

# ThermoCycle Moving Boundary Model

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## Abstract

The authors present a new moving boundary model that was integrated into the ThermoCycle package written in the Modelica language. Focussing on a seamless integration with existing components, this new component allows to calculate dynamic heat transfer in an efficient and robust way covering the full range of possible operating points in the liquid, two-phase, gas and supercritical domain. A basic validation performed with heat transfer data from two different experiments with evaporators shows that the model is able to reliably predict heat exchanger performance. The flexible implementation allows to compare different heat transfer correlations, which are made freely available as part of the ThermoCycle library.

## 1 Introduction and Motivation

Moving boundary (MB) models are established tools to calculate heat exchanger performance in both steady-state and dynamic operation. A fictitious heat transfer channel is split up into different sections and with each section accounting for a different fluid state. In the case of an evaporator the maximum number of sections  $N$  is 3 for a) subcooled, b) two-phase and c) superheated state. At higher pressures, the fluid might enter the supercritical state. Hence, there are four different sections out of which a maximum of three can occur simultaneously. The name moving boundary is derived from the fact that the interfaces between these sections do not have a fixed spatial position but merely a fixed thermodynamic location depending on the presence of liquid and gaseous fluid, respectively. The actual existence of a certain section and its length are determined based on the fluid state resulting in variable sectioning. A fixed total length superimposes the required boundary condition to calculate the length of each section.

Moving boundary formulations are a good compromise between computational efficiency, robustness and accuracy[2].

## 2 Formulation

In the following sections the moving boundary model is described. In section 2.1 the mathematical formulation of the general governing equations describing the fluid

flow through a control volume are reported together with the main assumptions adopted for the moving boundary model. Section 2.2 and ?? report the derivation process for the governing equations for the one phase and the two phase regions respectively. Section 2.4 deals with the wall energy balance. In section 2.5 the way in which the heat exchanger secondary fluid side is modelled.

## 2.1 Governing equations and assumptions

The fluid flow through a control volume of the heat exchanger is described with a mathematical formulation of the conservation laws of physics:

- The mass of the fluid is conserved.
- The rate of change in momentum equals the sum of the forces on a fluid particles (2nd law of Newton).
- The rate of change of energy is equal to the sum of the rate of thermal energy addition to and the rate of work done on a fluid particle (1st law of thermodynamics).

The fluid is considered as a continuum. The analysis of the fluid trend is carried out at macroscopic scale length (from 1  $\mu\text{m}$  and larger). As a consequence the fluid molecular structure and motion can be ignored. The evolution of the fluid is described in terms of macroscopic properties i.e. pressure, temperature, density and velocity and their space and time derivatives. The general form of mass energy and momentum balance are reported here under:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{u}) = 0 \quad (1)$$

$$\rho \frac{DE}{Dt} = -\text{div}(\rho h \mathbf{u}) + \text{div}(k \text{grad} T) + S_E \quad (2)$$

$$\text{General momentum balance} \quad (3)$$

### 2.1.1 Moving boundary model equations

In order to derive a low-order model to describe the behaviour of the fluid flowing through an heat exchanger using the moving boundary formulations, the following assumptions are considered:

- i. The Heat exchanger is considered as n 1-dimentional tubes (z-direction) through which the working fluid flows.
- ii. The tube is cylindrical with a constant cross sectional area.
- iii. The velocity of the fluid is uniform on the cross sectional area. Homogeneous two-phase flow.
- iv. Kinetic energy, gravitational forces and viscous stresses are neglected.
- v. No work is done on or generated by the control volume.
- vi. The enthalpy of the fluid is linear in each region of the tube (sub-cooled, two-phase, super-heated)
- vii. A static momentum balance is considered.

- viii. Pressure drop through the tube are neglected. Constant pressure.
- ix. The rate of thermal energy addition due to heat conduction is neglected.
- x. the rate of thermal energy addition due to heat convection is considered.
- xi. Thermal energy accumulation is considered for the metal wall of the tube.
- xii. Thermal energy conduction in the metal wall is neglected.
- xiii. The secondary fluid is treated as a constant heat capacity fluid.

These assumptions allow to reformulate the conservation equations expressed in equations 1 to 3 as:

$$A \cdot \frac{\partial \rho}{\partial t} + \frac{\partial \dot{m}}{\partial z} = 0 \quad (4)$$

$$A \cdot \frac{\partial(\rho \cdot h - p)}{\partial t} + \frac{\partial(h \cdot \dot{m})}{\partial z} = \dot{Q} \quad (5)$$

$$p_a = p_b \quad (6)$$

where A is the cross sectional area of the heat exchanger tube. The derivation process is described in detailed in [?]. In the developed model, the thermodynamic properties are calculated using Coolprop [1]. The state variables selected are  $p$  and  $\bar{h}$ . The convection heat transfer coefficients on primary side ( $U_{pf,1}, U_{pf,2}, U_{pf,3}$ ) and secondary side ( $U_{sf,1}, U_{sf,2}, U_{sf,3}$ ) can be calculated, using appropriate heat transfer model, or imposed as constant

## 2.2 One phase flow region

### 2.2.1 Mass balance derivation process

In order to get the mass balance for one zone of the moving boundary model, we need to integrate equation 4 over the length of the zone from  $a$  to  $b$ :

$$A \cdot \int_{l_a}^{l_b} \frac{\partial \rho}{\partial t} dz + \int_{l_a}^{l_b} \frac{\partial \dot{m}}{\partial z} dz = 0 \quad (7)$$

Applying Leibniz rule for the first term and solving the integral for the second term of equation 7, results:

$$A \cdot \left[ \frac{d}{dt} \cdot \int_{l_a}^{l_b} \rho dz - \rho_a \cdot \frac{dl_a}{dt} + \rho_b \cdot \frac{dl_b}{dt} \right] = \dot{m}_a - \dot{m}_b \quad (8)$$

Solving the first term of equation 8 results in:

$$\frac{d}{dt} \cdot \int_{l_a}^{l_b} \rho dz = \frac{d}{dt} \cdot (\bar{\rho} \cdot l) = \bar{\rho} \cdot \frac{dl}{dt} + l \cdot \frac{d\bar{\rho}}{dt} \quad (9)$$

The bar over the density indicates the mean value of the variable over the considered zone. Substituting equation 9 in equation 8 results in:

$$A \cdot \left[ \bar{\rho} \cdot \frac{dl}{dt} + l \cdot \frac{d\bar{\rho}}{dt} - \rho_a \cdot \frac{dl_a}{dt} + \rho_b \cdot \frac{dl_b}{dt} \right] = \dot{m}_a - \dot{m}_b \quad (10)$$

Equation 10 represents the mass balance for a one phase zone of the heat exchanger.

The enthalpy distribution in the zone is considered linear as specified in section 2.1.1:

$$\bar{h} = \frac{1}{2} \cdot (h_a + h_b) \quad (11)$$

$$\frac{d\bar{\rho}}{dt} = \frac{\partial \bar{\rho}}{\partial p} \cdot \frac{dp}{dt} + \frac{\partial \bar{\rho}}{\partial \bar{h}} \cdot \frac{d\bar{h}}{dt} = \frac{\partial \bar{\rho}}{\partial p} \cdot \frac{dp}{dt} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}}{\partial \bar{h}} \cdot \left( \frac{dh_a}{dt} + \frac{dh_b}{dt} \right) \quad (12)$$

where:

$$\frac{dh_b}{dt} = \frac{\partial h_b}{\partial p} \frac{dp}{dt} \quad , \quad \frac{dh_a}{dt} = \frac{\partial h_a}{\partial p} \frac{dp}{dt} \quad (13)$$

in case of subcooled and superheated zone respectively.

### 2.2.2 Energy Balance: derivation process

Equation 5 is integrated over the length to get the energy balance of one zone of the heat exchanger:

$$A \cdot \int_{l_a}^{l_b} \frac{\partial(\rho \cdot h)}{\partial t} dz - A \cdot l \cdot \frac{dp}{dt} + \int_{l_a}^{l_b} \frac{\partial(h \cdot \dot{m})}{\partial z} dz = \dot{Q} \quad (14)$$

Applying Leibniz rule for the first integral term and solving the second integral term results in:

$$A \cdot \left[ \frac{d}{dt} \int_{l_a}^{l_b} (\rho \cdot h) dz + (\rho_a h_a) \cdot \frac{dl_a}{dt} - (\rho_b h_b) \cdot \frac{dl_b}{dt} \right] - A \cdot l_a \cdot \frac{dp}{dt} = \dot{m}_a \cdot h_a - \dot{m}_b \cdot h_b + \dot{Q} \quad (15)$$

The rate of enthalpy change can be calculated using equation 16. Applying the mean-value theorem to get rid of the integral and approximating  $\bar{\rho h} = \bar{\rho} \bar{h}$  it results in:

$$\frac{d}{dt} \int_{l_a}^{l_b} (\rho \cdot h) dz = \frac{d}{dt} (\bar{\rho h} \cdot l) \approx \frac{d}{dt} (\bar{\rho} \cdot \bar{h} \cdot l) = \bar{\rho} \frac{dl}{dt} + \bar{h} l \frac{d\bar{\rho}}{dt} + \bar{\rho} l \frac{d\bar{h}}{dt} \quad (16)$$

Substituting equation 16 into equation 15 the energy balance for a one phase zone results in:

$$A \cdot \left[ \bar{\rho} \frac{dl}{dt} + \bar{h} l \frac{d\bar{\rho}}{dt} + \bar{\rho} l \frac{d\bar{h}}{dt} + (\rho_a h_a) \cdot \frac{dl_a}{dt} - (\rho_b h_b) \cdot \frac{dl_b}{dt} \right] - A \cdot l_a \cdot \frac{dp}{dt} = \dot{m}_a \cdot h_a - \dot{m}_b \cdot h_b + \dot{Q} \quad (17)$$

which represents the energy balance for the first zone of the heat exchanger. The same procedure can be applied to develop the energy balance for the second and the third zone of the heat exchanger.

## 2.3 Two Phase region

In this section the mass and energy balance for the two-phase zone are reported. A description of two-phase flow behaviour together with the definition of the primary variables characterising two-phase flow is reported in the appendix.

### 2.3.1 Mass balance

Assuming homogeneous two-phase flow condition, the average density is calculated based on equation 18:

$$\bar{\rho} = (1 - \bar{\gamma})\rho' + \bar{\gamma}\rho'' \quad (18)$$

where  $\bar{\gamma}$  is the average void fraction. Based on equation 18, the rate of mass change for a two-phase flow becomes:

$$\frac{d}{dt} \int_{l_a}^{l_b} \rho dz = \frac{d}{dt} (\bar{\rho} \cdot l) = \frac{d}{dt} (\bar{\rho} \cdot l) \frac{d}{dt} (l \cdot ((1 - \bar{\gamma})\rho' + \bar{\gamma}\rho'')) = \quad (19)$$

$$= \frac{dl}{dt} \cdot ((1 - \bar{\gamma})\rho' + \bar{\gamma}\rho'') + ll \cdot \left( \left( \frac{d\bar{\gamma}}{dt} \rho'' + \frac{d\rho''}{dt} \bar{\gamma} + \frac{d\rho'}{dt} - \bar{\gamma} \frac{d\rho'}{dt} - \rho' \frac{d\bar{\gamma}}{dt} \right) \right) = \quad (20)$$

$$= \frac{dl}{dt} \cdot ((1 - \bar{\gamma})\rho' + \bar{\gamma}\rho'') + ll \cdot \left( (\rho'' - \rho') \frac{d\bar{\gamma}}{dt} + \bar{\gamma} \frac{d\rho''}{dp} \frac{dp}{dt} + (1 - \bar{\gamma}) \frac{d\rho'}{dt} \right) \quad (21)$$

Substituting equation 21 in equation 8 the mass balance for a two-phase flow results in:

$$A \cdot \left[ \frac{dl}{dt} \cdot ((1 - \bar{\gamma})\rho' + \bar{\gamma}\rho'') + ll \cdot \left( (\rho'' - \rho') \frac{d\bar{\gamma}}{dt} + \bar{\gamma} \frac{d\rho''}{dp} \frac{dp}{dt} + (1 - \bar{\gamma}) \frac{d\rho'}{dt} \right) - \right. \quad (22)$$

$$\left. - \rho_a \cdot \frac{dl_a}{dt} + \rho_b \cdot \frac{dl_b}{dt} \right] = \dot{m}_a - \dot{m}_b \quad (23)$$

### 2.3.2 Energy balance

Considering equation 18, the rate of energy change for a two-phase flow fluid results in:

$$\frac{d}{dt} \int_{l_a}^{l_b} (\rho \cdot h) dz = \frac{d}{dt} (\bar{\rho} \bar{h} \cdot l) \approx \frac{d}{dt} (\bar{\rho} \cdot \bar{h} \cdot l) = \frac{d}{dt} (l \cdot ((1 - \bar{\gamma})\rho' h' + \bar{\gamma}\rho'' h'')) = \quad (24)$$

$$\frac{dl}{dt} \cdot ((1 - \bar{\gamma})\rho' h' + \bar{\gamma}\rho'' h'') + l \cdot \left( (\rho'' h'' - \rho' h') \frac{d\bar{\gamma}}{dt} + \bar{\gamma}\rho'' \frac{dh''}{dt} + \bar{\gamma} h'' \frac{d\rho''}{dt} + \right. \quad (25)$$

$$\left. + h' \frac{d\rho'}{dt} + \rho' \frac{dh'}{dt} - h' \rho' \frac{d\bar{\gamma}}{dt} - \bar{\gamma} h' \frac{d\rho'}{dt} - \bar{\gamma} \rho' \frac{dh'}{dt} \right) = \quad (26)$$

$$= \frac{dl}{dt} \cdot ((1 - \bar{\gamma})\rho' h' + \bar{\gamma}\rho'' h'') + l \cdot \left( (\rho'' h'' - \rho' h') \frac{d\bar{\gamma}}{dt} + \bar{\gamma} h'' \frac{\partial \rho''}{\partial p} \frac{dp}{dt} + \bar{\gamma} \rho'' \frac{\partial h''}{\partial p} \frac{dp}{dt} + \right. \quad (27)$$

$$\left. + (1 - \bar{\gamma}) h' \frac{\partial \rho'}{\partial p} \frac{dp}{dt} + (1 - \bar{\gamma}) \rho' \frac{\partial h'}{\partial p} \frac{dp}{dt} \right) \quad (28)$$

Substituting equation 28, in equation 15, the two-phase flow energy balance results:

$$A \cdot \left[ \frac{dl}{dt} \cdot ((1 - \bar{\gamma})\rho' h' + \bar{\gamma}\rho'' h'') + l \cdot ((\rho'' h'' - \rho' h') \frac{d\bar{\gamma}}{dt} + \bar{\gamma} h'' \frac{\partial \rho''}{\partial p} \frac{dp}{dt} + \bar{\gamma} \rho'' \frac{\partial h''}{\partial p} \frac{dp}{dt} + \right. \quad (29)$$

$$\left. + (1 - \bar{\gamma}) h' \frac{\partial \rho'}{\partial p} \frac{dp}{dt} + (1 - \gamma) \rho' \frac{\partial h p}{\partial p} \frac{dp}{dt} \right) + (\rho_a h_a) \cdot \frac{dl_a}{dt} - (\rho_b h_b) \cdot \frac{dl_b}{dt} \Big] - A \cdot l_a \cdot \frac{dp}{dt} = \quad (30)$$

$$= \dot{m}_a \cdot h_a - \dot{m}_b \cdot h_b + \dot{Q} \quad (31)$$

## 2.4 Metal wall

The energy accumulation in the metal wall can be expressed as:

$$\rho_w c_w A_w \frac{\partial T_w}{\partial t} = \dot{Q}_{pf} - \dot{Q}_{sf} \quad (32)$$

Integrating over the length of one zone it results:

$$\rho_w c_w A_w \int_{l_a}^{l_b} \frac{\partial T_w}{\partial t} dz = \dot{Q}_{pf} - \dot{Q}_{sf} \quad (33)$$

Applying Leibinz rule:

$$\rho_w c_w A_w \left[ \frac{d}{dt} \int_{l_a}^{l_b} \partial T_w dz + T_w(l_b) \frac{dl_b}{dt} - T_w(l_a) \frac{dl_a}{dt} \right] = \dot{Q}_{pf} - \dot{Q}_{sf} \quad (34)$$

Solving the integral the thermal energy balance for the metal wall is:

$$\rho_w c_w A_w \left[ \frac{d}{dt} \int_{l_a}^{l_b} \partial T_w dz + T_w(l_b) \frac{dl_b}{dt} - T_w(l_a) \frac{dl_a}{dt} \right] = \dot{Q}_{pf} - \dot{Q}_{sf} \quad (35)$$

## 2.5 Secondary fluid model

The secondary fluid model is a steady-state model where the thermal energy transfer with the metal wall is solved using the  $\epsilon$ -NTU method or the LMTD method.

## 2.6 Heat Transfer

Based on Nusselt number (Nu) from Reynolds number (Re) and Prandtl number (Pr) for a characteristic length  $L$ . Angles are usually calculated in radians or  $\pi$ .

## 3 Results and Discussion

Compared to [4], the model ....

[5]  
[6]

## 4 Conclusion

## A Two phase flow behaviour

The two phase flow regime is characterized by the flow patterns that the fluid assumes during evaporation or condensation. The flow configuration depends on the thermodynamic properties of the fluid, the mass flow, the thermal energy flux and the channel geometry. A zone characterized by a certain flow pattern is called flow region. Visual inspections is the easier approach to detect the flow patterns. Different probes have been developed during the years to accomplish this point. Based on the channel geometry we can distinguish among flow pattern in horizontal and in vertical pipes. The type of flow configurations are several ecc ecc. Since the gas and the liquid in a two-phase flow are characterized by different velocity, they can be treated as two distinct continua with an interface where the velocity is the same. The gas fraction of a two-phase flow can be measured at a local point of the channel by inserting a pipe perpendicular to the channel or by an electrical probe. This would allow to get instantaneous local gas fraction. Averaging over time we can get a local time averaged gas fraction. On the other hand an instantaneous volume fraction can be defined as the gas fraction of the total volume considered. Enclosing a pipe section of length  $\Delta z$ , we can measure the volume fraction. Then letting  $\Delta z$  goes to zero we get the area averaged gas fraction which is the most adopted definition of void fraction and is then defined as:

$$\bar{\gamma} = \frac{A''}{A} \quad (36)$$

### A.1 Average Void Fraction Estimation

The state variables are pressure and enthalpy and all other properties are functions of those two. The regularly used quantities at the phase boundaries at constant pressure are denoted with ' for the liquid phase and '' for the vapour phase. Hence, the void fraction  $\gamma$  can be expressed as a function of vapour fraction  $x$  by using the enthalpy-based formulations

$$x(p, h) = \frac{h - h'}{h'' - h'} \quad \text{and} \quad (37)$$

$$\gamma(p, h) = \frac{x\rho'}{x\rho' + (1 - x)\rho''}. \quad (38)$$

Integrating Eq. 38 over an enthalpy range  $\Delta h = h_b - h_a$  allows us to calculate the average void fraction  $\bar{\gamma}$  as a function of pressure and the two enthalpies as shown by Bonilla et al. [3] resulting in

$$\bar{\gamma}(p, h_a, h_b) = \int_{h_a}^{h_b} \gamma(p, h) dh (h_b - h_a)^{-1} \quad (39)$$

$$= \frac{\rho'^2 (h_a - h_b) + \rho' \rho'' \left( h_b - h_a + (h' - h'') \ln \left( \frac{\Gamma(h_a)}{\Gamma(h_b)} \right) \right)}{(h_a - h_b) (\rho' - \rho'')^2} \quad (40)$$

$$\text{with } \Gamma(h) = \rho'(h - h') + \rho''(h'' - h). \quad (41)$$

This  $\Gamma(h)$  can be regarded as a density-based fraction of the enthalpy of evaporation

$$\Gamma(x) = (x\rho' + (1 - x)\rho'') (h'' - h'). \quad (42)$$

The  $\bar{\gamma}$  obtained from Eq. 40 allows us to calculate the average density  $\bar{\rho}$  from

$$\bar{\rho} = (1 - \bar{\gamma})\rho' + \bar{\gamma}\rho'' \quad (43)$$

A computationally efficient dynamic model for the two-phase region requires an analytic derivative for the average void fraction. Please see Eq. 44 through Eq. 47 for an expression of  $d\bar{\gamma}/dt$  that is explicit in  $p$ ,  $h_a$  and  $h_b$ .

Average void fraction partial derivatives

$$\frac{d\bar{\gamma}}{dt} = + \frac{\partial\bar{\gamma}}{\partial p} \frac{dp}{dt} + \frac{\partial\bar{\gamma}}{\partial h_a} \frac{dh_a}{dt} + \frac{\partial\bar{\gamma}}{\partial h_b} \frac{dh_b}{dt} \quad (44)$$

$$\begin{aligned} \frac{\partial\bar{\gamma}}{\partial p} = & + \frac{\frac{d\rho'}{dp}}{\Delta h_{ab}\Delta\rho_{tp}^2} \{ \Delta h_{ab}\rho' + \rho''\Delta h_{ab,tp} \} \\ & - \frac{2\rho' \left( \frac{d\rho'}{dp} - \frac{d\rho''}{dp} \right)}{\Delta h_{ab}\Delta\rho_{tp}^3} \{ \Delta h_{ab}\rho' + \rho''\Delta h_{ab,tp} \} \\ & + \frac{\rho'}{\Delta h_{ab}\Delta\rho_{tp}^2} \left\{ \Delta h_{ab} \frac{d\rho'}{dp} + \frac{d\rho''}{dp} \Delta h_{ab,tp} \right. \\ & \left. + \rho'' \left[ \left( \frac{dh'}{dp} - \frac{dh''}{dp} \right) \ln(G) + \frac{\Delta h_{tp}}{\Gamma(h_a)} (\Theta(h_a) - G\Theta(h_b)) \right] \right\} \quad (45) \end{aligned}$$

$$\begin{aligned} \frac{\partial\bar{\gamma}}{\partial h_a} = & - \frac{\rho'}{\Delta h_{ab}^2\Delta\rho_{tp}^2} \{ \Delta h_{ab}\rho' + \rho''\Delta h_{ab,tp} \} \\ & + \frac{\rho'}{\Delta h_{ab}\Delta\rho_{tp}^2} \left\{ +\rho' + \rho'' \left( -1 + \frac{\Delta h_{tp}\Delta\rho_{tp}}{\Gamma(h_a)} \right) \right\} \quad (46) \end{aligned}$$

$$\begin{aligned} \frac{\partial\bar{\gamma}}{\partial h_b} = & + \frac{\rho'}{\Delta h_{ab}^2\Delta\rho_{tp}^2} \{ \Delta h_{ab}\rho' + \rho''\Delta h_{ab,tp} \} \\ & + \frac{\rho'}{\Delta h_{ab}\Delta\rho_{tp}^2} \left\{ -\rho' + \rho'' \left( +1 - \frac{\Delta h_{tp}\Delta\rho_{tp}}{\Gamma(h_b)} \right) \right\} \quad (47) \end{aligned}$$

with  $\Delta h_{tp} = h' - h''$ ,  $\Delta\rho_{tp} = \rho' - \rho''$ ,  $\Delta h_{ab} = h_a - h_b$ ,

$$\Theta(h) = (h - h') \frac{d\rho'}{dp} - \rho' \frac{dh'}{dp} + (h'' - h) \frac{d\rho''}{dp} + \rho'' \frac{dh''}{dp},$$

$$\Delta h_{ab,tp} = -\Delta h_{ab} + \Delta h_{tp} \ln(G), \text{ and } G = \Gamma(h_a)/\Gamma(h_b).$$



## B Explicit moving boundary formulation for a general evaporator model

### SUB-COOLED ZONE

#### Primary fluid

##### *Conservation equations*

Mass balance

$$A \left[ L_1 \cdot \frac{d\bar{\rho}_1}{dt} + (\bar{\rho}_1 - \rho_1) \cdot \frac{dL_1}{dt} \right] = \dot{m}_{\text{in}} - \dot{m}_{12} \quad (48)$$

Energy balance

$$AL_1 \left[ \bar{\rho}_1 \cdot \frac{d\bar{h}_1}{dt} + \bar{h}_1 \cdot \frac{d\bar{\rho}_1}{dt} - \frac{dp_1}{dt} \right] + A (\bar{\rho}_1 \bar{h}_1 - \rho_1 h_1) \frac{dL_1}{dt} = \dot{m}_{\text{in}} \dot{h}_{\text{in}} - \dot{m}_A \dot{h}_A + \dot{Q}_{\text{r1}} \quad (49)$$

$$\frac{d\bar{\rho}_1}{dt} = \left[ \frac{\partial \bar{\rho}_1}{\partial p_1} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}_1}{\partial \bar{h}_1} \cdot \frac{\partial h_1}{\partial p_1} \right] \frac{dp_1}{dt} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}_1}{\partial \bar{h}_1} \cdot \frac{dh_{\text{in}}}{dt} \quad (50)$$

$$\frac{d\bar{h}_1}{dt} = \frac{1}{2} \cdot \left[ \frac{\partial \bar{h}_1}{\partial p_1} \cdot \frac{dp_1}{dt} + \frac{dh_{\text{in}}}{dt} \right] \quad (51)$$

##### *Constitutive equations*

$$\bar{h}_1 = \frac{1}{2}(h_{\text{in}} + h_1) \quad (52)$$

$$\text{subcool} = \text{setState\_ph}(p_1, \bar{h}_1) \quad (53)$$

$$\text{sat} = \text{setSat\_p}(p_2) \quad (54)$$

$$\rho_1 = \text{bubbleDensity}(\text{sat}) \quad (55)$$

$$h_1 = \text{bubbleEnthalpy}(\text{sat}) \quad (56)$$

$$\frac{\partial \bar{h}_1}{\partial p_1} = \text{dBubbleEnthalpy\_dPressure}(\text{sat}) \quad (57)$$

$$\frac{\partial \bar{\rho}_1}{\partial \bar{h}_1} = \text{density\_derh\_p}(\text{subcool}) \quad (58)$$

$$\frac{\partial \bar{\rho}_1}{\partial p_1} = \text{density\_derp\_h}(\text{subcool}) \quad (59)$$

$$\bar{T}_1 = \text{temperature}(\text{subcool}) \quad (60)$$

$$\dot{Q}_{r,1} = \pi D L_1 U_{pf,1} (T_{w,1} - \bar{T}_1) \quad (61)$$

**Metal wall**

*Conservation equations*

Energy balance:

$$C_w (M_{tot} \cdot \frac{L_1}{L}) \cdot \frac{dT_{w,1}}{dt} + \frac{M_{tot}}{L} (T_{w,1} - T_{w,12}) \frac{dL_1}{dt} = \dot{Q}_{sf,1} - \dot{Q}_{r,1} \quad (62)$$

*Constitutive equation*

$$T_{w,12} = \frac{T_{w,1} L_2 + T_{w,2} L_1}{L_1 + L_2} \quad (63)$$

$$C_w = const. \quad (64)$$

$$M_{tot} = const. \quad (65)$$

$$L = const. \quad (66)$$

## TWO-PHASE ZONE

**Primary fluid**

*Conservation equations*

Mass balance:

$$A \left[ L_2 \cdot \frac{d\bar{\rho}_2}{dt} + (\bar{\rho}_2 - \rho_v) \cdot \frac{dL_2}{dt} + (\rho_l - \rho_v) \cdot \frac{dL_1}{dt} \right] = \dot{m}_A - \dot{m}_{23} \quad (67)$$

Energy balance:

$$A \left[ L_2 \cdot \frac{d\bar{\rho}_2 \bar{h}_2}{dt} + (\bar{\rho}_2 \bar{h}_2 - \rho_v h_v) \cdot \frac{dL_2}{dt} + (\rho_l h_l - \rho_v h_v) \cdot \frac{dL_1}{dt} - L_2 \cdot \frac{dp_2}{dt} \right] = \dot{m}_A h_A - \dot{m}_{23} h_{23} + \dot{Q}_{r,2} \quad (68)$$

$$\frac{d\bar{\rho}_2}{dt} = \left( \frac{\partial \rho_v}{\partial p_2} \cdot \bar{\alpha} + \frac{\partial \rho_l}{\partial p_2} \cdot (1 - \bar{\alpha}) \right) \cdot \frac{dp_2}{dt} + (\rho_v - \rho_l) \cdot \left[ \frac{\partial \bar{\alpha}}{\partial p_2} \cdot \frac{dp_2}{dt} + \frac{\partial \bar{\alpha}}{\partial h_{OUT,alpha}} \cdot \frac{dh_{OUT,alpha}}{dt} \right] \quad (69)$$

$$\begin{aligned} \frac{d\bar{\rho}_2 \bar{h}_2}{dt} = & [\bar{\alpha} \cdot \left( \frac{\partial \rho_v}{\partial p_2} \cdot h_v + \frac{\partial h_v}{\partial p_2} \cdot \rho_v \right) + (1 - \bar{\alpha}) \cdot \left( \frac{\partial \rho_l}{\partial p_2} \cdot h_l + \frac{\partial h_l}{\partial p_2} \cdot \rho_l \right)] \cdot \frac{dp_2}{dt} \\ & + (\rho_v h_v - \rho_l h_l) \cdot \left[ \frac{\partial \bar{\alpha}}{\partial p_2} \cdot \frac{dp_2}{dt} + \frac{\partial \bar{\alpha}}{\partial h_{OUT,alpha}} \cdot \frac{dh_{OUT,alpha}}{dt} \right] \end{aligned} \quad (70)$$

*Constitutive equations*

$$\bar{\rho}_2 = \rho_v \bar{\alpha} + \rho_l \cdot (1 - \bar{\alpha}) \quad (71)$$

$$\bar{\rho}_2 \bar{h}_2 = \rho_v h_v \bar{\alpha} + \rho_l h_l \cdot (1 - \bar{\alpha}) \quad (72)$$

$$\rho_v = \text{dewDensity}(\text{sat}) \quad (73)$$

$$h_v = \text{dewEnthalpy}(\text{sat}) \quad (74)$$

$$\frac{\partial \bar{h}_v}{\partial p_2} = \text{dDewEnthalpy_dPressure}(\text{sat}) \quad (75)$$

$$\bar{T}_2 = \text{temperature}(\text{sat}) \quad (76)$$

$$\dot{Q}_{r,2} = \pi D L_2 U_{pf,2} (T_{w,2} - \bar{T}_2) \quad (77)$$

## Metal Wall

*Conservation equation*

Energy balance:

$$C_w \cdot \frac{M_{\text{tot}}}{L} \cdot [L_2 \cdot \frac{dT_{w,2}}{dt} + (T_{w,12} - T_{w,23}) \frac{dL_1}{dt} + (\bar{T}_{w,2} - T_{w,23}) \frac{dL_2}{dt}] = \dot{Q}_{sf,2} - \dot{Q}_{r,2} \quad (78)$$

## SUPER-HEATED ZONE

### Primary fluid

*Conservation equations*

Mass balance:

$$A[L_3 \cdot \frac{d\bar{\rho}_3}{dt} + (\rho_v - \bar{\rho}_3) \cdot (\frac{dL_1}{dt} + \frac{dL_2}{dt})] = \dot{m}_{23} - \dot{m}_{\text{out}} \quad (79)$$

Energy balance:

$$AL_3[\bar{\rho}_3 \cdot \frac{d\bar{h}_3}{dt} + \bar{h}_3 \cdot \frac{d\bar{\rho}_3}{dt} - \frac{dp_3}{dt}] + A(\rho_v h_v - \bar{\rho}_3 \bar{h}_3) \cdot (\frac{dL_1}{dt} + \frac{dL_2}{dt}) = \dot{m}_{23} \dot{h}_v - \dot{m}_{\text{out}} \dot{h}_{\text{out}} + \dot{Q}_{r,3} \quad (80)$$

$$\frac{d\bar{\rho}_3}{dt} = [\frac{\partial \bar{\rho}_3}{\partial p_3} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}_3}{\partial \bar{h}_3} \cdot \frac{\partial \bar{h}_v}{\partial p_3}] \frac{dp_3}{dt} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}_3}{\partial \bar{h}_3} \cdot \frac{dh_{\text{out}}}{dt} \quad (81)$$

$$\frac{d\bar{h}_3}{dt} = \frac{1}{2} \cdot [\frac{\partial \bar{h}_v}{\partial p_3} \cdot \frac{dp_3}{dt} + \frac{dh_{\text{out}}}{dt}] \quad (82)$$

*Constitutive equations*

$$vap = setState(p_3, \bar{h}_3) \quad (83)$$

$$\bar{h}_3 = \frac{1}{2}(h_v - h_{out}) \quad (84)$$

$$\frac{\partial \bar{\rho}_3}{\partial \bar{h}_3} = density\_derh\_p(vap) \quad (85)$$

$$\frac{\partial \bar{\rho}_3}{\partial p_3} = density\_derp\_h(vap) \quad (86)$$

$$\bar{T}_3 = temperature(vap) \quad (87)$$

$$\dot{Q}_{r,3} = \pi D L_3 U_{pf,3} (T_{w,3} - \bar{T}_3) \quad (88)$$

**Metal wall**

*Conservation equations*

Energy balance:

$$C_w \frac{M_{tot}}{L} \cdot [L_3 \cdot \frac{dT_{w,3}}{dt} + (T_{w,23} - T_{w,3})(\frac{dL_1}{dt} + \frac{dL_2}{dt})] = \dot{Q}_{sf,3} - \dot{Q}_{r,3} \quad (89)$$

*Constitutive equations*

$$T_{w,23} = \frac{T_{w,3}L_2 + T_{w,2}L_3}{L_2 + L_3} \quad (90)$$

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## Glossary

$A$  Cross Sectional Area ( $\text{m}^2$ ).  
 $L$  Length (m).  
 $N$  Number of sections.  
 $U$  Heat transfer coefficient ( $\text{W m}^{-2} \text{K}^{-1}$ ).  
 $\bar{h}$  Average specific enthalpy ( $\text{J kg}^{-1}$ ).  
 $\dot{Q}$  Heat flow (W).  
 $\dot{m}$  Mass flow ( $\text{kg s}^{-1}$ ).  
 $h$  Specific enthalpy ( $\text{J kg}^{-1}$ ).  
 $l$  Length of a zone (m).  
 $p$  Pressure (Pa).  
 $t$  Time (s).

## Acronyms

**MB** moving boundary.

## Dimensionless Numbers

Nu Nusselt number:  $\frac{\text{convection}}{\text{conduction}} = \frac{U L}{\lambda}$ .  
Pr Prandtl number:  $\frac{\text{viscous diffusion}}{\text{thermal diffusion}} = \frac{c_p \mu}{\lambda}$ .  
Re Reynolds number:  $\frac{\text{inertia}}{\text{viscosity}} = \frac{\rho v L}{\mu}$ .

## Greek Symbols

$\bar{\alpha}$  Average void fraction (ND).

$\pi$  ratio of circumference of circle to its diameter.

$\bar{\rho}$  Average density ( $\text{kg m}^{-3}$ ).

## Subscripts

12 at the interface of zone 1 and 2.

1 in zone 1.

23 at the interface of zone 2 and 3.

2 in zone 2.

3 in zone 3.

**in** at the inlet.

**l** saturated liquid.

**out** at the outlet.

**pf** primary fluid.

**sf** secondary fluid.

**v** saturated vapour.

**w** Metal wall.

*a* control volume inlet.

*b* control volume outlet.