Thesis Title

Thesis Subtitle

Author Name

B.Sc. Final Year Dissertation

Cardiff School of Mathematics

CARDIFF UNIVERSITY

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Contents

List of Figures

Chapter 1

Introduction

1.1 Introduction

Chapter 2

Awesome theorems and stuff

2.1 Introduction

Note that I can refer to other chapters: see Chapter 1 and even specific equations in each chapter, this is an

Chapter 3

Conclusion

Chapter 4

Results

In this chapter, the implementation of a fingerprint function within the Axelrod-Python library will be examined with a walk through of the development process. This includes the addition of two strategy transformers, the Dual and JossAnn as defined in . Then several results will be presented where analytical fingerprints are compared with analytical ones. A discussion that compares different fingerprints of strategies within Axelrod-Python will also be given.

4.1 The Dual

The dual of a strategy is defined such that when the original strategy and the dual are presented with identical histories they will return opposite actions **??**. This means the dual relies on knowledge of how the original strategy would have behaved in a given situation, which is impractical to infer from the source code. However, the required behaviour can be achieved by having the original strategy as an attribute of the dual. Whenever the dual has to submit a move, it can first get the original strategy to suggest what move should it would have made, and then flip that action.

```
Data: A strategy
Result: The d of the strategy
if First Turn then
    create copy of original strategy;
end
simulate original strategy;
update original strategy's history/internal state;
return Flip of original strategy's move
```

Algorithm 1: The Dual of a Strategy

4.2 Implementation of Fingerprinting

As defined in section a fingerprint function is merely the expected score of a strategy when played against a Joss-Ann transformer of a probe with varying parameters. As part of this project, a numerical implementation has now been included in the Axelrod-Python library. It begins by taking a sample of the x, y values that may define the Joss-Ann Transformer. The strategy then plays a match against a transformer with each of the sampled values. The average score per turn can be calculated at the end of each match which corresponds to the expected score required by the analytical fingerprint function. The whole process can be repeated for reliability and the resulting scores plotted. The player interactions have been modelled as a spatial tournament within Axelrod-Python, where the strategy plays all of the probes and a probe only plays the strategy. For an example with 9 probes, see Figure 4.1.

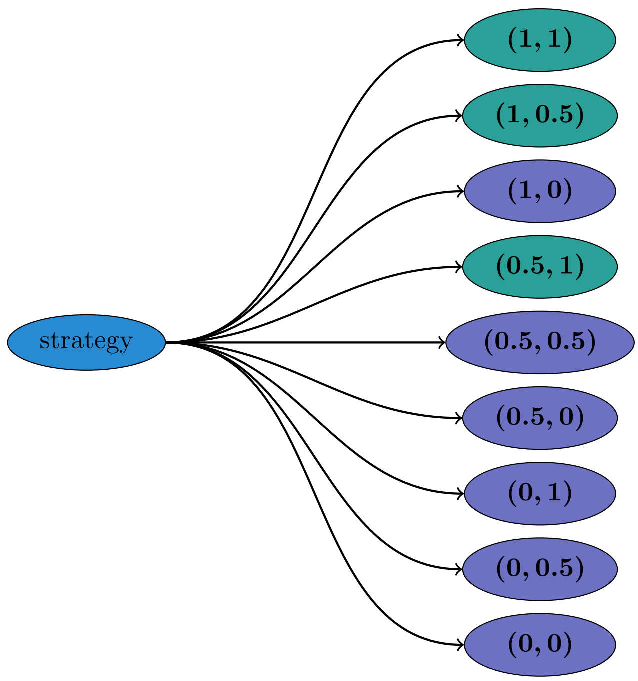


Figure 4.1: A spatial tournament for the strategy against 9 probes

Whether the numerical fingerprint matches the analytical one relies heavily on the choice of parameters. Specifically the `turns, repetetitons` and `step` variables. The `step` variable determines the number of x, y values taken. Listing 1 shows how a grid of points is constructed over the unit square where the distance between each point is taken as `step`. Therefore, a smaller `step` value means more points are created and so greater detail is included in the plot (similar to pixels).

The `turns` variable determines how many interactions there will be in a match. Enough turns must be selected to ensure that steady long term behaviour is reached otherwise the average score per turn can be wildly innacurate. However, once this state is reached, extending the number of turns has a minimal effect on the accuracy of the plot. The `repetitions` variable decides how many times the tournament would be repeated. The Axelrod-Python implementation of fingerprinting is a random process (due to the Joss-Ann)

```python
def create_points(step):
    """Creates a set of Points over the unit square.
    A Point has coordinates (x, y). This function constructs points that are
    separated by a step equal to `step`. The points are over the unit
    square which implies that the number created will be (1/`step` + 1)^2.
    Parameters
    ----------
    step : float
        The separation between each Point. Smaller steps will produce more
        Points with coordinates that will be closer together.
    Returns
    ----------
    points : list
        of Point objects with coordinates (x, y)
    """
    num = int((1 / step) // 1) + 1
    points = [Point(j, k) for j in np.linspace(0, 1, num)
              for k in np.linspace(0, 1, num)]

    return points
```

Listing 1: Axelrod-Python code to create a sample of x, y points

and high repetitions helps to reduce the effects of this.

4.3 Comparison of Analytical and Numerical Plots

In figure 4.2, several analytical fingerprints from previous literature are shown [1, 2]. Colourings or shadings are used to make certain features stand out, and an attempt to replicate this behaviour was implemented in Axelrod-Python. The popular plotting library, matplotlib, has many options for different colour maps which are demonstrated in Appendix .

Using the analytical fingerprints from previous literature [1, 2], and the fingerprint formulae provided alongside them, the most appropriate colour map was chosen. The colour map Seismic [3] was selected due to its divergent properties (although all colour maps are available within the library). With divergent colour maps, all extreme values (high or low) are coloured, whilst mid range values are left white [4]. This highlights areas of interest, and in Figure 4.3 it can be seen that this matches previous work well.

With the knowledge that the choice of colourmap is appropriate, a comparison can now be made between analytical fingerprints and numerical ones obtained via the Axelrod-Python library. Table 4.1 gives the analytical fingerprint functions of several well known strategies that will then be used to validate the numerical versions.

Figures 4.4 4.5 4.6 4.7 4.8 compare plots of known analytical fingerprint functions with numerical approximations obtained with the Axelrod-Python library. The analytical plots were created with the code seen in

```python
def fingerprint(self, turns=50, repetitions=10, step=0.01, processes=None,
                filename=None, in_memory=False, progress_bar=True):
    """Build and play the spatial tournament.

    Creates the probes and their edges then builds a spatial tournament.
    When the coordinates of the probe sum to more than 1, the dual of the
    probe is taken instead and then the Joss-Ann Transformer is applied. If
    the coordinates sum to less than 1 (or equal), then only the Joss-Ann is
    applied, a dual is not required.

    Parameters
    ----------
    turns : integer, optional
        The number of turns per match
    repetitions : integer, optional
        The number of times the round robin should be repeated
    step : float, optional
        The separation between each Point. Smaller steps will
        produce more Points that will be closer together.
    processes : integer, optional
        The number of processes to be used for parallel processing
    progress_bar : bool
        Whether or not to create a progress bar which will be updated

    Returns
    ----------
    self.data : dictionary
        A dictionary where the keys are coordinates of the form (x, y) and
        the values are the mean score for the corresponding interactions.
    """
```

Listing 2: Axelrod-Python docstring for the fingerprint function

Strategy	Analytical Fingerprint Function
TitForTat	$\dfrac{y^2+5xy+3x^2}{(x+y)^2}$
Psycho (Anti TitForTat	$\dfrac{4(y-1)(x-1)+5(y-1)^2}{2(y-1)(x-1)+(x-1)^2+(y-1)^2}$
WinStayLoseShit (Pavlov)	$\dfrac{(3x+y)(x-1)+5y(y-1)}{(x+2y)(x-1)+y(y-1)}$
AllC (Cooperator)	$3-3y$
AllD (Defector)	$4x+1$

Table 4.1: A selection of analytical fingerprint functions for well known strategies. The probe used is TitForTat.

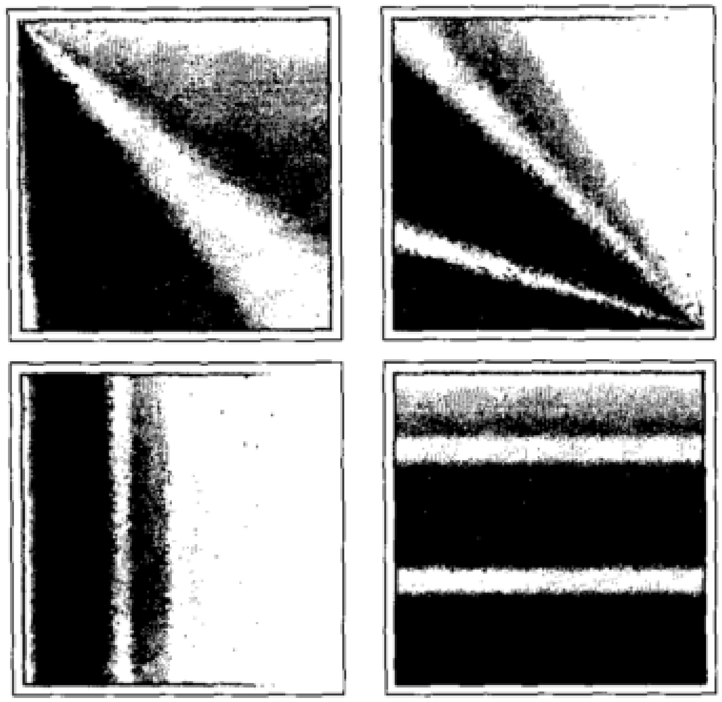
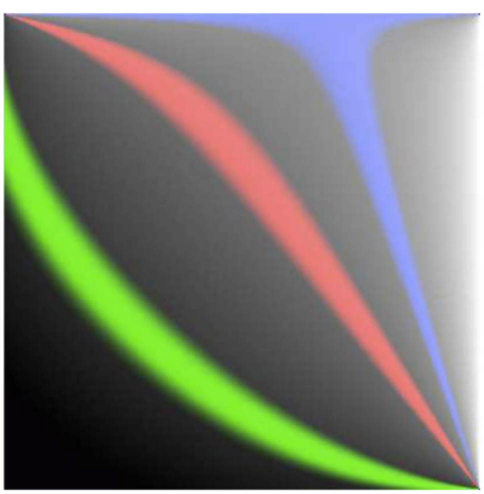
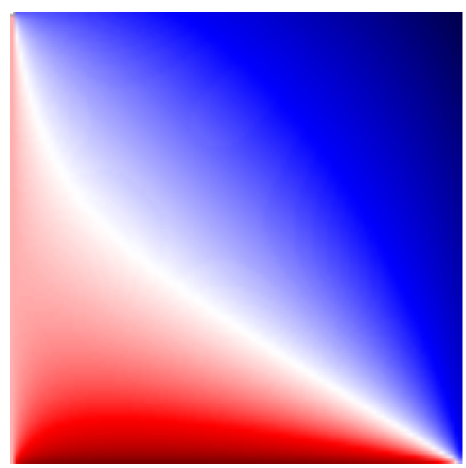


Figure 4.2: Shaded plots of the fingerprint functions for the strategies TitForTat, Psycho, AllD and AllC, in reading order from [1]



(a) WSLS fingerprint from previous literature [2]



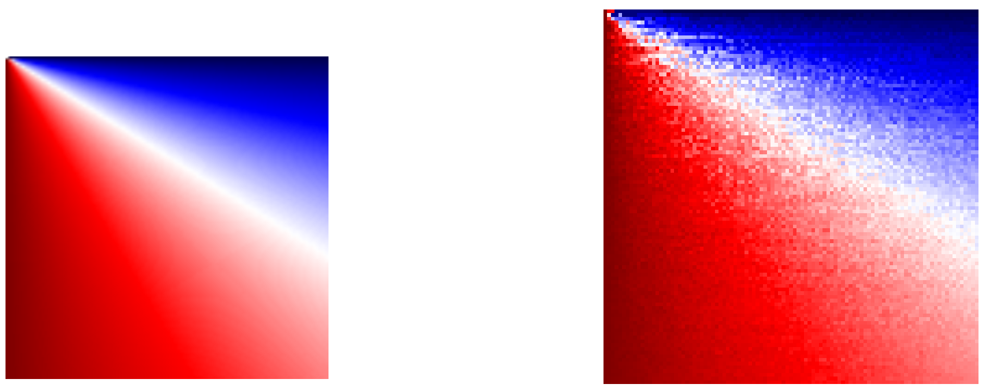
(b) Analytical WSLS fingerprint demonstrating Seismic colouring

Figure 4.3: A comparison of a fingerprint plot from previous literature to asses the suitability of the Seismic colour map [2]

listing 3. The parameters `turns=500`, `repetitions=200`, `step=0.01` are as described in section 4.2. The parameter `processes=0` ensures that the function will use the maximum number of cores available on the computer.

```python
import axelrod as axl
strats = [axl.TitForTat, axl.WinStayLoseShift, axl.AntiTitForTat,
          axl.Cooperator, axl.Defector]
for s in strats:
    probe = axl.TitForTat
    af = axl.AshlockFingerprint(s, probe)
    data = af.fingerprint(turns=500, repetitions=200, step=0.01, processes=0)
    p = af.plot()
    p.savefig('{}-Numerical.pdf'.format(s.name))
```

Listing 3: Code to create the numerical plots for several strategies

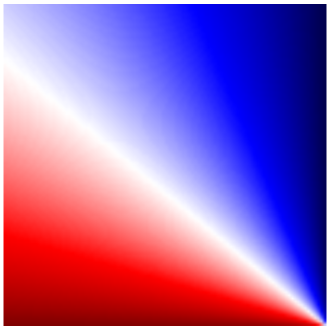


(a) Exact analytical fingerprint

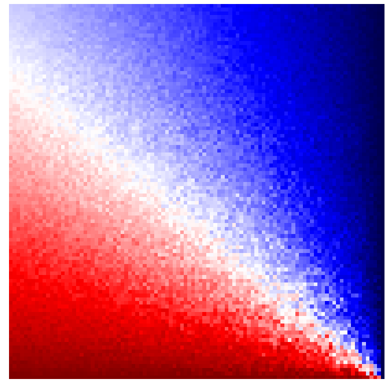
(b) Numerical Fingerprint

Figure 4.4: A comparison of the analytical fingerprint of TitForTat and the numerical version produced by Axelrod-Python library.

4.4 The Development Process

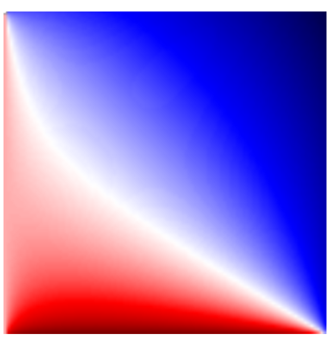


(a) Exact analytical fingerprint

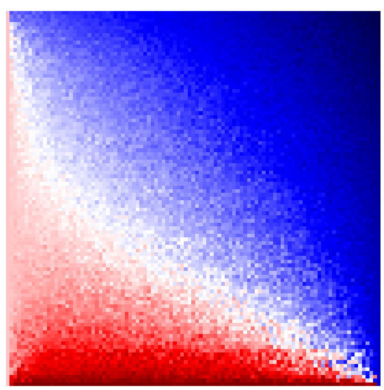


(b) Numerical Fingerprint

Figure 4.5: A comparison of the analytical fingerprint of Psycho and the numerical version produced by Axelrod-Python library.

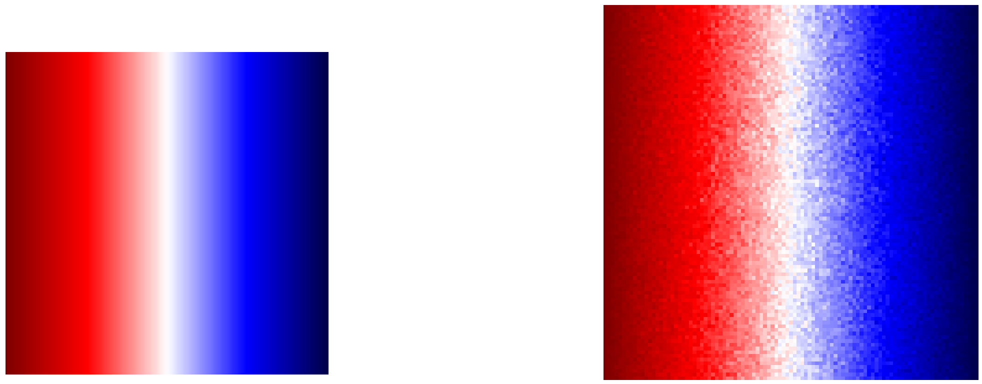


(a) Exact analytical fingerprint



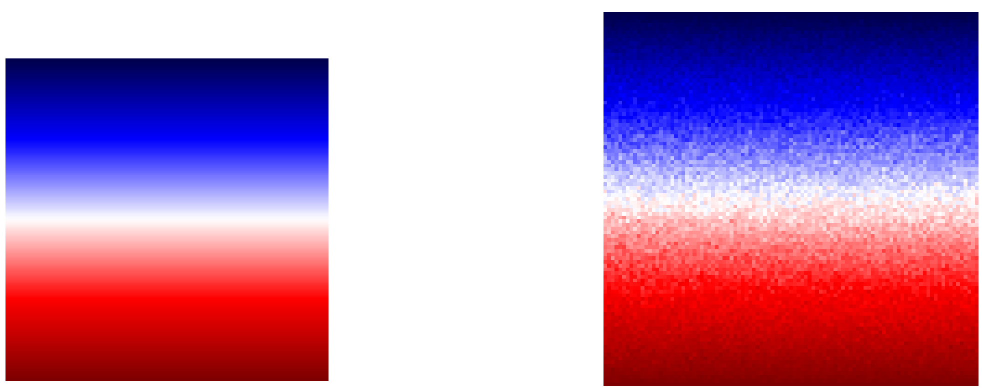
(b) Numerical Fingerprint

Figure 4.6: A comparison of the analytical fingerprint of WinStayLoseShit and the numerical version produced by Axelrod-Python library.



(a) Exact analytical fingerprint (b) Numerical Fingerprint

Figure 4.7: A comparison of the analytical fingerprint of Cooperator and the numerical version produced by Axelrod-Python library.



(a) Exact analytical fingerprint (b) Numerical Fingerprint

Figure 4.8: A comparison of the analytical fingerprint of Defector and the numerical version produced by Axelrod-Python library.

Chapter 5

Appendix

The end.

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