A Cooperative Positioning Enhancement Method Based on Doppler Effect for Vehicular Networks with GPS Availability

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***Abstract*— The Global Positioning System has already provided the positioning information for a variety of applications in vehicular navigation systems. However, the limited accuracy of pseudorange-based GPS for civilian usage makes it unsuitable for many safety applications. Cooperative positioning in vehicular networks is a relatively new concept for enhancing positioning in a group of vehicles which can communicate with each other. Sharing each node's data in the network of nodes and measuring distance between nodes are the main aspects of almost all cooperative positioning algorithms. However, distance measurement is very challenging and problematic for mobile networks and especially for the harsh environment of vehicular networks. The most common techniques of radio ranging such as Received Signal Strength (RSS), Time of Arrival (TOA), and Time Difference of Arrival (TDOA) are often considered in cooperative positioning algorithms without considering their essential shortcomings and limitations in vehicular networks. In this article, a new method for cooperative positioning is presented that is not based on the inter-node distance estimation. This technique uses the Doppler effect. In this new method, not only is the requirement for accurate distance estimation eliminated but an improvement of over 50% in accuracy is achieved. Unlike other methods, the relative mobility of the nodes is an advantage, making it a more suitable solution for vehicular applications. Unlike most cooperative positioning methods, which use a centralized algorithm, the proposed technique can be distributed in each node, which is also more feasible in vehicular networks.**

***Index Terms*— Cooperative Positioning, Doppler Effect, DSRC, Vehicular Network**

I. INTRODUCTION

Position data is an important and fundamental basis of many systems in civilian, military, and industry applications. Vehicular applications are among the most demanding systems for accurate position information. Nowadays, Global Navigation Satellite Systems (GNSS) such as GPS [1] are used for localization. While car navigation systems are the most comprehensive vehicular application of GPS, other emerging systems in vehicular networks may not be able to use GPS data. The limited accuracy of GPS signals and its availability do not meet the requirements of safety-related applications such as collision avoidance. Although some augmentation systems such as Differential GPS (DGPS) [1] can be used to improve performance, the cost and infrastructure requirements prevent the comprehensive use of such systems. Some innovative approaches have been presented in recent years for positioning accuracy enhancement within vehicular networks, based on communicating data among nodes of the network. This concept, called “Cooperative Positioning”, is not necessarily limited to vehicular applications and has been considered for many other sensor networks with or without mobility. An overview of cooperative localization techniques in wireless sensor networks is carried out in [2]. This comprehensive study covers dynamic large scale networks and time varying topologies. Distance-based techniques are introduced such as RSS and TOA but the details and limitations of these techniques are not discussed. The other aspect of this work is consideration of centralized and distributed algorithms.

One of the important factors in cooperative positioning is the communication between nodes of the network for sharing data. In 1999, addressing the safety issues and other potential applications in vehicular environments, the U.S. Federal Communication Commission [3] assigned a bandwidth of 75MHz from 5.85GHz to 5.925GHz for Dedicated Short Range Communication (DSRC), specifically for reliable, fast, and short range communication between vehicles or a vehicle and roadside infrastructure [3], [4]. This also provided a valuable opportunity for development of innovative ideas in cooperative positioning. Although there is a variety of techniques and methods proposed in the literature relating to cooperative positioning in vehicular networks, most are not compatible with DSRC specifications.

An important parameter in cooperative positioning is the distance between the nodes of the network. RSS, TOA, and TDOA are the most common techniques considered for radio ranging [5] in cooperative positioning algorithms. An omission in some related literature is the study of shortcomings and limitations of each method, especially when considering vehicular networks. Achieving the ranging accuracy required or assumed for cooperative positioning algorithms is mostly impossible using existing techniques. For example, RSS, a simple and cost effective method of ranging, is often assumed as the distance estimation technique in cooperative localization algorithms, while the accuracy of this vulnerable method is not sufficient for most applications [6]. An overview of challenges in radio ranging with DSRC is discussed in “to be published” [7].

Cooperative positioning has been considered for both localization and positioning accuracy improvement. In some methods such as those presented in [8],[9] position data of the nodes are available and accuracy improvement is achieved with cooperative positioning. The methods presented in [10], [11] use some GPS-equipped nodes for localizing the other nodes. In these works, some important issues such as specific ranging methods and their impacts on the algorithm or computational burden of centralized algorithms are not examined.

Although RSS is a poor ranging technique, especially in vehicular environment, it has been considered as the ranging method for the proposed algorithms of cooperative positioning in [12], [13] without considering the RSS shortcomings and their effect on the performance of the algorithms. In the methods and algorithms discussed, a variety of techniques are presented for cooperative positioning based on data fusion and optimization. Besides vehicular networks, cooperative positioning is presented for other mobile network applications such as robotic networks [14]-[18]. Although these works are not focused on vehicular environment, their methods may support vehicular applications.

The common fundamental parameter used in cooperative positioning in vehicular or other applications is distance between the nodes. Euclidean or hop distance is an essential input for the algorithms. Estimation of the distance between the nodes is challenging, especially in a vehicular environment and achieving the accuracy of ranging assumed in most of the algorithms is not actually possible.

In this article, considering the fundamental limitations of distance measurement with radio ranging, a new method is developed for cooperative positioning not using the distances between the nodes but the Doppler offset frequencies between a target node and its neighbours. It is noteworthy that frequency offset is measured in the physical layer of DSRC [19], so the Doppler effect on frequency can be estimated. Having GPS data at each node and communication between the nodes are two general assumptions in the new technique. The result of this work is a positioning accuracy improvement of more than 50% compared to what GPS alone can provide.

Section II defines the problem and explains the Doppler Effect on data content. In section III, the cooperative positioning algorithm using Doppler Effect is explained. Section IV is dedicated to simulation results analysis and discussion. In Section V, a comparison is made between the performance of the proposed algorithm and distance-based cooperative positioning algorithms. Section VI concludes the contribution of this work and outlines some further areas of related research.

II. PROBLEM DEFINITION AND ASSUMPTIONS

A. Problem Definition

Assume there are $n+1$ cars. Among these vehicles, one car is considered as the target vehicle and the other n cars, moving in approaching lanes, are neighbours. In section II.*C*, it is shown that relative speed for the vehicles travelling in the same direction does not result in effective Doppler shift. All of the vehicles are equipped with a GPS receiver and announce their GPS position and odometer-based velocity to the others through DSRC. Also assume that in the target vehicle, the GPS data of n neighbours is available through DSRC and the frequency offset of each signal from the neighbours can be measured in the target node. This frequency offset is the result of clock drifts in the receiver and transmitters and Doppler shift due to relative motion. The problem is finding an algorithm to estimate target node position with an accuracy better than that provided by GPS based on the estimated Doppler shift in each signal between the target node and its neighbours. Fig. 1 shows a schematic of the situation in 2D space. The presented method can be easily extended to a 3D situation. Before continuing to the solution, the available data from the Doppler Effect and its characteristics is discussed.

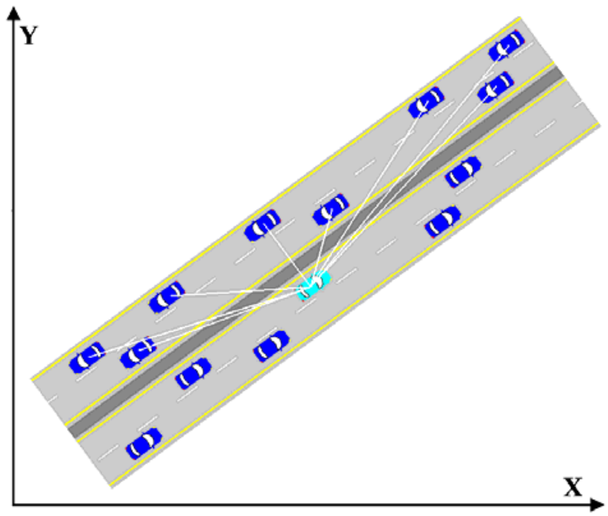

Fig. 1. Target and neighbour nodes in a four-lane two-way street

B. Available Data from Doppler Effect

Due to the Doppler effect, the frequency of the received signal differs from the transmitted signal frequency if there is relative movement between the receiver and transmitter. This difference depends on the relative speed and direction of movement. Consider two moving beacons B_1 and B_2 in random directions with speeds of v_1 and v_2 in time t depicted in Fig. 2.

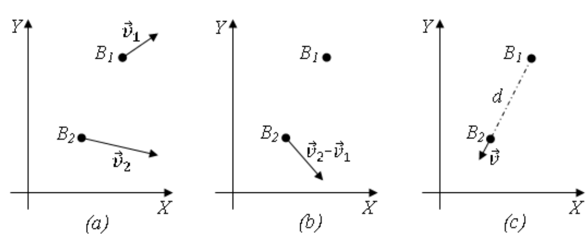

Fig. 2. (a) two moving beacons (b) relative velocity (c) velocity along the connecting line

In Fig. 2, (a) shows the two moving beacons. Equivalently, (b) shows B_2 moving with the relative velocity and B_1 is not moving and (c) shows the relative velocity of the two beacons along the line connecting the beacons at time t i.e. the rate of change of the distance vector connecting the two points. If B_2 is considered to be the transmitter and B_1 is the receiver, the Doppler shift Δf Hz in B_1 can be modeled as:

$$\frac{\Delta f}{f} = \frac{-v}{c} \qquad (1)$$

where Δf is Doppler shift, f is the carrier frequency, v is the relative speed along the connecting line, and c is the speed of light. From Eq. (1) it can be seen that if Doppler shift is measured, the relative speed v can be estimated. But the important point is that in the real world, the frequency offset in the receiver is not solely caused by the Doppler effect. Receiver and transmitter clock drifts are two sources of disturbance that should be considered in Eq. (1) and the effect of this disturbance on the accuracy of speed estimation must be found. Assume that the clock frequency drifts in the transmitter and receiver are γ and ε respectively. From Eq. (1) we have:

$$\frac{\Delta f}{f+\gamma} = \frac{-v}{c} \qquad (2)$$

and estimated velocity in the receiver is:

$$\hat{v} = \frac{-c(\Delta f + \varepsilon)}{f} \qquad (3)$$

and the error of velocity estimation can be calculated using Eq. (2) and Eq. (3):

$$\hat{v} - v = -\frac{c\varepsilon}{f} - \frac{c\gamma\Delta f}{f(f+\gamma)} \approx -\frac{c\varepsilon}{f} \qquad (4)$$

In Eq. (4), the effect of the transmitter clock drift on the error of estimated velocity is eliminated due to the magnitude of the DSRC carrier frequency, 5.9 *GHz*. However, the effect of the receiver clock drift is considerable. Considering a frequency drift of 1 ppm commercial crystals we assume ε varies between ±6 *KHz*, which results in velocity estimation error of about 300 *m/s*. From this, it is obvious that Doppler frequency shift must be estimated based on the measured frequency offset. Referring to Eq. (1), one can calculate Doppler shift if v is known. If at the beginning of communication, the distance between the target node and each neighbour is more than 100 *m*, due to the geometry of the road and lane width, the relative speed along the connecting line of the target node and its neighbors is approximately the difference of the odometer-based speeds which are communicated through DSRC. Regarding this, the Doppler shift can be estimated with Eq. (1) and assuming constant clock drifts during the link life of communication, ε can be estimated by subtracting the estimated Doppler shift from the measured frequency offset.

Referring to frequency offset estimation error [20]-[23], a Gaussian error with zero mean and standard deviation of 100*Hz* in DSRC carrier frequency is assumed. A Kalman filter is used for estimating both clock drifts and Doppler shift. Fig. 3 shows a sample simulation result of estimated Doppler shift by a Kalman filter based on estimated frequency offset.

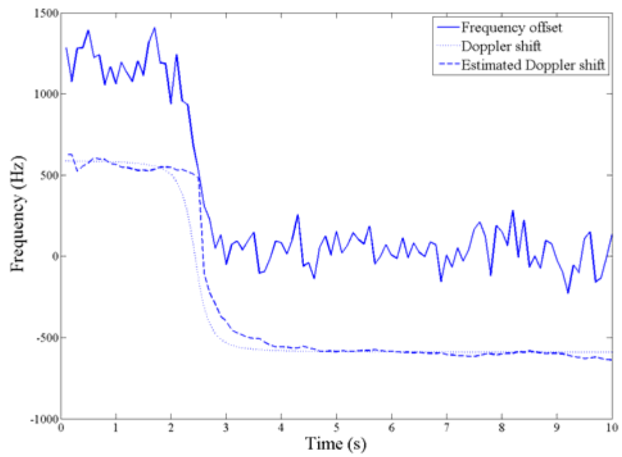

Fig. 3. Estimation of Doppler shift and clock drift effect removal

If Doppler shift is estimated at intervals of *T sec.*, and *T* is small enough with regard to acceleration of beacons there is:

$$d(k+1) \cong d(k) - \frac{c\Delta f}{f} T \qquad (5)$$

Equation (5) shows that distance variation between the beacons can be tracked by Doppler shift but the problem of distance estimation is not possible due to ambiguity of the initial distance between the nodes when the Doppler offset estimation begins. In a vehicular environment, simulations show that for *T*< 1 *sec*, the distance rate can be tracked using Doppler shift, while the relative speed between the target node and its neighbours is above 10 *m/s*.

C. *Feasibility of the Approach*

Regarding the issues presented in section II.*B* and the necessity for availability of a number of neighbour vehicles for cooperative positioning, it is important to consider the feasibility of Doppler-based cooperative positioning in a vehicular environment. For this, the logged speed data of the passing vehicles on the I-94 highway in Minnesota, USA, on June 4, 2009 have been considered. These data were gathered by speed cameras observing 4 lanes, 2 lanes in each direction, at Site#25 on the I-94 and provided by Minnesota Department of Transportation. Analyzing the data, the mean of the relative speeds between the vehicles in approaching lanes were estimated. Fig. 4 shows the results over 24 hours.

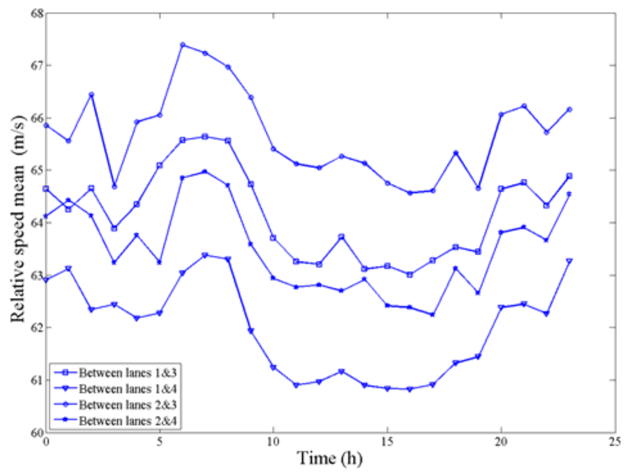

Fig. 4. The mean of relative speeds between the vehicles in approaching lanes over 24 hours

As can be seen, the mean of relative speeds is more than 60 *m/s*, which is equivalent to about 1200Hz Doppler shift for the 5.9 *GHz* DSRC carrier frequency. Regarding these data, the mean of the relative speed between the vehicles travelling in the same direction is less than 5 *m/s* which is equivalent to about 100 *Hz* in DSRC carrier frequency. Assuming 100 *Hz* as the error of frequency offset estimation, obviously, Doppler shift cannot be used for cooperative positioning with the neighbors traveling in the same direction with target vehicle.

The other important factor derived from the speed data is the average number of neighbors in approaching lanes. Considering a segment of the road around the target vehicle within the DSRC range, say ±500 *m*, Fig. 5 shows the average number of the neighbours over 24 hours. As can be seen, the maximum number of neighbours is less than 25. This constraint on the number of neighbours is very important for cooperative positioning algorithms and is mostly absent from the literature.

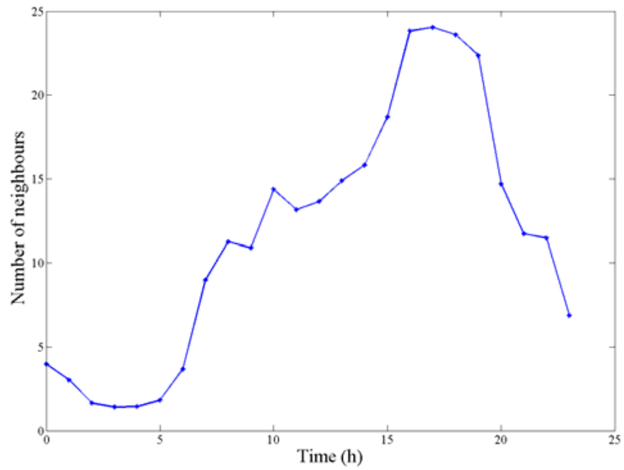

Fig. 5. The mean of the number of neighbours over 24 hours

III. Cooperative Positioning Algorithm

The algorithm being proposed to estimate position consists of three major steps. Fig. 6 shows the solution diagram. In the first step the distances between the target node and neighbours are estimated based on Doppler shift. Of course the estimated distances have ambiguities due to the unknown distances at the beginning of the algorithm.

Although GPS data can provide a rough estimate of the distances, it is not enough to eliminate the ambiguity over the limited lifetime of the vehicular communication links. The ambiguity of the estimated distance makes it impossible to use directly in the positioning algorithm but the rate of distance change will be useful. In the second step, the change rate of the estimated ambiguous distances and GPS data are used to estimate the target vehicle position more accurately than for GPS alone. The third step consists of fusing the results of the second step with GPS data of the target.

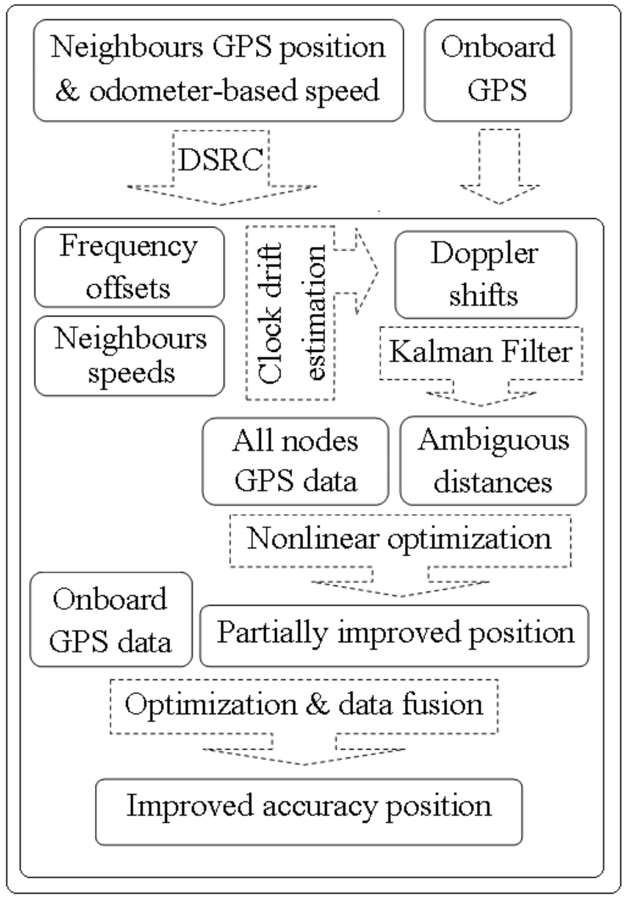

Fig. 6. Overall block diagram of proposed solution

A. Ambiguous Distance Estimation (step 1)

The first step of our cooperative positioning technique is the estimation of the ambiguous distances between the target node and its neighbours. The following state space and measurement models are defined:

System:

$$\begin{cases} d_i(t+1) = d_i(t) + v_{Di}(t) + \xi_{di}(t) \\ v_{Di}(t+1) = v_{Di}(t) + \xi_{vi}(t) \end{cases} \qquad (6)$$

Measurement (Doppler shift):

$$Z_i(t) = \frac{-f}{c} v_{Di}(t) + \xi_i(t) \qquad (7)$$

where t is the discrete time interval, d_i is the distance between the target vehicle and neighbour vehicle i ($i=1,..,n$), n is the number of neighbour vehicles, v_{Di} is the relative speed between the target vehicle and neighbour vehicle i along the line connecting them , f is the carrier frequency, c is speed of light, ξ_{di} and ξ_{vi}

are system noise, and ξ_i is measurement noise. Referring to section *II.B,* the measurement noise is considered a zero mean Gaussian random variable with 100*Hz* standard deviation, and the effect of clock drifts is considered a sample of Gaussian random variable with zero mean and 6 *kHz* standard deviation. Now using a Kalman filter, the clock drifts, the Doppler shift, and the ambiguous distance between target node and neighbours can be estimated. Fig. 7 is a sample of the estimated distance between the target vehicle and one of the neighbours over 100 time intervals. GPS data is used for forming the initial state of the system in Kalman filtering.

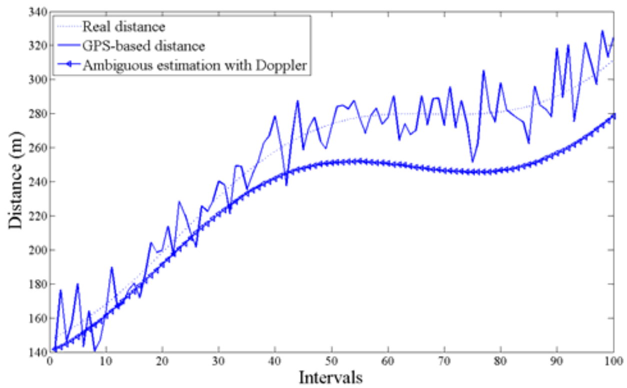

Fig. 7. Estimation of distance with ambiguity using Kalman filter

As can be seen, using Doppler and a Kalman filter results in a distance estimation which is much less noisy than what GPS provides but with an ambiguity over time. Although GPS position data can be used for removing the ambiguity to some extent, here the ambiguous distances are used in the cooperative positioning algorithm. The outcome of Kalman filtering provides the following vector:

$$\psi = [\tilde{d}_i(t) \quad \tilde{v}_{Di}(t)]^T \tag{8}$$

where $\tilde{d}_i(t)$ is the estimated ambiguous distance in *m* and $\tilde{v}_{Di}(t)$ is the estimated relative2 speed between the target node and the neighbour node *i* along the connecting line between them in *m/s.* Now having ψ from Eq. (8), the position of the target will be estimated using an optimization algorithm.

B. Target position estimation (steps 2 &3)

In this part of the algorithm the following cost function is defined at time *t*:

$$S(\omega) = \frac{1}{2}\beta(\omega)^T \beta(\omega) = \frac{1}{2}\sum_{i=1}^{n} \beta_i^2(\omega) \qquad (9)$$

$$\beta(\omega) = [\beta_1(\omega), \ldots, \beta_n(\omega)]^T \qquad (10)$$

$$\beta_i(\omega) = \tilde{v}_{Di}(t-1)T - d_i(t) + \tilde{d}_i(t-1) \qquad (11)$$

and $\omega = [x(t) \;\; y(t)]^T$ is the position of the vehicle to be estimated. In Eq. (11), $\tilde{v}_{Di}$ and $\tilde{d}_i$ are provided from Eq. (8) and:

$$d_i(t) = \sqrt{(x(t) - \breve{x}_i(t))^2 + (y(t) - \breve{y}_i(t))^2} \qquad (12)$$

In Eq. (12), $\breve{x}_i$ and $\breve{y}_i$ are the GPS position solution of neighbour node i in an arbitrary XY reference plane. Also x and y are coordinates of the target vehicle which must be estimated in that reference plane. Now the target position can be estimated by minimizing the cost function (9):

$$[\tilde{x}(t) \;\; \tilde{y}(t)]^T = \arg\min\{S(\omega)\} \qquad (13)$$

This optimization can be done by nonlinear Least Squares [24]. Here, the Gauss-Newton method was employed for optimization. Once the optimization is complete, the derived coordinates, $\tilde{x}$ and $\tilde{y}$, are still not entirely satisfactory. For more explanation refer to Fig. 8.

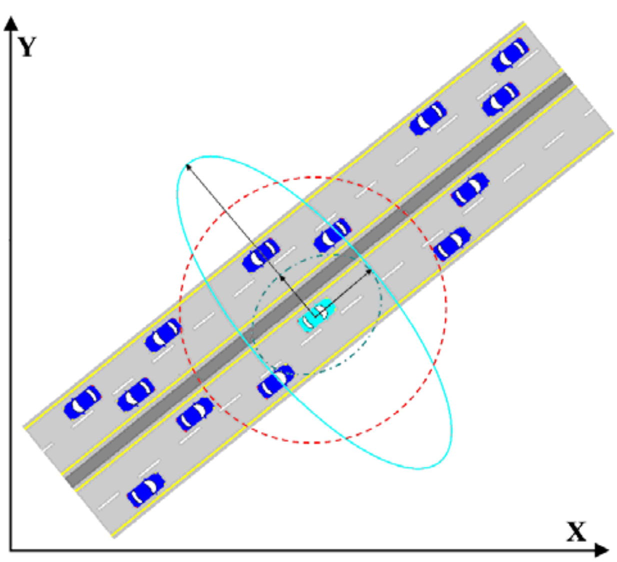

Fig. 8. Directional accuracy enhancement due to street constraint

As can be seen in this figure, because of the topology of the network, due to the cars being constrained to be on the street, the effectiveness of positioning using Doppler data is mostly along the street. In other words, there is not much information from Doppler in the direction orthogonal to target vehicle travel. Therefore, it is expected that the result of Eq. (13) has an improvement in accuracy only along the direction approximately parallel to the street. In Fig. 8, the dashed circle shows the error of GPS. The ellipse shows the error of estimated coordinates resulting from Eq. (13). As can be seen, the accuracy has got better in the along street direction. For many applications in vehicular networks such as detecting the lane of travel or whether the vehicle has left the road, the positioning accuracy cross to the street is very important. For achieving the accuracy near to the circle enclosed in the ellipsoid, averaging the GPS data over time in the direction orthogonal to the street can be useful due to the fact that cars have less movement orthogonal to the direction of the road.

Assume $\omega_0 = [x_0 \; y_0]^T$ is the real position of the target vehicle at time *t*. Linearization of cost function *S* defined by Eq. (9) around this point results in:

$$\begin{aligned} S(\omega) \approx \frac{1}{2}\beta(\omega_0)^T \beta(\omega_0) + (J^T \beta(\omega_0))^T (\omega - \omega_0) \\ + \frac{1}{2}(\omega - \omega_0)^T J^T J (\omega - \omega_0) \end{aligned} \tag{14}$$

where J is the Jacobian matrix of β in ω_0. Now using Eq. (14) for:

$$\frac{\partial S(\omega)}{\omega} = 0 \tag{15}$$

results in:

$$\omega = \omega_0 - (J^T J)^{-1} J^T \beta(\omega_0) \tag{16}$$

The error covariance of ω can be calculated as:

$$\mathrm{P} = E[(\omega - \omega_0)(\omega - \omega_0)^T] \tag{17}$$

Replacing Eq. (16) in Eq. (17) and considering the independence of $\beta(\omega_0)$ entries results in:

$$\mathrm{P} = (J^T J)^{-1} I_{n \times n} \sigma_\beta^2 \tag{18}$$

where σ_β^2 is the variance of β_i and $I_{n\times n}$ is an identity matrix. Now, regarding Fig.7, it is expected that the semi-minor axis of the estimation error ellipse corresponds to the smaller Eigenvalue of P and the semi-major axis corresponds to the larger Eigenvalue of P . From this point, the initial estimation of Eq. (13) is divided into components parallel and orthogonal to the error ellipse semi-minor axis. For this, $\tilde{\theta}$ is calculated as follows:

$$\tilde{\theta} = a\tan\left(\frac{\varphi_y(t)}{\varphi_x(t)}\right) \qquad (19)$$

where:

$$\Phi(t) = [\varphi_x(t) \quad \varphi_y(t)]^T \qquad (20)$$

and Φ is the Eigenvector corresponding to the smaller Eigenvalue of P. Rotating the initial estimate with $-\tilde{\theta}$ will results in:

$$\begin{bmatrix} \tilde{x}_r(t) \\ \tilde{y}_r(t) \end{bmatrix} = \begin{bmatrix} \tilde{x}(t)\cos(\tilde{\theta}) + \tilde{y}(t)\sin(\tilde{\theta}) \\ -\tilde{x}(t)\sin(\tilde{\theta}) + \tilde{y}(t)\cos(\tilde{\theta}) \end{bmatrix} \qquad (21)$$

Now, the orthogonal section of estimation, $\tilde{y}_r$, is replaced with the averaged orthogonal component of rotated GPS data, $\tilde{y}_{ra}$:

$$\tilde{y}_{ra}(t) = \frac{1}{m}\sum_{j=0}^{m}[-\breve{x}(t-j)\sin(\tilde{\theta}) + \breve{y}(t-j)\cos(\tilde{\theta})] \qquad (22)$$

In Eq. (22), m is the time window of averaging. Replacing Eq. (22) in Eq. (21) and rotating with $\tilde{\theta}$ results in the final estimation of target vehicle:

$$\begin{bmatrix} \hat{x}(t) \\ \hat{y}(t) \end{bmatrix} = \begin{bmatrix} \tilde{x}_r(t)\cos(\tilde{\theta}) - \tilde{y}_{ra}(t)\sin(\tilde{\theta}) \\ \tilde{x}_r(t)\sin(\tilde{\theta}) + \tilde{y}_{ra}(t)\cos(\tilde{\theta}) \end{bmatrix} \qquad (23)$$

Regarding Eq. (23), the estimate of target vehicle position based on GPS data of the nodes and Doppler shifts is complete.

IV. Simulations and Results

Two parameters are set for all simulations. One is sampling time T=0.2 s and σ_p^2=100 m^2 is the GPS error variance along the X and Y axes. The carrier frequency is set to 5.9GHz in the DSRC band and speed of light c=3×10^8 m/s. Referring to section *II.B*, the frequency offset resulting from transmitter and receiver clock drifts is considered a sample from a zero mean Gaussian random variable with standard deviation of 6 kHz. Also a zero mean Gaussian random variable with standard deviation of 100 Hz is considered for frequency measurement noise. For assigning the averaging window size in Eq. (22), m, a simulation was run to evaluate the effect of this parameter on estimation error. Assuming n=10, varying m between 1 and 20, and repeating the position estimation algorithm for 10000 times, Fig. 9 shows the effect of m on the trace of estimation error covariance. As can be seen, a value of m between 7 and 14 results in a better performance. Regarding this, m=10 is considered for the rest of simulations.

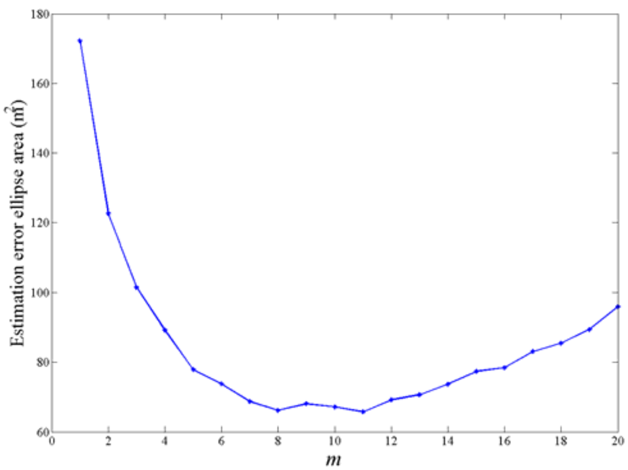

Fig. 9. Effect of averaging window size on estimation error

A. A Single Trial Simulation

For the first simulation, it is assumed there are 10 neighbour nodes, the street has a random slope in the XY plane which is the tangent of the angle between the street direction and the X axis. The simulation is run for 100 time intervals equivalent to 20 s and variable speeds are set as follows:

Target vehicle: $v = 15 + 5\sin(\frac{2\pi t}{100}) \quad m/s$ (24)

$$\textit{Neighbours speed: } v = a_1 + a_2 \sin(\frac{a_3 \pi t}{100}) \quad m/s \quad (25)$$

For each neighbour node, a_1, a_2, and a_3 are samples of random variables with uniform distribution over 10-30, 0-10, and 1-2 respectively. Fig. 10 shows the result of cooperative positioning. As can be seen, a considerable enhancement is achieved using cooperative positioning with the RMSE of error reduced by more than 50%. This shows the functionality of the new algorithm.

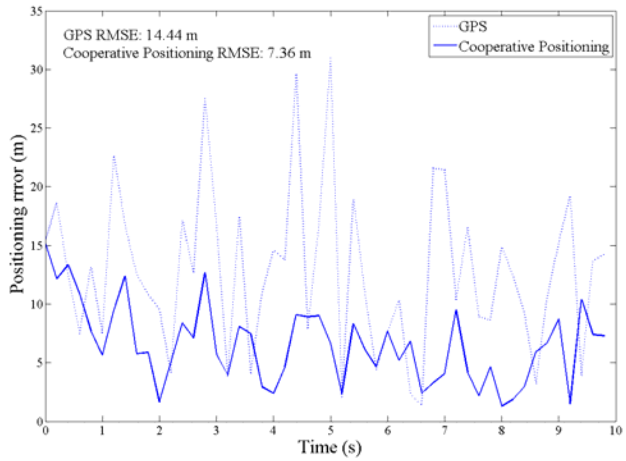

Fig. 10. Cooperative positioning error and GPS error for a single trial

B. *Statistical Performance Evaluation*

In this section the performance of the proposed algorithm is investigated statistically. Also, the effect of the number of neighbour nodes is considered. From Eq. (11) and Eq. (12), one can see that for the case of n=3 neighbours, trilateration can be used for target position estimation provided there is not any noise and uncertainty in the system. In our problem, optimization with n>3 is used for estimation of target position. For this, the number of neighbours is varied between 4 and 15 and for each instance of n, the simulation is repeated 5000 times. The street slope in the *XY* plane is considered constant during the 5000 trials. Each trial simulates the algorithms for 50 intervals and the output of the algorithm at an arbitrary interval, say 25, is considered for comparing with GPS and ideal position data. The speeds are set as in Eq. (24) and Eq. (25) at each trial. Fig. 11 shows the performance of the cooperative positioning algorithm and the effect of different number of neighbour nodes.

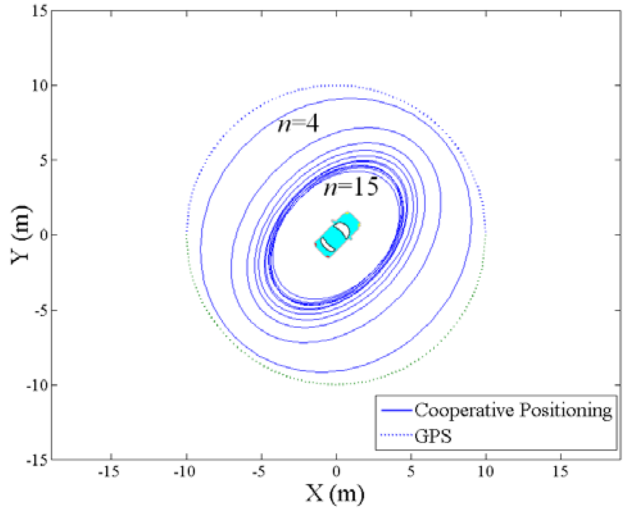

Fig. 11. Positioning enhancement and effect of number of neighbours

As can be seen in Fig. 11, increasing the number of neighbour nodes improves the efficiency of the proposed algorithm. The other point is the saturation of the improvement trend for high numbers of neighbour nodes.

Now for a better sense of the proposed algorithm functionality, the following parameter is defined as the improvement factor in percent.

$$\mu = (1 - \frac{drmsCP}{drmsGPS}) \times 100 \qquad (26)$$

In Eq. (26), *drmsCP* is *distance root mean square* [1] of cooperative positioning algorithm and *drmsGPS* is drms of GPS data. For *drmsCP* the 5000 estimated position of target vehicle in a certain interval t is considered and there is:

$$drmsCP = \sqrt{trace\left(\mathrm{cov}\left(\begin{bmatrix}\hat{x}(t)\\ \hat{y}(t)\end{bmatrix}\right)\right)} \qquad (27)$$

$$drmsGPS = \sqrt{2\sigma_p^2} = 10\sqrt{2} \qquad (28)$$

Fig. 12 shows the improvement factor versus number of neighbour nodes. As can be seen a considerable improvement of about 55% is gained with about 15 neighbour nodes. Although this improves with increasing the number of neighbour nodes, this improvement becomes incrementally smaller with higher numbers of neighbours.

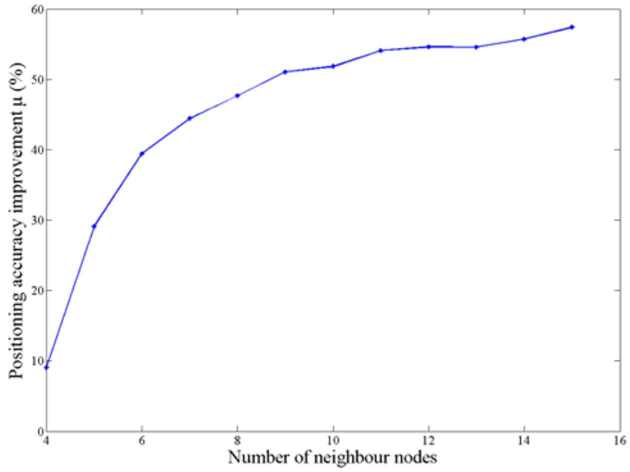

Fig. 12. Positioning accuracy improvement and number of neighbours

V. Evaluation Against Distance-Based Algorithms

In this section the efficiency of the proposed algorithm is analyzed against the performance of cooperative positioning algorithms which are based on distance measurement. For this, the efficiency of the proposed technique will be compared to the Cramer-Rao Lower Bound (CRLB) [25] of the distance based algorithms. In continue, the CRLB is calculated.

For a reasonable comparison, similar situations must be defined. The new algorithm was developed for the situation in which there are *n* neighbour vehicles and one target car. There is communication between the target car and each neighbour vehicle. All of the nodes have GPS data and Doppler shift is measured while the target node is communicating with neighbours. Now, for the distance-based cooperative positioning all of the same assumptions are taken into account except Doppler measurements. Instead of that, the distance between the target vehicle and its neighbours are assumed to be available using some sort of radio ranging during the communication.

With these assumptions, the following parameters are defined:

$$\begin{cases} W = [x_1, y_1, \cdots, x_{n+1}, y_{n+1}]^T \\ Z = [\breve{x}_1, \breve{y}_1, \cdots, \breve{x}_{n+1}, \breve{y}_{n+1}, \breve{r}_1, \cdots, \breve{r}_n]^T \\ \eta = [x_1, y_1, \cdots, x_{n+1}, y_{n+1}, r_1, \cdots, r_n]^T \end{cases} \quad (29)$$

where W is the vector of unknown real positions of the nodes, Z is measurement vector including GPS data and measured ranges, and η is the vector of unknown real positions and real ranges and:

$$r_i = \sqrt{(x_{n+1} - x_i)^2 + (y_{n+1} - y_i)^2} \quad , i = 1, \ldots, n \quad (30)$$

$$\Sigma = \begin{bmatrix} \sigma_p^2 I_{(2n+2)\times(2n+2)} & \mathrm{O} \\ \mathrm{O} & \sigma_r^2 I_{n\times n} \end{bmatrix} \quad (31)$$

where Σ is the covariance matrix of measurements, σ_r^2 is range measurement error variance, and σ_p^2 is GPS error variance along the X and Y axes. The conditional joint probability density function of measurements is:

$$f(Z|W) = \frac{(2\pi)^{-m/2}}{\sqrt{\det(\Sigma)}} \exp\left\{ -\frac{1}{2}(Z-\eta)^T \Sigma^{-1} (Z-\eta) \right\} \quad (32)$$

where $m=2n+3$.

The kl entry of Fisher information matrix of measurements is:

$$F_{kl} = -E\left\{ \frac{\partial^2 \ln(f(Z|W))}{\partial w_k \partial w_l} \right\}, k, l = 1, \ldots, 2n+2 \quad (33)$$

Replacing Eq. (32) in Eq. (33) results in:

$$F_{kk} = \begin{cases} \dfrac{1}{\sigma_p^2} + \dfrac{(x_{n+1} - x_i)^2}{\sigma_r^2 r_i^2} & , k = 2i-1 \\ \dfrac{1}{\sigma_p^2} + \dfrac{(y_{n+1} - y_i)^2}{\sigma_r^2 r_i^2} & , k = 2i \\ \dfrac{1}{\sigma_p^2} + \displaystyle\sum_{i=1}^{n} \dfrac{(x_{n+1} - x_i)^2}{\sigma_r^2 r_i^2} & , k = 2n+1 \\ \dfrac{1}{\sigma_p^2} + \displaystyle\sum_{i=1}^{n} \dfrac{(y_{n+1} - y_i)^2}{\sigma_r^2 r_i^2} & , k = 2n+2 \end{cases} \quad (34)$$

and $F_{kl}=F_{lk}$ are:

$$\begin{cases} -\dfrac{(x_{n+1}-x_i)^2}{\sigma_r^2 r_i^2} & ,k=2n+1,l=2i-1 \\ -\dfrac{(y_{n+1}-y_i)^2}{\sigma_r^2 r_i^2} & ,k=2n+2,l=2i \\ -\dfrac{(x_{n+1}-x_i)(y_{n+1}-y_i)}{\sigma_r^2 r_i^2} & ,k=2n+2,l=2i-1 \\ -\dfrac{(x_{n+1}-x_i)(y_{n+1}-y_i)}{\sigma_r^2 r_i^2} & ,k=2n+1,l=2i \\ \dfrac{(x_{n+1}-x_i)(y_{n+1}-y_i)}{\sigma_r^2 r_i^2} & ,k=2i-1,l=2i \\ \sum_{i=1}^{n}\dfrac{(x_{n+1}-x_i)(y_{n+1}-y_i)}{\sigma_r^2 r_i^2} & ,k=2n+1,l=2n+2 \\ 0 & ,otherwise \end{cases} \quad (35)$$

Having the Fisher information matrix, the CRLB of distance based cooperative positioning is defined as CR_{dbcp}:

$$CR_{dbcp} = F^{-1} \quad (36)$$

Now the same improvement factor as in Eq. (26) is defined for CR_{dbcp}:

$$\mu_{dbcp} = (1-\frac{drmsCR_{dbcp}}{drmsGPS})\times 100 \quad (37)$$

where:

$$drmsCR_{dbcp} = \sqrt{\sum_{i=2n+1}^{2n+2} CR_{dbcp}(i,i)} \quad (38)$$

and drmsGPS is the same as Eq. (28). Fig. 13 shows the improvement factor of the presented algorithm and the best performance of distance based algorithms which is achievable in CRLB. As can be seen in Fig. 13, for more than 5 neighbour nodes, the proposed algorithm is more accurate than distance-based best conditions according to the CRLB. The other important issue, illustrated in this figure, is the ineffectiveness of increased ranging accuracy in positioning enhancement. This is due to the GPS positioning error of the nodes which limits the accuracy improvement even with accurate inter-node ranging. The Doppler-based cooperative positioning technique would confront this problem, but the separation of optimization to along-street and cross-street in our algorithm, has caused the considerable jump over the distance-based performance at CRLB.

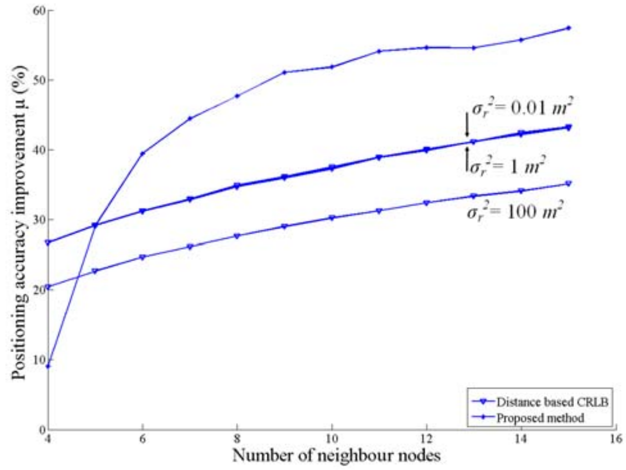

Fig. 13. Efficiency of proposed algorithm and distance based methods

VI. Conclusion

A novel method of cooperative positioning based on the use of the Doppler effect was introduced. It improves positioning enhancement in a network of GPS-equipped vehicles. Mobility of the nodes is a necessity of the new technique. The effect of the transmitter and receiver clock drifts, due to low accuracy crystals, on frequency offset was eliminated. It was shown that the algorithm can improve the accuracy by more than 50% compared to GPS positioning alone. The effect of the number of neighbour nodes was investigated, and between 10 and 15 nodes is shown to be a reasonable number. Achieving performance with a limited number of nodes is very important due to the communication constraints in a vehicular network. Fewer nodes should mean better quality of communication. Logged speed data implies that these numbers of the neighbours are consistent with real situations in vehicular environments. The performance of the proposed algorithm was compared to the CRLB of distance-based cooperative positioning techniques and it was shown to be better. The superior performance of the presented technique occurs even when the distance-based methods have accurate ranging. This is due to separation of optimization algorithm for along-street and cross-street directions. Availability of frequency offset data in the levels above the physical layer of DSRC is necessary for feasibility of implementing the Doppler-based applications. There is a variety of complementary future works with regard to the presented method including the effect of limited DSRC bandwidth, the delay in availability of neighbour nodes data, and accuracy of Doppler shift estimation in DSRC.

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