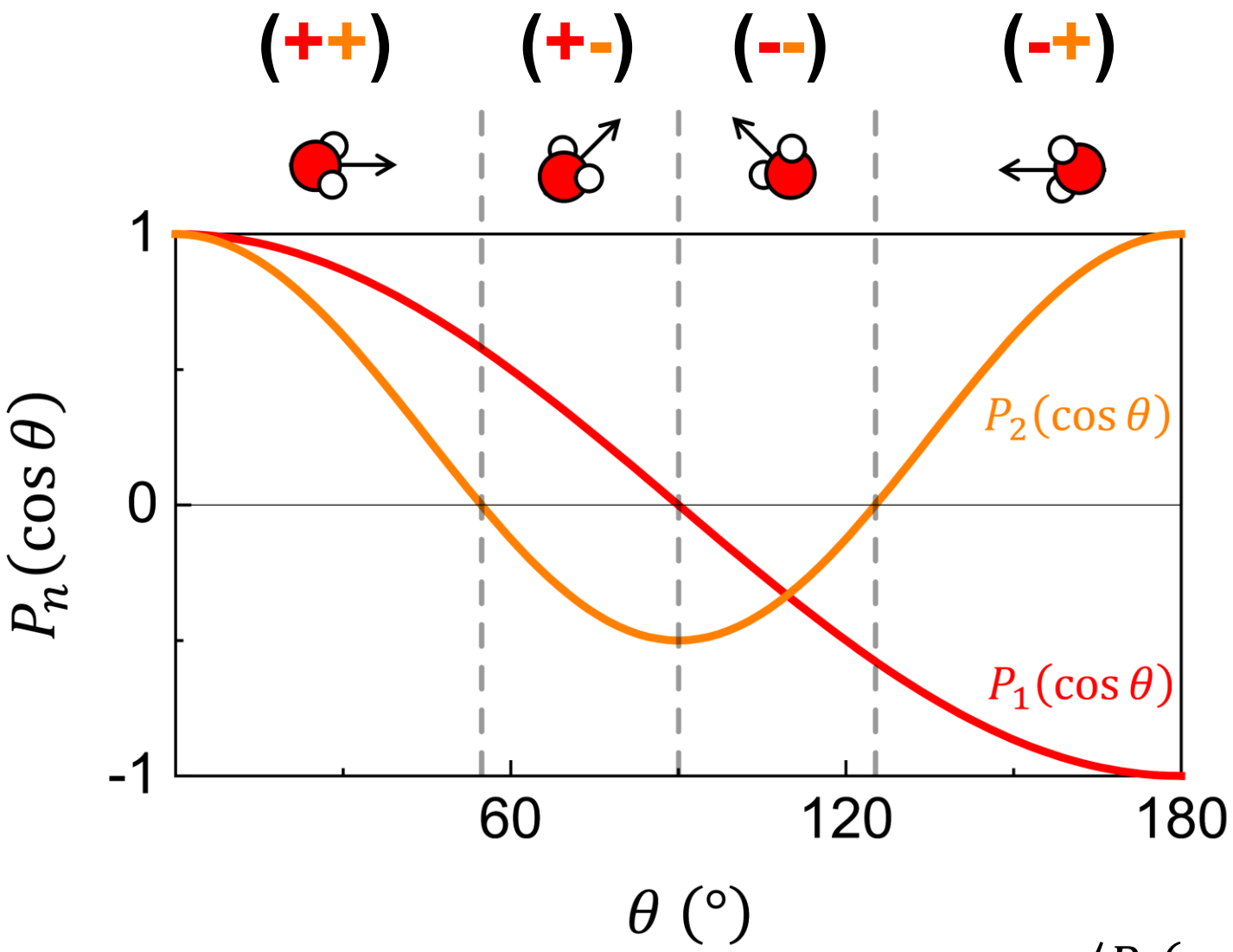
SI Result 1 (a): Legendre Polynomial



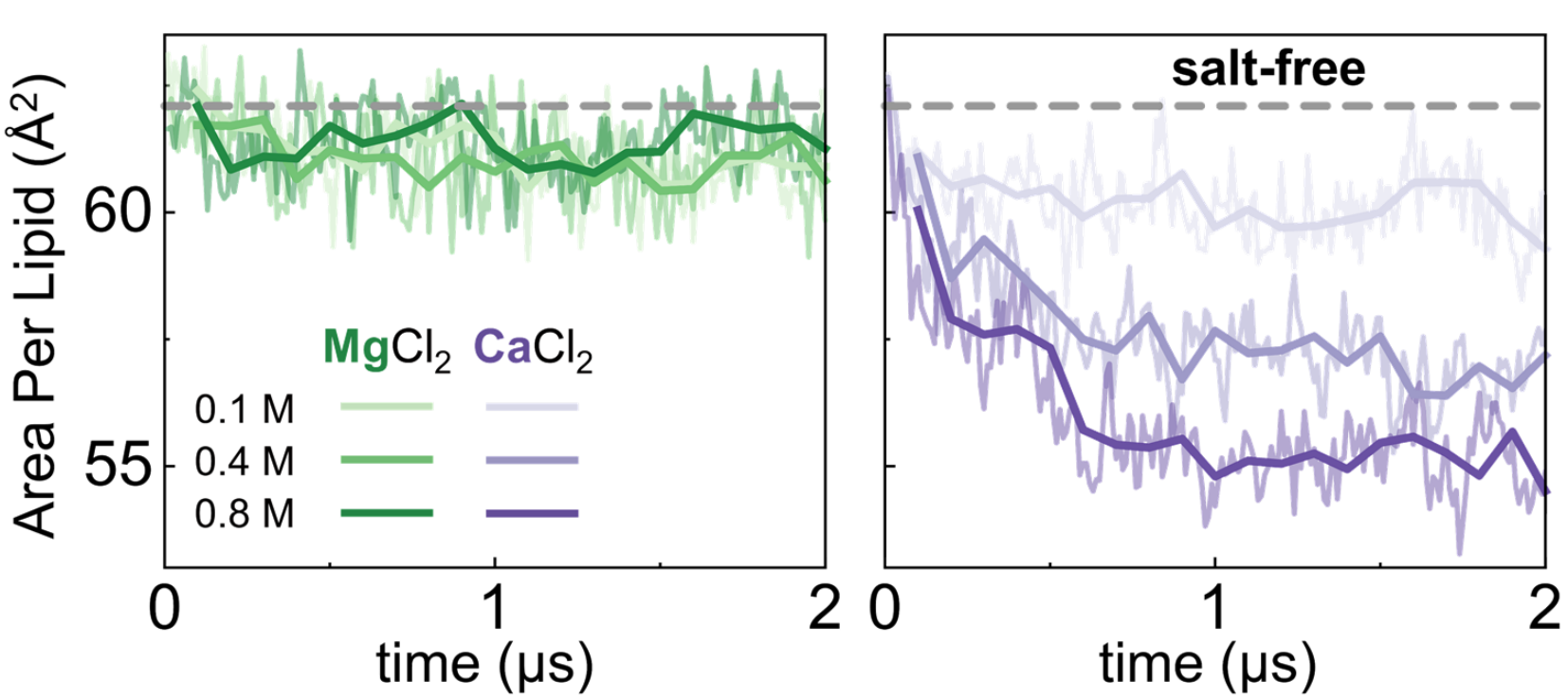
See **Fig. 2 (d)** and **Fig. 2 (e)**

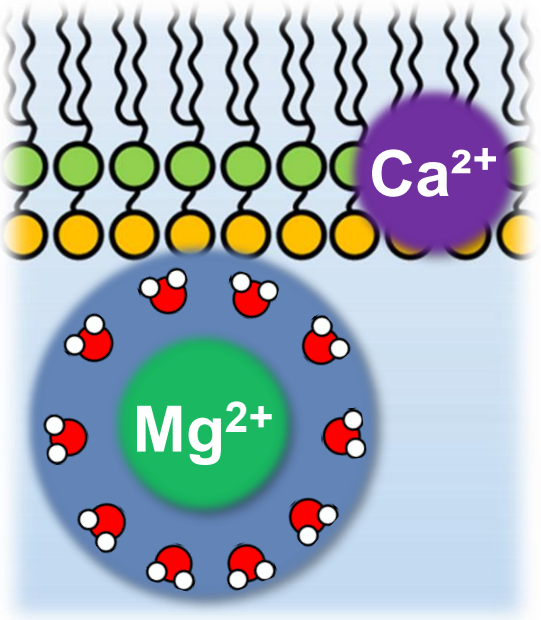
$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = (3\cos^2\theta - 1)/2$$

$$\langle P_n(\cos\theta)\rangle(z) = \int_0^{\pi} d\theta \sin\theta\, P_n(\cos\theta) p(\theta, z)$$

SI Result 1 (b): Area per lipid





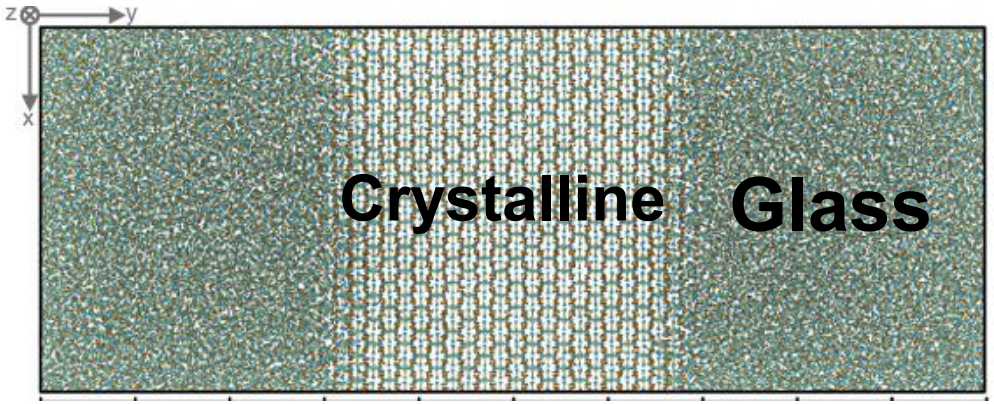
- Mg^{2+} cannot induce membrane condensation due to its **hydration shell** **[7]**.
- Ca^{2+} reduces repulsion between headgroups, inducing **membrane condensation** **[7]**.
- As Conc. of $CaCl_2$ increases, portion of interfacial water (**IW**) of $CaCl_2$ decreases.

(See $\rho_{H_2O}(z)$ in **Fig. 2 (f)**)

[7] Yi Dong, J. Phys. Chem. Lett., 13, 11237-11244 (2022)

SI Result 1 (c): Lateral Displacement Distribution

Li_3PS_4



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Article

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Disorder-induced enhancement of lithium-ion transport in solid-state electrolytes

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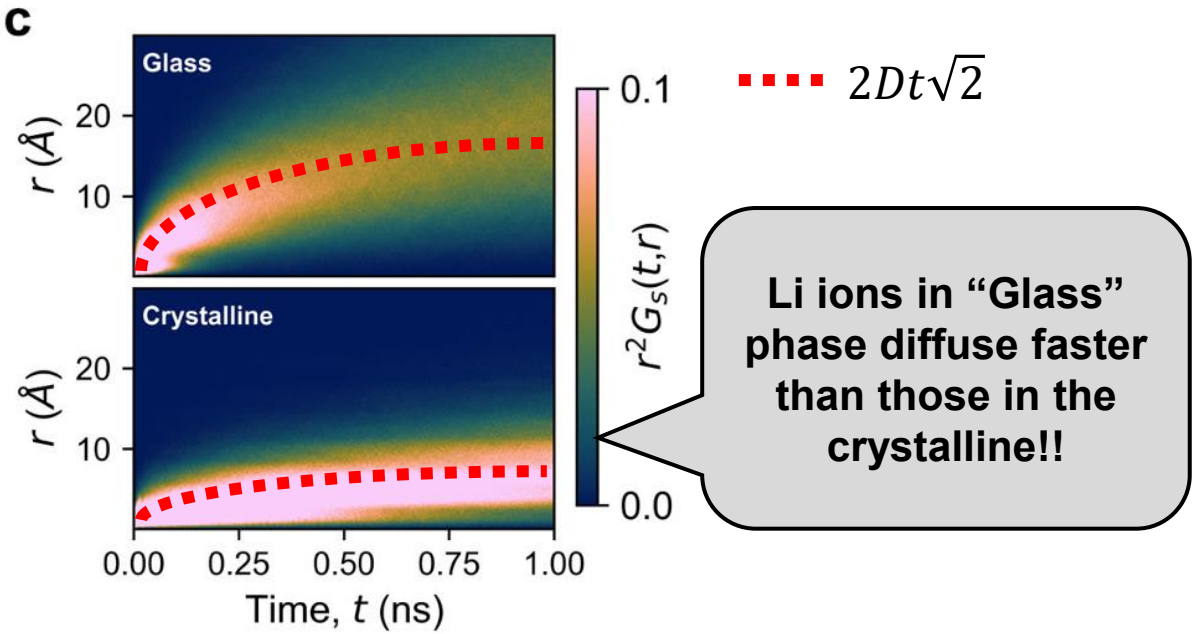
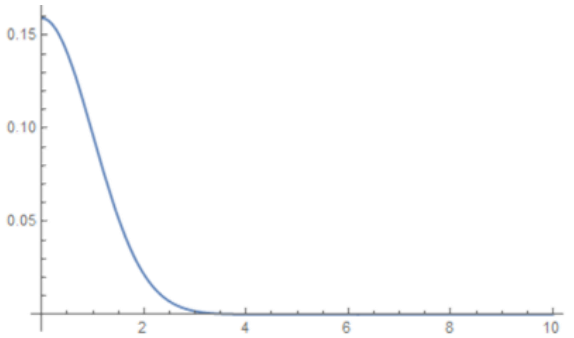


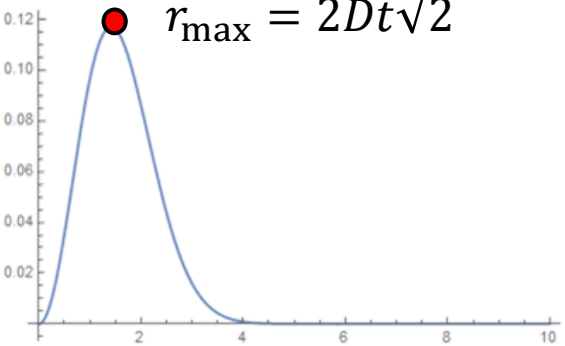
Li ions in "Glass" phase diffuse faster than those in the crystalline!!

$$G_s(\boldsymbol{r},t) = (4\pi Dt)^{-3/2} e^{-r^2/4Dt}$$



$$r^2 G_s(\boldsymbol{r},t)$$

$$r_{\max} = 2Dt\sqrt{2}$$



SI Result 1 (c): Lateral Displacement Distribution

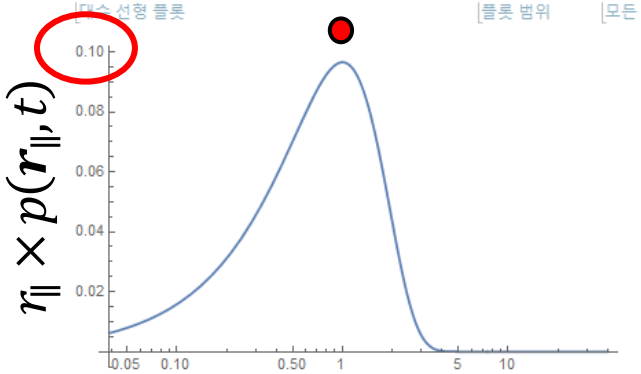
$$G_s(\boldsymbol{r}_\parallel, t) \cong \sum_n f_n G_{\mathcal{N}}\left(\boldsymbol{r}_\parallel, t | D_\parallel^{(n)}\right) \qquad \sum_n f_n = 1$$

$$G_{\mathcal{N}}(\boldsymbol{r}_\parallel, t | D_\parallel) \equiv (4\pi D_\parallel t)^{-1} e^{-r_\parallel^2 / 4 D_\parallel t}$$

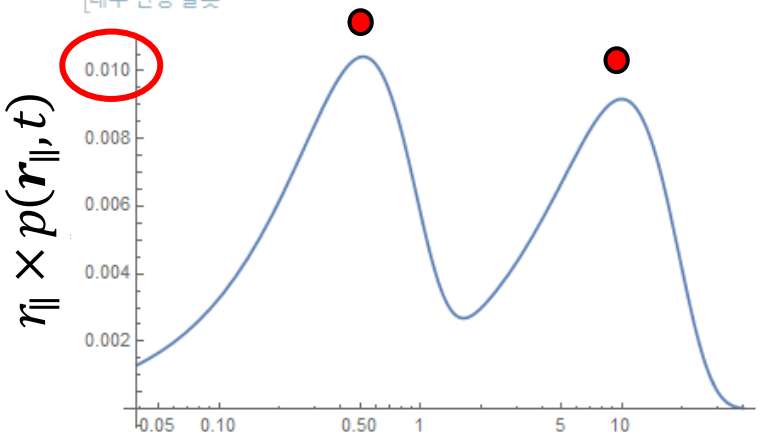
$$r_\parallel \times G_{\mathcal{N}}(\boldsymbol{r}_\parallel, t | D_\parallel) \rightarrow \text{ maximum at } r_\parallel = \sigma = \sqrt{2 D_\parallel t}$$

```
In[2]:= P[x_, σ_] := (2 π σ²)^-1 Exp[-x² / (2 σ²)]
                                  지수 함수
```

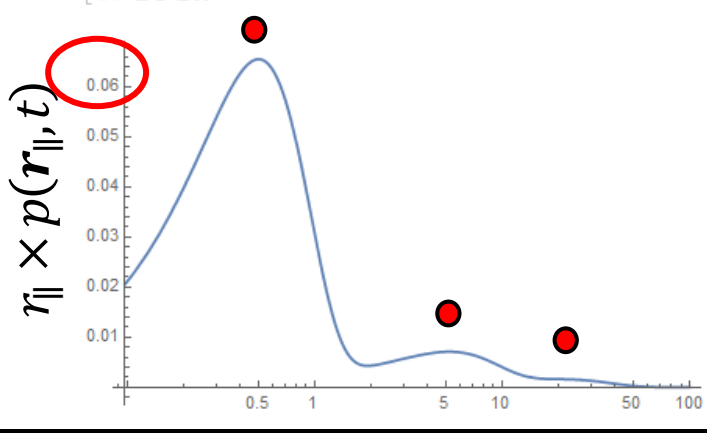
```
In[30]:= LogLinearPlot[x (P[x, 1]), {x, 0, 40}, PlotRange → All]
          대수 선형 플롯                          플롯 범위   모든
```




```
In[26]:= LogLinearPlot[x (0.05 P[x, 0.5] + 0.95 P[x, 10]), {x, 0, 40}, PlotRange → All]
          대수 선형 플롯                                                  플롯 범위   모든
```




```
In[29]:= LogLinearPlot[x (1/3 P[x, 0.5] + 1/3 P[x, 5] + 1/3 P[x, 20]), {x, 0, 100}, PlotRange → All]
          대수 선형 플롯                                                  플롯 범위   모든
```

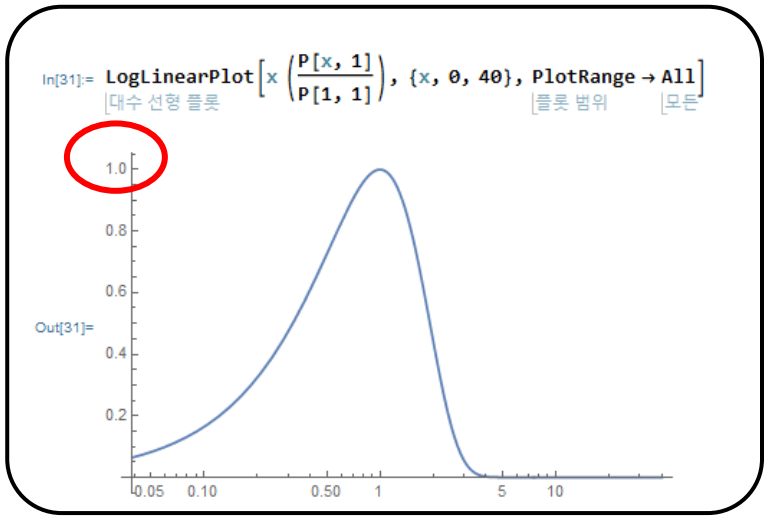


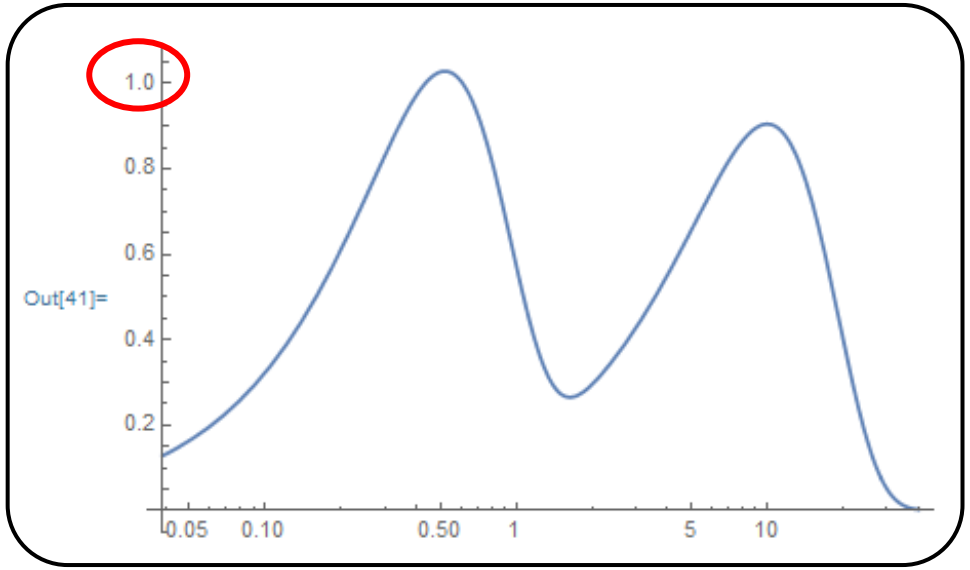
SI Result 1 (c): Lateral Displacement Distribution

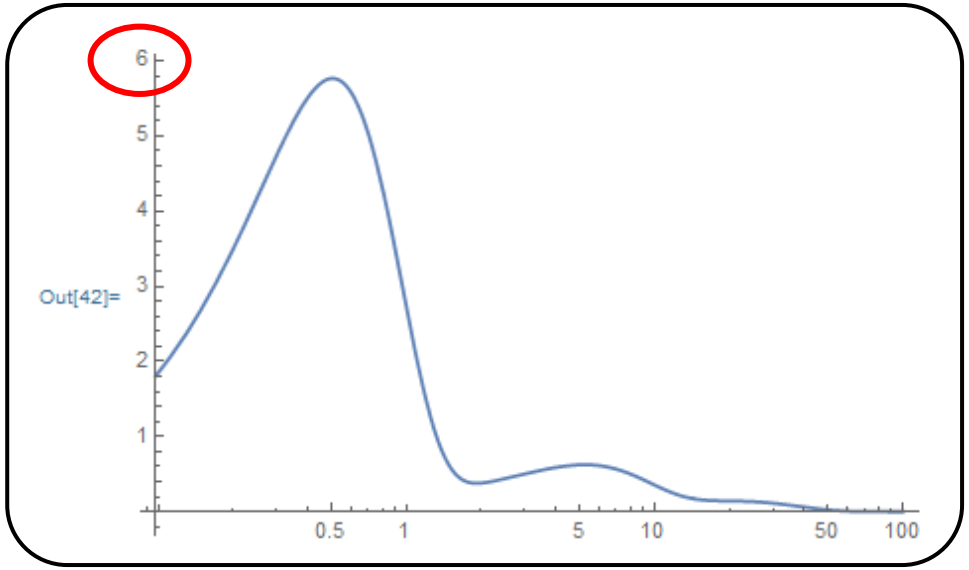
$$G_s(\boldsymbol{r}_\parallel, t) \cong \sum_n f_n G_{\mathcal{N}}\left(\boldsymbol{r}_\parallel, t | D_\parallel^{(n)}\right) \qquad \sum_n f_n = 1$$

$$G_{\mathcal{N}}(\boldsymbol{r}_\parallel, t | D_\parallel) \equiv (4\pi D_\parallel t)^{-1} e^{-r_\parallel^2 / 4 D_\parallel t}$$

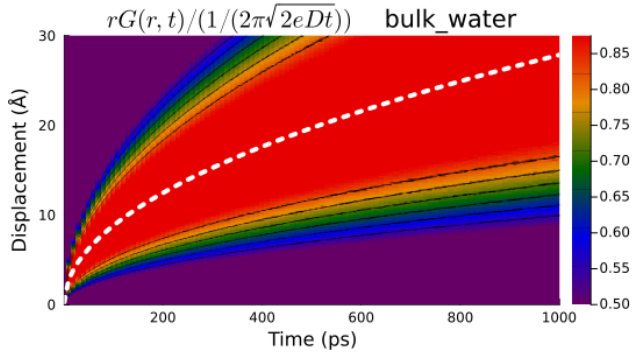
$$f(\boldsymbol{r}_\parallel, t) \equiv \left. \frac{r_\parallel G_s(\boldsymbol{r}_\parallel, t)}{\sigma\, p_{\mathcal{N}}(\sigma, t|\sigma)} \right|_{\sigma = \sqrt{2\langle D_\parallel \rangle t}}$$

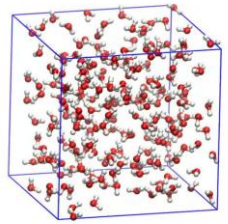


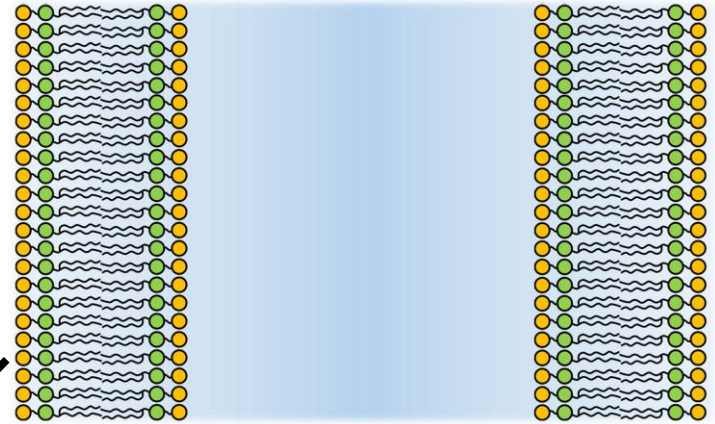


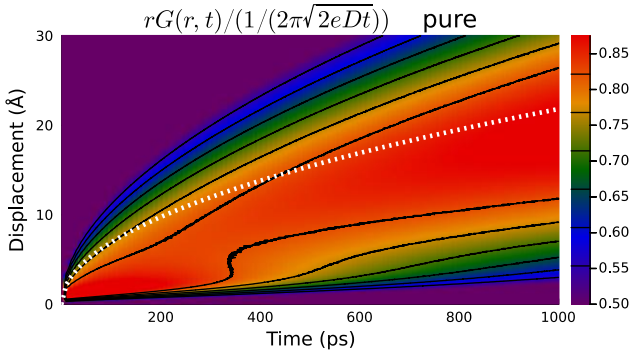


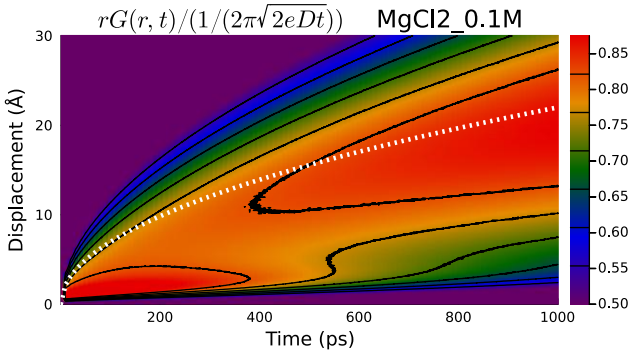
SI Result 1 (c): Lateral Displacement Distribution

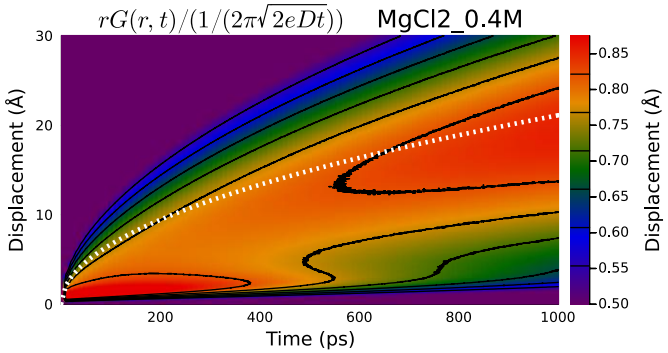


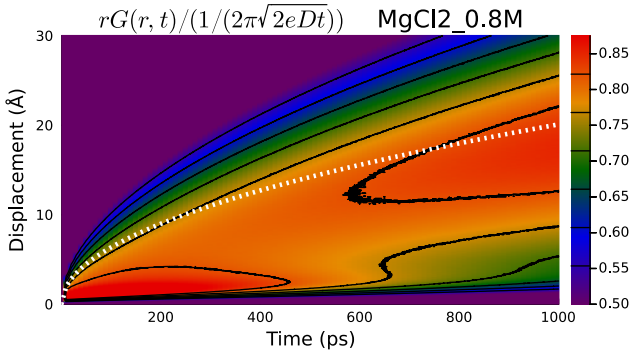


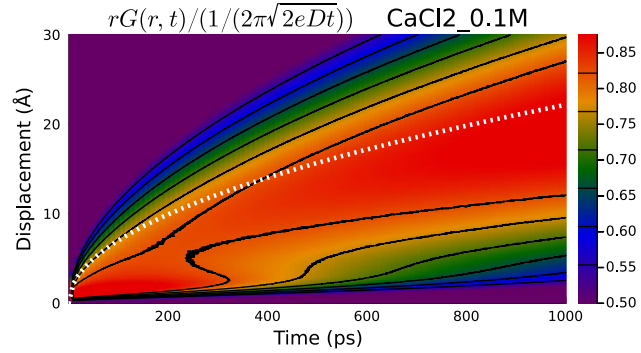


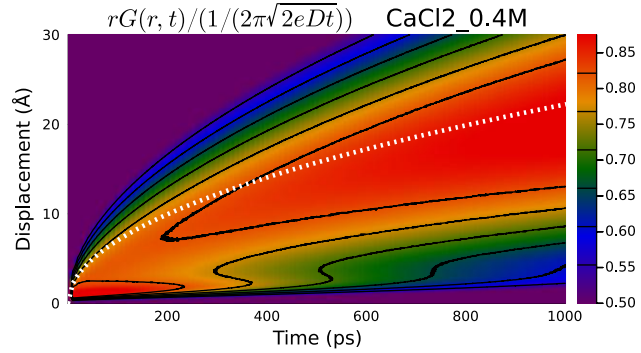


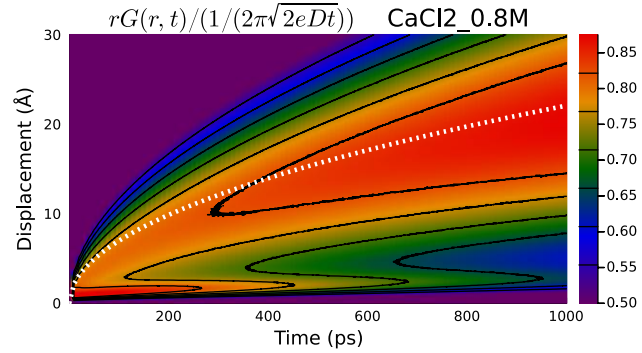


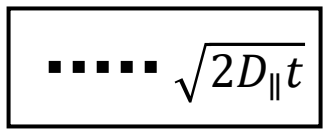




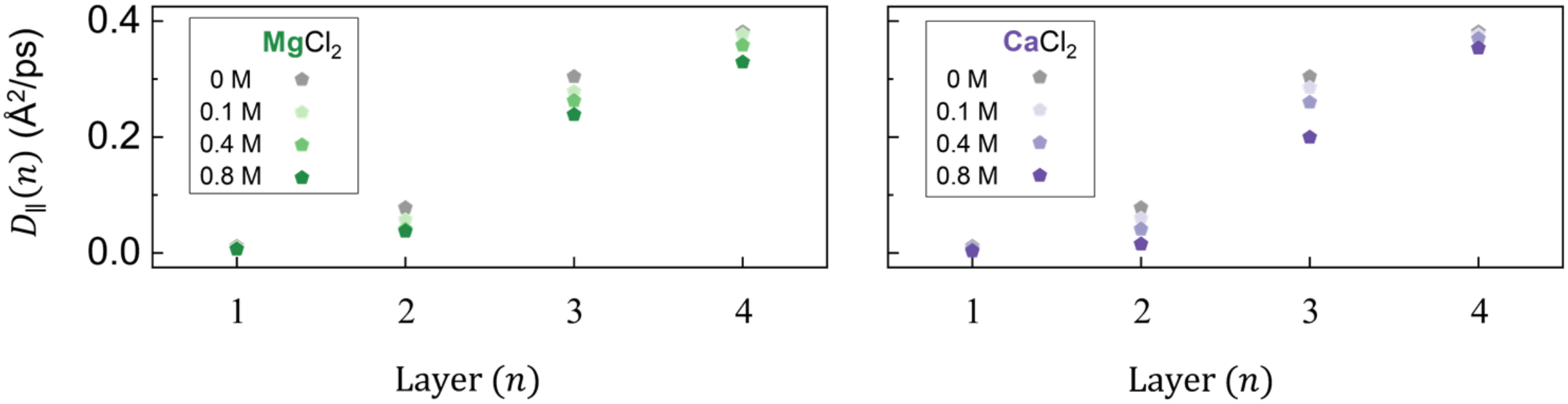








SI Result 1 (d): Region dependent lateral diffusion coefficient

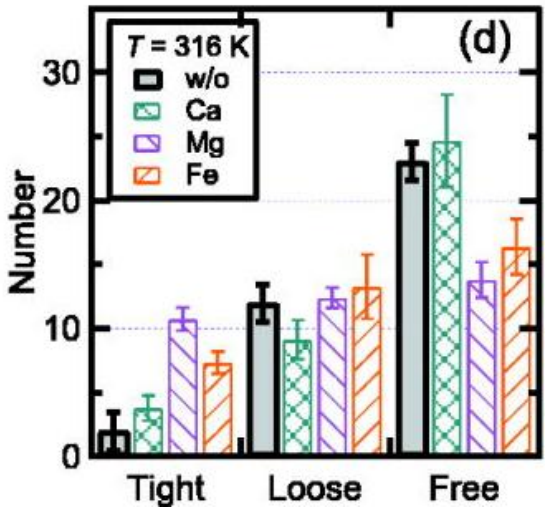


$D_{\parallel}(n)$: determined by umbrella sampling (1 μs-long)

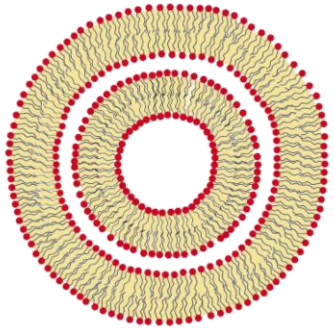
$\mathrm{Layer}(n)$: defined by the nodes of $\langle P_2(\cos\theta)\rangle(z)$ in the salt-free case. (see **Fig. 2 (e)**)

SI Result 1 (e): Experimental result

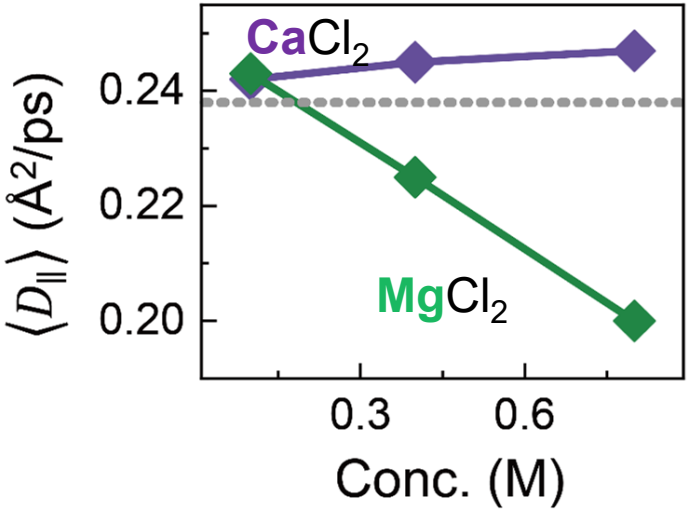
Fig. 3

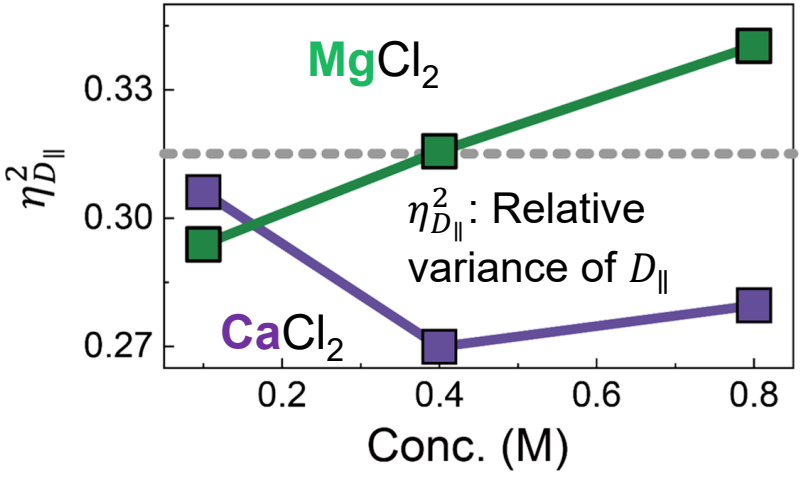


[Experiment]
DMPC
37 H_2O/lipid molecule.
0.45 M conc.



Multilamellar lipid vesicle





Quasi-elastic neutron scattering study of the effects of metal cations on the hydration water between phospholipid bilayers

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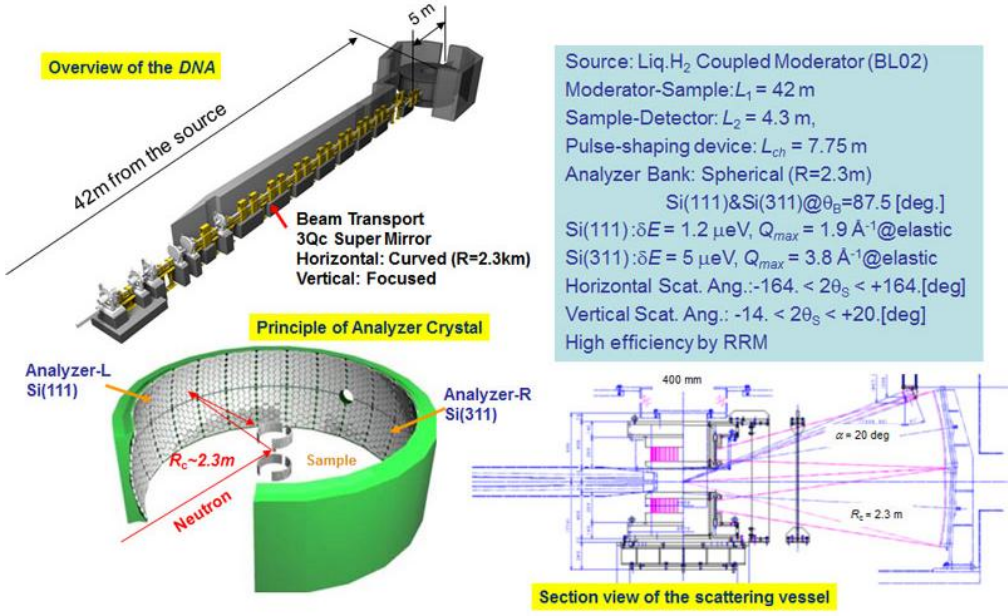

J-PARC MLF BL02 DNA: Dynamic Spectrometer