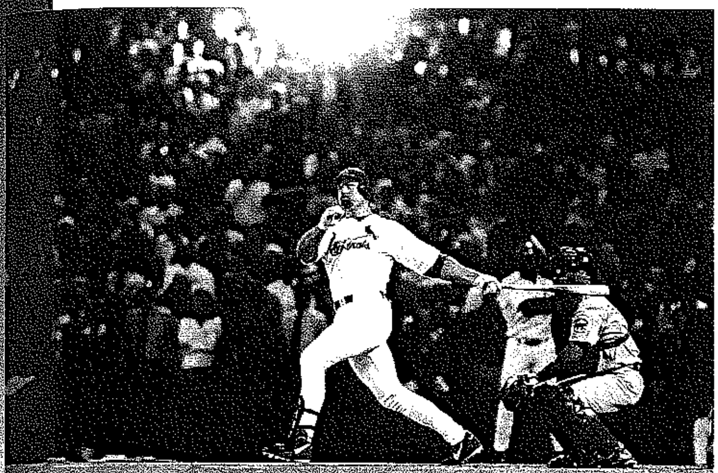
7.3 Random Thoughts

Are Coincidences as Truly Amazing as They First Appear?



During the great Sammy Sosa–Mark McGwire home-run race of 1998, Mark McGwire tied a home-run record of 61 home runs on his own father's 61st birthday. *What an amazing coincidence!*

> *How dare we speak of the laws of chance? Is not chance the antithesis of all law?*
>
> BERTRAND RUSSELL

Coincidences are so striking because any particular one is extremely improbable. However, what is even more improbable is that no coincidence will occur. We saw in the Birthday Question that finding, in a room of 50, two people who share the same birthday is extremely likely, even though the probability of any particular two people having the same birthday is extremely low. If you were one of a pair of people in that room with the same birthday as someone else, you would feel that a surprising coincidence had occurred—as indeed it had. But almost certainly some pair of people in the room would experience that coincidence. Let's now delve into the mysterious world of coincidences.

Coincidences and random happenings easily befuddle our intuition. To expose them for what they are, we must describe them clearly and analyze them quantitatively. Looking at simplified situations will help us understand whence misleading impressions arise. As usual, we start with concrete examples.

A Deadly Coincidence

We began working on a first draft of this section for the first edition of this text during a two-week period in late June and early July of 1997.

During that time, five famous people died—television celebrity Brian Keith (June 24), deep-sea diver Jacques Cousteau (June 25), actor Robert Mitchum (July 1), actor Jimmy Stewart (July 2), and news commentator Charles Kuralt (July 4). As we were writing about randomness, we began to ponder: What are the chances of five famous people dying during those two weeks? Isn't it strange that during the two weeks we were writing a section about coincidence such a public coincidence actually occurred?*

Having noticed this sad but interesting phenomenon, we decided to analyze and attempt to understand it. Is it strange or not that five famous people died during a two-week period? We know we should be sad, but should we be surprised? Contemplating this question brings up several of the main ideas associated with randomness, probability, and coincidence. The first is the meaning of *strange*. Presumably we mean that the event had a low probability of occurring. But to associate a probability with the event of five famous people dying, we are obliged to specify the total collection of possible occurrences with which to compare the death of the five. One possibility is to consider all deaths during that period and ask how likely it is that five of the people who died would be famous.

At least 52 million people died in 1997, which is an average of one million deaths per week worldwide. Thus, our question might be rephrased in a provocative way: Among the roughly two million people on Earth who die during any two-week period, what is the probability that five of them are famous? Already this phrasing of the question makes the fact of five famous deaths a little less surprising. But perhaps these five deaths would still be surprising if the total number of famous people is extremely low. How many famous people are there on Earth?

We just made some conservative guesses of the number of famous people in different categories: 300 singers; 600 actors; 600 sports figures; 1200 leaders; 150 scientists; 200 businesspeople; 200 artists and writers; and 750 miscellaneous people. So, let's say there are 4000 famous people in the world. Nearly all are famous for less than 40 years of their lives, so we estimate that at least 100 must die each year, which amounts to approximately two per week. Therefore, as a matter of fact, our five deaths are pretty much right on target for the average number of deaths of famous people for a two-week period. To check our figures, we looked in a world almanac that lists famous people who died between October 1995 and October 1996. The list consisted of 105 names, so our estimates appear to be reasonably accurate.

Actually, the coincidence of five famous deaths should probably be viewed in the context of all possible remarkable coincidences that might

*As an even more eery coincidence, while working on an early draft of this section for the 3rd edition back in June of 2009, within a matter of days, the following celebrities died: sidekick Ed McMahon, Queen of Pin-Up Farrah Fawcett, King of Pop Michael Jackson, Pitchman Billy Mays, Oscar Winner Karl Malden, 50s TV Star Gale Storm, Vegas Impressionist Fred Travalena. We're already concerned about the loss of (famous) lives as we move to future editions.

have happened during that two-week period. We didn't set out to look for a coincidence involving deaths of celebrities. The huge collection of all conceivable coincidences diminishes the significance of the five-death coincidence even further. When looked at in this way, many coincidences lose some of their luster.

Let's now see if we can come to understand the presence of coincidence in our own lives.

Personal Coincidence

Think of the most amazing coincidences in your life. Perhaps you were walking in the airport in Chicago and ran into an old friend you hadn't seen in 10 years. Perhaps you thought about a car crash, and the very next day, a relative was in an automobile accident. Perhaps you and a college classmate independently decided to open a Starbucks, rather than go to grad school to study math. Perhaps your birthday is the same as a string of numbers in your social security number. Some remarkable coincidences have occurred in your life.

How unlikely are any of these events? The answer is that each one of them is extremely unlikely. However, let's look at the same situations from a different point of view. Let's consider the probability that you will *avoid* remarkable coincidences. Often, to better understand a possibility, it is valuable to consider the opposite possibility.

To better understand a possibility, consider the opposite possibility.

Each day, suppose you wake up in the morning and think of an event that has a one-in-a-thousand chance of happening that day. In other words, imagine that 1000 equally likely things might happen that day, of which one of them is the rare event that you are considering. Let's compute the probability of not one of those coincidences coming to pass during a year. The first day, you have a 999/1000 probability of not experiencing that coincidence (pretty likely it will not happen). Using the ideas developed in the previous section, we see that your chance of missing rare coincidences both the first and second days is $992^2/1000^2$ since you have 999 times 999 possible ways of not experiencing the coincidence during the two days and 1000 times 1000 total things that could happen during those two days. Your chance of missing out for any number of days is simply 999 raised to the power of the number of days divided by 1000 raised to the power of the number of days, which is the same as 999/1000 raised to the number of days. Using a calculator and taking 999/1000 to the 365th power, we see that missing out every day for a year has a probability of 0.69. So, your chance of experiencing one of your one-in-a-thousand coincidences during one year is 0.31—nearly 1/3 (one in three chances—not that unlikely). During a three-year period, your chance of missing every single day is 999/1000 raised to the 1095th power, a mere 0.33. In other words, the probability that during a three-year period your one-in-a-thousand event will occur at least once is a whopping 2/3 (two in three chances).

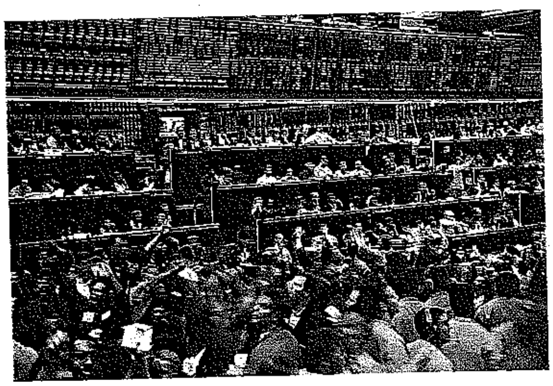
The probability that such a one-in-a-thousand event will happen at least once in 10 years is 0.97, and after 20 years the probability that at least one such unlikely event will happen to you is 0.9993. In other words, even if we select, in advance, each morning the one-in-a-thousand coincidence that we would count for that day, each of us is almost certain to experience that coincidence from time to time. Of course, in practice we would take note of a one-in-a-thousand coincidence even if it did not happen to be our particular coincidence *du jour*. Therefore, we see that it would be truly remarkable if we never experienced such a coincidence.

Moral: *Coincidence Happens.*

How to Get Rich Quick as a Stock Whiz

Predicting the future is a feat full of folly. One method of beating the odds is to make many predictions and then declare success if a few of them materialize (and hope that people will just forget those that don't). Another is to cover all bets. Let's take a look at how we can predict the future in the stock market with impressive accuracy, from some people's point of view.

Let's take a list of 1024 investors and send them a letter on Monday. To 512 investors we write, "IBM stock will go up next week"; to the other 512 we write, "IBM stock will go down next week." The following week, we send 512 letters to the group for whom we were correct and write to 256 of them, "IBM stock will go up next week," and to the other 256 we write, "IBM stock will go down next week." At the end of that week 256 people will begin to pay attention to our ability to predict the future. We continue the pattern. After nine weeks, two people will have seen us predict the future nine times in a row. Now we ask them to send us a large check requesting our next week's prediction. We then send an "up" letter to one, a "down" letter to the other, and, to assuage our conscience, refund the payment for the person whom we misled.



Another up and down day on the market

In real life, we presume that such a scam is illegal, but it does happen inadvertently in another way. There are thousands of people who predict stock market activity. Some are correct sometimes. Suppose these stock analysts were literally flipping coins to make their predictions. Still, we would expect that someone would have a good track record if enough of them were flipping coins. The moral of this story is beware of investment counsel that says, "This expert correctly predicted the big crash of 1987." That person may well have done so, and that person no doubt believes that the prediction was not based on randomness but was, instead, based on special insight. How are we going to determine whether the truth rests in randomness among a lot of predictors who really don't have any special insight, or in the incredible instincts of a few special people? This question is essentially impossible to answer. Or is it? We leave these conundrums for you to ponder but do caution you not to take advice blindly.

Hey, Hey, We're the Monkeys

As we have said, although it is rare to see a particular coincidence within a random event, it is even more improbable to *never* see a coincidence. We further illustrate this idea with perhaps the most famous example of randomness.

Suppose we have a very large number of monkeys, each banging away randomly on his or her own word processor. Their typing is completely random. If we let them type indefinitely, would one of them at some point randomly type out Shakespeare's entire *Hamlet?*

The answer is yes. If we have enough monkeys typing long enough, we are bound to get *Hamlet.* Why? Because if we perform a random event enough times, we would expect to see any possible outcome, no matter how unlikely it may be. This result is known as the *Infinite Monkey Theorem* or, as we like to refer to it, *Hamlet Happens.*

The Infinite Monkey Theorem.
If we put an army of monkeys at word processors, eventually one will bash out the script for Hamlet.

This observation was first made by the astronomer Sir Arthur Eddington in 1929 while describing some features of the second law of thermodynamics. He wrote: "If I let my fingers wander idly over the keys of a typewriter it might happen that my screed made an intelligible sentence. If an army of monkeys were strumming on typewriters they might write all the books in the British Museum. The chance of their doing so is decidedly more favorable than the chance of the molecules returning to one half of the vessel."

Don't be shocked if the improbable occurs from time to time.

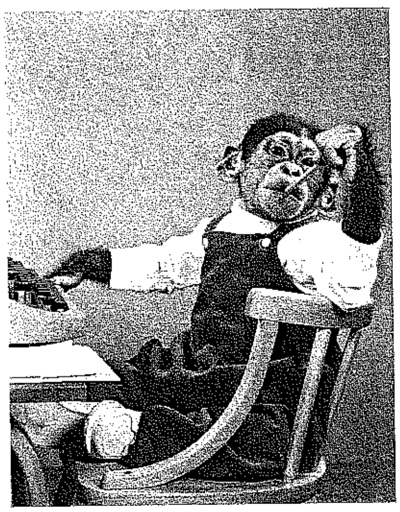
Although *Hamlet* does happen, it does not happen often. Suppose 1,000,000 monkeys randomly typed at standard 48-key computer keyboards, and each typed one character per second. We would expect to wait more than

1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000

years before one of them typed "To be or not to be: that is the question."

There are many literary references to this random fact about monkeys and Shakespeare. One is from Douglas Adams's book *The Hitchhiker's Guide to the Galaxy*: "'Ford!' he said, 'there's an infinite number of monkeys outside who want to talk to us about this script for *Hamlet* they've worked out.'"

Our favorite story on this subject is a 1960s routine from the button-down mind of comedian Bob Newhart. In the routine, a lab technician monitoring infinitely many monkeys typing on typewriters is interviewed on a news program. His words were roughly: "Scientists have claimed that if you have enough monkeys typing randomly for enough time, one of them will eventually produce *Hamlet.* To test this theory, we have brought in a pack of monkeys and have been letting them type away for 72 days now. Let's see how they're doing. [He reaches over and pulls out the paper from one of the typewriters and reads.] 'To be, or not to be: that is the gezortenblatt.' Well, I guess we're not quite there yet—back to the studio."

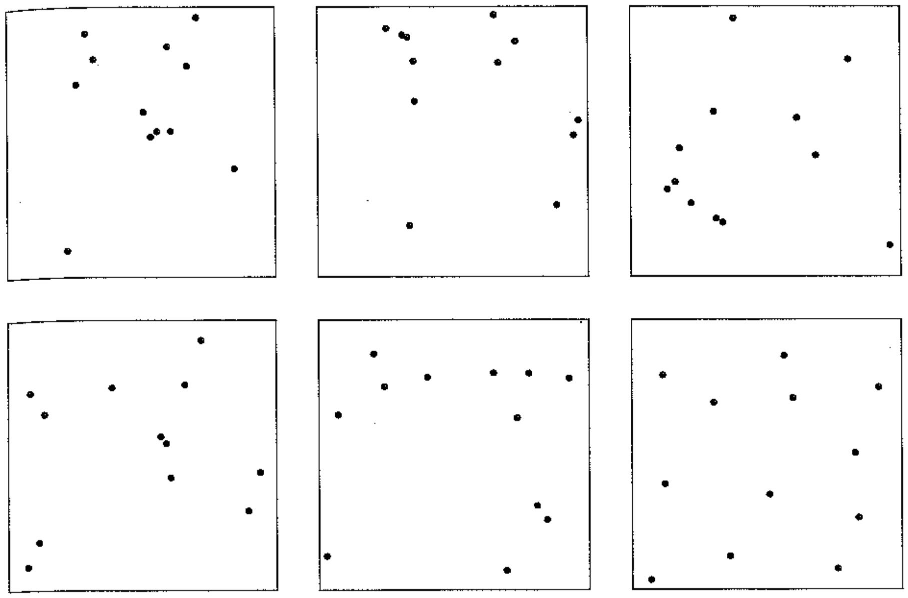


This classic comedy routine illustrates another important point. If we let those monkeys type away, we will not only see *Hamlet,* but also any possible variation of it. Basically, if we have enough monkeys and enough time, we could generate every book ever written—although for this book only two monkeys were required.

This monkey business has more than just mere entertainment value. In 1993, George Marsaglia and Arif Zaman from Florida State University used this monkey fact to detect errors in computer programs that generate random numbers. Their basic strategy was to convert the numbers to letters and, roughly speaking, determine how likely it would be for the random-number generator to type a particular phrase, such as "To be, or not to be." Building a computer program to generate random numbers is a challenging task. However, as we've seen with the monkeys, within the purely random we occasionally will see familiar patterns. As one may expect, we might have to wait a long, long time to stumble across a desired pattern. Thus, if you do attempt the monkey experiment, don't be discouraged if after typing away for 500 trillion quadrillion years, they only produce *Macbeth.*

Random Spots

Randomness often fools us. On page 611 are six pictures, each containing 12 spots. Five of those pictures were drawn randomly, meaning that we used a random process to choose the location of each spot within its square. However, one of the collections of spots was drawn by a person who had not studied randomness whom we asked to draw 12 spots randomly in the square. Can you guess which one was drawn by the person who was trying to be random rather than by an actually random process?



The answer is that the right bottom one was drawn by a person. Why would we guess that configuration was not randomly generated? Notice that in a sense the randomly located spots usually show clusters and areas with no spots at all. The bottom right image, on the other hand, has its spots spread evenly and carefully over the entire rectangle. Often people feel that random distributions should be evenly spread out. Of course, any distribution of spots could result from a random process; however, the randomly generated patterns usually exhibit more clustering (that is, "coincidences") and often have large empty spaces. A naive person simulating randomness tends to want to avoid having large open spaces.

Expect the Unexpected

If seeing random spots before your eyes is a bit disconcerting, then fear not, there is a terrific exercise that can help us retrain our intuition into randomness that only involves coin-flipping. This fantastic flipping experiment is one that you can try for yourself with a couple of friends or family members—who are unfamiliar with the subtleties of randomness. It's really great fun and in terms of randomness, really hits the spot (whoops, another spot before our eyes).

Give your two friends the following charge. You will leave the room and once you do, one of them will flip a coin 200 times and record the heads and tails in order as they occur. Independently (without consulting with the coin-flipper), the other person will pretend to flip a coin 200 times and just write down a random sequence of H's and T's that could have resulted. They are to decide which role each person will take (the real flipper or the fake flipper) but they are not to tell you who's who.

You now leave the room and let the flipping fly. When you are called back into the room, you are handed two sheets of paper each containing a run of 200 H's and T's. Your mission is to identify the genuine flipping sequence and expose the fraudulent flipping sequence. You can try this experiment yourself by looking at the following two sequences of H's and T's. Before reading on, which one do you think was generated by a person and which one records actual coin flips?

H	T	T	H	H	H	H	H	T	H
H	H	H	T	T	H	T	T	T	H
H	T	T	H	H	H	T	T	H	T
T	H	T	T	T	T	H	T	T	H
H	H	T	H	H	H	H	T	T	H
H	H	T	H	T	T	H	H	T	H
T	H	H	T	H	T	T	T	T	T
T	H	H	T	H	T	H	H	T	H
H	T	T	T	T	H	T	T	H	T
T	H	H	T	H	T	H	H	T	H
T	H	T	T	H	H	T	H	H	H
H	T	H	T	H	H	H	T	H	T
H	T	H	T	H	T	H	H	T	H
H	T	H	H	H	T	T	T	H	T
H	H	H	H	T	H	T	H	H	T
T	H	H	T	H	H	T	H	H	H
H	H	T	T	H	T	H	T	H	H
T	H	H	T	T	T	T	T	H	T
T	H	T	T	T	H	H	H	H	T
T	T	T	H	H	T	H	T	H	H

T	T	T	H	H	H	H	T	H	H
T	H	H	H	H	T	H	T	H	T
T	H	T	T	T	T	H	T	T	T
T	H	T	H	H	H	T	T	H	H
H	H	H	H	H	T	H	H	H	T
T	H	T	H	T	H	T	H	T	H
T	H	H	T	T	H	H	H	T	T
H	T	H	H	H	H	T	H	H	T
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T	H	H	H	T	T	H	H	H	T
H	T	H	H	T	H	T	T	H	H
T	T	T	H	H	T	H	T	H	H
H	H	H	H	T	T	T	H	H	T
H	H	H	T	H	T	H	H	T	H
T	T	H	T	T	T	H	T	H	H
T	T	T	H	H	H	T	H	T	H
H	H	T	T	T	H	T	T	T	H

A run of 200 H's and T's

H	T	T	T	H	T	T	H	H	H	T	T	T	H	T	T	H	T	T	T
H	H	T	H	H	H	T	T	H	T	H	H	H	T	H	T	H	T	H	T
T	T	H	H	T	H	T	T	T	H	H	T	T	H	T	T	H	T	H	T
H	T	T	T	H	H	T	T	H	H	T	H	T	T	T	T	H	H	T	H
H	H	H	T	T	T	H	H	T	T	T	T	H	T	H	T	H	H	H	H
T	H	H	H	H	T	H	H	T	H	H	H	H	H	T	T	H	T	H	H
T	H	H	T	H	H	H	T	T	H	T	T	H	T	H	H	T	H	H	H
H	T	T	T	H	T	T	H	H	H	T	H	T	T	T	H	T	T	T	H
H	T	T	T	H	H	H	H	H	T	T	T	H	T	T	H	H	H	H	T
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T	T	H	T	H	H	T	H	H	H	T	H	T	T	T	H	T	H	T	T
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H	H	T	H	H	T	H	H	H	H	T	T	H	T	H	T	H	H	H	H
T	H	H	T	H	H	H	T	H	H	T	T	T	H	T	T	T	H	T	H
T	T	H	H	H	H	T	H	T	H	H	H	H	T	H	T	T	T	H	T
H	T	H	T	H	H	T	T	T	T	H	H	H	T	H	T	T	H	T	T
T	T	H	T	H	H	H	T	T	T	H	T	H	H	H	T	T	H	H	T
T	T	T	H	H	H	T	H	T	H	H	T	T	T	H	H	H	H	T	H
H	H	H	T	H	H	H	T	T	H	T	T	H	H	H	T	T	H	H	T
T	T	H	T	H	H	T	T	H	T	H	H	H	T	H	T	H	T	H	H

Another run of 200 H's and T's

Can you tell which of the preceding is the random one? The answer is that the first one was generated with an actual coin and the second one was created by a person who deliberately attempted to write a random-looking list of H's and T's. How can you tell?

The answer is that we must learn to expect the unexpected. When probabilistically unsophisticated people attempt this challenge of acting randomly, they are generally reluctant to put too many H's in a row or T's in a row. It is natural to think that too many of one letter in a row is somehow unrandom; however, the coin has no such prejudice. Here are the same two sequences with runs of more than five H's in a row highlighted in red and runs of five or more T's highlighted in blue.

H	T	T	H	H	H	H	H	T	H	T	T	T	H	H	H	H	T	H	H
H	H	H	T	T	H	T	T	T	H	T	H	H	H	H	T	H	T	H	T
H	T	T	H	H	H	T	T	H	T	T	H	T	T	T	T	H	T	T	T
T	H	T	T	T	T	H	T	T	H	T	H	T	H	H	H	T	T	H	H
H	H	T	H	H	H	H	T	T	H	H	H	H	H	H	T	H	H	H	T
H	H	T	H	T	T	H	H	T	H	T	H	T	H	T	H	T	H	T	H
T	H	H	T	H	T	T	T	T	T	T	H	H	T	T	H	H	H	T	T
T	H	H	T	H	T	H	H	T	H	H	T	H	H	H	H	T	H	H	T
H	T	T	T	T	H	T	T	H	T	T	H	T	H	H	T	H	T	T	H
T	H	H	T	H	T	H	H	T	H	T	H	T	H	T	H	T	T	H	T
T	H	T	T	H	H	T	H	H	H	T	H	H	T	H	H	T	H	H	T
H	T	H	T	H	H	H	T	H	T	T	T	T	H	T	T	T	T	T	T
H	T	H	T	H	T	H	H	T	H	T	H	H	H	T	T	H	H	H	T
H	T	H	H	H	T	T	T	H	T	H	T	H	H	T	H	T	T	H	H
H	H	H	H	T	H	T	H	H	T	T	T	T	H	H	T	H	T	H	H
T	H	H	T	H	H	T	H	H	H	H	H	H	H	T	T	T	H	H	T
H	H	T	T	H	T	H	T	H	H	H	H	H	T	H	T	H	H	T	H
T	H	H	T	T	T	T	T	H	T	T	H	T	T	T	T	H	T	H	H
T	H	T	T	T	H	H	H	H	T	T	T	T	H	H	H	T	H	T	H
T	T	T	H	H	T	H	T	H	H	H	H	T	T	T	H	T	T	T	H

H	T	T	T	H	T	T	H	H	H	T	T	T	H	T	T	H	T	T	T
H	H	T	H	H	H	T	T	H	T	H	H	H	T	H	T	H	T	H	T
T	T	H	H	T	H	T	T	T	H	H	T	T	H	T	T	H	T	H	T
H	T	T	T	H	H	T	T	H	H	T	H	T	T	T	T	H	H	T	H
H	H	H	T	T	T	H	H	T	T	T	T	H	T	H	T	H	H	H	H
T	H	H	H	H	T	H	H	T	H	H	H	H	H	T	T	H	T	H	H
T	H	H	T	H	H	H	T	T	H	T	T	H	T	H	H	T	H	H	H
H	T	T	T	H	T	T	H	H	H	T	H	T	T	T	H	T	T	T	H
H	T	T	T	H	H	H	H	H	T	T	T	H	T	T	H	H	H	H	T
T	H	T	T	H	H	H	T	T	T	H	T	T	T	H	T	H	T	T	T
T	T	H	T	H	H	T	H	H	H	T	H	T	T	T	H	T	H	T	T
T	H	H	T	H	H	T	H	H	H	H	T	H	T	T	H	H	T	H	T
H	H	T	H	H	T	H	H	H	H	T	T	H	T	H	T	H	H	H	H
T	H	H	T	H	H	H	T	H	H	T	T	T	H	T	T	T	H	T	H
T	T	H	H	H	H	T	H	T	H	H	H	H	T	H	T	T	T	H	T
H	T	H	T	H	H	T	T	T	T	H	H	H	T	H	T	T	H	T	T
T	T	H	T	H	H	H	T	T	T	H	T	H	H	H	T	T	H	H	T
T	T	T	H	H	H	T	H	T	H	H	T	T	T	H	H	H	H	T	H
H	H	H	T	H	H	H	T	T	H	T	T	H	H	H	T	T	H	H	T
T	T	H	T	H	H	T	T	H	T	H	H	H	T	H	T	H	T	H	H

Notice that the coin-generated sequence has many streaks of five or more of the same letter in a row. The human-generated sequence had only two streaks with five in a row, and no streak with six in a row. The reality is that the probability when flipping a coin 200 times of having at least one streak of six or longer is 96%. The probability of having at least one streak of five or longer is 99.9%. So the moral of the story is, yet again, to expect the unexpected—surprise!

History Does Not Matter

Confusion about randomness can cause serious trouble for gamblers. One of the most common mathematical crimes that gamblers commit is thinking that their luck must change. Suppose a gambler is playing a game involving flipping a fair coin in which he wins if the coin comes up heads and loses if the coin comes up tails. Unfortunately, 10 tails have turned up in a row, so he says to himself, "Surely my luck will change; the coin's got to come up heads *this time*. I'll bet my life savings." That kind of reasoning *does* have a transformative effect on the life of our gullible gambler. He arrived at the casino in a small $5,000 Buick, but will travel home in a huge $500,000 bus.

To demonstrate the fact that random events are totally blind to history, we performed the following computer simulation. We simulated flipping a coin 11 times in a row. Then we repeated that simulation 1,024,000 times. In other words, we simulated more than 11 million coin flips. The reason we performed this "11-coin-flips" simulation 1,024,000 times is that with that enormous number of trials, on average the laws of probability imply that we would expect about 1000 of those simulations to have the first 10 flips all be tails. We ran this simulation three times. As a matter of fact, when we actually performed this repeated simulation on a computer for the first time, we found 1010 of those 1,024,000 simulations had their first 10 flips as all tails—very close to what the mathematics predicted.

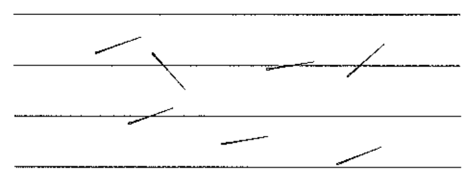
For each of these 1010 instances in which the first 10 of our 11 flips were all tails, we looked at the next flip, the 11th flip. Surely after 10 tails in a row, our luckless gambler would bet that a heads would almost surely appear in the next flip. Answer: no, and our simulation verified it. As you see in the table, in the first run we saw the 11th flip was tails 495 times and heads 515 times—almost equal. We ran the entire simulation two more times. The second time, the first 10 flips were all tails 1033 times, of which the 11th flip was tails 523 times and heads 510 times—again almost equal. In the third simulation, the first 10 flips were all tails on 955 occasions, of which the 11th flip was tails 491 times and heads 464 times—yet again nearly equal. When it comes to randomness, history does not matter, so heads-up!

	Number of Times 10 Tails	Number of Tails for 11th	Number of Heads for 11th
Trial 1	1010	495	515
Trial 2	1033	523	510
Trial 3	955	491	464

Flipped a coin 11 times and repeated 1,024,000 times. Repeated the whole experiment three times.

From Needle Droppings to an Approximation of π

Do random events ever lead to concrete results? Seems unlikely—after all, they're random. Let's consider the following random experiment. Suppose we have a sheet of lined notebook paper and a needle whose length is equal to the distance between consecutive lines on the paper. We now randomly drop the needle onto the paper. We notice that sometimes the needle crosses one of the lines, and sometimes the needle does not cross any line. What is the probability that the needle will cross a line?



Some needles cross lines—others do not.

This question was first raised and then answered by the 18th-century French scientist Georges Louis Leclerc Comte de Buffon (if you drop his name on the paper, it will definitely hit a line). The surprising answer is that the probability of the needle hitting a line can be computed and is exactly equal to $2/\pi$. Using this fact, Buffon was able to give estimates for π by, we kid you not, throwing French bread sticks over his shoulders numerous times onto a tiled floor and counting the number of times the bread sticks crossed the lines between the tiles. Although we are told not to play with food, Buffon's food tossing actually gave birth to an entire realm of mathematics now known as *geometric probability.* Hundreds of years after Buffon tossed his bread sticks, atomic scientists discovered that a similar needle-dropping model seems to accurately predict the chances that a neutron produced by the fission of an atomic nucleus would either be stopped or deflected by another nucleus near it—so, even nature appears to drop needles.

Since the probability of the needle hitting the line is equal to $2/\pi$, we can get a good approximation of π by just dropping a needle onto paper many, many times. Here's how: First drop the needle a good number of times and keep track of the number of line hits and the total number of drops. We know that the relative frequency of line hits, which equals the number of line hits divided by the total number of drops, will be approximately the probability of hitting the line. But the probability of hitting the line is $2/\pi$. Therefore, we can solve for π and see that

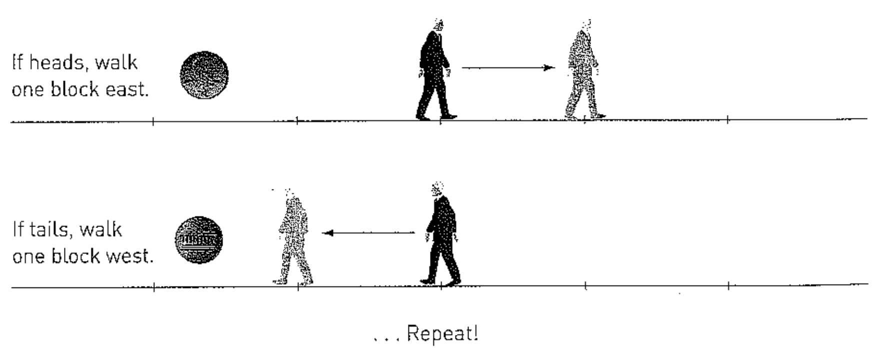
$$\pi = 2 \times \frac{\text{(total number of drops)}}{\text{(the number of line hits)}}.$$

In 1864, by making 1100 drops, the English scientist Fox repeated this experiment and used his findings to give an estimate for π of 3.1419. Today we can visit a number of Web sites and watch computers simulate this experiment with thousands of virtual drops. The occurrence of π in the needle-dropping experiment shows that even randomness has a rich and precise structure.

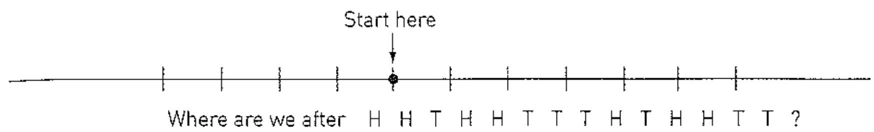
Our next random journey could be an opening scene from the classic 1960s TV series *Mission: Impossible.*

Random Journeys—Mission Impossible?

"Good morning, Mr. Phelps. The road you are standing on runs east and west and goes on forever in both directions. You are holding a penny. Somewhere, although you don't know where, along the side of the road is a small tape recorder that will self-destruct in five seconds after you play the message [thus it has an extremely limited manufacturer's warranty]. Your mission, should you decide to accept it, is to find the tape recorder by walking one block at a time as follows: You flip the penny. If the coin comes up heads, then you walk one block east; if it comes up tails, you walk one block west. You repeat this process indefinitely until you find the recorder. Question: Is this mission possible? As always, if you or any of your I.M. Force is caught or killed, or dies of old age, the secretary will disavow any knowledge of your actions. Good luck, Jim."



The preceding walking scenario is an illustration of a *random walk*—a walk whereby the direction we move at any particular stage is selected at random. In this case we are walking one block at a time, and it is equally likely that we walk east or west. For example, suppose our flipping gave a sequence of H H T H H T T T H T H H T T and we started at the marked spot. Draw what our path would look like below.



Although we have no idea how far away it lies, or even in which direction the poorly made recorder is, the surprising fact is that the probability of finding the recorder in this manner is equal to 1—we will find it with (probabilistic) certainty. Why? If we flipped the coin indefinitely, we should not be shocked to see long, long, long strings of heads. That string would result in a long, long, long journey eastward. This observation illustrates that it is possible and even reasonable for us to migrate far from our starting position (and similarly, eventually return back home). Although this informal observation certainly does not *prove* that the probability is, in fact, equal to 1, it does make the result plausible.

If we walk randomly on a grid in the plane where we move east, west, north, or south at each step, then the probability of finding the hidden recorder remains equal to 1, but we won't try to justify that fact here. In fact, you may not return any too soon. In fact, the following table records 30 simulations of random walks in the plane. In each case, we recorded how many steps were required before the random walk returned to the starting place.

4	36,374,154	28
4	2	2
28	56	>100,000,000
56	80,326	>100,000,000
6	74	>100,000,000
32	16	926
8,072,890	28	118
22	2	>100,000,000
6	2	99,276
4	300,658	2

Notice that in one case the first return did not occur until after more than 36 million steps were taken and in four cases we abandoned the simulation after we had taken 100 million steps without ever returning to our starting point. Nevertheless, with probability 1, we will eventually return to where we start when we are walking in the plane.

Surprisingly, however, if we consider walking randomly in *three* dimensions (we now have wings and can move up and down), then the probability that we will ever return back to where we start is only about 0.699 . . . , and we are not at all certain of finding a hidden tape. Why the change in likelihood? Well, as we add dimensions (and thus new directions to randomly journey), we move in a space with more degrees of freedom. When we have three or more dimensions, it turns out that it is possible for us to get lost in space and never return home.

Random walks actually occur in nature. The path of a liquid or gas molecule is determined by its knocking around and bumping into nearby molecules. This path is an example of a random walk, and such movement is known as *Brownian motion*. The theory of random walks has also been used to study and analyze other phenomena, including the behavior of the stock market.

A Look Back

COINCIDENCES and random behavior do occur, often with predictable frequency. A bit of careful thought reveals that coincidences are not as shocking as they may first appear. One of the most famous illustrations of randomness is the scenario of monkeys randomly typing *Hamlet—Hamlet Happens.* Randomness frequently boggles our intuition. We need to learn to expect the unexpected when dealing with random events. Buffon's needle shows how random behavior can be used to estimate numbers, such as π. The theory of random walks is filled with counterintuitive and surprising outcomes and appears in Brownian motion and the behavior of the stock market.

We can apply the principles of probability to understand coincidences and random behavior. The basic definition of probability and how to compute it allows us to gauge more meaningfully how rare or common a seemingly unlikely event really is. We can estimate the probability of no coincidences occurring. This opposite view helps us to understand how likely coincidences really are.

If an experience or an idea seems surprising or vague, often we do not fully understand it. We can understand it more deeply by looking at the opposite point of view or by analyzing it using familiar principles. Often such analyses not only solve the mystery but also lead to deeper insights into related issues.

Consider all points of view.

•

Apply reason to understand the mysterious or the unknown.

•

Expect the unexpected.

Mindscapes *Invitations to Further Thought*

In this section, Mindscapes marked **(H)** *have hints for solutions at the back of the book. Mindscapes marked* **(ExH)** *have expanded hints for solutions at the back of the book. Mindscapes marked* **(S)** *have solutions.*

I. Developing Ideas

1. **Daily deaths.** About 58 million people die every year. About how many die each day, on average?
2. **Wake-up call.** Suppose you wake up each morning and think of an event that has a one-in-a-thousand chance of happening that day. What are the chances the event you thought of in the morning will *not* happen that day? What are the chances you will not experience your event *du jour* two days in a row? What are the chances you will pass an entire year without experiencing one of these coincidences?
3. **More than 12 monkeys.** What does the Infinite Monkey Theorem assert?
4. **Get shorty.** Who was Georges Louis Leclerc Comte de Buffon? What was his role in the history of probability?
5. **Nothing but heads.** If you flip a fair coin three times, what are the chances you will get all heads? If you get all heads and you flip the coin once more, what is the probability that you will get heads on your fourth flip?

II. Solidifying Ideas

6. **Pick a number.** Pick a number from the following list: 1, 2, 3, 4. Write it down. If each student in a class of 50 were also to pick a number, how many would you guess had selected the same number as you did?

7. **Personal coincidences.** List three coincidences you have experienced in your life.

8. **No way.** It is the last Sunday of spring break and you are flying back to school. You have a connecting flight in Chicago. In the gate area, you see a bunch of friends from school. Are you shocked? Discuss this coincidence.

9. **Enquiring minds.** For a previous year, find the end-of-the-year issue of the *National Enquirer* or a similar fine publication and find all the predictions made by the psychics. How many of them actually happened? Based on your investigation, can psychics predict the future?

10. **Milestones.** Take a look at the obituary sections of two consecutive weeks of either *Time* or *Newsweek* magazine. How many famous people died? Use your answer to make your own estimate of the number of famous people who die in one year.

11. **Unlucky numbers.** Suppose you randomly picked 1000 people from the telephone book. What would you estimate the probability to be that one of them will die within the next year? Justify your estimate.

12. **A bad block (S).** Suppose 1054 people died in Datasville last year. Why must there be a two-week period during which 40 of those people died?

13. **Coin toss test.** Ask two friends to help you repeat the coin toss test. Ask one to toss a fair coin 100 times and record the sequence of H's and T's. Ask the other to create a "random" sequence of 100 H's and T's without tossing a coin or consulting the first friend. (Have them decide who does which task after you leave the room so that you don't know which friend does which task.) Return to the room and see if you can tell which sequence is which.

14. **Deceptive dice (ExH).** You asked three friends to do an experiment while you were out of the room. Two of them each tossed a fair die 40 times and recorded the results. The third friend generated a sequence of 40 digits from 1 to 6 with an effort to make it look like the random results of tossing an actual die. You return to see the results below. Which sequences do you think came from tossing an actual die? Why?

a. 2 2 6 3 5 4 4 4 6 2 3 2 5 2 6 6 2 1 3 3 1 4 3 4 1 1 1 5 4 3 2 5 1 3 3 2 6 5 1 2
b. 5 3 4 1 6 3 4 2 1 6 2 5 3 4 1 6 2 3 3 2 5 4 1 1 6 2 3 5 4 2 5 6 1 4 2 5 6 3 1 3
c. 5 6 2 5 1 6 2 3 1 6 1 4 4 6 3 4 6 2 3 3 3 3 2 6 6 2 5 1 3 6 5 1 6 4 6 2 4 5 3 6

15. **Murphy's Law.** *If something can go wrong, it will.* Given our discussion of randomness, do you agree or disagree with this law? Using examples, try to place it in the context of our discussion of coincidence.

16. **A striking deal.** Get two decks of ordinary playing cards, give one deck to a friend, and have the person arrange it in any order, either random or planned. Bet the person that you will be able to take the other deck and place it in an order so that, if you both turn over the cards one at a time, at least once both cards will come up exactly the same. Shuffle your deck randomly. Now turn one card from each deck over and continue until there is a match. If there is a match, you win; otherwise, you lose. Play the game six or more times and record the results. Given your data, what would you guess is the probability of your winning?

17. **Drop the needle.** Try Buffon's needle experiment by dropping a needle 50 times and recording your results. Use your numbers to give an approximation for π. Go to the Web and try the computer simulation. Record the results and the associated approximations of π.

18. **IBM again (H).** Suppose we try the IBM stock-prediction scheme of sending half (or as close to half as possible) of the group the "up" message and the other half the "down" message, but this time we start with only 600 people. How many weeks could we go until we are down to just two people seeing a perfect track record?

19. **The dart index.** Take a page of stock quotes from a newspaper. Mount it on a piece of cardboard and throw 10 darts randomly at the page. Record the 10 stocks you hit. Now get a newspaper that is exactly six months old and look up the prices of those 10 stocks. Suppose you bought 100 shares of each of those 10 stocks six months ago. How would you do if you sold them all today?

20. **Random walks.** Using a piece of graph paper, a penny, and a dime, embark on a random walk on the grid. Flip both coins. If the penny lands heads up, you move one unit to the right; if it lands tails up, you move one unit to the left. If the dime is heads, you move one unit up, and if it is tails, you move one unit down. Mark your trail on the graph paper as you make 50 flips of the coins.

21. **Random guesses (S).** A multiple-choice test has 100 questions; each question has four possible answers from which to choose. Each question is worth one point, and no points are taken off for choosing an incorrect answer. Someone decides to take the test by selecting answers randomly. What is the probability that a person taking this test gets 100%? Suppose now that the person actually reads the questions, is able to eliminate two of the incorrect choices, and then guesses randomly from the other two choices. What is the probability that the person gets 100%?

22. **Random dates.** There is a room filled with exotic people, any one of whom you would be happy to ask out on a date. Suppose that the probability that any particular person agrees to go on a date with you is 0.5. What is the probability that the first person you ask says yes? What is the probability that the first five people you ask say no?

23. **Random phones (H).** Suppose you roll a 10-sided die with sides marked from 0 to 9. If you continued to roll and recorded the outcomes, what do you think the probability would be that at some point you would see seven digits in a row that make up your telephone number? Explain your reasoning.

24. **Four-ever (ExH).** Suppose you roll a die repeatedly, forever. Is it possible that you would roll only 4's? Is it likely? What is the probability of rolling only 4's?

25. **Pick a number, revisited.** In Mindscape II.6, did you pick 3? Are you impressed? What is the probability that we correctly guessed your answer?

III. Creating New Ideas

26. **Good start (H).** Suppose the monkey is typing using only the 26 letter keys. What is the probability that the monkey will type "cat" right off the bat?

27. **Even moves.** Suppose you embark on a random walk on the real number line. Show that, if you return back to your starting point, you must have flipped the coins an even number of times.

28. **Playing the numbers.** Here is a numbers game. You choose a number from 000 to 999 each morning and compare it to the last three digits of the official attendance figures at the nearest racetrack. What is the probability of guessing the correct number at least once if you make a guess each day for three years? (*Hint:* Consider a related example from this section.)

29. **Random results.** Someone looks at a list of 10,000 numbers, each from 1 to 100, generated by a random-number generator and states that the program must not work correctly because there is a string of 17 numbers starting with the 2713th number that reads 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17. How do you respond to this conclusion?

30. **Monkey names.** Suppose we have a monkey typing on a word processor that has only 26 keys (only letters, no spaces, numbers, punctuation, and so on). The monkey types randomly for a long, long time. What is more likely to be seen: MICHAELSTARBIRD or EDWARDBURGER? Explain your answer.

31. **The streak.** Suppose you flip a fair coin 10 times on two different occasions. One time you see 10 heads, the other time you see H H T H H H T T H T. Is either one of these outcomes more likely than the other? Which one is random? Explain.

32. **Girl, Girl, . . . (S).** A couple has eight children. Suppose that the probability of having a girl is 0.5. What is the probability of producing eight girls? How does that answer compare with the probability of producing boy, girl, boy, girl, boy, girl, boy, girl? Explain. (*Note:* This does happen.)

What an amazing coincidence. . . . All the daughters have the same outfits!

33. **One mistake is okay.** Suppose we try the IBM stock-prediction scheme with only 128 people. Now, however, we are allowed one mistake. That is, if we send a batch of letters out saying that IBM stock will go down next week, and it actually goes up, we can keep sending letters to this group as long as we never make another mistake. After five weeks, how many people will have seen you make at most one wrong prediction?

34. **Picking and matching.** You and a friend individually and secretly pick a number from 1 2 3 4. What is the probability that you both picked 3? What is the probability that you both picked the same number?

35. **Picking and matching.** You and a friend individually and secretly pick a number from 1 2 3 4. What is the probability that you both picked 3? What is the probability that you both picked the same number? Why is this question here? Think about what this section is about.

36. **Dice are different (ExH).** In Mindscape II.14 you probably used ideas similar to those in the analogous text example on coin tosses. Yet how are the two experiments different? What qualities might you look for (or look for the absence of) in a sequence of alleged random die tosses that would not apply in the case of coin tosses? In particular, can you tell which of the sequences below was generated by rolling an actual die and which was created by a person writing down what they think of as random digits (before reading this text)?

a. 2 6 3 5 4 3 2 1 6 6 5 3 2 4 5 3 6 1 1 4
b. 4 3 1 1 6 4 4 5 6 5 1 6 5 1 6 3 3 1 3 2

IV. Further Challenges

37. **Death row (H).** You may have noticed that two pairs of the celebrity deaths we mentioned occurred one day after the other (Brian Keith on June 24 and Jacques Cousteau the next day; Robert Mitchum on July 1 and Jimmy Stewart the next day). Suppose that two celebrities die in one week (Monday through Sunday). What is the probability that they die on the same day? What is the probability that they die one day after the other in that same week?

38. **Striking again.** Consider the striking deal game described in Mindscape II.16. Compute the actual probability that you will win. (*Hint:* It might be easier to first compute the probability that your opponent will win.)

39. **Random returns.** Suppose we take a random walk on an infinitely long street. Show that, with probability 1, we walk past every point on the street *arbitrarily* often. (*Hint:* Once you land on a particular point, imagine that you are starting a random walk from scratch.)

40. **Random natural.** Suppose you have a 10-sided die with sides labeled from 0 to 9. You roll it 50 times and record the digits to create a 50-digit natural number. What is the probability that the digit 9 occurs at least once in your random 50-digit number? What do you conclude about the digits of very large random natural numbers?

41. **Ace of spades.** You randomly shuffle a deck of cards and then look at the first card. If it is the ace of spades, you win; if not, you lose. What is the probability that you will win after 36 tries?

V. In Your Own Words

42. **Personal perspectives.** Write a short essay describing the most interesting or surprising discovery you made in exploring the material in this section. If any material seemed puzzling or even unbelievable, address that as well. Explain why you chose the topics you did. Finally, comment on the aesthetics of the mathematics and ideas in this section.

43. **With a group of folks.** In a small group, discuss the ideas of coincidence and the Infinite Monkey Theorem. After your discussion, write a brief narrative describing these ideas in your own words.

44. **Creative writing.** Write an imaginative story (it can be humorous, dramatic, whatever you like) that involves or evokes the ideas of this section.

45. **Power beyond the mathematics.** Provide several real-life issues—ideally, from your own experience—that some of the strategies of thought presented in this section would effectively approach and resolve.