ACM 常用算法模板

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1 数据结构

1.1 并查集

```
struct dsu {
1
2
      int n;
3
      vector<int> fa;
4
      dsu(int _n) : n(_n) {
5
          fa.resize(n + 1);
6
          iota(fa.begin(), fa.end(), 0);
7
      }
8
      int find(int x) { return x == fa[x] ? x : fa[x] = find(fa[x]); }
9
      int merge(int x, int y) {
10
          int fax = find(x), fay = find(y);
          if (fax == fay) return 0;
                                     // 一个集合
11
12
          return fa[find(x)] = find(y); // 合并到哪个集合了
13
      }
14 };
```

1.2 树状数组

一维

```
template <class T>
 1
2
   struct Fenwick_tree {
3
       Fenwick_tree(int n) : n(n), tree(n + 1, 0) {}
4
       T query(int 1, int r) {
5
           auto query = [&](int pos) {
6
                T res = 0;
7
                while (pos) {
8
                    res += tree[pos];
9
                    pos -= lowbit(pos);
10
                }
11
                return res;
12
           };
13
           return query(r) - query(l - 1);
14
15
       void update(int pos, T num) {
16
           while (pos <= n) {</pre>
17
                tree[pos] += num;
18
                pos += lowbit(pos);
19
20
       }
21
  private:
22
       int n;
23
       vector<T> tree;
24 };
```

二维

```
1 template <class T>
2 struct Fenwick_tree_2 {
```

```
3
       Fenwick_tree_2(int n, int m) : n(n), m(m), tree(n + 1, vector<T>(m + 1)) {}
4
       T query(int l1, int r1, int l2, int r2) {
5
           auto query = [&](int 1, int r) {
6
               T res = 0;
7
               for (int i = 1; i; i -= lowbit(i))
                    for (int j = r; j; j -= lowbit(j)) res += tree[i][j];
8
9
10
           };
11
           return query(12, r2) - query(12, r1 - 1) - query(11 - 1, r2) +
12
                  query(11 - 1, r1 - 1);
13
       void update(int x, int y, T num) {
14
15
           for (int i = x; i <= n; i += lowbit(i))</pre>
16
               for (int j = y; j <= m; j += lowbit(j)) tree[i][j] += num;</pre>
17
       }
18
  private:
19
       int n, m;
20
       vector<vector<T>> tree;
21 };
```

三维

```
template <class T>
 1
2
   struct Fenwick_tree_3 {
3
       Fenwick_tree_3(int n, int m, int k)
 4
           : n(n),
5
             m(m),
6
             k(k),
7
             tree(n + 1, vector\langle T \rangle (m + 1, vector \langle T \rangle (k + 1))) {}
8
       T query(int a, int b, int c, int d, int e, int f) {
9
           auto query = [&](int x, int y, int z) {
10
               T res = 0;
11
               for (int i = x; i; i -= lowbit(i))
12
                    for (int j = y; j; j -= lowbit(j))
13
                        for (int p = z; p; p -= lowbit(p)) res += tree[i][j][p];
14
               return res;
15
           };
16
           T res = query(d, e, f);
           res -= query(a - 1, e, f) + query(d, b - 1, f) + query(d, e, c - 1);
17
           res += query(a - 1, b - 1, f) + query(a - 1, e, c - 1) +
18
19
                   query(d, b - 1, c - 1);
           res -= query(a - 1, b - 1, c - 1);
20
21
           return res;
22
23
       void update(int x, int y, int z, T num) {
24
           for (int i = x; i <= n; i += lowbit(i))</pre>
25
               for (int j = y; j <= m; j += lowbit(j))</pre>
                    for (int p = z; p <= k; p += lowbit(p)) tree[i][j][p] += num;</pre>
26
27
       }
28
  private:
29
       int n, m, k;
30
       vector<vector<T>>> tree;
31 };
```

1.3 线段树

```
template <class Data, class Num>
2
  struct Segment_Tree {
3
       inline void update(int 1, int r, Num x) { update(1, 1, r, x); }
4
       inline Data query(int 1, int r) { return query(1, 1, r); }
5
       Segment_Tree(vector<Data>& a) {
6
           n = a.size();
7
           tree.assign(n * 4 + 1, {});
8
           build(a, 1, 1, n);
9
       }
10
  private:
11
       int n;
12
       struct Tree {
13
           int 1, r;
14
           Data data;
15
       };
16
       vector<Tree> tree;
17
       inline void pushup(int pos) {
18
           tree[pos].data = tree[pos << 1].data + tree[pos << 1 | 1].data;</pre>
19
       }
20
       inline void pushdown(int pos) {
21
           tree[pos << 1].data = tree[pos << 1].data + tree[pos].data.lazytag;</pre>
22
           tree[pos << 1 | 1].data =
23
               tree[pos << 1 | 1].data + tree[pos].data.lazytag;</pre>
24
           tree[pos].data.lazytag = Num::zero();
25
26
       void build(vector<Data>& a, int pos, int 1, int r) {
27
           tree[pos].l = 1;
28
           tree[pos].r = r;
29
           if (1 == r) {
30
               tree[pos].data = a[l - 1];
31
               return;
32
           }
33
           int mid = (tree[pos].l + tree[pos].r) >> 1;
           build(a, pos << 1, 1, mid);</pre>
34
35
           build(a, pos << 1 | 1, mid + 1, r);
36
           pushup(pos);
37
       }
       void update(int pos, int& 1, int& r, Num& x) {
38
39
           if (1 > tree[pos].r || r < tree[pos].1) return;</pre>
40
           if (1 <= tree[pos].1 && tree[pos].r <= r) {</pre>
               tree[pos].data = tree[pos].data + x;
41
42
               return;
43
           pushdown(pos);
44
45
           update(pos << 1, 1, r, x);
46
           update(pos << 1 | 1, 1, r, x);
47
           pushup(pos);
```

```
48
49
       Data query(int pos, int& 1, int& r) {
50
           if (1 > tree[pos].r || r < tree[pos].l) return Data::zero();</pre>
           if (1 <= tree[pos].1 && tree[pos].r <= r) return tree[pos].data;</pre>
51
52
           pushdown(pos);
53
           return query(pos << 1, 1, r) + query(pos << 1 | 1, 1, r);
54
       }
55
  };
56
  struct Num {
57
       11 add;
58
       inline static Num zero() { return {0}; }
59
       inline Num operator+(Num b) { return {add + b.add}; }
60 };
61
  struct Data {
62
       11 sum, len;
63
       Num lazytag;
64
       inline static Data zero() { return {0, 0, Num::zero()}; }
65
       inline Data operator+(Num b) {
66
           return {sum + len * b.add, len, lazytag + b};
67
68
       inline Data operator+(Data b) {
69
           return {sum + b.sum, len + b.len, Num::zero()};
70
       }
71
  };
```

1.4 可持久化线段树

```
constexpr int MAXN = 200000;
2
  vector<int> root(MAXN << 5);</pre>
3
  struct Persistent_seg {
4
       int n;
5
       struct Data {
6
           int ls, rs;
 7
           int val;
8
       };
9
       vector<Data> tree;
10
       Persistent_seg(int n, vector<int>& a) : n(n) { root[0] = build(1, n, a); }
11
       int build(int 1, int r, vector<int>& a) {
12
           if (1 == r) {
13
               tree.push_back({0, 0, a[1]});
14
               return tree.size() - 1;
15
           }
16
           int mid = 1 + r \gg 1;
17
           int ls = build(l, mid, a), rs = build(mid + 1, r, a);
           tree.push_back({ls, rs, tree[ls].val + tree[rs].val});
18
19
           return tree.size() - 1;
20
21
       int update(int rt, const int& idx, const int& val, int l, int r) {
22
           if (1 == r) {
23
               tree.push_back({0, 0, tree[rt].val + val});
24
               return tree.size() - 1;
```

```
25
26
           int mid = 1 + r >> 1, ls = tree[rt].ls, rs = tree[rt].rs;
27
           if (idx <= mid) ls = update(ls, idx, val, l, mid);</pre>
           else rs = update(rs, idx, val, mid + 1, r);
28
29
           tree.push_back({ls, rs, tree[ls].val + tree[rs].val});
30
           return tree.size() - 1;
31
       }
32
       int query(int rt1, int rt2, int k, int l, int r) {
33
           if (1 == r) return 1;
           int mid = l + r \gg 1;
34
           int lcnt = tree[tree[rt2].ls].val - tree[tree[rt1].ls].val;
35
           if (k <= lcnt) return query(tree[rt1].ls, tree[rt2].ls, k, l, mid);</pre>
36
37
           else return query(tree[rt1].rs, tree[rt2].rs, k - lcnt, mid + 1, r);
38
       }
39 };
```

1.5 st 表

```
auto lg = []() {
 1
       array<int, 10000001> lg;
3
       lg[1] = 0;
4
       for (int i = 2; i \leftarrow 10000000; i++) lg[i] = lg[i >> 1] + 1;
5
       return lg;
6
  }();
7
   template <typename T>
8
   struct st {
9
       int n;
10
       vector<vector<T>> a;
11
       st(vector<T>& _a) : n(_a.size()) {
12
           a.assign(lg[n] + 1, vector<int>(n));
           for (int i = 0; i < n; i++) a[0][i] = _a[i];</pre>
13
14
           for (int j = 1; j <= lg[n]; j++)</pre>
                for (int i = 0; i + (1 << j) - 1 < n; i++)</pre>
15
16
                    a[j][i] = max(a[j - 1][i], a[j - 1][i + (1 << (j - 1))]);
17
18
       T query(int 1, int r) {
19
           int k = lg[r - l + 1];
20
           return max(a[k][1], a[k][r - (1 << k) + 1]);</pre>
21
       }
22 };
```

2 图论

存图

```
1 struct Graph {
2    int n;
3    struct Edge {
4       int to, w;
5    };
```

```
6     vector<vector<Edge>> graph;
7     Graph(int _n) {
8          n = _n;
9          graph.assign(n + 1, vector<Edge>());
10     };
11     void add(int u, int v, int w) { graph[u].push_back({v, w}); }
12 };
```

2.1 最短路

dijkstra

```
void dij(Graph& graph, vector<int>& dis, int t) {
2
       vector<int> visit(graph.n + 1, 0);
3
       priority_queue<pair<int, int>> que;
 4
       dis[t] = 0;
5
       que.emplace(0, t);
 6
       while (!que.empty()) {
7
           int u = que.top().second;
8
           que.pop();
9
           if (visit[u]) continue;
10
           visit[u] = 1;
11
           for (auto& [to, w] : graph.graph[u]) {
12
               if (dis[to] > dis[u] + w) {
                   dis[to] = dis[u] + w;
13
14
                   que.emplace(-dis[to], to);
15
               }
16
           }
17
       }
18
```

2.2 树上问题

2.2.1 最近公公祖先

倍增法

```
vector<int> dep;
  vector<array<int, 21>> fa;
3
  dep.assign(n + 1, 0);
  fa.assign(n + 1, array<int, 21>{});
5
  void binary_jump(int root) {
6
       function<void(int)> dfs = [&](int t) {
7
           dep[t] = dep[fa[t][0]] + 1;
8
           for (auto& [to] : graph[t]) {
9
               if (to == fa[t][0]) continue;
10
               fa[to][0] = t;
               dfs(to);
11
12
           }
13
       };
14
       dfs(root);
15
       for (int j = 1; j <= 20; j++)</pre>
```

```
16
           for (int i = 1; i <= n; i++) fa[i][j] = fa[fa[i][j - 1]][j - 1];</pre>
17
18
  int lca(int x, int y) {
19
       if (dep[x] < dep[y]) swap(x, y);</pre>
       for (int i = 20; i >= 0; i--)
20
21
           if (dep[fa[x][i]] >= dep[y]) x = fa[x][i];
22
       if (x == y) return x;
23
       for (int i = 20; i >= 0; i--) {
24
           if (fa[x][i] != fa[y][i]) {
25
                x = fa[x][i];
26
                y = fa[y][i];
27
           }
28
       }
29
       return fa[x][0];
30 }
```

树剖

```
int lca(int x, int y) {
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]]) swap(x, y);
        x = fa[top[x]];
    }
    if (dep[x] < dep[y]) swap(x, y);
    return y;
}</pre>
```

2.2.2 树链剖分

```
1 vector<int> fa, siz, dep, son, dfn, rnk, top;
2 fa.assign(n + 1, 0);
3 \text{ siz.assign}(n + 1, 0);
4 | dep.assign(n + 1, 0);
5 \mid son.assign(n + 1, 0);
6 dfn.assign(n + 1, 0);
7
  rnk.assign(n + 1, 0);
8
  top.assign(n + 1, 0);
9
   void hld(int root) {
       function<void(int)> dfs1 = [&](int t) {
10
11
           dep[t] = dep[fa[t]] + 1;
12
           siz[t] = 1;
13
           for (auto& [to, w] : graph[t]) {
                if (to == fa[t]) continue;
14
15
                fa[to] = t;
16
                dfs1(to);
17
                if (siz[son[t]] < siz[to]) son[t] = to;</pre>
                siz[t] += siz[to];
18
19
           }
20
       };
21
       dfs1(root);
22
       int dfn_tail = 0;
23
       for (int i = 1; i <= n; i++) top[i] = i;</pre>
```

```
24
       function<void(int)> dfs2 = [&](int t) {
25
           dfn[t] = ++dfn_tail;
           rnk[dfn_tail] = t;
26
27
           if (!son[t]) return;
28
           top[son[t]] = top[t];
29
           dfs2(son[t]);
30
           for (auto& [to, w] : graph[t]) {
               if (to == fa[t] || to == son[t]) continue;
31
32
               dfs2(to);
33
           }
34
       };
35
       dfs2(root);
36 }
```

2.3 强连通分量

```
void tarjan(Graph& g1, Graph& g2) {
2
       int dfn_tail = 0, cnt = 0;
3
       vector < int > dfn(g1.n + 1, 0), low(g1.n + 1, 0), exist(g1.n + 1, 0),
4
           belong(g1.n + 1, 0);
5
       stack<int> sta;
6
       function<void(int)> dfs = [&](int t) {
7
           dfn[t] = low[t] = ++dfn_tail;
8
           sta.push(t);
9
           exist[t] = 1;
10
           for (auto& [to] : g1.graph[t])
               if (!dfn[to]) {
11
12
                    dfs(to);
13
                    low[t] = min(low[t], low[to]);
14
               } else if (exist[to]) low[t] = min(low[t], dfn[to]);
           if (dfn[t] == low[t]) {
15
16
               cnt++;
               while (int temp = sta.top()) {
17
18
                    belong[temp] = cnt;
19
                    exist[temp] = 0;
20
                    sta.pop();
21
                    if (temp == t) break;
22
               }
           }
23
24
       };
25
       for (int i = 1; i <= g1.n; i++)</pre>
26
           if (!dfn[i]) dfs(i);
27
       g2 = Graph(cnt);
       for (int i = 1; i <= g1.n; i++) g2.w[belong[i]] += g1.w[i];</pre>
28
29
       for (int i = 1; i <= g1.n; i++)</pre>
30
           for (auto& [to] : g1.graph[i])
31
               if (belong[i] != belong[to]) g2.add(belong[i], belong[to]);
32
```

2.4 拓扑排序

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```
void toposort(Graph& g, vector<int>& dis) {
 2
       vector < int > in(g.n + 1, 0);
3
       for (int i = 1; i <= g.n; i++)</pre>
4
           for (auto& [to] : g.graph[i]) in[to]++;
5
       queue<int> que;
6
       for (int i = 1; i <= g.n; i++)</pre>
7
           if (!in[i]) {
8
                que.push(i);
9
                dis[i] = g.w[i]; // dp
10
           }
       while (!que.empty()) {
11
12
           int u = que.front();
13
           que.pop();
           for (auto& [to] : g.graph[u]) {
14
15
                in[to]--;
16
                dis[to] = max(dis[to], dis[u] + g.w[to]); // dp
17
                if (!in[to]) que.push(to);
18
           }
19
       }
20 }
```

3 字符串

3.1 kmp

```
vector<int> kmp(string&& s) {
    vector<int> next(s.size(), -1);

for (int i = 1, j = -1; i < s.size(); i++) {
    while (j >= 0 && s[i] != s[j + 1]) j = next[j];

    if (s[i] == s[j + 1]) j++;
    next[i] = j;

}
return next;

9 }
```

3.2 哈希

```
1 constexpr int N = 2e6;
  constexpr ll mod[2] = {2000000011, 2000000033}, base[2] = {20011, 20033};
3
  vector<array<11, 2>> pow_base(N);
5
  pow_base[0][0] = pow_base[0][1] = 1;
 6
  for (int i = 1; i < N; i++) {
      pow_base[i][0] = pow_base[i - 1][0] * base[0] % mod[0];
8
       pow_base[i][1] = pow_base[i - 1][1] * base[1] % mod[1];
9
10
11
  struct Hash {
12
      int size;
```

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```
13
       vector<array<11, 2>> hash;
14
       Hash() {}
15
       Hash(const string& s) {
16
           size = s.size();
17
           hash.resize(size);
18
           hash[0][0] = hash[0][1] = s[0];
19
           for (int i = 1; i < size; i++) {</pre>
               hash[i][0] = (hash[i - 1][0] * base[0] + s[i]) % mod[0];
20
21
               hash[i][1] = (hash[i - 1][1] * base[1] + s[i]) % mod[1];
22
           }
23
       }
24
       array<11, 2> operator[](const array<int, 2>& range) const {
25
           int 1 = range[0], r = range[1];
26
           if (1 == 0) return hash[r];
           auto single_hash = [&](bool flag) {
27
28
               return (hash[r][flag] -
29
                        hash[l - 1][flag] * pow_base[r - l + 1][flag] % mod[flag] +
30
                        mod[flag]) %
31
                       mod[flag];
32
33
           return {single_hash(0), single_hash(1)};
34
       }
35 };
```

3.3 manacher

```
void manacher(const string& _s, vector<int>& r) {
2
       string s(_s.size() * 2 + 1, '$');
3
       for (int i = 0; i < _s.size(); i++) s[2 * i + 1] = _s[i];</pre>
4
       r.resize(_s.size() * 2 + 1);
5
       for (int i = 0, maxr = 0, mid = 0; i < s.size(); i++) {</pre>
6
           if (i < maxr) r[i] = min(r[mid * 2 - i], maxr - i);</pre>
7
           while (i - r[i] - 1 >= 0 \&\& i + r[i] + 1 < s.size() \&\&
8
                   s[i - r[i] - 1] == s[i + r[i] + 1])
9
10
           if (i + r[i] > maxr) maxr = i + r[i], mid = i;
11
       }
12 }
```

4 数学

4.1 线性筛法

```
auto [min_prime, prime] = []() {
   constexpr int N = 10000000;

   vector<int> min_prime(N + 1, 0), prime;

for (int i = 2; i <= N; i++) {
    if (min_prime[i] == 0) {
        min_prime[i] = i;
   }
}</pre>
```

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```
7
               prime.push_back(i);
8
           }
9
           for (auto& j : prime) {
               if (j > min_prime[i] || j > N / i) break;
10
11
               min_prime[j * i] = j;
12
           }
13
       }
14
       return tuple{min_prime, prime};
15 }();
```

4.2 分解质因数

```
1
  auto num_prime(int num) {
2
       vector<array<int, 2>> res;
3
       for (auto& i : prime) {
4
           if (i > num / i) break;
5
           if (num % i == 0) {
6
               res.push_back({i, 0});
7
               while (num % i == 0) {
8
                    num /= i;
9
                    res.back()[1]++;
10
               }
11
           }
12
13
       if (num > 1) res.push_back({num, 1});
14 }
```

4.3 组合数

4.4 盒子与球

n 个球, m 个盒

4.5 线性基

```
1 // 线性基
2 struct basis {
    array<unsigned ll, 64> p{};
4
5 // 将x插入此线性基中
```

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| 球同 | 盒同 | 可空 | 公式 |
|----|----|----|---|
| ✓ | ✓ | ✓ | $f_{n,m} = f_{n-1,m-1} + f_{n-m,m}$ |
| ✓ | ✓ | × | $f_{n-m,m}$ |
| × | ✓ | ✓ | $\sum_{i=1}^{m} g_{n,i}$ |
| × | ✓ | × | $g_{n,m} = g_{n-1,m-1} + m * g_{n-1,m}$ |
| ✓ | × | ✓ | C_{n+m-1}^{m-1} |
| ✓ | × | × | C_{n-1}^{m-1} |
| × | × | ✓ | m^n |
| × | × | × | $m!*g_{n,m}$ |

```
6
      void insert(unsigned ll x) {
7
           for (int i = 63; i >= 0; i--) {
8
               if ((x >> i) & 1) {
9
                   if (p[i]) x ^= p[i];
10
                   else {
11
                       p[i] = x;
12
                       break;
13
                   }
14
               }
           }
15
16
      }
17
18
      // 将另一个线性基插入此线性基中
19
      void insert(basis other) {
20
           for (int i = 0; i <= 63; i++) {</pre>
21
               if (!other.p[i]) continue;
22
               insert(other.p[i]);
23
           }
24
      }
25
26
      // 最大异或值
27
      unsigned ll max_basis() {
28
           unsigned 11 res = 0;
29
           for (int i = 63; i >= 0; i--)
30
               if ((res ^ p[i]) > res) res ^= p[i];
31
           return res;
32
      }
33 };
```

4.6 矩阵快速幂

```
constexpr ll mod = 2147493647;
struct Mat {
   int n, m;
   vector<vector<ll>>> mat;
```

```
5
       Mat(int n, int m) : n(n), m(n), mat(n, vector<ll>(m, 0)) {}
       Mat(vector<vector<ll>>> mat) : n(mat.size()), m(mat[0].size()), mat(mat) {}
6
7
       Mat operator*(const Mat& other) {
8
           assert(m == other.n);
9
           Mat res(n, other.m);
           for (int i = 0; i < res.n; i++)</pre>
10
11
                for (int j = 0; j < res.m; j++)</pre>
                    for (int k = 0; k < m; k++)
12
13
                         res.mat[i][j] =
                             (res.mat[i][j] + mat[i][k] * other.mat[k][j] \% mod) \%
14
15
                             mod;
16
           return res;
17
       }
18
  };
19
  Mat ksm(Mat a, 11 b) {
20
       assert(a.n == a.m);
21
       Mat res(a.n, a.m);
       for (int i = 0; i < res.n; i++) res.mat[i][i] = 1;</pre>
22
23
       while (b) {
24
           if (b & 1) res = res * a;
25
           b >>= 1;
           a = a * a;
26
27
28
       return res;
29 }
```

```
1 const double PI = acos(-1);
  constexpr double eps = 1e-8;
3 using T = 11;
4
5 template <typename T>
6
  bool equal(T a, T b) {
7
      return a == b;
8
9
  // 两浮点数是否相等
10 bool equal(double a, double b) { return abs(a - b) < eps; }
11
12 // 向量
13 struct vec {
14
      T x, y;
      vec(T_x = 0, T_y = 0) : x(_x), y(_y) {}
15
16
      // 模
17
18
      double length() const { return sqrt(x * x + y * y); }
19
20
      // 与x轴正方向的夹角
21
      double angle() const {
          double angle = atan2(y, x);
22
```

```
23
          if (angle < 0) angle += 2 * PI;</pre>
24
           return angle;
25
      }
26
      // 逆时针旋转
27
28
      void rotate(const double &theta) {
29
          double temp = x;
          x = x * cos(theta) - y * sin(theta);
30
31
          y = y * cos(theta) + temp * sin(theta);
32
      }
33
34
      bool operator==(const vec &other) const {
35
           return equal(x, other.x) && equal(y, other.y);
36
      bool operator<(const vec &other) const {</pre>
37
38
           return equal(angle(), other.angle()) ? x < other.x</pre>
39
                                                 : angle() < other.angle();</pre>
40
      }
41
      vec operator+(const vec &other) const { return {x + other.x, y + other.y}; }
42
43
      vec operator-() const { return {-x, -y}; }
44
      vec operator-(const vec &other) const { return -other + (*this); }
45
      vec operator*(const T &other) const { return {x * other, y * other}; }
46
      vec operator/(const T &other) const { return {x / other, y / other}; }
47
      T operator*(const vec &other) const { return x * other.x + y * other.y; }
48
      // 叉积 结果大于0,a在b的顺时针,小于0,a在b的逆时针,
49
      // 等于0共线,可能同向或反向,结果绝对值表示 a b形成的平行四边行的面积
50
51
      T operator^(const vec &other) const { return x * other.y - y * other.x; }
52
53
      friend istream &operator>>(istream &input, vec &data) {
54
           input >> data.x >> data.y;
55
           return input;
56
      }
57
      friend ostream &operator<<(ostream &output, const vec &data) {</pre>
           output << fixed << setprecision(6);</pre>
58
59
          output << data.x << " " << data.y;</pre>
60
          return output;
61
      }
62 };
63
  // 两点间的距离
64
65 T distance(const vec &a, const vec &b) {
66
      return sqrt((a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y));
67 }
68
69 // 两向量夹角
70
  double angle(const vec &a, const vec &b) {
71
      double theta = abs(a.angle() - b.angle());
72
      if (theta > PI) theta = 2 * PI - theta;
73
      return theta;
74
```

```
75
   // 多边形的面积
76
77
   double polygon_area(vector<vec> &p) {
78
        T area = 0;
79
        for (int i = 1; i < p.size(); i++) area += p[i - 1] ^ p[i];</pre>
80
        area += p.back() ^ p[0];
81
        return abs(area / 2.0);
82
83
   // 多边形的周长
84
85
   double polygon_length(vector<vec> &p) {
86
        double length = 0;
        for (int i = 1; i < p.size(); i++) length += (p[i - 1] - p[i]).length();</pre>
87
88
        length += (p.back() - p[0]).length();
89
        return length;
90
91
   // 多边形直径的两个端点
92
93
   auto polygon_dia(vector<vec> &p) {
94
        int n = p.size();
95
        array<vec, 2> res{};
        if (n <= 1) return res;</pre>
96
97
        if (n == 2) return res = {p[0], p[1]};
98
        T mx = 0;
99
        for (int i = 0, j = 2; i < n; i++) {
100
            while (abs((p[i] - p[j]) ^ (p[(i + 1) % n] - p[j])) <=</pre>
101
                   abs((p[i] - p[(j + 1) % n]) ^ (p[(i + 1) % n] - p[(j + 1) % n])))
102
                j = (j + 1) \% n;
103
            if (auto tmp = distance(p[i], p[j]); tmp > mx) {
104
                mx = tmp;
105
                res = {p[i], p[j]};
106
107
            if (auto tmp = distance(p[(i + 1) % n], p[j]); tmp > mx) {
108
                mx = tmp;
109
                res = \{p[(i + 1) \% n], p[j]\};
110
            }
111
112
        return res;
113 }
114
115
   // 凸包
116
   auto convex_hull(vector<vec> &p) {
117
        sort(p.begin(), p.end(), [](vec &a, vec &b) {
118
            return equal(a.x, b.x) ? a.y < b.y : a.x < b.x;</pre>
119
        });
120
121
        vector<int> sta(p.size() + 1, 0);
122
        vector<bool> v(p.size(), false);
123
        int tp = -1;
124
        sta[++tp] = 0;
125
126
        auto update_convex_hull = [&](int lim, int i) {
```

```
127
           while (tp > lim &&
                   ((p[sta[tp]] - p[sta[tp - 1]]) ^ (p[i] - p[sta[tp]])) <= 0)
128
129
                v[sta[tp--]] = 0;
130
            sta[++tp] = i;
131
           v[i] = 1;
132
       };
133
       for (int i = 1; i < p.size(); i++) update_convex_hull(0, i);</pre>
134
135
136
       int cnt = tp;
137
       for (int i = p.size() - 1; i >= 0; i--) {
138
           if (v[i]) continue;
139
           update_convex_hull(cnt, i);
140
       }
141
142
       vector<vec> res(tp);
143
       for (int i = 0; i < tp; i++) res[i] = p[sta[i]];</pre>
144
       return res;
145
146
147
   // 以整点为顶点的线段上的整点个数
   T count(const vec &a, const vec &b) {
148
149
       vec c = a - b;
150
       return gcd(abs(c.x), abs(c.y)) + 1;
151
152
   // 以整点为顶点的多边形边上整点个数
153
154
   T count(vector<vec> &p) {
155
       T cnt = 0;
156
       for (int i = 1; i < p.size(); i++) cnt += count(p[i - 1], p[i]);</pre>
157
       cnt += count(p.back(), p[0]);
158
       return cnt - p.size();
159
160
161 // 直线
162
   struct line {
163
       vec point, direction;
164
       line(const vec &p, const vec &d) : point(p), direction(d) {}
165 };
166
167
   // 点到直线距离
168
   double distance(const vec &a, const line &b) {
169
       return abs((b.point - a) ^ (b.point + b.direction - a)) /
170
              b.direction.length();
171
172
173 // 两直线是否垂直
174
   bool perpendicular(const line &a, const line &b) {
175
       return equal(a.direction * b.direction, 0);
176
   }
177
178 // 两直线是否平行
```

```
179 bool parallel(const line &a, const line &b) {
180
       return equal(a.direction ^ b.direction, 0);
181
182
   // 两直线交点
183
184
   vec intersection(T A, T B, T C, T D, T E, T F) {
       return {(B * F - C * E) / (A * E - B * D),
185
                (C * D - A * F) / (A * E - B * D);
186
187
188
   // 两直线交点
189
   vec intersection(const line &a, const line &b) {
190
       return intersection(a.direction.y, -a.direction.x,
191
192
                            a.direction.x * a.point.y - a.direction.y * a.point.x,
193
                            b.direction.y, -b.direction.x,
                            b.direction.x * b.point.y - b.direction.y * b.point.x);
194
195
```

6 杂项

6.1 高精度

```
struct bignum {
 1
2
       string num;
3
 4
       bignum() : num("0") {}
5
       bignum(const string& num) : num(num) {
6
           reverse(this->num.begin(), this->num.end());
7
       }
8
       bignum(ll num) : num(to_string(num)) {
9
           reverse(this->num.begin(), this->num.end());
10
       }
11
12
       bignum operator+(const bignum& other) {
13
           bignum res;
14
           res.num.pop_back();
15
           res.num.reserve(max(num.size(), other.num.size()) + 1);
16
           for (int i = 0, j = 0, x; i < num.size() || i < other.num.size() || j;
17
                i++) {
               x = j;
18
19
               j = 0;
20
               if (i < num.size()) x += num[i] - '0';</pre>
21
               if (i < other.num.size()) x += other.num[i] - '0';</pre>
               if (x >= 10) j = 1, x -= 10;
22
               res.num.push_back(x + '0');
23
24
           }
25
           res.num.capacity();
26
           return res;
27
       }
28
```

```
29
       bignum operator*(const bignum& other) {
30
            vector<int> res(num.size() + other.num.size() - 1, 0);
31
            for (int i = 0; i < num.size(); i++)</pre>
                for (int j = 0; j < other.num.size(); j++)</pre>
32
                    res[i + j] += (num[i] - '0') * (other.num[j] - '0');
33
34
            int g = 0;
            for (int i = 0; i < res.size(); i++) {</pre>
35
36
                res[i] += g;
37
                g = res[i] / 10;
38
                res[i] %= 10;
39
40
           while (g) {
41
                res.push_back(g % 10);
42
                g /= 10;
43
44
            int lim = res.size();
45
           while (lim > 1 && res[lim - 1] == 0) lim--;
46
           bignum res2;
47
            res2.num.resize(lim);
            for (int i = 0; i < lim; i++) res2.num[i] = res[i] + '0';</pre>
48
49
            return res2;
50
       }
51
52
       bool operator<(const bignum& other) {</pre>
53
           if (num.size() == other.num.size())
54
                for (int i = num.size() - 1; i >= 0; i--)
55
                    if (num[i] == other.num[i]) continue;
56
                    else return num[i] < other.num[i];</pre>
57
            return num.size() < other.num.size();</pre>
58
       }
59
60
       friend istream& operator>>(istream& in, bignum& a) {
61
           in >> a.num;
62
            reverse(a.num.begin(), a.num.end());
63
           return in;
64
65
       friend ostream& operator<<(ostream& out, bignum a) {</pre>
66
            reverse(a.num.begin(), a.num.end());
67
            return out << a.num;</pre>
68
       }
69 };
```

6.2 扫描线

```
struct Line {
8
9
           11 x, y_up, y_down;
10
           int pd;
11
12
       vector<Line> line(rec.size() * 2);
13
       vector<ll> y_set(rec.size() * 2);
14
       for (int i = 0; i < rec.size(); i++) {</pre>
15
           y_set[i * 2] = rec[i].y_l;
16
           y_{set[i * 2 + 1] = rec[i].y_r;}
17
           line[i * 2] = {rec[i].x_l, rec[i].y_r, rec[i].y_l, 1};
18
           line[i * 2 + 1] = {rec[i].x_r, rec[i].y_r, rec[i].y_l, -1};
19
20
       sort(y_set.begin(), y_set.end());
21
       y_set.erase(unique(y_set.begin(), y_set.end()), y_set.end());
22
       sort(line.begin(), line.end(), [](Line a, Line b) { return a.x < b.x; });</pre>
23
       struct Data {
24
           int 1, r;
25
           ll len, cnt, raw_len;
26
       };
27
       vector<Data> tree(4 * y_set.size());
28
       function<void(int, int, int)> build = [&](int pos, int 1, int r) {
29
           tree[pos].l = 1;
30
           tree[pos].r = r;
31
           if (1 == r) {
32
               tree[pos].raw_len = y_set[r + 1] - y_set[l];
33
               tree[pos].cnt = tree[pos].len = 0;
34
               return;
35
36
           build(ls, 1, mid);
37
           build(rs, mid + 1, r);
38
           tree[pos].raw_len = tree[ls].raw_len + tree[rs].raw_len;
39
       };
40
       function < void(int, int, int, int) > update = [&](int pos, int 1, int r,
41
                                                          int num) {
42
           if (1 <= tree[pos].1 && tree[pos].r <= r) {</pre>
43
               tree[pos].cnt += num;
44
               tree[pos].len = tree[pos].cnt ? tree[pos].raw_len
45
                                 : tree[pos].l == tree[pos].r
46
47
                                     : tree[ls].len + tree[rs].len;
48
               return;
49
50
           if (1 <= mid) update(ls, 1, r, num);</pre>
51
           if (r > mid) update(rs, l, r, num);
52
           tree[pos].len =
53
               tree[pos].cnt ? tree[pos].raw_len : tree[ls].len + tree[rs].len;
54
       };
       build(1, 0, y_set.size() - 2);
55
       auto find_pos = [&](ll num) {
56
57
           return lower_bound(y_set.begin(), y_set.end(), num) - y_set.begin();
58
       };
59
       11 \text{ res} = 0;
```

6.3 模运算

```
class modint {
2
       11 num;
3
  public:
 4
       modint(ll num = 0) : num(num % mod) {}
5
       modint pow(modint other) {
6
           modint res(1), temp = *this;
7
           while (other.num) {
8
               if (other.num & 1) res = res * temp;
9
               temp = temp * temp;
10
               other.num >>= 1;
11
12
           return res;
13
       }
14
       modint inv() { return this->pow(mod - 2); }
15
       modint operator+(modint other) { return modint(this->num + other.num); }
16
       modint operator-() { return {-this->num}; }
17
       modint operator-(modint other) { return modint(-other + *this); }
18
       modint operator*(modint other) { return modint(this->num * other.num); }
19
       modint operator/(modint other) { return *this * other.inv(); }
20
       friend istream& operator>>(istream& is, modint& other) {
21
           is >> other.num;
22
           other.num %= mod;
23
           return is;
24
       }
25
       friend ostream& operator<<(ostream& os, modint other) {</pre>
26
           other.num = (other.num + mod) % mod;
27
           return os << other.num;</pre>
28
       }
29 };
```

6.4 分数

```
1
  struct frac {
2
      11 a, b;
3
      frac() : a(0), b(1) {}
4
      frac(ll _a, ll _b) : a(_a), b(_b) {
5
          assert(b);
6
          if (a) {
7
              int tmp = gcd(a, b);
8
              a /= tmp;
```

```
9
               b /= tmp;
10
           } else *this = frac();
11
12
       frac operator+(const frac& other) {
           return frac(a * other.b + other.a * b, b * other.b);
13
14
15
       frac operator-() const {
16
           frac res = *this;
17
           res.a = -res.a;
18
           return res;
19
       }
       frac operator-(const frac& other) const { return -other + *this; }
20
       frac operator*(const frac& other) const {
21
           return frac(a * other.a, b * other.b);
22
23
       }
24
       frac operator/(const frac& other) const {
25
           assert(other.a);
           return *this * frac(other.b, other.a);
26
27
       }
       bool operator<(const frac& other) const { return (*this - other).a < 0; }</pre>
28
29
       bool operator<=(const frac& other) const { return (*this - other).a <= 0; }</pre>
       bool operator>=(const frac& other) const { return (*this - other).a >= 0; }
30
31
       bool operator>(const frac& other) const { return (*this - other).a > 0; }
32
       bool operator == (const frac& other) const {
33
           return a == other.a && b == other.b;
34
       }
35
       bool operator!=(const frac& other) const { return !(*this == other); }
36 };
```

6.5 表达式求值

```
// 格式化表达式
2
  string format(const string& s1) {
3
      stringstream ss(s1);
4
      string s2;
5
      char ch;
6
      while ((ch = ss.get()) != EOF) {
7
          if (ch == ' ') continue;
8
          if (isdigit(ch)) s2 += ch;
9
          else {
10
               if (s2.back() != ' ') s2 += ' ';
               s2 += ch;
11
               s2 += ' ';
12
13
          }
14
      }
15
      return s2;
16 }
17
  // 中缀表达式转后缀表达式
18
19 string convert(const string& s1) {
20
      unordered_map<char, int> rank{
```

```
21
           {'+', 2}, {'-', 2}, {'*', 1}, {'/', 1}, {'^', 0}};
22
       stringstream ss(s1);
23
       string s2, temp;
24
       stack<char> op;
25
       while (ss >> temp) {
26
           if (isdigit(temp[0])) s2 += temp + ' ';
           else if (temp[0] == '(') op.push('(');
27
           else if (temp[0] == ')') {
28
29
               while (op.top() != '(') {
30
                    s2 += op.top();
                    s2 += ' ';
31
32
                    op.pop();
33
               }
34
               op.pop();
35
           } else {
               while (!op.empty() && op.top() != '(' &&
36
                       (temp[0] != '^' && rank[op.top()] <= rank[temp[0]] ||</pre>
37
38
                        rank[op.top()] < rank[temp[0]])) {</pre>
39
                    s2 += op.top();
                    s2 += ' ';
40
41
                    op.pop();
42
43
               op.push(temp[0]);
44
           }
45
46
       while (!op.empty()) {
47
           s2 += op.top();
           s2 += ' ';
48
49
           op.pop();
50
51
       return s2;
52
53
  // 计算后缀表达式
54
  int calc(const string& s) {
56
       stack<int> num;
57
       stringstream ss(s);
58
       string temp;
59
       while (ss >> temp) {
60
           if (isdigit(temp[0])) num.push(stoi(temp));
61
           else {
62
               int b = num.top();
63
               num.pop();
64
               int a = num.top();
65
               num.pop();
               if (temp[0] == '+') a += b;
66
67
               else if (temp[0] == '-') a -= b;
               else if (temp[0] == '*') a *= b;
68
69
               else if (temp[0] == '/') a /= b;
               else if (temp[0] == '^') a = ksm(a, b);
70
71
               num.push(a);
72
```

```
73 }
74 return num.top();
75 }
```

6.6 对拍

linux/Mac

```
g++ a.cpp -o program/a -O2 -std=c++17
  g++ b.cpp -o program/b -02 -std=c++17
  g++ suiji.cpp -o program/suiji -O2 -std=c++17
5
  cnt=0
6
7
  while true; do
8
      let cnt++
9
       echo TEST:$cnt
10
       ./program/suiji > in
11
12
       ./program/a < in > out.a
       ./program/b < in > out.b
13
14
15
       diff out.a out.b
16
      if [ $? -ne 0 ]; then break; fi
17
  done
```

windows

```
@echo off
 1
2
3
  g++ a.cpp -o program/a -O2 -std=c++17
  g++ b.cpp -o program/b -02 -std=c++17
  g++ suiji.cpp -o program/suiji -O2 -std=c++17
5
6
7
  set cnt=0
8
9
  :again
10
       set /a cnt=cnt+1
       echo TEST:%cnt%
11
12
       .\program\suiji > in
13
       .\program\a < in > out.a
14
       .\program\b < in > out.b
15
16
       fc output.a output.b
17 if not errorlevel 1 goto again
```

6.7 开栈

任选一种

```
1 -Wl,--stack=0x10000000
2 -Wl,-stack_size -Wl,0x10000000
```

```
3 -Wl,-z,stack-size=0x10000000
```

6.8 日期

```
1 int month[] = {0, 31, 28, 31, 30, 31, 30, 31, 30, 31, 30, 31};
2 int pre[13];
3
  vector<int> leap;
  struct Date {
5
       int y, m, d;
6
       bool operator<(const Date& other) const {</pre>
7
           return array<int, 3>{y, m, d} <</pre>
8
                  array<int, 3>{other.y, other.m, other.d};
9
       }
10
       Date(const string& s) {
11
           stringstream ss(s);
12
           char ch;
13
           ss >> y >> ch >> m >> ch >> d;
14
       }
15
       int dis() const {
16
           int yd = (y - 1) * 365 +
17
                     (upper_bound(leap.begin(), leap.end(), y - 1) - leap.begin());
18
19
               pre[m - 1] + (m > 2 && (y % 4 == 0 && y % 100 || y % 400 == 0));
20
           return yd + md + d;
21
22
       int dis(const Date& other) const { return other.dis() - dis(); }
23 };
24 for (int i = 1; i \leftarrow 12; i++) pre[i] = pre[i - 1] + month[2];
25
  for (int i = 1; i <= 1000000; i++)</pre>
       if (i % 4 == 0 && i % 100 || i % 400 == 0) leap.push_back(i);
```