

ACM 常用算法模板

therehello

2023 年 8 月 17 日

目录

1	数据结构	3
1.1	并查集	3
1.2	树状数组	3
1.3	线段树	5
1.4	可持久化线段树	6
1.5	st 表	7
2	图论	8
2.1	最短路	8
2.2	树上问题	8
2.2.1	最近公公祖先	8
2.2.2	树链剖分	9
2.3	强连通分量	10
2.4	拓扑排序	11
3	字符串	12
3.1	kmp	12
3.2	哈希	12
3.3	manacher	13
4	数学	14
4.1	扩展欧几里得	14
4.2	线性筛法	14
4.3	分解质因数	15
4.4	pollard rho	15
4.5	组合数	16
4.6	数论分块	17
4.7	积性函数	17
4.7.1	定义	17
4.7.2	例子	17
4.8	狄利克雷卷积	18
4.8.1	性质	18
4.8.2	例子	18
4.9	莫比乌斯反演	18
4.9.1	莫比乌斯函数性质	18
4.9.2	莫比乌斯变换/反演	18
4.10	杜教筛	18
4.10.1	示例	19
4.11	盒子与球	19
4.12	线性基	19
4.13	矩阵快速幂	20
5	计算几何	22
5.1	扫描线	28

6 杂项	31
6.1 高精度	31
6.2 模运算	32
6.3 分数	33
6.4 表达式求值	33
6.5 日期	35
6.6 对拍	36
6.7 编译常用选项	36
6.8 开栈	36
6.9 clang-format	37

1 数据结构

1.1 并查集

```

1 struct dsu {
2     int n;
3     vector<int> fa;
4     dsu(int _n) : n(_n) {
5         fa.resize(n + 1);
6         iota(fa.begin(), fa.end(), 0);
7     }
8     int find(int x) { return x == fa[x] ? x : fa[x] = find(fa[x]); }
9     int merge(int x, int y) {
10         int fax = find(x), fay = find(y);
11         if (fax == fay) return 0; // 一个集合
12         return fa[find(x)] = find(y); // 合并到哪个集合了
13     }
14 };

```

1.2 树状数组

一维

```

1 template <class T>
2 struct Fenwick_tree {
3     Fenwick_tree(int n) : n(n), tree(n + 1, 0) {}
4     T query(int l, int r) {
5         auto query = [&](int pos) {
6             T res = 0;
7             while (pos) {
8                 res += tree[pos];
9                 pos -= lowbit(pos);
10            }
11            return res;
12        };
13        return query(r) - query(l - 1);
14    }
15    void update(int pos, T num) {
16        while (pos <= n) {
17            tree[pos] += num;
18            pos += lowbit(pos);
19        }
20    }
21 private:
22     int n;
23     vector<T> tree;
24 };

```

二维

```

1 template <class T>
2 struct Fenwick_tree_2 {

```

```

3 Fenwick_tree_2(int n, int m) : n(n), m(m), tree(n + 1, vector<T>(m + 1)) {}
4 T query(int l1, int r1, int l2, int r2) {
5     auto query = [&](int l, int r) {
6         T res = 0;
7         for (int i = l; i; i -= lowbit(i))
8             for (int j = r; j; j -= lowbit(j)) res += tree[i][j];
9         return res;
10    };
11    return query(l2, r2) - query(l2, r1 - 1) - query(l1 - 1, r2) +
12           query(l1 - 1, r1 - 1);
13 }
14 void update(int x, int y, T num) {
15     for (int i = x; i <= n; i += lowbit(i))
16         for (int j = y; j <= m; j += lowbit(j)) tree[i][j] += num;
17 }
18 private:
19     int n, m;
20     vector<vector<T>> tree;
21 };

```

三维

```

1 template <class T>
2 struct Fenwick_tree_3 {
3     Fenwick_tree_3(int n, int m, int k)
4         : n(n),
5           m(m),
6           k(k),
7           tree(n + 1, vector<vector<T>>(m + 1, vector<T>(k + 1))) {}
8     T query(int a, int b, int c, int d, int e, int f) {
9         auto query = [&](int x, int y, int z) {
10             T res = 0;
11             for (int i = x; i; i -= lowbit(i))
12                 for (int j = y; j; j -= lowbit(j))
13                     for (int p = z; p; p -= lowbit(p)) res += tree[i][j][p];
14             return res;
15         };
16         T res = query(d, e, f);
17         res -= query(a - 1, e, f) + query(d, b - 1, f) + query(d, e, c - 1);
18         res += query(a - 1, b - 1, f) + query(a - 1, e, c - 1) +
19               query(d, b - 1, c - 1);
20         res -= query(a - 1, b - 1, c - 1);
21         return res;
22     }
23     void update(int x, int y, int z, T num) {
24         for (int i = x; i <= n; i += lowbit(i))
25             for (int j = y; j <= m; j += lowbit(j))
26                 for (int p = z; p <= k; p += lowbit(p)) tree[i][j][p] += num;
27     }
28 private:
29     int n, m, k;
30     vector<vector<vector<T>>> tree;
31 };

```

1.3 线段树

```

1 template <class Data, class Num>
2 struct Segment_Tree {
3     inline void update(int l, int r, Num x) { update(1, l, r, x); }
4     inline Data query(int l, int r) { return query(1, l, r); }
5     Segment_Tree(vector<Data>& a) {
6         n = a.size();
7         tree.assign(n * 4 + 1, {});
8         build(a, 1, 1, n);
9     }
10 private:
11     int n;
12     struct Tree {
13         int l, r;
14         Data data;
15     };
16     vector<Tree> tree;
17     inline void pushup(int pos) {
18         tree[pos].data = tree[pos << 1].data + tree[pos << 1 | 1].data;
19     }
20     inline void pushdown(int pos) {
21         tree[pos << 1].data = tree[pos << 1].data + tree[pos].data.lazytag;
22         tree[pos << 1 | 1].data =
23             tree[pos << 1 | 1].data + tree[pos].data.lazytag;
24         tree[pos].data.lazytag = Num::zero();
25     }
26     void build(vector<Data>& a, int pos, int l, int r) {
27         tree[pos].l = l;
28         tree[pos].r = r;
29         if (l == r) {
30             tree[pos].data = a[l - 1];
31             return;
32         }
33         int mid = (tree[pos].l + tree[pos].r) >> 1;
34         build(a, pos << 1, l, mid);
35         build(a, pos << 1 | 1, mid + 1, r);
36         pushup(pos);
37     }
38     void update(int pos, int& l, int& r, Num& x) {
39         if (l > tree[pos].r || r < tree[pos].l) return;
40         if (l <= tree[pos].l && tree[pos].r <= r) {
41             tree[pos].data = tree[pos].data + x;
42             return;
43         }
44         pushdown(pos);
45         update(pos << 1, l, r, x);
46         update(pos << 1 | 1, l, r, x);
47         pushup(pos);

```

```

48     }
49     Data query(int pos, int& l, int& r) {
50         if (l > tree[pos].r || r < tree[pos].l) return Data::zero();
51         if (l <= tree[pos].l && tree[pos].r <= r) return tree[pos].data;
52         pushdown(pos);
53         return query(pos << 1, l, r) + query(pos << 1 | 1, l, r);
54     }
55 };
56 struct Num {
57     ll add;
58     inline static Num zero() { return {0}; }
59     inline Num operator+(Num b) { return {add + b.add}; }
60 };
61 struct Data {
62     ll sum, len;
63     Num lazytag;
64     inline static Data zero() { return {0, 0, Num::zero()}; }
65     inline Data operator+(Num b) {
66         return {sum + len * b.add, len, lazytag + b};
67     }
68     inline Data operator+(Data b) {
69         return {sum + b.sum, len + b.len, Num::zero()};
70     }
71 };

```

1.4 可持久化线段树

```

1  constexpr int MAXN = 200000;
2  vector<int> root(MAXN << 5);
3  struct Persistent_seg {
4      int n;
5      struct Data {
6          int ls, rs;
7          int val;
8      };
9      vector<Data> tree;
10     Persistent_seg(int n, vector<int>& a) : n(n) { root[0] = build(1, n, a); }
11     int build(int l, int r, vector<int>& a) {
12         if (l == r) {
13             tree.push_back({0, 0, a[l]});
14             return tree.size() - 1;
15         }
16         int mid = l + r >> 1;
17         int ls = build(l, mid, a), rs = build(mid + 1, r, a);
18         tree.push_back({ls, rs, tree[ls].val + tree[rs].val});
19         return tree.size() - 1;
20     }
21     int update(int rt, const int& idx, const int& val, int l, int r) {
22         if (l == r) {
23             tree.push_back({0, 0, tree[rt].val + val});
24             return tree.size() - 1;

```



```

25     }
26     int mid = l + r >> 1, ls = tree[rt].ls, rs = tree[rt].rs;
27     if (idx <= mid) ls = update(ls, idx, val, l, mid);
28     else rs = update(rs, idx, val, mid + 1, r);
29     tree.push_back({ls, rs, tree[ls].val + tree[rs].val});
30     return tree.size() - 1;
31 }
32 int query(int rt1, int rt2, int k, int l, int r) {
33     if (l == r) return l;
34     int mid = l + r >> 1;
35     int lcnt = tree[tree[rt2].ls].val - tree[tree[rt1].ls].val;
36     if (k <= lcnt) return query(tree[rt1].ls, tree[rt2].ls, k, l, mid);
37     else return query(tree[rt1].rs, tree[rt2].rs, k - lcnt, mid + 1, r);
38 }
39 };

```

1.5 st 表

```

1 auto lg = []() {
2     array<int, 10000001> lg;
3     lg[1] = 0;
4     for (int i = 2; i <= 10000000; i++) lg[i] = lg[i >> 1] + 1;
5     return lg;
6 }();
7 template <typename T>
8 struct st {
9     int n;
10    vector<vector<T>>> a;
11    st(vector<T>& _a) : n(_a.size()) {
12        a.assign(lg[n] + 1, vector<int>(n));
13        for (int i = 0; i < n; i++) a[0][i] = _a[i];
14        for (int j = 1; j <= lg[n]; j++)
15            for (int i = 0; i + (1 << j) - 1 < n; i++)
16                a[j][i] = max(a[j - 1][i], a[j - 1][i + (1 << (j - 1))]);
17    }
18    T query(int l, int r) {
19        int k = lg[r - l + 1];
20        return max(a[k][l], a[k][r - (1 << k) + 1]);
21    }
22 };

```

2 图论

存图

```

1 struct Graph {
2     int n;
3     struct Edge {
4         int to, w;
5     };
6     vector<vector<Edge>> graph;
7     Graph(int _n) {
8         n = _n;
9         graph.assign(n + 1, vector<Edge>());
10    };
11    void add(int u, int v, int w) { graph[u].push_back({v, w}); }
12 };

```

2.1 最短路

dijkstra

```

1 void dij(Graph& graph, vector<int>& dis, int t) {
2     vector<int> visit(graph.n + 1, 0);
3     priority_queue<pair<int, int>> que;
4     dis[t] = 0;
5     que.emplace(0, t);
6     while (!que.empty()) {
7         int u = que.top().second;
8         que.pop();
9         if (visit[u]) continue;
10        visit[u] = 1;
11        for (auto& [to, w] : graph.graph[u]) {
12            if (dis[to] > dis[u] + w) {
13                dis[to] = dis[u] + w;
14                que.emplace(-dis[to], to);
15            }
16        }
17    }
18 }

```

2.2 树上问题

2.2.1 最近公公祖先

倍增法

```

1 vector<int> dep;
2 vector<array<int, 21>> fa;
3 dep.assign(n + 1, 0);
4 fa.assign(n + 1, array<int, 21>{});
5 void binary_jump(int root) {
6     function<void(int)> dfs = [&](int t) {

```

```

7     dep[t] = dep[fa[t][0]] + 1;
8     for (auto& [to] : graph[t]) {
9         if (to == fa[t][0]) continue;
10        fa[to][0] = t;
11        dfs(to);
12    }
13 };
14 dfs(root);
15 for (int j = 1; j <= 20; j++)
16     for (int i = 1; i <= n; i++) fa[i][j] = fa[fa[i][j - 1]][j - 1];
17 }
18 int lca(int x, int y) {
19     if (dep[x] < dep[y]) swap(x, y);
20     for (int i = 20; i >= 0; i--)
21         if (dep[fa[x][i]] >= dep[y]) x = fa[x][i];
22     if (x == y) return x;
23     for (int i = 20; i >= 0; i--) {
24         if (fa[x][i] != fa[y][i]) {
25             x = fa[x][i];
26             y = fa[y][i];
27         }
28     }
29     return fa[x][0];
30 }

```

树剖

```

1 int lca(int x, int y) {
2     while (top[x] != top[y]) {
3         if (dep[top[x]] < dep[top[y]]) swap(x, y);
4         x = fa[top[x]];
5     }
6     if (dep[x] < dep[y]) swap(x, y);
7     return y;
8 }

```

2.2.2 树链剖分

```

1 vector<int> fa, siz, dep, son, dfn, rnk, top;
2 fa.assign(n + 1, 0);
3 siz.assign(n + 1, 0);
4 dep.assign(n + 1, 0);
5 son.assign(n + 1, 0);
6 dfn.assign(n + 1, 0);
7 rnk.assign(n + 1, 0);
8 top.assign(n + 1, 0);
9 void hld(int root) {
10     function<void(int)> dfs1 = [&](int t) {
11         dep[t] = dep[fa[t]] + 1;
12         siz[t] = 1;
13         for (auto& [to, w] : graph[t]) {
14             if (to == fa[t]) continue;

```

```

15         fa[to] = t;
16         dfs1(to);
17         if (siz[son[t]] < siz[to]) son[t] = to;
18         siz[t] += siz[to];
19     }
20 };
21 dfs1(root);
22 int dfn_tail = 0;
23 for (int i = 1; i <= n; i++) top[i] = i;
24 function<void(int)> dfs2 = [&](int t) {
25     dfn[t] = ++dfn_tail;
26     rnk[dfn_tail] = t;
27     if (!son[t]) return;
28     top[son[t]] = top[t];
29     dfs2(son[t]);
30     for (auto& [to, w] : graph[t]) {
31         if (to == fa[t] || to == son[t]) continue;
32         dfs2(to);
33     }
34 };
35 dfs2(root);
36 }

```

2.3 强连通分量

```

1 void tarjan(Graph& g1, Graph& g2) {
2     int dfn_tail = 0, cnt = 0;
3     vector<int> dfn(g1.n + 1, 0), low(g1.n + 1, 0), exist(g1.n + 1, 0),
4         belong(g1.n + 1, 0);
5     stack<int> sta;
6     function<void(int)> dfs = [&](int t) {
7         dfn[t] = low[t] = ++dfn_tail;
8         sta.push(t);
9         exist[t] = 1;
10        for (auto& [to] : g1.graph[t])
11            if (!dfn[to]) {
12                dfs(to);
13                low[t] = min(low[t], low[to]);
14            } else if (exist[to]) low[t] = min(low[t], dfn[to]);
15        if (dfn[t] == low[t]) {
16            cnt++;
17            while (int temp = sta.top()) {
18                belong[temp] = cnt;
19                exist[temp] = 0;
20                sta.pop();
21                if (temp == t) break;
22            }
23        }
24    };
25    for (int i = 1; i <= g1.n; i++)
26        if (!dfn[i]) dfs(i);

```

```
27 g2 = Graph(cnt);
28 for (int i = 1; i <= g1.n; i++) g2.w[belong[i]] += g1.w[i];
29 for (int i = 1; i <= g1.n; i++)
30     for (auto& [to] : g1.graph[i])
31         if (belong[i] != belong[to]) g2.add(belong[i], belong[to]);
32 }
```

2.4 拓扑排序

```
1 void toposort(Graph& g, vector<int>& dis) {
2     vector<int> in(g.n + 1, 0);
3     for (int i = 1; i <= g.n; i++)
4         for (auto& [to] : g.graph[i]) in[to]++;
5     queue<int> que;
6     for (int i = 1; i <= g.n; i++)
7         if (!in[i]) {
8             que.push(i);
9             dis[i] = g.w[i]; // dp
10        }
11    while (!que.empty()) {
12        int u = que.front();
13        que.pop();
14        for (auto& [to] : g.graph[u]) {
15            in[to]--;
16            dis[to] = max(dis[to], dis[u] + g.w[to]); // dp
17            if (!in[to]) que.push(to);
18        }
19    }
20 }
```

3 字符串

3.1 kmp

```

1 auto kmp(string& s) {
2     vector next(s.size(), -1);
3     for (int i = 1, j = -1; i < s.size(); i++) {
4         while (j >= 0 && s[i] != s[j + 1]) j = next[j];
5         if (s[i] == s[j + 1]) j++;
6         next[i] = j;
7     }
8     // next 意为长度
9     for (auto& i : next) i++;
10    return next;
11 }

```

3.2 哈希

```

1 constexpr int N = 2e6;
2 constexpr ll mod[2] = {2000000011, 2000000033}, base[2] = {20011, 20033};
3 vector<array<ll, 2>> pow_base(N);
4
5 pow_base[0][0] = pow_base[0][1] = 1;
6 for (int i = 1; i < N; i++) {
7     pow_base[i][0] = pow_base[i - 1][0] * base[0] % mod[0];
8     pow_base[i][1] = pow_base[i - 1][1] * base[1] % mod[1];
9 }
10
11 struct Hash {
12     int size;
13     vector<array<ll, 2>> hash;
14     Hash() {}
15     Hash(const string& s) {
16         size = s.size();
17         hash.resize(size);
18         hash[0][0] = hash[0][1] = s[0];
19         for (int i = 1; i < size; i++) {
20             hash[i][0] = (hash[i - 1][0] * base[0] + s[i]) % mod[0];
21             hash[i][1] = (hash[i - 1][1] * base[1] + s[i]) % mod[1];
22         }
23     }
24     array<ll, 2> operator[] (const array<int, 2>& range) const {
25         int l = range[0], r = range[1];
26         if (l == 0) return hash[r];
27         auto single_hash = [&](bool flag) {
28             return (hash[r][flag] -
29                     hash[l - 1][flag] * pow_base[r - l + 1][flag] % mod[flag] +
30                     mod[flag]) %
31                     mod[flag];
32         };
33         return {single_hash(0), single_hash(1)};

```

```
34     }  
35 };
```

3.3 manacher

```
1 void manacher(const string& _s, vector<int>& r) {  
2     string s(_s.size() * 2 + 1, '$');  
3     for (int i = 0; i < _s.size(); i++) s[2 * i + 1] = _s[i];  
4     r.resize(_s.size() * 2 + 1);  
5     for (int i = 0, maxr = 0, mid = 0; i < s.size(); i++) {  
6         if (i < maxr) r[i] = min(r[mid * 2 - i], maxr - i);  
7         while (i - r[i] - 1 >= 0 && i + r[i] + 1 < s.size() &&  
8             s[i - r[i] - 1] == s[i + r[i] + 1])  
9             ++r[i];  
10        if (i + r[i] > maxr) maxr = i + r[i], mid = i;  
11    }  
12 }
```

4 数学

4.1 扩展欧几里得

$$x = x + k * dx, y = y - k * dy$$

$$\text{若要求 } x > 0, k > -\frac{x}{dx} \Rightarrow k \geq \lceil \frac{-x+1}{dx} \rceil$$

$$\text{若要求 } x \geq 0, k > -\frac{x}{dx} \Rightarrow k \geq \lceil -\frac{x}{dx} \rceil$$

$$\text{若要求 } y > 0, k < \frac{y}{dy} \Rightarrow k \leq \lfloor \frac{y-1}{dy} \rfloor$$

$$\text{若要求 } y \geq 0, k < \frac{y}{dy} \Rightarrow k \leq \lfloor \frac{y}{dy} \rfloor$$

```

1 int __exgcd(int a, int b, int& x, int& y) {
2     if (!b) {
3         x = 1;
4         y = 0;
5         return a;
6     }
7     int d = __exgcd(b, a % b, x, y);
8     int t = x;
9     x = y;
10    y = t - (a / b) * y;
11    return d;
12 }
13 array<int, 2> exgcd(int a, int b, int c) {
14     int x, y;
15     int gcd_a_b = __exgcd(a, b, x, y);
16     if (c % gcd_a_b) return {INT_MAX, INT_MAX};
17     x *= c / gcd_a_b;
18     y *= c / gcd_a_b;
19     int dx = b / gcd_a_b;
20     int dy = a / gcd_a_b;
21     // x = x + k* dx y = y - k* dy
22     // 调整为 x >= 0 的最小解
23     int k = ceil(-1.0 * x / dx);
24     x += k * dx;
25     y -= k * dy;
26     return {x, y};
27 }

```

4.2 线性筛法

```

1 constexpr int N = 10000000;
2 array<int, N + 1> min_prime;
3 vector<int> primes;
4 bool ok = []() {
5     for (int i = 2; i <= N; i++) {
6         if (min_prime[i] == 0) {
7             min_prime[i] = i;
8             primes.push_back(i);
9         }
10        for (auto& j : primes) {
11            if (j > min_prime[i] || j > N / i) break;

```



```

12         min_prime[j * i] = j;
13     }
14 }
15 return 1;
16 }();

```

4.3 分解质因数

```

1 auto getprimes(int n) {
2     vector<array<int, 2>> res;
3     for (auto& i : primes) {
4         if (i > n / i) break;
5         if (n % i == 0) {
6             res.push_back({i, 0});
7             while (n % i == 0) {
8                 n /= i;
9                 res.back()[1]++;
10            }
11        }
12    }
13    if (n > 1) res.push_back({n, 1});
14    return res;
15 }

```

4.4 pollard rho

```

1 using LL = __int128_t;
2
3 random_device rd;
4 mt19937 seed(rd());
5
6 ll power(ll a, ll b, ll mod) {
7     ll res = 1;
8     while (b) {
9         if (b & 1) res = (LL)res * a % mod;
10        a = (LL)a * a % mod;
11        b >>= 1;
12    }
13    return res;
14 }
15
16 bool isprime(ll n) {
17     static array primes{2, 3, 5, 7, 11, 13, 17, 19, 23};
18     static unordered_map<ll, bool> S;
19     if (n < 2) return 0;
20     if (S.count(n)) return S[n];
21     ll d = n - 1, r = 0;
22     while (!(d & 1)) {
23         r++;
24         d >>= 1;

```

```

25     }
26     for (auto& a : primes) {
27         if (a == n) return S[n] = 1;
28         ll x = power(a, d, n);
29         if (x == 1 || x == n - 1) continue;
30         for (int i = 0; i < r - 1; i++) {
31             x = (LL)x * x % n;
32             if (x == n - 1) break;
33         }
34         if (x != n - 1) return S[n] = 0;
35     }
36     return S[n] = 1;
37 }
38
39 ll pollard_rho(ll n) {
40     ll s = 0, t = 0;
41     ll c = seed() % (n - 1) + 1;
42     ll val = 1;
43     for (int goal = 1;; goal *= 2, s = t, val = 1) {
44         for (int step = 1; step <= goal; step++) {
45             t = ((LL)t * t + c) % n;
46             val = (LL)val * abs(t - s) % n;
47             if (step % 127 == 0) {
48                 ll g = gcd(val, n);
49                 if (g > 1) return g;
50             }
51         }
52         ll g = gcd(val, n);
53         if (g > 1) return g;
54     }
55 }
56 auto getprimes(ll n) {
57     unordered_set<ll> S;
58     auto get = [&](auto self, ll n) {
59         if (n < 2) return;
60         if (isprime(n)) {
61             S.insert(n);
62             return;
63         }
64         ll mx = pollard_rho(n);
65         self(self, n / mx);
66         self(self, mx);
67     };
68     get(get, n);
69     return S;
70 }

```

4.5 组合数

```

1 array<modint, N + 1> fac, ifac;
2 fac[0] = ifac[0] = 1;

```

```

3 for (int i = 1; i <= N; i++) {
4     fac[i] = fac[i - 1] * i;
5     ifac[i] = fac[i].inv();
6 }
7 modint C(int n, int m) {
8     if (n < m) return 0;
9     if (m == 0) return 1;
10    if (n <= mod) return fac[n] * ifac[m] * ifac[n - m];
11    // n >= mod 时需要这个
12    return C(n % mod, m % mod) * C(n / mod, m / mod);
13 }

```

4.6 数论分块

求解形如 $\sum_{i=1}^n f(i)g(\lfloor \frac{n}{i} \rfloor)$ 的合式

$$s(n) = \sum_{i=1}^n f(i)$$

```

1 modint sqrt_decomposition(int n) {
2     auto s = [&](int x) { return x; };
3     auto g = [&](int x) { return x; };
4     modint res = 0;
5     while (l <= R) {
6         int r = n / (n / l);
7         res = res + (s(r) - s(l - 1)) * g(n / l);
8         l = r + 1;
9     }
10    return res;
11 }

```

4.7 积性函数

4.7.1 定义

函数 $f(n)$ 满足 $f(1) = 1$ 且 $\forall x, y \in \mathbf{N}^*, \gcd(x, y) = 1$ 都有 $f(xy) = f(x)f(y)$, 则 $f(n)$ 为积性函数。

函数 $f(n)$ 满足 $f(1) = 1$ 且 $\forall x, y \in \mathbf{N}^*$ 都有 $f(xy) = f(x)f(y)$, 则 $f(n)$ 为完全积性函数。

4.7.2 例子

- 单位函数: $\varepsilon(n) = [n = 1]$ 。(完全积性)
- 恒等函数: $\text{id}_k(n) = n^k$ 。(完全积性)
- 常数函数: $1(n) = 1$ 。(完全积性)
- 除数函数: $\sigma_k(n) = \sum_{d|n} d^k$ 。 $\sigma_0(n)$ 通常简记作 $d(n)$ 或 $\tau(n)$, $\sigma_1(n)$ 通常简记作 $\sigma(n)$ 。
- 欧拉函数: $\varphi(n) = \sum_{i=1}^n [\gcd(i, n) = 1]$ 。
- 莫比乌斯函数: $\mu(n) = \begin{cases} 1 & n = 1 \\ 0 & \exists d > 1, d^2 \mid n, \text{ 其中 } \omega(n) \text{ 表示 } n \text{ 的本质不同质因子个数, 它是} \\ (-1)^{\omega(n)} & \text{otherwise} \end{cases}$
一个加性函数。

4.8 狄利克雷卷积

对于两个数论函数 $f(x)$ 和 $g(x)$, 则它们的狄利克雷卷积得到的结果 $h(x)$ 定义为:

$$h(x) = \sum_{d|x} f(d)g\left(\frac{x}{d}\right) = \sum_{ab=x} f(a)g(b)$$

可以简记为: $h = f * g$ 。

4.8.1 性质

交换律: $f * g = g * f$ 。

结合律: $(f * g) * h = f * (g * h)$ 。

分配律: $(f + g) * h = f * h + g * h$ 。

等式的性质: $f = g$ 的充要条件是 $f * h = g * h$, 其中数论函数 $h(x)$ 要满足 $h(1) \neq 0$ 。

4.8.2 例子

- $\varepsilon = \mu * 1 \iff \varepsilon(n) = \sum_{d|n} \mu(d)$
- $id = \varphi * 1 \iff id(n) = \sum_{d|n} \varphi(d)$
- $d = 1 * 1 \iff d(n) = \sum_{d|n} 1$
- $\sigma = id * 1 \iff \sigma(n) = \sum_{d|n} d$
- $\varphi = \mu * id \iff \varphi(n) = \sum_{d|n} d \cdot \mu\left(\frac{n}{d}\right)$

4.9 莫比乌斯反演

4.9.1 莫比乌斯函数性质

- $\sum_{d|n} \mu(d) = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}$, 即 $\sum_{d|n} \mu(d) = \varepsilon(n)$, $\mu * 1 = \varepsilon$
- $[\gcd(i, j) = 1] = \sum_{d|\gcd(i, j)} \mu(d)$

4.9.2 莫比乌斯变换/反演

$f(n) = \sum_{d|n} g(d)$, 那么有 $g(n) = \sum_{d|n} \mu(d)f\left(\frac{n}{d}\right) = \sum_{n|d} \mu\left(\frac{d}{n}\right)f(d)$ 。

用狄利克雷卷积表示则为 $f = g * 1$, 有 $g = f * \mu$ 。

$f \rightarrow g$ 称为莫比乌斯反演, $g \rightarrow f$ 称为莫比乌斯反演。

4.10 杜教筛

杜教筛被用于处理一类数论函数的前缀和问题。对于数论函数 f , 杜教筛可以在低于线性时间的复杂度内计算 $S(n) = \sum_{i=1}^n f(i)$ 。

$$S(n) = \frac{\sum_{i=1}^n (f * g)(i) - \sum_{i=2}^n g(i)S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)}{g(1)}$$

可以构造恰当的数论函数 g 使得:

- 可以快速计算 $\sum_{i=1}^n (f * g)(i)$ 。
- 可以快速计算 g 的单点值, 用数论分块求解 $\sum_{i=2}^n g(i)S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$ 。

4.10.1 示例

```
1 ll sum_phi(ll n) {
2     if (n <= N) return sp[n];
3     if (sp2.count(n)) return sp2[n];
4     ll res = 0, l = 2;
5     while (l <= n) {
6         ll r = n / (n / l);
7         res = res + (r - l + 1) * sum_phi(n / l);
8         l = r + 1;
9     }
10    return sp2[n] = (ll)n * (n + 1) / 2 - res;
11 }
12 ll sum_miu(ll n) {
13     if (n <= N) return sm[n];
14     if (sm2.count(n)) return sm2[n];
15     ll res = 0, l = 2;
16     while (l <= n) {
17         ll r = n / (n / l);
18         res = res + (r - l + 1) * sum_miu(n / l);
19         l = r + 1;
20     }
21     return sm2[n] = 1 - res;
22 }
```

4.11 盒子与球

n 个球, m 个盒

球同	盒同	可空	公式
✓	✓	✓	$f_{n,m} = f_{n-1,m-1} + f_{n-m,m}$
✓	✓	✗	$f_{n-m,m}$
✗	✓	✓	$\sum_{i=1}^m g_{n,i}$
✗	✓	✗	$g_{n,m} = g_{n-1,m-1} + m * g_{n-1,m}$
✓	✗	✓	C_{n+m-1}^{m-1}
✓	✗	✗	C_{n-1}^{m-1}
✗	✗	✓	m^n
✗	✗	✗	$m! * g_{n,m}$

4.12 线性基

```
1 // 线性基
2 struct basis {
3     int rnk = 0;
4     array<ull, 64> p{};
5 }
```

```

6 // 将x插入此线性基中
7 void insert(ull x) {
8     for (int i = 63; i >= 0; i--) {
9         if ((x >> i) & 1) {
10             if (p[i] x ^= p[i];
11             else {
12                 for (int j = 0; j < i; j++)
13                     if (x >> j & 1) x ^= p[j];
14                 for (int j = i + 1; j <= 63; j++)
15                     if (p[j] >> i & 1) p[j] ^= x;
16                 p[i] = x;
17                 rnk++;
18                 break;
19             }
20         }
21     }
22 }
23
24 // 将另一个线性基插入此线性基中
25 void insert(basis other) {
26     for (int i = 0; i <= 63; i++) {
27         if (!other.p[i]) continue;
28         insert(other.p[i]);
29     }
30 }
31
32 // 最大异或值
33 ull max_basis() {
34     ull res = 0;
35     for (int i = 63; i >= 0; i--)
36         if ((res ^ p[i]) > res) res ^= p[i];
37     return res;
38 }
39 };

```

4.13 矩阵快速幂

```

1 constexpr ll mod = 2147493647;
2 struct Mat {
3     int n, m;
4     vector<vector<ll>> mat;
5     Mat(int n, int m) : n(n), m(m), mat(n, vector<ll>(m, 0)) {}
6     Mat(vector<vector<ll>> mat) : n(mat.size()), m(mat[0].size()), mat(mat) {}
7     Mat operator*(const Mat& other) {
8         assert(m == other.n);
9         Mat res(n, other.m);
10        for (int i = 0; i < res.n; i++)
11            for (int j = 0; j < res.m; j++)
12                for (int k = 0; k < m; k++)
13                    res.mat[i][j] =
14                        (res.mat[i][j] + mat[i][k] * other.mat[k][j] % mod) %

```

```
15         mod;
16     return res;
17 }
18 };
19 Mat ksm(Mat a, ll b) {
20     assert(a.n == a.m);
21     Mat res(a.n, a.m);
22     for (int i = 0; i < res.n; i++) res.mat[i][i] = 1;
23     while (b) {
24         if (b & 1) res = res * a;
25         b >>= 1;
26         a = a * a;
27     }
28     return res;
29 }
```

5 计算几何

```

1 double eps = 1e-8;
2 const double PI = acos(-1);
3 using T = ll;
4
5 template <typename T>
6 int cmp(T a, T b) {
7     return a != b ? a < b ? -1 : 1 : 0;
8 }
9
10 int cmp(double a, double b) {
11     double c = a - b;
12     if (abs(c) < eps) return 0;
13     return c < 0 ? -1 : 1;
14 }
15
16 // 向量
17 struct vec {
18     T x, y;
19     vec(T _x = 0, T _y = 0) : x(_x), y(_y) {}
20
21     // 模
22     double length2() const { return x * x + y * y; }
23     double length() const { return sqrt(x * x + y * y); }
24
25     // 与x轴正方向的夹角
26     double angle() const {
27         double angle = atan2(y, x);
28         if (angle < 0) angle += 2 * PI;
29         return angle;
30     }
31
32     // 逆时针旋转
33     vec &rotate(const double &theta) {
34         double tmp = x;
35         x = x * cos(theta) - y * sin(theta);
36         y = y * cos(theta) + tmp * sin(theta);
37         return *this;
38     }
39
40     bool operator==(const vec &other) const {
41         return !cmp(x, other.x) && !cmp(y, other.y);
42     }
43     bool operator<(const vec &other) const {
44         int tmp = cmp(angle(), other.angle());
45         if (tmp) return tmp == -1 ? 0 : 1;
46         tmp = cmp(x, other.x);
47         return tmp == -1 ? 0 : 1;
48     }
49
50     vec operator+(const vec &other) const { return {x + other.x, y + other.y}; }

```



```

51     vec operator-() const { return {-x, -y}; }
52     vec operator-(const vec &other) const { return -other + (*this); }
53     vec operator*(const T &other) const { return {x * other, y * other}; }
54     vec operator/(const T &other) const { return {x / other, y / other}; }
55     T operator*(const vec &other) const { return x * other.x + y * other.y; }
56
57     // 叉积 结果大于0, a到b为逆时针, 小于0, a到b顺时针,
58     // 等于0共线, 可能同向或反向, 结果绝对值表示 a b 形成的平行四边形的面积
59     T operator^(const vec &other) const { return x * other.y - y * other.x; }
60
61     friend istream &operator>>(istream &input, vec &data) {
62         input >> data.x >> data.y;
63         return input;
64     }
65     friend ostream &operator<<(ostream &output, const vec &data) {
66         output << fixed << setprecision(6);
67         output << data.x << " " << data.y;
68         return output;
69     }
70 };
71
72 bool xycmp(const vec &a, const vec &b) {
73     int tmp = cmp(a.x, b.x);
74     if (tmp) return tmp == -1 ? 0 : 1;
75     tmp = cmp(a.y, b.y);
76     return tmp == -1 ? 0 : 1;
77 }
78
79 T cross(const vec &a, const vec &b, const vec &c) { return (a - c) ^ (b - c); }
80
81 // 两点间的距离
82 T distance(const vec &a, const vec &b) {
83     return (a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y);
84 }
85
86 // 两向量夹角
87 double angle(const vec &a, const vec &b) {
88     double theta = abs(a.angle() - b.angle());
89     if (theta > PI) theta = 2 * PI - theta;
90     return theta;
91 }
92
93 // 判断点是否在凸包内
94 bool in_polygon(const vec &a, vector<vec> &p) {
95     int n = p.size();
96     if (n == 1) return a == p[0];
97     if (cross(a, p[1], p[0]) > 0 || cross(p.back(), a, p[0]) > 0) return 0;
98     auto cmp = [&p](vec &x, const vec &y) { return ((x - p[0]) ^ y) >= 0; };
99     int i = lower_bound(p.begin() + 2, p.end(), a - p[0], cmp) - p.begin() - 1;
100    return cross(p[(i + 1) % n], a, p[i]) >= 0;
101 }
102

```

```

103 // 多边形的面积
104 double polygon_area(vector<vec> &p) {
105     T area = 0;
106     for (int i = 1; i < p.size(); i++) area += p[i - 1] ^ p[i];
107     area += p.back() ^ p[0];
108     return abs(area / 2.0);
109 }
110
111 // 多边形的周长
112 double polygon_length(vector<vec> &p) {
113     double length = 0;
114     for (int i = 1; i < p.size(); i++) length += (p[i - 1] - p[i]).length();
115     length += (p.back() - p[0]).length();
116     return length;
117 }
118
119 // 以整点为顶点的线段上的整点个数
120 T count(const vec &a, const vec &b) {
121     vec c = a - b;
122     return gcd(abs(c.x), abs(c.y)) + 1;
123 }
124
125 // 以整点为顶点的多边形边上整点个数
126 T count(vector<vec> &p) {
127     T cnt = 0;
128     for (int i = 1; i < p.size(); i++) cnt += count(p[i - 1], p[i]);
129     cnt += count(p.back(), p[0]);
130     return cnt - p.size();
131 }
132
133 // 凸包直径的两个端点
134 auto polygon_dia(vector<vec> &p) {
135     int n = p.size();
136     array<vec, 2> res{};
137     if (n == 1) return res;
138     if (n == 2) return res = {p[0], p[1]};
139     T mx = 0;
140     for (int i = 0, j = 2; i < n; i++) {
141         while (abs(cross(p[i], p[(i + 1) % n], p[j])) <=
142             abs(cross(p[i], p[(i + 1) % n], p[(j + 1) % n])))
143             j = (j + 1) % n;
144         if (T tmp = distance(p[i], p[j]); tmp > mx) {
145             mx = tmp;
146             res = {p[i], p[j]};
147         }
148         if (T tmp = distance(p[(i + 1) % n], p[j]); tmp > mx) {
149             mx = tmp;
150             res = {p[(i + 1) % n], p[j]};
151         }
152     }
153     return res;
154 }

```

```

155
156 // 凸包
157 auto convex_hull(vector<vec> &p) {
158     sort(p.begin(), p.end(), xycmp);
159     int n = p.size();
160     vector sta(n + 1, 0);
161     vector v(n, false);
162     int tp = -1;
163     sta[++tp] = 0;
164     auto update = [&](int lim, int i) {
165         while (tp > lim &&
166             ((p[sta[tp]] - p[sta[tp - 1]]) ^ (p[i] - p[sta[tp]])) <= 0)
167             v[sta[tp--]] = 0;
168         sta[++tp] = i;
169         v[i] = 1;
170     };
171     for (int i = 1; i < n; i++) update(0, i);
172     int cnt = tp;
173     for (int i = n - 1; i >= 0; i--) {
174         if (v[i]) continue;
175         update(cnt, i);
176     }
177     vector<vec> res(tp);
178     for (int i = 0; i < tp; i++) res[i] = p[sta[i]];
179     return res;
180 }
181
182 // 闵可夫斯基和 两个点集的和构成一个凸包
183 auto minkowski(vector<vec> &a, vector<vec> &b) {
184     rotate(a.begin(), min_element(a.begin(), a.end(), xycmp), a.end());
185     rotate(b.begin(), min_element(b.begin(), b.end(), xycmp), b.end());
186     int n = a.size(), m = b.size();
187     vector<vec> c{a[0] + b[0]};
188     c.reserve(n + m);
189     int i = 0, j = 0;
190     while (i < n && j < m) {
191         vec x = a[(i + 1) % n] - a[i];
192         vec y = b[(j + 1) % m] - b[j];
193         c.push_back(c.back() + ((x ^ y) >= 0 ? (i++, x) : (j++, y)));
194     }
195     while (i + 1 < n) {
196         c.push_back(c.back() + a[(i + 1) % n] - a[i]);
197         i++;
198     }
199     while (j + 1 < m) {
200         c.push_back(c.back() + b[(j + 1) % m] - b[j]);
201         j++;
202     }
203     return c;
204 }
205
206 // 过凸多边形外一点求凸多边形的切线，返回切点下标

```

```

207 auto tangent(const vec &a, vector<vec> &p) {
208     int n = p.size();
209     int l = -1, r = -1;
210     for (int i = 0; i < n; i++) {
211         T tmp1 = cross(p[i], p[(i - 1 + n) % n], a);
212         T tmp2 = cross(p[i], p[(i + 1) % n], a);
213         if (l == -1 && tmp1 <= 0 && tmp2 <= 0) l = i;
214         else if (r == -1 && tmp1 >= 0 && tmp2 >= 0) r = i;
215     }
216     return array{l, r};
217 }
218
219 // 直线
220 struct line {
221     vec point, direction;
222     line(const vec &p = vec(), const vec &d = vec()) : point(p), direction(d) {}
223 };
224
225 // 点到直线距离
226 double distance(const vec &a, const line &b) {
227     return abs((b.point - a) ^ (b.point + b.direction - a)) /
228         b.direction.length();
229 }
230
231 // 判断点在直线哪边, 大于0在左边, 等于0在线上, 小于0在右边
232 T side_line(const vec &a, const line &b) { return b.direction ^ (a - b.point); }
233
234 // 两直线是否垂直
235 bool perpendicular(const line &a, const line &b) {
236     return !cmp(a.direction * b.direction, 0);
237 }
238
239 // 两直线是否平行
240 bool parallel(const line &a, const line &b) {
241     return !cmp(a.direction ^ b.direction, 0);
242 }
243
244 // 点的垂线是否与线段有交点
245 bool perpendicular(const vec &a, const line &b) {
246     vec perpen(-b.direction.y, b.direction.x);
247     bool cross1 = (perpen ^ (b.point - a)) > 0;
248     bool cross2 = (perpen ^ (b.point + b.direction - a)) > 0;
249     return cross1 != cross2;
250 }
251
252 // 点到线段距离
253 double distance_seg(const vec &a, const line &b) {
254     if (perpendicular(a, b)) return distance(a, b);
255     return min(distance(a, b.point), distance(a, b.point + b.direction));
256 }
257
258 // 两直线交点

```

```

259 vec intersection(T A, T B, T C, T D, T E, T F) {
260     return {(B * F - C * E) / (A * E - B * D),
261             (C * D - A * F) / (A * E - B * D)};
262 }
263
264 // 两直线交点
265 vec intersection(const line &a, const line &b) {
266     return intersection(a.direction.y, -a.direction.x,
267                         a.direction.x * a.point.y - a.direction.y * a.point.x,
268                         b.direction.y, -b.direction.x,
269                         b.direction.x * b.point.y - b.direction.y * b.point.x);
270 }
271
272 struct circle {
273     vec o;
274     double r;
275     circle(const vec &o, T _r) : o(_o), r(_r){};
276     // 点与圆的关系 -1在圆内, 0在圆上, 1在圆外
277     int relation(const vec &other) const {
278         double len = (other - o).length();
279         return cmp(len, r);
280     }
281     double area() { return PI * r * r; }
282 };
283
284 // 圆与直线交点
285 auto intersection(const circle &c, const line &l) {
286     double d = distance(c.o, l);
287     vector<vec> res;
288     double len = l.direction.length();
289     vec mid = l.point + l.direction * ((c.o - l.point) * l.direction / len);
290     if (!cmp(d, c.r)) res.push_back(mid);
291     else if (d < c.r) {
292         d = sqrt(c.r * c.r - d * d) / len;
293         res.push_back(mid + l.direction * d);
294         res.push_back(mid - l.direction * d);
295     }
296     return res;
297 }
298
299 // oab三角形与圆相交的面积
300 double area(const circle &c, const vec &a, const vec &b) {
301     vec oa = a - c.o, ob = b - c.o;
302     T cab = oa ^ ob;
303     if (!cmp(cab, 0)) return 0;
304     if (c.relation(a) != 1 && c.relation(b) != 1) return cab / 2.0;
305     vec ba = a - b, bo = -ob;
306     vec ab = -ba, ao = -oa;
307     auto r = c.r;
308     double ang;
309     double loa = oa.length(), lob = ob.length(), lab = ab.length();
310     double x =

```

```

311     (ba * bo + sqrt(r * r * lab * lab - (ba ^ bo) * (ba ^ bo))) / lab;
312     double y =
313         (ab * ao + sqrt(r * r * lab * lab - (ab ^ ao) * (ab ^ ao))) / lab;
314     if (cmp(lob, r) == -1 && cmp(loa, r) != -1) {
315         ang = cab * (1 - x / lab) / (r * loa);
316         ang = min(max((double)-1, ang), (double)1);
317         return (asin(ang) * r * r + cab * x / lab) / 2;
318     }
319     if (cmp(lob, r) != -1 && cmp(loa, r) == -1) {
320         ang = cab * (1 - y / lab) / (r * lob);
321         ang = min(max((double)-1, ang), (double)1);
322         return (asin(ang) * r * r + cab * y / lab) / 2;
323     }
324     if (cmp(abs(cab), r * lab) != -1 || cmp(ab * ao, 0) != 1 ||
325         cmp(ba * bo, 0) != 1) {
326         ang = cab / (loa * lob);
327         ang = min(max((double)-1, ang), (double)1);
328         double tmp = -asin(ang);
329         if (cmp(oa * ob, 0) == -1)
330             if (cmp(cab, 0) == -1) tmp -= PI;
331             else tmp += PI;
332         else tmp = -tmp;
333         return tmp * r * r / 2;
334     }
335     ang = cab * (1 - x / lab) / (r * loa);
336     ang = min(max((double)-1, ang), (double)1);
337     double ang2 = cab * (1 - y / lab) / (r * lob);
338     ang2 = min(max((double)-1, ang2), (double)1);
339     return ((asin(ang) + asin(ang2)) * r * r + cab * ((x + y) / lab - 1)) / 2;
340 }
341
342 // 多边形与圆相交的面积
343 double area(vector<vec> &p, circle c) {
344     double res = 0;
345     for (int i = 1; i < p.size(); i++) res += area(c, p[i - 1], p[i]);
346     res += area(c, p.back(), p[0]);
347     return abs(res);
348 }

```

5.1 扫描线

```

1 #define ls (pos << 1)
2 #define rs (ls | 1)
3 #define mid ((tree[pos].l + tree[pos].r) >> 1)
4 struct Rectangle {
5     ll x_l, y_l, x_r, y_r;
6 };
7 ll area(vector<Rectangle>& rec) {
8     struct Line {
9         ll x, y_up, y_down;
10        int pd;

```

```

11     };
12     vector<Line> line(rec.size() * 2);
13     vector<ll> y_set(rec.size() * 2);
14     for (int i = 0; i < rec.size(); i++) {
15         y_set[i * 2] = rec[i].y_l;
16         y_set[i * 2 + 1] = rec[i].y_r;
17         line[i * 2] = {rec[i].x_l, rec[i].y_r, rec[i].y_l, 1};
18         line[i * 2 + 1] = {rec[i].x_r, rec[i].y_r, rec[i].y_l, -1};
19     }
20     sort(y_set.begin(), y_set.end());
21     y_set.erase(unique(y_set.begin(), y_set.end()), y_set.end());
22     sort(line.begin(), line.end(), [](Line a, Line b) { return a.x < b.x; });
23     struct Data {
24         int l, r;
25         ll len, cnt, raw_len;
26     };
27     vector<Data> tree(4 * y_set.size());
28     function<void(int, int, int)> build = [&](int pos, int l, int r) {
29         tree[pos].l = l;
30         tree[pos].r = r;
31         if (l == r) {
32             tree[pos].raw_len = y_set[r + 1] - y_set[l];
33             tree[pos].cnt = tree[pos].len = 0;
34             return;
35         }
36         build(ls, l, mid);
37         build(rs, mid + 1, r);
38         tree[pos].raw_len = tree[ls].raw_len + tree[rs].raw_len;
39     };
40     function<void(int, int, int, int)> update = [&](int pos, int l, int r,
41                                                     int num) {
42         if (l <= tree[pos].l && tree[pos].r <= r) {
43             tree[pos].cnt += num;
44             tree[pos].len = tree[pos].cnt ? tree[pos].raw_len
45                             : tree[pos].l == tree[pos].r
46                             ? 0
47                             : tree[ls].len + tree[rs].len;
48             return;
49         }
50         if (l <= mid) update(ls, l, r, num);
51         if (r > mid) update(rs, l, r, num);
52         tree[pos].len =
53             tree[pos].cnt ? tree[pos].raw_len : tree[ls].len + tree[rs].len;
54     };
55     build(1, 0, y_set.size() - 2);
56     auto find_pos = [&](ll num) {
57         return lower_bound(y_set.begin(), y_set.end(), num) - y_set.begin();
58     };
59     ll res = 0;
60     for (int i = 0; i < line.size() - 1; i++) {
61         update(1, find_pos(line[i].y_down), find_pos(line[i].y_up) - 1,
62               line[i].pd);

```

```
63     res += (line[i + 1].x - line[i].x) * tree[1].len;  
64 }  
65 return res;  
66 }
```


6 杂项

6.1 高精度

```

1 struct bignum {
2     string num;
3
4     bignum() : num("0") {}
5     bignum(const string& num) : num(num) {
6         reverse(this->num.begin(), this->num.end());
7     }
8     bignum(ll num) : num(to_string(num)) {
9         reverse(this->num.begin(), this->num.end());
10    }
11
12    bignum operator+(const bignum& other) {
13        bignum res;
14        res.num.pop_back();
15        res.num.reserve(max(num.size(), other.num.size()) + 1);
16        for (int i = 0, j = 0, x; i < num.size() || i < other.num.size() || j;
17             i++) {
18            x = j;
19            j = 0;
20            if (i < num.size()) x += num[i] - '0';
21            if (i < other.num.size()) x += other.num[i] - '0';
22            if (x >= 10) j = 1, x -= 10;
23            res.num.push_back(x + '0');
24        }
25        res.num.capacity();
26        return res;
27    }
28
29    bignum operator*(const bignum& other) {
30        vector<int> res(num.size() + other.num.size() - 1, 0);
31        for (int i = 0; i < num.size(); i++)
32            for (int j = 0; j < other.num.size(); j++)
33                res[i + j] += (num[i] - '0') * (other.num[j] - '0');
34        int g = 0;
35        for (int i = 0; i < res.size(); i++) {
36            res[i] += g;
37            g = res[i] / 10;
38            res[i] %= 10;
39        }
40        while (g) {
41            res.push_back(g % 10);
42            g /= 10;
43        }
44        int lim = res.size();
45        while (lim > 1 && res[lim - 1] == 0) lim--;
46        bignum res2;
47        res2.num.resize(lim);
48        for (int i = 0; i < lim; i++) res2.num[i] = res[i] + '0';

```

```

49     return res2;
50 }
51
52 bool operator<(const bignum& other) {
53     if (num.size() == other.num.size())
54         for (int i = num.size() - 1; i >= 0; i--)
55             if (num[i] == other.num[i]) continue;
56             else return num[i] < other.num[i];
57     return num.size() < other.num.size();
58 }
59
60 friend istream& operator>>(istream& in, bignum& a) {
61     in >> a.num;
62     reverse(a.num.begin(), a.num.end());
63     return in;
64 }
65 friend ostream& operator<<(ostream& out, bignum a) {
66     reverse(a.num.begin(), a.num.end());
67     return out << a.num;
68 }
69 };

```

6.2 模运算

```

1 constexpr int N = 1e5;
2 constexpr int mod = 1e9 + 7;
3 struct modint {
4     int x;
5     modint(ll _x = 0) : x(_x % mod) {}
6     modint pow(ll b) const {
7         modint res(1), a = *this;
8         while (b) {
9             if (b & 1) res = res * a;
10            a = a * a;
11            b >>= 1;
12        }
13        return res;
14    }
15    modint inv() const { return pow(mod - 2); }
16    modint operator+(const modint& other) { return modint(x + other.x); }
17    modint operator-(const modint& other) { return modint(-other.x); }
18    modint operator-(const modint& other) { return modint(-other + *this); }
19    modint operator*(const modint& other) { return modint((ll)x * other.x); }
20    modint operator/(const modint& other) { return *this * other.inv(); }
21    friend istream& operator>>(istream& is, modint& other) {
22        ll _x;
23        is >> _x;
24        other = modint(_x);
25        return is;
26    }
27    friend ostream& operator<<(ostream& os, modint other) {

```

```

28     other.x = (other.x + mod) % mod;
29     return os << other.x;
30 }
31 };

```

6.3 分数

```

1 struct frac {
2     ll a, b;
3     frac() : a(0), b(1) {}
4     frac(ll _a, ll _b) : a(_a), b(_b) {
5         assert(b);
6         if (a) {
7             int tmp = gcd(a, b);
8             a /= tmp;
9             b /= tmp;
10        } else *this = frac();
11    }
12    frac operator+(const frac& other) {
13        return frac(a * other.b + other.a * b, b * other.b);
14    }
15    frac operator-() const {
16        frac res = *this;
17        res.a = -res.a;
18        return res;
19    }
20    frac operator-(const frac& other) const { return -other + *this; }
21    frac operator*(const frac& other) const {
22        return frac(a * other.a, b * other.b);
23    }
24    frac operator/(const frac& other) const {
25        assert(other.a);
26        return *this * frac(other.b, other.a);
27    }
28    bool operator<(const frac& other) const { return (*this - other).a < 0; }
29    bool operator<=(const frac& other) const { return (*this - other).a <= 0; }
30    bool operator>=(const frac& other) const { return (*this - other).a >= 0; }
31    bool operator>(const frac& other) const { return (*this - other).a > 0; }
32    bool operator==(const frac& other) const {
33        return a == other.a && b == other.b;
34    }
35    bool operator!=(const frac& other) const { return !(*this == other); }
36 };

```

6.4 表达式求值

```

1 // 格式化表达式
2 string format(const string& s1) {
3     stringstream ss(s1);
4     string s2;

```

```

5   char ch;
6   while ((ch = ss.get()) != EOF) {
7       if (ch == ' ') continue;
8       if (isdigit(ch)) s2 += ch;
9       else {
10          if (s2.back() != ' ') s2 += ' ';
11          s2 += ch;
12          s2 += ' ';
13      }
14  }
15  return s2;
16 }
17
18 // 中缀表达式转后缀表达式
19 string convert(const string& s1) {
20     unordered_map<char, int> rank{
21         {'+', 2}, {'-', 2}, {'*', 1}, {'/', 1}, {'^', 0}};
22     stringstream ss(s1);
23     string s2, temp;
24     stack<char> op;
25     while (ss >> temp) {
26         if (isdigit(temp[0])) s2 += temp + ' ';
27         else if (temp[0] == '(') op.push('(');
28         else if (temp[0] == ')') {
29             while (op.top() != '(') {
30                 s2 += op.top();
31                 s2 += ' ';
32                 op.pop();
33             }
34             op.pop();
35         } else {
36             while (!op.empty() && op.top() != '(' &&
37                 (temp[0] != '^' && rank[op.top()] <= rank[temp[0]] ||
38                 rank[op.top()] < rank[temp[0]])) {
39                 s2 += op.top();
40                 s2 += ' ';
41                 op.pop();
42             }
43             op.push(temp[0]);
44         }
45     }
46     while (!op.empty()) {
47         s2 += op.top();
48         s2 += ' ';
49         op.pop();
50     }
51     return s2;
52 }
53
54 // 计算后缀表达式
55 int calc(const string& s) {
56     stack<int> num;

```

```

57     stringstream ss(s);
58     string temp;
59     while (ss >> temp) {
60         if (isdigit(temp[0])) num.push(stoi(temp));
61         else {
62             int b = num.top();
63             num.pop();
64             int a = num.top();
65             num.pop();
66             if (temp[0] == '+') a += b;
67             else if (temp[0] == '-') a -= b;
68             else if (temp[0] == '*') a *= b;
69             else if (temp[0] == '/') a /= b;
70             else if (temp[0] == '^') a = ksm(a, b);
71             num.push(a);
72         }
73     }
74     return num.top();
75 }

```

6.5 日期

```

1  int month[] = {0, 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
2  int pre[13];
3  vector<int> leap;
4  struct Date {
5      int y, m, d;
6      bool operator<(const Date& other) const {
7          return array<int, 3>{y, m, d} <
8              array<int, 3>{other.y, other.m, other.d};
9      }
10     Date(const string& s) {
11         stringstream ss(s);
12         char ch;
13         ss >> y >> ch >> m >> ch >> d;
14     }
15     int dis() const {
16         int yd = (y - 1) * 365 +
17             (upper_bound(leap.begin(), leap.end(), y - 1) - leap.begin());
18         int md =
19             pre[m - 1] + (m > 2 && (y % 4 == 0 && y % 100 || y % 400 == 0));
20         return yd + md + d;
21     }
22     int dis(const Date& other) const { return other.dis() - dis(); }
23 };
24 for (int i = 1; i <= 12; i++) pre[i] = pre[i - 1] + month[2];
25 for (int i = 1; i <= 1000000; i++)
26     if (i % 4 == 0 && i % 100 || i % 400 == 0) leap.push_back(i);

```

6.6 对拍

linux/Mac

```

1 g++ a.cpp -o program/a -O2 -std=c++17
2 g++ b.cpp -o program/b -O2 -std=c++17
3 g++ suiji.cpp -o program/suiji -O2 -std=c++17
4
5 cnt=0
6
7 while true; do
8     let cnt++
9     echo TEST:$cnt
10
11     ./program/suiji > in
12     ./program/a < in > out.a
13     ./program/b < in > out.b
14
15     diff out.a out.b
16     if [ $? -ne 0 ];then break;fi
17 done

```

windows

```

1 @echo off
2
3 g++ a.cpp -o program/a -O2 -std=c++17
4 g++ b.cpp -o program/b -O2 -std=c++17
5 g++ suiji.cpp -o program/suiji -O2 -std=c++17
6
7 set cnt=0
8
9 :again
10     set /a cnt=cnt+1
11     echo TEST:%cnt%
12     .\program\suiji > in
13     .\program\a < in > out.a
14     .\program\b < in > out.b
15
16     fc output.a output.b
17 if not errorlevel 1 goto again

```

6.7 编译常用选项

```
1 -Wall -Woverflow -Wextra -Wpedantic -Wfloat-equal -Wshadow -fsanitize=address,undefined
```

6.8 开栈

不同的编译器可能命令不一样

```

1 -Wl,--stack=0x10000000
2 -Wl,-stack_size -Wl,0x10000000
3 -Wl,-z,stack-size=0x10000000

```

6.9 clang-format

```
1 BasedOnStyle: Google
2 IndentWidth: 4
3 ColumnLimit: 80
4 AllowShortIfStatementsOnASingleLine: AllIfsAndElse
5 AccessModifierOffset: -4
6 EmptyLineBeforeAccessModifier: Leave
7 RemoveBracesLLVM: true
```