ThermoCycle Moving Boundary Model

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Abstract

The authors present a new moving boundary model that was integrated into the ThermoCycle package written in the Modelica language. Focussing on a seamless integration with existing components, this new component allows to calculate dynamic heat transfer in an efficient and robust way covering the full range of possible operating points in the liquid, two-phase, gas and supercritical domain. A basic validation performed with heat transfer data from two different experiments with evaporators shows that the model is able to reliably predict heat exchanger performance. The flexible implementation allows to compare different heat transfer correlations, which are made freely available as part of the ThermoCycle library.

1 Introduction and Motivation

Moving boundary (MB) models are established tools to calculate heat exchanger performance in both steady-state and dynamic operation. A fictitious heat transfer channel is split up into different sections and with each section accounting for a different fluid state. In the case of an evaporator the maximum number of sections N is 3 for a) subcooled, b) two-phase and c) superheated state. At higher pressures, the fluid might enter the supercritical state. Hence, there are four different sections out of which a maximum of three can occur simultaneously. The name moving boundary is derived from the fact that the interfaces between these sections do not have a fixed spatial position but merely a fixed thermodynamic location depending on the presence of liquid and gaseous fluid, respectively. The actual existence of a certain section and its length are determined based on the fluid state resulting in variable sectioning. A fixed total length superimposes the required boundary condition to calculate the length of each section.

Moving boundary formulations are a good compromise between computational efficiency, robustness and accuracy[2].

2 Formulation

2.1 Assumptions

i. The tube is cylindrical
ii. The velocity of the fluid is uniform on the cross sectional area
iii. The enthalpy of the fluid is linear in each region of the tube (sub-cooled, two-phase, super-heated)
iv. Pressure is considered constant (at least for now)
v. The secondary fluid is treated as an incompressible fluid

2.2 Equations

Thermodynamic properties are calculated using Coolprop [1]. The state variable selected are p and $\bar{h}$. The heat transfer coefficients ($U_{\mathrm{pf},1}, U_{\mathrm{pf},2}, U_{\mathrm{pf},3}$ - $U_{\mathrm{sf},1}, U_{\mathrm{sf},2}, U_{\mathrm{sf},3}$) are calculated using appropriate heat transfer models. Different model of void fraction ($\bar{\alpha}$) have been also implemented.

SUB-COOLED ZONE

Primary fluid

Conservation equations

Mass balance

$$\mathrm{A}[\mathrm{L}_1 \cdot \frac{d\bar{\rho}_1}{dt} + (\bar{\rho}_1 - \rho_\mathrm{l}) \cdot \frac{d\mathrm{L}_1}{dt}] = \dot{m}_{\mathrm{IN}} - \dot{m}_{\mathrm{A}} \tag{1}$$

Energy balance

$$\mathrm{AL}_1[\bar{\rho}_1 \cdot \frac{d\bar{h}_1}{dt} + \bar{h}_1 \cdot \frac{d\bar{\rho}_1}{dt} - \frac{dp_1}{dt}] + \mathrm{A}(\bar{\rho}_1\bar{h}_1 - \rho_\mathrm{l}h_\mathrm{l})\frac{d\mathrm{L}_1}{dt} = \dot{m}_{\mathrm{IN}}\dot{h}_{\mathrm{IN}} - \dot{m}_{\mathrm{A}}\dot{h}_{\mathrm{A}} + \dot{Q}_{\mathrm{r}1} \tag{2}$$

$$\frac{d\bar{\rho}_1}{dt} = [\frac{\partial\bar{\rho}_1}{\partial p_1} + \frac{1}{2} \cdot \frac{\partial\bar{\rho}_1}{\partial\bar{h}_1} \cdot \frac{\partial h_\mathrm{l}}{\partial p_1}]\frac{dp_1}{dt} + \frac{1}{2} \cdot \frac{\partial\bar{\rho}_1}{\partial\bar{h}_1} \cdot \frac{dh_{\mathrm{IN}}}{dt} \tag{3}$$

$$\frac{d\bar{h}_1}{dt} = \frac{1}{2} \cdot [\frac{\partial\bar{h}_\mathrm{l}}{\partial p_1} \cdot \frac{dp_1}{dt} + \frac{dh_{\mathrm{IN}}}{dt}] \tag{4}$$

Constitutive equations

$$\bar{h}_1 = \frac{1}{2}(h_{\mathrm{IN}} + h_\mathrm{l}) \tag{5}$$

$$subcool = setState\_ph(p_1, \bar{h}_1) \tag{6}$$

$$sat = setSat\_p(p_2) \tag{7}$$

$$\rho_{\mathrm{l}} = bubbleDensity(sat) \tag{8}$$

$$h_{\mathrm{l}} = bubbleEnthalpy(sat) \tag{9}$$

$$\frac{\partial \bar{h}_{\mathrm{l}}}{\partial p_{1}} = dBubbleEnthalpy\_dPressure(sat) \tag{10}$$

$$\frac{\partial \bar{\rho}_{1}}{\partial \bar{h}_{1}} = density\_derh\_p(subcool) \tag{11}$$

$$\frac{\partial \bar{\rho}_{1}}{\partial p_{1}} = density\_derp\_h(subcool) \tag{12}$$

$$\bar{T}_{1} = temperature(subcool) \tag{13}$$

$$\dot{Q}_{\mathrm{r},1} = \pi \mathrm{D}\mathrm{L}_{1} U_{\mathrm{pf},1}(T_{\mathrm{w},1} - \bar{T}_{1}) \tag{14}$$

Metal wall

Conservation equations

Energy balance:

$$C_{\mathrm{w}}(M_{\mathrm{tot}} \cdot \frac{\mathrm{L}_{1}}{\mathrm{L}}) \cdot \frac{d\mathrm{T}_{\mathrm{w},1}}{dt} + \frac{M_{\mathrm{tot}}}{\mathrm{L}}(\mathrm{T}_{\mathrm{w},1} - \mathrm{T}_{\mathrm{w},12})\frac{d\mathrm{L}_{1}}{dt} = \dot{Q}_{\mathrm{sf},1} - \dot{Q}_{\mathrm{r},1} \tag{15}$$

Constitutive equation

$$\mathrm{T}_{\mathrm{w},12} = \frac{\mathrm{T}_{\mathrm{w},1}\mathrm{L}_{2} + \mathrm{T}_{\mathrm{w},2}\mathrm{L}_{1}}{\mathrm{L}_{1} + \mathrm{L}_{2}} \tag{16}$$

$$C_{\mathrm{w}} = const. \tag{17}$$

$$M_{\mathrm{tot}} = const. \tag{18}$$

$$L = const. \tag{19}$$

Secondary fluid

Conservation equation

Energy balance:

$$\rho_{\mathrm{sf}} A_{\mathrm{sf}}[\mathrm{L}_{1} \cdot \frac{dh_{\mathrm{sf},1}}{dt} + (\mathrm{h}_{\mathrm{sf},1} - \mathrm{h}_{\mathrm{sf},12})\frac{d\mathrm{L}_{1}}{dt}] = \dot{m}_{\mathrm{sf}}(h_{\mathrm{sf,in}} - h_{\mathrm{sf},12}) + \dot{Q}_{\mathrm{sf},1} \tag{20}$$

Constitutive equations

$$h_{\mathrm{sf,12}} = \frac{\mathrm{h_{sf,1}L_2 + h_{sf,2}L_1}}{\mathrm{L_1 + L_2}} \tag{21}$$

$$Q_{\mathrm{sf,1}} == \pi \mathrm{DL_1} U_{\mathrm{sf,1}}(T_{\mathrm{w,1}} - \bar{T}_{\mathrm{sf,1}}) \tag{22}$$

$$sfzone1 = setState(p1, h_{sf,1}) \tag{23}$$

$$T_{\mathrm{sf,1}} = temperature(sfzone1) \tag{24}$$

TWO-PHASE ZONE

Primary fluid

Conservation equations

Mass balance:

$$\mathrm{A}[\mathrm{L_2} \cdot \frac{d\bar{\rho}_2}{dt} + (\bar{\rho}_2 - \rho_{\mathrm{g}}) \cdot \frac{d\mathrm{L_2}}{dt} + (\rho_{\mathrm{l}} - \rho_{\mathrm{g}}) \cdot \frac{d\mathrm{L_1}}{dt}] = \dot{m}_{\mathrm{A}} - \dot{m}_{\mathrm{B}} \tag{25}$$

Energy balance:

$$\mathrm{A}[\mathrm{L_2} \cdot \frac{d\bar{\rho}_2\bar{h}_2}{dt} + (\bar{\rho}_2\bar{h}_2 - \rho_{\mathrm{g}}h_{\mathrm{g}}) \cdot \frac{d\mathrm{L_2}}{dt} + (\rho_{\mathrm{l}}h_{\mathrm{l}} - \rho_{\mathrm{g}}h_{\mathrm{g}}) \cdot \frac{d\mathrm{L_1}}{dt} - \mathrm{L_2} \cdot \frac{dp_2}{dt}] = \dot{m}_{\mathrm{A}}\dot{h}_{\mathrm{A}} - \dot{m}_{\mathrm{B}}\dot{h}_{\mathrm{B}} + \dot{Q}_{\mathrm{r,2}} \tag{26}$$

$$\frac{d\bar{\rho}_2}{dt} = (\frac{\partial \rho_{\mathrm{g}}}{\partial p_2} \cdot \bar{\alpha} + \frac{\partial \rho_{\mathrm{l}}}{\partial p_2} \cdot (1 - \bar{\alpha})) \cdot \frac{dp_2}{dt} + (\rho_{\mathrm{g}} - \rho_{\mathrm{l}}) \cdot [\frac{\partial \bar{\alpha}}{\partial p_2} \cdot \frac{dp_2}{dt} + \frac{\partial \bar{\alpha}}{\partial h_{\mathrm{OUT,alpha}}} \cdot \frac{dh_{\mathrm{OUT,alpha}}}{dt}] \tag{27}$$

$$\begin{aligned}\frac{d\bar{\rho}_2\bar{h}_2}{dt} &= [\bar{\alpha} \cdot (\frac{\partial \rho_{\mathrm{g}}}{\partial p_2} \cdot h_{\mathrm{g}} + \frac{\partial h_{\mathrm{g}}}{\partial p_2} \cdot \rho_{\mathrm{g}}) + (1 - \bar{\alpha}) \cdot (\frac{\partial \rho_{\mathrm{l}}}{\partial p_2} \cdot h_{\mathrm{l}} + \frac{\partial h_{\mathrm{l}}}{\partial p_2} \cdot \rho_{\mathrm{l}})] \cdot \frac{dp_2}{dt} \\ &+ (\rho_{\mathrm{g}}h_{\mathrm{g}} - \rho_{\mathrm{l}}h_{\mathrm{l}}) \cdot [\frac{\partial \bar{\alpha}}{\partial p_2} \cdot \frac{dp_2}{dt} + \frac{\partial \bar{\alpha}}{\partial h_{\mathrm{OUT,alpha}}} \cdot \frac{dh_{\mathrm{OUT,alpha}}}{dt}]\end{aligned} \tag{28}$$

Constitutive equations

$$\bar{\rho}_2 = \rho_{\mathrm{g}}\bar{\alpha} + \rho_{\mathrm{l}} \cdot (1 - \bar{\alpha}) \tag{29}$$

$$\bar{\rho}_2\bar{h}_2 = \rho_{\mathrm{g}}h_{\mathrm{g}}\bar{\alpha} + \rho_{\mathrm{l}}h_{\mathrm{l}} \cdot (1 - \bar{\alpha}) \tag{30}$$

$$\rho_{\mathrm{g}} = dewDensity(sat) \tag{31}$$

$$h_{\mathrm{g}} = dewEnthalpy(sat) \tag{32}$$

$$\frac{\partial \bar{h}_{\mathrm{g}}}{\partial p_2} = dDewEnthalpy\_dPressure(sat) \tag{33}$$

$$\bar{T}_2 = temperature(sat) \tag{34}$$

$$\dot{Q}_{\mathrm{r},2} = \pi \mathrm{DL}_2 U_{\mathrm{pf},2}(T_{\mathrm{w},2} - \bar{T}_2) \tag{35}$$

Metal Wall

Conservation equation

Energy balance:

$$C_{\mathrm{w}} \cdot \frac{M_{\mathrm{tot}}}{\mathrm{L}} \cdot [\mathrm{L}_2 \cdot \frac{d\mathrm{T}_{\mathrm{w},2}}{dt} + (\mathrm{T}_{\mathrm{w},12} - \mathrm{T}_{\mathrm{w},23})\frac{d\mathrm{L}_1}{dt} + (\bar{\mathrm{T}}_2 - \mathrm{T}_{\mathrm{w},23})\frac{d\mathrm{L}_2}{dt}] = \dot{Q}_{\mathrm{sf},2} - \dot{Q}_{\mathrm{r},2} \tag{36}$$

Secondary fluid

Conservation equation

Energy balance:

$$\rho_{\mathrm{sf}} A_{\mathrm{sf}}[\mathrm{L}_2 \cdot \frac{dh_{\mathrm{sf},2}}{dt} + (\mathrm{h}_{\mathrm{sf},12} - \mathrm{h}_{\mathrm{sf},23})\frac{d\mathrm{L}_1}{dt}(\mathrm{h}_{\mathrm{sf},2} - \mathrm{h}_{\mathrm{sf},23})\frac{d\mathrm{L}_2}{dt}] = \dot{m}_{\mathrm{sf}}(h_{\mathrm{sf},12} - h_{\mathrm{sf},23}) + \dot{Q}_{\mathrm{sf},1} \tag{37}$$

Constitutive equations

$$Q_{\mathrm{sf},2} == \pi \mathrm{DL}_2 U_{\mathrm{sf},2}(T_{\mathrm{w},2} - \bar{T}_{\mathrm{sf},2}) \tag{38}$$

$$sfzone2 = setState(p2, h_{sf,2}) \tag{39}$$

$$\bar{T}_{\mathrm{sf},2} = temperature(sfzone2) \tag{40}$$

SUPER-HEATED ZONE

Primary fluid

Conservation equations

Mass balance:

$$\mathrm{A}[\mathrm{L}_3 \cdot \frac{d\bar{\rho}_3}{dt} + (\rho_{\mathrm{g}} - \bar{\rho}_3) \cdot (\frac{d\mathrm{L}_1}{dt} + \frac{d\mathrm{L}_2}{dt})] = \dot{m}_{\mathrm{B}} - \dot{m}_{\mathrm{OUT}} \tag{41}$$

Energy balance:

$$\mathrm{AL}_3[\bar{\rho}_3 \cdot \frac{d\bar{h}_3}{dt} + \bar{h}_3 \cdot \frac{d\bar{\rho}_3}{dt} - \frac{dp_3}{dt}] + \mathrm{A}(\rho_\mathrm{g} h_\mathrm{g} - \bar{\rho}_3 \bar{h}_3) \cdot (\frac{d\mathrm{L}_1}{dt} + \frac{d\mathrm{L}_2}{dt}) = \dot{m}_\mathrm{B} \dot{h}_\mathrm{g} - \dot{m}_\mathrm{OUT} \dot{h}_\mathrm{OUT} + \dot{Q}_{\mathrm{r},3} \tag{42}$$

$$\frac{d\bar{\rho}_3}{dt} = [\frac{\partial \bar{\rho}_3}{\partial p_3} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}_3}{\partial \bar{h}_3} \cdot \frac{\partial \bar{h}_\mathrm{g}}{\partial p_3}] \frac{dp_3}{dt} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}_3}{\partial \bar{h}_3} \cdot \frac{dh_\mathrm{OUT}}{dt} \tag{43}$$

$$\frac{d\bar{h}_3}{dt} = \frac{1}{2} \cdot [\frac{\partial \bar{h}_\mathrm{g}}{\partial p_3} \cdot \frac{dp_3}{dt} + \frac{dh_\mathrm{OUT}}{dt}] \tag{44}$$

Constitutive equations

$$vap = setState(p_3, \bar{h}_3) \tag{45}$$

$$\bar{h}_3 = \frac{1}{2}(h_\mathrm{g} - h_\mathrm{OUT}) \tag{46}$$

$$\frac{\partial \bar{\rho}_3}{\partial \bar{h}_3} = density\_derh\_p(vap) \tag{47}$$

$$\frac{\partial \bar{\rho}_3}{\partial p_3} = density\_derp\_h(vap) \tag{48}$$

$$\bar{T}_3 = temperature(vap) \tag{49}$$

$$\dot{Q}_{\mathrm{r},3} = \pi \mathrm{DL}_3 U_{\mathrm{pf},3}(T_{\mathrm{w},3} - \bar{T}_3) \tag{50}$$

Metal wall

Conservation equations

Energy balance:

$$C_\mathrm{w} \frac{M_\mathrm{tot}}{L} \cdot [\mathrm{L}_3 \cdot \frac{d\mathrm{T}_{\mathrm{w},3}}{dt} + (\mathrm{T}_{\mathrm{w},23} - \mathrm{T}_{\mathrm{w},3})(\frac{d\mathrm{L}_1}{dt} + \frac{d\mathrm{L}_2}{dt})] = \dot{Q}_{\mathrm{sf},3} - \dot{Q}_{\mathrm{r},3} \tag{51}$$

Constitutive equations

$$\mathrm{T}_{\mathrm{w},23} = \frac{\mathrm{T}_{\mathrm{w},3}\mathrm{L}_2 + \mathrm{T}_{\mathrm{w},2}\mathrm{L}_3}{\mathrm{L}_2 + \mathrm{L}_3} \tag{52}$$

Secondary fluid

Conservation equations

Energy balance:

$$\rho_\mathrm{sf} A_\mathrm{sf}[\mathrm{L}_3 \cdot \frac{d\bar{h}_{\mathrm{sf},3}}{dt} + (\mathrm{h}_{\mathrm{sf},23} - \mathrm{h}_{\mathrm{sf},3})(\frac{d\mathrm{L}_1}{dt} + \frac{d\mathrm{L}_2}{dt})] = \dot{m}_\mathrm{sf}(h_{\mathrm{sf},23} - h_{\mathrm{sf,OUT}}) + \dot{Q}_{\mathrm{sf},3} \tag{53}$$

$$h_{\mathrm{sf},23} = \frac{\mathrm{h}_{\mathrm{sf},3}\mathrm{L}_2 + \mathrm{h}_{\mathrm{sf},2}\mathrm{L}_3}{\mathrm{L}_2 + \mathrm{L}_3} \tag{54}$$

$$Q_{\mathrm{sf},3} == \pi \mathrm{D} \mathrm{L}_3 U_{\mathrm{sf},3}(T_{\mathrm{w},3} - \bar{T}_{\mathrm{sf},3}) \tag{55}$$

$$sfzone3 = setState(p3, h_{sf,3}) \tag{56}$$

$$\bar{T}_{\mathrm{sf},3} = temperature(sfzone3) \tag{57}$$

2.3 Heat Transfer

Based on Nusselt number (Nu) from Reynolds number (Re) and Prandtl number (Pr) for a characteristic length L. Angles are usually calculated in radians or π.

2.4 Pressure Drop

3 Results and Discussion

Compared to [3], the model ....

[4]

[5]

4 Conclusion

References

[1] Ian H. Bell, Sylvain Quoillin, Jorrit Wronski, and Vincent Lemort. Coolprop: An open-source reference-quality thermophysical property library. In *submitted abstract to ASME-ORC 2013 – 2nd International Seminar on ORC Power Systems*, 2013.

[2] Satyam Bendapudi, James E. Braun, and Eckhard A. Groll. A comparison of moving-boundary and finite-volume formulations for transients in centrifugal chillers. *International Journal of Refrigeration*, 31(8):1437–1452, 2008.

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