

# ACM 常用算法模板

therehello

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# 1 数据结构

## 1.1 并查集

```

1 struct dsu {
2     int n;
3     vector<int> fa;
4     dsu(int _n) : n(_n) {
5         fa.resize(n + 1);
6         iota(fa.begin(), fa.end(), 0);
7     }
8     int find(int x) { return x == fa[x] ? x : fa[x] = find(fa[x]); }
9     int merge(int x, int y) {
10         int fax = find(x), fay = find(y);
11         if (fax == fay) return 0; // 一个集合
12         return fa[find(x)] = find(y); // 合并到哪个集合了
13     }
14 };

```

## 1.2 树状数组

一维

```

1 template <class T>
2 struct Fenwick_tree {
3     Fenwick_tree(int n) : n(n), tree(n + 1, 0) {}
4     T query(int l, int r) {
5         auto query = [&](int pos) {
6             T res = 0;
7             while (pos) {
8                 res += tree[pos];
9                 pos -= lowbit(pos);
10            }
11            return res;
12        };
13        return query(r) - query(l - 1);
14    }
15    void update(int pos, T num) {
16        while (pos <= n) {
17            tree[pos] += num;
18            pos += lowbit(pos);
19        }
20    }
21
22 private:
23     int n;
24     vector<T> tree;
25 };

```

二维

```

1 template <class T>

```

```

2 struct Fenwick_tree_2 {
3     Fenwick_tree_2(int n, int m) : n(n), m(m), tree(n + 1, vector<T>(m + 1)) {}
4     T query(int l1, int r1, int l2, int r2) {
5         auto query = [&](int l, int r) {
6             T res = 0;
7             for (int i = l; i; i -= lowbit(i))
8                 for (int j = r; j; j -= lowbit(j)) res += tree[i][j];
9             return res;
10        };
11        return query(l2, r2) - query(l2, r1 - 1) - query(l1 - 1, r2) +
12            query(l1 - 1, r1 - 1);
13    }
14    void update(int x, int y, T num) {
15        for (int i = x; i <= n; i += lowbit(i))
16            for (int j = y; j <= m; j += lowbit(j)) tree[i][j] += num;
17    }
18
19 private:
20     int n, m;
21     vector<vector<T>> tree;
22 };

```

### 三维

```

1 template <class T>
2 struct Fenwick_tree_3 {
3     Fenwick_tree_3(int n, int m, int k)
4         : n(n),
5           m(m),
6           k(k),
7           tree(n + 1, vector<vector<T>>(m + 1, vector<T>(k + 1))) {}
8     T query(int a, int b, int c, int d, int e, int f) {
9         auto query = [&](int x, int y, int z) {
10             T res = 0;
11             for (int i = x; i; i -= lowbit(i))
12                 for (int j = y; j; j -= lowbit(j))
13                     for (int p = z; p; p -= lowbit(p)) res += tree[i][j][p];
14             return res;
15        };
16        T res = query(d, e, f);
17        res -= query(a - 1, e, f) + query(d, b - 1, f) + query(d, e, c - 1);
18        res += query(a - 1, b - 1, f) + query(a - 1, e, c - 1) +
19            query(d, b - 1, c - 1);
20        res -= query(a - 1, b - 1, c - 1);
21        return res;
22    }
23    void update(int x, int y, int z, T num) {
24        for (int i = x; i <= n; i += lowbit(i))
25            for (int j = y; j <= m; j += lowbit(j))
26                for (int p = z; p <= k; p += lowbit(p)) tree[i][j][p] += num;
27    }
28
29 private:

```

```

30     int n, m, k;
31     vector<vector<vector<T>>> tree;
32 };

```

### 1.3 线段树

```

1  template <class Data, class Num>
2  struct Segment_Tree {
3      inline void update(int l, int r, Num x) { update(1, l, r, x); }
4      inline Data query(int l, int r) { return query(1, l, r); }
5      Segment_Tree(vector<Data>& a) {
6          n = a.size();
7          tree.assign(n * 4 + 1, {});
8          build(a, 1, 1, n);
9      }
10
11  private:
12      int n;
13      struct Tree {
14          int l, r;
15          Data data;
16      };
17      vector<Tree> tree;
18      inline void pushup(int pos) {
19          tree[pos].data = tree[pos << 1].data + tree[pos << 1 | 1].data;
20      }
21      inline void pushdown(int pos) {
22          tree[pos << 1].data = tree[pos << 1].data + tree[pos].data.lazytag;
23          tree[pos << 1 | 1].data =
24              tree[pos << 1 | 1].data + tree[pos].data.lazytag;
25          tree[pos].data.lazytag = Num::zero();
26      }
27      void build(vector<Data>& a, int pos, int l, int r) {
28          tree[pos].l = l;
29          tree[pos].r = r;
30          if (l == r) {
31              tree[pos].data = a[l - 1];
32              return;
33          }
34          int mid = (tree[pos].l + tree[pos].r) >> 1;
35          build(a, pos << 1, l, mid);
36          build(a, pos << 1 | 1, mid + 1, r);
37          pushup(pos);
38      }
39      void update(int pos, int& l, int& r, Num& x) {
40          if (l > tree[pos].r || r < tree[pos].l) return;
41          if (l <= tree[pos].l && tree[pos].r <= r) {
42              tree[pos].data = tree[pos].data + x;
43              return;
44          }
45          pushdown(pos);

```

```

46     update(pos << 1, l, r, x);
47     update(pos << 1 | 1, l, r, x);
48     pushup(pos);
49 }
50 Data query(int pos, int& l, int& r) {
51     if (l > tree[pos].r || r < tree[pos].l) return Data::zero();
52     if (l <= tree[pos].l && tree[pos].r <= r) return tree[pos].data;
53     pushdown(pos);
54     return query(pos << 1, l, r) + query(pos << 1 | 1, l, r);
55 }
56 };
57 struct Num {
58     ll add;
59     inline static Num zero() { return {0}; }
60     inline Num operator+(Num b) { return {add + b.add}; }
61 };
62 struct Data {
63     ll sum, len;
64     Num lazytag;
65     inline static Data zero() { return {0, 0, Num::zero()}; }
66     inline Data operator+(Num b) {
67         return {sum + len * b.add, len, lazytag + b};
68     }
69     inline Data operator+(Data b) {
70         return {sum + b.sum, len + b.len, Num::zero()};
71     }
72 };

```

## 1.4 可持久化线段树

```

1 constexpr int MAXN = 200000;
2 vector<int> root(MAXN << 5);
3 struct Persistent_seg {
4     int n;
5     struct Data {
6         int ls, rs;
7         int val;
8     };
9     vector<Data> tree;
10    Persistent_seg(int n, vector<int>& a) : n(n) { root[0] = build(1, n, a); }
11    int build(int l, int r, vector<int>& a) {
12        if (l == r) {
13            tree.push_back({0, 0, a[l]});
14            return tree.size() - 1;
15        }
16        int mid = l + r >> 1;
17        int ls = build(l, mid, a), rs = build(mid + 1, r, a);
18        tree.push_back({ls, rs, tree[ls].val + tree[rs].val});
19        return tree.size() - 1;
20    }
21    int update(int rt, const int& idx, const int& val, int l, int r) {

```

```

22     if (l == r) {
23         tree.push_back({0, 0, tree[rt].val + val});
24         return tree.size() - 1;
25     }
26     int mid = l + r >> 1, ls = tree[rt].ls, rs = tree[rt].rs;
27     if (idx <= mid)
28         ls = update(ls, idx, val, l, mid);
29     else
30         rs = update(rs, idx, val, mid + 1, r);
31     tree.push_back({ls, rs, tree[ls].val + tree[rs].val});
32     return tree.size() - 1;
33 }
34 int query(int rt1, int rt2, int k, int l, int r) {
35     if (l == r) return l;
36     int mid = l + r >> 1;
37     int lcnt = tree[tree[rt2].ls].val - tree[tree[rt1].ls].val;
38     if (k <= lcnt)
39         return query(tree[rt1].ls, tree[rt2].ls, k, l, mid);
40     else
41         return query(tree[rt1].rs, tree[rt2].rs, k - lcnt, mid + 1, r);
42 }
43 };

```

## 1.5 st 表

```

1 auto lg = []() {
2     array<int, 10000001> lg;
3     lg[1] = 0;
4     for (int i = 2; i <= 10000000; i++) lg[i] = lg[i >> 1] + 1;
5     return lg;
6 }();
7 template <typename T>
8 struct st {
9     int n;
10    vector<vector<T>> a;
11    st(vector<T>& _a) : n(_a.size()) {
12        a.assign(lg[n] + 1, vector<int>(n));
13        for (int i = 0; i < n; i++) a[0][i] = _a[i];
14        for (int j = 1; j <= lg[n]; j++)
15            for (int i = 0; i + (1 << j) - 1 < n; i++)
16                a[j][i] = max(a[j - 1][i], a[j - 1][i + (1 << (j - 1))]);
17    }
18    T query(int l, int r) {
19        int k = lg[r - l + 1];
20        return max(a[k][l], a[k][r - (1 << k) + 1]);
21    }
22 };

```

## 2 图论

存图

```

1 struct Graph {
2     int n;
3     struct Edge {
4         int to, w;
5     };
6     vector<vector<Edge>> graph;
7     Graph(int _n) {
8         n = _n;
9         graph.assign(n + 1, vector<Edge>());
10    };
11    void add(int u, int v, int w) { graph[u].push_back({v, w}); }
12 };

```

### 2.1 最短路

dijkstra

```

1 void dij(Graph& graph, vector<int>& dis, int t) {
2     vector<int> visit(graph.n + 1, 0);
3     priority_queue<pair<int, int>> que;
4     dis[t] = 0;
5     que.emplace(0, t);
6     while (!que.empty()) {
7         int u = que.top().second;
8         que.pop();
9         if (visit[u]) continue;
10        visit[u] = 1;
11        for (auto& [to, w] : graph.graph[u]) {
12            if (dis[to] > dis[u] + w) {
13                dis[to] = dis[u] + w;
14                que.emplace(-dis[to], to);
15            }
16        }
17    }
18 }

```

### 2.2 树上问题

#### 2.2.1 最近公公祖先

倍增法

```

1 vector<int> dep;
2 vector<array<int, 21>> fa;
3 dep.assign(n + 1, 0);
4 fa.assign(n + 1, array<int, 21>{});
5 void binary_jump(int root) {
6     function<void(int)> dfs = [&](int t) {

```



```

7      dep[t] = dep[fa[t][0]] + 1;
8      for (auto& [to] : graph[t]) {
9          if (to == fa[t][0]) continue;
10         fa[to][0] = t;
11         dfs(to);
12     }
13 };
14 dfs(root);
15 for (int j = 1; j <= 20; j++)
16     for (int i = 1; i <= n; i++) fa[i][j] = fa[fa[i][j - 1]][j - 1];
17 }
18 int lca(int x, int y) {
19     if (dep[x] < dep[y]) swap(x, y);
20     for (int i = 20; i >= 0; i--) {
21         if (dep[fa[x][i]] >= dep[y]) x = fa[x][i];
22     }
23     if (x == y) return x;
24     for (int i = 20; i >= 0; i--) {
25         if (fa[x][i] != fa[y][i]) {
26             x = fa[x][i];
27             y = fa[y][i];
28         }
29     }
30     return fa[x][0];
31 }

```

树剖

```

1 int lca(int x, int y) {
2     while (top[x] != top[y]) {
3         if (dep[top[x]] < dep[top[y]]) swap(x, y);
4         x = fa[top[x]];
5     }
6     if (dep[x] < dep[y]) swap(x, y);
7     return y;
8 }

```

### 2.2.2 树链剖分

```

1 vector<int> fa, siz, dep, son, dfn, rnk, top;
2 fa.assign(n + 1, 0);
3 siz.assign(n + 1, 0);
4 dep.assign(n + 1, 0);
5 son.assign(n + 1, 0);
6 dfn.assign(n + 1, 0);
7 rnk.assign(n + 1, 0);
8 top.assign(n + 1, 0);
9 void hld(int root) {
10     function<void(int)> dfs1 = [&](int t) {
11         dep[t] = dep[fa[t]] + 1;
12         siz[t] = 1;
13         for (auto& [to, w] : graph[t]) {

```

```

14         if (to == fa[t]) continue;
15         fa[to] = t;
16         dfs1(to);
17         if (siz[son[t]] < siz[to]) son[t] = to;
18         siz[t] += siz[to];
19     }
20 };
21 dfs1(root);
22 int dfn_tail = 0;
23 for (int i = 1; i <= n; i++) top[i] = i;
24 function<void(int)> dfs2 = [&](int t) {
25     dfn[t] = ++dfn_tail;
26     rnk[dfn_tail] = t;
27     if (!son[t]) return;
28     top[son[t]] = top[t];
29     dfs2(son[t]);
30     for (auto& [to, w] : graph[t]) {
31         if (to == fa[t] || to == son[t]) continue;
32         dfs2(to);
33     }
34 };
35 dfs2(root);
36 }

```

## 2.3 强连通分量

```

1 void tarjan(Graph& g1, Graph& g2) {
2     int dfn_tail = 0, cnt = 0;
3     vector<int> dfn(g1.n + 1, 0), low(g1.n + 1, 0), exist(g1.n + 1, 0),
4         belong(g1.n + 1, 0);
5     stack<int> sta;
6     function<void(int)> dfs = [&](int t) {
7         dfn[t] = low[t] = ++dfn_tail;
8         sta.push(t);
9         exist[t] = 1;
10        for (auto& [to] : g1.graph[t]) {
11            if (!dfn[to]) {
12                dfs(to);
13                low[t] = min(low[t], low[to]);
14            } else if (exist[to])
15                low[t] = min(low[t], dfn[to]);
16        }
17        if (dfn[t] == low[t]) {
18            cnt++;
19            while (int temp = sta.top()) {
20                belong[temp] = cnt;
21                exist[temp] = 0;
22                sta.pop();
23                if (temp == t) break;
24            }
25        }
26    }
27 }

```

```

26     };
27     for (int i = 1; i <= g1.n; i++)
28         if (!dfn[i]) dfs(i);
29     g2 = Graph(cnt);
30     for (int i = 1; i <= g1.n; i++) g2.w[belong[i]] += g1.w[i];
31     for (int i = 1; i <= g1.n; i++)
32         for (auto& [to] : g1.graph[i])
33             if (belong[i] != belong[to]) g2.add(belong[i], belong[to]);
34 }

```

## 2.4 拓扑排序

```

1 void toposort(Graph& g, vector<int>& dis) {
2     vector<int> in(g.n + 1, 0);
3     for (int i = 1; i <= g.n; i++)
4         for (auto& [to] : g.graph[i]) in[to]++;
5     queue<int> que;
6     for (int i = 1; i <= g.n; i++)
7         if (!in[i]) {
8             que.push(i);
9             dis[i] = g.w[i]; // dp
10        }
11    while (!que.empty()) {
12        int u = que.front();
13        que.pop();
14        for (auto& [to] : g.graph[u]) {
15            in[to]--;
16            dis[to] = max(dis[to], dis[u] + g.w[to]); // dp
17            if (!in[to]) que.push(to);
18        }
19    }
20 }

```

## 3 字符串

### 3.1 kmp

```

1 vector<int> kmp(string&& s) {
2     vector<int> next(s.size(), -1);
3     for (int i = 1, j = -1; i < s.size(); i++) {
4         while (j >= 0 && s[i] != s[j + 1]) j = next[j];
5         if (s[i] == s[j + 1]) j++;
6         next[i] = j;
7     }
8     return next;
9 }

```

### 3.2 哈希

```

1 constexpr int N = 2e6;
2 constexpr ll mod[2] = {2000000011, 2000000033}, base[2] = {20011, 20033};
3 vector<array<ll, 2>> pow_base(N);
4
5 pow_base[0][0] = pow_base[0][1] = 1;
6 for (int i = 1; i < N; i++) {
7     pow_base[i][0] = pow_base[i - 1][0] * base[0] % mod[0];
8     pow_base[i][1] = pow_base[i - 1][1] * base[1] % mod[1];
9 }
10
11 struct Hash {
12     int size;
13     vector<array<ll, 2>> hash;
14     Hash() {}
15     Hash(const string& s) {
16         size = s.size();
17         hash.resize(size);
18         hash[0][0] = hash[0][1] = s[0];
19         for (int i = 1; i < size; i++) {
20             hash[i][0] = (hash[i - 1][0] * base[0] + s[i]) % mod[0];
21             hash[i][1] = (hash[i - 1][1] * base[1] + s[i]) % mod[1];
22         }
23     }
24     array<ll, 2> operator[] (const array<int, 2>& range) const {
25         int l = range[0], r = range[1];
26         if (l == 0) return hash[r];
27         auto single_hash = [&](bool flag) {
28             return (hash[r][flag] -
29                     hash[l - 1][flag] * pow_base[r - l + 1][flag] % mod[flag] +
30                     mod[flag]) %
31                     mod[flag];
32         };
33         return {single_hash(0), single_hash(1)};
34     }
35 };

```

### 3.3 manacher

```

1 void manacher(const string& _s, vector<int>& r) {
2     string s(_s.size() * 2 + 1, '$');
3     for (int i = 0; i < _s.size(); i++) s[2 * i + 1] = _s[i];
4     r.resize(_s.size() * 2 + 1);
5     for (int i = 0, maxr = 0, mid = 0; i < s.size(); i++) {
6         if (i < maxr) r[i] = min(r[mid * 2 - i], maxr - i);
7         while (i - r[i] - 1 >= 0 && i + r[i] + 1 < s.size() &&
8                 s[i - r[i] - 1] == s[i + r[i] + 1])
9             ++r[i];
10        if (i + r[i] > maxr) maxr = i + r[i], mid = i;
11    }
12 }

```

## 4 数学

### 4.1 线性筛法

```

1 auto [min_prime, primes] = []() {
2     constexpr int N = 10000000;
3     vector<int> min_prime(N + 1, 0), primes;
4     for (int i = 2; i <= N; i++) {
5         if (min_prime[i] == 0) {
6             min_prime[i] = i;
7             primes.push_back(i);
8         }
9         for (auto& prime : primes) {
10             if (prime > min_prime[i] || prime > N / i) break;
11             min_prime[prime * i] = prime;
12         }
13     }
14     return tuple{min_prime, primes};
15 }();

```

### 4.2 分解质因数

```

1 void num_primes(int num, vector<int>& ans) {
2     for (auto& prime : primes) {
3         if (prime > num / prime) break;
4         if (num % prime == 0) {
5             while (num % prime == 0) num /= prime;
6             ans.push_back(prime);
7         }
8     }
9     if (num > 1) ans.push_back(num);
10 }

```

### 4.3 组合数

```

1 modint C(int n, int m) {
2     if (m == 0) return 1;
3     if (n <= mod)
4         return factorial[n] * factorial[m].inv() * factorial[n - m].inv();
5     else
6         return C(n % mod, m % mod) *
7             C(n / mod, m / mod); // n >= mod 时需要这个
8 }

```

### 4.4 盒子与球

$n$  个球,  $m$  个盒

球同	盒同	可空	公式
✓	✓	✓	$f_{n,m} = f_{n-1,m-1} + f_{n-m,m}$
✓	✓	✗	$f_{n-m,m}$
✗	✓	✓	$\sum_{i=1}^m g_{n,i}$
✗	✓	✗	$g_{n,m} = g_{n-1,m-1} + m * g_{n-1,m}$
✓	✗	✓	$C_{n+m-1}^{m-1}$
✓	✗	✗	$C_{n-1}^{m-1}$
✗	✗	✓	$m^n$
✗	✗	✗	$m! * g_{n,m}$

## 4.5 线性基

```

1 // 线性基
2 struct basis {
3     array<unsigned ll, 64> p{};
4
5     // 将x插入此线性基中
6     void insert(unsigned ll x) {
7         for (int i = 63; i >= 0; i--) {
8             if ((x >> i) & 1) {
9                 if (p[i])
10                     x ^= p[i];
11                 else {
12                     p[i] = x;
13                     break;
14                 }
15             }
16         }
17     }
18
19     // 将另一个线性基插入此线性基中
20     void insert(basis other) {
21         for (int i = 0; i <= 63; i++) {
22             if (!other.p[i]) continue;
23             insert(other.p[i]);
24         }
25     }
26
27     // 最大异或值
28     unsigned ll max_basis() {
29         unsigned ll res = 0;
30         for (int i = 63; i >= 0; i--) {
31             if ((res ^ p[i]) > res) res ^= p[i];
32         }
33         return res;
34     }

```

```
35 };
```

## 4.6 矩阵快速幂

```
1 constexpr ll mod = 2147493647;
2 struct Mat {
3     int n, m;
4     vector<vector<ll>> mat;
5     Mat(int n, int m) : n(n), m(m), mat(n, vector<ll>(m, 0)) {}
6     Mat(vector<vector<ll>> mat) : n(mat.size()), m(mat[0].size()), mat(mat) {}
7     Mat operator*(const Mat& other) {
8         assert(m == other.n);
9         Mat res(n, other.m);
10        for (int i = 0; i < res.n; i++)
11            for (int j = 0; j < res.m; j++)
12                for (int k = 0; k < m; k++)
13                    res.mat[i][j] =
14                        (res.mat[i][j] + mat[i][k] * other.mat[k][j] % mod) %
15                        mod;
16        return res;
17    }
18 };
19 Mat ksm(Mat a, ll b) {
20     assert(a.n == a.m);
21     Mat res(a.n, a.m);
22     for (int i = 0; i < res.n; i++) res.mat[i][i] = 1;
23     while (b) {
24         if (b & 1) res = res * a;
25         b >>= 1;
26         a = a * a;
27     }
28     return res;
29 }
```

## 5 计算几何

```
1 #define PI M_PI
2 constexpr double eps = 1e-8;
3 using T = int;
4
5 template <typename T>
6 bool equal(T a, T b) {
7     return a == b;
8 }
9 // 两浮点数是否相等
10 bool equal(double a, double b) { return abs(a - b) < eps; }
11
12 // 向量
13 struct vec {
```

```

14  T x, y;
15  vec(T _x = 0, T _y = 0) : x(_x), y(_y) {}
16
17  // 模
18  double length() const { return sqrt(x * x + y * y); }
19
20  // 与x轴正方向的夹角
21  double angle() const {
22      double angle = atan2(y, x);
23      if (angle < 0) angle += 2 * PI;
24      return angle;
25  }
26
27  // 逆时针旋转
28  void rotate(const double &theta) {
29      double temp = x;
30      x = x * cos(theta) - y * sin(theta);
31      y = y * cos(theta) + temp * sin(theta);
32  }
33
34  bool operator==(const vec &other) const {
35      return equal(x, other.x) && equal(y, other.y);
36  }
37  bool operator<(const vec &other) const {
38      return equal(angle(), other.angle()) ? x < other.x
39          : angle() < other.angle();
40  }
41
42  vec operator+(const vec &other) const { return {x + other.x, y + other.y}; }
43  vec operator-() const { return {-x, -y}; }
44  vec operator-(const vec &other) const { return -other + (*this); }
45  vec operator*(const T &other) const { return {x * other, y * other}; }
46  vec operator/(const T &other) const { return {x / other, y / other}; }
47  T operator*(const vec &other) const { return x * other.x + y * other.y; }
48
49  // 叉积 结果大于0, a在b的顺时针, 小于0, a在b的逆时针,
50  // 等于0共线, 可能同向或反向, 结果绝对值表示 a b形成的平行四边形的面积
51  T operator^(const vec &other) const { return x * other.y - y * other.x; }
52
53  friend istream &operator>>(istream &input, vec &data) {
54      input >> data.x >> data.y;
55      return input;
56  }
57  friend ostream &operator<<(ostream &output, const vec &data) {
58      output << fixed << setprecision(6);
59      output << data.x << " " << data.y;
60      return output;
61  }
62 };
63
64 // 两点间的距离
65 T distance(const vec &a, const vec &b) {

```



```

66     return sqrt((a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y));
67 }
68
69 // 两向量夹角
70 double angle(const vec &a, const vec &b) {
71     double theta = abs(a.angle() - b.angle());
72     if (theta > PI) theta = 2 * PI - theta;
73     return theta;
74 }
75
76 // 多边形的面积
77 double polygon_area(vector<vec> &p) {
78     T area = 0;
79     for (int i = 1; i < p.size(); i++) area += p[i - 1] ^ p[i];
80     area += p.back() ^ p[0];
81     return abs(area / 2.0);
82 }
83
84 // 多边形的周长
85 double polygon_length(vector<vec> &p) {
86     double length = 0;
87     for (int i = 1; i < p.size(); i++) length += (p[i - 1] - p[i]).length();
88     length += (p.back() - p[0]).length();
89     return length;
90 }
91
92 // 多边形直径的两个端点
93 auto polygon_dia(vector<vec> &p) {
94     int n = p.size();
95     array<vec, 2> res{};
96     if (n <= 1) return res;
97     if (n == 2) return res = {p[0], p[1]};
98     T mx = 0;
99     for (int i = 0, j = 3; i < n; i++) {
100         while (abs((p[i] - p[j]) ^ (p[(i + 1) % n] - p[j])) <=
101             abs((p[i] - p[(j + 1) % n]) ^ (p[(i + 1) % n] - p[(j + 1) % n])))
102             j = (j + 1) % n;
103         if (auto tmp = distance(p[i], p[j]); tmp > mx) {
104             mx = tmp;
105             res = {p[i], p[j]};
106         }
107         if (auto tmp = distance(p[(i + 1) % n], p[j]); tmp > mx) {
108             mx = tmp;
109             res = {p[(i + 1) % n], p[j]};
110         }
111     }
112     return res;
113 }
114
115 // 凸包
116 auto convex_hull(vector<vec> &p) {
117     sort(p.begin(), p.end(), [](vec &a, vec &b) {

```

```

118     return equal(a.x, b.x) ? a.y < b.y : a.x < b.x;
119 });
120
121 vector<int> sta(p.size() + 1, 0);
122 vector<bool> v(p.size(), false);
123 int tp = -1;
124 sta[++tp] = 0;
125
126 auto update_convex_hull = [&](int lim, int i) {
127     while (tp > lim &&
128         ((p[sta[tp]] - p[sta[tp - 1]]) ^ (p[i] - p[sta[tp]])) <= 0)
129         sta[++tp] = i;
130     v[i] = true;
131 };
132
133 for (int i = 1; i < p.size(); i++) update_convex_hull(0, i);
134
135 int cnt = tp;
136 for (int i = p.size() - 1; i >= 0; i--) {
137     if (v[i]) continue;
138     update_convex_hull(cnt, i);
139 }
140
141 vector<vec> res(tp);
142 for (int i = 0; i < tp; i++) res[i] = p[sta[i]];
143 return res;
144 }
145
146 // 以整点为顶点的线段上的整点个数
147 T count(const vec &a, const vec &b) {
148     vec c = a - b;
149     return gcd(abs(c.x), abs(c.y)) + 1;
150 }
151
152 // 以整点为顶点的多边形边上整点个数
153 T count(vector<vec> &p) {
154     T cnt = 0;
155     for (int i = 1; i < p.size(); i++) cnt += count(p[i - 1], p[i]);
156     cnt += count(p.back(), p[0]);
157     return cnt - p.size();
158 }
159
160 // 直线
161 struct line {
162     vec point, direction;
163     line(const vec &p, const vec &d) : point(p), direction(d) {}
164 };
165
166 // 点到直线距离
167 double distance(const vec &a, const line &b) {
168     return abs((b.point - a) ^ (b.point + b.direction - a)) /
169         b.direction.length();

```

```

170 }
171
172 // 两直线是否垂直
173 bool perpendicular(const line &a, const line &b) {
174     return equal(a.direction * b.direction, 0);
175 }
176
177 // 两直线是否平行
178 bool parallel(const line &a, const line &b) {
179     return equal(a.direction ^ b.direction, 0);
180 }
181
182 // 两直线交点
183 vec intersection(T A, T B, T C, T D, T E, T F) {
184     return {(B * F - C * E) / (A * E - B * D),
185             (C * D - A * F) / (A * E - B * D)};
186 }
187
188 // 两直线交点
189 vec intersection(const line &a, const line &b) {
190     return intersection(a.direction.y, -a.direction.x,
191                         a.direction.x * a.point.y - a.direction.y * a.point.x,
192                         b.direction.y, -b.direction.x,
193                         b.direction.x * b.point.y - b.direction.y * b.point.x);
194 }

```

## 6 杂项

### 6.1 高精度

```

1 struct bignum {
2     string num;
3
4     bignum() : num("0") {}
5     bignum(const string& num) : num(num) {
6         reverse(this->num.begin(), this->num.end());
7     }
8     bignum(ll num) : num(to_string(num)) {
9         reverse(this->num.begin(), this->num.end());
10    }
11
12    bignum operator+(const bignum& other) {
13        bignum res;
14        res.num.pop_back();
15        res.num.reserve(max(num.size(), other.num.size()) + 1);
16        for (int i = 0, j = 0, x; i < num.size() || i < other.num.size() || j;
17             i++) {
18            x = j;
19            j = 0;
20            if (i < num.size()) x += num[i] - '0';

```

```

21         if (i < other.num.size()) x += other.num[i] - '0';
22         if (x >= 10) j = 1, x -= 10;
23         res.num.push_back(x + '0');
24     }
25     res.num.capacity();
26     return res;
27 }
28
29 bignum operator*(const bignum& other) {
30     vector<int> res(num.size() + other.num.size() - 1, 0);
31     for (int i = 0; i < num.size(); i++) {
32         for (int j = 0; j < other.num.size(); j++) {
33             res[i + j] += (num[i] - '0') * (other.num[j] - '0');
34         }
35     }
36     int g = 0;
37     for (int i = 0; i < res.size(); i++) {
38         res[i] += g;
39         g = res[i] / 10;
40         res[i] %= 10;
41     }
42     while (g) {
43         res.push_back(g % 10);
44         g /= 10;
45     }
46     int lim = res.size();
47     while (lim > 1 && res[lim - 1] == 0) lim--;
48     bignum res2;
49     res2.num.resize(lim);
50     for (int i = 0; i < lim; i++) res2.num[i] = res[i] + '0';
51     return res2;
52 }
53
54 bool operator<(const bignum& other) {
55     if (num.size() == other.num.size())
56         for (int i = num.size() - 1; i >= 0; i--)
57             if (num[i] == other.num[i])
58                 continue;
59             else
60                 return num[i] < other.num[i];
61     return num.size() < other.num.size();
62 }
63
64 friend istream& operator>>(istream& in, bignum& a) {
65     in >> a.num;
66     reverse(a.num.begin(), a.num.end());
67     return in;
68 }
69 friend ostream& operator<<(ostream& out, bignum a) {
70     reverse(a.num.begin(), a.num.end());
71     return out << a.num;
72 }

```

73 | };

## 6.2 扫描线

[illegible]

```

48         return;
49     }
50     if (l <= mid) update(ls, l, r, num);
51     if (r > mid) update(rs, l, r, num);
52     tree[pos].len =
53         tree[pos].cnt ? tree[pos].raw_len : tree[ls].len + tree[rs].len;
54 };
55 build(1, 0, y_set.size() - 2);
56 auto find_pos = [&](ll num) {
57     return lower_bound(y_set.begin(), y_set.end(), num) - y_set.begin();
58 };
59 ll res = 0;
60 for (int i = 0; i < line.size() - 1; i++) {
61     update(1, find_pos(line[i].y_down), find_pos(line[i].y_up) - 1,
62         line[i].pd);
63     res += (line[i + 1].x - line[i].x) * tree[1].len;
64 }
65 return res;
66 }

```

### 6.3 模运算

```

1 class modint {
2     ll num;
3
4 public:
5     modint(ll num = 0) : num(num % mod) {}
6     modint pow(modint other) {
7         modint res(1), temp = *this;
8         while (other.num) {
9             if (other.num & 1) res = res * temp;
10            temp = temp * temp;
11            other.num >>= 1;
12        }
13        return res;
14    }
15    modint inv() { return this->pow(mod - 2); }
16    modint operator+(modint other) { return modint(this->num + other.num); }
17    modint operator-(modint other) { return {-this->num}; }
18    modint operator-(modint other) { return modint(-other + *this); }
19    modint operator*(modint other) { return modint(this->num * other.num); }
20    modint operator/(modint other) { return *this * other.inv(); }
21    friend istream& operator>>(istream& is, modint& other) {
22        is >> other.num;
23        other.num %= mod;
24        return is;
25    }
26    friend ostream& operator<<(ostream& os, modint other) {
27        other.num = (other.num + mod) % mod;
28        return os << other.num;
29    }

```

```
30 };
```

## 6.4 分数

```
1 struct frac {
2     ll a, b;
3     frac() : a(0), b(1) {}
4     frac(ll a, ll b = 1) {
5         if (a) {
6             ll temp = gcd(a, b);
7             this->a = a / temp;
8             this->b = b / temp;
9         } else {
10            this->a = 0;
11            this->b = 1;
12        }
13    }
14    frac operator+(const frac& other) {
15        return frac(a * other.b + other.a * b, b * other.b);
16    }
17    frac operator-() const {
18        frac res = *this;
19        res.a = -res.a;
20        return res;
21    }
22    frac operator-(const frac& other) { return -other + *this; }
23    frac operator*(const frac& other) { return frac(a * other.a, b * other.b); }
24    frac operator/(const frac& other) {
25        assert(other.a);
26        return *this * frac(other.b, other.a);
27    }
28    bool operator<(const frac& other) { return (*this - other).a < 0; }
29    bool operator<=(const frac& other) { return (*this - other).a <= 0; }
30    bool operator>(const frac& other) { return (*this - other).a > 0; }
31    bool operator>=(const frac& other) { return (*this - other).a >= 0; }
32    bool operator==(const frac& other) { return a == other.a && b == other.b; }
33    bool operator!=(const frac& other) { return !(*this == other); }
34 };
```

## 6.5 表达式求值

```
1 // 格式化表达式
2 string format(const string& s1) {
3     stringstream ss(s1);
4     string s2;
5     char ch;
6     while ((ch = ss.get()) != EOF) {
7         if (ch == ' ') continue;
8         if (isdigit(ch))
9             s2 += ch;
```

```

10     else {
11         if (s2.back() != ' ') s2 += ' ';
12         s2 += ch;
13         s2 += ' ';
14     }
15 }
16 return s2;
17 }
18
19 // 中缀表达式转后缀表达式
20 string convert(const string& s1) {
21     unordered_map<char, int> rank{
22         {'+', 2}, {'-', 2}, {'*', 1}, {'/', 1}, {'^', 0}};
23     stringstream ss(s1);
24     string s2, temp;
25     stack<char> op;
26     while (ss >> temp) {
27         if (isdigit(temp[0]))
28             s2 += temp + ' ';
29         else if (temp[0] == '(')
30             op.push('(');
31         else if (temp[0] == ')') {
32             while (op.top() != '(') {
33                 s2 += op.top();
34                 s2 += ' ';
35                 op.pop();
36             }
37             op.pop();
38         } else {
39             while (!op.empty() && op.top() != '(' &&
40                 (temp[0] != '^' && rank[op.top()] <= rank[temp[0]] ||
41                 rank[op.top()] < rank[temp[0]])) {
42                 s2 += op.top();
43                 s2 += ' ';
44                 op.pop();
45             }
46             op.push(temp[0]);
47         }
48     }
49     while (!op.empty()) {
50         s2 += op.top();
51         s2 += ' ';
52         op.pop();
53     }
54     return s2;
55 }
56
57 // 计算后缀表达式
58 int calc(const string& s) {
59     stack<int> num;
60     stringstream ss(s);
61     string temp;

```



```

62 while (ss >> temp) {
63     if (isdigit(temp[0]))
64         num.push(stoi(temp));
65     else {
66         int b = num.top();
67         num.pop();
68         int a = num.top();
69         num.pop();
70         if (temp[0] == '+')
71             a += b;
72         else if (temp[0] == '-')
73             a -= b;
74         else if (temp[0] == '*')
75             a *= b;
76         else if (temp[0] == '/')
77             a /= b;
78         else if (temp[0] == '^')
79             a = ksm(a, b);
80         num.push(a);
81     }
82 }
83 return num.top();
84 }

```

## 6.6 对拍

linux/Mac

```

1 g++ a.cpp -o program/a -O2 -std=c++17
2 g++ b.cpp -o program/b -O2 -std=c++17
3 g++ suiji.cpp -o program/suiji -O2 -std=c++17
4
5 cnt=0
6
7 while true; do
8     let cnt++
9     echo TEST:$cnt
10
11     ./program/suiji > in
12     ./program/a < in > out.a
13     ./program/b < in > out.b
14
15     diff out.a out.b
16     if [ $? -ne 0 ];then break;fi
17 done

```

windows

```

1 @echo off
2
3 g++ a.cpp -o program/a -O2 -std=c++17
4 g++ b.cpp -o program/b -O2 -std=c++17
5 g++ suiji.cpp -o program/suiji -O2 -std=c++17

```

```

6
7 set cnt=0
8
9 :again
10     set /a cnt=cnt+1
11     echo TEST:%cnt%
12     .\program\suiji > in
13     .\program\a < in > out.a
14     .\program\b < in > out.b
15
16     fc output.a output.b
17 if not errorlevel 1 goto again

```

## 6.7 开栈

任选一种

```

1 -Wl,--stack=0x10000000
2 -Wl,-stack_size -Wl,0x10000000
3 -Wl,-z,stack-size=0x10000000

```

## 6.8 日期

```

1 int month[] = {0, 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
2 int pre[13];
3 vector<int> leap;
4 struct Date {
5     int y, m, d;
6     bool operator<(const Date& other) const {
7         return array<int, 3>{y, m, d} <
8             array<int, 3>{other.y, other.m, other.d};
9     }
10    Date(const string& s) {
11        stringstream ss(s);
12        char ch;
13        ss >> y >> ch >> m >> ch >> d;
14    }
15    int dis() const {
16        int yd = (y - 1) * 365 +
17            (upper_bound(leap.begin(), leap.end(), y - 1) - leap.begin());
18        int md =
19            pre[m - 1] + (m > 2 && (y % 4 == 0 && y % 100 || y % 400 == 0));
20        return yd + md + d;
21    }
22    int dis(const Date& other) const { return other.dis() - dis(); }
23 };
24 for (int i = 1; i <= 12; i++) pre[i] = pre[i - 1] + month[2];
25 for (int i = 1; i <= 1000000; i++)
26     if (i % 4 == 0 && i % 100 || i % 400 == 0) leap.push_back(i);

```