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From Public-Key Cryptography to PKI: Reflections on Standardizing the RSA Algorithm

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RSA Public-Key Cryptosystem: Review

Key Pairs

- Public key: (n, e)
- Private key: (n, d)
 - where *modulus* n is product of two large primes p, q , and *exponents* e, d satisfy $e \cdot d \equiv 1 \pmod{\text{lcm}(p-1, q-1)}$

Public-Key Cryptosystem

- Encryption of message m with public key: $c = m^e \pmod n$
- Decryption of ciphertext c with private key: $m = c^d \pmod n$

Digital Signature Scheme

- Signature on message m with private key: $s = m^d \pmod n$
- Verification of signature s (and recovery of m) with public key: $m = s^e \pmod n$

About PKCS

“The *Public-Key Cryptography Standards* are specifications produced by RSA Laboratories in cooperation with secure systems developers worldwide for the purpose of accelerating the deployment of public-key cryptography.”
(PKCS #1 v2.2, 2016 [RFC8017])

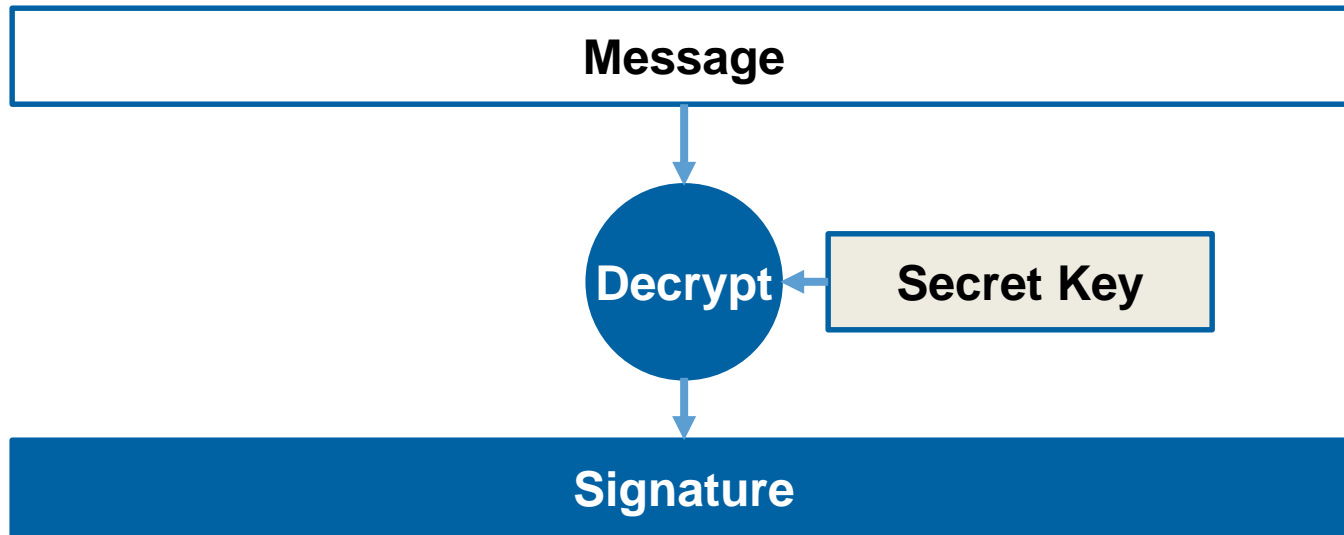
Outline

- Part I: RSA Signatures
- Part II: RSA Encryption
- Part III: Lessons Learned

Part I: RSA Signatures

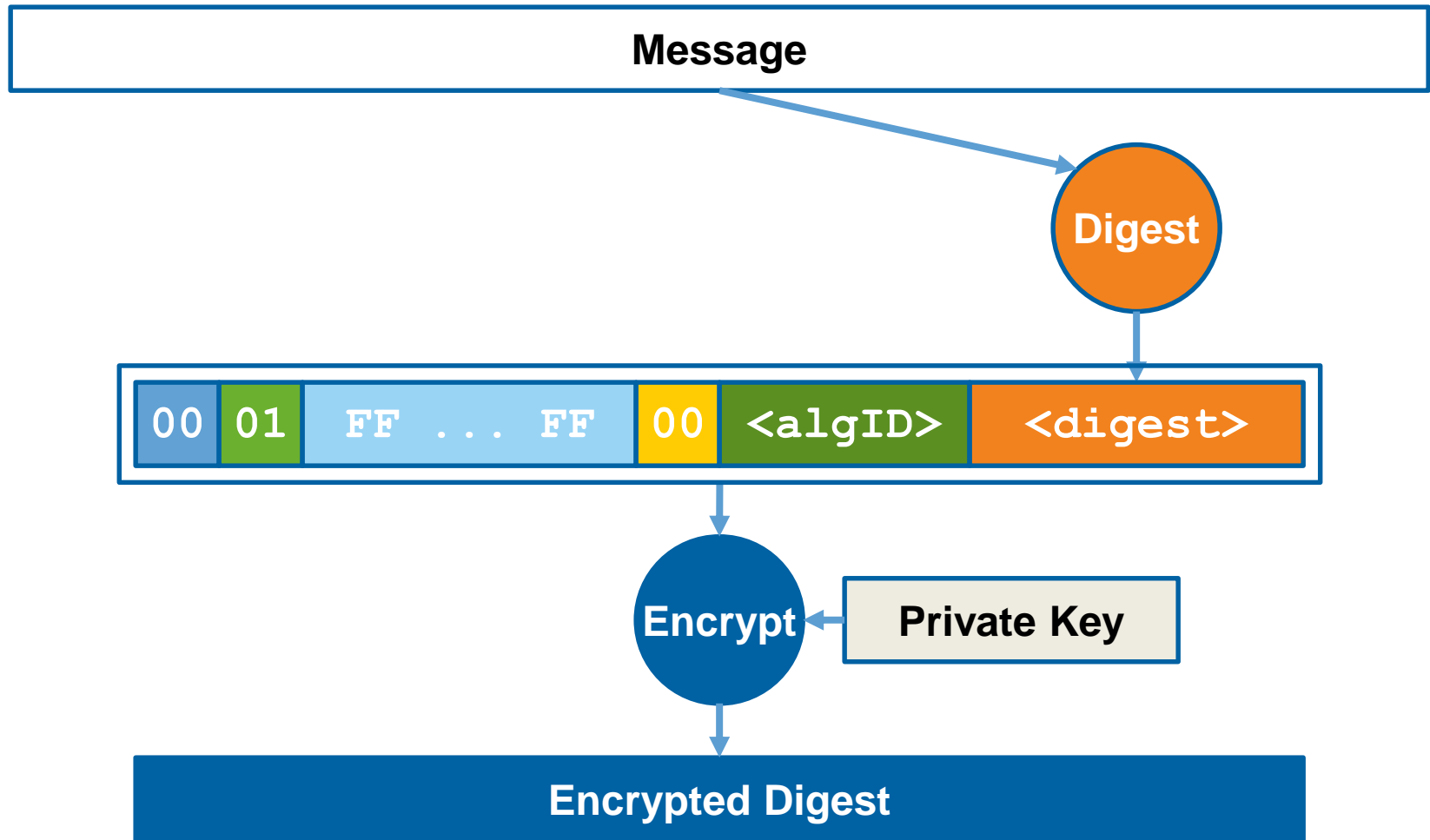
RSA Signatures: Original Model

Diffie-Hellman (1976) and RSA (1978)

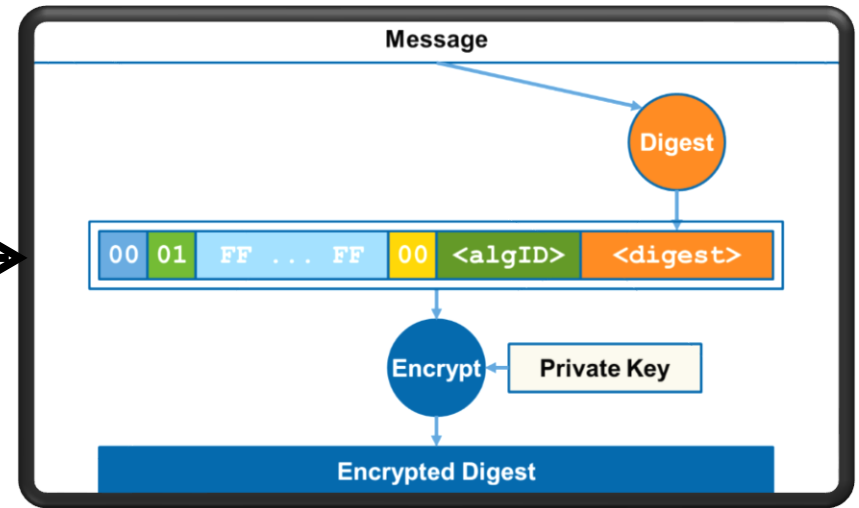
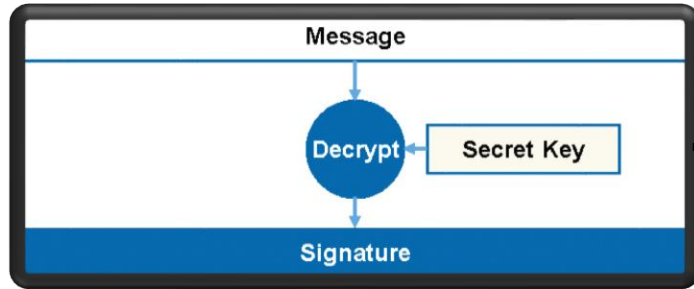


RSA Signatures: “Standard” Model

PKCS #1 (1991)

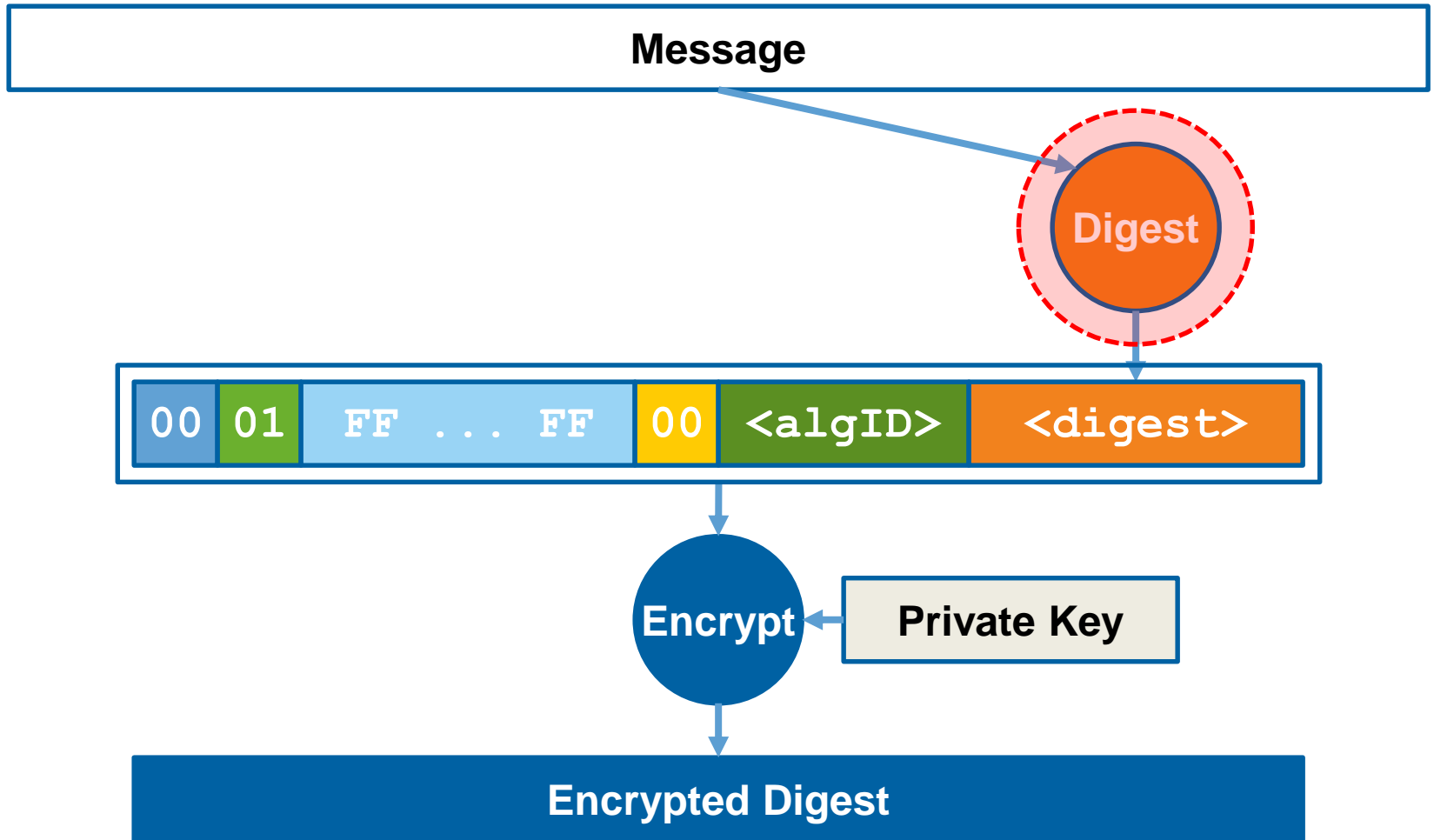


How Did Original Model Change to Standard?

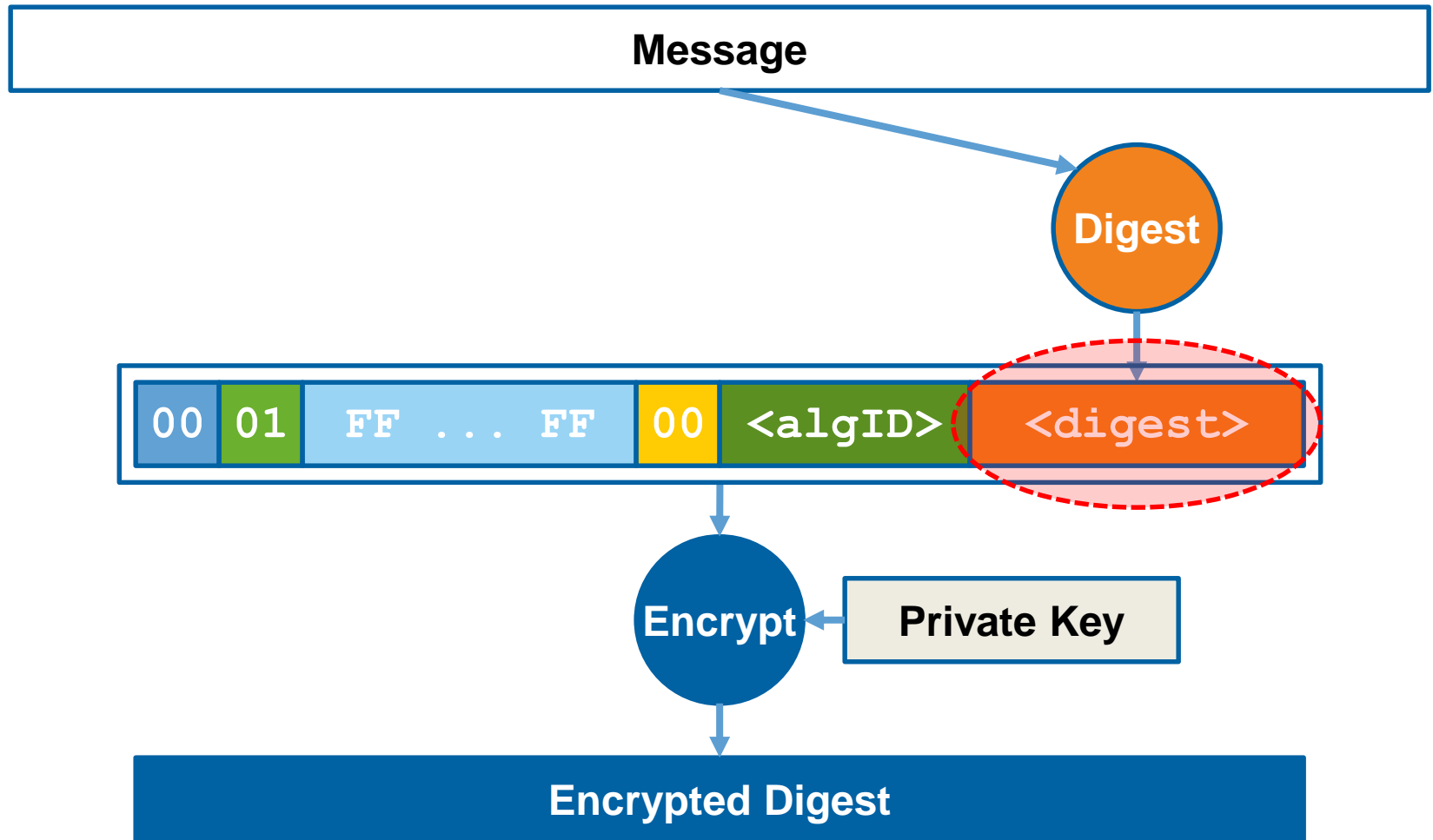


1. Hash-then-sign paradigm
2. Partial domain “digests”
3. Algorithm identifiers
4. Fixed padding
5. Block type
6. Encrypting with private key
7. Encrypted digest

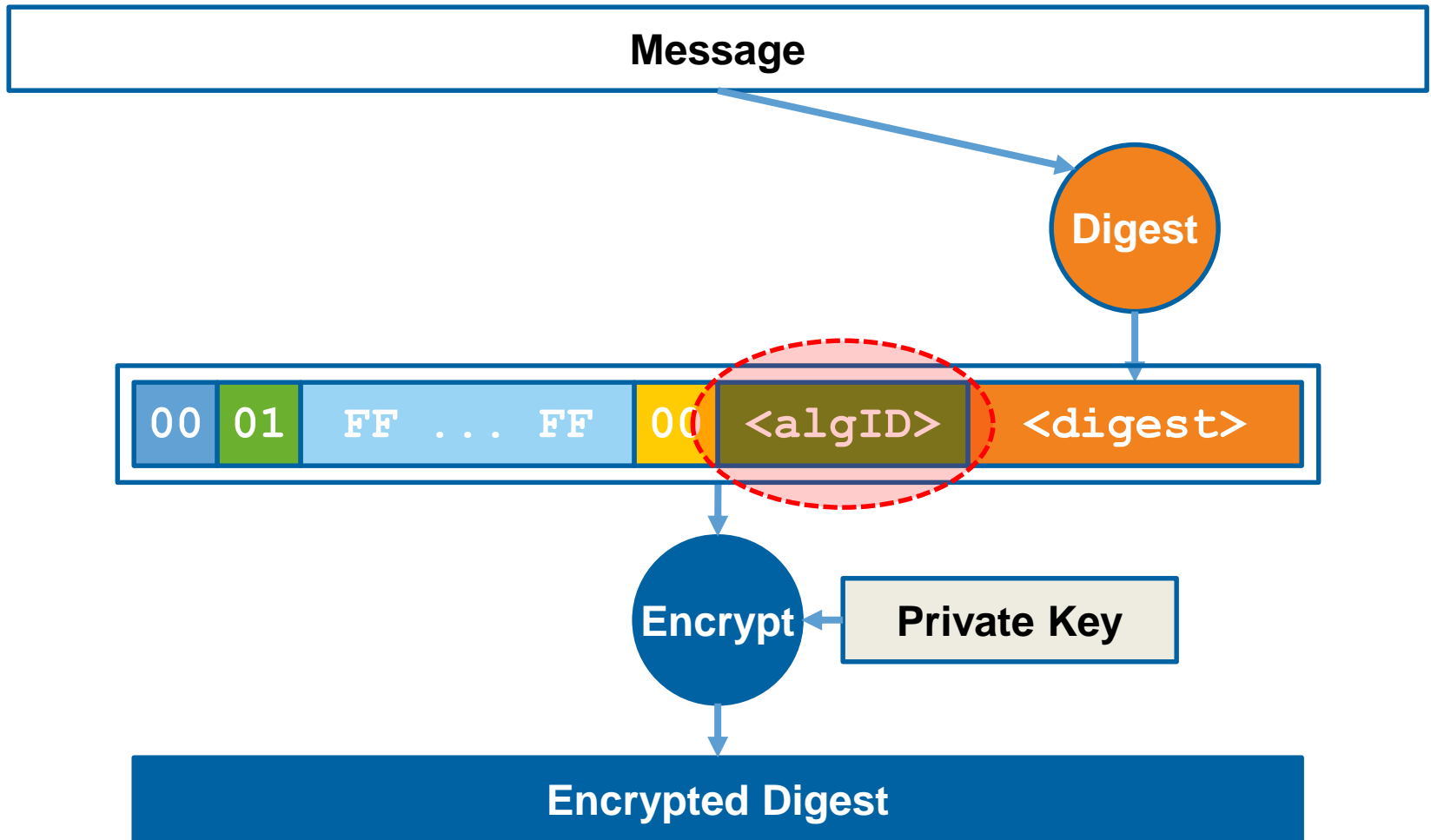
1. Hash-then-Sign Paradigm



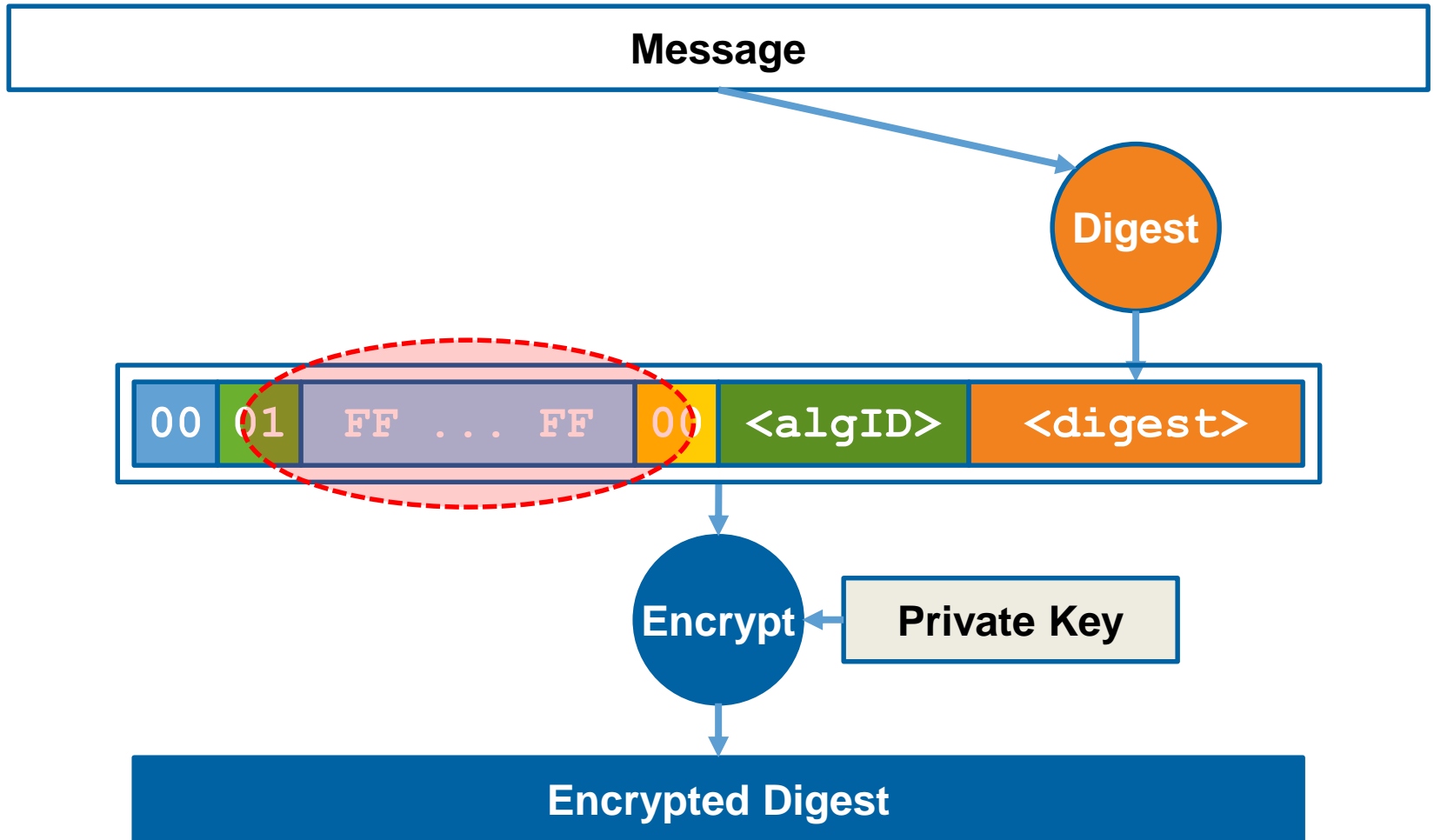
2. Partial Domain “Digests”



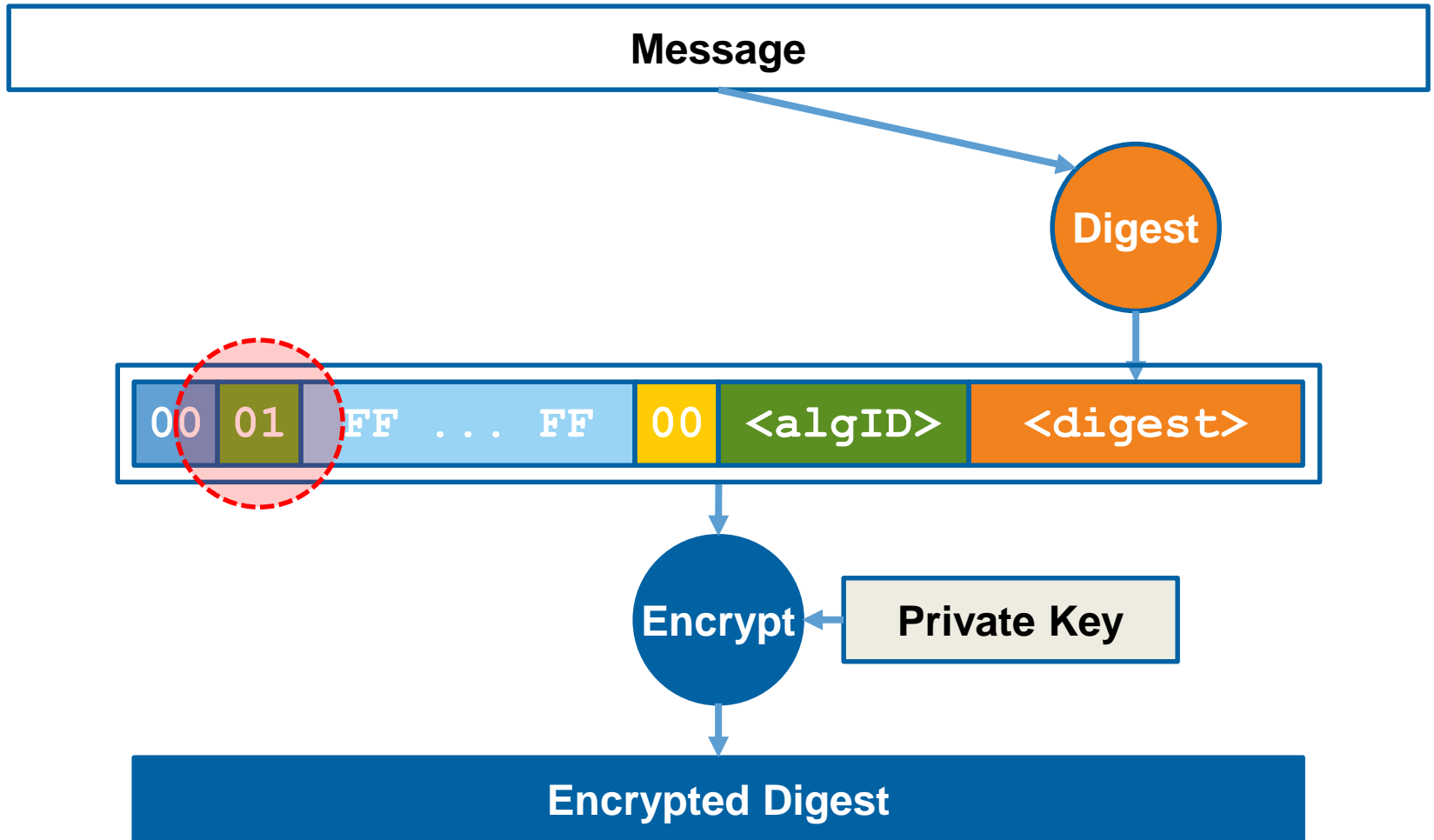
3. Algorithm Identifier



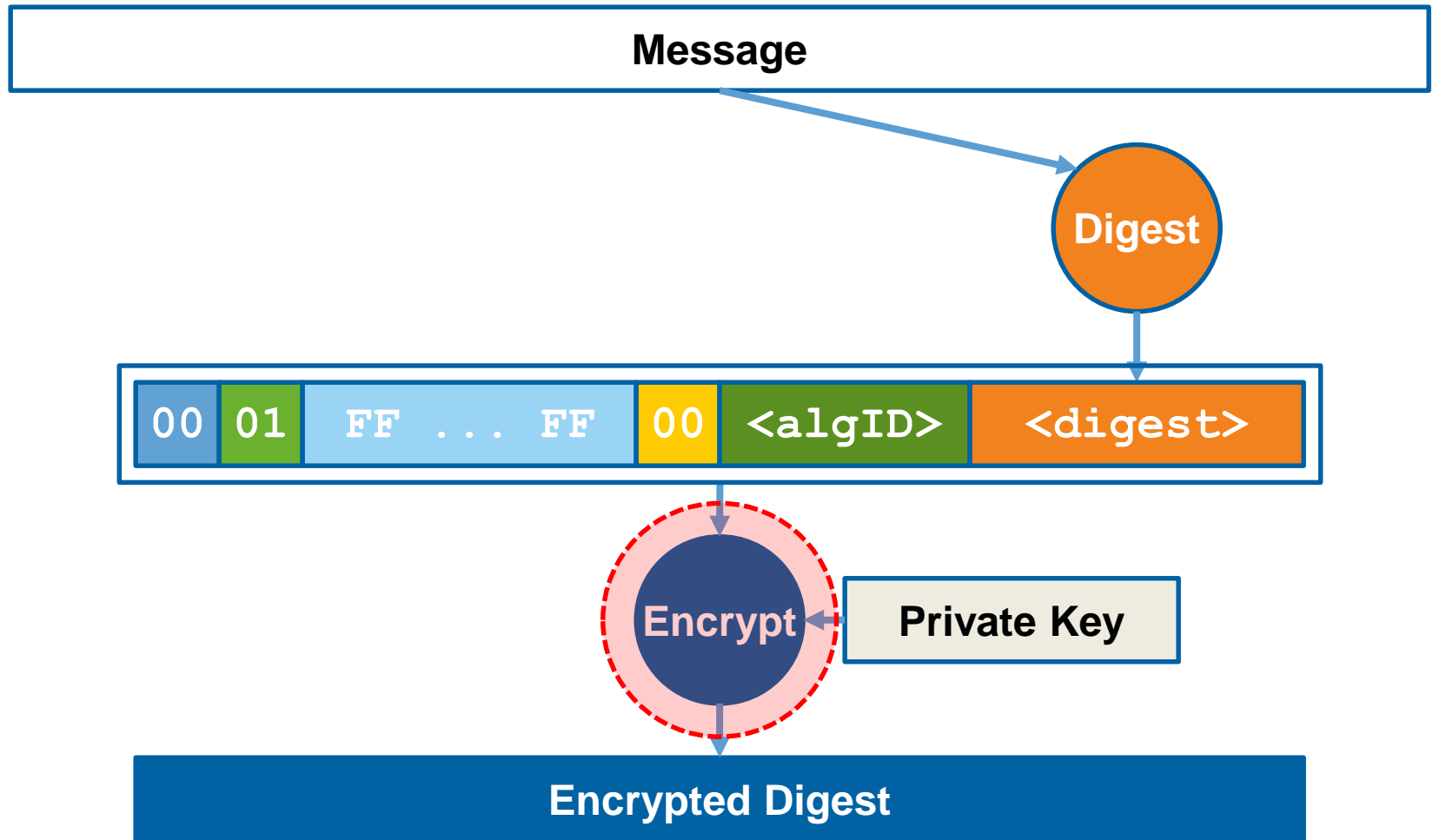
4. Fixed Padding



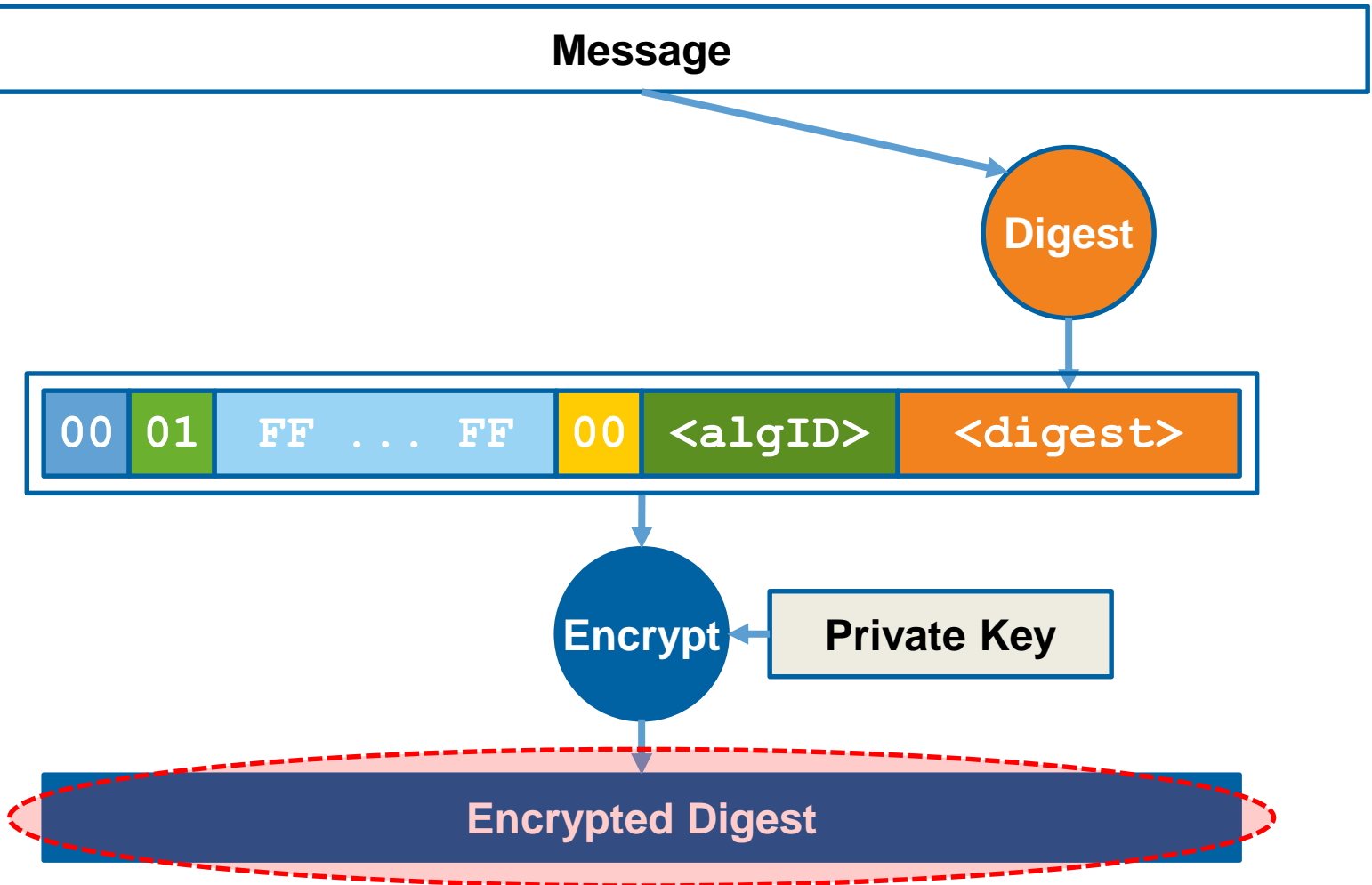
5. Block Type



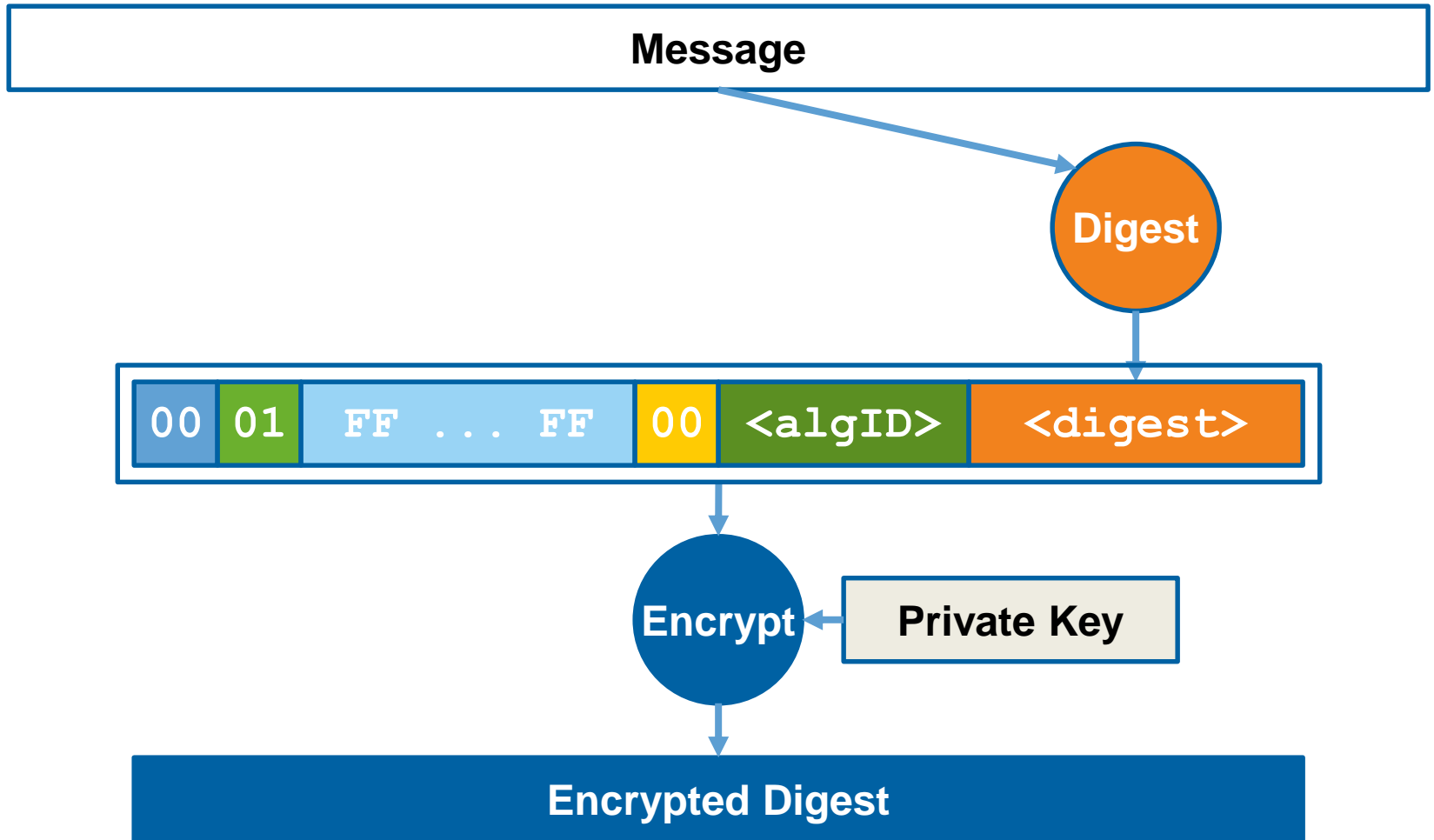
6. Encrypting with Private Key



7. Encrypted Digest



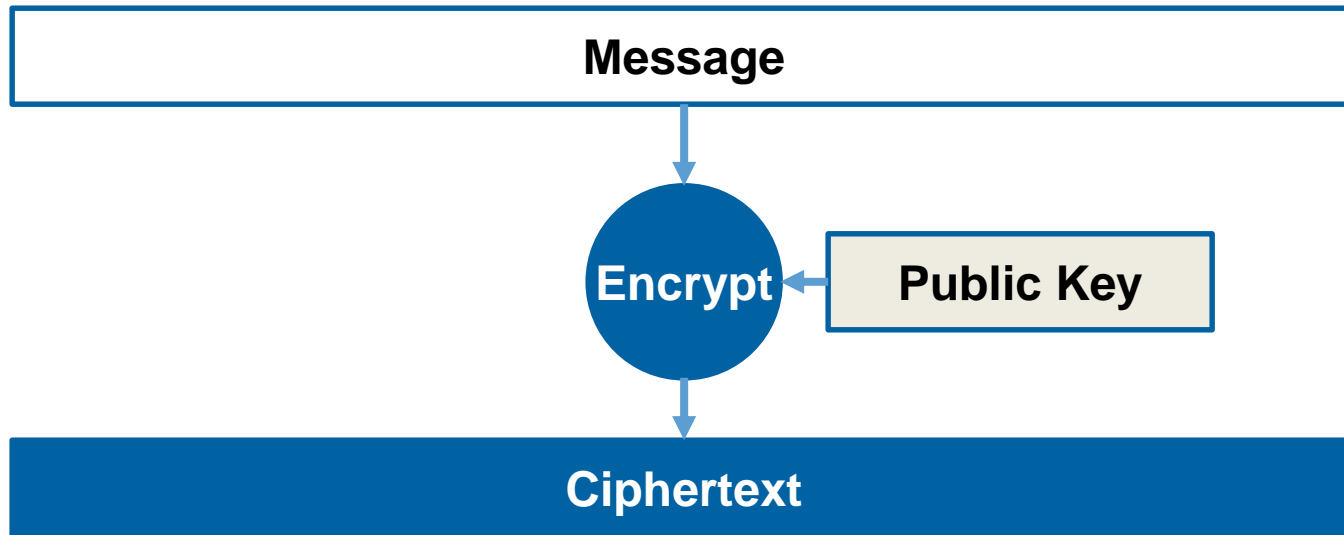
PKCS RSA Signatures: Summary



Part II: RSA Encryption

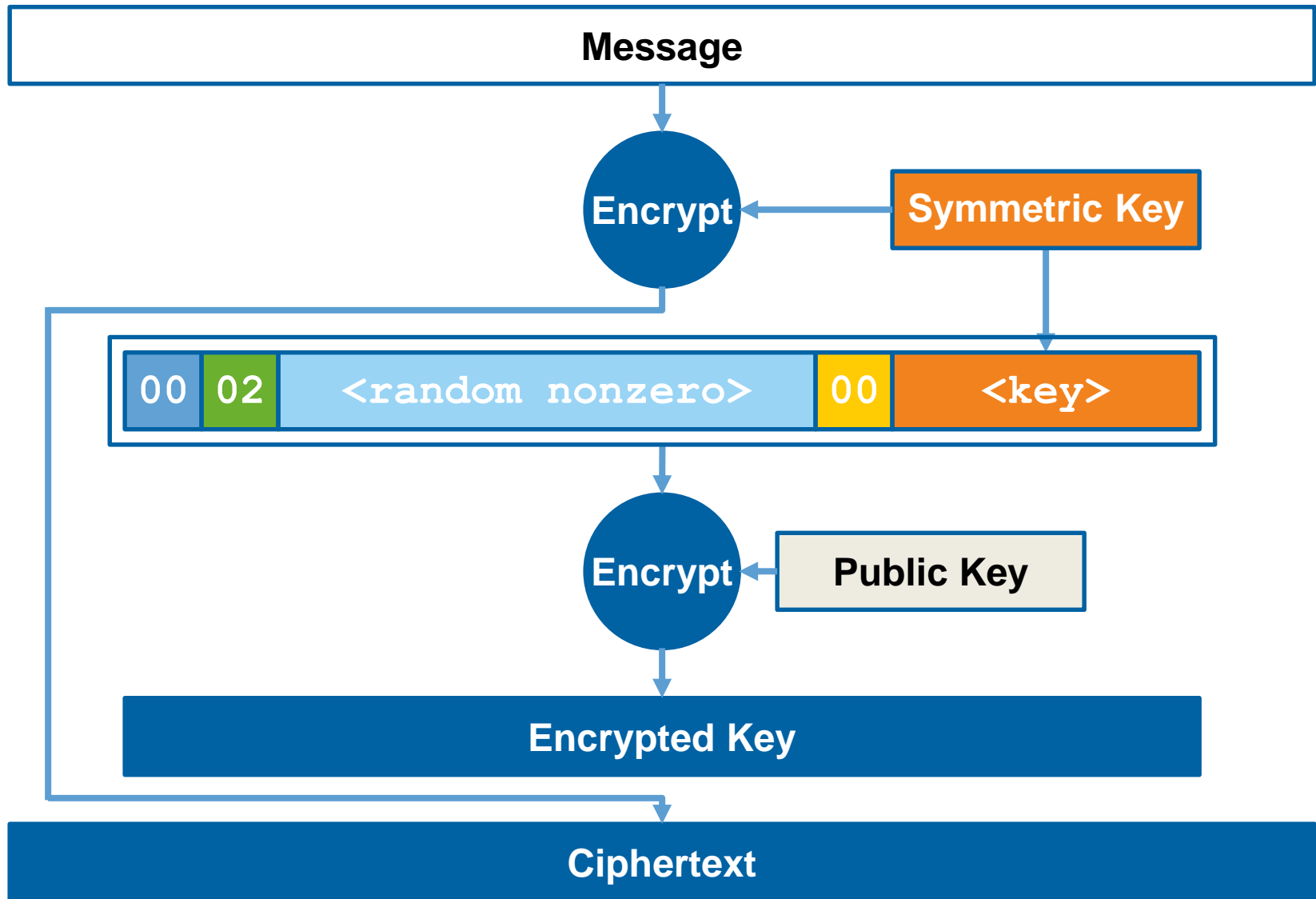
RSA Encryption: Original Model

Diffie-Hellman (1976) and RSA (1978)

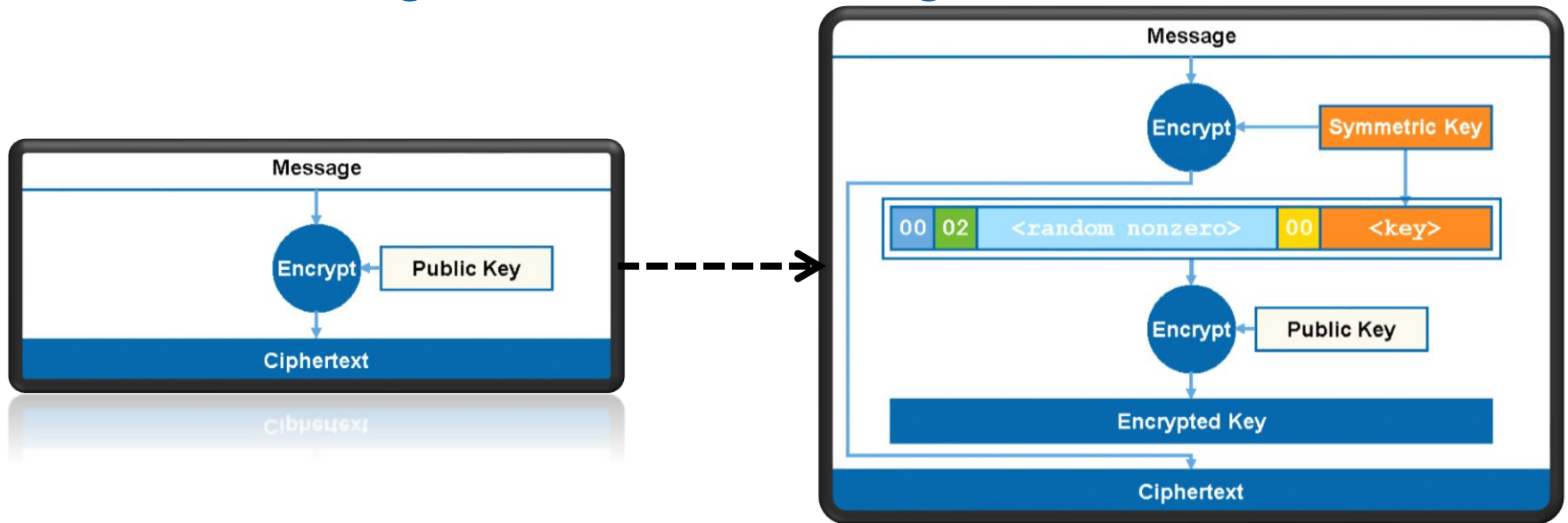


RSA Encryption: “Standard” Model

PKCS #1 and #7 (1991)

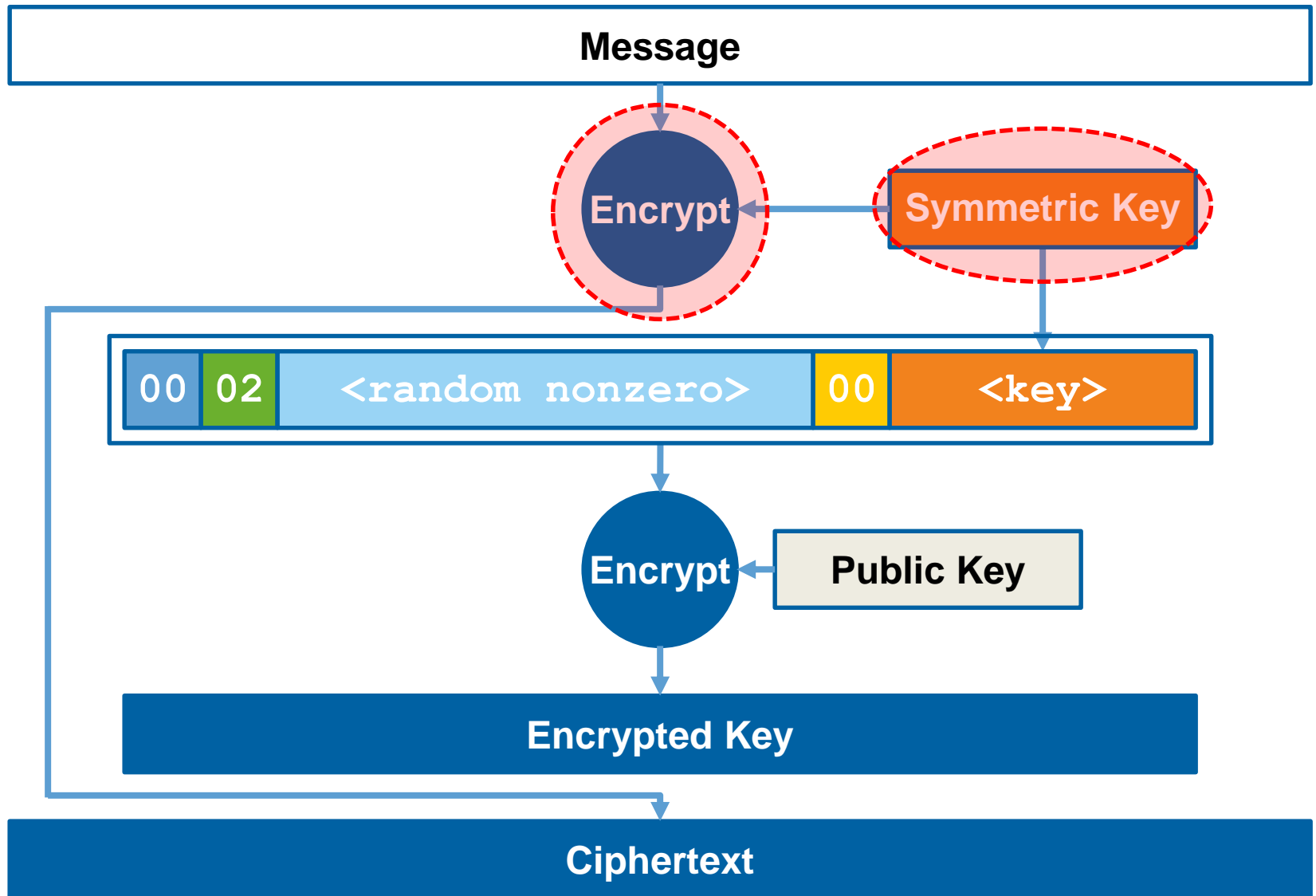


How Did Original Model Change to Standard?

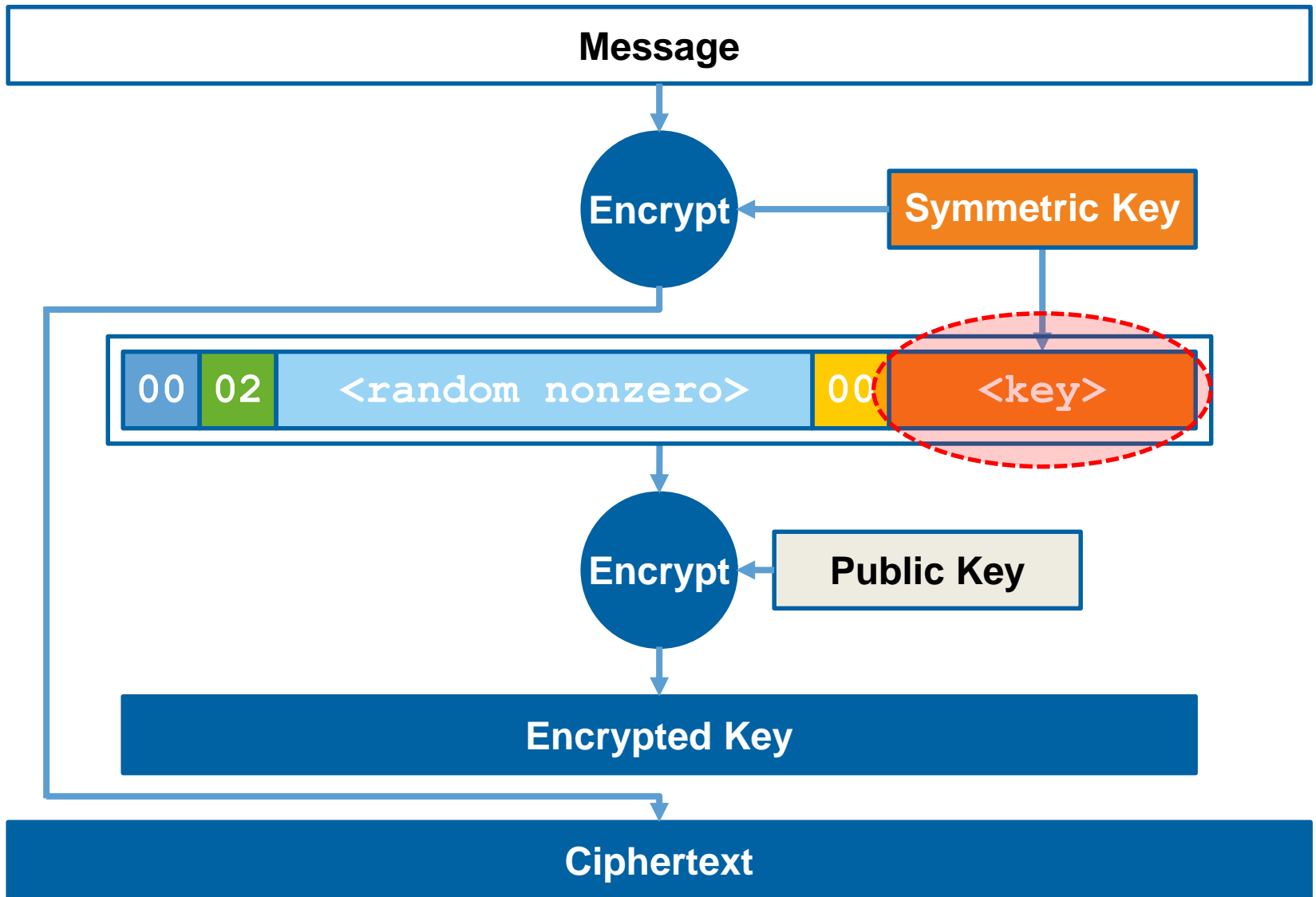


1. Encrypt-then-wrap paradigm
2. Partial domain encryption keys
3. Random padding
4. Block type

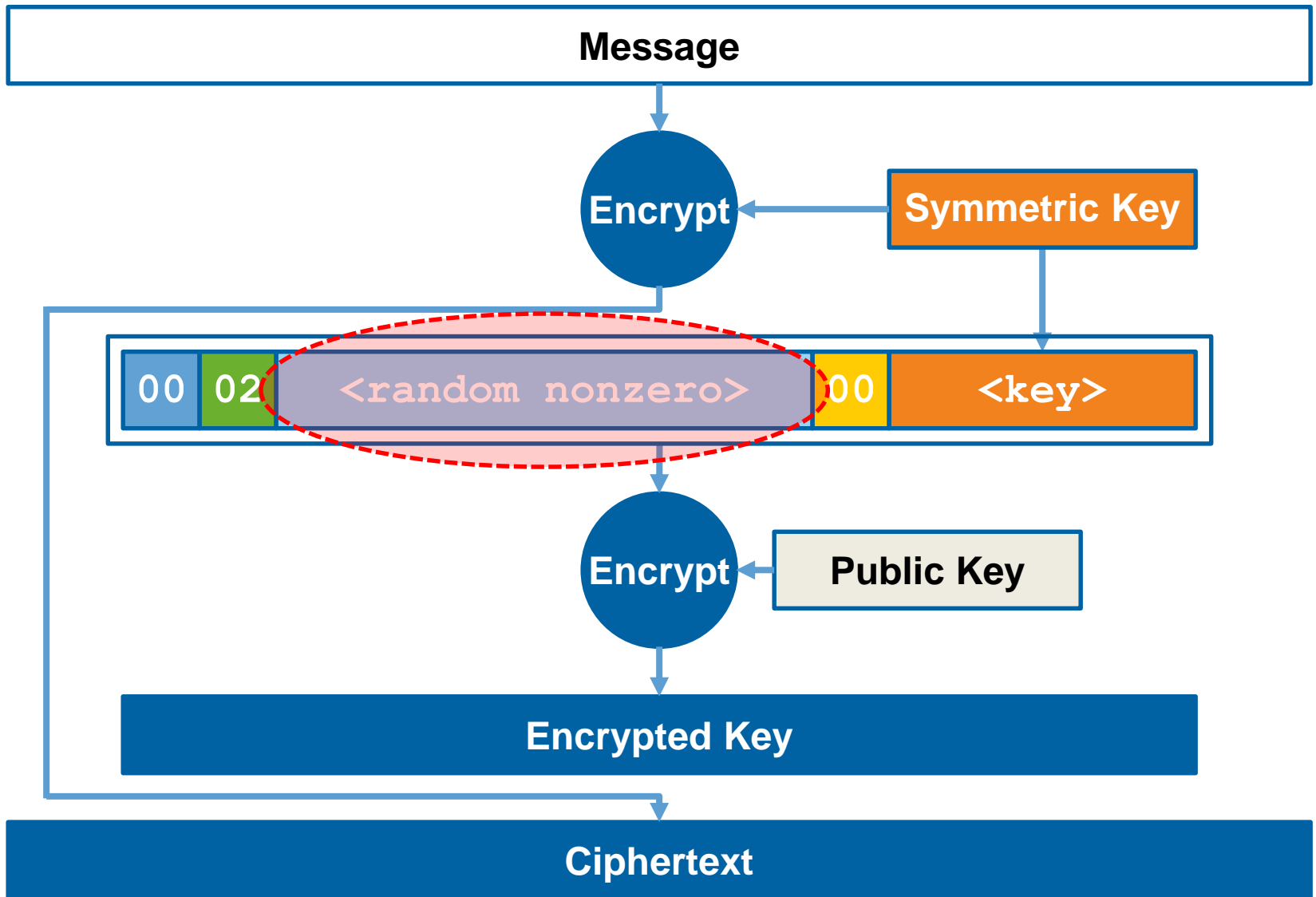
1. Encrypt-then-Wrap Paradigm



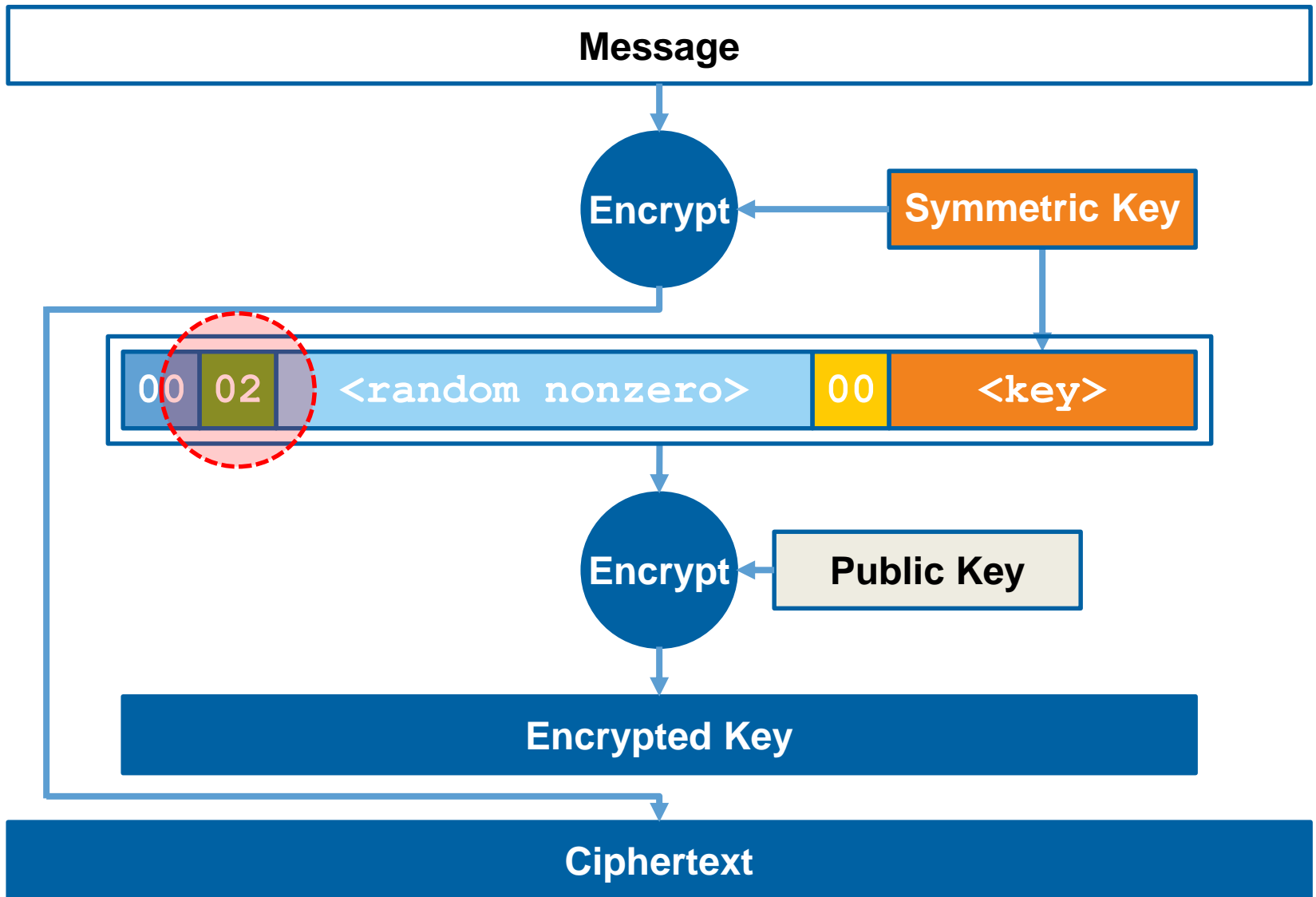
2. Partial Domain Encryption Keys



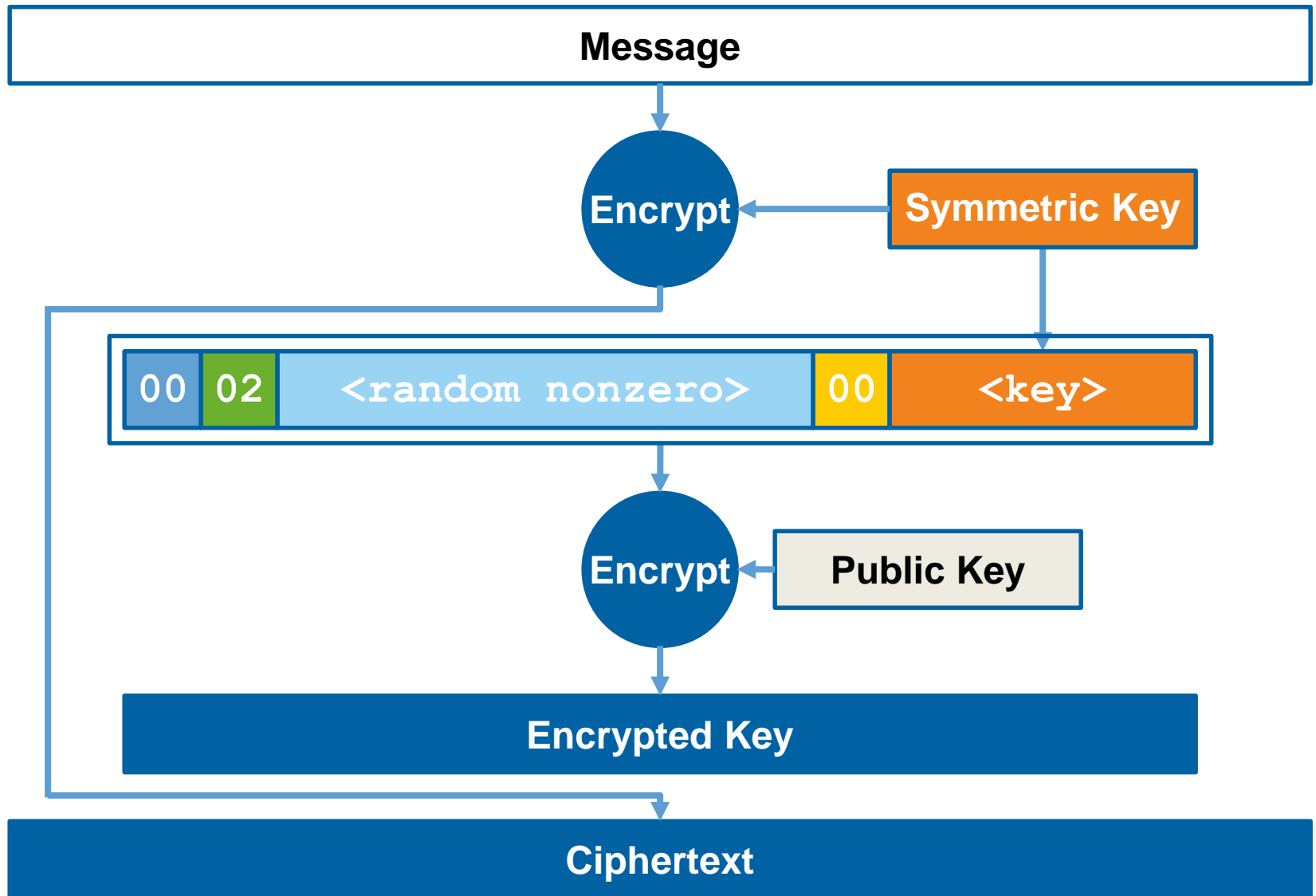
3. Random Padding



4. Block Type

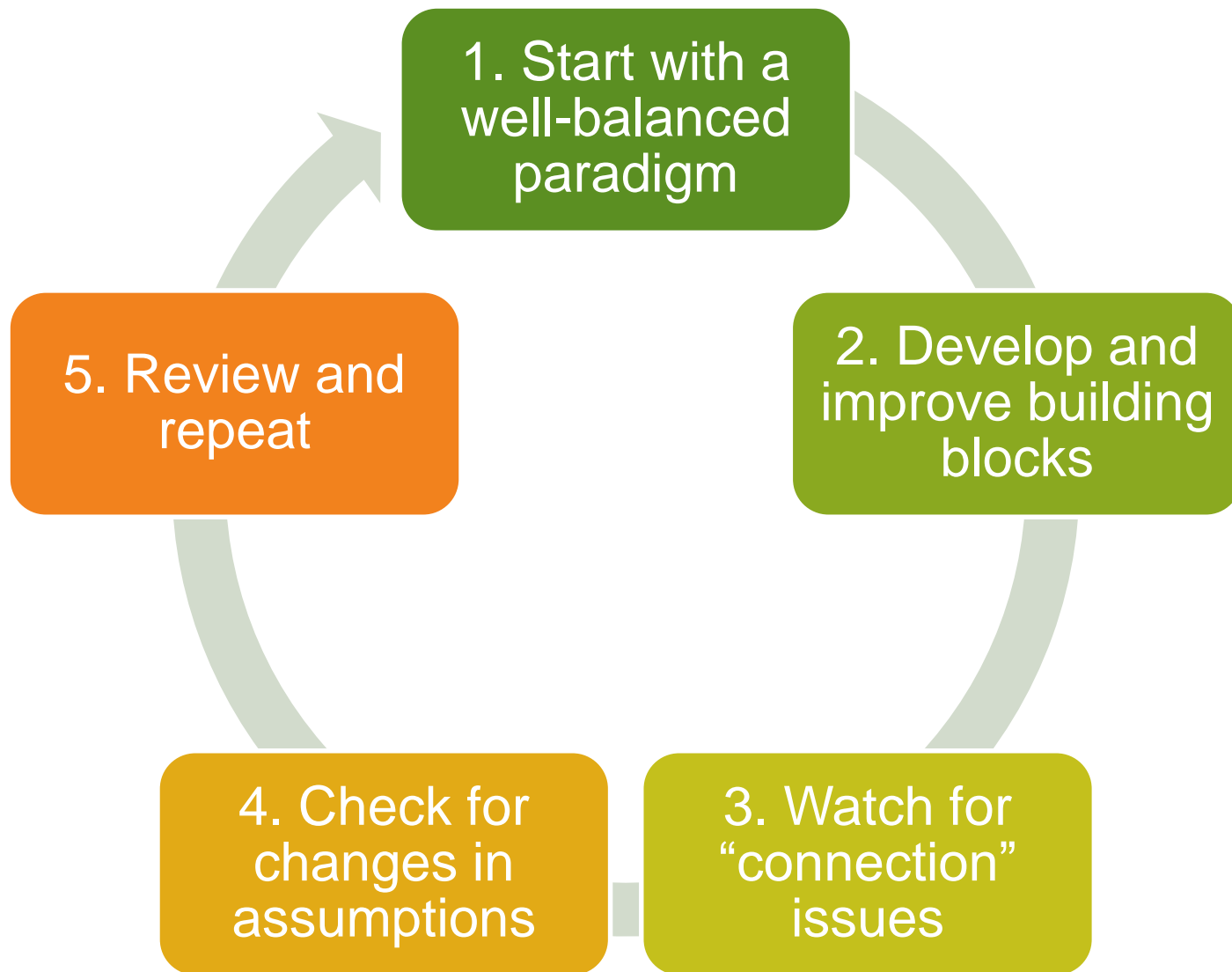


PKCS RSA Encryption: Summary

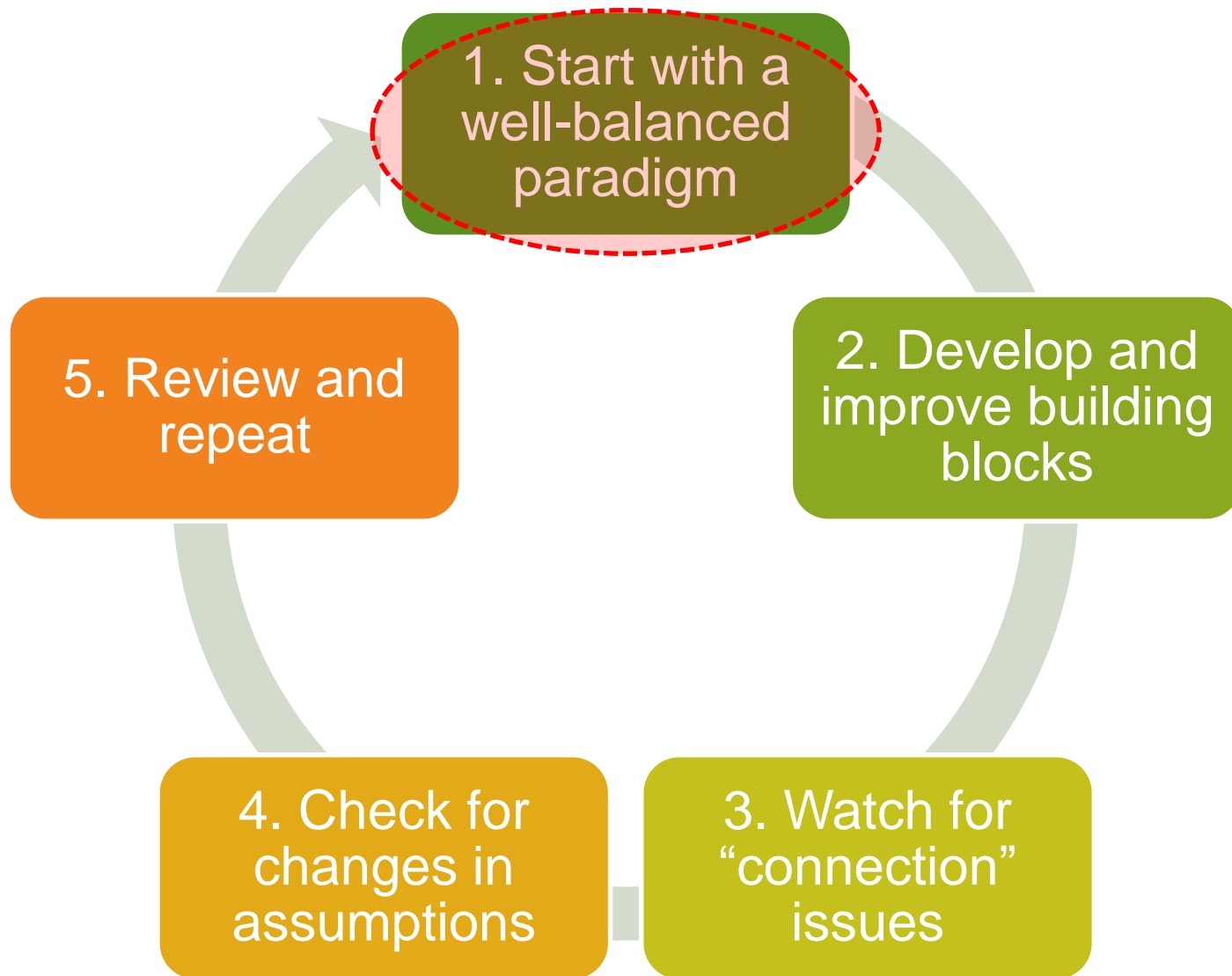


Conclusion: Five Lessons Learned about Standardizing Cryptography

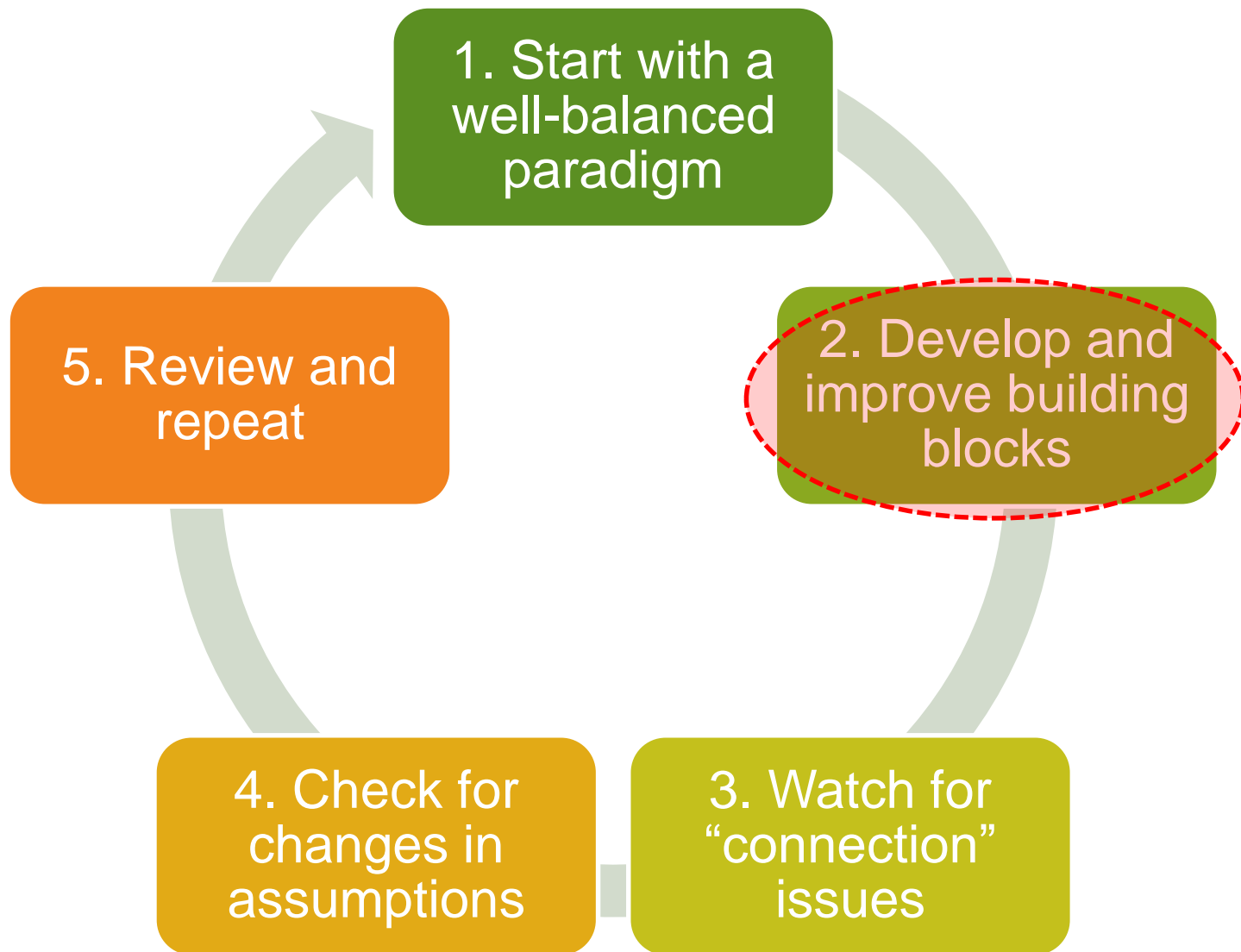
Five Lessons Learned about Standardizing Cryptography



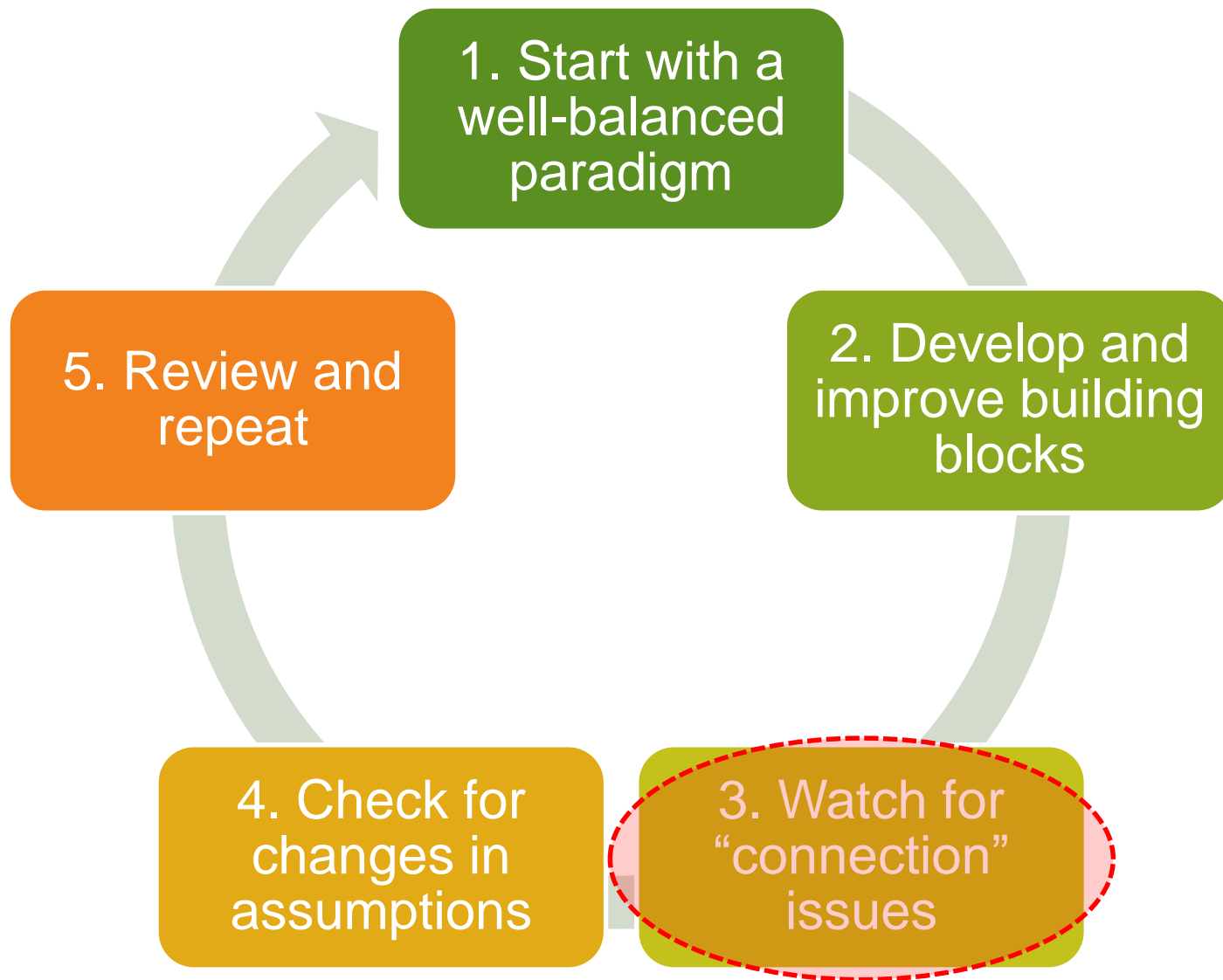
1. Start with a Well-Balanced Paradigm



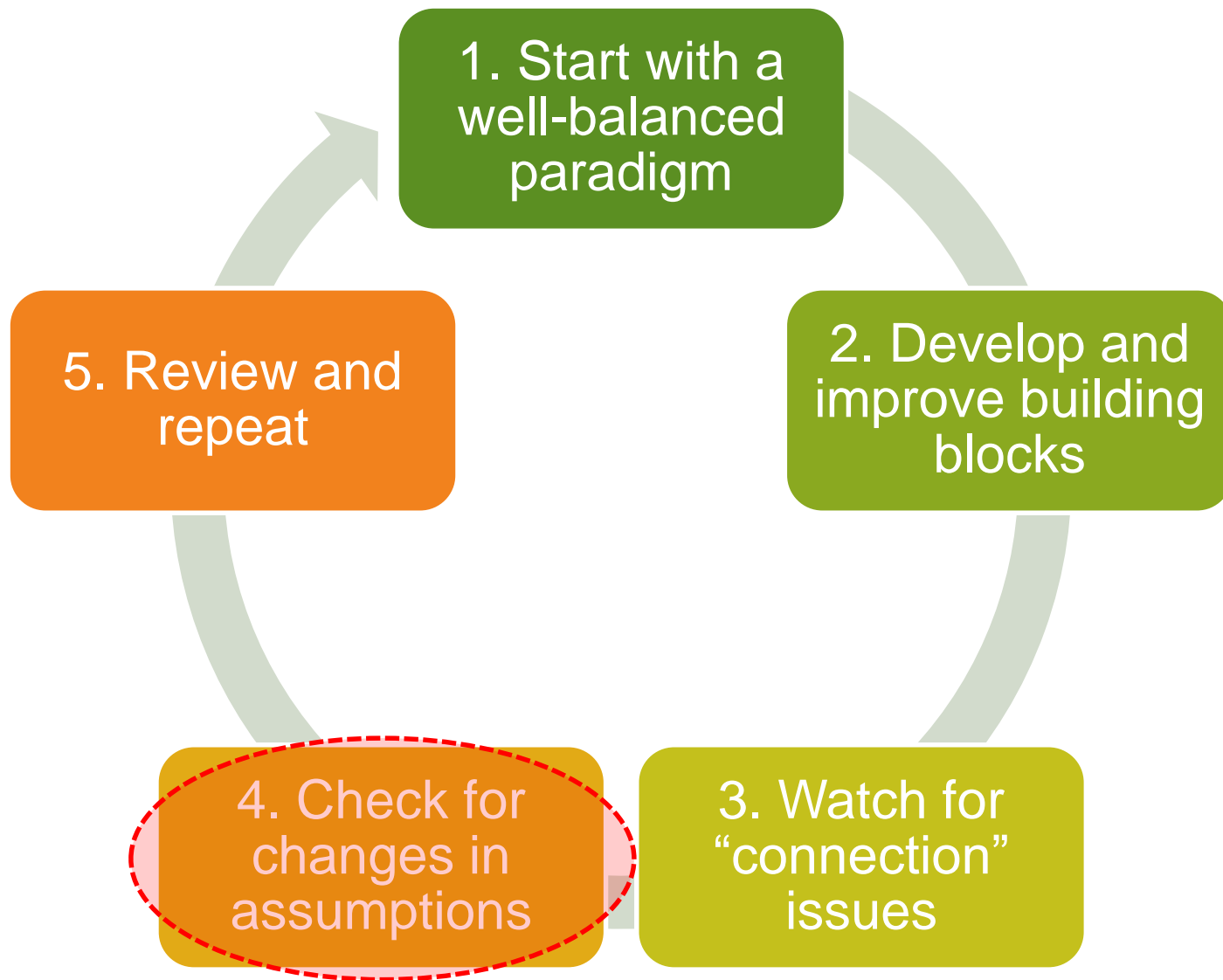
2. Develop and Improve Building Blocks



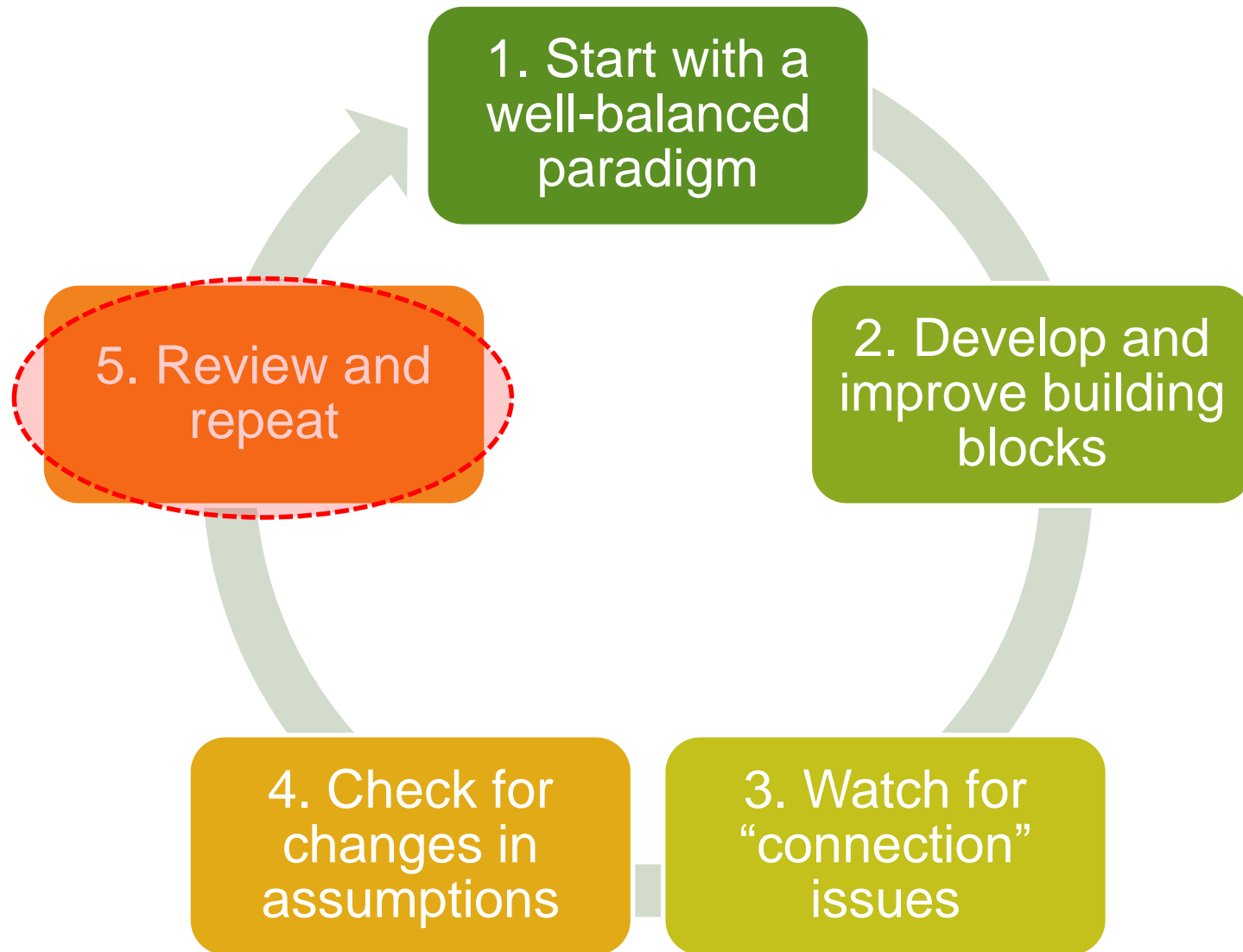
3. Watch for “Connection” Issues



4. Check for Changes in Assumptions



5. Review and Repeat



Questions?

Appendix: Other RSA Standardization Issues (overview)

Other RSA Standardization Issues

Public / private key pairs

- Modulus size
- Public exponent values
- Key pair (and prime) generation
- Public key validity
- Public key syntax
- Private key syntax

Message syntax

- Signed messages
- Enveloped (encrypted) messages

Key management

- Certificate syntax
- Certificate request syntax
- Certificate revocation list syntax
- Certificate lifecycle management
- Certificate status protocols
- Private key containers

Cryptographic APIs

and more ...

Selected References for Further Reading

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