Assignment 1

Motion Planning with Aerospace Applications

3-25

1. Assumptions on initial conditions & dimensions:-

Lets picture 2 footballs colliding (Robot shape)

$R_1 = 10cm = 0.1m$ $R_2 = 0.1m.$

(a) To find the distance of closest approach:-

Let A be robot 1 & B be robot 2.

V_{AO} is initial vel of robot 1 } in global coordinates.

V_{BO} is " " " " 2

Let $V_{AO} = 2m/s$ $V_{BO} = 4m/s$

$\alpha_{AO} = 40^\circ$ $\alpha_{BO} = 80^\circ$ $\theta_0 = 10^\circ$

Let initial separation $R_0 = 1m$

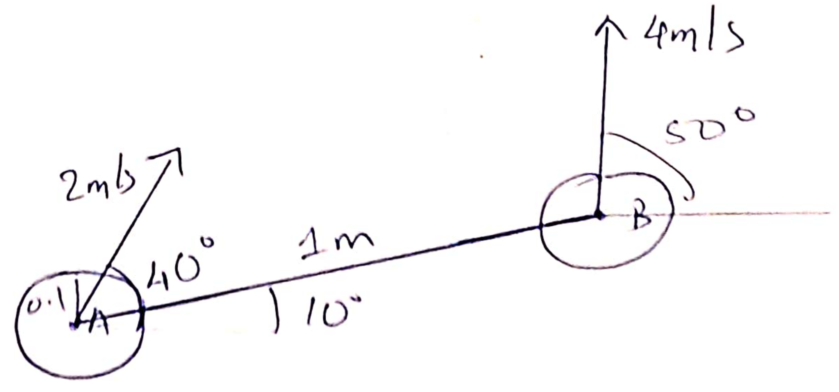
We assume $V_A, V_B, \alpha_A, \alpha_B$ dont change with time

Lets find relative velocity along line joining A & B

$$V_R = \dot{R} = V_B \cos(\alpha_B - \theta_0) - V_A \cos(\alpha_A - \theta_0)$$

$$V_\theta = R\dot{\theta} = V_B \sin(\alpha_B - \theta_0) - V_A \sin(\alpha_A - \theta_0)$$

~~YR~~



$V_{\theta 0} = 4 \times \sin(80-10) - 2\sin(40-10)$

$V_{\theta} = 2.75877 \, m/s$

$V_{R0} = 4\cos(80-10) - 2\cos(40-10)$

$= \cancel{+.332\, m/s} \; -0.364$

$R_0 = 1m.$

~~Obser~~ Expression for Rmiss & time to reach Rmiss

Observe that $V_R^2 + V_\theta^2 = V_T^2 + V_M^2 - 2V_T V_M \cos(\alpha_T - \alpha_M)$

Here $V_T, V_M, \alpha_T, \alpha_M$ are const. (doesnt change with time).

Hence $V_R^2 + V_\theta^2 = c^2$

$(\dot{R})^2 + (R\dot{\theta})^2 = c^2$ [Differentiate R, we get $\dot{V_R} = (\dot{\theta} V_\theta)$ & $V_\theta^2 = R\dot{V_R} = R\ddot{R}$]

$(\dot{R})^2 + R\ddot{R} = c^2$

$\frac{d}{dt}[R\dot{R}] = c^2$

$R\dot{R} = c^2 t + b$ $\quad b = R_0 V_{R0}$

At pt. of closest approach, $V_R = 0$

$0 = c^2 t_c + R_0 V_{R0}$

$t_c = \frac{-R_0 V_{R0}}{V_{R0}^2 + V_{\theta 0}^2}$

$$R V_R = c^2 t + b$$

$$R \dot{R} = c^2 t + b$$

$$\frac{1}{2} \frac{d}{dt} R^2 = c^2 + b$$

$$R^2 = c^2 t^2 + 2bt + a$$

At $t = 0$, $R = R_0$, $a = R_0^2$

$$R^2 = c^2 t^2 + 2 R_0 V_{R0} t + R_0^2 \quad - (1)$$

At B_1, $t_c = \dfrac{-R_0 V_{R0}}{V_{R0}^2 + V_{\theta 0}^2}$

$$R_c = R_0 \sqrt{\frac{V_{\theta 0}^2}{V_{R0}^2 + V_{\theta 0}^2}}$$

Substituting our assumed values:-

$$R_c = 1 \times \sqrt{\frac{2.78777^2}{0.36^2 + 2.75877^2}} = 0.9914 m.$$

$t_c = \cancel{0.3145}$ $+ 0.047 sec.$

So the 2 robots come closest at 0.314s with a separation of 0.763m b/w there centres.

This is not a collision because

$0.9914 > 0.1 + 0.1$ (sum of radii)

(b) Changing the initial parameters to allow collision

$\cancel{\alpha_B = 4}$ $V_B = 1 \, m/s$ $\quad V_A = 2 \, m/s$

$V_{\theta 0} = 1 \cdot \sin(80-10) - 2 \cdot \sin(40-10)$

$= -0.0603 \, m/s$

$\cancel{V_{R0}}$

$V_{R0} = 1 \cdot \cos(80-10) - 2 \cdot \cos(40-10)$

$= -1.39 \, m/s$

To confirm collision:-

$R_c = 1 \cdot \sqrt{\dfrac{0.0603^2}{0.0603^2 + 1.39^2}} = 0.0433 < 0.2$

Hence collision occurs.

But when does it 1st occur?

From previous derivation we know

$R^2 = c^2 t^2 + 2b\,t + a$

or

$R^2 = (V_{R0}^2 + V_{\theta 0}^2) t^2 + 2 R_0 V_{R0} t + R_0^2$

Lets find t when $R = 0.2$

$0.2^2 = (1.39^2 + 0.0603^2) t^2 + 2 \cdot 1 \cdot (-1.39) t + 1^2$

Solving for t.

$$1.936t^2 + -2.78t + 0.96 = 0$$

$$t = \frac{+2.78 \pm \sqrt{2.78^2 - 4 \times 1.936 \times 0.96}}{2 \times 1.936}$$

$$= 0.858 \text{ or } 0.5572$$

So the time of 1st collision = 0.5572s and
time at which they get out of collision = $\underline{0.858s}$.

2. The distance of closest approach (when they dont collide, does not depend on the radii ~~or~~ size as it is the distance b/w centers.

Hence part a gives the same answer.
~~& Major axis = 0.~~ which is equal to
For part b, lets define the elipse.

Major axis = 0.1
Minor axis = 0.01 for both elipse.

Since major axis is 0.1, which is equal to circle in the previous question, we can say that this is a conservative estimate for size of ellipse,

The $R_c = 0.991m > 0.1 + 0.1$

Hence $R_c = 0.991m$ is the closest distance b/w the elipses at $t = \underline{0.47s}$

(2) Find collision time of ellipses :-

We know the trajectory of the elipse' centers

We know the eqn. of ellipse w.r.t the center

We can check when the eqn. of ellipse can be equated & solved. The min 't' at which this is possible will be the collision time.

Eqn for centre :-

$x_A(t) = 2\cos(40^\circ)t$

$y_A(t) = 2\sin(40^\circ)t$

$x_B(t) = 1\cos(10^\circ) + 1\cos(80^\circ)t$

$y_B(t) = 1\sin(10^\circ) + 1\sin(80^\circ)t$

Eqn for elipse A :-

$$\Big[[x - x_A(t)]\cos(40^\circ) + [y - y_A(t)]\sin(40^\circ)\Big]^2 / a^2$$

$$+ \Big[-[x - x_A(t)]\sin(40^\circ) + [y - y_A(t)]\cos(40^\circ)\Big]^2 / b^2$$

$$= 1$$

Substituting eqn of centre.

$$\frac{\left[(x - 2(\cos 40^\circ)t)\cos 40^\circ + (y - 2\sin 40^\circ t)\sin 40^\circ\right]^2}{0.1^2}$$

$$+ \frac{\left[-(x - 2\cos 40^\circ t)\sin 40^\circ t + (y - 2\sin 40^\circ t)\cos 40^\circ\right]^2}{0.02^2}$$

This has 3 variables x, y, t.

IIIly we write for elipse B :-

$[(x - \cos 10^\circ - \cos 80^\circ t)\cos 80^\circ + (y - \sin 10^\circ - \sin 80^\circ t)$
$\sin 80^\circ]^2 / 0.1^2 + [-(x - \cos 10^\circ - \cos 80^\circ t)\cos 80^\circ$
$+ (y - \sin 10^\circ - \sin 80^\circ t)\cos 80^\circ]^2 / 0.02^2$

This has 3 variables too, x, y, t.

We find the t that gives same x & y for both eqn.

We can solve this using numerically with MATLAB.

For this we parametrise the elipse equation

$x = x_c + 0.1\cos(\phi)\cos(\alpha) - b\sin(\phi)\sin(\alpha)$

$y = y_c + 0.1\cos(\phi)\sin(\alpha) - b\sin(\phi)\cos(\alpha)$

Ans

This has been solved on python :-

Initial time = 0.6861s

Final time = 0.28s

This has been verified with an animation & comparing points manually.