ABSTRACT SURFACES

DIFFERENTIAL GEOMETRY, SPRING 2015

Central Theme

Explore Riemann's generalization of the concept of a surface to a type of geometric space that **need not be embedded in any ambient space**.

Minimum Requirements

Write a paper exploring the basics of abstract Riemannian surfaces.

- 7-10 pages, in LaTeX, with attention paid to standard English grammar, spelling and usage.
- Give a clear definition of the concept of a *manifold*, but in the case of dimension equal to 2. Pay particular attention to the way that overlap maps work, and think about why they are necessary.
- Work out several examples, including at least these: the sphere, the torus.
- Include images where appropriate.
- Discuss how the geometry on an abstract surface can be defined by starting with the *metric tensor* g_{ij}, which is a generalization of our first fundamental form.

Extensions to Explore

Have you ever really considered non-orientable surfaces?

Resources

Just about any book on differential geometry at a level "higher" than Shifrin will discuss the construction of abstract manifolds. Take a look at do Carmo's *Differential Geometry of Curves and Surfaces*, or Milman and Parker's *Elements of Differential Geometry* for a reasonable start.

If you are interested in Riemann's ground-breaking lecture, see Spivak's *A Comprehensive Introduction to Differential Geometry, Vol. II.*