Satellite-to-ground coincidence matching under Doppler effects

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1 Introduction

Entanglement-based quantum key distribution (QKD) relies on a steady source of photon pairs. The first step in establishing a key in such a scheme is the assignment of photodetection events to entangled photon pairs. Ho et al. (2009) [1] introduced an algorithm for coincidence matching under a constant time offset (ΔT) and a constant frequency difference (Δu). However, for satellite-based photon pair sources, an element of Doppler shift causes the relative clock frequency between the satellite and groundstation to vary in time ($\Delta T(t), \Delta u(t)$). The complexity from Doppler shift rates at different angles of elevation can cause this cross correlation to be spread out over thousands of time bins. This note introduces two methods for correcting Doppler effects in satellite-to-ground coincidence matching.

1.1 Time stamping a satellite SPDC source

We consider a setup where a spontaneous parametric down conversion (SPDC) source on a satellite is time-stamped by two individual time-stamp cards with a relative clock drift of 165μs/s (Fig 1).

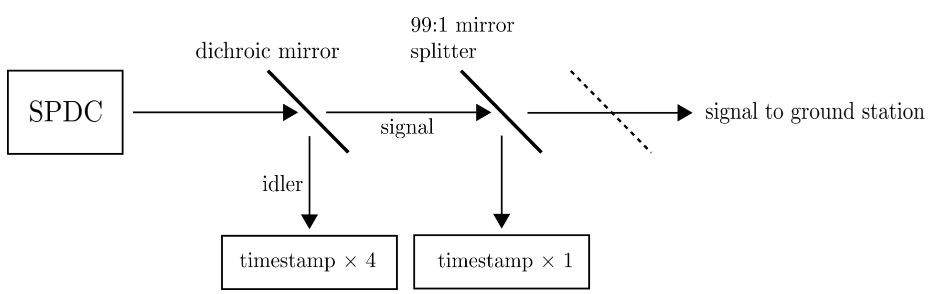
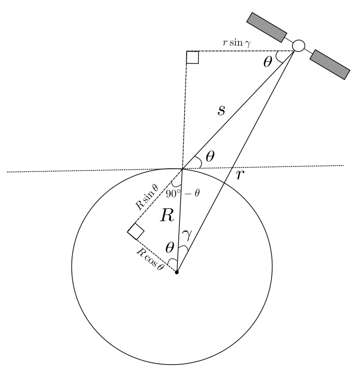


Figure 1: Idlers and 1% of signal photons are time-stamped on the satellite. The rest of the signal photons are transmitted to ground.

1.2 Propagation delay

We introduce a propagation delay for each time-stamp based on pair generation time. When the elevation angle is at its maximum 90°, we define that t is 0 seconds. Assuming that the satellite is in a circular orbit at altitude $h = 500$km, with a carrier frequency of $f_c = 1$MHz, a simplified expression for the carrier propagation delay can be given by:



$$V = \sqrt{\frac{R^2}{r} \cdot g}, \tag{1}$$

$$\gamma = \frac{V \cdot t}{r}, \tag{2}$$

$$\theta = \arccos\left(\frac{r \sin\gamma}{s}\right), \tag{3}$$

$$s = \sqrt{R^2 + r^2 - 2Rr\cos(\gamma)}, \tag{4}$$

$$f_d = f_c\left(\frac{V}{c}\cos\theta\right). \tag{5}$$

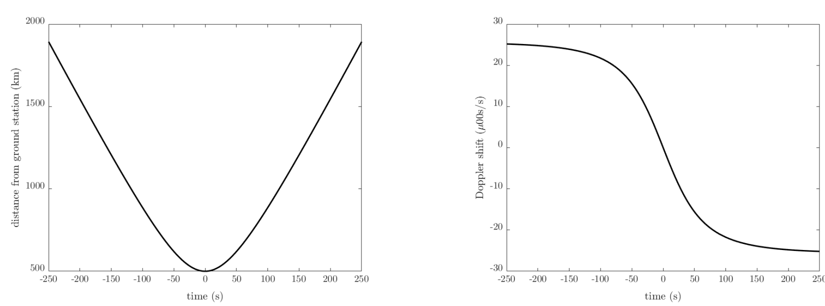
Figure 2: A simplified schematic of the satellite's orbit.

where R is the Earth's radius, $r = R+h$, g is gravitational acceleration, and c is the speed of light. Equation (1) is the velocity of the satellite; (2) is its orbital phase angle γ; (3) is its angle of elevation θ; (4) is its distance from the ground station s; and (5) is the Doppler shift of the carrier signal from the satellite.

Using this model, we specify an absolute time offset of

$$\Delta t = \frac{s(t)}{c}, \tag{6}$$

where distance is calculated for each pair generation time in orbit (Fig 3(a)).



(a) (b)

Figure 3: (a) Distance to ground station, (b) Doppler shift of carrier signal.

1.3 Doppler shift

The Earth station's clock can have an offset t_s from true UTC and a drift of this offset $\frac{dt_s}{dt}$ with time. Since the satellite is moving at a different speed relative to the ground-station, the relative frequency difference between satellite and ground-station clocks is Doppler shifted (Fig 5). This is a more significant source of error than the propagation delay Δt (eq. 6), by almost one order of magnitude (Fig 4(a) and 4(b)).

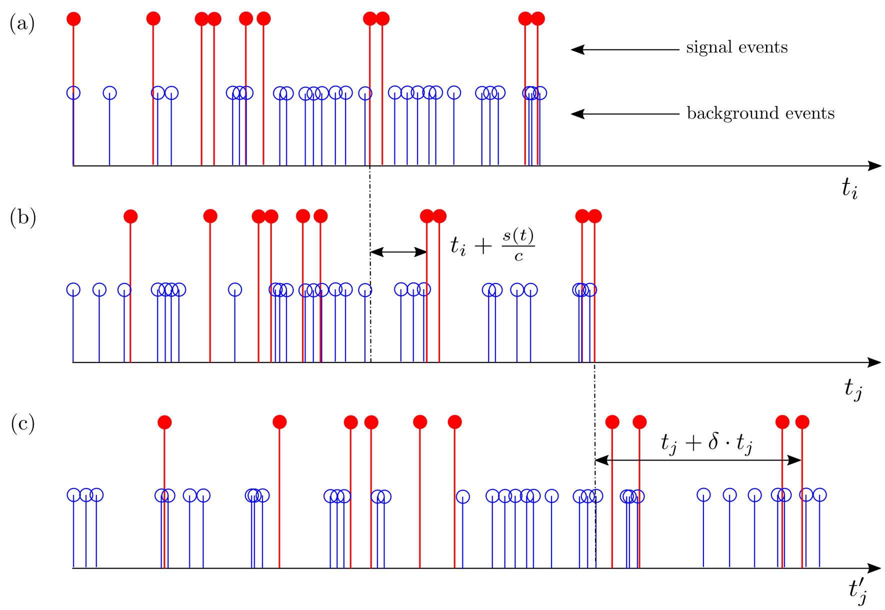


Figure 4: Effect of Doppler shift and Doppler shift rate on photoevent sets. Trace (a) represents the event set $\{t_i\}$ on side A, trace (b) an event set $\{t_j\}$ on side B with a time offset $\frac{s(t)}{c}$, but the same reference clock frequency. Trace (c) illustrates a set $\{t'_j\}$ with an additional relative frequency difference δ between both reference clocks. Image adapted from [1].

Since the stream of time stamps $\{t_i\}$ and $\{t_j\}$ on each side has no intrinsic time structure, it is difficult to distinguish a signal event from a background event in the frequency-shifted $\{t'_j\}$ (Fig 4(c)).

The change in the Doppler shift over time is highest at 90° elevation angle and can reach up to nearly 400 ns/s^2 (Fig 5). This causes a change in relative clock drift over time. This stretch is of the order of microseconds per second (23 μs/s for Doppler, 165 μs/s for clock drifts) and it causes the cross correlation to be spread out thousands of time bins, making it difficult to distinguish a significant peak in the cross correlation.

The former subsections used a simplified model of the Earth's orbit to introduce Doppler shift concepts. Numerically accurate calculations can be done using data from the Python `pyephem` package.

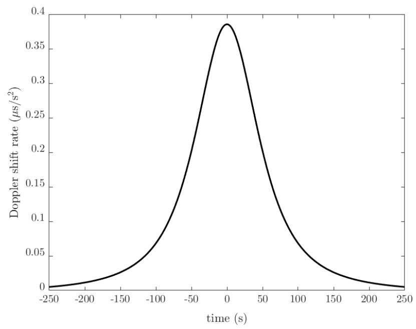


Figure 5: The change in Doppler shift.

To correct for this, we introduce a second order term to our time shift equation:

$$\Delta t = \frac{s(t)}{c} + \delta \cdot t, \tag{7}$$

where the Doppler shift δ is defined by (cf. eq. 5)

$$\delta = \frac{V}{c} \cos\theta. \tag{8}$$

2 Correction using timing beacon on satellite

The timing beacon correction method installs a beacon with regular pulses on the satellite. We time-stamp the beacon signal both on the satellite and at the ground station (Fig 6). If we find the cross correlation between the beacon pulses registered on the satellite timestamp and those registered on the ground station, we can apply the same transformation to the idler and signal photoevents. We find that even a high jitter beacon with poor detection efficiency is sufficient to correct for all clock drifts including the change of the Doppler shift for high elevation angles.

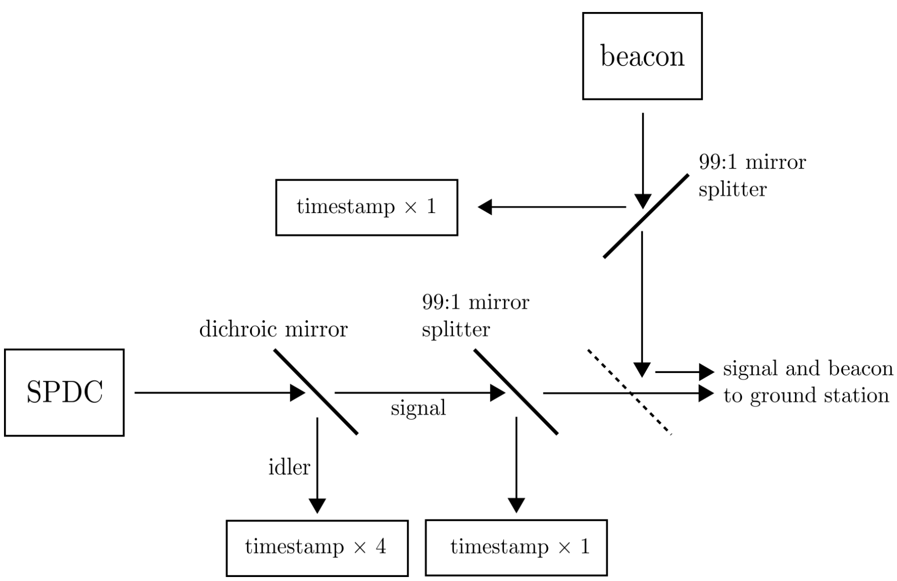
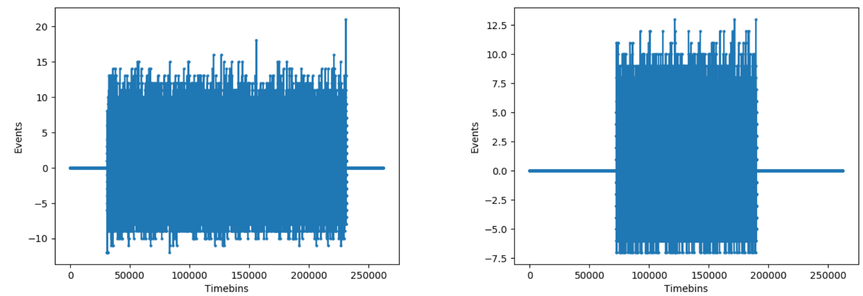


Figure 6: The time-stamping units and beacon generator use the same clock reference.

2.1 Experimental setup

`TODO: describe atomic clock tabletop demo setup`



(a) (b)

Figure 7: Timebins of 10000ns on (a) Alice tabletop atomic clock, (b) Bob tabletop atomic clock

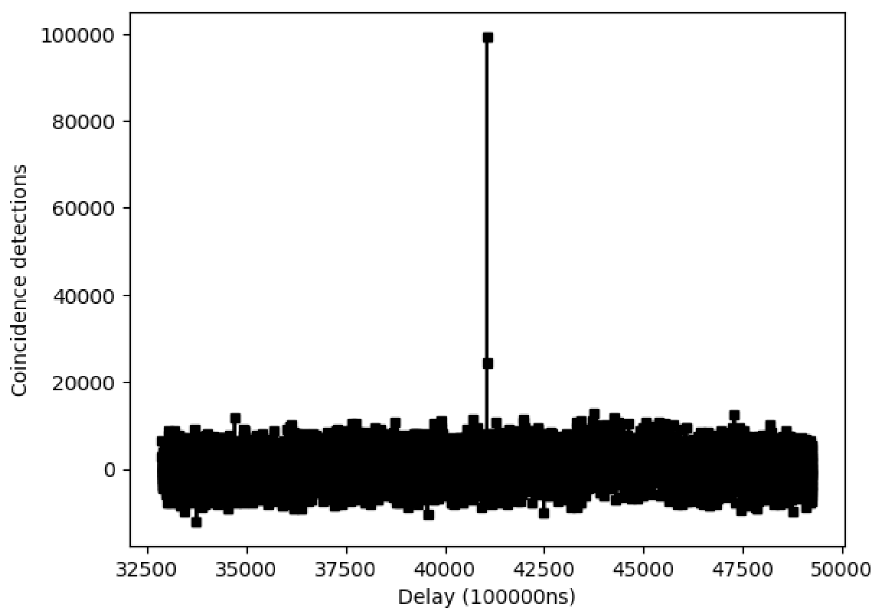
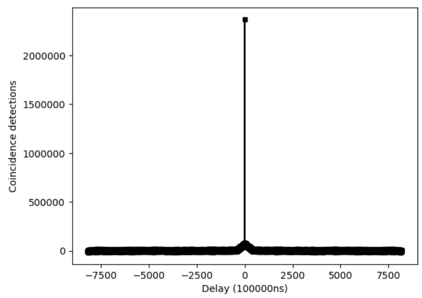


Figure 8: Cross-correlation on Fig 7.

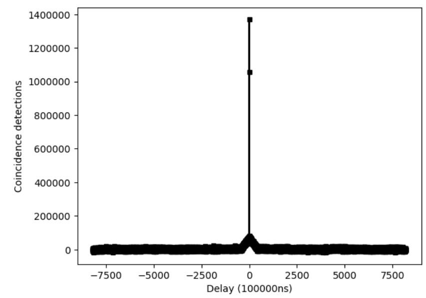
2.2 Cross-correlation using FFT

The second-order correlation function $g^{(2)}(\tau)$ is the intensity analogue of the first-order correlation function $g^{(1)}(\tau)$ that determines the visibility of interference fringes. $g^{(1)}(\tau)$ quantifies the way in which the electric field fluctuates in time, whereas $g^{(2)}(\tau)$ quantifies the intensity fluctuations.

3 Correction with entanglement



(a)



(b)

Figure 9: Autocorrelation of signal with (a) no shift (b) propagation delay

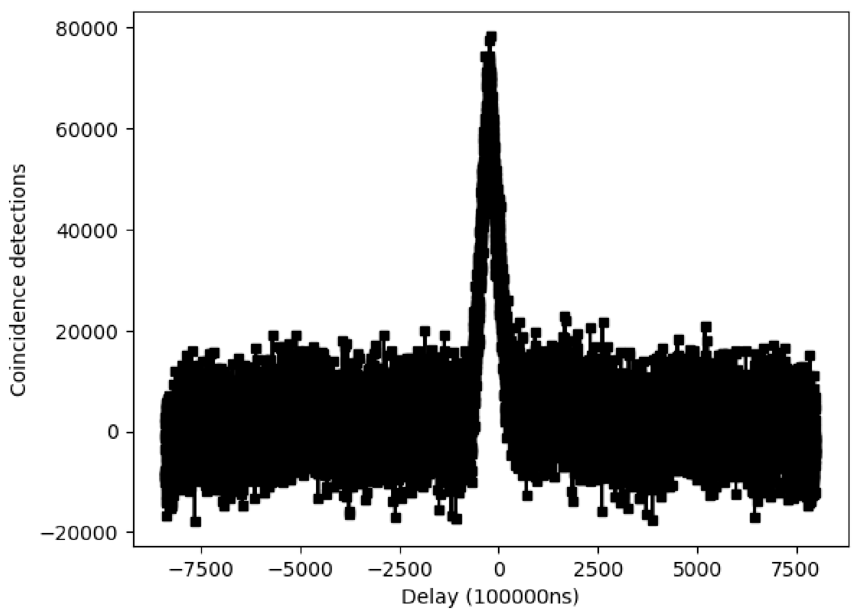


Figure 10: Autocorrelation of signal with delay and Doppler shift

3.1 Ambiguity function

The two-dimensional ambiguity function is commonly used in radar signal analysis as the most complete statement of the waveform's inherent performance. It reveals the range-Doppler position of ambiguous responses and defines the range and Doppler resolution. It is defined as

$$A(\nu,\tau) \equiv \int_{-\infty}^{\infty} \tilde{s}\big(t+\frac{\tau}{2}\big)\tilde{s}^{*}\big(t-\frac{\tau}{2}\big)e^{2i\pi\nu t}dt \tag{9}$$

3.2 Beat function

In this case, a radio signal transmitted at a nominal frequency off0from a satellite in motion will appear at the receiver with aslightly different frequency offR. This is significant because the amount of shift from thenominal frequency is mathematically related to the relative change in position of the satelliteover time.There are two methods by which GPS receivers typically measure this effect. The firstand most common method arises simply from the way in which a GPS receiver maintainsits satellite locks. It has been shown that satellite locks are achieved by maximizing anautocorrelation function between the received PRN code on the L1 signal and the samelocally generated code shifted through time. It is apparent that the autocorrelation functionmust be maximized by aligning the received and generated code sequences over a significantamount of time (at least one full code cycle). This is only possible if the code being generatedhas the same frequency as the code being received.When a receiver is carrying out its search-and-acquire procedure for the various PRNcodes, it must try to align the codes not only in time, but also in frequency. As such,most receivers will scale the generated code according to a set of frequency bins centeredaround the nominal L1 frequency [42]. Upon finding a frequency value (and alignment) thatresults in a successful autocorrelation, the frequency can then be fine-tuned to maximize theautocorrelation result. The exact frequency that does so should be equal to the frequencyof the received signal. As such, the instantaneous Doppler shift can be found by simplysubtracting the nominal frequency from the received frequency:fD=fRf0.14

It should be noted that the satellites and receivers are not moving relative to one anotherin a constant fashion; therefore, the Doppler shifts change over time. The receiver is con-stantly tracking this shift to maintain its satellite lock, with the result that every Dopplerobservation is aninstantaneousobservation that changes with each consecutive epoch.The second method by which a receiver might calculate the Doppler shift to a satelliteis by determining thebeat fre-quencyresulting from mixing the received GPS signal with thelocally-generated one [5]. When two sine waves with slightly different frequencies are mixedto-gether, a wave is produced with two frequency components equal to the sum and differenceof the original waves frequencies. Knowing thatfd(t)dt, this can be written as:S1(t)S2(t) =A1sin21(t)A2sin22(t)=A1A22[cos2(2(t)1(t))cos2(2(t) +1(t))](3)By passing this resulting wave through a low-pass filter to remove the high-frequency com-ponent, a pure sine wave containing only the beat frequency (i.e. the Doppler shift) willremain:SB(t) =Bsin2B(t)(4)whereBis the amplitude of the resulting beat signal andB(t) is the resulting phasedifference which changes as a function of time (a.k.a. the beat frequency). Once the Dopplershift is known, it can be used to determine user velocity, signal anomalies, or even userposition with the aid of additional observables

`TODO: fill in machine learning method`

1. find middle part with no shift
2. try to shift middle part and find differential equation (order 2)

References

[1] Caleb Ho, Antia Lamas-Linares, and Christian Kurtsiefer. Clock synchronization by remote detection of correlated photon pairs. *New Journal of Physics*, 11(4):045011, 2009.