PROJECT TITLE

DIFFERENTIAL GEOMETRY, SPRING 2015

Central Theme

There is a beautiful connection between the *total curvature* of a closed space curve and the topology of that closed space curve when considered as a knot. The first step in this direction is a theorem due to Fenchel, which relies on the Cauchy-Crofton formula. This was later extended by Fáry and Milnor (independently). There is even some sense as to using total curvature to measure how complicated a knot is!

Minimum Requirements

Write a paper exploring the basics of the relationship between the total curvature of a knot and its topology.

- 7-10 pages, in LaTeX, with attention paid to standard English grammar, spelling and usage.
- Give a clear definition of the total curvature of a space curve.
- Compute several examples of knots of varying complexity.
- Include images where appropriate.
- Prove the theorems of Cauchy-Crofton, Fenchel, and Fáry-Milnor carefully and completely.

Extensions to Explore

Read the original literature by Milnor and Fáry (in translation) and explore how the total curvature of a knot measures the level of knottiness. Knot theory has advanced much since the 1950's, so maybe a bit of poking around on the internet will help settle things. You may run across the term *bridge number*. Find and show us something cool.

Resources

This material is newer than Struik's book! (Well, Fenchel's theorem gets a mention on page 204.) You can find some of the basics here in §1.3 of Shifrin, and the exercises in that section, particularly Proposition 3.2, Theorem 3.3 and Theorem 3.4 and Exercise 11.

Here are three of the original papers, which are short and reasonably accessible:

(1) A translation of Fáry's original paper into English: `https://www.cs.duke.edu/~brittany/research/fary.pdf`
(2) Milnor's first paper on the subject (jstor link should be available on campus) `http://www.jstor.org/stable/1969467`
(3) Milnor's second paper, exploring the deeper question `http://www.mscand.dk/article/viewFile/10387/8408`