these findings also apply to tumor samples and extend to a three-level classification system.

Gene: SECTM1

Data: Colon adenocarcinomas from TCGA

Pubmed source= TCGA COAD exp HiSeqV2

EXAMPLE: MICROSATELLITE INSTABILITY IN COLON CANCER

[Edited] Raw Data Summary:

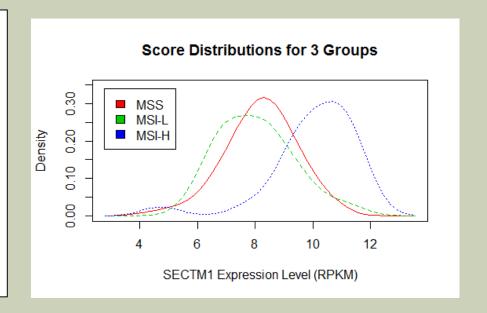
		<u>n</u>	mu	sd
Level 1:	MSS	98	8.21	1.17
Level 2:	MSI-L	25	8.03	1.29
Level 3:	MSI-H	25	10.05	1.49

VUS=0.3419, 95% CI=[0.2258, 0.457]

Best cut-points: lower(c_1)=8.0439, upper(c_2)=9.3773

The group correct classification probabilities are

MSS MSH-L MSH-H 0.4184 0.6429 0.8000



VUS = Probability that if we choose at random one MSS, MSI-L and MSI-H sample: $SECTM1_{MSI-L} < SECTM1_{MSI-H}$



EXAMPLE: MICROSATELLITE INSTABILITY IN COLON CANCER

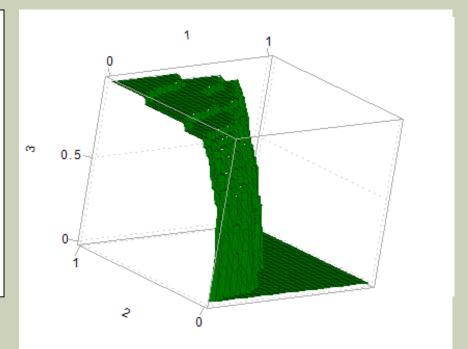


			illu	<u> </u>
Level 1:	MSI-L	3 5	8.03	1.29
Level 2:	MSS	98	8.21	1.17
Level 3:	MSI-H	25	10.05	1.49

VUS(new)=0.4692, 95% CI=[0.3289, 0.6017] VUS(old)=0.3419

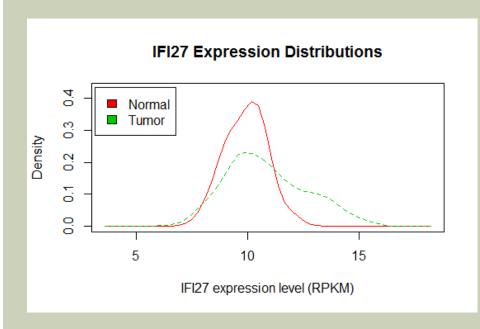
Best cut-points: lower(c_1)=7.4214, upper(c_2)=9.3773

The group correct classification probabilities are MSS MSH-L MSH-H 0.4184 0.4400 0.8000



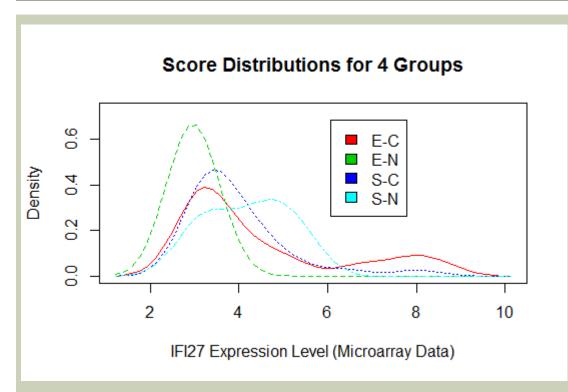
VUS = Probability that if we choose at random one MSS, MSI-L and MSI-H sample: $SECTM1_{MSI-L} < SECTM1_{MSI-H}$

SOMETHING TO CONSIDER: REMEMBER IFI27?



- The original AUC was not very impressive, but levels are moderately higher in tumors
 - Brief literature review yields info that IFI27 upregulation has been implicated in epithelial cancer cells.
 - Can we gain further insight if we stratify by cell type?
 - We can't do this with the TCGA data, but Genomics Portal has a different data set I can use
 - Tumor and normal tissue samples classified as either epithelial or squamous
 - PubMed source= GSE10797,

SOMETHING TO CONSIDER: REMEMBER IFI27?



- Heterogeneity of BC adds 'noise' to the ROC analysis
 - Keeping in mind small sample size, a few sources in this case include:
 - Higher levels of IFI27 measured in normal stromal cells
 - An 'island' of cancerous epithelial cells with marked IFI27 upregulation

J Surg Res. 2015 Jan;193(1):255-64. doi: 10.1016/j.jss.2014.06.055. Epub 2014 Jul 5.

Interferon alpha-inducible protein 27 promotes epithelial-mesenchymal transition and induces ovarian tumorigenicity and stemness.

Li S1, Xie Y2, Zhang W3, Gao J4, Wang M1, Zheng G1, Yin X1, Xia H5, Tao X8.

SUMMARY OF ROC ANALYSIS

- Flexible tool for evaluating classification models
 - Provides statistical and graphical measures of overall accuracy
 - Yields optimal cutoff values for prediction
- Some considerations for use in functional genomics:
 - Heterogeneity of biological data
 - We are using relatively simple methods but dealing with appreciably complex phenomena
 - Unequal sample sizes for groups
 - Generally more cancer samples than normal samples
 - More genes are non-DE vs DE (when considering ranking systems)
- This is only the tip of the iceberg!
 - Bayesian methods, semi-parametric methods, misclassification costs, training/validation issues, multivariable models...

QUESTIONS?

