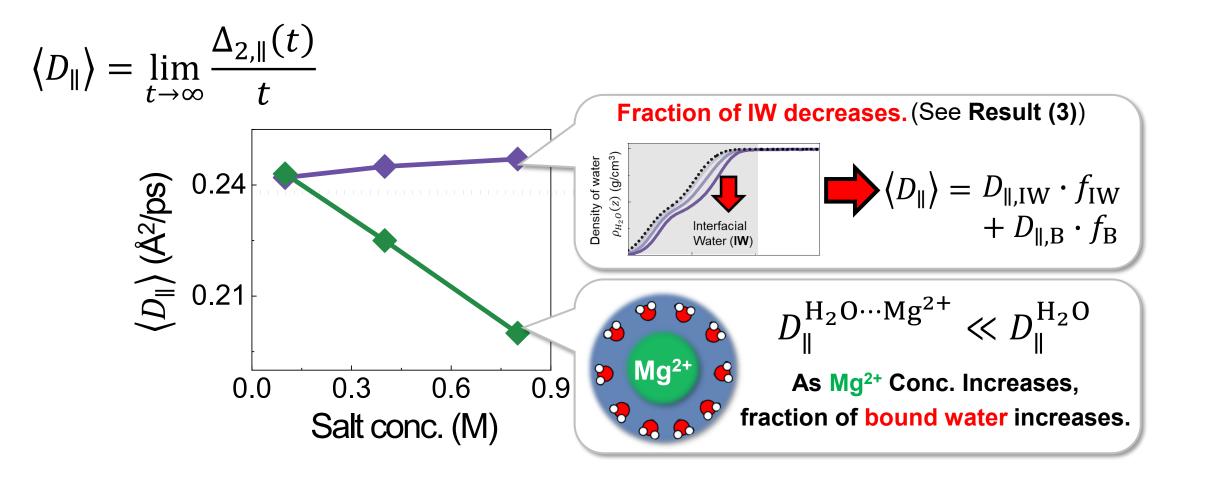
### SI Result 1. Mean Diffusion Coefficient

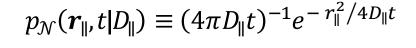


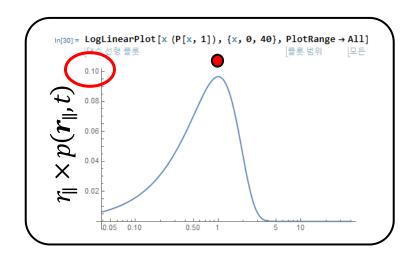
# SI Result 2. Lateral Displacement Distribution

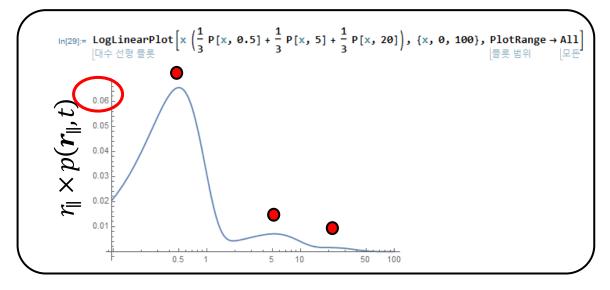
$$p(\mathbf{r}_{\parallel},t) \cong \sum_{n} f_{n} p_{\mathcal{N}} \left(\mathbf{r}_{\parallel},t|D_{\parallel}^{(n)}\right) \qquad \sum_{n} f_{n} = 1$$

$$r_{\parallel} \times p_{\mathcal{N}}(\boldsymbol{r}_{\parallel}, t|D_{\parallel}) \rightarrow \text{maximum at } r_{\parallel} = \sigma = \sqrt{2D_{\parallel}t}$$

$$In[2] = P[x_{\_}, \sigma_{\_}] := (2 \pi \sigma^2)^{-1} \exp[-x^2/(2 \sigma^2)]$$



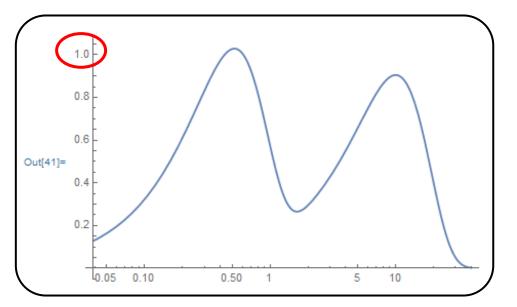




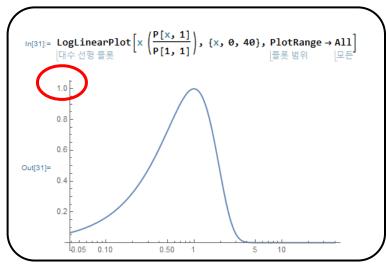
# SI Result 2. Lateral Displacement Distribution

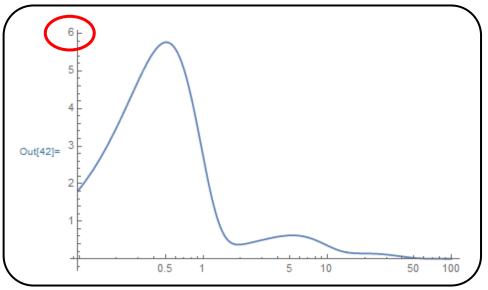
$$p(\mathbf{r}_{\parallel},t) \cong \sum_{n} f_{n} p_{\mathcal{N}} \left(\mathbf{r}_{\parallel},t|D_{\parallel}^{(n)}\right) \qquad \sum_{n} f_{n} = 1$$

$$f(\mathbf{r}_{\parallel},t) \equiv \frac{r_{\parallel}p(\mathbf{r}_{\parallel},t)}{\sigma p_{\mathcal{N}}(\sigma,t|\sigma)}\Big|_{\sigma=\sqrt{2\langle D_{\parallel}\rangle t}}$$

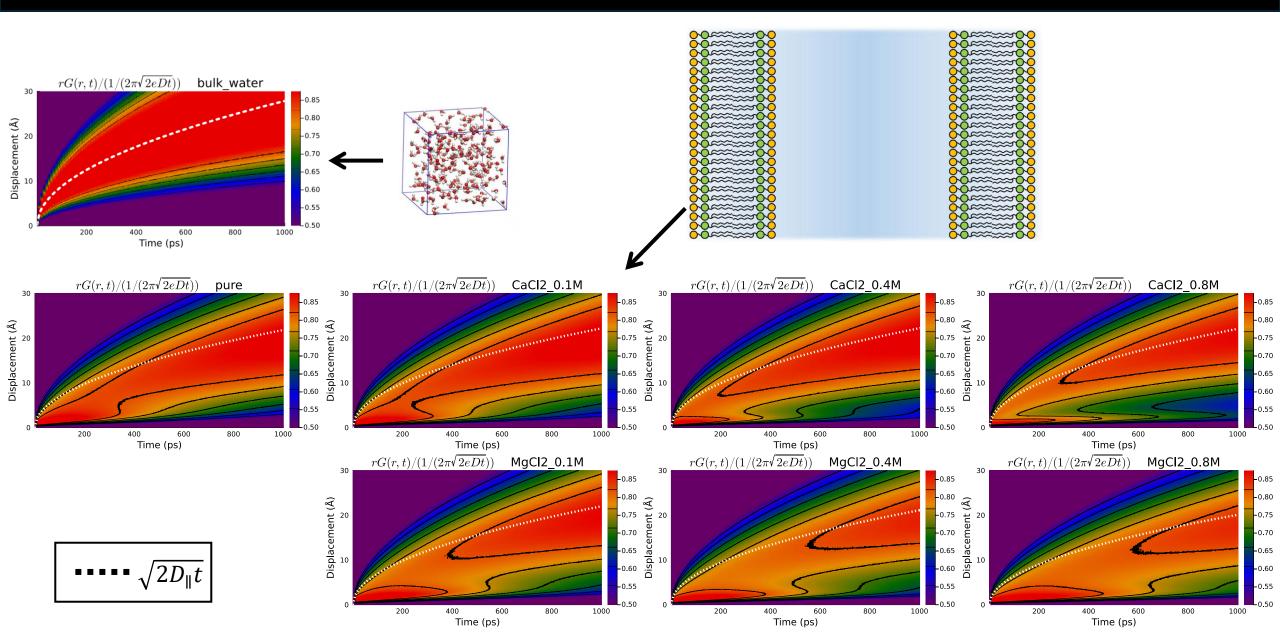


$$p_{\mathcal{N}}(\mathbf{r}_{\parallel}, t|D_{\parallel}) \equiv (4\pi D_{\parallel}t)^{-1}e^{-r_{\parallel}^2/4D_{\parallel}t}$$





## SI Result 2. Lateral Displacement Distribution



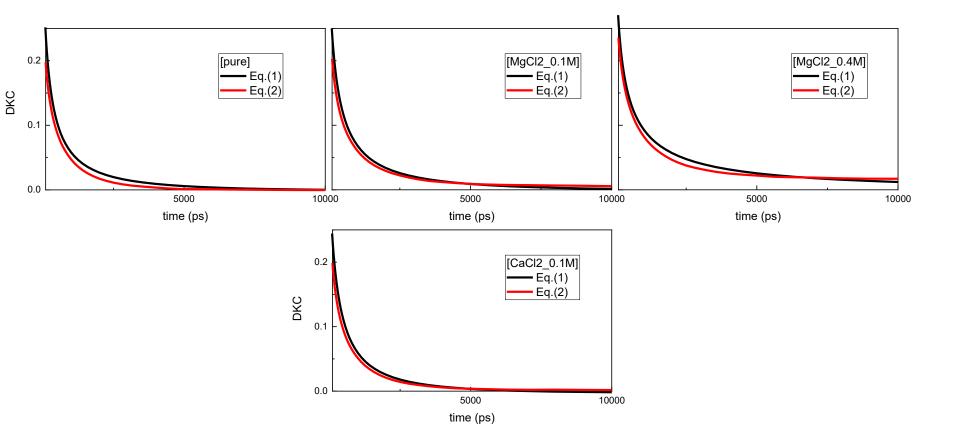
#### SI Result 4. Diffusion Kernel Correlation

$$C_{\mathcal{D}_{\parallel}}(t) = \int d\mathbf{\Gamma} \int d\mathbf{\Gamma}_{0} \frac{\delta \widehat{\mathcal{D}}_{\parallel}(\mathbf{\Gamma}, s)}{\langle \widehat{\mathcal{D}}_{\parallel}(s) \rangle} \, \widehat{G}(\Gamma, s | \Gamma_{0}) \frac{\delta \widehat{\mathcal{D}}_{\parallel}(\mathbf{\Gamma}_{0}, s)}{\langle \widehat{\mathcal{D}}_{\parallel}(s) \rangle} P_{st}(\mathbf{\Gamma}_{0}) \quad (1)$$

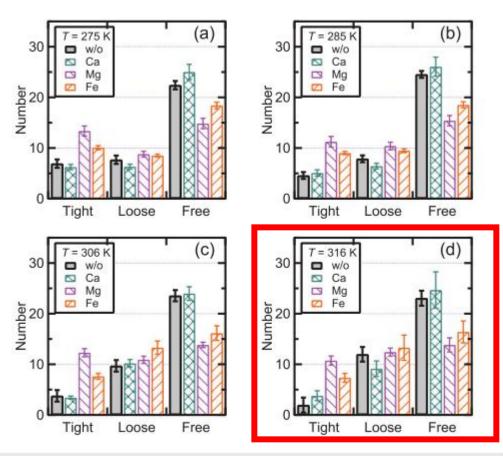
$$C_{\mathcal{D}_{\parallel}}(t) = \int d\mathbf{\Gamma} \int d\mathbf{\Gamma}_{0} \frac{\delta \widehat{\mathcal{D}}_{\parallel}(\mathbf{\Gamma}, s)}{\left\langle \widehat{\mathcal{D}}_{\parallel}(s) \right\rangle} \widehat{G}(\mathbf{\Gamma}, s | \mathbf{\Gamma}_{0}) \frac{\delta \widehat{\mathcal{D}}_{\parallel}(\mathbf{\Gamma}_{0}, s)}{\left\langle \widehat{\mathcal{D}}_{\parallel}(s) \right\rangle} P_{st}(\mathbf{\Gamma}_{0}) \quad (1) \qquad C_{\mathcal{D}_{\parallel}}(t) = \sum_{n} \sum_{m} \frac{\delta D_{\parallel}(n)}{\left\langle D_{\parallel} \right\rangle} G^{(d)}(n, t | m) \frac{\delta D_{\parallel}(m)}{\left\langle D_{\parallel} \right\rangle} P_{st}(m) \quad (2)$$

#### From MSD & NGP

From  $G^{(d)}(n,t|m)$  and  $D_{\parallel}(n)$  (Umbrella Sampling)

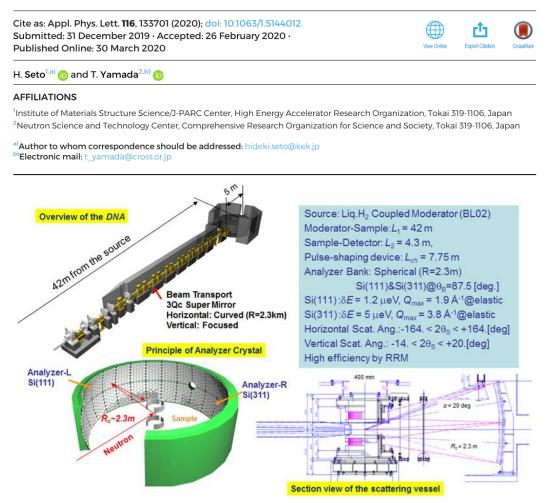


# SI Result 5. Experimental result



**FIG. 3.** Temperature dependence of the numbers of free water molecules, loosely bound water molecules, and tightly bound water molecules for  $d67DMPC-37H_2O$  (gray),  $d67DMPC-37H_2O-0.25CaCl_2$  (green),  $d67DMPC-37H_2O-0.25MgCl_2$  (purple), and  $d67DMPC-37H_2O-0.25FeCl_2$  (orange).

# Quasi-elastic neutron scattering study of the effects of metal cations on the hydration water between phospholipid bilayers



J-PARC MLF BL02 DNA: Dynamic Spectrometer

# SI Result 6. PDF of cation, anion

