



# Zero-knowledge on the blockchain

Privacy-preserving cryptocurrencies

Privacy-preserving smart contracts

Proof of regulatory compliance

Blockchain-based sovereign identity

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see [ZKProof Standardization – Applications Track]

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Need zero-knowledge non-interactive arguments of knowledge, ideally succinct ones: zk-SNARK.

QAP-based

[GGPR13][PGHR13][BCGTV13][DFGK14] [Groth16][GM17][BG18]...

Fastest verification. Widely used.

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Standardization

Reference String (SRS)

QAP-based

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### Generating an SRS:

Trusted setup

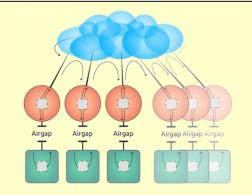
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Require a Structured Reference String (SRS)

### Generating an SRS:

- Trusted setup
- MPC + destruction
- Updatable SRS





[BCGTV15][BGG17]

[GKMMM18, MBKM19]

Scalable SRS generation ("Powers of Tau")

[BGM18]

### **Avoiding a Structured Reference String**

Other zk-SNARKs

PCP-based (e.g., libSTARK)

[Micali94][BCGT13][BCS16][BBCGGHPRSTV17][BBHR18]

Asymptotically succinct but large constants.

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Non-succinct ZK:

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#### Non-succinct ZK:

• Aurora [BCRSVW19]

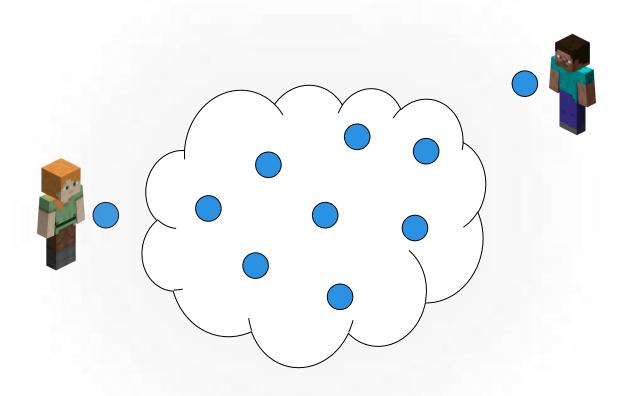
• Bulletproofs [BCCGP16][BBBPWM17]

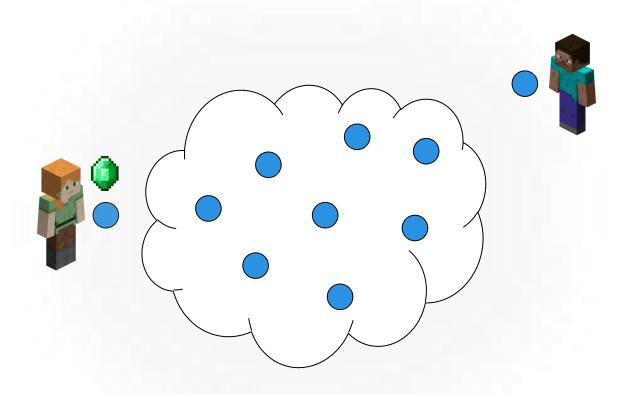
• Hyrax [WTSTW17]

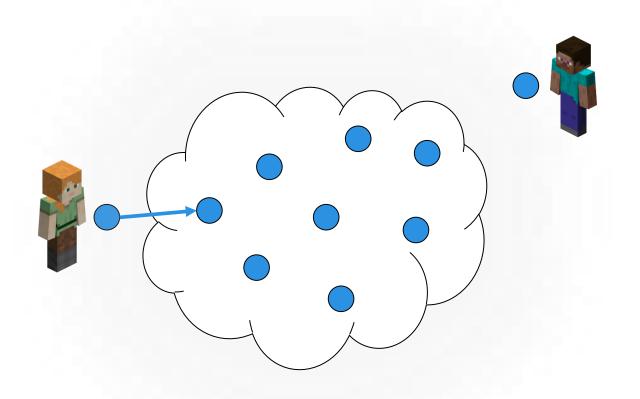
• Ligero [AHIV17]

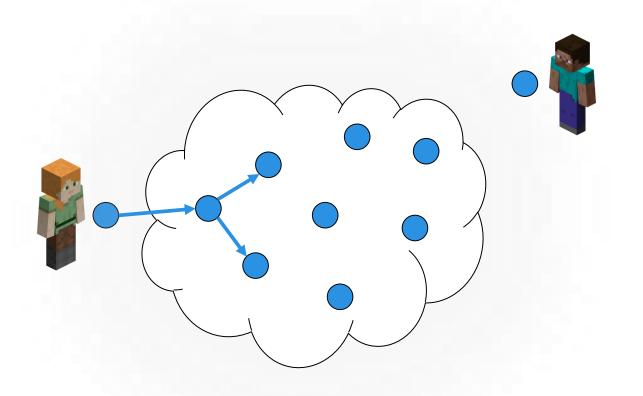
• ZKBoo(++) [GMO16][CDGORRSZ17]

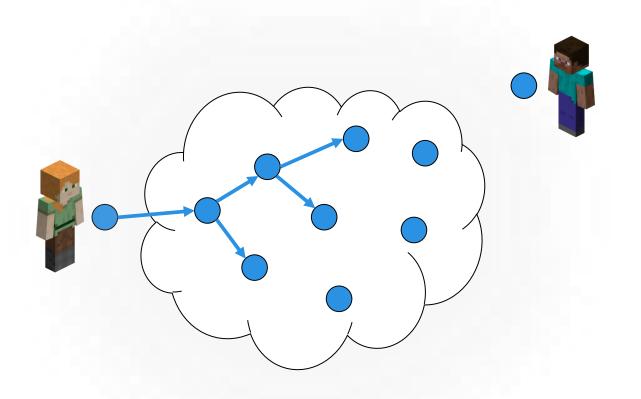
Slow verification and/or large proofs (as statement grows).

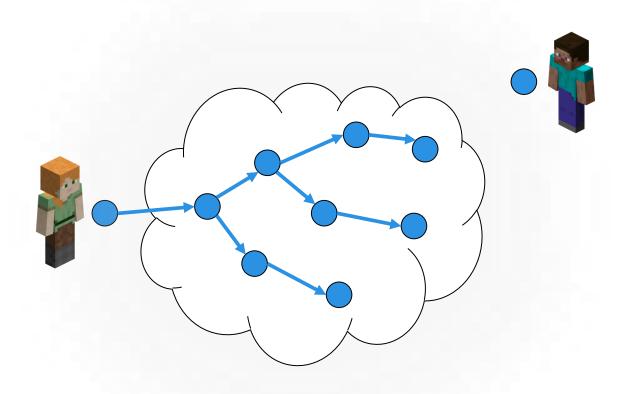


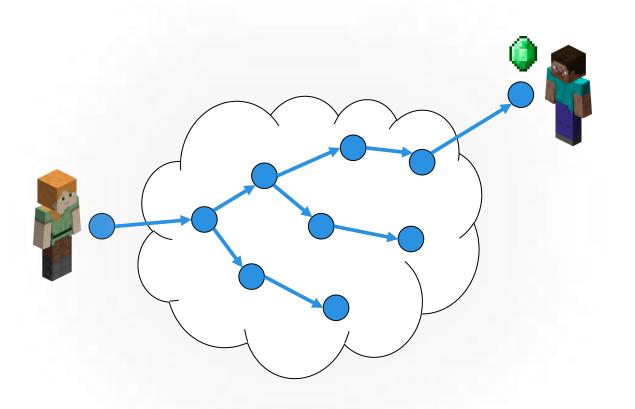


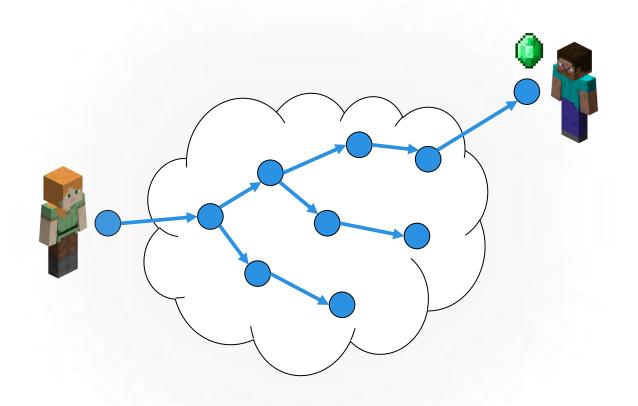




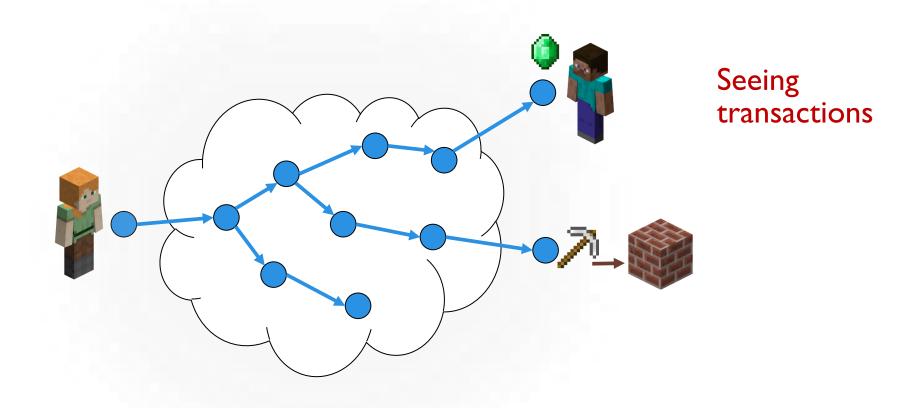


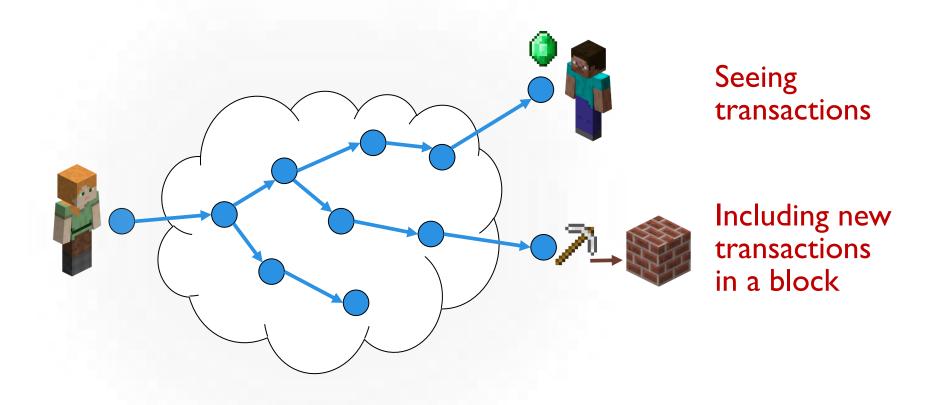


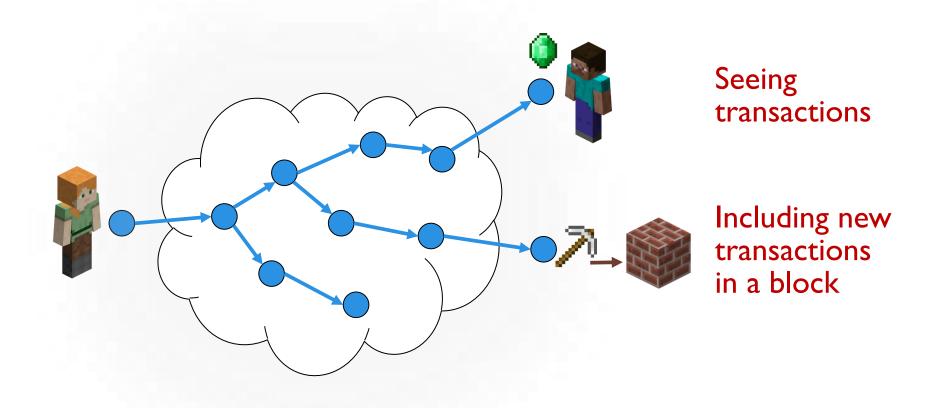




Seeing transactions



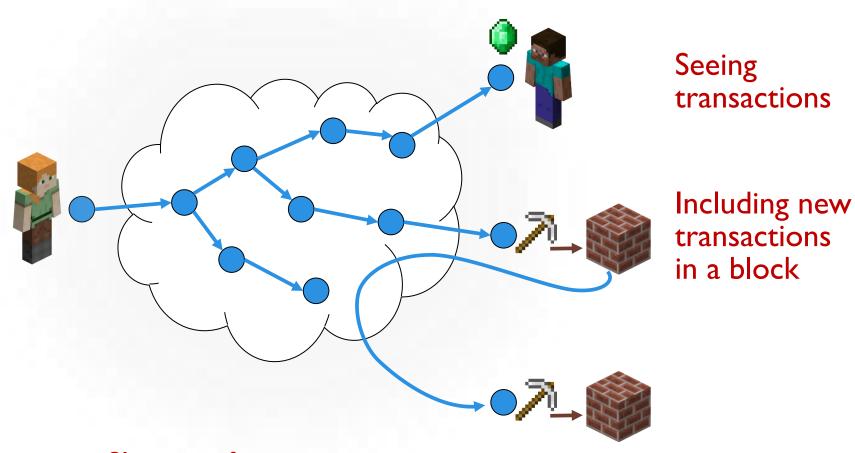






#### Slow verification

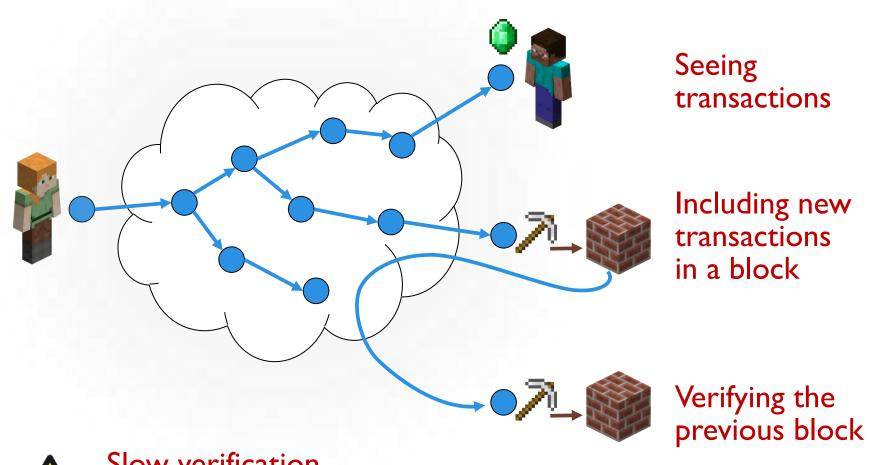
- → miners get a head start by not validating
- → double-spends and chain splits [July 2015 Bitcoin fork]





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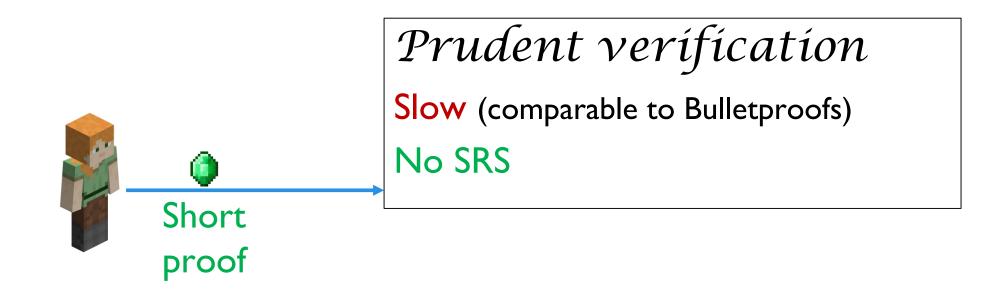
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zero-knowledge succinct <u>hybrid</u> argument of knowledge

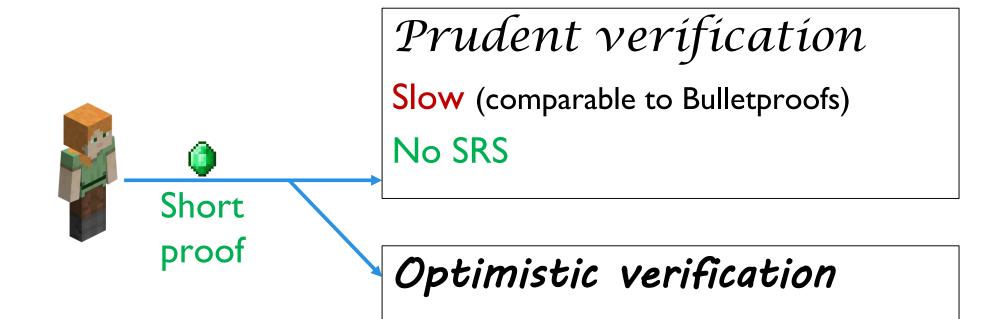
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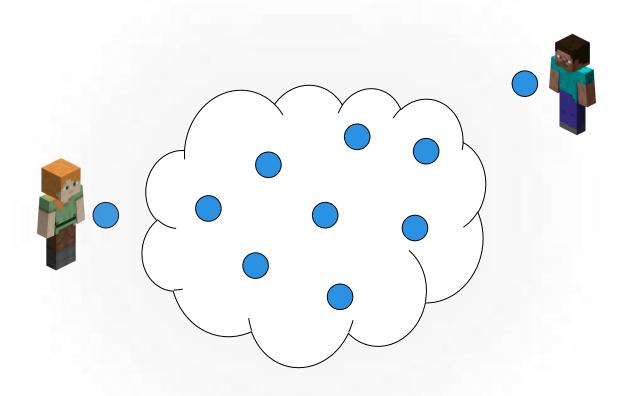


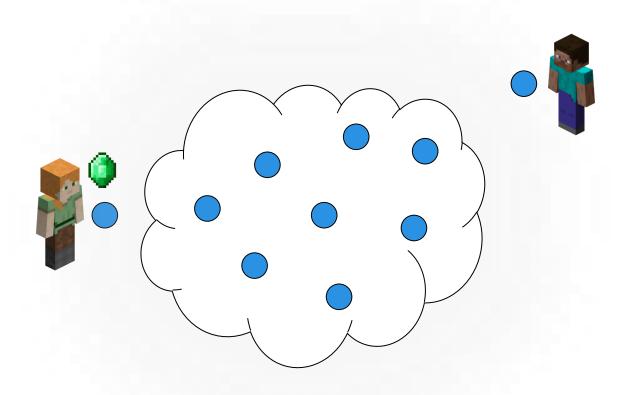
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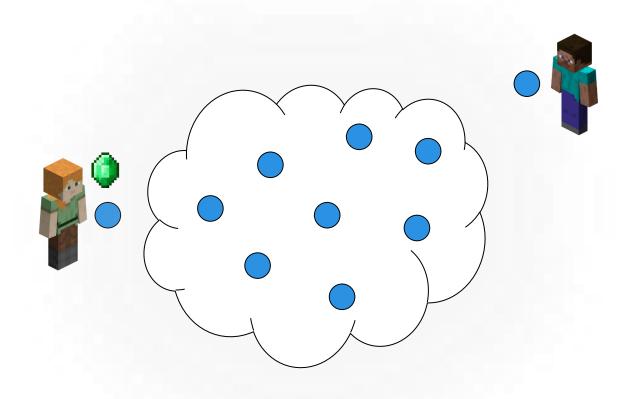


Relies on SRS

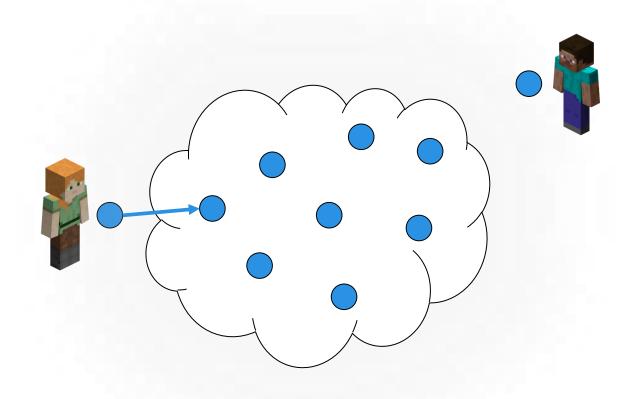
Fast (comparable to QAP-based SNARK)



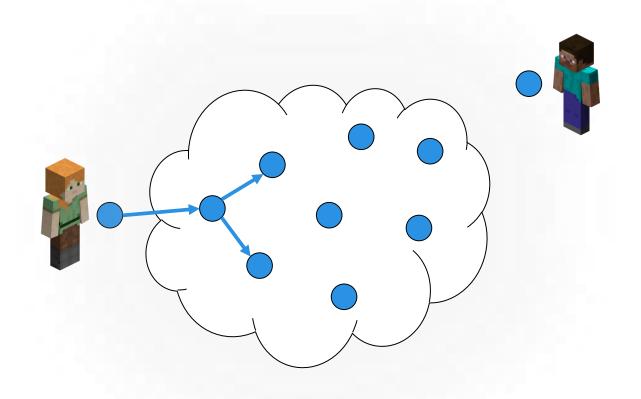


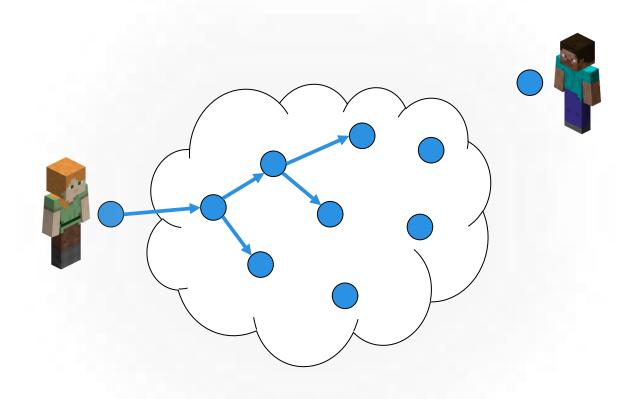


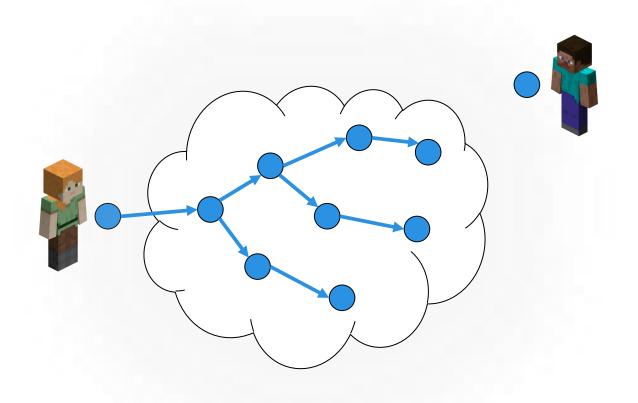
Optimistically verify during propagation.

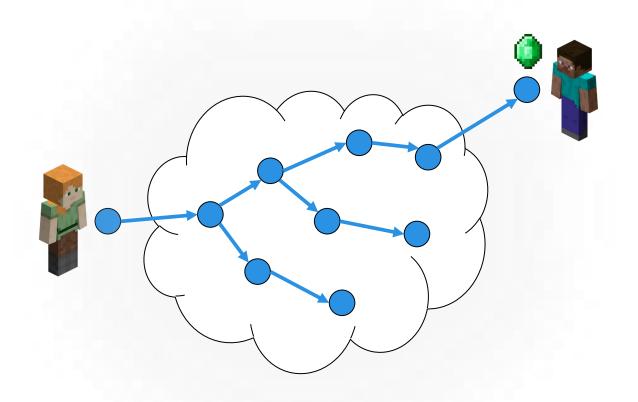


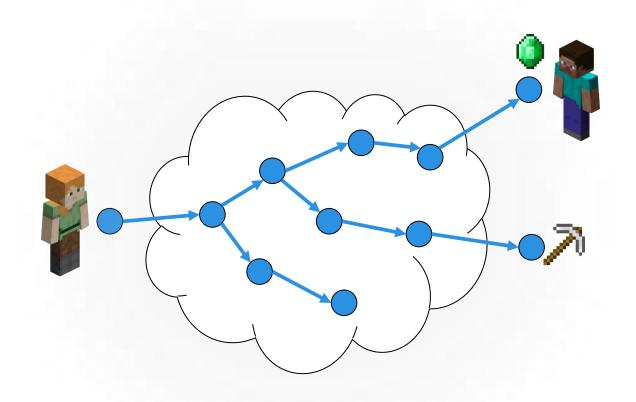
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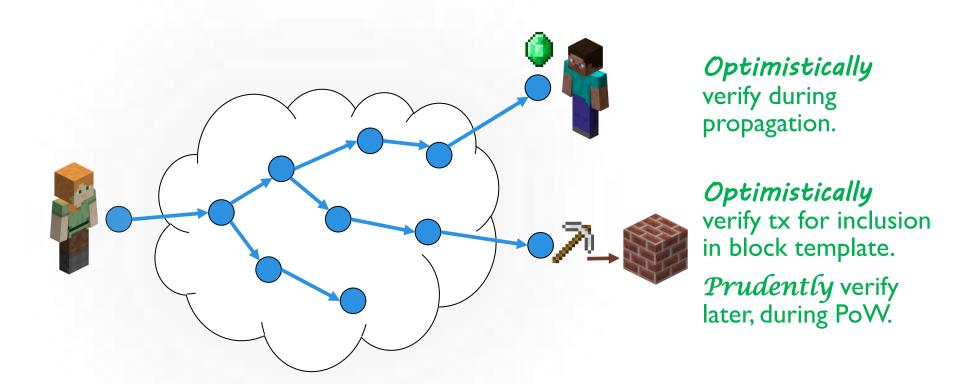


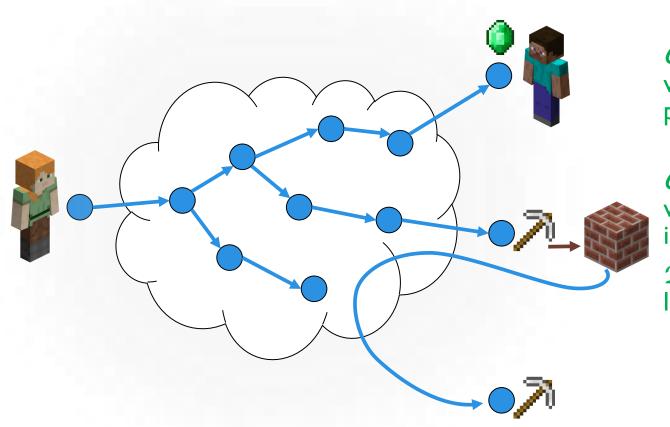








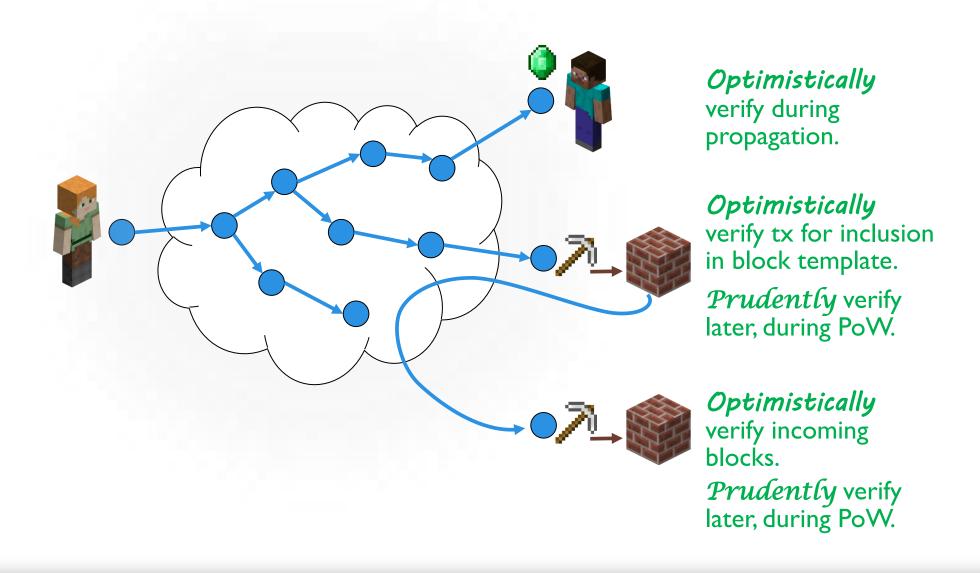


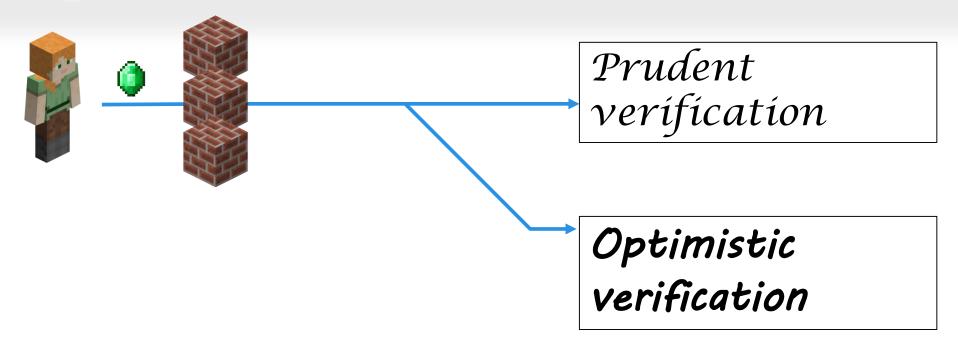


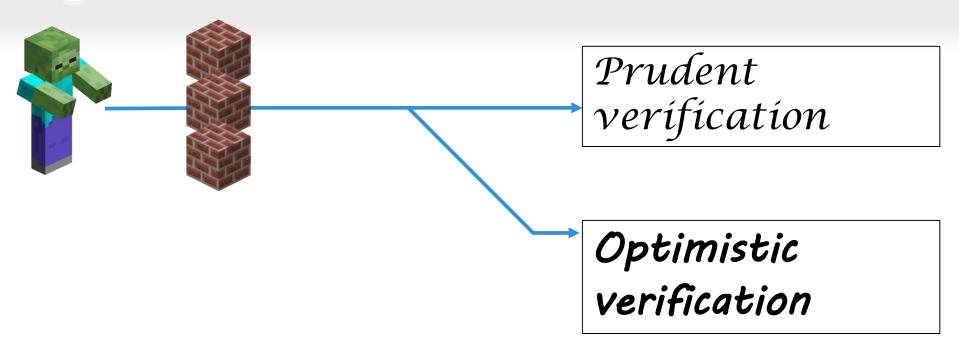
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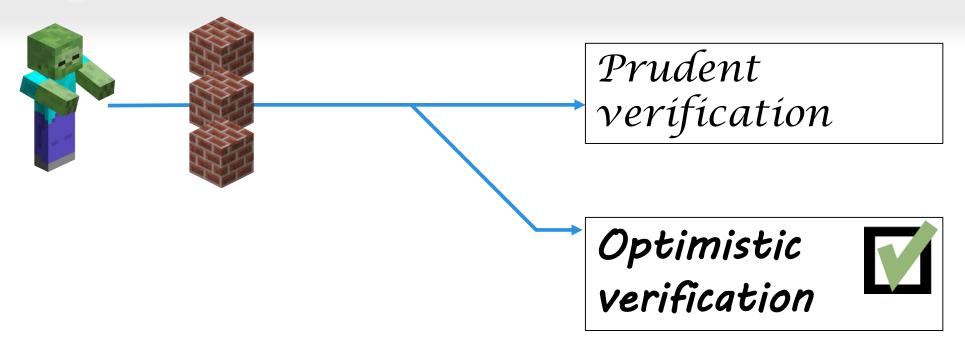
Optimistically verify tx for inclusion in block template.

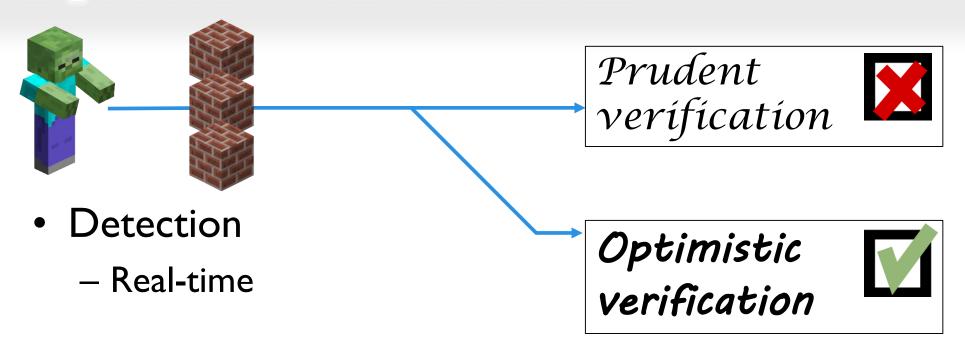
**Prudently** verify later, during PoW.

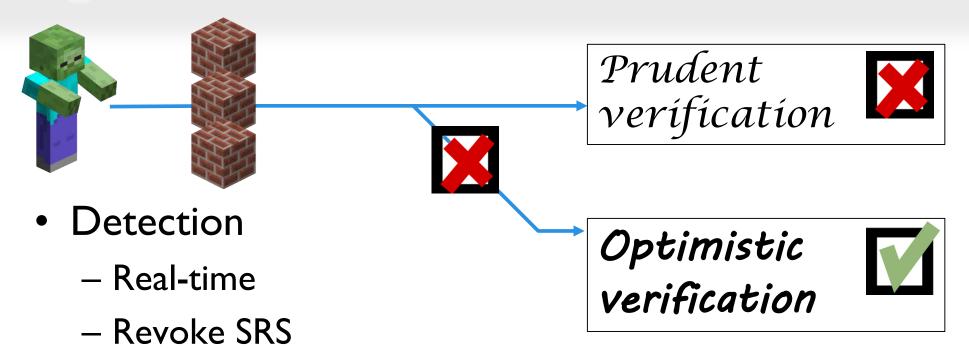




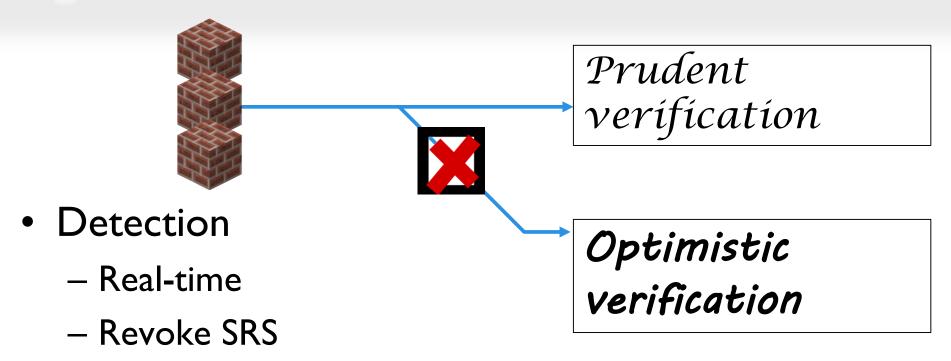




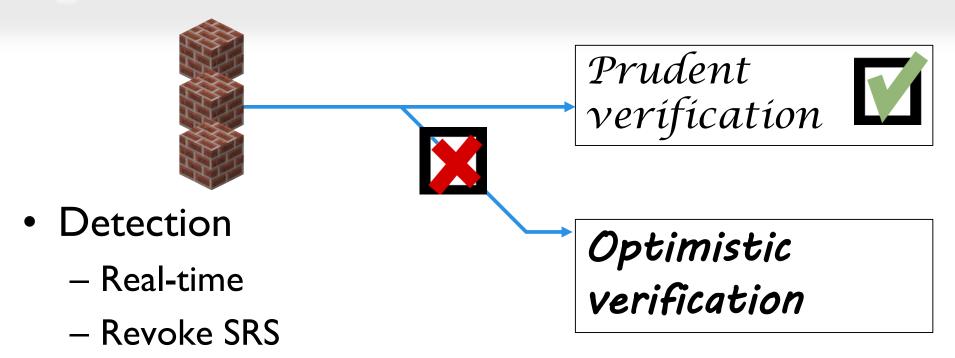




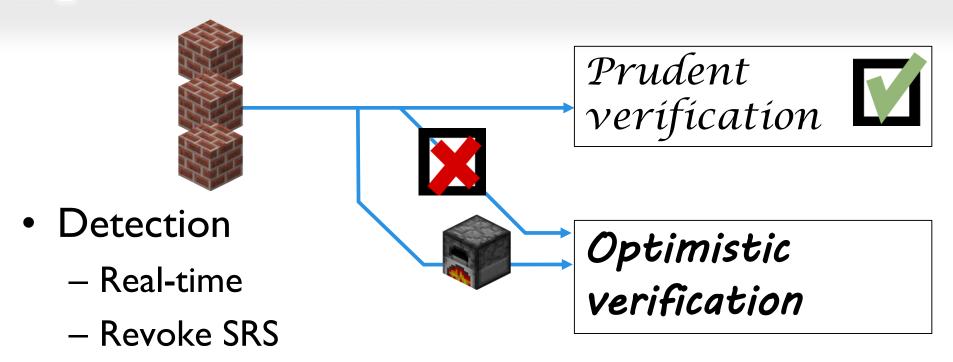
– Such pair of proofs is a fraud proof for compromised SRS!



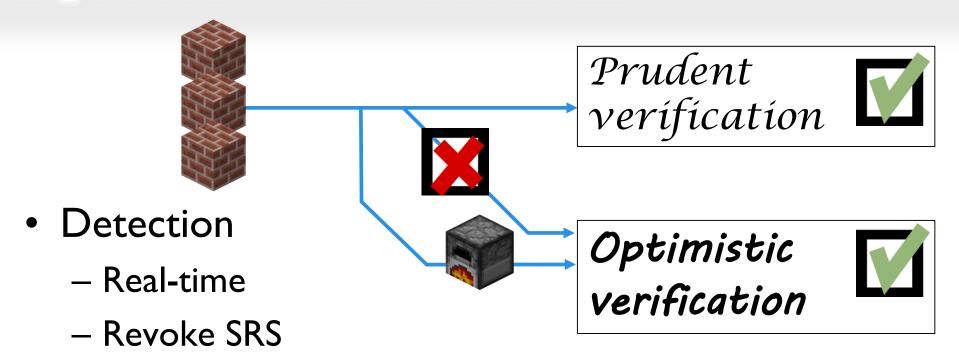
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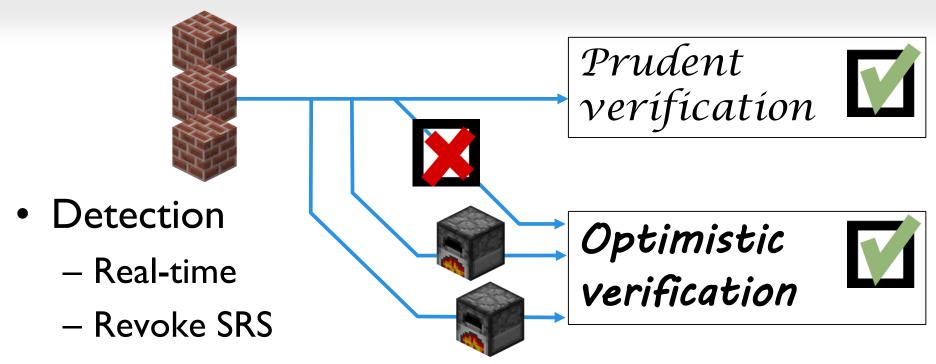
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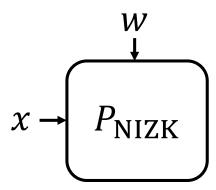
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SHARK requirement: anyone can refresh, without original sender.

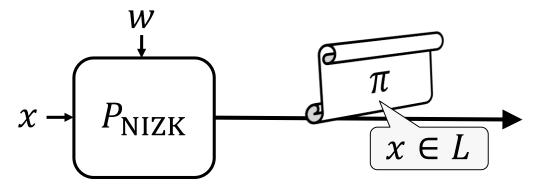
Attempt – parallel composition:

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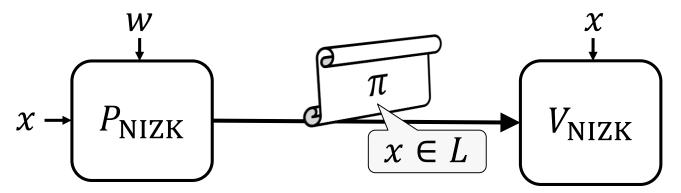
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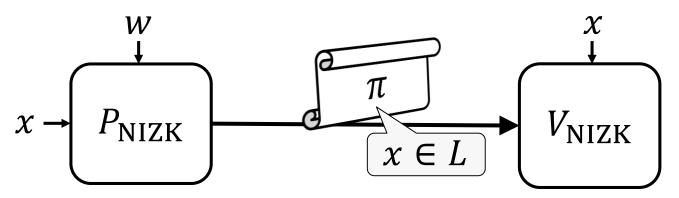


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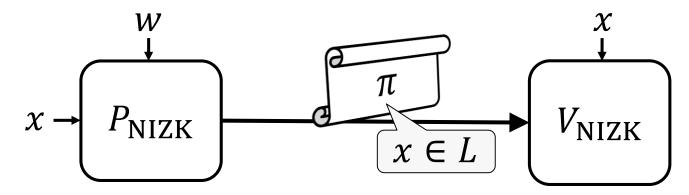
1) Fix a language L and construct a NIZK for it:



Call  $\pi$  "prudent proof"

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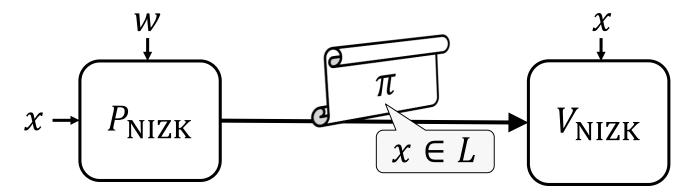


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(2) Construct a SNARK for the same L:

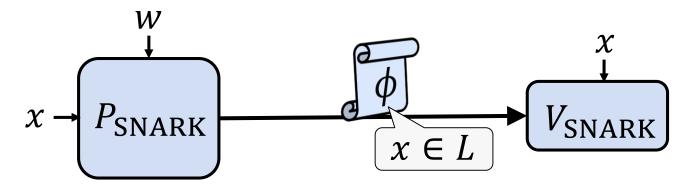
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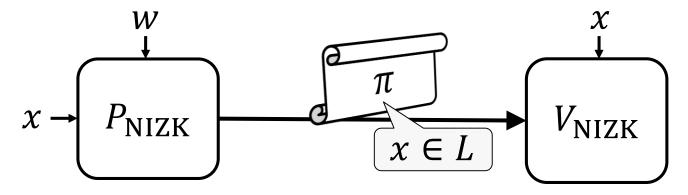
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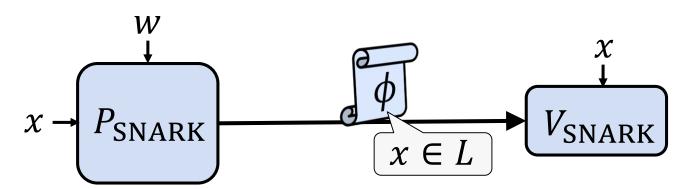
Not pictured:  $G_{NIZK}$ ,  $G_{SNARK}$ 

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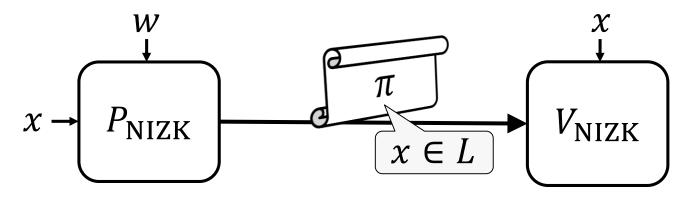
Optimistic proofs should be refreshable without *w* 

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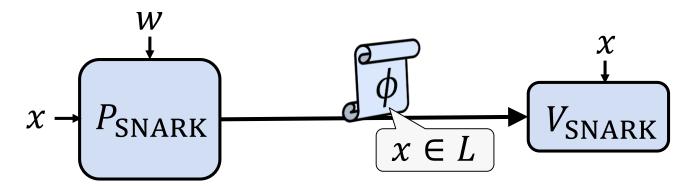
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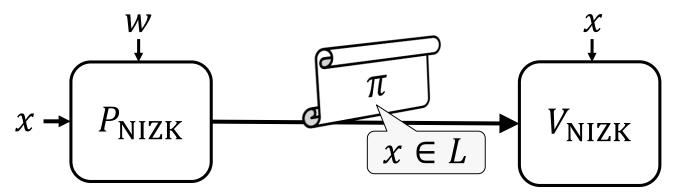
 $\Rightarrow \phi$  is **not** a SHARK optimistic proof

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Construction:

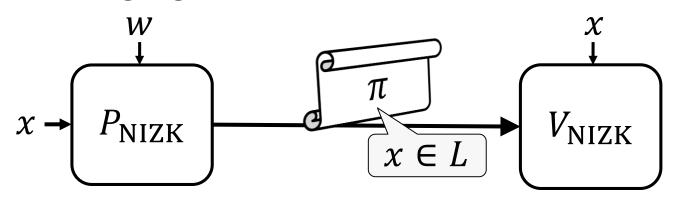
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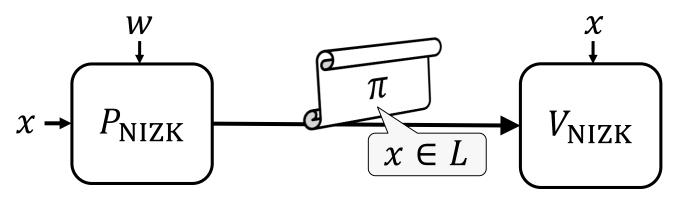
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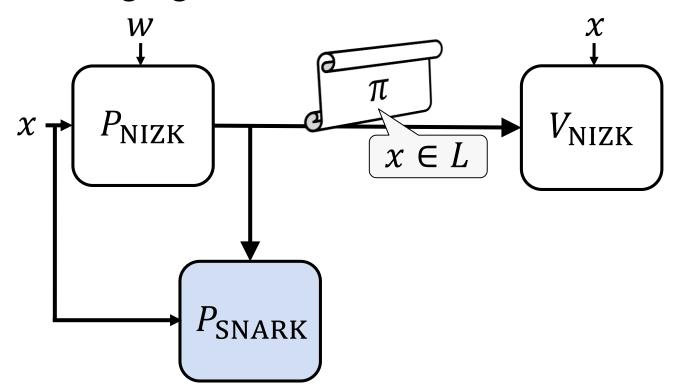


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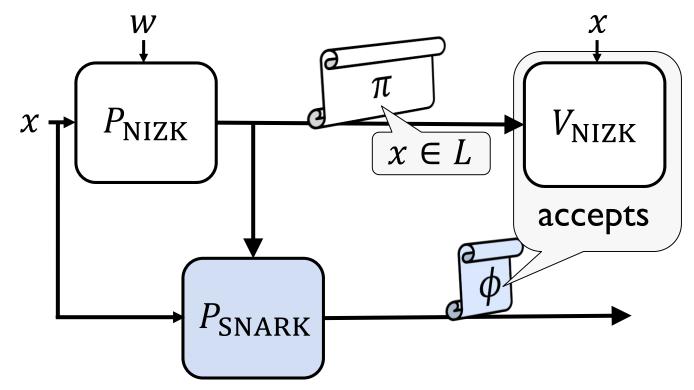


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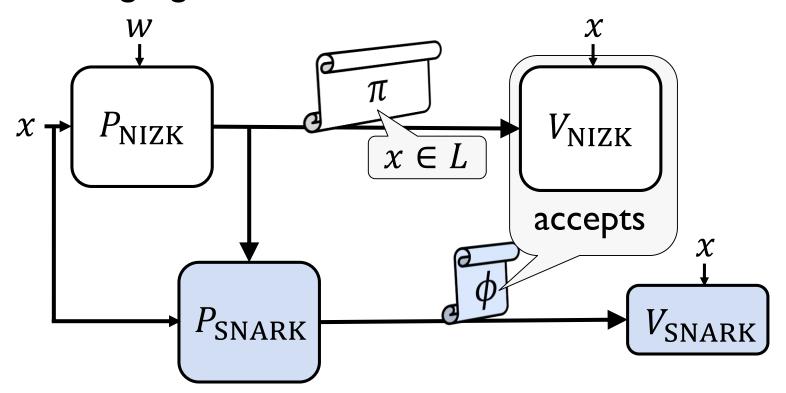


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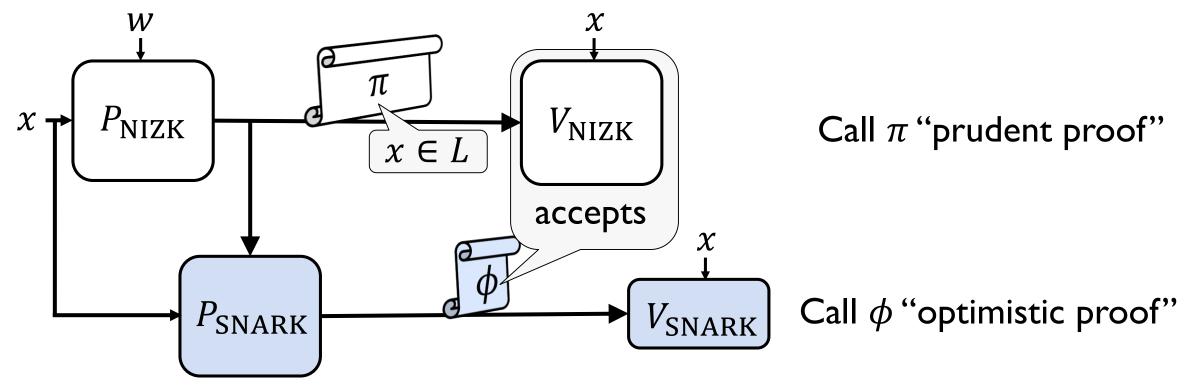


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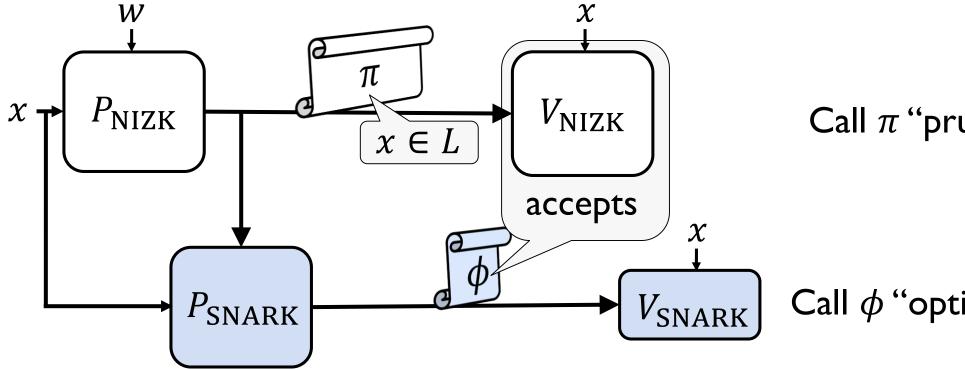
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Call  $\phi$  "optimistic proof"

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Generically combining state-of-the-art components:

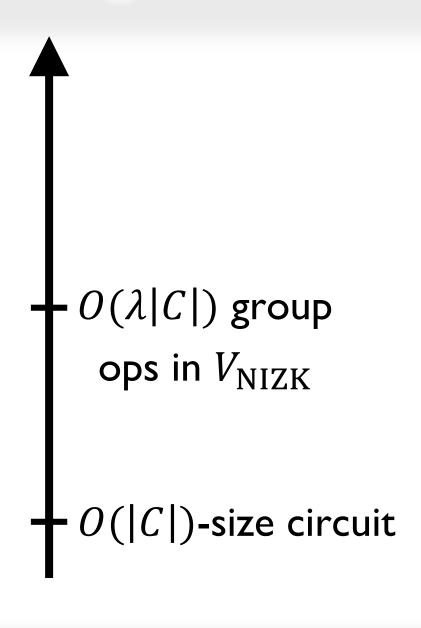
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Bulletproofs NIZK + Groth I 6 SNARK

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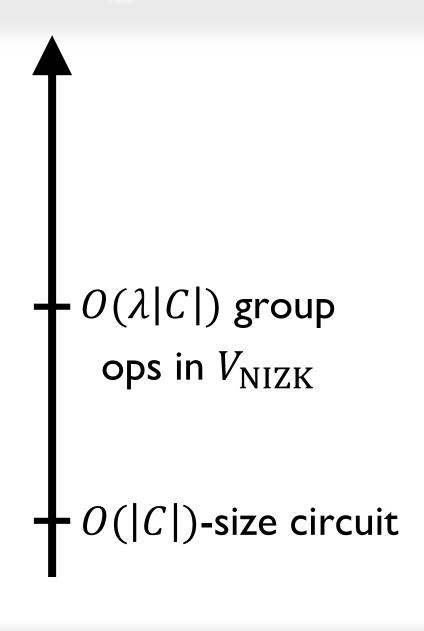
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+O(|C|)-size circuit



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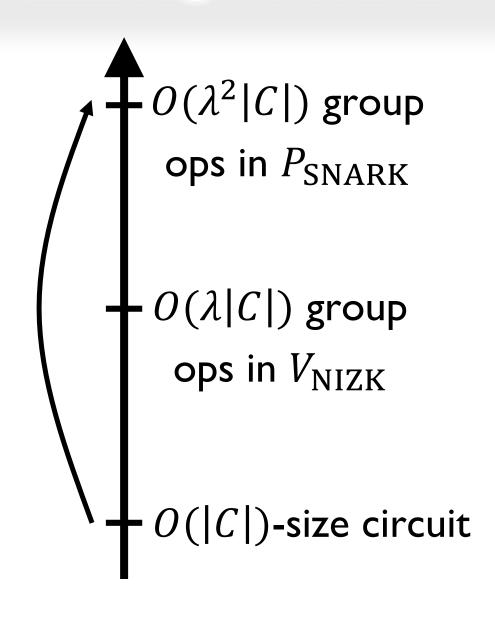
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 $+ O(\lambda |C|)$  group ops in  $V_{\text{NIZK}}$ 

+O(|C|)-size circuit

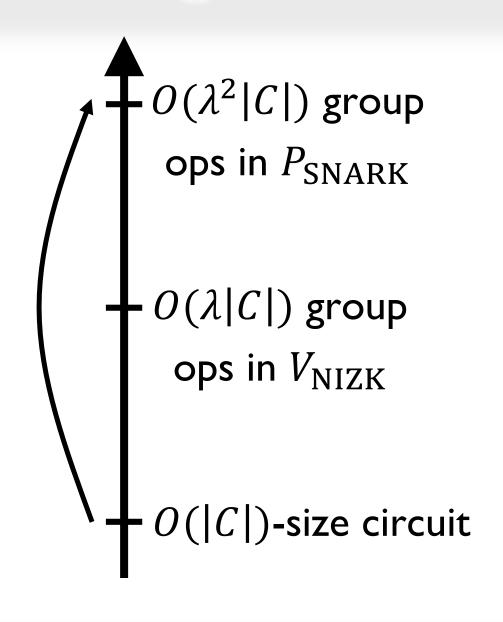
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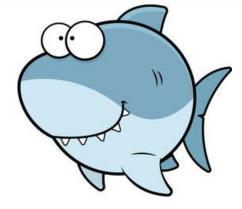
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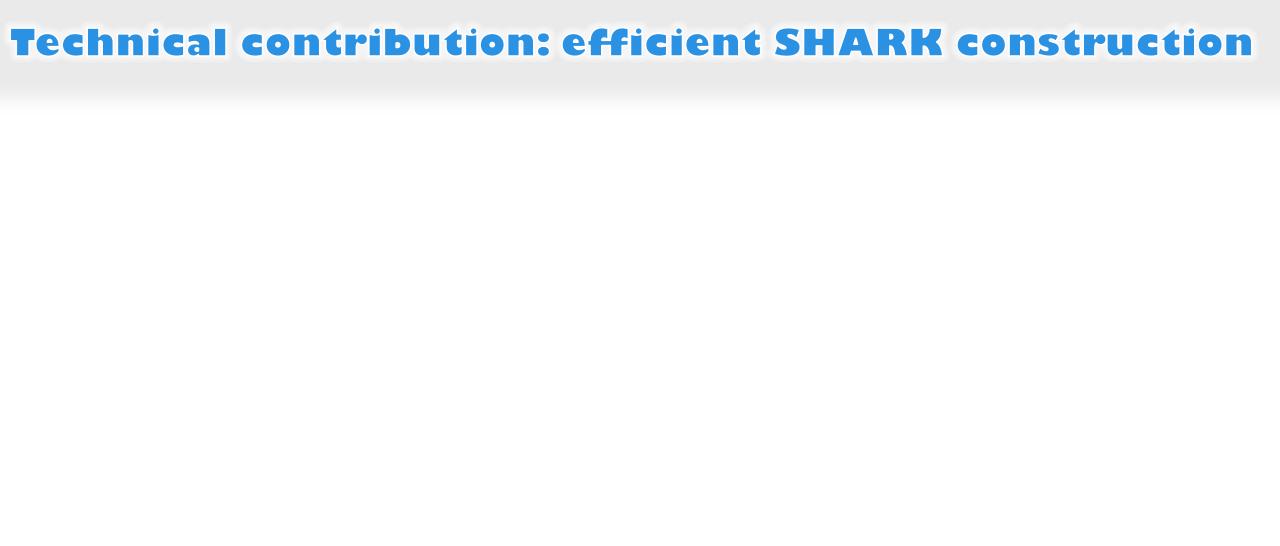
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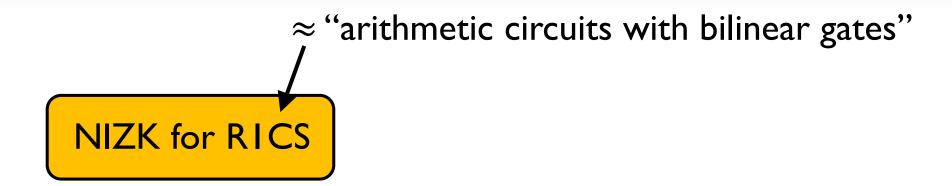


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NIZK for RICS



"arithmetic circuits with bilinear gates"
 a new compilation technique for linear PCPs
 NIZK for RICS

 $\approx$  "arithmetic circuits with bilinear gates"

NIZK for RICS

- a new compilation technique for linear PCPs
- public coin NIZK from LPCPs ⇒ prudent mode

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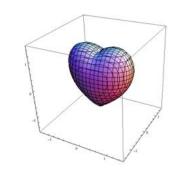
 an optimized variant of Bulletproofs' inner product argument

≈ "arithmetic circuits with bilinear gates"

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 $\Rightarrow V_{\text{NIZK}}$  has an "algebraic heart"

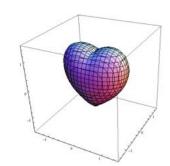


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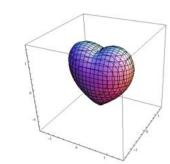


A special-purpose SNARK

≈ "arithmetic circuits with bilinear gates"

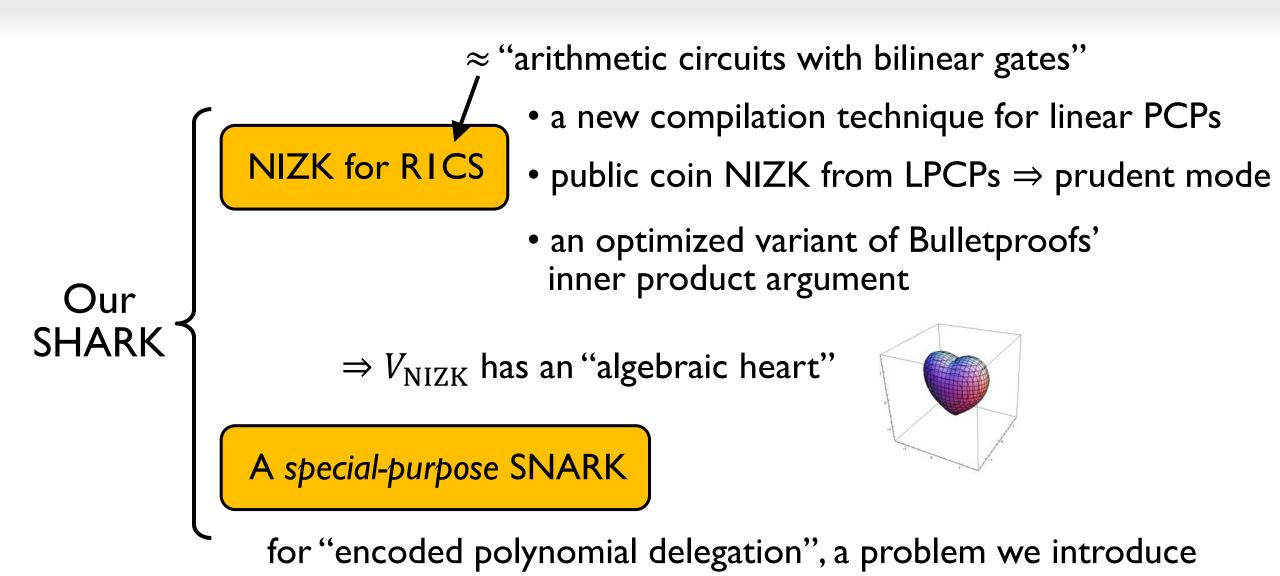
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A special-purpose SNARK

for "encoded polynomial delegation", a problem we introduce



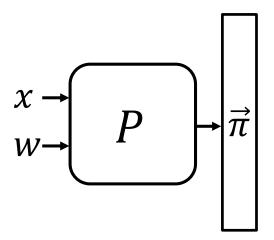
[GGPR12] [BCIOP13]

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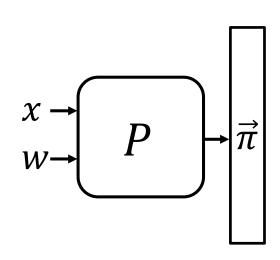
1 Design a proof system sound against linear provers

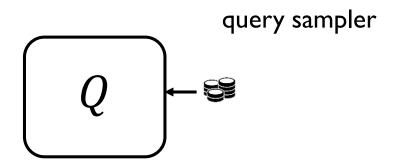
- 1) Design a proof system sound against linear provers
- 2 Force prover to be linear using a cryptographic encoding

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- 2 Force prover to be linear using a cryptographic encoding

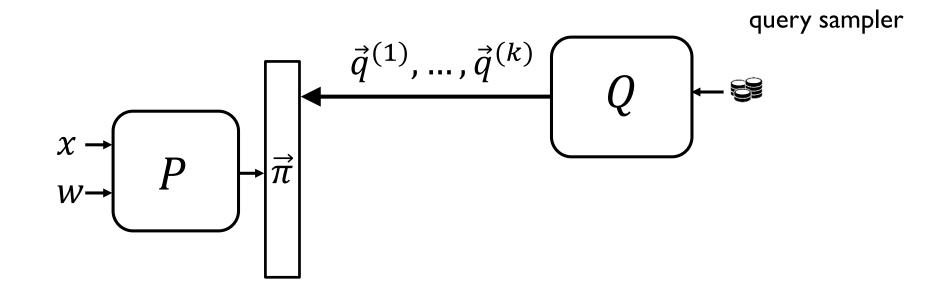


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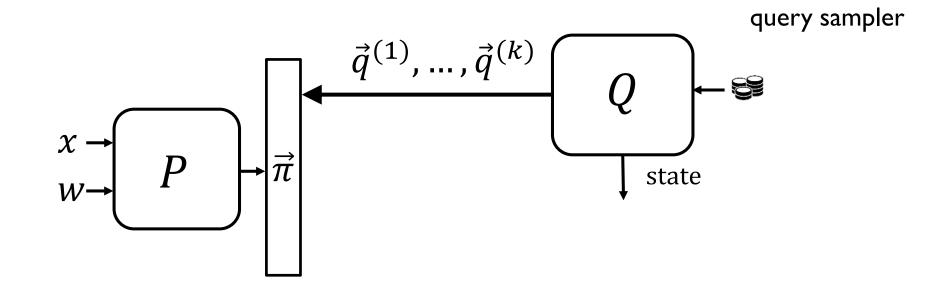




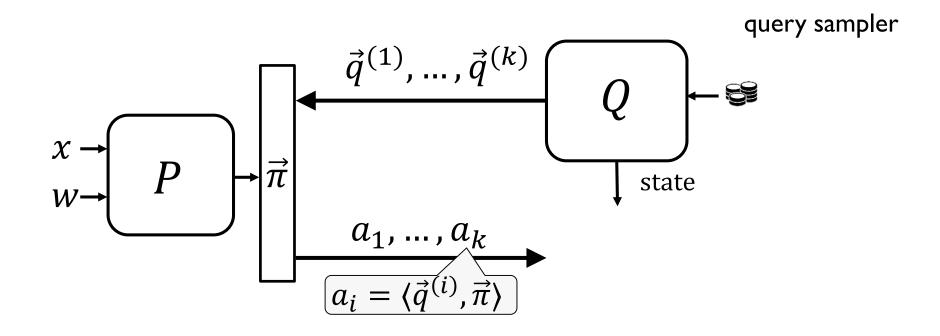
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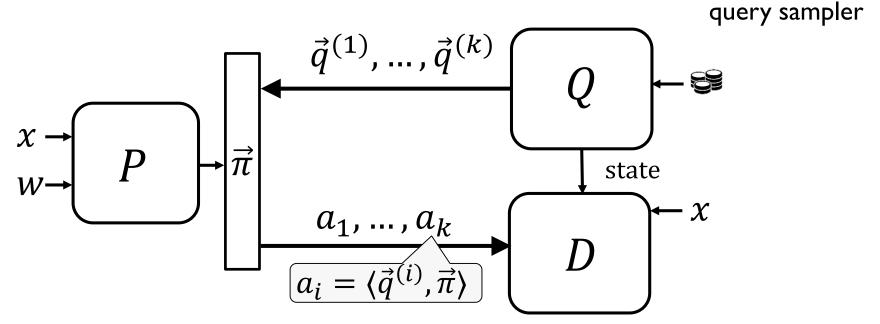
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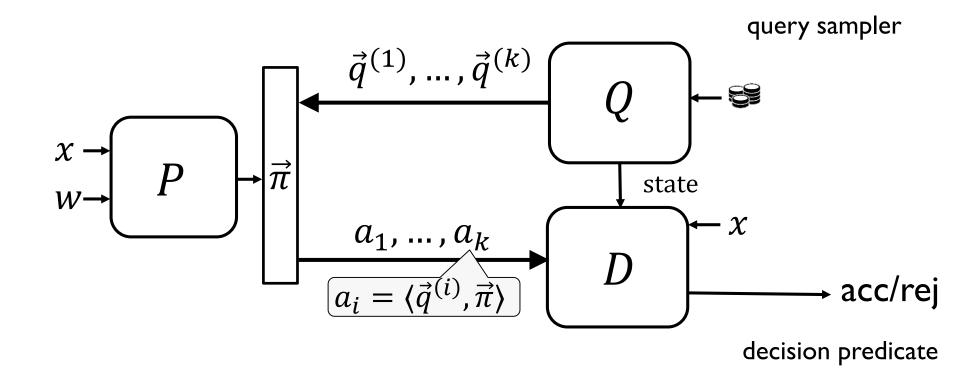


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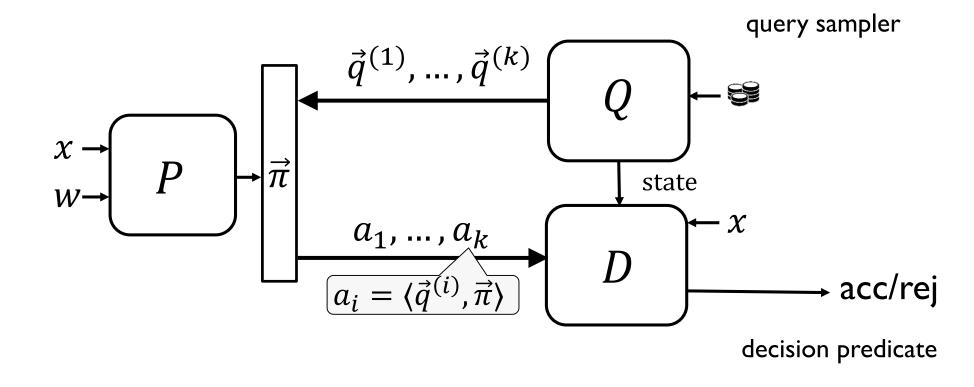


decision predicate

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Can define natural notions of completeness, PoK, ZK.



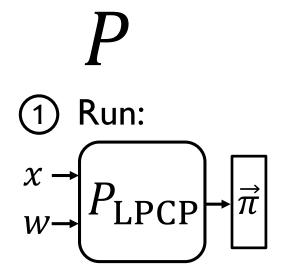
# Our compilation technique

P

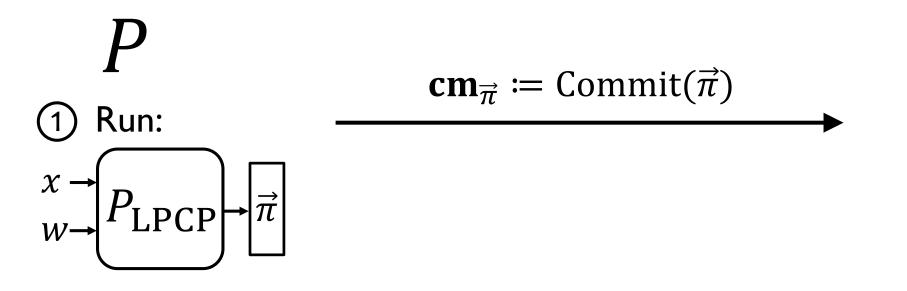
V

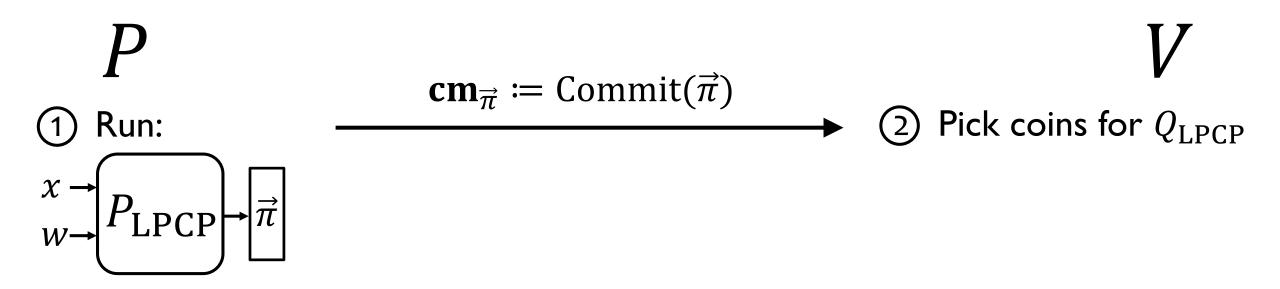
Observe that  $P_{LPCP}$  does not need to know queries a priori.

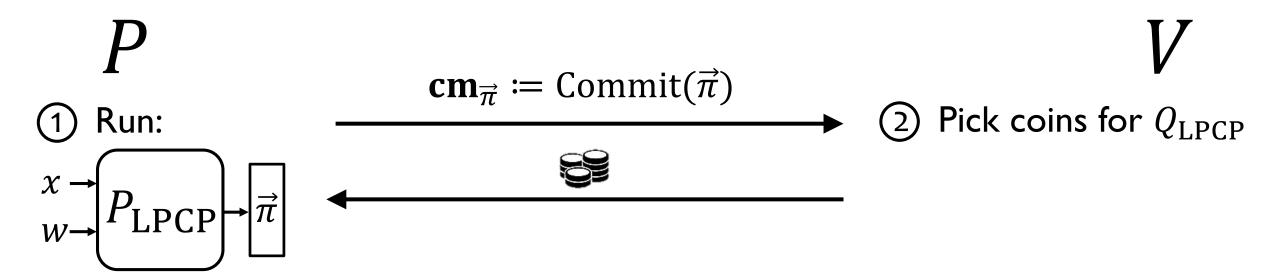
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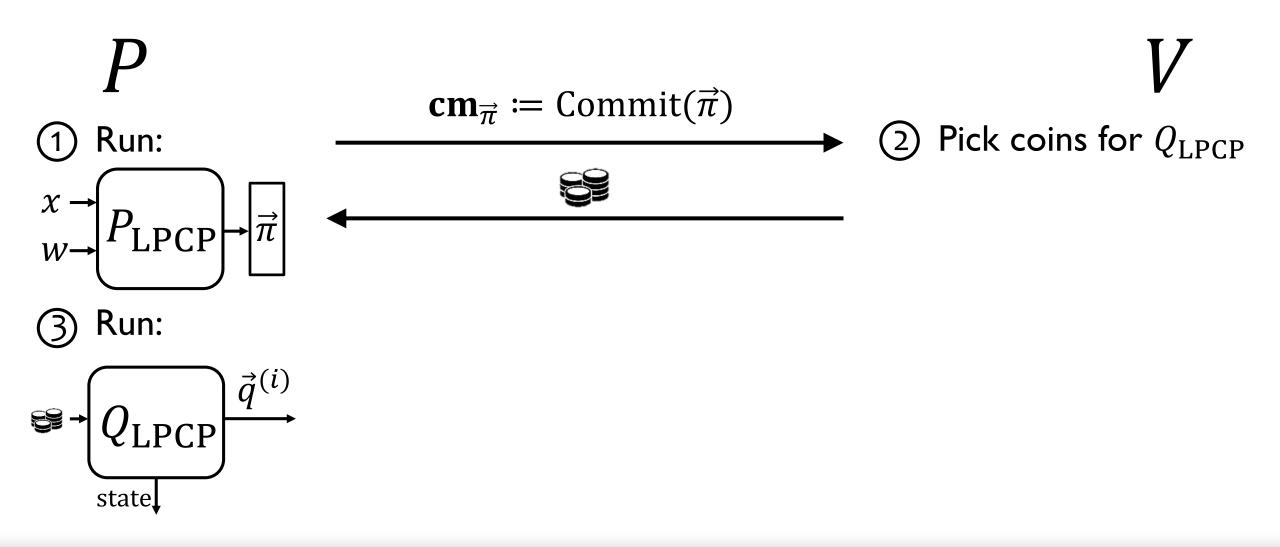




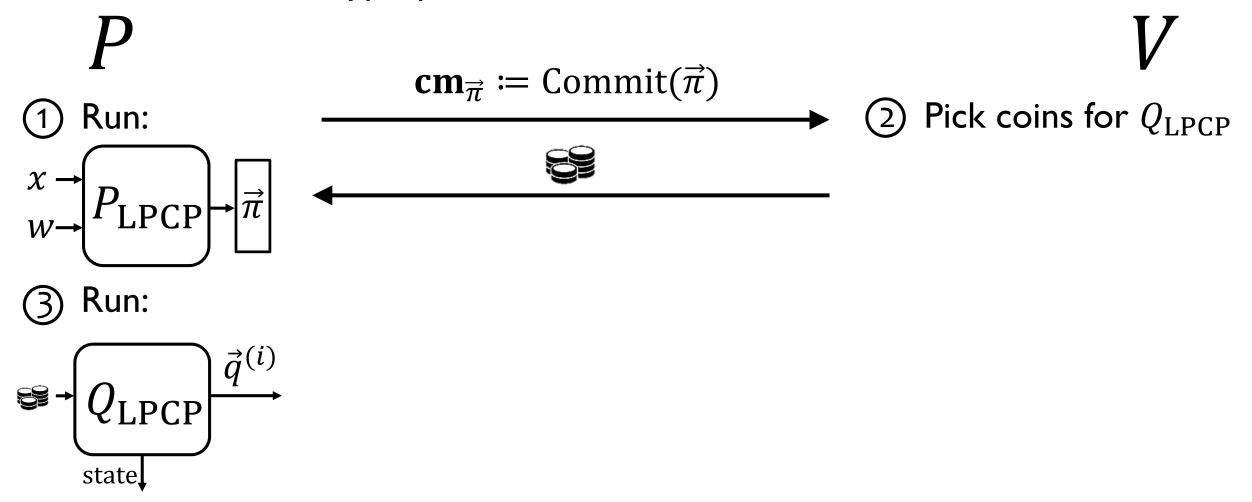




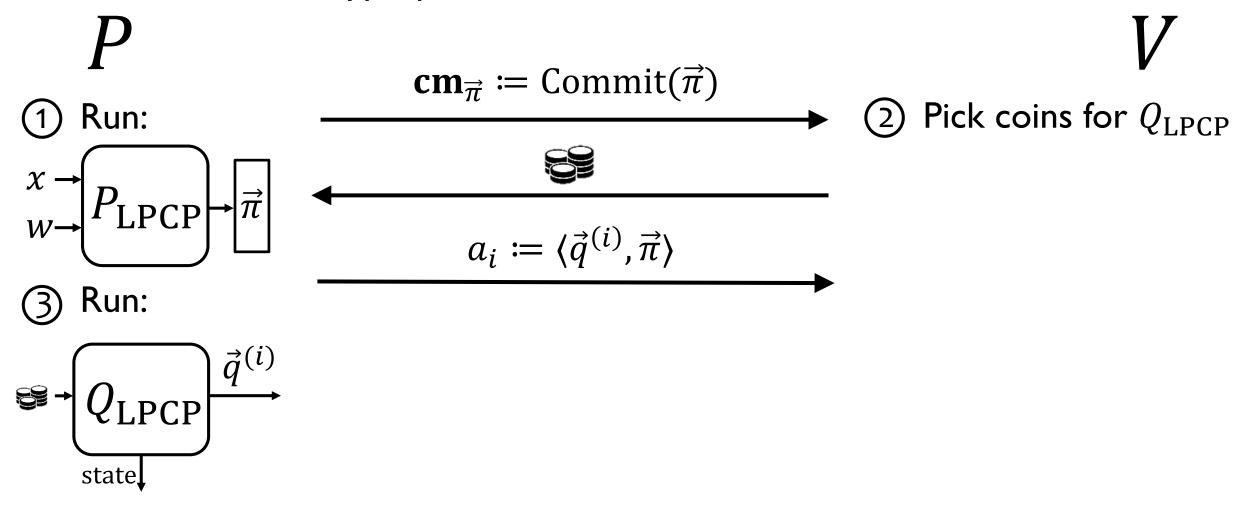




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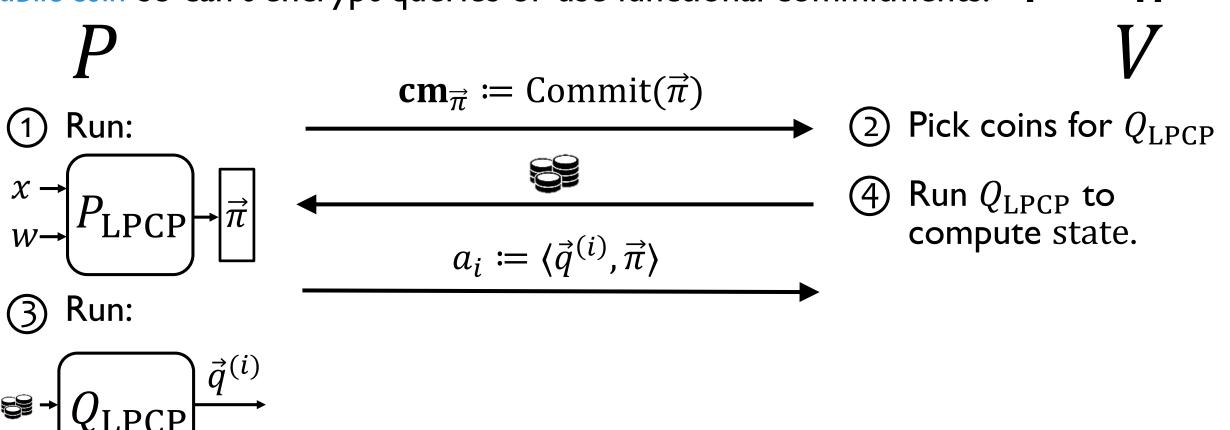


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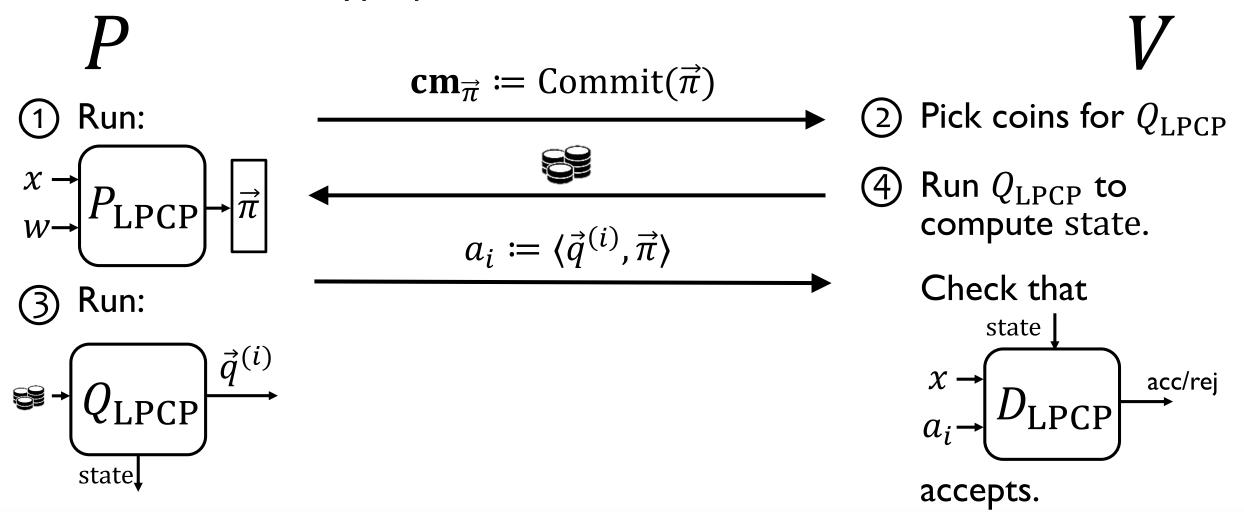


state,

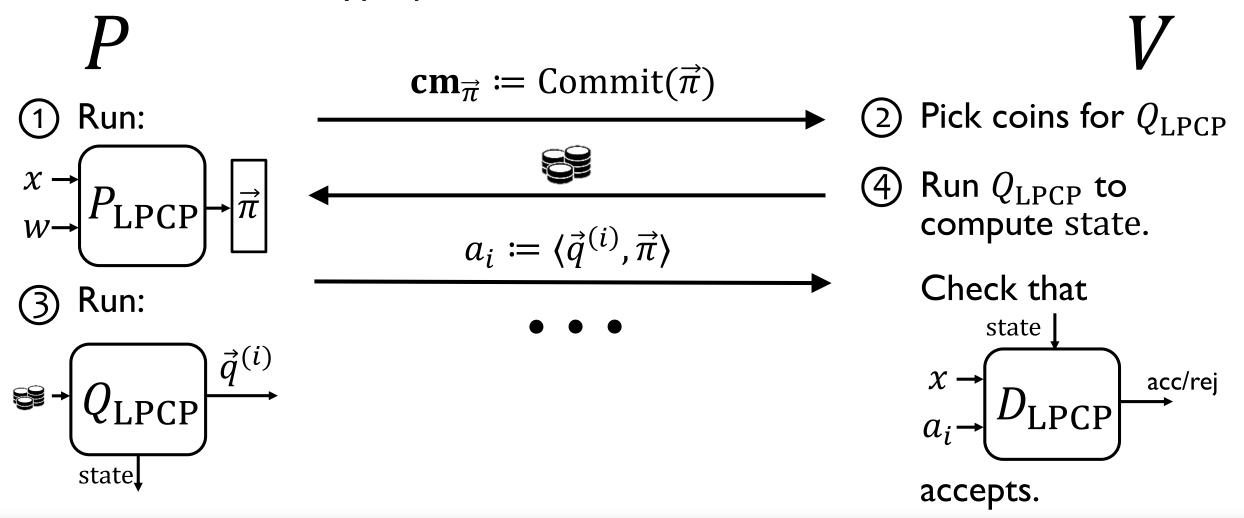
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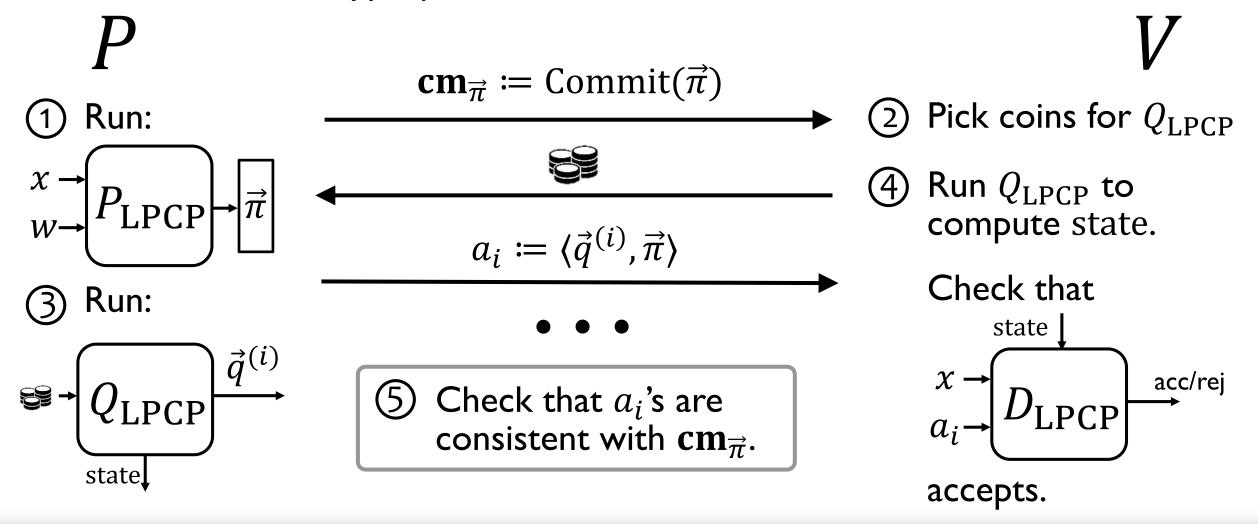
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Verifier knows:  $\Re$ , a's, and commitment to proof  $\mathbf{cm}_{\vec{\pi}} \coloneqq \mathrm{Commit}(\vec{\pi})$ 

Goal: for every query  $\vec{q}$  check  $a = \langle \vec{q}, \vec{\pi} \rangle$  for a pre-committed  $\vec{\pi}$ 

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[BCCGP16,BBBPWM18]

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Result: NIZK from linear PCPs!

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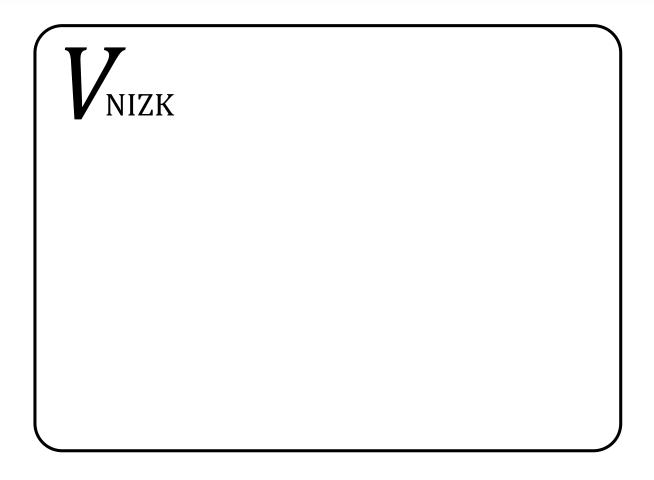
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SHARK prudent proofs





# $V_{\scriptscriptstyle{ m NIZK}}$

 check that LPCP decision predicate accepts (cheap)

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A new building block: "encoded polynomial delegation"

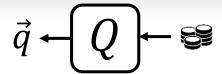
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Most efficient linear PCP: quadratic arithmetic programs of [GGPR I 2]

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$$\vec{q} \leftarrow Q \leftarrow \Theta$$

Most efficient linear PCP: quadratic arithmetic programs of [GGPR I 2]

Each query  $\vec{q}$  has nice algebraic structure:

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Most efficient linear PCP: quadratic arithmetic programs of [GGPR I2]

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Most efficient linear PCP: quadratic arithmetic programs of [GGPR I 2]

$$\tau = 3$$

$$\vec{q} = (p_1(\tau), p_2(\tau), ..., p_n(\tau))$$

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$$\tau^d \cdot (p_{1,d}, p_{2,d}, \dots, p_{n,d})$$

$$p_i(\tau) = p_{i,0} + p_{i,1}\tau + \dots + p_{i,d}\tau^d$$

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$$\begin{split} \vec{q} &= (p_{1}(\tau), p_{2}(\tau), \dots, p_{n}(\tau)) \\ &= ( & p_{1,0}, & p_{2,0}, \dots, & p_{n,0}) \\ & \tau \cdot ( & p_{1,1}, & p_{2,1}, \dots, & p_{n,1}) \\ & \cdots \\ & \tau^{d} \cdot ( & p_{1,d}, & p_{2,d}, \dots, & p_{n,d}) \end{split}$$

$$cm_{\vec{q}} &= \operatorname{Commit}(p_{1}(\tau), p_{2}(\tau), \dots, p_{n}(\tau))$$

$$\tau^{d} \cdot ( & p_{1,d}, & p_{2,d}, \dots, & p_{n,d})$$

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Most efficient linear PCP: quadratic arithmetic programs of [GGPR I 2]

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$$\vec{q} = (p_{1}(\tau), p_{2}(\tau), \dots, p_{n}(\tau))$$

$$= ( p_{1,0}, p_{2,0}, \dots, p_{n,0}) +$$

$$\tau \cdot ( p_{1,1}, p_{2,1}, \dots, p_{n,1}) +$$

$$\cdots$$

$$\tau^{d} \cdot ( p_{1,d}, p_{2,d}, \dots, p_{n,d})$$

$$cm_{\vec{q}} = Commit(p_{1}(\tau), p_{2}(\tau), \dots, p_{n}(\tau))$$

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$$\tau^{d} \cdot ( p_{1,d}, p_{2,d}, \dots, p_{n,d})$$

Goal: compute 
$$\mathbf{cm}_{\vec{q}} \coloneqq \operatorname{Commit}(\vec{q})$$

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Most efficient linear PCP: quadratic arithmetic programs of [GGPR I 2]

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$$\vec{q} = (p_{1}(\tau), p_{2}(\tau), \dots, p_{n}(\tau))$$

$$= (p_{1,0}, p_{2,0}, \dots, p_{n,0}) + \tau \cdot (p_{1,1}, p_{2,1}, \dots, p_{n,1}) + \tau \cdot (p_{1,1}, p_{2,1}, \dots, p_{n,1}) + \tau \cdot (p_{1,d}, p_{2,d}, \dots, p_{n,d})$$

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Goal: outsource computation of 
$$\mathbf{cm}_{\vec{q}} = \mathrm{Com}(p_1(\tau), p_2(\tau), \dots, p_n(\tau))$$

$$= \mathrm{Com}(p_{1,0}, p_{2,0}, \dots, p_{n,0}) +$$

$$\tau \cdot \mathrm{Com}(p_{1,1}, p_{2,1}, \dots, p_{n,1}) +$$

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$$\tau^d \cdot \mathrm{Com}(p_{1,d}, p_{2,d}, \dots, p_{n,d})$$

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Com()'s are fully determined by L!

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Fixed parameters  $U_0, U_1, \dots, U_d \in \mathbb{G}$ 

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$$U \coloneqq U_0 + \tau \cdot U_1 + \dots + \tau^d \cdot U_d \ .$$

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$$U \coloneqq U_0 + \tau \cdot U_1 + \dots + \tau^d \cdot U_d .$$

For outsourcing  $\mathbf{cm}_{\vec{q}}$  set  $U_k = \text{Com}(p_{1,k}, p_{2,k}, ..., p_{n,k})$ 

**New building block:** SNARK for encoded polynomial delegation in pairing groups

Goal: outsource computation of 
$$\mathbf{cm}_{\vec{q}} = \mathrm{Com}(p_1(\tau), p_2(\tau), \dots, p_n(\tau))$$

$$= \mathrm{Com}(p_{1,0}, p_{2,0}, \dots, p_{n,0}) +$$

$$\tau \cdot \mathrm{Com}(p_{1,1}, p_{2,1}, \dots, p_{n,1}) +$$

$$\dots$$

$$\tau^d \cdot \mathrm{Com}(p_{1,d}, p_{2,d}, \dots, p_{n,d})$$

Com( )'s are fully determined by L!

#### **Encoded polynomial delegation**

Fixed parameters  $U_0, U_1, \dots, U_d \in \mathbb{G}$ 

Goal: given input  $\tau \in \mathbb{F}$ , outsource this computation:

$$U \coloneqq U_0 + \tau \cdot U_1 + \dots + \tau^d \cdot U_d \ .$$

For outsourcing  $\mathbf{cm}_{\vec{q}}$  set  $U_k = \text{Com}(p_{1,k}, p_{2,k}, ..., p_{n,k})$ 

**New building block:** SNARK for encoded polynomial delegation in pairing groups + multilinear variant for our optimized IP argument.



SHARK optimistic proofs

Goal: outsource computation of

$$\mathbf{cm}_{\vec{q}} = \mathrm{Com}(p_1(\tau), p_2(\tau), \dots, p_n(\tau))$$

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...

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Com( )'s are fully determined by L!

### **Encoded polynomial delegation**

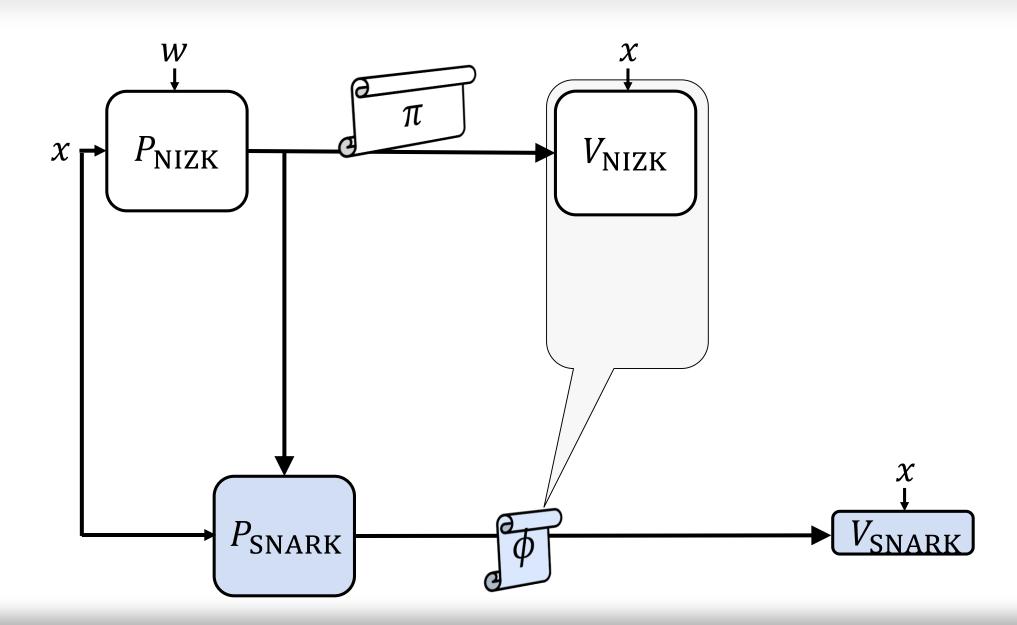
Fixed parameters  $U_0, U_1, \dots, U_d \in \mathbb{G}$ 

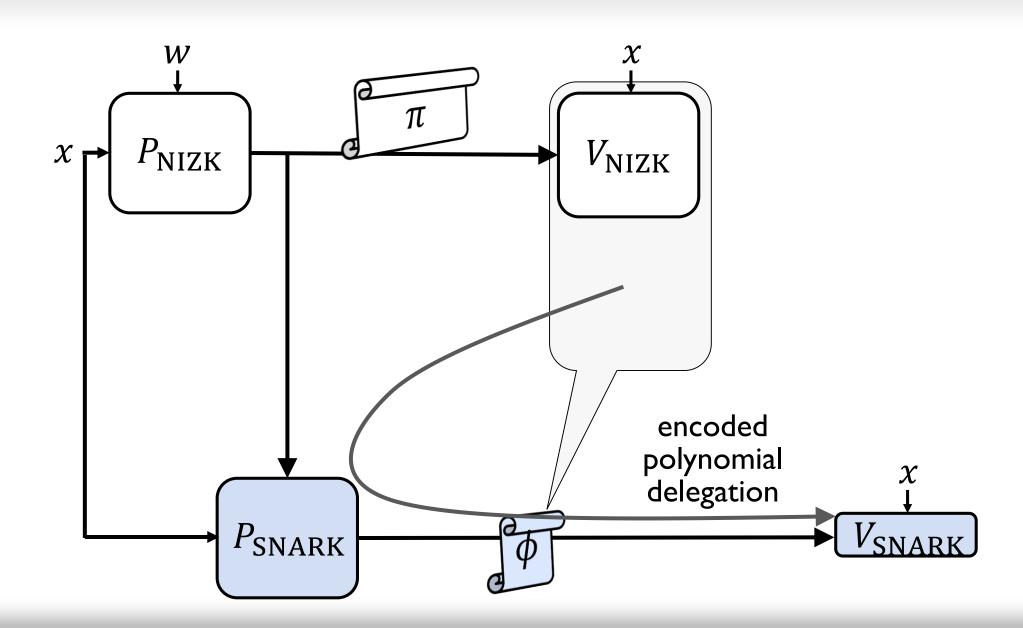
Goal: given input  $\tau \in \mathbb{F}$ , outsource this computation:

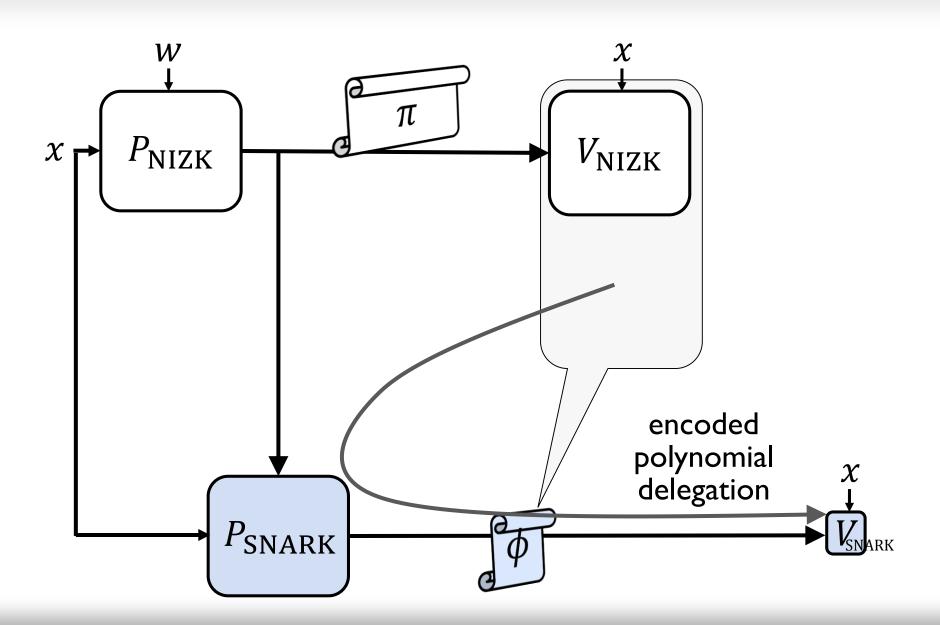
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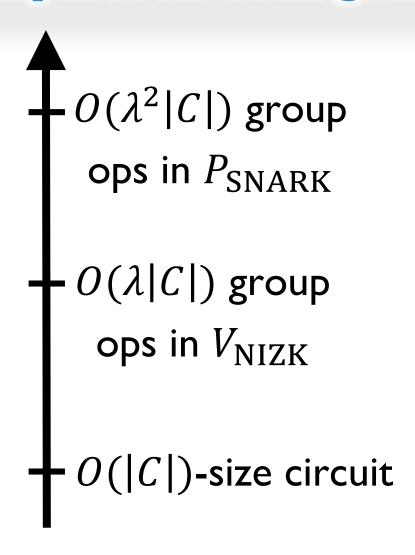
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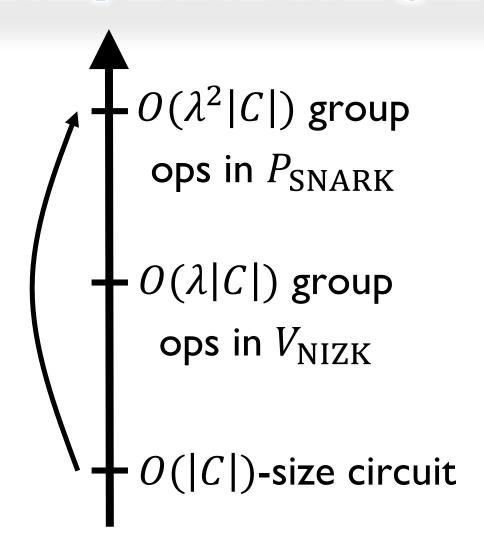




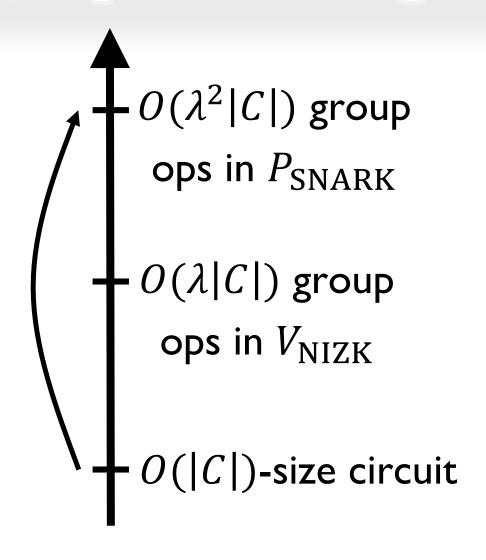




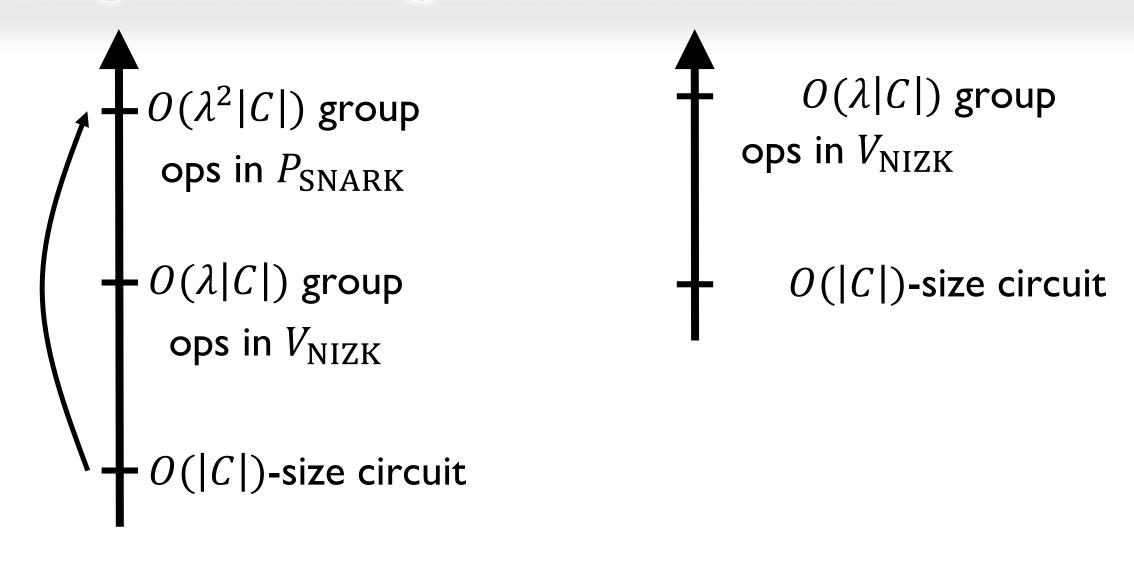
**Generic instantiation** 



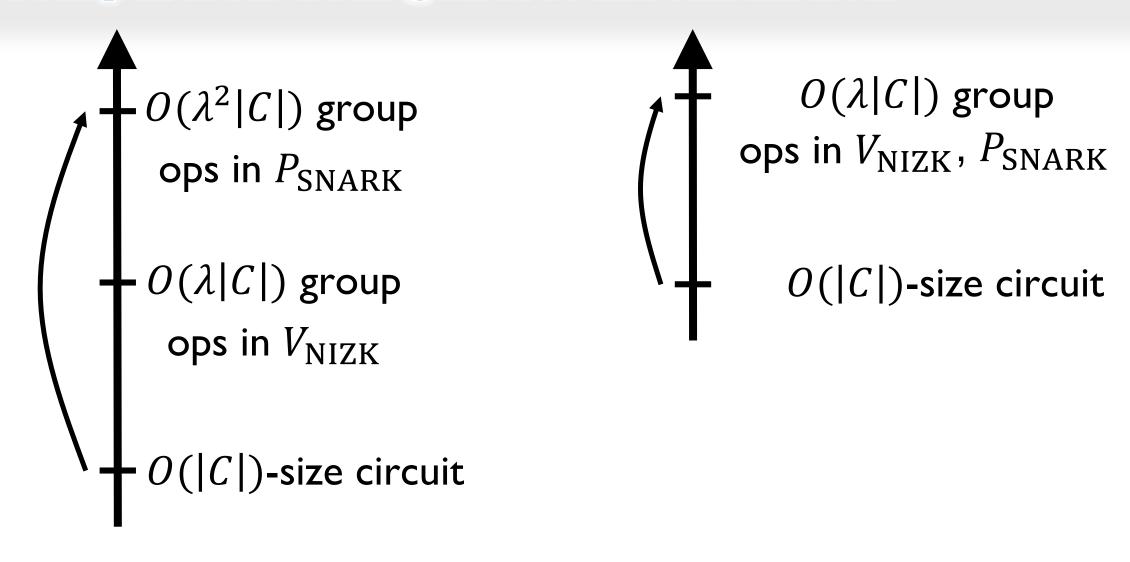
**Generic instantiation** 



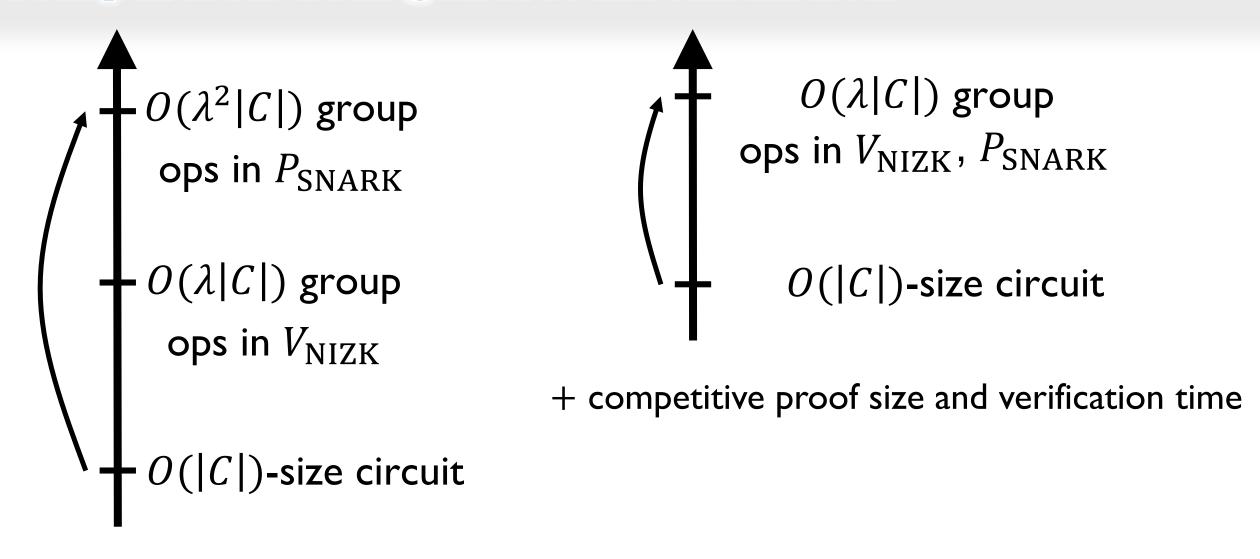
**Generic instantiation** 



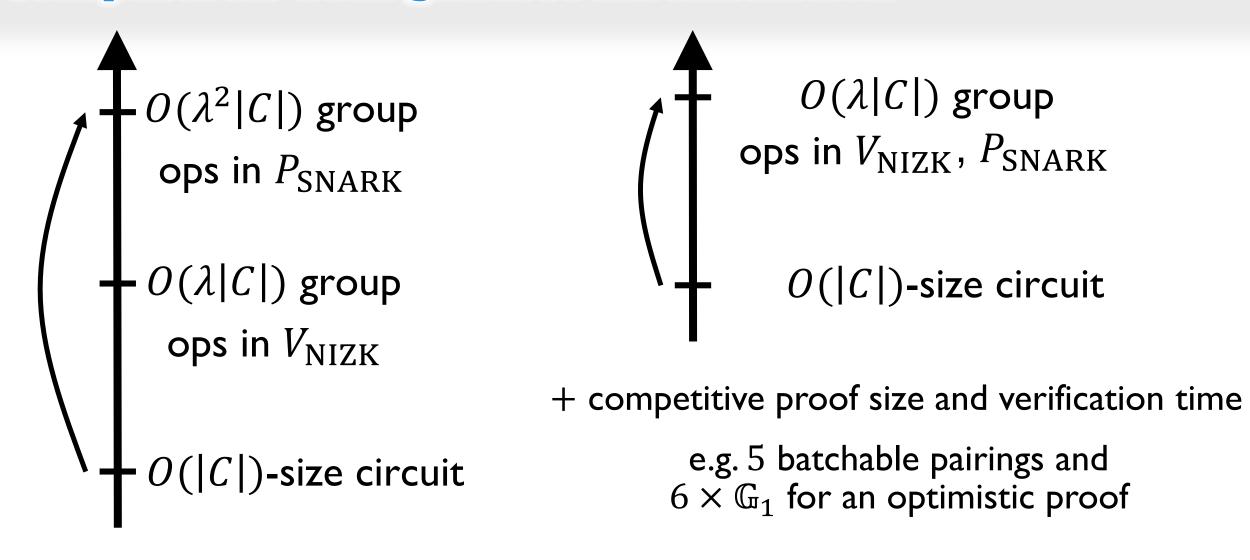
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**Generic instantiation** 

 New primitive: private-coin setup needed for performance but not for soundness or ZK



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- New building blocks along the way:
  - Optimized inner product argument
  - SNARK for encoded polynomial delegation



