ThermoCycle Moving Boundary Model

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Abstract

The authors present a new moving boundary model that was integrated into the ThermoCycle package written in the Modelica language. Focussing on a seamless integration with existing components, this new component allows to calculate dynamic heat transfer in an efficient and robust way covering the full range of possible operating points in the liquid, two-phase, gas and supercritical domain. A basic validation performed with heat transfer data from two different experiments with evaporators shows that the model is able to reliably predict heat exchanger performance. The flexible implementation allows to compare different heat transfer correlations, which are made freely available as part of the ThermoCycle library.

1 Introduction and Motivation

Moving boundary (MB) models are established tools to calculate heat exchanger performance in both steady-state and dynamic operation. A fictitious heat transfer channel is split up into different sections and with each section accounting for a different fluid state. In the case of an evaporator the maximum number of sections N is 3 for a) subcooled, b) two-phase and c) superheated state. At higher pressures, the fluid might enter the supercritical state. Hence, there are four different sections out of which a maximum of three can occur simultaneously. The name moving boundary is derived from the fact that the interfaces between these sections do not have a fixed spatial position but merely a fixed thermodynamic location depending on the presence of liquid and gaseous fluid, respectively. The actual existence of a certain section and its length are determined based on the fluid state resulting in variable sectioning. A fixed total length superimposes the required boundary condition to calculate the length of each section.

Moving boundary formulations are a good compromise between computational efficiency, robustness and accuracy[2].

2 Formulation

2.1 Assumptions

i. The tube is cylindrical

ii. The velocity of the fluid is uniform on the cross sectional area

iii. The enthalpy of the fluid is linear in each region of the tube (sub-cooled, two-phase, super-heated)

iv. Pressure is considered constant (at least for now)

v. The secondary fluid is treated as an incompressible fluid

2.2 Equations

Thermodynamic properties are calculated using Coolprop [1]. The state variable selected are p and $\bar{h}$. The heat transfer coefficients on primary side ($U_{\mathrm{pf},1}$,$U_{\mathrm{pf},2}$,$U_{\mathrm{pf},3}$) and secondary side ($U_{\mathrm{sf},1}$,$U_{\mathrm{sf},2}$,$U_{\mathrm{sf},3}$) are calculated using appropriate heat transfer models. Different model of void fraction $\bar{\alpha}$ have been also implemented.

<div align="center">

SUB-COOLED ZONE

</div>

Primary fluid

<div align="center">Conservation equations</div>

Mass balance

$$A \left[L_1 \cdot \frac{d\bar{\rho}_1}{dt} + (\bar{\rho}_1 - \rho_\mathrm{l}) \cdot \frac{dL_1}{dt} \right] = \dot{m}_\mathrm{in} - \dot{m}_{12} \tag{1}$$

Energy balance

$$AL_1 \left[\bar{\rho}_1 \cdot \frac{d\bar{h}_1}{dt} + \bar{h}_1 \cdot \frac{d\bar{\rho}_1}{dt} - \frac{dp_1}{dt} \right] + A \left(\bar{\rho}_1 \bar{h}_1 - \rho_\mathrm{l} h_\mathrm{l} \right) \frac{dL_1}{dt} = \dot{m}_\mathrm{in} \dot{h}_\mathrm{in} - \dot{m}_\mathrm{A} \dot{h}_\mathrm{A} + \dot{Q}_\mathrm{r1} \tag{2}$$

$$\frac{d\bar{\rho}_1}{dt} = \left[\frac{\partial \bar{\rho}_1}{\partial p_1} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}_1}{\partial \bar{h}_1} \cdot \frac{\partial h_\mathrm{l}}{\partial p_1} \right] \frac{dp_1}{dt} + \frac{1}{2} \cdot \frac{\partial \bar{\rho}_1}{\partial \bar{h}_1} \cdot \frac{dh_\mathrm{in}}{dt} \tag{3}$$

$$\frac{d\bar{h}_1}{dt} = \frac{1}{2} \cdot \left[\frac{\partial \bar{h}_\mathrm{l}}{\partial p_1} \cdot \frac{dp_1}{dt} + \frac{dh_\mathrm{in}}{dt} \right] \tag{4}$$

<div align="center">Constitutive equations</div>

$$\bar{h}_1 = \frac{1}{2}(h_\mathrm{in} + h_\mathrm{l}) \tag{5}$$

$$subcool = setState\_ph(p_1, \bar{h}_1) \tag{6}$$

$$sat = setSat\_p(p_2) \tag{7}$$

<div align="center">2</div>

$$\rho_l = bubbleDensity(sat) \tag{8}$$

$$h_l = bubbleEnthalpy(sat) \tag{9}$$

$$\frac{\partial \bar{h}_l}{\partial p_1} = dBubbleEnthalpy\_dPressure(sat) \tag{10}$$

$$\frac{\partial \bar{\rho}_1}{\partial \bar{h}_1} = density\_derh\_p(subcool) \tag{11}$$

$$\frac{\partial \bar{\rho}_1}{\partial p_1} = density\_derp\_h(subcool) \tag{12}$$

$$\bar{T}_1 = temperature(subcool) \tag{13}$$

$$\dot{Q}_{r,1} = \pi D L_1 U_{pf,1}(T_{w,1} - \bar{T}_1) \tag{14}$$

Metal wall

Conservation equations

Energy balance:

$$C_w(M_{tot} \cdot \frac{L_1}{L}) \cdot \frac{dT_{w,1}}{dt} + \frac{M_{tot}}{L}(T_{w,1} - T_{w,12})\frac{dL_1}{dt} = \dot{Q}_{sf,1} - \dot{Q}_{r,1} \tag{15}$$

Constitutive equation

$$T_{w,12} = \frac{T_{w,1}L_2 + T_{w,2}L_1}{L_1 + L_2} \tag{16}$$

$$C_w = const. \tag{17}$$

$$M_{tot} = const. \tag{18}$$

$$L = const. \tag{19}$$

Secondary fluid

Conservation equation

Energy balance:

$$\rho_{sf}A_{sf}[L_1 \cdot \frac{dh_{sf,1}}{dt} + (h_{sf,1} - h_{sf,12})\frac{dL_1}{dt}] = \dot{m}_{sf}(h_{sf,in} - h_{sf,12}) + \dot{Q}_{sf,1} \tag{20}$$

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$$h_{\mathrm{sf},12} = \frac{\mathrm{h}_{\mathrm{sf},1}L_2 + \mathrm{h}_{\mathrm{sf},2}L_1}{L_1 + L_2} \tag{21}$$

$$Q_{\mathrm{sf},1} == \pi \mathrm{D}L_1 U_{\mathrm{sf},1}(T_{\mathrm{w},1} - \bar{T}_{\mathrm{sf},1}) \tag{22}$$

$$sfzone1 = setState(p1, h_{sf,1}) \tag{23}$$

$$T_{\mathrm{sf},1} = temperature(sfzone1) \tag{24}$$

TWO-PHASE ZONE

Primary fluid

Conservation equations

Mass balance:

$$A\left[L_2 \cdot \frac{d\bar{\rho}_2}{dt} + (\bar{\rho}_2 - \rho_{\mathrm{v}}) \cdot \frac{dL_2}{dt} + (\rho_{\mathrm{l}} - \rho_{\mathrm{v}}) \cdot \frac{dL_1}{dt}\right] = \dot{m}_{\mathrm{A}} - \dot{m}_{23} \tag{25}$$

Energy balance:

$$A\left[L_2 \cdot \frac{d\bar{\rho}_2\bar{h}_2}{dt} + (\bar{\rho}_2\bar{h}_2 - \rho_{\mathrm{v}}h_{\mathrm{v}}) \cdot \frac{dL_2}{dt} + (\rho_{\mathrm{l}}h_{\mathrm{l}} - \rho_{\mathrm{v}}h_{\mathrm{v}}) \cdot \frac{dL_1}{dt} - L_2 \cdot \frac{dp_2}{dt}\right] = \dot{m}_{\mathrm{A}}\dot{h}_{\mathrm{A}} - \dot{m}_{23}\dot{h}_{23} + \dot{Q}_{\mathrm{r},2} \tag{26}$$

$$\frac{d\bar{\rho}_2}{dt} = \left(\frac{\partial\rho_{\mathrm{v}}}{\partial p_2} \cdot \bar{\alpha} + \frac{\partial\rho_{\mathrm{l}}}{\partial p_2} \cdot (1 - \bar{\alpha})\right) \cdot \frac{dp_2}{dt} + (\rho_{\mathrm{v}} - \rho_{\mathrm{l}}) \cdot \left[\frac{\partial\bar{\alpha}}{\partial p_2} \cdot \frac{dp_2}{dt} + \frac{\partial\bar{\alpha}}{\partial h_{\mathrm{OUT,alpha}}} \cdot \frac{dh_{\mathrm{OUT,alpha}}}{dt}\right] \tag{27}$$

$$\frac{d\bar{\rho}_2\bar{h}_2}{dt} = \left[\bar{\alpha} \cdot \left(\frac{\partial\rho_{\mathrm{v}}}{\partial p_2} \cdot h_{\mathrm{v}} + \frac{\partial h_{\mathrm{v}}}{\partial p_2} \cdot \rho_{\mathrm{v}}\right) + (1 - \bar{\alpha}) \cdot \left(\frac{\partial\rho_{\mathrm{l}}}{\partial p_2} \cdot h_{\mathrm{l}} + \frac{\partial h_{\mathrm{l}}}{\partial p_2} \cdot \rho_{\mathrm{l}}\right)\right] \cdot \frac{dp_2}{dt}$$
$$+ (\rho_{\mathrm{v}}h_{\mathrm{v}} - \rho_{\mathrm{l}}h_{\mathrm{l}}) \cdot \left[\frac{\partial\bar{\alpha}}{\partial p_2} \cdot \frac{dp_2}{dt} + \frac{\partial\bar{\alpha}}{\partial h_{\mathrm{OUT,alpha}}} \cdot \frac{dh_{\mathrm{OUT,alpha}}}{dt}\right] \tag{28}$$

Constitutive equations

$$\bar{\rho}_2 = \rho_{\mathrm{v}}\bar{\alpha} + \rho_{\mathrm{l}} \cdot (1 - \bar{\alpha}) \tag{29}$$

$$\bar{\rho}_2\bar{h}_2 = \rho_{\mathrm{v}}h_{\mathrm{v}}\bar{\alpha} + \rho_{\mathrm{l}}h_{\mathrm{l}} \cdot (1 - \bar{\alpha}) \tag{30}$$

$$\rho_{\mathrm{v}} = dewDensity(sat) \tag{31}$$

$$h_{\mathrm{v}} = dewEnthalpy(sat) \tag{32}$$

$$\frac{\partial \bar{h}_{\mathrm{v}}}{\partial p_2} = dDewEnthalpy\_dPressure(sat) \tag{33}$$

$$\bar{T}_2 = temperature(sat) \tag{34}$$

$$\dot{Q}_{\mathrm{r},2} = \pi \mathrm{D} L_2 U_{\mathrm{pf},2}(T_{\mathrm{w},2} - \bar{T}_2) \tag{35}$$

Metal Wall

Conservation equation

Energy balance:

$$C_{\mathrm{w}} \cdot \frac{M_{\mathrm{tot}}}{L} \cdot [L_2 \cdot \frac{d\mathrm{T}_{\mathrm{w},2}}{dt} + (\mathrm{T}_{\mathrm{w},12} - \mathrm{T}_{\mathrm{w},23})\frac{dL_1}{dt} + (\bar{\mathrm{T}}_2 - \mathrm{T}_{\mathrm{w},23})\frac{dL_2}{dt}] = \dot{Q}_{\mathrm{sf},2} - \dot{Q}_{\mathrm{r},2} \tag{36}$$

Secondary fluid

Conservation equation

Energy balance:

$$\rho_{\mathrm{sf}} A_{\mathrm{sf}} \left[L_2 \cdot \frac{dh_{\mathrm{sf},2}}{dt} + (\mathrm{h}_{\mathrm{sf},12} - \mathrm{h}_{\mathrm{sf},23})\frac{dL_1}{dt}(\mathrm{h}_{\mathrm{sf},2} - \mathrm{h}_{\mathrm{sf},23})\frac{dL_2}{dt} \right] = \dot{m}_{\mathrm{sf}}(h_{\mathrm{sf},12} - h_{\mathrm{sf},23}) + \dot{Q}_{\mathrm{sf},1} \tag{37}$$

Constitutive equations

$$Q_{\mathrm{sf},2} == \pi \mathrm{D} L_2 U_{\mathrm{sf},2}(T_{\mathrm{w},2} - \bar{T}_{\mathrm{sf},2}) \tag{38}$$

$$sfzone2 = setState(p2, h_{sf,2}) \tag{39}$$

$$\bar{T}_{\mathrm{sf},2} = temperature(sfzone2) \tag{40}$$

SUPER-HEATED ZONE

Primary fluid

Conservation equations

Mass balance:

$$\mathrm{A}[L_3 \cdot \frac{d\bar{\rho}_3}{dt} + (\rho_{\mathrm{v}} - \bar{\rho}_3) \cdot (\frac{dL_1}{dt} + \frac{dL_2}{dt})] = \dot{m}_{23} - \dot{m}_{\mathrm{out}} \tag{41}$$

Energy balance:

$$\mathrm{A}L_3[\bar{\rho}_3 \cdot \frac{d\bar{h}_3}{dt} + \bar{h}_3 \cdot \frac{d\bar{\rho}_3}{dt} - \frac{dp_3}{dt}] + \mathrm{A}(\rho_\mathrm{v} h_\mathrm{v} - \bar{\rho}_3 \bar{h}_3) \cdot (\frac{dL_1}{dt} + \frac{dL_2}{dt}) = \dot{m}_{23}\dot{h}_\mathrm{v} - \dot{m}_\mathrm{out}\dot{h}_\mathrm{out} + \dot{Q}_\mathrm{r,3} \tag{42}$$

$$\frac{d\bar{\rho}_3}{dt} = [\frac{\partial\bar{\rho}_3}{\partial p_3} + \frac{1}{2} \cdot \frac{\partial\bar{\rho}_3}{\partial\bar{h}_3} \cdot \frac{\partial\bar{h}_\mathrm{v}}{\partial p_3}]\frac{dp_3}{dt} + \frac{1}{2} \cdot \frac{\partial\bar{\rho}_3}{\partial\bar{h}_3} \cdot \frac{dh_\mathrm{out}}{dt} \tag{43}$$

$$\frac{d\bar{h}_3}{dt} = \frac{1}{2} \cdot [\frac{\partial\bar{h}_\mathrm{v}}{\partial p_3} \cdot \frac{dp_3}{dt} + \frac{dh_\mathrm{out}}{dt}] \tag{44}$$

Constitutive equations

$$vap = setState(p_3, \bar{h}_3) \tag{45}$$

$$\bar{h}_3 = \frac{1}{2}(h_\mathrm{v} - h_\mathrm{out}) \tag{46}$$

$$\frac{\partial\bar{\rho}_3}{\partial\bar{h}_3} = density\_derh\_p(vap) \tag{47}$$

$$\frac{\partial\bar{\rho}_3}{\partial p_3} = density\_derp\_h(vap) \tag{48}$$

$$\bar{T}_3 = temperature(vap) \tag{49}$$

$$\dot{Q}_\mathrm{r,3} = \pi\mathrm{D}L_3 U_\mathrm{pf,3}(T_\mathrm{w,3} - \bar{T}_3) \tag{50}$$

Metal wall

Conservation equations

Energy balance:

$$C_\mathrm{w}\frac{M_\mathrm{tot}}{L} \cdot [L_3 \cdot \frac{d\mathrm{T}_\mathrm{w,3}}{dt} + (\mathrm{T}_\mathrm{w,23} - \mathrm{T}_\mathrm{w,3})(\frac{dL_1}{dt} + \frac{dL_2}{dt})] = \dot{Q}_\mathrm{sf,3} - \dot{Q}_\mathrm{r,3} \tag{51}$$

Constitutive equations

$$\mathrm{T}_\mathrm{w,23} = \frac{\mathrm{T}_\mathrm{w,3}L_2 + \mathrm{T}_\mathrm{w,2}L_3}{L_2 + L_3} \tag{52}$$

Secondary fluid

Conservation equations

Energy balance:

$$\rho_\mathrm{sf}A_\mathrm{sf}[L_3 \cdot \frac{d\bar{h}_\mathrm{sf,3}}{dt} + (\mathrm{h}_\mathrm{sf,23} - \mathrm{h}_\mathrm{sf,3})(\frac{dL_1}{dt} + \frac{dL_2}{dt})] = \dot{m}_\mathrm{sf}(h_\mathrm{sf,23} - h_\mathrm{sf,OUT}) + \dot{Q}_\mathrm{sf,3} \tag{53}$$

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$$h_{\text{sf},23} = \frac{h_{\text{sf},3}L_2 + h_{\text{sf},2}L_3}{L_2 + L_3} \tag{54}$$

$$Q_{\text{sf},3} = \pi \text{D}L_3 U_{\text{sf},3}(T_{\text{w},3} - \bar{T}_{\text{sf},3}) \tag{55}$$

$$sfzone3 = setState(p3, h_{sf,3}) \tag{56}$$

$$\bar{T}_{\text{sf},3} = temperature(sfzone3) \tag{57}$$

2.3 Heat Transfer

Based on Nusselt number (Nu) from Reynolds number (Re) and Prandtl number (Pr) for a characteristic length L. Angles are usually calculated in radians or π.

2.4 Pressure Drop

3 Results and Discussion

Compared to [3], the model ....

[4]

[5]

4 Conclusion

References

[1] Ian H. Bell, Sylvain Quoillin, Jorrit Wronski, and Vincent Lemort. Coolprop: An open-source reference-quality thermophysical property library. In *submitted abstract to ASME-ORC 2013 – 2nd International Seminar on ORC Power Systems*, 2013.

[2] Satyam Bendapudi, James E. Braun, and Eckhard A. Groll. A comparison of moving-boundary and finite-volume formulations for transients in centrifugal chillers. *International Journal of Refrigeration*, 31(8):1437–1452, 2008.

[3] Martin Ryhl Kærn. *Analysis of Flow Maldistribution in Fin-and-tube Evaporators for Residential Air-conditioning Systems*. Phd thesis, Technical University of Denmark, 2011.

[4] Wei-Jiang Zhang and Chun-Lu Zhang. A generalized moving-boundary model for transient simulation of dry-expansion evaporators under larger disturbances. *International Journal of Refrigeration*, 29(7):1119–1127, November 2006.

[5] Wei-Jiang Zhang, Chun-Lu Zhang, and Guo-Liang Ding. On three forms of momentum equation in transient modeling of residential refrigeration systems. *International Journal of Refrigeration*, 32(5):938–944, August 2009.

Glossary

A Area (m^2).

c Specific heat capacity ($J\,kg^{-1}\,K^{-1}$).

$\bar{h}$ Average specific enthalpy ($J\,kg^{-1}$).

L Length (m).

$\dot{m}$ Mass flow ($kg\,s^{-1}$).

N Number of sections.

p Pressure (Pa).

$\dot{Q}$ Heat flow (W).

T Temperature (K).
t Time (s).

U Heat transfer coefficient ($W\,m^{-2}\,K^{-1}$).

v Velocity ($m\,s^{-1}$).

Acronyms

MB moving boundary.

Dimensionless Numbers

Nu Nusselt number: $\frac{\text{convection}}{\text{conduction}} = \frac{U\,L}{\lambda}$.
Pr Prandtl number: $\frac{\text{viscous diffusion}}{\text{thermal diffusion}} = \frac{c_p\,\mu}{\lambda}$.
Re Reynolds number: $\frac{\text{inertia}}{\text{viscosity}} = \frac{\rho\,v\,L}{\mu}$.

Greek Symbols

$\bar{\alpha}$ Average void fraction (ND).

λ Thermal conductivity ($W\,m^{-1}\,K^{-1}$).

μ Dynamic viscosity ($Pa\,s$).

π ratio of circumference of circle to its diameter.

$\bar{\rho}$ Average density ($kg\,m^{-3}$).

Subscripts

1 in zone 1.

12 at the interface of zone 1 and 2.

2 in zone 2.

23 at the interface of zone 2 and 3.

3 in zone 3.

in at the inlet.

l saturated liquid.

out at the outlet.

p at constant pressure.

pf primary fluid.

sf secondary fluid.

v saturated vapour.

5   Modelling

5.1   Modelica Characteristics

- Enthalpy controls switching
- Only length gets defined, no zones get disabled. Their respective length might be set to 10^{-10} though.
- Solid wall with constant properties is included in the cell of the primary fluid.
- A secondary fluid is assumed to be incompressible, for now. This is our second component.
- Should length L be treated as input, output or as a variable in the connectors?
- Implementation of heat transfer and void fractions models starting with [3] and Sylvain's EES file.