PARAMETRIZATION $\alpha(t)$

$$\alpha(t) = \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \qquad \alpha'(t) = \begin{pmatrix} \frac{du}{dt} \\ \frac{dv}{dt} \end{pmatrix} \qquad \alpha'(0) = \begin{pmatrix} \frac{du}{dt}\big|_{t=0} \\ \frac{dv}{dt}\big|_{t=0} \end{pmatrix} = \begin{pmatrix} \alpha u \\ \alpha v \end{pmatrix}$$

$$\Phi(u,v) = (x(u,v),\ y(u,v),\ z(u,v))$$

$$d\Phi\begin{pmatrix} \alpha u \\ \partial v \end{pmatrix} = \frac{d}{dt}(\Phi \circ \alpha(t))\Big|_{t=0} = \frac{d}{dt}\left(\Phi(u(t), v(t))\right)\Big|_{t=0}$$

$$= \begin{pmatrix} \frac{d}{dt} x(u(t),v(t))\big|_{t=0} \\ \frac{d}{dt} y(u(t),v(t))\big|_{t=0} \\ \frac{d}{dt} z(u(t),v(t))\big|_{T=0} \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial u}\alpha u + \frac{\partial x}{\partial v}\alpha v \\ \frac{\partial y}{\partial u}\alpha u + \frac{\partial y}{\partial v}\partial v \\ \frac{\partial z}{\partial x}\partial u + \frac{\partial z}{\partial v}\alpha v \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{pmatrix}}_{\text{JACOBIAN}} \begin{pmatrix} \alpha u \\ \alpha v \end{pmatrix}$$