

$u, v \in V$, $\{e_i\}$ ST-BASIS

$$\begin{aligned}\langle u, v \rangle &= \left\langle \sum_i u_i e_i, \sum_j v_j e_j \right\rangle = \sum_{i,j} \langle u_i e_i, v_j e_j \rangle = \sum_{i,j} u_i v_j \underbrace{\langle e_i, e_j \rangle}_{\neq 0 \text{ only for } i=j} \\ &= \sum_i u_i v_i = U^T V\end{aligned}$$

$\{w_i\}$ GENERIC BASIS

$$\begin{aligned}\langle u, v \rangle &= \sum_{i,j} u_i v_j \underbrace{\langle w_i, w_j \rangle}_{w_i^T w_j} = U^T W^T W V \\ &= (W U)^T (W V)\end{aligned}$$