KNOT THEORY: EXAM ONE

MATH 4159/5159, SPRING 2017

Instructions: Please write your responses on the blank paper provided. Put your name on each sheet of paper you use, and label your work clearly with the task numbers.

Task 1. What is our technical definition of the term *knot*? What are the important parts of this definition, and why are they there?

Task 2. What does it mean for a knot to be *composite*? Draw an example.

Task 3. Describe R-equivalence of knot diagrams. Include in your discussion a description of what R-equivalence is and why it is important to us.

Task 4. Draw two knot diagrams which are different, but represent the same knot. How do you know they are the same?

Task 5. For each of the two links described below, (a) draw the link, and (b) say how many components it has.
   (1) The pretzel link $P_{-2,4,3}$.
   (2) The torus link $T_{4,8}$.

Task 6. Draw a knot projection of a non-trivial knot which has 5 crossings, so that one crossing change is enough to turn the knot into the unknot. Describe how you know your example works. *Bonus:* Can you find an infinite family of such knots, including one with m crossings for each natural number $m \geq 3$?
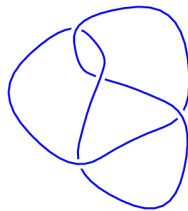

FIGURE 1. The Figure Eight Knot, 4_1.

Task 7. Show that the knot in Figure 1 is *achiral*. *Bonus:* Is it invertible?