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Tutorial Sheet Module 1

MAT1011-Calculus for Engineers

1. Show that the rectangle that has maximum area for a given perimeter is a square.

2. The height of an object moving vertically is given by $s = -16t^2 + 96t + 112$ with s in feet and t in second. Find
(a) its velocity when $t = 0$
(b) Its maximum height
(c) Its velocity when s = 0.

3. Determine that constant a and b in order that function $f(x) = x^3 + ax^2 + bx + c$ a relative minima at $x = 4$ and a point of inflection at $x = 1$.

4. A vertical line passing through the point (1,2) intersects the X axis at $A(a, 0)$ and Y axis at $B(0, b)$. Find area of triangle of least area if a and b are positive.

5. Determine the absolute extrema of the $f(x) = 8x^3 + 81x^2 - 42x - 8$ on the $[-8, 2]$

6. Give the intervals where the given functions are increasing and decreasing
(a) $f(x) = \frac{x^2-3x}{x+1}$
(b) $f(x) = x^2e^{-3x}$
(c) $f(x) = x^3 - x$
(d) $f(x) = x\log x$

7. Without solving the equations, show that equations
(a) $f(x) = 2x^3 - 3x^2 - 12x - 6$ on [-1,0]
(b) $f(x) = x^4 + 2x^3 - 2$ on [0,1] have exactly one real root.

8. A bike drove 30 miles during a one hour trip. Show that bike speed was equal to 30 mile / hour at least once during the trip.

9. Find the point at the curves $f(x)$ where tangent is parallel to the chord joining end points of curves.
(a) $f(x) = x^2 + 2x - 1$ on $\begin{bmatrix} 0 & 1 \end{bmatrix}$.
(b) $f(x) = x + \frac{1}{x}$ on $\left[\frac{1}{2}2\right]$
(c) $f(x) = \sqrt{x-1}$ on $\begin{bmatrix} 1 & 3 \end{bmatrix}$.

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10. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

11. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$.

12. Find the area bounded on the right by $x + y = 2$, on the left by $y = x^2$ and below by the X axis.

13. Find the area of the region bounded by the curve $y = xe^{-x}$ and the x -axis from $x = 0$ to $x = 4$

14. Find the volume generated by revolving the given region about the given axis.
(a) The region bounded by $y = x^4, x = 1$ and $y = 0$ about Y axis.
(b) The triangle with vertices $(1,1),(1,2)(2,2)$ about X axis.
(c) The region in the first quadrant bounded by $x = y - y^3, x = 1$ and $y = 1$ about X axis.

15. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$

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Tutorial Sheet Module 2

MAT1011-Calculus for Engineers

1. Find (a) $L\left[\cos^4 t\right]$ (b) $L[\cos(3t-4)]$ (c) $L\left[t^2e^{-t}\cos t\right]$

   (d) $L\left[\frac{e^{-3t}\sin 3t}{t}\right]$ (e) $L\left[te^{-t}\cosh t\right]$ (f) $L\left[\frac{\sin^2 t}{t}\right]$

2. Find $L[f(t)]$ if

   (a) $f(t)=\begin{cases} e^t, & 0\le t<1 \\ 0 & t>1\end{cases}$ (b) $f(t)=\begin{cases} \cos t, & 0\le t<\pi \\ \sin t, & t>\pi\end{cases}$.

   (c) $L[f(t)]$ if $f(t)=\begin{cases} \sin t, & 0\le t<\pi \\ 0, & t>\pi\end{cases}$

3. Find the Laplace transform for the square wave given by

   $f(t)=\begin{cases} k & ,0\le t\le \frac{T}{2} \\ -k & ,\frac{T}{2}\le t\le T\end{cases}, f(t+T)=f(t)$

4. Find the Laplace transform of the Half-sine wave rectifier function

   $f(t)=\begin{cases} \sin\omega t & ,0\le t\le \frac{\pi}{\omega} \\ 0 & ,\frac{\pi}{\omega}\le t\le \frac{2\pi}{\omega}\end{cases}$

5. Find the Laplace transform of the periodic function $f(t)=\begin{cases} \sin t & ,0\le t\le \pi \\ 0 & ,\pi\le t\le 2\pi\end{cases}$.

6. Find (a) $L^{-1}\left[\frac{s-3}{(s^2+4s+13)}\right]$ (b) $L^{-1}\left[\frac{s-2}{(6s^2+20)}\right]$

   (c) $L^{-1}\left[\frac{s}{(s+2)^2+4)}\right]$ (d) $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$

7. Using convolution theorem,find

   (a) $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$ (b) $L^{-1}\left[\frac{2}{(s+1)(s^2+4)}\right]$ (c) $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$

   (d) $\frac{s^2}{(s^2+a^2)^2}$ (e) $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$

8. Find (a) $L\left[e^{-2t}\int_0^t\left(\frac{1-\cos u}{u}\right)du\right]$ (b) $L\left[e^{-t}\int_0^t t^2\cos t\,dt\right]$

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(c) $L\left[\int_0^t \frac{\sin x}{x} dx + te^{-t}\cos^2(2t)\right]$ (d) L $\left[\frac{\cos(at)-\cos(bt)}{t}\right]$ if it exists.

9. Find $L[f(t)]$ if $f(t) = |t-1| + |t+1|$ for $t \geq 0$

10. Find (a) $L\left[t^2 u(t-1) + t\delta(t-1)\right]$ (b) $L\left[e^{-t}\left(1+3t+t^{\frac{1}{2}}\right)u(t-2)\right]$

11. Find Laplace transform of f(t) = tan(t) if it exists, otherwise justify your answer.

12. Find (a) $L\left[e^{-2t}\cosh^3(t)\right]$ (b) $L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right]$

13. Express the following function in terms of Unit Step function and hence find its Laplace transform.

$$\text{(a)}\ f(t) = \begin{cases} \cos(t), 0 < t < \pi \\ 1, \pi < t < 2\pi \\ \sin(t), t > 2\pi \end{cases} \qquad \text{(b)}\ f(t) = \begin{cases} \cos(t), 0 < t < \pi \\ \cos(2t), \pi < t < 2\pi \\ \cos(3t), t > 2\pi \end{cases}$$

14. Find the Laplace transform of the function $f(t)$ defined by

$$f(t) = |\cos(\omega t)|, t \geq 0$$

15. Find (a) $L^{-1}\left\{\log\left(1+\frac{\omega^2}{s^2}\right\}\right.$

(b) $L^{-1}\left\{\tan^{-1}\left(\frac{1}{S}\right)\right\}$

(c) $L^{-1}\left[s\log\left(\frac{s-a}{s+a}\right)+a\right]$

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Tutorial Sheet Module 3

MAT1011-Calculus for Engineers

1. Determine the limits if they exist.
   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x}{\sqrt{x^2+y^2}}$ (b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{1-x-y}{x^2+y^2}$
   (c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{x^3+y^3}$ (d) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x}{x^2+y^2}$
   (e) $\lim\limits_{(x,y)\to(2,4)} \dfrac{(x-2)^2(y-4)^2}{(x-2)^3+(y-4)^3}$.
2. Show that the following limits exist.
   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{x^2+y^2}$ (b) $\lim\limits_{(x,y)\to(0,0)} \left(x^3+y^3\right)\cos\dfrac{1}{xy}$
   (c) $\lim\limits_{(x,y)\to(0,0)} y+x\cos\dfrac{1}{y}$ (d) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\left(x^3+y^3\right)e^{-x^2y^2}}{x^2+y^2}$
   (e) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\left(x^4-y^4\right)\left(1-x^2\right)}{x^2+y^2}$.
3. Discuss the continuity of the following functions at (0,0) .
   (a)
   $$f(x,y)=\begin{cases}\frac{(x-y)^2}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$
   (b)
   $$f(x,y)=\begin{cases}\frac{(x-y)xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$
   (c)
   $$f(x,y)=\begin{cases}\frac{x^4y^4}{(x^2+y^2)^3} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$
   (d)
   $$f(x,y)=\begin{cases}\frac{x^4y^5}{x^5+y^5} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$
4. Let z=log $\left(u^2+v\right), u=\exp\left(x^2+y^2\right), v=x^2+y$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial u}$.

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5. Let $u = f(r, s), r = x + at, s = y + bt$, where x, y, t are independent variables and, a and b are constants. Show that $\frac{\partial u}{\partial t} = a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y}$
6. Find the equation of the tangent plane and normal line to the surface $x^2 + 2y^2 + 3z^2 = 12$ at $(1, 2, -1)$.
7. If $z = \sin^{-1}(x - y), \quad x = 3t, \quad y = 4t^3$; show that $\frac{dz}{dt} = \frac{3}{\sqrt{1-t^2}}$.
8. If $z = f(x, y)$, where $x = e^u + e^{-v}, y = e^{-u} - e^v$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

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Tutorial Sheet Module 4

MAT1011-Calculus for Engineers

1. Find the relative maxima and minima of the following functions (a) $f(x, y) = xy + \frac{9}{x} + \frac{3}{y}$
   (b) $f(x, y) = x^2 + y^2 + xy + \frac{1}{x} + \frac{1}{y}$
   (c) $f(x, y) = x^2 + \frac{2}{x^2 y} + y^2$

2. Find the relative and absolute maxima and minima of the following functions in the given domain. (a) $f(x, y) = x^2 - y^2 - 2y, x^2 + y^2 \leq 1$
   (b) $f(x, y) = xy, x^2 + y^2 \leq 1$
   (c) $f(x, y) = x + y, 4x^2 + 9y^2 \leq 36$
   (d) $f(x, y) = 4x^2 + y^2 - 2x + 1, 2x^2 + y^2 \leq 1$
   (e) $f(x, y) = x^2 + y^2 - x - y + 1, 0 \leq x \leq 2, 0 \leq y \leq 2$
   (f) $f(x, y) = x^3 + y^3 - xy$ over the triangular region bounded by the lines $x = 0, y = 0$, and $y = 2x$

3. Using the Lagrange method of multipliers, solve the following problems.
   (a) Find the smallest and the largest values of xy on the line segment $x + 2y = 2, x \geq 0, y \geq 0$
   (b) Find the smallest and the largest values of $x + 2y$ on the circle $x^2 + y^2 = 1$
   (c) Find the points on the curve $x^2 + xy + y^2 = 16$ which are nearest and farthest form the origin.
   (d) Find the triangle whose perimeter is constant and has largest area.
   (e) Find the extreme value of xyz when $x + y + z = a, a > 0$.
   (f) Find the extreme value of $a^3x^2 + b^3y^2 + c^3z^2$ such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, where
   $$a > 0, b > 0, c > 0$$

4. Expand $f(x, y) = \sin(x + 2y)$ in Taylor's series upto third degree terms about $(1, 3)$. Hence, approximate $f(0.99, 3.01)$

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MULTIPLE INTEGRALS

PART- A

1. Evaluate the following :

(i) $\int_4^3\int_1^2 (x+y)^{-2}dxdy$ (ii) $\int_2^a\int_2^b \frac{dxdy}{xy}$ (iii) $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin(x+y)\,dxdy$ (iv) $\int_0^1\int_x^{\sqrt{x}} xy\,(x+y)\,dxdy$

(v) $\int_0^{\pi}\int_0^{a\cos\theta} r\sin\theta\,drd\theta$ (vi) $\int_0^{\frac{\pi}{2}}\int_0^{\sin\theta} r\,d\theta\,dr$ (vii) $\int_1^2\int_0^{x^2} x\,dxdy$ (viii) $\int_0^a\int_0^{\sqrt{a^2-x^2}} dydx$ (ix) $\int_1^2\int_0^y \frac{dxdy}{x^2+y^2}$

(x) $\int_0^1\int_0^{\sqrt{1+y^2}} \frac{dxdy}{1+x^2+y^2}$.

[Ans.: (i) log (24/25) (ii) log (a/2) log (b/2) (iii) 2 (iv) 3/56 (v) $a^2/3$ (vi) $\pi/8$ (vii) 15/4
(viii) $\pi a^2/4$ (ix) $\frac{\pi}{4}\log 2$ (x) $\frac{\pi}{4}\log(1+\sqrt{2})$]

2. Find $\iint dxdy$ over the region bounded by $x \geq 0, y \geq 0, x+y \leq 1$. [Ans.: 1/2]

3. Find the limits of integration in the double integral $\iint f(x,y)dxdy$ over the region bounded by $x=1, y=0\, and\, y^2=4x$ in the first quadrant.

4. Sketch the region of integration for (i) $\int_0^1\int_0^x f(x,y)\,dydx$ (ii) $\int_0^b\int_0^{\frac{a}{b}(b-y)} f(x,y)dxdy$.

5. Shade the region of integration $\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} dxdy$.

6. Change the order of integration in $\int_0^1\int_y^{2-y} xy\,dxdy$

7. Find the area of a circle of radius 'a' by double integration in polar coordinates. [Ans.: πa^2]

8. Transform the following integrations (i) $\int_0^{\infty}\int_0^y dxdy$ (ii) $\int_{-a}^a\int_{-\sqrt{a\ -x^2}}^{\sqrt{a\ -x^2}} dydx$ to polar coordinates.

9. Express the region x $\geq$0, y $\geq$0, z $\geq$0, $x^2 + y^2 + z^2 \leq 1$ by triple integration.

10. Evaluate (i) $\int_0^{2\pi}\int_0^{\pi}\int_0^a r^4\, Sin\phi\, dr\, d\phi\, d\theta$ (ii) $\int_{\rho=0}^1\int_{z=\rho^2}^{\rho}\int_{\theta=0}^{2\pi} \rho\, d\rho\, dz\, d\theta$ (iii) $\int_0^1\int_0^{1-x}\int_0^{x+y} e^z\,dx\,dy\,dz$

(iv) $\int_0^1\int_0^2\int_0^3 xyz\,dxdydz$. [Ans.: (i) $\frac{4\pi a^5}{5}$ (ii) $\frac{\pi}{6}$ (iii) ½ (iv) 9/2]

PART-B

1. Evaluate $\iint (1+xy)dxdy$ in the region bounded by the line $y = x-1$ and the parabola $y^2 = 2x+6$.
[Ans.: 54]

2. Evaluate $\iint xy\,dxdy$ over the region bounded by the x-axis, $x = 2a$ and $x^2 = 4ay$. [Ans.: $4a^3/3$]

3. Evaluate $\iint xy\,dxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. [Ans.: $a^4/8$]

4. Find the value of $\iint xy\,dxdy$ taken over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.[Ans.: $a^2b^2/8$]

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5. Evaluate $\iint r^3 dr d\theta$ over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.[Ans.: $45\pi/2$]

6. Evaluate $\iint \frac{r dr d\theta}{\sqrt{a^2 + r^2}}$ over the one loop of the lemniscates $r^2 = a^2 \cos 2\theta$. [Ans.: $\frac{a}{2}(4-\pi)$]

7. Show that $\iint_R r^2 \sin\theta dr d\theta = \frac{2a^3}{3}$, where R is the semicircle $r = 2a\cos\theta$ above the initial line.

8. Change the order the integration and hence evaluate the following:

i) $\int_0^a \int_{x^2/a}^{2a-x} xy\, dy\, dx$ (ii) $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\, dx$ (iii) $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y\, dx\, dy$ (iv) $\int_0^b \int_0^{a/b\sqrt{b^2-y^2}} xy\, dx\, dy$ (v) $\int_0^1 \int_y^{\sqrt{y}} \frac{x}{x^2+y^2} dxdy$

(vi) $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$ (vii) $\int_0^a \int_y^a \frac{x}{\sqrt{x^2+y^2}} dxdy$. [Ans.: (i) $\frac{3a^4}{8}$ (ii) 16/3 (iii) $a^3/6$ (iv) $\frac{a^2b^2}{8}$

(v) $\log\sqrt{2}$ (vi) 1 (vii) $\frac{a^2}{2}\log(1+\sqrt{2})$]

9. Using double integration , find the area bounded by y = x and y = x^2 [Ans: 1/ 6]

10. Find, by double integration, the area between the parabola $y^2 = 4ax$ and the line $y = x$. [Ans.: $8a^2/3$]

11. Find the area common to $y^2 = 4ax$ and $x^2 = 4ay$ using double integration. [Ans.: $16a^3/3$]

12. Find the smaller area bounded by y = 2-x and $x^2 + y^2 = 4$. [Ans: π -2]

13. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [Ans : π ab]

14. Find the area of the cardioid r = a (1 + cosθ) by using double integration.. [Ans: $\frac{3\pi a^2}{2}$]

15. Find the area which is inside the circle $r = 3a\cos\theta$ and outside the cardioid r = a (1 + cosθ) .[Ans.: πa^2]

16. By converting in to polar coordinates and evaluate the following:

(i) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2} dx\, dy$ (ii) $\int_0^a \int_y^a \frac{x}{x^2+y^2} dx\, dy$ (iii) $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dxdy$ (iv) $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2)\, dydx$

(v) $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}} dxdy$. [Ans.: (i) $\pi/2$, (ii) $\frac{\pi a}{4}$, (iii) $\frac{a^3}{3}\log(\sqrt{2}+1)$, (iv) $\frac{3\pi a^4}{4}$ (v) $\frac{a}{\sqrt{2}}$]

17. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing in to polar coordinates. Hence show that $\int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2$.

18. Evaluate the integration $\iiint_V dx\, dy\, dz$, where V is finite region of space formed by the planes

x = 0, y = 0, z = 0 and 2x +3y +4z = 12. [Ans.:12]

19. Evaluate the integration $\iiint x\, y\, z\, dx\, dy\, dz$ taken throughout the volume for which x, y, z $\geq$ 0 an $x^2 + y^2 + z^2 \leq 9$. [Ans.: 243/16]

20. Evaluate $\iiint_V (x+y+z)\, dx\, dy\, dz$, where V is the region of space inside the cylinder $x^2 + y^2 = a^2$

that is bounded by the planes z = 0 and z = h. [Ans.: $\frac{\pi a^2 h^2}{2}$]

21. Evaluate $\iiint \frac{dxdydz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$. [Ans.: $\pi^2 a^2/8$]

22. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ by triple integration. [Ans.: $\frac{4\pi a^3}{3}$]

23. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ triple integration. [Ans.: $\frac{4\pi\, abc}{3}$]

24. Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

[Ans.: $abc/6$]

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Tutorial Sheet Module 6

MAT1011-Calculus for Engineers

1. Find the divergence and curl of $e^{xy} \sin z\vec{j} + y \tan^{-1}(x/z)\vec{k}$
2. Show that $\vec{F} = y^2z^3\vec{i} + 2xyz^3\vec{j} + 3xy^2z^2\vec{k}$ is irrotational and find its scalar potential.
3. The force in an electrostatic field $f(x, y, z)$ has the direction of the gradient of f. Find ∇f and its value at P.
   (a) $f = (x-1)^2 - (y+1)^2, P : (4, -3)$
   (b) $f = y/\left(x^2 + y^2\right), P : (5, 3)$
   (c) $f = x^2 - 2x - y^2, P : (-2, 6)$
4. Find the directional derivative of f at P in the direction of $\mathbf{a}$.
   (a) $f = x^2 + y^2 - z, P : (1, 1, -2), \mathbf{a} = [1, 1, 2]$
   (b) $f = x^2 + y^2 + z^2, P : (2, -2, 1), \mathbf{a} = [-1, -1, 0]$
   (c) $f = xyz, P : (-1, 1, 3), \mathbf{a} = [1, -2, 2]$
5. Let $\mathbf{v} = [x, \quad y, \quad v_3]$. Find a v_3 such that (a) $\operatorname{div} \mathbf{v} > 0$ everywhere, (b) $\operatorname{div} \mathbf{v} > 0$ if $|z| < 1$ and $\operatorname{div} \mathbf{v} < 0$ if $|z| > 1$
6. (i) Prove the following vector identities
   (a) $\operatorname{div}(k\mathbf{v}) = k \operatorname{div} \mathbf{v} \quad (k \text{ constant })$
   (b) $\operatorname{div}(f\mathbf{v}) = f \operatorname{div} \mathbf{v} + \mathbf{v} \cdot \nabla f$
   (c) $\operatorname{div}(f\nabla g) = f\nabla^2 g + \nabla f \cdot \nabla g$
   (d) $\operatorname{div}(f\nabla g) - \operatorname{div}(g\nabla f) = f\nabla^2 g - g\nabla^2 f$.
   (ii) Verify (b) for $f = e^{xyz}$ and $\mathbf{v} = ax\mathbf{i} + by\mathbf{j} + cz\mathbf{k}$
7. Show that the flow with velocity vector $\mathbf{v} = y\vec{i}$ is incompressible.
8. Let $\mathbf{v} = [x, \quad y, \quad v_3]$. Find a v_3 such that (a) $\operatorname{div} \mathbf{v} > 0$ everywhere, (b) $\operatorname{div} \mathbf{v} > 0$ if $|z| < 1$ and $\operatorname{div} \mathbf{v} < 0$ if $|z| > 1$.
9. Granted sufficient differentiability of a scalar function f and a vector function $\mathbf{v}$, which of the following make sense? $\operatorname{grad} f, f \operatorname{grad} f, \mathbf{v} \operatorname{grad} f, \mathbf{v} \cdot \operatorname{grad} f, \operatorname{div} f$, $\operatorname{div} \mathbf{v}, \operatorname{div}(f\mathbf{v}), \operatorname{curl}(f\mathbf{v}), \operatorname{curl} f, f \operatorname{curl} \mathbf{v}, \mathbf{v} \operatorname{curl} f$.
10. What direction does curl $\mathbf{v}$ have if $\mathbf{v}$ is a vector parallel to the xz -plane?
11. Find the tangent plane and normal line to $x^2 + y^2 + z^2 = 30$ at the point (1,-2,5).

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Tutorial Sheet Module 7

MAT1011-Calculus for Engineers

1. Compute the line integral of $\mathbf{F} = \langle -y, x \rangle$ along the line segment from (0,0) to (1,1).
2. Is the vector field $\mathbf{F} = \langle 0, x \rangle$ conservative? Give reasons to support your answer?
3. C is the curve $y = e^x$ from $(2, e^2)$ to (0,1) and $\mathbf{F} = \langle x^2, -y \rangle$, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.
4. C is the part of the circle of radius 3 in the first quadrant from (3,0) to (0,3) and $\mathbf{F} = \langle 1, -y \rangle$, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.
5. C is the part of the curve $x = \cos(y)$ from $(1, 2\pi)$ to (1,0) and $\mathbf{F} = \langle y, 2x \rangle$, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.
6. Evaluate $\displaystyle\iint_S z dS$, where S is the surface whose:

   Sides S_1 are given by the cylinder $x^2 + y^2 = 1$,
   Bottom S_2 is the disk $x^2 + y^2 \leq 1$ in the plane $z = 0$,
   Top S_3 is the part of the plane $z = 1 + x$ that lies above S_2.
7. Evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$ where: $\vec{F}(x, y, z) = y\vec{i} + x\vec{j} + z\vec{k}$, S is the boundary of the solid region E enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.
8. Let S be the closed surface that consists of the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$, and its base $x^2 + y^2 \leq 1, z = 0$. Let $\mathbf{E}$ be the electric field defined by $\mathbf{E}(x, y, z) = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$. Find the electric flux across S.
9. Evaluate $\displaystyle\iiint_V (\nabla \times \bar{F}) \mathrm{d}V$, where $\bar{F} = (2x^2 - 3z)\,\hat{i} - 2xy\hat{j} - 4x\hat{k}$ and V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.
10. Use Green's theorem to evaluate the line integral $\displaystyle\int_C \left(1 + xy^2\right) dx - x^2 y dy$, where C consists of the arc of the parabola $y = x^2$ from (-1,1) to (1,1) and the line joining the two points.

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11. Evaluate $\oint_C y^3 dx - x^3$ dy where C are the two circles of radius 2 and radius 1 centered at the origin with positive orientation.

12. Let $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$. Show that the clockwise circulation of the field $\mathbf{F} = \nabla f$ around the circle $x^2 + y^2 = a^2$ in the xy -plane is zero
(a) by taking $\mathbf{r} = (a \cos t)\mathbf{i} + (a \sin t)\mathbf{j}, 0 \leq t \leq 2\pi$, and
integrating $\mathbf{F} \cdot d\mathbf{r}$ over the circle. (b) by applying Stokes' Theorem.

13. Let S be the cylinder $x^2 + y^2 = a^2, 0 \leq z \leq h$, together with its top, $x^2 + y^2 \leq a^2, z = h$. Let $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + x^2\mathbf{k}$. Use Stokes' Theorem to find the flux of $\nabla \times \mathbf{F}$ outward through S.

14. Evaluate $\iint_S (7x\vec{i} - z\vec{k}) \cdot \mathbf{n} dA$ over the sphere $S : x^2 + y^2 + z^2 = 4$
(a) by divergence theorem, (b) directly.

15. Verify the divergence theorem.
(a) $\mathbf{v} = 3r^2\hat{\mathbf{e}}_r - r\dot{\mathbf{e}}_\theta + 2\hat{\mathbf{e}}_z, \quad \mathcal{V}$: the cylinder $r \leq 4$ $0 \leq \theta < 2\pi.0 \leq z \leq 5$,
(b) $\mathbf{v} = z\hat{\mathbf{e}}_z, \quad \mathcal{V}$: the cylinder $r \leq 2, 0 \leq \theta < 2\pi.$ $-3 \leq z \leq 6$.