FUNDAMENTAL GROUPS OF SURFACES

DIFFERENTIAL GEOMETRY, SPRING 2015

Central Theme

Explore some basic algebraic topology by starting with Poincaré's notion of the **fundamental group** of a topological space.

Minimum Requirements

Write a paper exploring the basics of fundamental groups.

- 7-10 pages, in LaTeX, with attention paid to standard English grammar, spelling and usage.
- Give a clear definition of the fundamental group of a surface.
- Compute the fundamental group for some examples, including: a sphere, a torus, a cylinder, a genus 2 surface
- Include images where appropriate.
- Prove that the fundamental group is, in fact, a group by showing that concatentation of continuous paths is a well-defined, associative binary operation. Prove that a continuous function between two spaces will induce a group homomorphism between those two spaces.

Extensions to Explore

There is a tricky theorem called the Seifert-Van Kampen theorem...

Resources

Armstrong's *Basic Topology*, Massey's *Algebraic Topology: An Introduction*, and especially Allen Hatcher's Algebraic Topology available from his web site for free as a pdf.