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Angular momentum.

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$= \hat{e}_x (\underbrace{x p_y - y p_x}_{L_x}) + \hat{e}_y (\underbrace{z p_x - x p_z}_{L_y}) + \hat{e}_z (\underbrace{x p_y - y p_x}_{L_z})$$

Operator identities

① $$\underbrace{[\hat{x}, \hat{p}_x]}_{\text{Commutator.}} \psi = (\hat{x} \underbrace{\hat{p}_x}_{-i\hbar\partial_x} - \hat{p}_x \hat{x}) \psi$$

$$= -i\hbar (x \partial_x \psi - \underbrace{\partial_x x \psi}_{= \psi + x \partial_x \psi})$$

$$= i\hbar \psi$$

$$\Rightarrow [\hat{x}, \hat{p}_x] = i\hbar$$

$$[\hat{y}, \hat{p}_y] = i\hbar$$

$$[\hat{z}, \hat{p}_z] = i\hbar$$

② $$[\hat{L}_x, \hat{L}_y] = \hat{L}_x \hat{L}_y - \hat{L}_y \hat{L}_x$$

$$= -(y\hat{p}_z - z\hat{p}_y)(x\hat{p}_z - z\hat{p}_x) + (x\hat{p}_z - z\hat{p}_x)(y\hat{p}_z - z\hat{p}_y)$$

$$= i\hbar \hat{L}_z$$

$$= -y\hat{p}_z x\hat{p}_z + y\hat{p}_z z\hat{p}_x + z\hat{p}_y x\hat{p}_z - z\hat{p}_y z\hat{p}_x + x\hat{p}_z y\hat{p}_z - x\hat{p}_z z\hat{p}_y - z\hat{p}_x y\hat{p}_z + z\hat{p}_x z\hat{p}_y$$

$$= y\hat{p}_z z\hat{p}_x - x\hat{p}_z z\hat{p}_y + z\hat{p}_y x\hat{p}_z - z\hat{p}_x y\hat{p}_z$$

$$= y\hat{p}_x\hat{p}_z z - x\hat{p}_y\hat{p}_z z + x\hat{p}_y z\hat{p}_z - y\hat{p}_x z\hat{p}_z$$

$$= y\hat{p}_x[\hat{p}_z, z] - x\hat{p}_y[\hat{p}_z, z]$$

Likewise, $[\hat{L}_y, \hat{L}_z] = i\hbar \hat{L}_x$)

$$= yp_xp_zz - xp_yp_zz + xp_yzp_z - yp_xzp_z$$

$$= y\hat{p}_x[\hat{p}_z, z] - x\hat{p}_y[\hat{p}_z, z]$$

$$= i\hbar(x\hat{p}_y - y\hat{p}_x)$$

Likewise, $[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x$

$[\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y$

Cyclic permutation.

③ $\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$: Ladder Operator.

$+$: raising / $-$: lowering.

$$[\hat{L}_z, \hat{L}_\pm] = [\hat{L}_z, \hat{L}_x] \pm i[\hat{L}_z, \hat{L}_y]$$

$$= i\hbar\hat{L}_y \pm \hbar\hat{L}_x$$

$$= \pm\hbar(\hat{L}_x \pm i\hat{L}_y)$$

$$= \pm\hbar\hat{L}_\pm$$

④ $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$

$$[\hat{L}^2, \hat{L}_z] = [\hat{L}_x^2, \hat{L}_z] + [\hat{L}_y^2, \hat{L}_z] + [\hat{L}_z^2, \hat{L}_z]$$

$$= \hat{L}_x[\hat{L}_x, \hat{L}_z] + [\hat{L}_x, \hat{L}_z]\hat{L}_x + \hat{L}_y[\hat{L}_y, \hat{L}_z] + [\hat{L}_y, \hat{L}_z]\hat{L}_y$$

$$= -i\hbar(\hat{L}_x\hat{L}_y + \hat{L}_y\hat{L}_x) + i\hbar(\hat{L}_y\hat{L}_x + \hat{L}_x\hat{L}_y)$$

$$= 0.$$

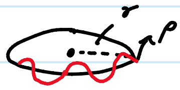
⑤ $\hat{L}_+\hat{L}_- = (\hat{L}_x + i\hat{L}_y)(\hat{L}_x - i\hat{L}_y)$

$$= \hat{L}_x^2 + \hat{L}_y^2 - i[\hat{L}_x, \hat{L}_y]$$

$$= \hat{L}_x^2 + \hat{L}_y^2 + \hbar\hat{L}_z$$

$$= \hat{L}^2 - \hat{L}_z^2 + \hbar\hat{L}_z$$

$\rightsquigarrow \hat{L}^2 = \hat{L}_+\hat{L}_- + \hat{L}_z^2 + \hbar\hat{L}_z$



$2\pi r = m\lambda.$

$L = rp = \frac{m\lambda}{2\pi} \cdot \frac{h}{\lambda} = m\hbar$

$\hat{L}_z \varphi_m = m\hbar \varphi_m$

$\hat{L}_z (\hat{L}_+ \varphi_m) = \left([\hat{L}_z, \hat{L}_+] + \hat{L}_+ \hat{L}_z \right) \varphi_m$

$= (m+1)\hbar \, (\hat{L}_+ \varphi_m)$

$\hat{L}_z (\hat{L}_- \varphi_m) = (m-1)\hbar \, (\hat{L}_- \varphi_m)$

: Ladder Operator...

m value → limitation..?!

⇓

Total angular momentum Conservation.

$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 = \hbar^2 K^2$

$\rightsquigarrow \quad m^2 \le K^2$

$-K \le m \le K \longrightarrow m_{min}$ & m_{max}

If $m = m_{max}$ If $m = m_{min}$

$\hat{L}_+ \varphi_{m_{max}} = 0$ $\hat{L}_- \varphi_{m_{min}} = 0.$