ACM 常用算法模板

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目录

1 数据结构

1.1 并查集

```cpp
struct dsu {
    int n;
    vector<int> fa, sz;
    dsu(int _n) : n(_n), fa(n + 1), sz(n + 1, 1) { iota(fa.begin(), fa.end(), 0); }
    int find(int x) { return x == fa[x] ? x : fa[x] = find(fa[x]); }
    int merge(int x, int y) {
        int fax = find(x), fay = find(y);
        if (fax == fay) return 0;  // 一个集合
        sz[fay] += sz[fax];
        return fa[fax] = fay;  // 合并到哪个集合了
    }
    int size(int x) { return sz[find(x)]; }
};
```

1.2 树状数组

1.2.1 一维

```cpp
template <class T>
struct fenwick {
    int n;
    vector<T> t;
    fenwick(int _n) : n(_n), t(n + 1) {}
    T query(int l, int r) {
        auto query = [&](int pos) {
            T res = 0;
            while (pos) {
                res += t[pos];
                pos -= lowbit(pos);
            }
            return res;
        };
        return query(r) - query(l - 1);
    }
    void add(int pos, T num) {
        while (pos <= n) {
            t[pos] += num;
            pos += lowbit(pos);
        }
    }
};
```

1.2.2 二维

```cpp
template <class T>
struct Fenwick_tree_2 {
    Fenwick_tree_2(int n, int m) : n(n), m(m), tree(n + 1, vector<T>(m + 1)) {}
    T query(int l1, int r1, int l2, int r2) {
        auto query = [&](int l, int r) {
            T res = 0;
            for (int i = l; i; i -= lowbit(i))
```

```
 8                for (int j = r; j; j -= lowbit(j)) res += tree[i][j];
 9            return res;
10        };
11        return query(l2, r2) - query(l2, r1 - 1) - query(l1 - 1, r2) + query(l1 - 1, r1 - 1);
12    }
13    void update(int x, int y, T num) {
14        for (int i = x; i <= n; i += lowbit(i))
15            for (int j = y; j <= m; j += lowbit(j)) tree[i][j] += num;
16    }
17 private:
18    int n, m;
19    vector<vector<T>> tree;
20 };
```

1.2.3 三维

```
 1 template <class T>
 2 struct Fenwick_tree_3 {
 3    Fenwick_tree_3(int n, int m, int k)
 4        : n(n), m(m), k(k), tree(n + 1, vector<vector<T>>(m + 1, vector<T>(k + 1))) {}
 5    T query(int a, int b, int c, int d, int e, int f) {
 6        auto query = [&](int x, int y, int z) {
 7            T res = 0;
 8            for (int i = x; i; i -= lowbit(i))
 9                for (int j = y; j; j -= lowbit(j))
10                    for (int p = z; p; p -= lowbit(p)) res += tree[i][j][p];
11            return res;
12        };
13        T res = query(d, e, f);
14        res -= query(a - 1, e, f) + query(d, b - 1, f) + query(d, e, c - 1);
15        res += query(a - 1, b - 1, f) + query(a - 1, e, c - 1) + query(d, b - 1, c - 1);
16        res -= query(a - 1, b - 1, c - 1);
17        return res;
18    }
19    void update(int x, int y, int z, T num) {
20        for (int i = x; i <= n; i += lowbit(i))
21            for (int j = y; j <= m; j += lowbit(j))
22                for (int p = z; p <= k; p += lowbit(p)) tree[i][j][p] += num;
23    }
24 private:
25    int n, m, k;
26    vector<vector<vector<T>>> tree;
27 };
```

1.3 线段树

```
 1 template <class Data, class Num>
 2 struct Segment_Tree {
 3    inline void update(int l, int r, Num x) { update(1, l, r, x); }
 4    inline Data query(int l, int r) { return query(1, l, r); }
 5    Segment_Tree(vector<Data>& a) {
 6        n = a.size();
 7        tree.assign(n * 4 + 1, {});
 8        build(a, 1, 1, n);
 9    }
10 private:
```

```cpp
    int n;
    struct Tree {
        int l, r;
        Data data;
    };
    vector<Tree> tree;
    inline void pushup(int pos) { tree[pos].data = tree[pos << 1].data + tree[pos << 1 | 1].data; }
    inline void pushdown(int pos) {
        tree[pos << 1].data = tree[pos << 1].data + tree[pos].data.lazytag;
        tree[pos << 1 | 1].data = tree[pos << 1 | 1].data + tree[pos].data.lazytag;
        tree[pos].data.lazytag = Num::zero();
    }
    void build(vector<Data>& a, int pos, int l, int r) {
        tree[pos].l = l;
        tree[pos].r = r;
        if (l == r) {
            tree[pos].data = a[l - 1];
            return;
        }
        int mid = (tree[pos].l + tree[pos].r) >> 1;
        build(a, pos << 1, l, mid);
        build(a, pos << 1 | 1, mid + 1, r);
        pushup(pos);
    }
    void update(int pos, int& l, int& r, Num& x) {
        if (l > tree[pos].r || r < tree[pos].l) return;
        if (l <= tree[pos].l && tree[pos].r <= r) {
            tree[pos].data = tree[pos].data + x;
            return;
        }
        pushdown(pos);
        update(pos << 1, l, r, x);
        update(pos << 1 | 1, l, r, x);
        pushup(pos);
    }
    Data query(int pos, int& l, int& r) {
        if (l > tree[pos].r || r < tree[pos].l) return Data::zero();
        if (l <= tree[pos].l && tree[pos].r <= r) return tree[pos].data;
        pushdown(pos);
        return query(pos << 1, l, r) + query(pos << 1 | 1, l, r);
    }
};
struct Num {
    ll add;
    inline static Num zero() { return {0}; }
    inline Num operator+(Num b) { return {add + b.add}; }
};
struct Data {
    ll sum, len;
    Num lazytag;
    inline static Data zero() { return {0, 0, Num::zero()}; }
    inline Data operator+(Num b) { return {sum + len * b.add, len, lazytag + b}; }
    inline Data operator+(Data b) { return {sum + b.sum, len + b.len, Num::zero()}; }
};
```

1.4 普通平衡树

1.4.1 树状数组实现

需要预先处理出来所有可能的数。

```cpp
int lowbit(int x) { return x & -x; }

template <typename T>
struct treap {
    int n, size;
    vector<int> t;
    vector<T> t2, S;
    treap(const vector<T>& a) : S(a) {
        sort(S.begin(), S.end());
        S.erase(unique(S.begin(), S.end()), S.end());
        n = S.size();
        size = 0;
        t = vector<int>(n + 1);
        t2 = vector<T>(n + 1);
    }
    int pos(T x) { return lower_bound(S.begin(), S.end(), x) - S.begin() + 1; }
    int sum(int pos) {
        int res = 0;
        while (pos) {
            res += t[pos];
            pos -= lowbit(pos);
        }
        return res;
    }

    // 插入cnt个x
    void insert(T x, int cnt) {
        size += cnt;
        int i = pos(x);
        assert(i <= n && S[i - 1] == x);
        for (; i <= n; i += lowbit(i)) {
            t[i] += cnt;
            t2[i] += cnt * x;
        }
    }

    // 删除cnt个x
    void erase(T x, int cnt) {
        assert(cnt <= count(x));
        insert(x, -cnt);
    }

    // x的排名
    int rank(T x) {
        assert(count(x));
        return sum(pos(x) - 1) + 1;
    }

    // 统计出现次数
    int count(T x) { return sum(pos(x)) - sum(pos(x) - 1); }

    // 第k小
    T kth(int k) {
```

```
54          assert(0 < k && k <= size);
55          int cnt = 0, x = 0;
56          for (int i = __lg(n); i >= 0; i--) {
57              x += 1 << i;
58              if (x >= n || cnt + t[x] >= k) x -= 1 << i;
59              else cnt += t[x];
60          }
61          return S[x];
62      }
63
64      // 前k小的数之和
65      T pre_sum(int k) {
66          assert(0 < k && k <= size);
67          int cnt = 0, x = 0;
68          T res = 0;
69          for (int i = __lg(n); i >= 0; i--) {
70              x += 1 << i;
71              if (x >= n || cnt + t[x] >= k) x -= 1 << i;
72              else {
73                  cnt += t[x];
74                  res += t2[x];
75              }
76          }
77          return res + (k - cnt) * S[x];
78      }
79
80      // 小于x，最大的数
81      optional<T> prev(T x) {
82          int k = pos(x) - 1;
83          if (k == 0) return nullopt;
84          return kth(sum(k));
85      }
86
87      // 大于x，最小的数
88      optional<T> next(T x) {
89          int k = sum(pos(x)) + 1;
90          if (k == size + 1) return nullopt;
91          return kth(sum(k));
92      }
93 };
```

1.4.2 集合平衡树

```
 1 template <typename T = ull>
 2 struct treap_set {
 3     static constexpr int w = 64;
 4     static constexpr T bit(int i) { return (T)1 << i; }
 5     int n;
 6     vector<vector<T>> nodes;
 7
 8     treap_set(int _n) : n(_n) {
 9         do {
10             nodes.emplace_back(_n = (_n + w - 1) / w);
11         } while (_n > 1);
12     }
13     treap_set(const string &s) : n(s.size()) {
14         int _n = n;
```

```
15          do {
16              nodes.emplace_back(_n = (_n + w - 1) / w);
17          } while (_n > 1);
18          for (int i = 0; i < n; i++)
19              if (s[i] == '1') nodes[0][i / w] |= bit(i % w);
20          for (int i = 1; i < nodes.size(); i++) {
21              for (int j = 0; j < nodes[i - 1].size(); j++)
22                  if (nodes[i - 1][j]) nodes[i][j / w] |= bit(j % w);
23          }
24      }
25      void clear() {
26          for (auto &i : nodes) fill(i.begin(), i.end(), 0);
27      }
28      void insert(int k) {
29          for (auto &node : nodes) {
30              node[k / w] |= bit(k % w);
31              k /= w;
32          }
33      }
34      void erase(int k) {
35          for (auto &node : nodes) {
36              node[k / w] &= ~bit(k % w);
37              k /= w;
38              if (node[k]) break;
39          }
40      }
41      bool contains(int k) { return nodes[0][k / w] & bit(k % w); }
42      // Find the smallest key greater than k.
43      optional<int> next(int k) {
44          for (int i = 0; i < nodes.size(); i++, k /= w) {
45              if (k % w == w - 1) continue;
46              T keys = nodes[i][k / w] & ~(bit(k % w + 1) - 1);
47              if (keys == 0) continue;
48              k = k / w * w + __countr_zero(keys);
49              for (int j = i - 1; j >= 0; j--) k = k * w + __countr_zero(nodes[j][k]);
50              return k;
51          }
52          return nullopt;
53      }
54      // Find the largest key samller than k.
55      optional<int> prev(int k) {
56          for (int i = 0; i < nodes.size(); i++, k /= w) {
57              if (k % w == 0) continue;
58              T keys = nodes[i][k / w] & (bit(k % w) - 1);
59              if (keys == 0) continue;
60              k = k / w * w + w - 1 - __countl_zero(keys);
61              for (int j = i - 1; j >= 0; j--) k = k * w + w - 1 - __countl_zero(nodes[j][k]);
62              return k;
63          }
64          return nullopt;
65      }
66 };
```

1.5 可持久化线段树

```
1 constexpr int MAXN = 200000;
2 vector<int> root(MAXN << 5);
```

```
 3  struct Persistent_seg {
 4      int n;
 5      struct Data {
 6          int ls, rs;
 7          int val;
 8      };
 9      vector<Data> tree;
10      Persistent_seg(int n, vector<int>& a) : n(n) { root[0] = build(1, n, a); }
11      int build(int l, int r, vector<int>& a) {
12          if (l == r) {
13              tree.push_back({0, 0, a[l]});
14              return tree.size() - 1;
15          }
16          int mid = l + r >> 1;
17          int ls = build(l, mid, a), rs = build(mid + 1, r, a);
18          tree.push_back({ls, rs, tree[ls].val + tree[rs].val});
19          return tree.size() - 1;
20      }
21      int update(int rt, const int& idx, const int& val, int l, int r) {
22          if (l == r) {
23              tree.push_back({0, 0, tree[rt].val + val});
24              return tree.size() - 1;
25          }
26          int mid = l + r >> 1, ls = tree[rt].ls, rs = tree[rt].rs;
27          if (idx <= mid) ls = update(ls, idx, val, l, mid);
28          else rs = update(rs, idx, val, mid + 1, r);
29          tree.push_back({ls, rs, tree[ls].val + tree[rs].val});
30          return tree.size() - 1;
31      }
32      int query(int rt1, int rt2, int k, int l, int r) {
33          if (l == r) return l;
34          int mid = l + r >> 1;
35          int lcnt = tree[tree[rt2].ls].val - tree[tree[rt1].ls].val;
36          if (k <= lcnt) return query(tree[rt1].ls, tree[rt2].ls, k, l, mid);
37          else return query(tree[rt1].rs, tree[rt2].rs, k - lcnt, mid + 1, r);
38      }
39  };
```

1.6 st 表

```
 1  auto lg = []() {
 2      array<int, 10000001> lg;
 3      lg[1] = 0;
 4      for (int i = 2; i <= 10000000; i++) lg[i] = lg[i >> 1] + 1;
 5      return lg;
 6  }();
 7  template <typename T>
 8  struct st {
 9      int n;
10      vector<vector<T>> a;
11      st(vector<T>& _a) : n(_a.size()) {
12          a.assign(lg[n] + 1, vector<int>(n));
13          for (int i = 0; i < n; i++) a[0][i] = _a[i];
14          for (int j = 1; j <= lg[n]; j++)
15              for (int i = 0; i + (1 << j) - 1 < n; i++)
16                  a[j][i] = max(a[j - 1][i], a[j - 1][i + (1 << (j - 1))]);
17      }
```

```
18      T query(int l, int r) {
19          int k = lg[r - l + 1];
20          return max(a[k][l], a[k][r - (1 << k) + 1]);
21      }
22  };
```

2 图论

存图

```cpp
struct Graph {
    int n;
    struct Edge {
        int to, w;
    };
    vector<vector<Edge>> graph;
    Graph(int _n) {
        n = _n;
        graph.assign(n + 1, vector<Edge>());
    };
    void add(int u, int v, int w) { graph[u].push_back({v, w}); }
};
```

2.1 最短路

2.1.1 dijkstra

```cpp
void dij(Graph& graph, vector<int>& dis, int t) {
    vector<int> visit(graph.n + 1, 0);
    priority_queue<pair<int, int>> que;
    dis[t] = 0;
    que.emplace(0, t);
    while (!que.empty()) {
        int u = que.top().second;
        que.pop();
        if (visit[u]) continue;
        visit[u] = 1;
        for (auto& [to, w] : graph.graph[u]) {
            if (dis[to] > dis[u] + w) {
                dis[to] = dis[u] + w;
                que.emplace(-dis[to], to);
            }
        }
    }
}
```

2.2 树上问题

2.2.1 最近公公祖先

倍增法

```cpp
vector<int> dep;
vector<array<int, 21>> fa;
dep.assign(n + 1, 0);
fa.assign(n + 1, array<int, 21>{});
void binary_jump(int root) {
    function<void(int)> dfs = [&](int t) {
        dep[t] = dep[fa[t][0]] + 1;
        for (auto& [to] : graph[t]) {
            if (to == fa[t][0]) continue;
            fa[to][0] = t;
            dfs(to);
```

```
12          }
13      };
14      dfs(root);
15      for (int j = 1; j <= 20; j++)
16          for (int i = 1; i <= n; i++) fa[i][j] = fa[fa[i][j - 1]][j - 1];
17  }
18  int lca(int x, int y) {
19      if (dep[x] < dep[y]) swap(x, y);
20      for (int i = 20; i >= 0; i--)
21          if (dep[fa[x][i]] >= dep[y]) x = fa[x][i];
22      if (x == y) return x;
23      for (int i = 20; i >= 0; i--) {
24          if (fa[x][i] != fa[y][i]) {
25              x = fa[x][i];
26              y = fa[y][i];
27          }
28      }
29      return fa[x][0];
30  }
```

树剖

```
1  int lca(int x, int y) {
2      while (top[x] != top[y]) {
3          if (dep[top[x]] < dep[top[y]]) swap(x, y);
4          x = fa[top[x]];
5      }
6      if (dep[x] < dep[y]) swap(x, y);
7      return y;
8  }
```

2.2.2　树链剖分

```
1  vector<int> fa, siz, dep, son, dfn, rnk, top;
2  fa.assign(n + 1, 0);
3  siz.assign(n + 1, 0);
4  dep.assign(n + 1, 0);
5  son.assign(n + 1, 0);
6  dfn.assign(n + 1, 0);
7  rnk.assign(n + 1, 0);
8  top.assign(n + 1, 0);
9  void hld(int root) {
10     function<void(int)> dfs1 = [&](int t) {
11         dep[t] = dep[fa[t]] + 1;
12         siz[t] = 1;
13         for (auto& [to, w] : graph[t]) {
14             if (to == fa[t]) continue;
15             fa[to] = t;
16             dfs1(to);
17             if (siz[son[t]] < siz[to]) son[t] = to;
18             siz[t] += siz[to];
19         }
20     };
21     dfs1(root);
22     int dfn_tail = 0;
23     for (int i = 1; i <= n; i++) top[i] = i;
24     function<void(int)> dfs2 = [&](int t) {
25         dfn[t] = ++dfn_tail;
```

```
26          rnk[dfn_tail] = t;
27          if (!son[t]) return;
28          top[son[t]] = top[t];
29          dfs2(son[t]);
30          for (auto& [to, w] : graph[t]) {
31              if (to == fa[t] || to == son[t]) continue;
32              dfs2(to);
33          }
34      };
35      dfs2(root);
36  }
```

2.3 强连通分量

```
1  void tarjan(Graph& g1, Graph& g2) {
2      int dfn_tail = 0, cnt = 0;
3      vector<int> dfn(g1.n + 1, 0), low(g1.n + 1, 0), exist(g1.n + 1, 0), belong(g1.n + 1, 0);
4      stack<int> sta;
5      function<void(int)> dfs = [&](int t) {
6          dfn[t] = low[t] = ++dfn_tail;
7          sta.push(t);
8          exist[t] = 1;
9          for (auto& [to] : g1.graph[t])
10              if (!dfn[to]) {
11                  dfs(to);
12                  low[t] = min(low[t], low[to]);
13              } else if (exist[to]) low[t] = min(low[t], dfn[to]);
14          if (dfn[t] == low[t]) {
15              cnt++;
16              while (int temp = sta.top()) {
17                  belong[temp] = cnt;
18                  exist[temp] = 0;
19                  sta.pop();
20                  if (temp == t) break;
21              }
22          }
23      };
24      for (int i = 1; i <= g1.n; i++)
25          if (!dfn[i]) dfs(i);
26      g2 = Graph(cnt);
27      for (int i = 1; i <= g1.n; i++) g2.w[belong[i]] += g1.w[i];
28      for (int i = 1; i <= g1.n; i++)
29          for (auto& [to] : g1.graph[i])
30              if (belong[i] != belong[to]) g2.add(belong[i], belong[to]);
31  }
```

2.4 拓扑排序

```
1  void toposort(Graph& g, vector<int>& dis) {
2      vector<int> in(g.n + 1, 0);
3      for (int i = 1; i <= g.n; i++)
4          for (auto& [to] : g.graph[i]) in[to]++;
5      queue<int> que;
6      for (int i = 1; i <= g.n; i++)
7          if (!in[i]) {
8              que.push(i);
```

```
 9            dis[i] = g.w[i];  // dp
10        }
11    while (!que.empty()) {
12        int u = que.front();
13        que.pop();
14        for (auto& [to] : g.graph[u]) {
15            in[to]--;
16            dis[to] = max(dis[to], dis[u] + g.w[to]);  // dp
17            if (!in[to]) que.push(to);
18        }
19    }
20 }
```

3 字符串

3.1 kmp

```cpp
auto kmp(string& s) {
    vector next(s.size(), -1);
    for (int i = 1, j = -1; i < s.size(); i++) {
        while (j >= 0 && s[i] != s[j + 1]) j = next[j];
        if (s[i] == s[j + 1]) j++;
        next[i] = j;
    }
    // next 意为长度
    for (auto& i : next) i++;
    return next;
}
```

3.2 哈希

```cpp
constexpr int N = 1e6;
int pow_base[N + 1][2];
constexpr ll mod[2] = {(int)2e9 + 11, (int)2e9 + 33}, base[2] = {(int)2e5 + 11, (int)2e5 + 33};

struct Hash {
    int size;
    vector<array<int, 2>> a;
    Hash() {}
    Hash(const string& s) {
        size = s.size();
        a.resize(size);
        a[0][0] = a[0][1] = s[0];
        for (int i = 1; i < size; i++) {
            a[i][0] = (a[i - 1][0] * base[0] + s[i]) % mod[0];
            a[i][1] = (a[i - 1][1] * base[1] + s[i]) % mod[1];
        }
    }
    array<int, 2> get(int l, int r) const {
        if (l == 0) return a[r];
        auto getone = [&](bool f) {
            int x = (a[r][f] - 1ll * a[l - 1][f] * pow_base[r - l + 1][f]) % mod[f];
            if (x < 0) x += mod[f];
            return x;
        };
        return {getone(0), getone(1)};
    }
};

auto _ = []() {
    pow_base[0][0] = pow_base[0][1] = 1;
    for (int i = 1; i <= N; i++) {
        pow_base[i][0] = pow_base[i - 1][0] * base[0] % mod[0];
        pow_base[i][1] = pow_base[i - 1][1] * base[1] % mod[1];
    }
    return true;
}();
```

3.3 manacher

```
auto manacher(const string& _s) {
    string s(_s.size() * 2 + 1, '$');
    for (int i = 0; i < _s.size(); i++) s[2 * i + 1] = _s[i];
    vector r(s.size(), 0);
    for (int i = 0, maxr = 0, mid = 0; i < s.size(); i++) {
        if (i < maxr) r[i] = min(r[mid * 2 - i], maxr - i);
        while (i - r[i] - 1 >= 0 && i + r[i] + 1 < s.size() && s[i - r[i] - 1] == s[i + r[i] + 1])
            ++r[i];
        if (i + r[i] > maxr) maxr = i + r[i], mid = i;
    }
    return r;
}
```

4 数学

4.1 扩展欧几里得

需保证 $a, b >= 0$

$x = x + k * dx, y = y - k * dy$

若要求 $x \geq p$，$k \geq \lceil \frac{p-x}{dx} \rceil$

若要求 $x \leq q$，$k \leq \lfloor \frac{q-x}{dx} \rfloor$

若要求 $y \geq p$，$k \leq \lfloor \frac{y-p}{dy} \rfloor$

若要求 $y \leq q$，$k \geq \lceil \frac{y-q}{dy} \rceil$

```cpp
int __exgcd(int a, int b, int& x, int& y) {
    if (!b) {
        x = 1;
        y = 0;
        return a;
    }
    int g = __exgcd(b, a % b, y, x);
    y -= a / b * x;
    return g;
}

array<int, 2> exgcd(int a, int b, int c) {
    int x, y;
    int g = __exgcd(a, b, x, y);
    if (c % g) return {INT_MAX, INT_MAX};
    int dx = b / g;
    int dy = a / g;
    x = c / g % dx * x % dx;
    if (x < 0) x += dx;
    y = (c - a * x) / b;
    return {x, y};
}
```

4.2 线性代数

4.2.1 向量公约数

```cpp
// 将这两个向量组转化为b.y=0的形式
array<vec, 2> gcd(vec a, vec b) {
    while (b.y != 0) {
        int t = a.y / b.y;
        a = a - b * t;
        swap(a, b);
    }
    return {a, b};
}

array<vec, 2> gcd(array<vec, 2> g, vec a) {
    auto [b, c] = gcd(g[0], a);
    g[0] = b;
    g[1] = vec(gcd(g[1].x, c.x), 0);
    if (g[1].x != 0) g[0].x %= g[1].x;
    return g;
}
```

4.3 筛法

primes

```cpp
constexpr int N = 1e7;
bitset<N + 1> ispr;
vector<int> primes;
bool _ = []() {
    ispr.set();
    ispr[0] = ispr[1] = 0;
    for (int i = 2; i <= N; i++) {
        if (!ispr[i]) continue;
        primes.push_back(i);
        for (int j = 2 * i; j <= N; j += i) ispr[j] = 0;
    }
    return 1;
}();
```

φ

```cpp
constexpr int N = 1e7;
array<int, N + 1> phi;
auto _ = []() {
    iota(phi.begin() + 1, phi.end(), 1);
    for (int i = 2; i <= N; i++) {
        if (phi[i] == i)
            for (int j = i; j <= N; j += i) phi[j] = phi[j] / i * (i - 1);
    }
    return true;
}();
```

μ

```cpp
constexpr int N = 1e7;
bitset<N + 1> ispr;
array<int, N + 1> mu;
auto _ = []() {
    mu.fill(1);
    ispr.set();
    mu[0] = ispr[0] = ispr[1] = 0;
    for (int i = 2; i <= N; i++) {
        if (!ispr[i]) continue;
        mu[i] = -1;
        for (int j = 2 * i; j <= N; j += i) {
            ispr[j] = 0;
            if (j / i % i == 0) mu[j] = 0;
            else mu[j] *= -1;
        }
    }
    return true;
}();
```

prime φ

```cpp
constexpr int N = 1e7;
bitset<N + 1> ispr;
array<int, N + 1> phi;
vector<int> primes;
bool _ = []() {
    ispr.set();
```

```
 7      ispr[0] = ispr[1] = 0;
 8      iota(phi.begin() + 1, phi.end(), 1);
 9      for (int i = 2; i <= N; i++) {
10          if (!ispr[i]) continue;
11          phi[i] = i - 1;
12          primes.push_back(i);
13          for (int j = 2 * i; j <= N; j += i) {
14              ispr[j] = 0;
15              phi[j] = phi[j] / i * (i - 1);
16          }
17      }
18      return 1;
19  }();
```

prime μ

```
 1  constexpr int N = 1e7;
 2  bitset<N + 1> ispr;
 3  array<int, N + 1> mu;
 4  vector<int> primes;
 5  bool _ = []() {
 6      mu.fill(1);
 7      ispr.set();
 8      mu[0] = ispr[0] = ispr[1] = 0;
 9      for (int i = 2; i <= N; i++) {
10          if (!ispr[i]) continue;
11          mu[i] = -1;
12          primes.push_back(i);
13          for (int j = 2 * i; j <= N; j += i) {
14              ispr[j] = 0;
15              if (j / i % i == 0) mu[j] = 0;
16              else mu[j] *= -1;
17          }
18      }
19      return 1;
20  }();
```

prime μ φ

```
 1  constexpr int N = 1e7;
 2  bitset<N + 1> ispr;
 3  array<int, N + 1> mu, phi;
 4  vector<int> primes;
 5  bool _ = []() {
 6      mu.fill(1);
 7      ispr.set();
 8      mu[0] = ispr[0] = ispr[1] = 0;
 9      iota(phi.begin() + 1, phi.end(), 1);
10      for (int i = 2; i <= N; i++) {
11          if (!ispr[i]) continue;
12          mu[i] = -1;
13          phi[i] = i - 1;
14          primes.push_back(i);
15          for (int j = 2 * i; j <= N; j += i) {
16              ispr[j] = 0;
17              if (j / i % i == 0) mu[j] = 0;
18              else mu[j] *= -1;
19              phi[j] = phi[j] / i * (i - 1);
20          }
```

```
21        }
22        return 1;
23 }();
```

```
 1 constexpr int N = 1e7;
 2 array<int, N + 1> minpr, mu, phi;
 3 vector<int> primes;
 4 bool _ = []() {
 5     phi[1] = mu[1] = 1;
 6     for (int i = 2; i <= N; i++) {
 7         if (minpr[i] == 0) {
 8             minpr[i] = i;
 9             mu[i] = -1;
10             phi[i] = i - 1;
11             primes.push_back(i);
12         }
13         for (auto& j : primes) {
14             if (i * j > N) break;
15             minpr[i * j] = j;
16             if (j < minpr[i]) {
17                 phi[i * j] = phi[i] * phi[j];
18                 mu[i * j] = -mu[i];
19             } else {
20                 mu[i * j] = 0;
21                 phi[i * j] = phi[i] * j;
22                 break;
23             }
24         }
25     }
26     return 1;
27 }();
```

4.4 分解质因数

```
 1 auto getprimes(int n) {
 2     vector<array<int, 2>> res;
 3     for (auto& i : primes) {
 4         if (i > n / i) break;
 5         if (n % i == 0) {
 6             res.push_back({i, 0});
 7             while (n % i == 0) {
 8                 n /= i;
 9                 res.back()[1]++;
10             }
11         }
12     }
13     if (n > 1) res.push_back({n, 1});
14     return res;
15 }
```

4.5 pollard rho

```
 1 using LL = __int128_t;
 2
 3 random_device rd;
```

```
4  mt19937 seed(rd());
5
6  ll power(ll a, ll b, ll mod) {
7      ll res = 1;
8      while (b) {
9          if (b & 1) res = (LL)res * a % mod;
10         a = (LL)a * a % mod;
11         b >>= 1;
12     }
13     return res;
14 }
15
16 bool isprime(ll n) {
17     static array primes{2, 3, 5, 7, 11, 13, 17, 19, 23};
18     static unordered_map<ll, bool> S;
19     if (n < 2) return 0;
20     if (S.count(n)) return S[n];
21     ll d = n - 1, r = 0;
22     while (!(d & 1)) {
23         r++;
24         d >>= 1;
25     }
26     for (auto& a : primes) {
27         if (a == n) return S[n] = 1;
28         ll x = power(a, d, n);
29         if (x == 1 || x == n - 1) continue;
30         for (int i = 0; i < r - 1; i++) {
31             x = (LL)x * x % n;
32             if (x == n - 1) break;
33         }
34         if (x != n - 1) return S[n] = 0;
35     }
36     return S[n] = 1;
37 }
38
39 ll pollard_rho(ll n) {
40     ll s = 0, t = 0;
41     ll c = seed() % (n - 1) + 1;
42     ll val = 1;
43     for (int goal = 1;; goal *= 2, s = t, val = 1) {
44         for (int step = 1; step <= goal; step++) {
45             t = ((LL)t * t + c) % n;
46             val = (LL)val * abs(t - s) % n;
47             if (step % 127 == 0) {
48                 ll g = gcd(val, n);
49                 if (g > 1) return g;
50             }
51         }
52         ll g = gcd(val, n);
53         if (g > 1) return g;
54     }
55 }
56 auto getprimes(ll n) {
57     unordered_set<ll> S;
58     auto get = [&](auto self, ll n) {
59         if (n < 2) return;
60         if (isprime(n)) {
61             S.insert(n);
```

```
62          return;
63      }
64      ll mx = pollard_rho(n);
65      self(self, n / mx);
66      self(self, mx);
67  };
68  get(get, n);
69  return S;
70 }
```

4.6   组合数

```
1  constexpr int N = 1e6;
2  array<modint, N + 1> fac, ifac;
3
4  modint C(int n, int m) {
5      if (m < 0 || m > n) return 0;
6      if (n <= mod) return fac[n] * ifac[m] * ifac[n - m];
7      // n >= mod 时需要这个
8      return C(n % mod, m % mod) * C(n / mod, m / mod);
9  }
10
11 auto _ = []() {
12     fac[0] = 1;
13     for (int i = 1; i <= N; i++) fac[i] = fac[i - 1] * i;
14     ifac[N] = fac[N].inv();
15     for (int i = N - 1; i >= 0; i--) ifac[i] = ifac[i + 1] * (i + 1);
16     return true;
17 }();
```

4.6.1   常用式子

- $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$

- $\binom{n}{k} = \frac{n-k}{k}\binom{n}{k-1}$

- $\sum_{i=0}^{n}(-1)^i\binom{n}{i} = [n = 0]$

- $\sum_{i=0}^{m}\binom{n}{i}\binom{m}{i} = \binom{m+n}{m}$

- $\sum_{i=0}^{n}\binom{n}{i}^2 = \binom{2n}{n}$

- $\sum_{i=0}^{n}i\binom{n}{i} = n2^{n-1}$

- $\sum_{i=0}^{n}i^2\binom{n}{i} = n(n+1)2^{n-2}$

- $\sum_{l=0}^{n}\binom{l}{k} = \binom{n+1}{k+1}$

- $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$

- $\sum_{i=0}^{n}\binom{n-i}{i} = F_{n+1}$，其中 F 是斐波那契数列。

- $\sum_{i=0}^{k}\binom{n}{i}\binom{m}{k-i} = \binom{n+m}{k}$

- $\sum_{i=1}^{n}\binom{n}{i}\binom{n}{i-1} = \binom{2n}{n+1}$

- $m^n = \sum_{i=0}^{m}\left\{{n \atop i}\right\}\binom{m}{i}i!$

4.7 数论分块

求解形如 $\sum_{i=1}^{n} f(i)g(\lfloor \frac{n}{i} \rfloor)$ 的合式

$s(n) = \sum_{i=1}^{n} f(i)$

```cpp
modint sqrt_decomposition(int n) {
    auto s = [&](int x) { return x; };
    auto g = [&](int x) { return x; };
    modint res = 0;
    while (l <= R) {
        int r = n / (n / l);
        res = res + (s(r) - s(l - 1)) * g(n / l);
        l = r + 1;
    }
    return res;
}
```

4.8 积性函数

4.8.1 定义

函数 $f(n)$ 满足 $f(1) = 1$ 且 $\forall x, y \in \mathbf{N}^*, \gcd(x, y) = 1$ 都有 $f(xy) = f(x)f(y)$，则 $f(n)$ 为积性函数。

函数 $f(n)$ 满足 $f(1) = 1$ 且 $\forall x, y \in \mathbf{N}^*$ 都有 $f(xy) = f(x)f(y)$，则 $f(n)$ 为完全积性函数。

4.8.2 例子

- 单位函数：$\varepsilon(n) = [n = 1]$。（完全积性）

- 恒等函数：$\mathrm{id}_k(n) = n^k$。（完全积性）

- 常数函数：$1(n) = 1$。（完全积性）

- 除数函数：$\sigma_k(n) = \sum_{d|n} d^k$。$\sigma_0(n)$ 通常简记作 $d(n)$ 或 $\tau(n)$，$\sigma_1(n)$ 通常简记作 $\sigma(n)$。

- 欧拉函数：$\varphi(n) = \sum_{i=1}^{n} [\gcd(i, n) = 1]$。

- 莫比乌斯函数：$\mu(n) = \begin{cases} 1 & n = 1 \\ 0 & \exists d > 1, d^2 \mid n \\ (-1)^{\omega(n)} & \text{otherwise} \end{cases}$，其中 $\omega(n)$ 表示 n 的本质不同质因子个数，它是一个加性函数。

4.9 狄利克雷卷积

对于两个数论函数 $f(x)$ 和 $g(x)$，则它们的狄利克雷卷积得到的结果 $h(x)$ 定义为：

$h(x) = \sum_{d|x} f(d)g\left(\frac{x}{d}\right) = \sum_{ab=x} f(a)g(b)$

可以简记为：$h = f * g$。

4.9.1 性质

交换律： $f * g = g * f$。

结合律： $(f * g) * h = f * (g * h)$。

分配律： $(f + g) * h = f * h + g * h$。

等式的性质： $f = g$ 的充要条件是 $f * h = g * h$，其中数论函数 $h(x)$ 要满足 $h(1) \neq 0$。

4.9.2 例子

- $\varepsilon = \mu * 1 \iff \varepsilon(n) = \sum_{d|n} \mu(d)$

- $id = \varphi * 1 \iff id(n) = \sum_{d|n} \varphi(d)$

- $d = 1 * 1 \iff d(n) = \sum_{d|n} 1$

- $\sigma = \mathrm{id} * 1 \iff \sigma(n) = \sum_{d|n} d$

- $\varphi = \mu * \mathrm{id} \iff \varphi(n) = \sum_{d|n} d \cdot \mu(\frac{n}{d})$

4.10 欧拉函数

```cpp
constexpr int N = 1e6;
array<int, N + 1> phi;
auto _ = []() {
    iota(phi.begin() + 1, phi.end(), 1);
    for (int i = 2; i <= N; i++) {
        if (phi[i] == i)
            for (int j = i; j <= N; j += i) phi[j] = phi[j] / i * (i - 1);
    }
    return true;
}();
```

4.11 莫比乌斯反演

4.11.1 莫比乌斯函数性质

- $\sum_{d|n} \mu(d) = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}$, 即 $\sum_{d|n} \mu(d) = \varepsilon(n)$, $\mu * 1 = \varepsilon$

- $[\gcd(i,j) = 1] = \sum_{d|\gcd(i,j)} \mu(d)$

```cpp
constexpr int N = 1e6;
array<int, N + 1> miu;
array<bool, N + 1> ispr;

auto _ = []() {
    miu.fill(1);
    ispr.fill(1);
    for (int i = 2; i <= N; i++) {
        if (!ispr[i]) continue;
        miu[i] = -1;
        for (int j = 2 * i; j <= N; j += i) {
            ispr[j] = 0;
            if ((j / i) % i == 0) miu[j] = 0;
            else miu[j] *= -1;
        }
    }
    return true;
}();
```

4.11.2 莫比乌斯变换/反演

$f(n) = \sum_{d|n} g(d)$，那么有 $g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d}) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$ 。

用狄利克雷卷积表示则为 $f = g * 1$，有 $g = f * \mu$。

$f \to g$ 称为莫比乌斯反演，$g \to f$ 称为莫比乌斯反演。

4.12 杜教筛

杜教筛被用于处理一类数论函数的前缀和问题。对于数论函数 f，杜教筛可以在低于线性时间的复杂度内计算 $S(n) = \sum_{i=1}^{n} f(i)$。

$$S(n) = \frac{\sum_{i=1}^{n} (f*g)(i) - \sum_{i=2}^{n} g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)}{g(1)}$$

可以构造恰当的数论函数 g 使得：

- 可以快速计算 $\sum_{i=1}^{n} (f*g)(i)$。

- 可以快速计算 g 的单点值，用数论分块求解 $\sum_{i=2}^{n} g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$。

4.12.1 示例

```cpp
ll sum_phi(ll n) {
    if (n <= N) return sp[n];
    if (sp2.count(n)) return sp2[n];
    ll res = 0, l = 2;
    while (l <= n) {
        ll r = n / (n / l);
        res = res + (r - l + 1) * sum_phi(n / l);
        l = r + 1;
    }
    return sp2[n] = (ll)n * (n + 1) / 2 - res;
}

ll sum_miu(ll n) {
    if (n <= N) return sm[n];
    if (sm2.count(n)) return sm2[n];
    ll res = 0, l = 2;
    while (l <= n) {
        ll r = n / (n / l);
        res = res + (r - l + 1) * sum_miu(n / l);
        l = r + 1;
    }
    return sm2[n] = 1 - res;
}
```

4.13 多项式

```cpp
#define countr_zero(n) __builtin_ctz(n)
constexpr int N = 1e6;
array<int, N + 1> inv;

int power(int a, int b) {
    int res = 1;
    while (b) {
```

```
 8          if (b & 1) res = 1ll * res * a % mod;
 9          a = 1ll * a * a % mod;
10          b >>= 1;
11      }
12      return res;
13 }
14
15 namespace NFTS {
16 int g = 3;
17 vector<int> rev, roots{0, 1};
18 void dft(vector<int> &a) {
19     int n = a.size();
20     if (rev.size() != n) {
21         int k = countr_zero(n) - 1;
22         rev.resize(n);
23         for (int i = 0; i < n; ++i) rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
24     }
25     if (roots.size() < n) {
26         int k = countr_zero(roots.size());
27         roots.resize(n);
28         while ((1 << k) < n) {
29             int e = power(g, (mod - 1) >> (k + 1));
30             for (int i = 1 << (k - 1); i < (1 << k); ++i) {
31                 roots[2 * i] = roots[i];
32                 roots[2 * i + 1] = 1ll * roots[i] * e % mod;
33             }
34             ++k;
35         }
36     }
37     for (int i = 0; i < n; ++i)
38         if (rev[i] < i) swap(a[i], a[rev[i]]);
39     for (int k = 1; k < n; k *= 2) {
40         for (int i = 0; i < n; i += 2 * k) {
41             for (int j = 0; j < k; ++j) {
42                 int u = a[i + j];
43                 int v = 1ll * a[i + j + k] * roots[k + j] % mod;
44                 int x = u + v, y = u - v;
45                 if (x >= mod) x -= mod;
46                 if (y < 0) y += mod;
47                 a[i + j] = x;
48                 a[i + j + k] = y;
49             }
50         }
51     }
52 }
53 void idft(vector<int> &a) {
54     int n = a.size();
55     reverse(a.begin() + 1, a.end());
56     dft(a);
57     int inv_n = power(n, mod - 2);
58     for (int i = 0; i < n; ++i) a[i] = 1ll * a[i] * inv_n % mod;
59 }
60 }  // namespace NFTS
61
62 struct poly {
63     poly &format() {
64         while (!a.empty() && a.back() == 0) a.pop_back();
65         return *this;
```

```
66         }
67         poly &reverse() {
68             ::reverse(a.begin(), a.end());
69             return *this;
70         }
71         vector<int> a;
72         poly() {}
73         poly(int x) {
74             if (x) a = {x};
75         }
76         poly(const vector<int> &_a) : a(_a) {}
77         int size() const { return a.size(); }
78         int &operator[](int id) { return a[id]; }
79         int at(int id) const {
80             if (id < 0 || id >= (int)a.size()) return 0;
81             return a[id];
82         }
83         poly operator-() const {
84             auto A = *this;
85             for (auto &x : A.a) x = (x == 0 ? 0 : mod - x);
86             return A;
87         }
88         poly mulXn(int n) const {
89             auto b = a;
90             b.insert(b.begin(), n, 0);
91             return poly(b);
92         }
93         poly modXn(int n) const {
94             if (n > size()) return *this;
95             return poly({a.begin(), a.begin() + n});
96         }
97         poly divXn(int n) const {
98             if (size() <= n) return poly();
99             return poly({a.begin() + n, a.end()});
100        }
101        poly &operator+=(const poly &rhs) {
102            if (size() < rhs.size()) a.resize(rhs.size());
103            for (int i = 0; i < rhs.size(); ++i)
104                if ((a[i] += rhs.a[i]) >= mod) a[i] -= mod;
105            return *this;
106        }
107        poly &operator-=(const poly &rhs) {
108            if (size() < rhs.size()) a.resize(rhs.size());
109            for (int i = 0; i < rhs.size(); ++i)
110                if ((a[i] -= rhs.a[i]) < 0) a[i] += mod;
111            return *this;
112        }
113        poly &operator*=(poly rhs) {
114            int n = size(), m = rhs.size(), tot = max(1, n + m - 1);
115            int sz = 1 << __lg(tot * 2 - 1);
116            a.resize(sz);
117            rhs.a.resize(sz);
118            NFTS::dft(a);
119            NFTS::dft(rhs.a);
120            for (int i = 0; i < sz; ++i) a[i] = 1ll * a[i] * rhs.a[i] % mod;
121            NFTS::idft(a);
122            return *this;
123        }
```

```cpp
124      poly &operator/=(poly rhs) {
125          int n = size(), m = rhs.size();
126          if (n < m) return (*this) = poly();
127          reverse();
128          rhs.reverse();
129          (*this) *= rhs.inv(n - m + 1);
130          a.resize(n - m + 1);
131          reverse();
132          return *this;
133      }
134      poly &operator%=(poly rhs) { return (*this) -= (*this) / rhs * rhs; }
135      poly operator+(const poly &rhs) const { return poly(*this) += rhs; }
136      poly operator-(const poly &rhs) const { return poly(*this) -= rhs; }
137      poly operator*(poly rhs) const { return poly(*this) *= rhs; }
138      poly operator/(poly rhs) const { return poly(*this) /= rhs; }
139      poly operator%(poly rhs) const { return poly(*this) %= rhs; }
140      poly powModPoly(int n, poly p) {
141          poly r(1), x(*this);
142          while (n) {
143              if (n & 1) (r *= x) %= p;
144              (x *= x) %= p;
145              n >>= 1;
146          }
147          return r;
148      }
149      int inner(const poly &rhs) {
150          int r = 0, n = min(size(), rhs.size());
151          for (int i = 0; i < n; ++i) r = (r + 1ll * a[i] * rhs.a[i]) % mod;
152          return r;
153      }
154      poly derivation() const {
155          if (a.empty()) return poly();
156          int n = size();
157          vector<int> r(n - 1);
158          for (int i = 1; i < n; ++i) r[i - 1] = 1ll * a[i] * i % mod;
159          return poly(r);
160      }
161      poly integral() const {
162          if (a.empty()) return poly();
163          int n = size();
164          vector<int> r(n + 1);
165          for (int i = 0; i < n; ++i) r[i + 1] = 1ll * a[i] * ::inv[i + 1] % mod;
166          return poly(r);
167      }
168      poly inv(int n) const {
169          assert(a[0] != 0);
170          poly x(power(a[0], mod - 2));
171          int k = 1;
172          while (k < n) {
173              k *= 2;
174              x *= (poly(2) - modXn(k) * x).modXn(k);
175          }
176          return x.modXn(n);
177      }
178      // 需要保证首项为 1
179      poly log(int n) const { return (derivation() * inv(n)).integral().modXn(n); }
180      // 需要保证首项为 0
181      poly exp(int n) const {
```

```
182            poly x(1);
183            int k = 1;
184            while (k < n) {
185                k *= 2;
186                x = (x * (poly(1) - x.log(k) + modXn(k))).modXn(k);
187            }
188            return x.modXn(n);
189        }
190        // 需要保证首项为 1，开任意次方可以先 ln 再 exp 实现。
191        poly sqrt(int n) const {
192            poly x(1);
193            int k = 1;
194            while (k < n) {
195                k *= 2;
196                x += modXn(k) * x.inv(k);
197                x = x.modXn(k) * inv2;
198            }
199            return x.modXn(n);
200        }
201        // 减法卷积，也称转置卷积 {\rm MULT}(F(x),G(x))=\sum_{i\ge0}(\sum_{j\ge
202        // 0}f_{i+j}g_j)x^i
203        poly mulT(poly rhs) const {
204            if (rhs.size() == 0) return poly();
205            int n = rhs.size();
206            ::reverse(rhs.a.begin(), rhs.a.end());
207            return ((*this) * rhs).divXn(n - 1);
208        }
209        int eval(int x) {
210            int r = 0, t = 1;
211            for (int i = 0, n = size(); i < n; ++i) {
212                r = (r + 1ll * a[i] * t) % mod;
213                t = 1ll * t * x % mod;
214            }
215            return r;
216        }
217        // 多点求值新科技: https://jkloverdcoi.github.io/2020/08/04/转置原理及其应用/
218        // 模板例题: https://www.luogu.com.cn/problem/P5050
219        auto evals(vector<int> &x) const {
220            if (size() == 0) return vector(x.size(), 0);
221            int n = x.size();
222            vector ans(n, 0);
223            vector<poly> g(4 * n);
224            auto build = [&](auto self, int l, int r, int p) -> void {
225                if (r - l == 1) {
226                    g[p] = poly({1, x[l] ? mod - x[l] : 0});
227                } else {
228                    int m = (l + r) / 2;
229                    self(self, l, m, 2 * p);
230                    self(self, m, r, 2 * p + 1);
231                    g[p] = g[2 * p] * g[2 * p + 1];
232                }
233            };
234            build(build, 0, n, 1);
235            auto solve = [&](auto self, int l, int r, int p, poly f) -> void {
236                if (r - l == 1) {
237                    ans[l] = f[0];
238                } else {
239                    int m = (l + r) / 2;
```

```
240                self(self, l, m, 2 * p, f.mulT(g[2 * p + 1]).modXn(m - l));
241                self(self, m, r, 2 * p + 1, f.mulT(g[2 * p]).modXn(r - m));
242            }
243        };
244        solve(solve, 0, n, 1, mulT(g[1].inv(size())).modXn(n));
245        return ans;
246    }
247 }; // 全家桶测试: https://www.luogu.com.cn/training/3015#information
248
249 auto _ = []() {
250    inv[0] = inv[1] = 1;
251    for (int i = 2; i < inv.size(); i++) inv[i] = 1ll * (mod - mod / i) * inv[mod % i] % mod;
252    return true;
253 }();
```

4.14　盒子与球

n 个球，m 个盒

球同	盒同	可空	公式
✓	✓	✓	$f_{n,m} = f_{n,m-1} + f_{n-m,m}$ 或 $[x^n]e^{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{\infty}\frac{x^{ij}}{j}}$
✓	✓	✗	$f_{n-m,m}$
✗	✓	✓	$\Sigma_{i=1}^{m}g_{n,i}$ 或 $\sum\limits_{i=1}^{m}\sum\limits_{j=0}^{i}\frac{j^n}{j!}\frac{(-1)^{i-j}}{(i-j)!}$
✗	✓	✗	$g_{n,m} = g_{n-1,m-1} + m * g_{n-1,m}$ 或 $\frac{1}{m!}\sum\limits_{i=0}^{m}(-1)^i \binom{m}{i}(m-i)^n$
✓	✗	✓	C_{n+m-1}^{m-1}
✓	✗	✗	C_{n-1}^{m-1}
✗	✗	✓	m^n
✗	✗	✗	$m! * g_{n,m}$ 或 $\sum\limits_{i=0}^{m}(-1)^i\binom{m}{i}(m-i)^n$

4.14.1　球同，盒同，可空

```
1 int solve(int n, int m) {
2    vector a(n + 1, 0);
3    for (int i = 1; i <= m; i++)
4        for (int j = i, k = 1; j <= n; j += i, k++) a[j] = (a[j] + inv[k]) % mod;
5    auto p = poly(a).exp(n + 1);
6    return (p.a[n] + mod) % mod;
7 }
```

若要求不超过 k 个，答案为 $[x^n y^m] \prod\limits_{i=0}^{k} \left(\sum\limits_{j=0}^{m} x^{ij} y^j \right)$。

4.14.2 球不同，盒同，可空

```
1  int solve(int n, int m) {
2      vector a(n + 1, 0);
3      vector b(n + 1, 0);
4      for (int i = 0; i <= n; i++) {
5          a[i] = ifac[i];
6          if (i & 1) a[i] = -a[i];
7          b[i] = 1ll * power(i, n) * ifac[i] % mod;
8      }
9      auto p = poly(a) * poly(b);
10     int ans = 0;
11     for (int i = 1; i <= min(n, m); i++) ans = (ans + p.a[i]) % mod;
12     return (ans + mod) % mod;
13 }
```

若要求不超过 k 个，答案为 $m! \cdot [x^n y^m] \prod\limits_{i=0}^{k} \left(\sum\limits_{j=0}^{n} \frac{1}{i!^j} x^{ij} y^j \right)$。

4.14.3 球同，盒不同，可空

若要求不超过 k 个，答案为 $[x^n] \left(\sum\limits_{i=0}^{k} x^i \right)^m = [x^n] \frac{(x^{k+1}-1)^m}{(x-1)^m}$。

也可以考虑容斥，令 $f(i)$ 表示至少有 i 个盒子装了 $> k$ 个球方案数，$f(i) = \binom{m}{i} \binom{n-(k+1)i+m-1}{m-1}$。

总方案数则为 $\sum\limits_{i=0}^{m} (-1)^i f(i)$。

4.14.4 球同，盒不同，不可空

若要求不超过 k 个，答案为 $[x^n] \left(\sum\limits_{i=1}^{k} x^i \right)^m = [x^n] \frac{(x^k-1)^m x^m}{(x-1)^m}$。

也可以考虑容斥，令 $f(i)$ 表示至少有 i 个盒子装了 $> k$ 个球方案数，$f(i) = \binom{m}{i} \binom{n-ki-1}{m-1}$。

总方案数则为 $\sum\limits_{i=0}^{m} (-1)^i f(i)$。

4.14.5 球不同，盒不同，可空

若要求不超过 k 个，答案为 $m! \cdot [x^n] \left(\sum\limits_{i=0}^{k} \frac{1}{i!} x^i \right)^m$。

4.14.6 球不同，盒不同，不可空

若要求不超过 k 个，答案为 $m! \cdot [x^n] \left(\sum\limits_{i=1}^{k} \frac{1}{i!} x^i \right)^m$。

4.15 线性基

```
1  // 线性基
2  struct basis {
3      int rnk = 0;
4      array<ull, 64> p{};
5
6      // 将x插入此线性基中
7      void insert(ull x) {
```

```
8        for (int i = 63; i >= 0; i--) {
9            if (!(x >> i & 1)) continue;
10           if (p[i]) x ^= p[i];
11           else {
12               p[i] = x;
13               rnk++;
14               break;
15           }
16       }
17   }
18
19   // 将另一个线性基插入此线性基中
20   void insert(basis other) {
21       for (int i = 0; i <= 63; i++) {
22           if (!other.p[i]) continue;
23           insert(other.p[i]);
24       }
25   }
26
27   // 最大异或值
28   ull max_basis() {
29       ull res = 0;
30       for (int i = 63; i >= 0; i--)
31           if ((res ^ p[i]) > res) res ^= p[i];
32       return res;
33   }
34 };
```

4.16　矩阵快速幂

```
1  constexpr ll mod = 2147493647;
2  struct Mat {
3      int n, m;
4      vector<vector<ll>> mat;
5      Mat(int n, int m) : n(n), m(n), mat(n, vector<ll>(m, 0)) {}
6      Mat(vector<vector<ll>> mat) : n(mat.size()), m(mat[0].size()), mat(mat) {}
7      Mat operator*(const Mat& other) {
8          assert(m == other.n);
9          Mat res(n, other.m);
10         for (int i = 0; i < res.n; i++)
11             for (int j = 0; j < res.m; j++)
12                 for (int k = 0; k < m; k++)
13                     res.mat[i][j] = (res.mat[i][j] + mat[i][k] * other.mat[k][j] % mod) % mod;
14         return res;
15     }
16 };
17 Mat ksm(Mat a, ll b) {
18     assert(a.n == a.m);
19     Mat res(a.n, a.m);
20     for (int i = 0; i < res.n; i++) res.mat[i][i] = 1;
21     while (b) {
22         if (b & 1) res = res * a;
23         b >>= 1;
24         a = a * a;
25     }
26     return res;
27 }
```

5 计算几何

5.1 整数

```cpp
constexpr double inf = 1e100;

// 向量
struct vec {
    static bool cmp(const vec &a, const vec &b) { return tie(a.x, a.y) < tie(b.x, b.y); }

    ll x, y;
    vec() : x(0), y(0) {}
    vec(ll _x, ll _y) : x(_x), y(_y) {}

    vec rotleft() const { return {-y, x}; }
    vec rotright() const { return {y, -x}; }

    // 模
    ll len2() const { return x * x + y * y; }
    double len() const { return sqrt(x * x + y * y); }

    // 是否在上半轴
    bool up() const { return y > 0 || y == 0 && x >= 0; }

    bool operator==(const vec &b) const { return tie(x, y) == tie(b.x, b.y); }
    // 极角排序
    bool operator<(const vec &b) const {
        if (up() != b.up()) return up() > b.up();
        ll tmp = (*this) ^ b;
        return tmp ? tmp > 0 : cmp(*this, b);
    }

    vec operator+(const vec &b) const { return {x + b.x, y + b.y}; }
    vec operator-() const { return {-x, -y}; }
    vec operator-(const vec &b) const { return -b + (*this); }
    vec operator*(ll b) const { return {x * b, y * b}; }
    ll operator*(const vec &b) const { return x * b.x + y * b.y; }

    // 叉积 结果大于0, a到b为逆时针, 小于0, a到b顺时针,
    // 等于0共线, 可能同向或反向, 结果绝对值表示 a b 形成的平行四边行的面积
    ll operator^(const vec &b) const { return x * b.y - y * b.x; }

    friend istream &operator>>(istream &in, vec &data) {
        in >> data.x >> data.y;
        return in;
    }
    friend ostream &operator<<(ostream &out, const vec &data) {
        out << fixed << setprecision(6);
        out << data.x << " " << data.y;
        return out;
    }
};

ll cross(const vec &a, const vec &b, const vec &c) { return (a - c) ^ (b - c); }

// 多边形的面积a
double polygon_area(vector<vec> &p) {
    ll area = 0;
```

```cpp
55        for (int i = 1; i < p.size(); i++) area += p[i - 1] ^ p[i];
56        area += p.back() ^ p[0];
57        return abs(area / 2.0);
58    }
59
60    // 多边形的周长
61    double polygon_len(vector<vec> &p) {
62        double len = 0;
63        for (int i = 1; i < p.size(); i++) len += (p[i - 1] - p[i]).len();
64        len += (p.back() - p[0]).len();
65        return len;
66    }
67
68    // 以整点为顶点的线段上的整点个数
69    ll count(const vec &a, const vec &b) {
70        vec c = a - b;
71        return gcd(abs(c.x), abs(c.y)) + 1;
72    }
73
74    // 以整点为顶点的多边形边上整点个数
75    ll count(vector<vec> &p) {
76        ll cnt = 0;
77        for (int i = 1; i < p.size(); i++) cnt += count(p[i - 1], p[i]);
78        cnt += count(p.back(), p[0]);
79        return cnt - p.size();
80    }
81
82    // 判断点是否在凸包内，凸包必须为逆时针顺序
83    bool in_polygon(const vec &a, vector<vec> &p) {
84        int n = p.size();
85        if (n == 0) return 0;
86        if (n == 1) return a == p[0];
87        if (n == 2) return cross(a, p[1], p[0]) == 0 && (p[0] - a) * (p[1] - a) <= 0;
88        if (cross(a, p[1], p[0]) > 0 || cross(p.back(), a, p[0]) > 0) return 0;
89        auto cmp = [&](vec &x, const vec &y) { return ((x - p[0]) ^ y) >= 0; };
90        int i = lower_bound(p.begin() + 2, p.end() - 1, a - p[0], cmp) - p.begin() - 1;
91        return cross(p[(i + 1) % n], a, p[i]) >= 0;
92    }
93
94    // 凸包直径的两个端点
95    auto polygon_dia(vector<vec> &p) {
96        int n = p.size();
97        array<vec, 2> res{};
98        if (n == 1) return res;
99        if (n == 2) return res = {p[0], p[1]};
100       ll mx = 0;
101       for (int i = 0, j = 2; i < n; i++) {
102           while (abs(cross(p[i], p[(i + 1) % n], p[j])) <=
103                   abs(cross(p[i], p[(i + 1) % n], p[(j + 1) % n])))
104               j = (j + 1) % n;
105           ll tmp = (p[i] - p[j]).len2();
106           if (tmp > mx) {
107               mx = tmp;
108               res = {p[i], p[j]};
109           }
110           tmp = (p[(i + 1) % n] - p[j]).len2();
111           if (tmp > mx) {
112               mx = tmp;
```

```
113            res = {p[(i + 1) % n], p[j]};
114        }
115    }
116    return res;
117 }
118
119 // 凸包
120 auto convex_hull(vector<vec> &p) {
121    sort(p.begin(), p.end(), vec::cmp);
122    int n = p.size();
123    vector sta(n + 1, 0);
124    vector v(n, false);
125    int tp = -1;
126    sta[++tp] = 0;
127    auto update = [&](int lim, int i) {
128        while (tp > lim && cross(p[i], p[sta[tp]], p[sta[tp - 1]]) >= 0) v[sta[tp--]] = 0;
129        sta[++tp] = i;
130        v[i] = 1;
131    };
132    for (int i = 1; i < n; i++) update(0, i);
133    int cnt = tp;
134    for (int i = n - 1; i >= 0; i--) {
135        if (v[i]) continue;
136        update(cnt, i);
137    }
138    vector<vec> res(tp);
139    for (int i = 0; i < tp; i++) res[i] = p[sta[i]];
140    return res;
141 }
142
143 // 闵可夫斯基和，两个点集的和构成一个凸包
144 auto minkowski(vector<vec> &a, vector<vec> &b) {
145    rotate(a.begin(), min_element(a.begin(), a.end(), vec::cmp), a.end());
146    rotate(b.begin(), min_element(b.begin(), b.end(), vec::cmp), b.end());
147    int n = a.size(), m = b.size();
148    vector<vec> c{a[0] + b[0]};
149    c.reserve(n + m);
150    int i = 0, j = 0;
151    while (i < n && j < m) {
152        vec x = a[(i + 1) % n] - a[i];
153        vec y = b[(j + 1) % m] - b[j];
154        c.push_back(c.back() + ((x ^ y) >= 0 ? (i++, x) : (j++, y)));
155    }
156    while (i + 1 < n) {
157        c.push_back(c.back() + a[(i + 1) % n] - a[i]);
158        i++;
159    }
160    while (j + 1 < m) {
161        c.push_back(c.back() + b[(j + 1) % m] - b[j]);
162        j++;
163    }
164    return c;
165 }
166
167 // 过凸多边形外一点求凸多边形的切线，返回切点下标
168 auto tangent(const vec &a, vector<vec> &p) {
169    int n = p.size();
170    int l = -1, r = -1;
```

```
171         for (int i = 0; i < n; i++) {
172             ll tmp1 = cross(p[i], p[(i - 1 + n) % n], a);
173             ll tmp2 = cross(p[i], p[(i + 1) % n], a);
174             if (l == -1 && tmp1 <= 0 && tmp2 <= 0) l = i;
175             else if (r == -1 && tmp1 >= 0 && tmp2 >= 0) r = i;
176         }
177         return array{l, r};
178     }
179
180     // 直线
181     struct line {
182         vec p, d;
183         line() {}
184         line(const vec &a, const vec &b) : p(a), d(b - a) {}
185     };
186
187     // 点到直线距离
188     double dis(const vec &a, const line &b) { return abs((b.p - a) ^ (b.p + b.d - a)) / b.d.len(); }
189
190     // 点在直线哪边，大于0在左边，等于0在线上，小于0在右边
191     ll side_line(const vec &a, const line &b) { return b.d ^ (a - b.p); }
192
193     // 两直线是否垂直
194     bool perpen(const line &a, const line &b) { return a.d * b.d == 0; }
195
196     // 两直线是否平行
197     bool parallel(const line &a, const line &b) { return (a.d ^ b.d) == 0; }
198
199     // 点的垂线是否与线段有交点
200     bool perpen(const vec &a, const line &b) {
201         vec p(-b.d.y, b.d.x);
202         bool cross1 = (p ^ (b.p - a)) > 0;
203         bool cross2 = (p ^ (b.p + b.d - a)) > 0;
204         return cross1 != cross2;
205     }
206
207     // 点到线段距离
208     double dis_seg(const vec &a, const line &b) {
209         if (perpen(a, b)) return dis(a, b);
210         return min((b.p - a).len(), (b.p + b.d - a).len());
211     }
212
213     // 点到凸包距离
214     double dis(const vec &a, vector<vec> &p) {
215         double res = inf;
216         for (int i = 1; i < p.size(); i++) res = min(dis_seg(a, line(p[i - 1], p[i])), res);
217         res = min(dis_seg(a, line(p.back(), p[0] - p.back())), res);
218         return res;
219     }
220
221     // 两直线交点
222     vec intersection(ll A, ll B, ll C, ll D, ll E, ll F) {
223         return {(B * F - C * E) / (A * E - B * D), (C * D - A * F) / (A * E - B * D)};
224     }
225
226     // 两直线交点
227     vec intersection(const line &a, const line &b) {
228         return intersection(a.d.y, -a.d.x, a.d.x * a.p.y - a.d.y * a.p.x, b.d.y, -b.d.x,
```

```
229                         b.d.x * b.p.y - b.d.y * b.p.x);
230 }
```

5.2 浮点数

```cpp
1  using lf = double;
2
3  constexpr lf eps = 1e-8;
4  constexpr lf inf = 1e100;
5  const lf PI = acos(-1);
6
7  int sgn(lf a, lf b) {
8      lf c = a - b;
9      return c < -eps ? -1 : c > eps ? 1 : 0;
10 }
11
12 // 向量
13 struct vec {
14     static bool cmp(const vec &a, const vec &b) {
15         return sgn(a.x, b.x) ? a.x < b.x : sgn(a.y, b.y) < 0;
16     }
17
18     lf x, y;
19     vec() : x(0), y(0) {}
20     vec(lf _x, lf _y) : x(_x), y(_y) {
21         if (sgn(y, 0) == 0) y = 0;
22     }
23
24     // 模
25     lf len2() const { return x * x + y * y; }
26     lf len() const { return sqrt(x * x + y * y); }
27
28     // 与x轴正方向的夹角
29     lf angle() const {
30         lf angle = atan2(y, x);
31         if (angle < 0) angle += 2 * PI;
32         return angle;
33     }
34
35     // 逆时针旋转
36     vec rotate(const lf &theta) const {
37         lf sint = sin(theta);
38         lf cost = cos(theta);
39         return {x * cost - y * sint, x * sint + y * cost};
40     }
41
42     vec rotleft() const { return {-y, x}; }
43
44     vec rotright() const { return {y, -x}; }
45
46     vec e() const {
47         lf tmp = len();
48         return {x / tmp, y / tmp};
49     }
50
51     // 是否在上半轴
52     bool up() const { return sgn(y, 0) > 0 || sgn(y, 0) == 0 && sgn(x, 0) >= 0; }
```

```
53
54        bool operator==(const vec &other) const { return sgn(x, other.x) == 0 && sgn(y, other.y) == 0; }
55
56        // 极角排序
57        bool operator<(const vec &b) const {
58            if (up() != b.up()) return up() > b.up();
59            lf tmp = (*this) ^ b;
60            return sgn(tmp, 0) ? tmp > 0 : cmp(*this, b);
61        }
62
63        vec operator+(const vec &b) const { return {x + b.x, y + b.y}; }
64        vec operator-() const { return {-x, -y}; }
65        vec operator-(const vec &b) const { return -b + (*this); }
66        vec operator*(lf b) const { return {x * b, y * b}; }
67        vec operator/(lf b) const { return {x / b, y / b}; }
68        lf operator*(const vec &b) const { return x * b.x + y * b.y; }
69
70        // 叉积 结果大于0，a到b为逆时针，小于0，a到b顺时针，
71        // 等于0共线，可能同向或反向，结果绝对值表示 a b 形成的平行四边行的面积
72        lf operator^(const vec &b) const { return x * b.y - y * b.x; }
73
74        friend istream &operator>>(istream &in, vec &data) {
75            in >> data.x >> data.y;
76            return in;
77        }
78        friend ostream &operator<<(ostream &out, const vec &data) {
79            out << fixed << setprecision(6);
80            out << data.x << " " << data.y;
81            return out;
82        }
83 };
84
85 lf cross(const vec &a, const vec &b, const vec &c) { return (a - c) ^ (b - c); }
86
87 lf angle(const vec &a, const vec &b) { return atan2(abs(a ^ b), a * b); }
88
89 // 多边形的面积
90 lf polygon_area(vector<vec> &p) {
91     lf area = 0;
92     for (int i = 1; i < p.size(); i++) area += p[i - 1] ^ p[i];
93     area += p.back() ^ p[0];
94     return abs(area / 2.0);
95 }
96
97 // 多边形的周长
98 lf polygon_len(vector<vec> &p) {
99     lf len = 0;
100    for (int i = 1; i < p.size(); i++) len += (p[i - 1] - p[i]).len();
101    len += (p.back() - p[0]).len();
102    return len;
103 }
104
105 // 判断点是否在凸包内，凸包必须为逆时针顺序
106 bool in_polygon(const vec &a, vector<vec> &p) {
107    int n = p.size();
108    if (n == 0) return 0;
109    if (n == 1) return a == p[0];
110    if (n == 2) return sgn(cross(a, p[1], p[0]), 0) == 0 && sgn((p[0] - a) * (p[1] - a), 0) <= 0;
```

```
111        if (sgn(cross(a, p[1], p[0]), 0) > 0 || sgn(cross(p.back(), a, p[0]), 0) > 0) return 0;
112        auto cmp = [&](vec &x, const vec &y) { return sgn((x - p[0]) ^ y, 0) >= 0; };
113        int i = lower_bound(p.begin() + 2, p.end() - 1, a - p[0], cmp) - p.begin() - 1;
114        return sgn(cross(p[(i + 1) % n], a, p[i]), 0) >= 0;
115 }
116
117 // 凸包直径的两个端点
118 auto polygon_dia(vector<vec> &p) {
119        int n = p.size();
120        array<vec, 2> res{};
121        if (n == 1) return res;
122        if (n == 2) return res = {p[0], p[1]};
123        lf mx = 0;
124        for (int i = 0, j = 2; i < n; i++) {
125            while (sgn(abs(cross(p[i], p[(i + 1) % n], p[j])),
126                       abs(cross(p[i], p[(i + 1) % n], p[(j + 1) % n]))) <= 0)
127                j = (j + 1) % n;
128            lf tmp = (p[i] - p[j]).len();
129            if (tmp > mx) {
130                mx = tmp;
131                res = {p[i], p[j]};
132            }
133            tmp = (p[(i + 1) % n] - p[j]).len();
134            if (tmp > mx) {
135                mx = tmp;
136                res = {p[(i + 1) % n], p[j]};
137            }
138        }
139        return res;
140 }
141
142 // 凸包
143 auto convex_hull(vector<vec> &p) {
144        sort(p.begin(), p.end(), vec::cmp);
145        int n = p.size();
146        vector sta(n + 1, 0);
147        vector v(n, false);
148        int tp = -1;
149        sta[++tp] = 0;
150        auto update = [&](int lim, int i) {
151            while (tp > lim && sgn(cross(p[i], p[sta[tp]], p[sta[tp - 1]]), 0) >= 0) v[sta[tp--]] = 0;
152            sta[++tp] = i;
153            v[i] = 1;
154        };
155        for (int i = 1; i < n; i++) update(0, i);
156        int cnt = tp;
157        for (int i = n - 1; i >= 0; i--) {
158            if (v[i]) continue;
159            update(cnt, i);
160        }
161        vector<vec> res(tp);
162        for (int i = 0; i < tp; i++) res[i] = p[sta[i]];
163        return res;
164 }
165
166 // 闵可夫斯基和，两个点集的和构成一个凸包
167 auto minkowski(vector<vec> &a, vector<vec> &b) {
168        rotate(a.begin(), min_element(a.begin(), a.end(), vec::cmp), a.end());
```

```
169        rotate(b.begin(), min_element(b.begin(), b.end(), vec::cmp), b.end());
170        int n = a.size(), m = b.size();
171        vector<vec> c{a[0] + b[0]};
172        c.reserve(n + m);
173        int i = 0, j = 0;
174        while (i < n && j < m) {
175            vec x = a[(i + 1) % n] - a[i];
176            vec y = b[(j + 1) % m] - b[j];
177            c.push_back(c.back() + (sgn(x ^ y, 0) >= 0 ? (i++, x) : (j++, y)));
178        }
179        while (i + 1 < n) {
180            c.push_back(c.back() + a[(i + 1) % n] - a[i]);
181            i++;
182        }
183        while (j + 1 < m) {
184            c.push_back(c.back() + b[(j + 1) % m] - b[j]);
185            j++;
186        }
187        return c;
188    }
189
190    // 过凸多边形外一点求凸多边形的切线，返回切点下标
191    auto tangent(const vec &a, vector<vec> &p) {
192        int n = p.size();
193        int l = -1, r = -1;
194        for (int i = 0; i < n; i++) {
195            lf tmp1 = cross(p[i], p[(i - 1 + n) % n], a);
196            lf tmp2 = cross(p[i], p[(i + 1) % n], a);
197            if (l == -1 && sgn(tmp1, 0) <= 0 && sgn(tmp2, 0) <= 0) l = i;
198            else if (r == -1 && sgn(tmp1, 0) >= 0 && sgn(tmp2, 0) >= 0) r = i;
199        }
200        return array{l, r};
201    }
202
203    // 直线
204    struct line {
205        vec p, d;
206        line() {}
207        line(const vec &a, const vec &b) : p(a), d(b - a) {}
208    };
209
210    // 点到直线距离
211    lf dis(const vec &a, const line &b) { return abs((b.p - a) ^ (b.p + b.d - a)) / b.d.len(); }
212
213    // 点在直线哪边，大于0在左边，等于0在线上，小于0在右边
214    int side_line(const vec &a, const line &b) { return sgn(b.d ^ (a - b.p), 0); }
215
216    // 两直线是否垂直
217    bool perpen(const line &a, const line &b) { return sgn(a.d * b.d, 0) == 0; }
218
219    // 两直线是否平行
220    bool parallel(const line &a, const line &b) { return sgn(a.d ^ b.d, 0) == 0; }
221
222    // 点的垂线是否与线段有交点
223    bool perpen(const vec &a, const line &b) {
224        vec p(-b.d.y, b.d.x);
225        bool cross1 = sgn(p ^ (b.p - a), 0) > 0;
226        bool cross2 = sgn(p ^ (b.p + b.d - a), 0) > 0;
```

```
227        return cross1 != cross2;
228  }
229
230  // 点到线段距离
231  lf dis_seg(const vec &a, const line &b) {
232        if (perpen(a, b)) return dis(a, b);
233        return min((b.p - a).len(), (b.p + b.d - a).len());
234  }
235
236  // 点到凸包距离
237  lf dis(const vec &a, vector<vec> &p) {
238        lf res = inf;
239        for (int i = 1; i < p.size(); i++) res = min(dis_seg(a, line(p[i - 1], p[i] - p[i - 1])), res);
240        res = min(dis_seg(a, line(p.back(), p[0] - p.back())), res);
241        return res;
242  }
243
244  // 两直线交点
245  vec intersection(lf A, lf B, lf C, lf D, lf E, lf F) {
246        return {(B * F - C * E) / (A * E - B * D), (C * D - A * F) / (A * E - B * D)};
247  }
248
249  // 两直线交点
250  vec intersection(const line &a, const line &b) {
251        return intersection(a.d.y, -a.d.x, a.d.x * a.p.y - a.d.y * a.p.x, b.d.y, -b.d.x,
252                            b.d.x * b.p.y - b.d.y * b.p.x);
253  }
254
255  struct circle {
256        vec o;
257        lf r;
258        circle(const vec &_o, lf _r) : o(_o), r(_r){};
259        circle(const vec &a, const vec &b, const vec &c) {
260            line u((a + b) / 2, (a + b) / 2 + (b - a).rotleft());
261            line v((b + c) / 2, (b + c) / 2 + (c - b).rotleft());
262            o = intersection(u, v);
263            r = (o - a).len();
264        }
265        // 内切圆
266        circle(const vec &a, const vec &b, const vec &c, bool t) {
267            line u, v;
268            double m = atan2(b.y - a.y, b.x - a.x), n = atan2(c.y - a.y, c.x - a.x);
269            u.p = a;
270            u.d = vec(cos((n + m) / 2), sin((n + m) / 2));
271            v.p = b;
272            m = atan2(a.y - b.y, a.x - b.x), n = atan2(c.y - b.y, c.x - b.x);
273            v.d = vec(cos((n + m) / 2), sin((n + m) / 2));
274            o = intersection(u, v);
275            r = dis_seg(o, line(a, b));
276        }
277
278        // 点与圆的关系 -1在圆内, 0在圆上, 1在圆外
279        int relation(const vec &a) const { return sgn((a - o).len(), r); }
280
281        // 圆与圆的关系 -3包含, -2内切, -1相交, 0外切, 1相离
282        int relation(const circle &a) const {
283            lf l = (a.o - o).len();
284            if (sgn(l, abs(r - a.r)) < 0) return -3;
```

```
285            if (sgn(l, abs(r - a.r)) == 0) return -2;
286            if (sgn(l, abs(r + a.r)) < 0) return -1;
287            if (sgn(l, abs(r + a.r)) == 0) return 0;
288            return 1;
289        }
290
291        lf area() { return PI * r * r; }
292    };
293
294    // 圆与圆交点
295    auto intersection(const circle &a, const circle &b) {
296        int rel = a.relation(b);
297        vector<vec> res;
298        if (rel == -3 || rel == 1) return res;
299        vec o = b.o - a.o;
300        lf l = (o.len2() + a.r * a.r - b.r * b.r) / (2 * o.len());
301        lf h = sqrt(a.r * a.r - l * l);
302        o = o.e();
303        vec tmp = a.o + o * l;
304        if (rel == -2 || rel == 0) res.push_back(tmp);
305        else {
306            res.push_back(tmp + o.rotleft() * h);
307            res.push_back(tmp + o.rotright() * h);
308        }
309        return res;
310    }
311
312    // 圆与直线交点
313    auto intersection(const circle &c, const line &l) {
314        lf d = dis(c.o, l);
315        vector<vec> res;
316        vec mid = l.p + l.d.e() * ((c.o - l.p) * l.d / l.d.len());
317        if (sgn(d, c.r) == 0) res.push_back(mid);
318        else if (sgn(d, c.r) < 0) {
319            d = sqrt(c.r * c.r - d * d);
320            res.push_back(mid + l.d.e() * d);
321            res.push_back(mid - l.d.e() * d);
322        }
323        return res;
324    }
325
326    // oab三角形与圆相交的面积
327    lf area(const circle &c, const vec &a, const vec &b) {
328        if (sgn(cross(a, b, c.o), 0) == 0) return 0;
329        vector<vec> p;
330        p.push_back(a);
331        line l(a, b);
332        auto tmp = intersection(c, l);
333        if (tmp.size() == 2) {
334            for (auto &i : tmp)
335                if (sgn((a - i) * (b - i), 0) < 0) p.push_back(i);
336        }
337        p.push_back(b);
338        if (p.size() == 4 && sgn((p[0] - p[1]) * (p[2] - p[1]), 0) > 0) swap(p[1], p[2]);
339        lf res = 0;
340        for (int i = 1; i < p.size(); i++)
341            if (c.relation(p[i - 1]) == 1 || c.relation(p[i]) == 1) {
342                lf ang = angle(p[i - 1] - c.o, p[i] - c.o);
```

```
343            res += c.r * c.r * ang / 2;
344        } else res += abs(cross(p[i - 1], p[i], c.o)) / 2.0;
345    return res;
346 }
347
348 // 多边形与圆相交的面积
349 lf area(vector<vec> &p, circle c) {
350    lf res = 0;
351    for (int i = 0; i < p.size(); i++) {
352        int j = i + 1 == p.size() ? 0 : i + 1;
353        if (sgn(cross(p[i], p[j], c.o), 0) <= 0) res += area(c, p[i], p[j]);
354        else res -= area(c, p[i], p[j]);
355    }
356    return abs(res);
357 }
```

三维

```
1 constexpr lf eps = 1e-8;
2
3 int sgn(lf a, lf b) {
4    lf c = a - b;
5    return c < -eps ? -1 : c < eps ? 0 : 1;
6 }
7
8 // 向量
9 struct vec3 {
10    lf x, y, z;
11    vec3() : x(0), y(0), z(0) {}
12    vec3(lf _x, lf _y, lf _z) : x(_x), y(_y), z(_z) {}
13
14    // 模
15    lf len2() const { return x * x + y * y + z * z; }
16    lf len() const { return hypot(x, y, z); }
17
18    bool operator==(const vec3 &b) const {
19        return sgn(x, b.x) == 0 && sgn(y, b.y) == 0 && sgn(z, b.z) == 0;
20    }
21    bool operator!=(const vec3 &b) const { return !(*this == b); }
22
23    vec3 operator+(const vec3 &b) const { return {x + b.x, y + b.y, z + b.z}; }
24    vec3 operator-() const { return {-x, -y, -z}; }
25    vec3 operator-(const vec3 &b) const { return -b + (*this); }
26    vec3 operator*(lf b) const { return {b * x, b * y, b * z}; }
27    lf operator*(const vec3 &b) const { return x * b.x + y * b.y + z * b.z; }
28
29    // 叉积 结果大于0，a到b为逆时针，小于0，a到b顺时针，
30    // 等于0共线，可能同向或反向，结果绝对值表示 a b 形成的平行四边行的面积
31    vec3 operator^(const vec3 &b) const {
32        return {y * b.z - z * b.y, z * b.x - x * b.z, x * b.y - y * b.x};
33    }
34
35    friend istream &operator>>(istream &in, vec3 &a) {
36        in >> a.x >> a.y >> a.z;
37        return in;
38    }
39    friend ostream &operator<<(ostream &out, const vec3 &a) {
40        out << fixed << setprecision(6);
41        out << a.x << " " << a.y << " " << a.z;
```

```
42          return out;
43      }
44  };
45
46  struct line3 {
47      vec3 p, d;
48      line3() {}
49      line3(const vec3 &a, const vec3 &b) : p(a), d(b - a) {}
50  };
51
52  struct plane {
53      vec3 p, d;
54      plane() {}
55      plane(const vec3 &a, const vec3 &b, const vec3 &c) : p(a) {
56          d = (b - a) ^ (c - a);
57          assert(d != vec3());
58      }
59  };
60
61  // 线面是否垂直
62  bool perpen(const line3 &a, const plane &b) { return (a.d ^ b.d) == vec3(); }
63
64  // 线面是否平行
65  bool parallel(const line3 &a, const plane &b) { return sgn(a.d * b.d, 0) == 0; }
66
67  // 线面交点
68  vec3 intersection(const line3 &a, const plane &b) {
69      assert(!parallel(a, b));
70      double t = (b.p - a.p) * b.d / (a.d * b.d);
71      return a.p + a.d * t;
72  }
```

5.3 扫描线

```
1   #define ls (pos << 1)
2   #define rs (ls | 1)
3   #define mid ((tree[pos].l + tree[pos].r) >> 1)
4   struct Rectangle {
5       ll x_l, y_l, x_r, y_r;
6   };
7   ll area(vector<Rectangle>& rec) {
8       struct Line {
9           ll x, y_up, y_down;
10          int pd;
11      };
12      vector<Line> line(rec.size() * 2);
13      vector<ll> y_set(rec.size() * 2);
14      for (int i = 0; i < rec.size(); i++) {
15          y_set[i * 2] = rec[i].y_l;
16          y_set[i * 2 + 1] = rec[i].y_r;
17          line[i * 2] = {rec[i].x_l, rec[i].y_r, rec[i].y_l, 1};
18          line[i * 2 + 1] = {rec[i].x_r, rec[i].y_r, rec[i].y_l, -1};
19      }
20      sort(y_set.begin(), y_set.end());
21      y_set.erase(unique(y_set.begin(), y_set.end()), y_set.end());
22      sort(line.begin(), line.end(), [](Line a, Line b) { return a.x < b.x; });
23      struct Data {
```

```
24          int l, r;
25          ll len, cnt, raw_len;
26      };
27      vector<Data> tree(4 * y_set.size());
28      function<void(int, int, int)> build = [&](int pos, int l, int r) {
29          tree[pos].l = l;
30          tree[pos].r = r;
31          if (l == r) {
32              tree[pos].raw_len = y_set[r + 1] - y_set[l];
33              tree[pos].cnt = tree[pos].len = 0;
34              return;
35          }
36          build(ls, l, mid);
37          build(rs, mid + 1, r);
38          tree[pos].raw_len = tree[ls].raw_len + tree[rs].raw_len;
39      };
40      function<void(int, int, int, int)> update = [&](int pos, int l, int r, int num) {
41          if (l <= tree[pos].l && tree[pos].r <= r) {
42              tree[pos].cnt += num;
43              tree[pos].len = tree[pos].cnt                  ? tree[pos].raw_len
44                              : tree[pos].l == tree[pos].r ? 0
45                                                           : tree[ls].len + tree[rs].len;
46              return;
47          }
48          if (l <= mid) update(ls, l, r, num);
49          if (r > mid) update(rs, l, r, num);
50          tree[pos].len = tree[pos].cnt ? tree[pos].raw_len : tree[ls].len + tree[rs].len;
51      };
52      build(1, 0, y_set.size() - 2);
53      auto find_pos = [&](ll num) {
54          return lower_bound(y_set.begin(), y_set.end(), num) - y_set.begin();
55      };
56      ll res = 0;
57      for (int i = 0; i < line.size() - 1; i++) {
58          update(1, find_pos(line[i].y_down), find_pos(line[i].y_up) - 1, line[i].pd);
59          res += (line[i + 1].x - line[i].x) * tree[1].len;
60      }
61      return res;
62 }
```

6 杂项

6.1 快读

```cpp
namespace IO {
constexpr int N = (1 << 20) + 1;
char Buffer[N];
int p = N;

char& get() {
    if (p == N) {
        fread(Buffer, 1, N, stdin);
        p = 0;
    }
    return Buffer[p++];
}

template <typename T = int>
T read() {
    T x = 0;
    bool f = 1;
    char c = get();
    while (!isdigit(c)) {
        f = c != '-';
        c = get();
    }
    while (isdigit(c)) {
        x = x * 10 + c - '0';
        c = get();
    }
    return f ? x : -x;
}
}  // namespace IO
using IO::read;
```

6.2 高精度

```cpp
struct bignum {
    string num;

    bignum() : num("0") {}
    bignum(const string& num) : num(num) { reverse(this->num.begin(), this->num.end()); }
    bignum(ll num) : num(to_string(num)) { reverse(this->num.begin(), this->num.end()); }

    bignum operator+(const bignum& other) {
        bignum res;
        res.num.pop_back();
        res.num.reserve(max(num.size(), other.num.size()) + 1);
        for (int i = 0, j = 0, x; i < num.size() || i < other.num.size() || j; i++) {
            x = j;
            j = 0;
            if (i < num.size()) x += num[i] - '0';
            if (i < other.num.size()) x += other.num[i] - '0';
            if (x >= 10) j = 1, x -= 10;
            res.num.push_back(x + '0');
        }
        res.num.capacity();
```

```
21         return res;
22     }
23
24     bignum operator*(const bignum& other) {
25         vector<int> res(num.size() + other.num.size() - 1, 0);
26         for (int i = 0; i < num.size(); i++)
27             for (int j = 0; j < other.num.size(); j++)
28                 res[i + j] += (num[i] - '0') * (other.num[j] - '0');
29         int g = 0;
30         for (int i = 0; i < res.size(); i++) {
31             res[i] += g;
32             g = res[i] / 10;
33             res[i] %= 10;
34         }
35         while (g) {
36             res.push_back(g % 10);
37             g /= 10;
38         }
39         int lim = res.size();
40         while (lim > 1 && res[lim - 1] == 0) lim--;
41         bignum res2;
42         res2.num.resize(lim);
43         for (int i = 0; i < lim; i++) res2.num[i] = res[i] + '0';
44         return res2;
45     }
46
47     bool operator<(const bignum& other) {
48         if (num.size() == other.num.size())
49             for (int i = num.size() - 1; i >= 0; i--)
50                 if (num[i] == other.num[i]) continue;
51                 else return num[i] < other.num[i];
52         return num.size() < other.num.size();
53     }
54
55     friend istream& operator>>(istream& in, bignum& a) {
56         in >> a.num;
57         reverse(a.num.begin(), a.num.end());
58         return in;
59     }
60     friend ostream& operator<<(ostream& out, bignum a) {
61         reverse(a.num.begin(), a.num.end());
62         return out << a.num;
63     }
64 };
```

6.3 离散化

```
1  template <typename T>
2  struct Hash {
3      vector<int> S;
4      vector<T> a;
5      Hash(const vector<int>& b) : S(b) {
6          sort(S.begin(), S.end());
7          S.erase(unique(S.begin(), S.end()), S.end());
8          a = vector<T>(S.size());
9      }
10     T& operator[](int i) const {
```

```
11          auto pos = lower_bound(S.begin(), S.end(), i) - S.begin();
12          assert(pos != S.size() && S[pos] == i);
13          return a[pos];
14      }
15 };
```

6.4   模运算

```
1 constexpr int mod = 998244353;
2
3 template <typename T>
4 T power(T a, int b) {
5      T res = 1;
6      while (b) {
7          if (b & 1) res = res * a;
8          a = a * a;
9          b >>= 1;
10     }
11     return res;
12 }
13
14 struct modint {
15     int x;
16     modint(int _x = 0) : x(_x) {
17         if (x < 0) x += mod;
18         else if (x >= mod) x -= mod;
19     }
20     modint inv() const { return power(*this, mod - 2); }
21     modint operator+(const modint& b) { return x + b.x; }
22     modint operator-() const { return -x; }
23     modint operator-(const modint& b) { return -b + *this; }
24     modint operator*(const modint& b) { return (ll)x * b.x % mod; }
25     modint operator/(const modint& b) { return *this * b.inv(); }
26     friend istream& operator>>(istream& is, modint& other) {
27         ll _x;
28         is >> _x;
29         other = modint(_x);
30         return is;
31     }
32     friend ostream& operator<<(ostream& os, modint other) { return os << other.x; }
33 };
```

6.5   分数

```
1 struct frac {
2      ll a, b;
3      frac() : a(0), b(1) {}
4      frac(ll _a, ll _b) : a(_a), b(_b) {
5          assert(b);
6          if (a) {
7              int tmp = gcd(a, b);
8              a /= tmp;
9              b /= tmp;
10         } else *this = frac();
11     }
12     frac operator+(const frac& other) { return frac(a * other.b + other.a * b, b * other.b); }
```

```cpp
13      frac operator-() const {
14          frac res = *this;
15          res.a = -res.a;
16          return res;
17      }
18      frac operator-(const frac& other) const { return -other + *this; }
19      frac operator*(const frac& other) const { return frac(a * other.a, b * other.b); }
20      frac operator/(const frac& other) const {
21          assert(other.a);
22          return *this * frac(other.b, other.a);
23      }
24      bool operator<(const frac& other) const { return (*this - other).a < 0; }
25      bool operator<=(const frac& other) const { return (*this - other).a <= 0; }
26      bool operator>=(const frac& other) const { return (*this - other).a >= 0; }
27      bool operator>(const frac& other) const { return (*this - other).a > 0; }
28      bool operator==(const frac& other) const { return a == other.a && b == other.b; }
29      bool operator!=(const frac& other) const { return !(*this == other); }
30  };
```

6.6　表达式求值

```cpp
1  // 格式化表达式
2  string format(const string& s1) {
3      stringstream ss(s1);
4      string s2;
5      char ch;
6      while ((ch = ss.get()) != EOF) {
7          if (ch == ' ') continue;
8          if (isdigit(ch)) s2 += ch;
9          else {
10              if (s2.back() != ' ') s2 += ' ';
11              s2 += ch;
12              s2 += ' ';
13          }
14      }
15      return s2;
16  }
17
18  // 中缀表达式转后缀表达式
19  string convert(const string& s1) {
20      unordered_map<char, int> rank{{'+', 2}, {'-', 2}, {'*', 1}, {'/', 1}, {'^', 0}};
21      stringstream ss(s1);
22      string s2, temp;
23      stack<char> op;
24      while (ss >> temp) {
25          if (isdigit(temp[0])) s2 += temp + ' ';
26          else if (temp[0] == '(') op.push('(');
27          else if (temp[0] == ')') {
28              while (op.top() != '(') {
29                  s2 += op.top();
30                  s2 += ' ';
31                  op.pop();
32              }
33              op.pop();
34          } else {
35              while (!op.empty() && op.top() != '(' &&
36                      (temp[0] != '^' && rank[op.top()] <= rank[temp[0]] ||
```

```
37                    rank[op.top()] < rank[temp[0]])) {
38                s2 += op.top();
39                s2 += ' ';
40                op.pop();
41            }
42            op.push(temp[0]);
43        }
44    }
45    while (!op.empty()) {
46        s2 += op.top();
47        s2 += ' ';
48        op.pop();
49    }
50    return s2;
51 }
52
53 // 计算后缀表达式
54 int calc(const string& s) {
55    stack<int> num;
56    stringstream ss(s);
57    string temp;
58    while (ss >> temp) {
59        if (isdigit(temp[0])) num.push(stoi(temp));
60        else {
61            int b = num.top();
62            num.pop();
63            int a = num.top();
64            num.pop();
65            if (temp[0] == '+') a += b;
66            else if (temp[0] == '-') a -= b;
67            else if (temp[0] == '*') a *= b;
68            else if (temp[0] == '/') a /= b;
69            else if (temp[0] == '^') a = ksm(a, b);
70            num.push(a);
71        }
72    }
73    return num.top();
74 }
```

6.7 日期

```
1 int month[] = {0, 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
2 int pre[13];
3 vector<int> leap;
4 struct Date {
5    int y, m, d;
6    bool operator<(const Date& other) const {
7        return array<int, 3>{y, m, d} < array<int, 3>{other.y, other.m, other.d};
8    }
9    Date(const string& s) {
10        stringstream ss(s);
11        char ch;
12        ss >> y >> ch >> m >> ch >> d;
13    }
14    int dis() const {
15        int yd = (y - 1) * 365 + (upper_bound(leap.begin(), leap.end(), y - 1) - leap.begin());
16        int md = pre[m - 1] + (m > 2 && (y % 4 == 0 && y % 100 || y % 400 == 0));
```

```
17        return yd + md + d;
18    }
19    int dis(const Date& other) const { return other.dis() - dis(); }
20 };
21 for (int i = 1; i <= 12; i++) pre[i] = pre[i - 1] + month[2];
22 for (int i = 1; i <= 1000000; i++)
23     if (i % 4 == 0 && i % 100 || i % 400 == 0) leap.push_back(i);
```

6.8 builtin 函数

如果是 long long 型，记得函数后多加个 ll。

- ctz，从最低位连续的 0 的个数，如果传入 0 则行为未定义。

- clz，从最高位连续的 0 的个数，如果传入 0 则行为未定义。

- popcount，二进制 1 的个数。

- parity，二进制 1 的个数奇偶性。

6.9 对拍

linux/Mac

```
1 #!/bin/bash
2
3 g++ $1 -o a -O2
4 g++ $2 -o b -O2
5 g++ random.cpp -o random -O2
6
7 cnt=0
8 while true; do
9     let cnt++
10     echo TEST:$cnt
11     ./random > in
12     ./a < in > out.a
13     ./b < in > out.b
14     if ! diff out.a out.b; then break; fi
15 done
```

windows

```
1 @echo off
2
3 g++ %1 -o a -O2
4 g++ %2 -o b -O2
5 g++ random.cpp -o random -O2
6
7 set cnt=0
8
9 :again
10     set /a cnt=cnt+1
11     echo TEST:%cnt%
12     .\random > in
13     .\a < in > out.a
14     .\b < in > out.b
15     fc out.a out.b > nul
16 if not errorlevel 1 goto again
```

6.10　编译常用选项

```
1  -fsanitize=address,undefined -Woverflow -Wshadow -Wall -Wextra -Wpedantic -Wfloat-equal
```

6.11　开栈

不同的系统/编译器可能命令不一样

```
1  ulimit -s
2  -Wl,--stack=0x10000000
3  -Wl,-stack_size -Wl,0x10000000
4  -Wl,-z,stack-size=0x10000000
```

6.12　clang-format

转储配置

```
1  clang-format -style=Google -dump-config > ./.clang-format
```

.clang-format

```
1  BasedOnStyle: Google
2  IndentWidth: 4
3  AllowShortIfStatementsOnASingleLine: AllIfsAndElse
4  ColumnLimit: 100
```