# Satellite-to-ground coincidence matching under Doppler effects

August 6, 2019

## 1 Introduction

Entanglement-based quantum key distribution (QKD) relies on a steady source of photon pairs. The first step in establishing a key in such a scheme is the assignment of photodetection events to entangled photon pairs. Ho et al. (2009) [1] introduced an algorithm for coincidence matching under a constant time offset ($\Delta T$) and a constant frequency difference ($\Delta u$). However, for satellite-based photon pair sources, an element of Doppler shift causes the time offset and frequency difference between the two time-stamps to vary in time ($\Delta T(t), \Delta u(t)$). The complexity from Doppler shift rates at different angles of elevation can cause this cross correlation to be spread out over thousands of time bins. This note introduces two methods for correcting Doppler effects in satellite-to-ground coincidence matching.

### 1.1 Time stamping a satellite SPDC source

We consider a setup where a spontaneous parametric down conversion (SPDC) source on a satellite is time-stamped by two individual time-stamp cards with a relative clock drift of $165\mu$s/s (Fig 1).

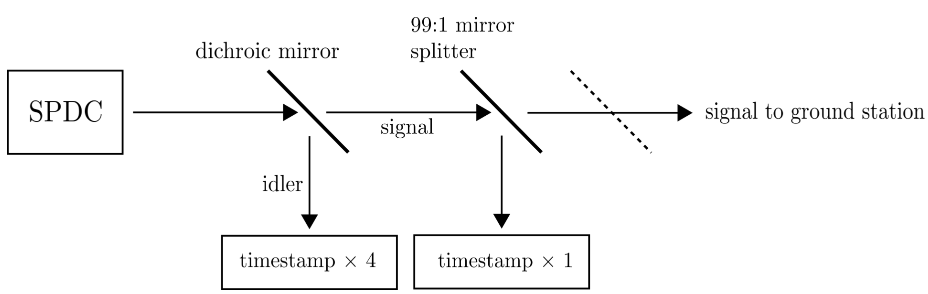
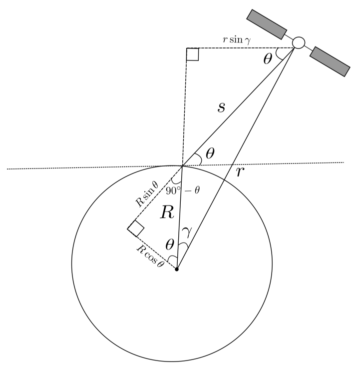



Figure 1: Idlers and 1% of signal photons are time-stamped on the satellite. The rest of the signal photons are transmitted to ground.

## 1.2 First order Doppler shift

We introduce a Doppler shift for each time-stamp based on pair generation time. When the elevation angle is at its maximum 90°, we define that $t$ is 0 seconds. Assuming that the satellite is in a circular orbit at altitude $h = 500$km, with a carrier frequency of $f_c = 1$MHz, a simplified expression for the carrier Doppler shift can be given by:




$$V = \sqrt{\frac{R^2}{r} \cdot g}, \tag{1}$$

$$\gamma = \frac{V \cdot t}{r}, \tag{2}$$

$$\theta = \arccos\left(\frac{r \sin\gamma}{s}\right), \tag{3}$$

$$s = \sqrt{R^2 + r^2 - 2Rr\cos(\gamma)}, \tag{4}$$

$$f_d = f_c\left(\frac{V}{c}\cos\theta\right). \tag{5}$$

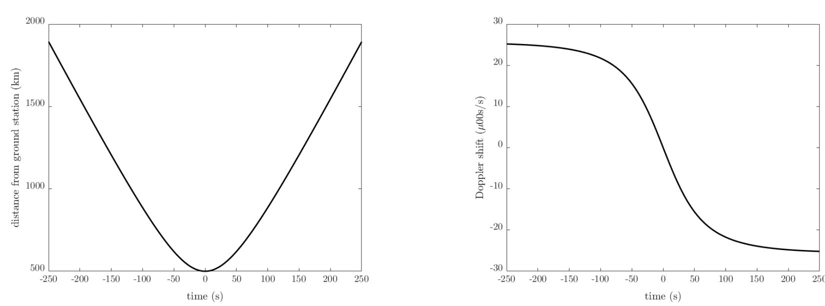
Figure 2: A simplified schematic of the satellite's orbit.

where $R$ is the Earth's radius, $r = R+h$, $g$ is gravitational acceleration, and $c$ is the speed of light. Equation (1) is the velocity of the satellite; (2) is its orbital phase angle $\gamma$; (3) is its angle of elevation $\theta$; (4) is its distance from the ground station $s$; and (5) is the Doppler shift of the carrier signal from the satellite.

Using this model, we specify an absolute time offset of

$$\Delta t = \frac{s(t)}{c}, \tag{6}$$

where distance is calculated for each pair generation time in orbit (Fig 3(a)).




(a) (b)

Figure 3: (a) Distance to ground station, (b) Doppler shift of carrier signal.

## 1.3 Complications from clock drift

The Earth station's clock can have an offset $t_s$ from true UTC and a drift of this offset $\frac{dt_s}{dt}$ with time. This relative frequency difference between the satellite and ground-station clocks is also Doppler shifted (Fig 5). This is a more significant source of error than the first order Doppler shift $\Delta t$ (eq. 6), by almost one order of magnitude (Fig 4(a) and 4(b)).

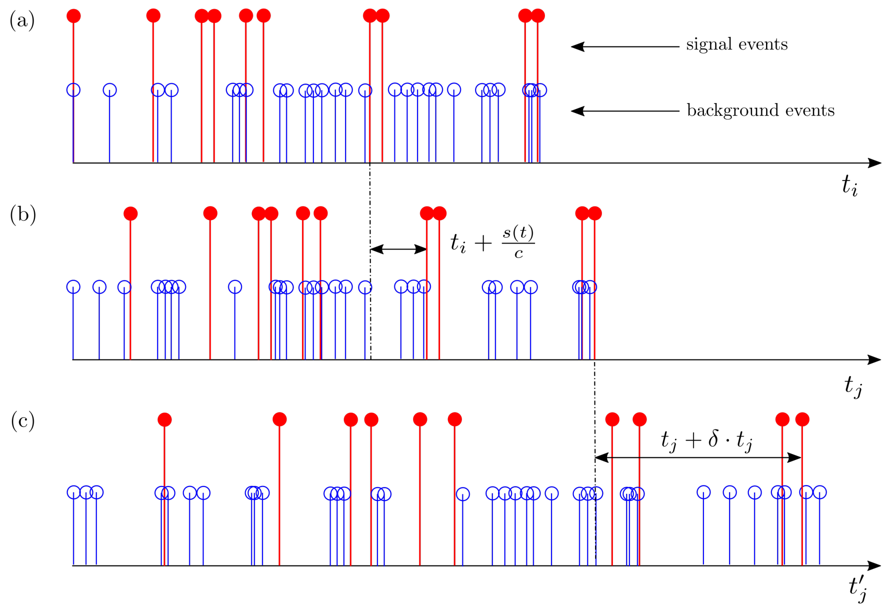



Figure 4: Effect of Doppler shift and Doppler shift rate on photoevent sets. Trace (a) represents the event set $\{t_i\}$ on side A, trace (b) an event set $\{t_j\}$ on side B with a time offset $\frac{s(t)}{c}$ , but the same reference clock frequency. Trace (c) illustrates a set $\{t'_j\}$ with an additional relative frequency difference $\delta$ between both reference clocks. Image adapted from [1].

Since the stream of time stamps $\{t_i\}$ and $\{t_j\}$ on each side has no intrinsic time structure, it is difficult to distinguish a signal event from a background event in the frequency-shifted $\{t'_j\}$ (Fig 4(c)).

The change in the Doppler shift over time is highest at 90° elevation angle and can reach up to nearly 400 ns/s$^2$ (Fig 5). This causes a change in relative clock drift over time. This stretch is of the order of microseconds per second (23 $\mu$s/s for Doppler, 165 $\mu$s/s for clock drifts) and it causes the cross correlation to be spread out thousands of time bins, making it difficult to distinguish a significant peak in the cross correlation.

The former subsections used a simplified model of the Earth's orbit to introduce Doppler shift concepts. Numerically accurate calculations can be done using data from the Python `pyephem` package.

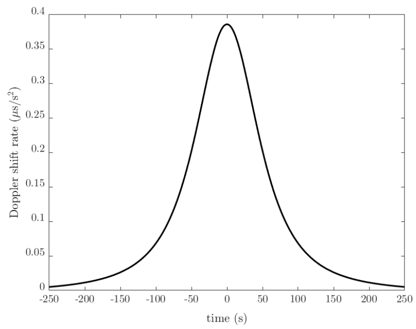



Figure 5: The change in Doppler shift.

To correct for this, we introduce a second order term to our time shift equation:

$$\Delta t = \frac{s(t)}{c} + \delta \cdot t, \tag{7}$$

where the Doppler shift $\delta$ is defined by (cf. eq. 5)

$$\delta = \frac{V}{c}\cos\theta. \tag{8}$$

## 2 Correction using timing beacon on satellite

The timing beacon correction method installs a beacon with regular pulses on the satellite. We time-stamp the beacon signal both on the satellite and at the ground station (Fig 6). If we find the cross correlation between the beacon pulses registered on the satellite timestamp and those registered on the ground station, we can apply the same transformation to the idler and signal photoevents. We find that even a high jitter beacon with poor detection efficiency is sufficient to correct for all clock drifts including the change of the Doppler shift for high elevation angles.

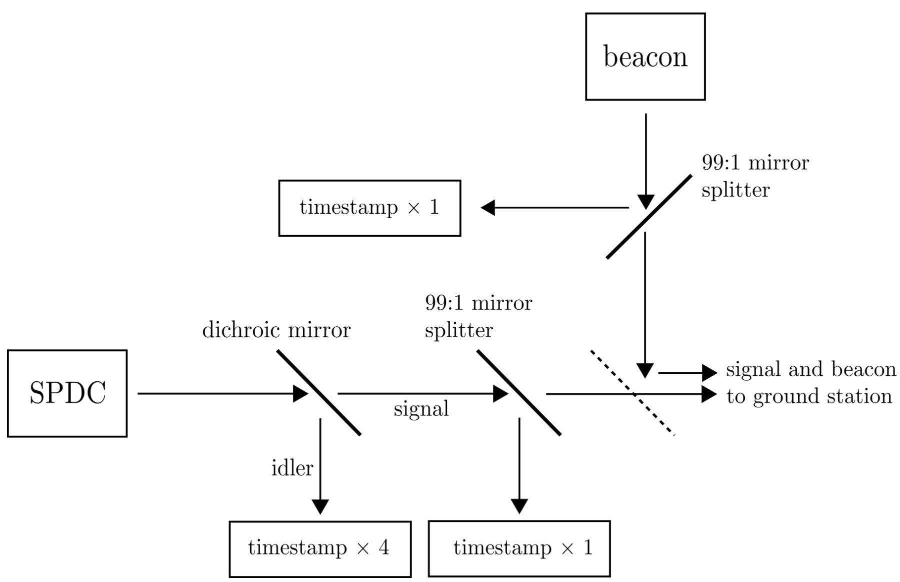
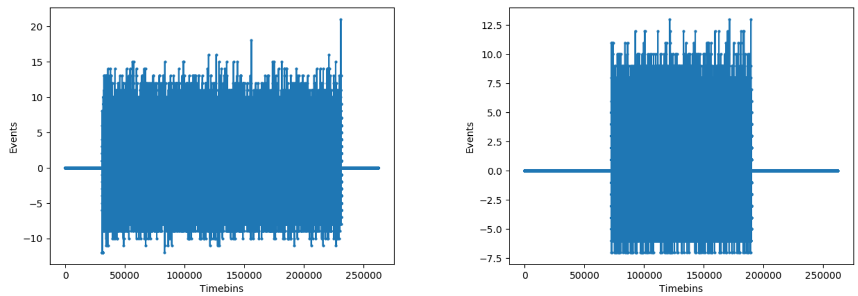



Figure 6: The time-stamping units and beacon generator use the same clock reference.

## 2.1 Experimental setup

`TODO: describe atomic clock tabletop demo setup`




(a) (b)

Figure 7: Timebins of 10000ns on (a) Alice tabletop atomic clock, (b) Bob tabletop atomic clock

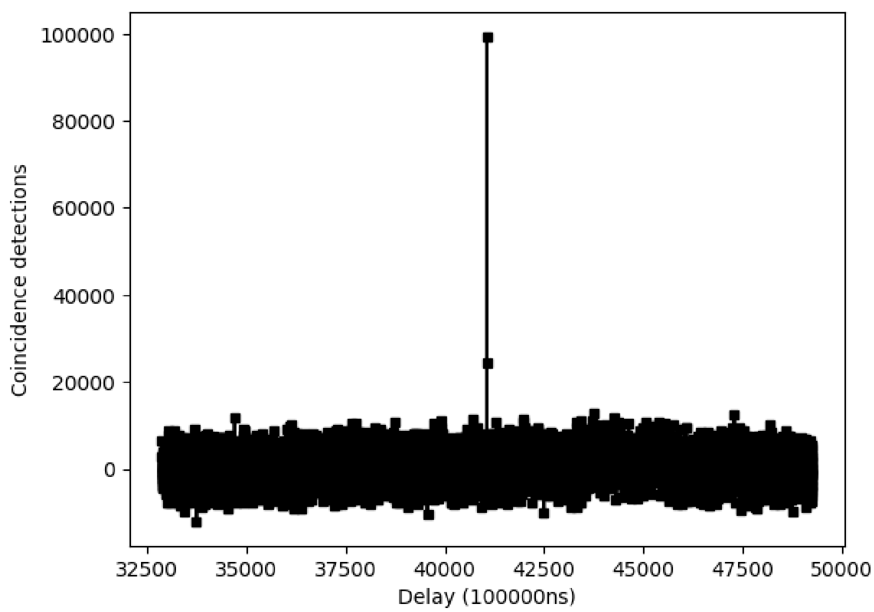
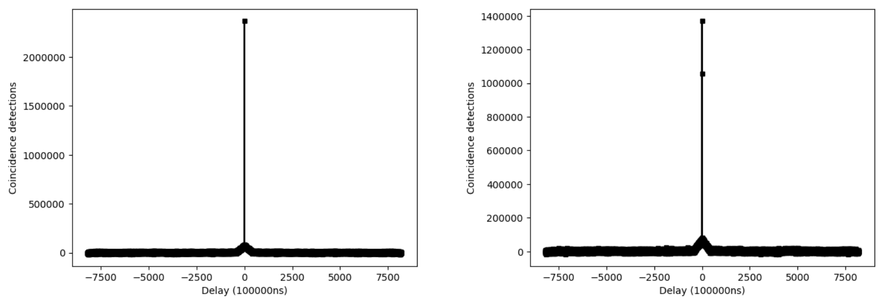



Figure 8: Cross-correlation on Fig 7.

## 2.2 Cross-correlation using FFT

The second-order correlation function $g^{(2)}(\tau)$ is the intensity analogue of the first-order correlation function $g^{(1)}(\tau)$ that determines the visibility of interference fringes. $g^{(1)}(\tau)$ quantifies the way in which the electric field fluctuates in time, whereas $g^{(2)}(\tau)$ quantifies the intensity fluctuations.

# 3 Correction with entanglement




(a) (b)

Figure 9: Autocorrelation of signal with (a) no shift (b) propagation delay

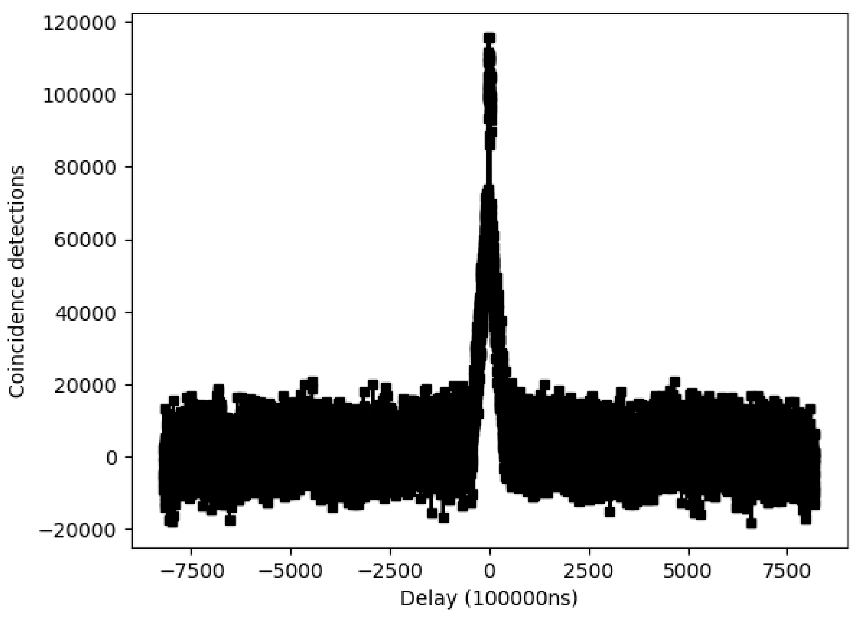



Figure 10: Autocorrelation of signal with delay and Doppler shift

### 3.1 Ambiguity function

The two-dimensional ambiguity function is commonly used in radar signal analysis as the most complete statement of the waveform's inherent performance. It reveals the range-Doppler position of ambiguous re-sponses and defines the range and Doppler resolution. It is defined as

$$A(\nu,\tau) \equiv \int_{-\infty}^{\infty} \tilde{s}\big(t+\frac{\tau}{2}\big)\tilde{s}^*\big(t-\frac{\tau}{2}\big)e^{2i\pi\nu t}dt \tag{9}$$

`TODO: fill in machine learning method`

1. find middle part with no shift
2. try to shift middle part and find differential equation (order 2)

# References


[1] Caleb Ho, Antia Lamas-Linares, and Christian Kurtsiefer. Clock synchronization by remote detection of correlated photon pairs. *New Journal of Physics*, 11(4):045011, 2009.