Planar diagram codes and Reidemeister moves of type 1

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Identifying that a knot can be simplified by a Reidemeister move of type 1 based on its planar diagram code

A knot is said to be "simplifiable" by a Reidemeister move of type 1 if a given crossing of the knot can be eliminated by un-twisting the crossing without creating any new crossings.

Claim. The planar diagram (PD) code tuple corresponding to a crossing of a knot indicates that the knot can be simplified by a Reidemeister move of type 1 if and only if the tuple contains at most three unique values.

Proof. We begin by showing that if a knot can be simplified by a Reidemeister move of type 1, then the PD code tuple representing the crossing to be simplified contains at most three unique values.

Consider all possible diagrams of a knot segment which can be simplified by a Reidemeister move of type 1. Depending on the orientation of the knot and whether we go over or under at the crossing and their corresponding PD codes there are four diagrams to consider, which are shown in Figure 1.

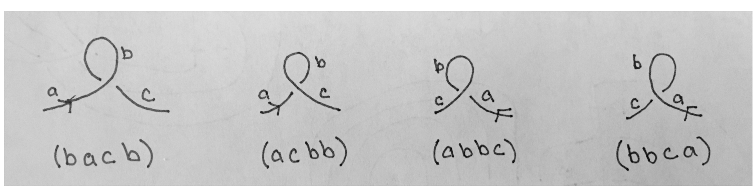

Figure 1: Knot segments which can be simplified by a Reidemeister move of type 1 and their corresponding PD code tuples

Note that for each of these knot segments, the corresponding PD code tuple includes the value of the loop segment, b, twice.

Hence if a knot can be simplified by a Reidemeister move of type 1, then the PD code tuple representing the crossing to be simplified contains at most three unique values.

We will finish by showing that every PD code tuple that has at most three unique values corresponds to a knot segment which can be simplified by a Reidemeister move of type 1.

Let us consider the diagrams that correspond to PD codes which contain at most three unique values, $(a,b,c,b), (a,c,b,b), (b,b,a,c), (b,a,c,b), (a,b,b,c)$ and (b,a,b,c), with $b \neq a, c$.

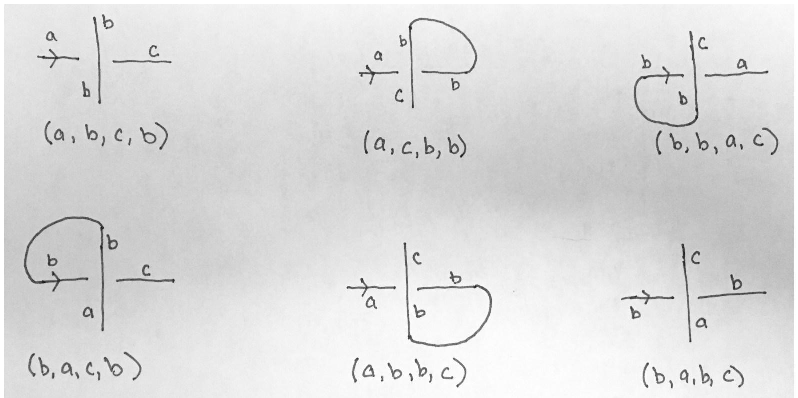

Figure 2: Diagrams of PD codes with at most 3 unique values

Notice in Figure 2 that for the tuples (a,b,c,b) and (b,a,b,c) we cannot connect the two segments labeled b without crossing over or under either segment a or segment c. This indicates that these two tuples are not valid PD code tuples. All of the other diagrams can be simplified by an un-twisting motion, which is a Reidemeister move of type 1.

Hence every PD code tuple with at most three unique entires indicates that the corresponding segment of the knot can be simplified by a Reidemeister move of type 1.

□

How the planar diagram code of a knot changes when the knot is simplified by a Reidemeister move of type 1

Suppose that (a,b,c,d) is a PD code tuple which indicates that a knot can be simplified by a Reidemeister move of type 1. Without loss of generality, suppose that b is the value in the tuple which is not unique. To adjust the PD code of the knot by performing a Reidemeister move of type 1 by un-twisting the crossing (a,b,c,d):

1. Remove the tuple (a,b,c,d) from the PD code.

2. Apply the function f to each element of the remaining tuples, where f is defined by

$$f(x) = \begin{cases} x & x < b \\ x-2 & x > b \end{cases}$$