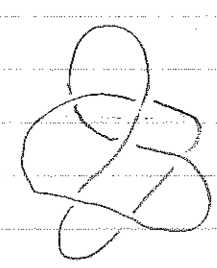
David Tarrow

1. Basic foundation of knots
   a. How to represent them
      i. Projections
      ii. Diagrams
   b. What we can do with projections
      i. Mirrors (chiral/achiral)
      ii Orientation (reverse, invertibility)
      iii Composition
2. Equivalence
   a. Δ-equivalence
      i. Closed polygonal curves
      ii. Elementary moves
   b. R-equivalence
      i. Smooth curves
      ii. R-moves and planar isotopies
3. Classification
   a. Families
      i. Twist knots
      ii. Pretzel links
      iii. Torus links
   b. Distinctness of knot families

Questions:
1. Explain the process of knot composition. Can two knots be composed in more than one way? Why or why not?
2. Show that the following knot is alternating using R-moves:
   Label each move



Definitions

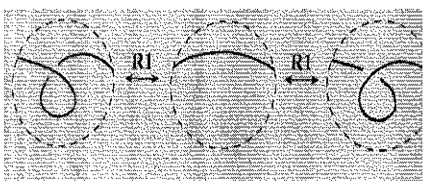
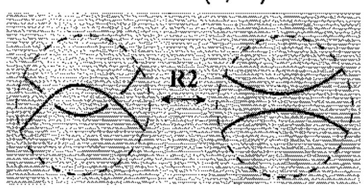
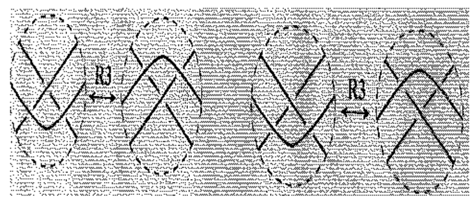
L - Link, D - Diagram of L.

Definition (section, page)

Chapter 1

Crossing Number (2, 12) The minimum number of crossings necessary to make a diagram of L.	**Unknotting Number** (2, 14) The minimum number of crossing to change on L to make the unknot	**Mirror image** (5, 17) Denoted D^m, obtained by changing all the crossings of D.
Chiral/A(mpi)chiral (5, 17) L is chiral if it is not equivalent to its mirror image, and achiral if it is.	**Composition/composite** (6, 18) The composition of two or more non-trivial knots is made by removing an arc of each knot and attaching the knots via the ends. The result is a composite knot.	**Factor knots/Prime knots** (6, 19) The knots non-trivial knots forming a composite knot. A knot with exactly one factor is prime.

Chapter 2

Elementary move (1, 28) Replacing the line segment(s) between adjacent points on L by the line (segment)s of an elementary triangle.	**Delta/R equivalence** (1, 28) (2,37) Two diagrams are delta equivalent if they differ by a sequence of links obtained from applying planar isotopies. Add R-moves for R equivalence.	**Planar isotopy** (2, 31) Elementary triangles coinciding with **(i)** no / **(ii)** exactly one / **(iii)** exactly two non-adjacent lines.
	**R1 - R3** (3,39) 	

Chapter 3

Twist knot (1,46) Denoted by T_n. Obtained by twisting parallel strands together and hooking the ends together.	**Pretzel link** (2,47) Denoted by $P_{p,q,r}$. From the bottom, twist clockwise for negative values and counter-clockwise for positive values	**Torus link** (3,49) Denoted by $T_{p,q}$ where p is the number of horizontal strands and q is the number of crossings. For q<0, inner->outer. For q>0, outer->inner.

Lemma 2.2.6-2.2.7

Theorem 2.3.1

Number of components in a pretzel knot

Proposition 2.2.14

given K, L
show K#L is
unknot
(impossible)

Number of components in a torus knot

Treasure Divis

Knot Theory

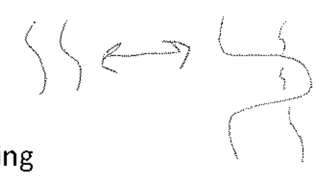
Things We Have Studied Before Test 1

- What is a link/knot (components)
  - Knot – a circle twisted and "knotted" in space — a simple closed polygonal curve in $\mathbb{R}^3$
  - Link – 1 or more interconnected knots
- Knot diagram → Regular projection (add crossing info)
- Unknotting number and crossing number
- Alternating
- Orientation (arrow)
- Chiral
  - When a knot is inequivalent to its mirror image
- Reverse
  - The other orientation
- Invertible
  - Changing the knot to get the reverse
- Composition
  - Prime
    - A knot is called "prime" when it cannot be realized as the composite of two non-trivial knots (no unknot)
  - Depends on orientation
- Equivalence — Δ-equivalence — R-equivalence

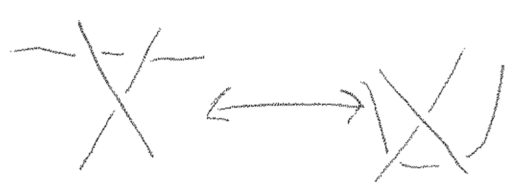
  same
- Isotopies
- R-1, R-2, R-3 moves
- Families of Links
  - Twist Knots
    - Invertible
    - T1 is trefoil
  - Pretzel Links
    - How many components
      - All odd = 1 component
      - 2 odd = 1 component
      - 1 odd = 2 components
      - All even = 3 components
    - If all 3 have same sign, then the pretzel is alternating
    - Pretzel links can be twist knots
    - Can rearrange the 3 segments into any order
      - Rotate them by 1

R-1

R-2



R-3



            - Can flip all upside down and they are still the same (reverses order)
    - Torus Links
        - First number is number of strands, second number is number of twists.
        - How many components
            - Greatest common divisor of two numbers
        - Can be a pretzel link
        - Can be a twist knot
            - T2,3 is a trefoil

Potential Exam Questions

1. Give us a Torus link in the form of Tp,q and ask us to a)draw it b)determine how many components it has and why we think that and c)state whether it is a knot.
2. Given a picture of a knot (of your choice based on the time allotted and length of the exam), find the unknotting number.
3. You could always ask for definitions, since you normally do that. I would suggest the definition of a knot, since that is kind of the foundation of this class.

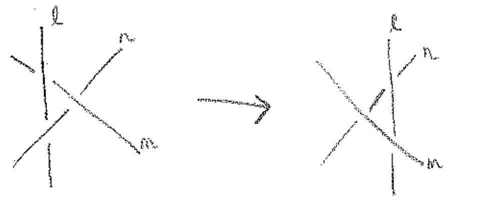
Seth Howard

Outline

- Explored common questions and basic properties of knots
- Explored ways of representing knots
- Defined knots and links
- Defined two concepts of equality of knots and links
- Sketched the outline of a proof that these two concepts of equality are the equal
- Discussed examples of classes of knots
- Informally proved a number of minor results about some of these classes of knots.

Possible test questions

1. Classify the move in the diagram.



a) R1
b) R2
c) R3
d) Isotopie
e) Illegal move

2. Find a series of Delta moves that demonstrates that the two diagrams are equivalent. (where the box represents some general knotted structure).

Denya Christoffer

★ week one : 01/09 - 01/13

- explored playing with knots found out how to draw a correct diagram of a knot and links without leaving any detail out.
- learned various definitions and new concepts and orientation.

★ week two : 01/18 - 01/20

- learned about achiral & chiral
- through an activity we found out that there aren't many ways to have useful notations for knots.

★ week three : 01/23 - 01/27

- learned to manipulate link diagrams by elementary triangles and moves.
- learned Δ-equivalence and practiced changing links to see if they are the same links by smaller elementary moves, R-equivalence.
- learned of planar isotopics, R1, R2 and R3 moves
- Defined the definition of a knot - a simple, closed polygonal curve in space.
- We found that by making a point we can make an elementary triangle.
- learned of lemma 2.2.6 and used it to prove lemma 2.2.7

★ week four : 01/30 - 02/03

- if a multiple point is not involved and there's only two segments in each sub-triangles then we can make an elementary triangle from just planar isotopies, R1 and R2 moves.

★ week five : 02/06 - 02/10

- Introduced to different families of knots, twist, pretzles, and torus links.
- The family of torus links and family of twist knots are not distinct.
- For all pretzle knots,

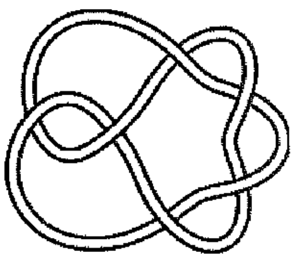
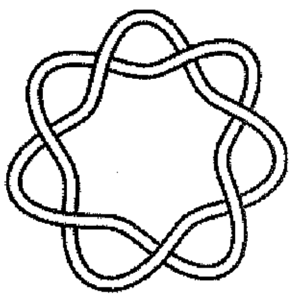
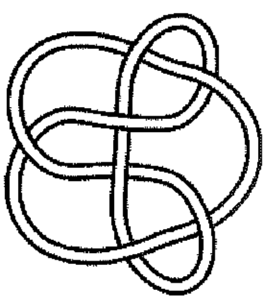
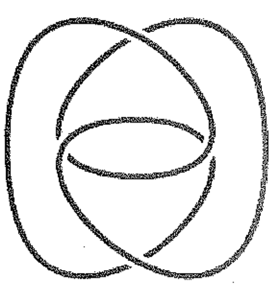
$abc = bca = cab$

Questions for knot theory exam.

- Chapter 1
    - Projections, Diagrams, and equivalence
        - Differences between diagrams and projections
        - How can we tell when knots are equivalent.
    - Crossing and Unknotting numbers
        - Determine crossing numbers of a knot.
        - What is an unknotting number?
    - Alternating Knots
        - When is a knot alternating?
        - The Unknot is alternating?
    - Games
    - mirrors , orientation and inverses
        - What is the mirror image of a knot?
        - When is a knot chiral?
        - Orientated diagram does it matter for a particular knot?
    - Knot composition and prime knots
        - When is knot a composite knot
        - When is a knot considered prime
    - Knot notation
        - Useful for describing knots to others when pictures are not an option.
    - Questions in knot theory
        - How might one determine if any 2 given knots are equivalent.
        - How can we prove that 2 knots are not equivalent
        - How can we tell if a knot is prime
        - How can we determine the unknotting number
- Chapter 2
    - Polygonal curves and Delta equivalence
        - Knots as closed polygonal curves
        - Delta moves with triangles.
        - Delta equivalent if set of delta moves
        - Elementary moves.
    - Diagram equivalence via R-moves
        - Planar isotopies
        - Subdividing a triangle
        - Reidemeister moves
            - 3 types
    - The equivalence of delta and r equivalence
        - If a set of knots are delta equivalent then they are r equivalent as well.
        - Transform a knot from one to another
        - ~~Dope~~ knot that is all knots 6 crossing or fewers.
          Rad

- Nonequivalence and Invariance
- Families of links and braids
  - Twist knots
  - Pretzel links
  - Torus links

3. Determine the following for each knot below
    a. The unknotting number
    b. The crossing number
    c. Is the knot alternating?

Knot	Unknotting number	Crossing number	Alternating
			
			
			
			

4. Using planar isotopies and R moves how that P 5,-1,-1, is equivalent to 6-1

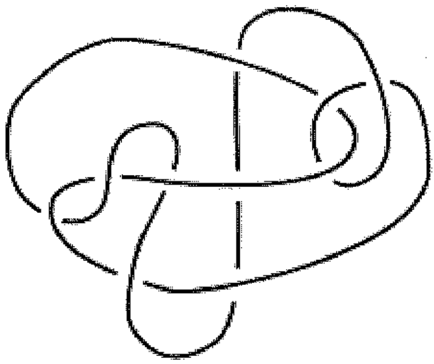
5. Using elementary moves, demonstrate that figure 1 can be transformed into figure 2.

5. Why is it best to use a link diagram instead of a link projection?

6. Show that the following knot is or is not chiral.

Questions

1. Show that via a series of R-Moves and planar isotopies that the following knot can be transformed into the unknot.



2. For the link T 4,10 do the following
    a. Draw T 4,10
    b. Determine the number of components in the link. Is it a knot?