QUADRATIC STRESS $\sum_{i>j} \left| d_\chi(x_i,x_j) - \|g(x_i)-g(x_j)\|_2 \right|^2$

• using $z_i = g(x_i)$ $\quad d_{ij}(Z) = \| z_i - z_j \|_2$

$$\sum_{i>j} \left| d_\chi(x_i,x_j) - d_{ij}(Z) \right|^2$$

$$= \sum_{i>j} \underbrace{d_\chi(x_i,x_j)^2}_{\text{easy to compute}} - \underbrace{2 d_\chi(x_i,x_j) d_{ij}(Z)}_{T2} + \underbrace{d_{ij}(Z)^2}_{T1}$$

T1 $\displaystyle\sum_{i>j} d_{ij}(Z)^2 = \sum_{i>j} \|z_i - z_j\|_2^2 = \sum_{i>j} \sum_{d=1}^{K} (z_i^d - z_j^d)^2$

$$= \sum_{i>j} \sum_{d=1}^{K} (z_i^d)^2 - 2 z_i^d z_j^d + (z_j^d)^2$$

dot products are defined for vectors

$$= \sum_{i>j} \langle z_i, z_i \rangle + \langle z_j, z_j \rangle - 2 \langle z_i, z_j \rangle$$

$$= \sum_{i>j} \langle z_i, z_i \rangle + \langle z_j, z_j \rangle - 2 \sum_{i>j} \langle z_i, z_j \rangle$$

$$= (m-1) \sum_{i=1}^{m} \langle z_i, z_i \rangle - \left(\sum_{i,j} \langle z_i, z_j \rangle - \sum_{i=1}^{m} \langle z_i, z_i \rangle \right)$$

$$= m \sum_{i=1}^{m} \langle z_i, z_i \rangle - \sum_{i,j} \langle z_i, z_j \rangle = \underbrace{m \cdot tr(ZZ^T) - tr(11^T ZZ^T)}_{\text{FORM 1}}$$

• if $V = mI - 11^T$

$$\Rightarrow \underbrace{tr(VZZ^T)}_{\text{FORM 2}} \quad \underbrace{tr(Z^T V Z)}_{\text{FORM 3}}$$

T2

post multiply

$$2\sum_{i>j} d_{ij}(Z)\, d_X(x_i,x_j) = \sum_{i>j} \underbrace{d_X(x_i,x_j)\, d_{ij}^{-1}(Z)}_{a_{ij}=a_{ji}}\, \underbrace{d_{ij}(Z)^2}_{T1}$$

$d_{ij}(Z)$ can be easily computed, is just a dist. matrix

$$tr(BZZ^T) = tr(Z^T B Z)$$

where $b_{ij} = \begin{cases} -a_{ij} & i \neq j \\ -\sum_{i\neq l} b_{il} & i = j \end{cases}$ $\quad B = -D_X \oslash D_Z + diag((D_X \oslash D_Z)\mathbf{1})$

$\boxed{\text{TOTAL FORMULA}}$

depend on Z

$$\sigma(Z) = tr(Z^T V Z) - 2\,tr(Z^T B_Z Z) + \sum_{i>j} dx_{ij}^2$$

$\boxed{\text{GRADIENT}}$

$$\nabla \sigma(Z) = 2VZ - 2B_Z Z$$

iteration: $Z^{t+1} = Z^t - 2\alpha\,(VZ^t - B_{Z^t} Z^t)$