Airy Differential Equation

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I chose the Airy differential equation,

$$\frac{d^2y}{dx^2} - xy = 0 \tag{1}$$

for any real number x.

In analyzing this differential equation, we notice that we only take a derivative with respect to one variable, so the equation is in fact an ordinary differential equation. Furthermore, we have to take a second derivative, so this is a second order differential equation. This is also a linear differential equation, since we do not take powers of any derivatives (or do anything else weird to them).

One reason this differential equation is interesting is because it can be solved. There are actually two linearly independent solutions to (1), denoted $\mathrm{Ai}(x)$ and $\mathrm{Bi}(x)$, where

$$Ai(x) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt,$$

and

$$Bi(x) = \frac{1}{\pi}\int_0^\infty \left[e^{-\frac{t^3}{3}+xt} + \sin\left(\frac{t^3}{3} + xt\right)\right] dt.$$

$\mathrm{Ai}(x)$ is referred to as the Airy function, and $\mathrm{Bi}(x)$ is referred to as the Airy function *of the second kind.* $\mathrm{Ai}(x)$ and $\mathrm{Bi}(x)$ have the same amplitude of oscillation, but their phases differ by $\pi/2$.

A less superficial reason why the Airy differential equation is interesting is that it has several applications. For example, the Airy functions can be used to describe the pattern resulting from diffraction and interference produced by a light source. As I understand it, this means that if a light source shines on a surface, the Airy functions can be used to describe the pattern the light makes on the surface, taking into account environmental disturbances. This would make the Airy equation and functions useful in a field like astronomy, where it is helpful to understand how light from a star might behave.

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Differential Equations

Dr. Hitchman

The differential equation I have chosen is one that I have seen before in some previous Physics courses, but I still find interesting. The equation is known as Hooke's law which describes simple harmonic motion. An example of simple harmonic motion could be an object with mass attached to a spring with an initial displacement to create a force on the object. Once released, the object undergoes simple harmonic motion. Hooke's law for simple harmonic motion has the form

$$m\frac{d^2x(t)}{dt^2} = -kx(t). \tag{1}$$

In equation 1, m is the mass of the object, k is the spring constant, x(t) is the function that depicts the motion of the object as time progresses, and $\frac{d^2x(t)}{dt^2}$ is the second derivative of the position function with respect to time (the acceleration of the object attached to the spring).

I find this differential equation interesting because it can be applied to a simple problem, like a single mass on a single spring, but it can also be applied to a more complicated problems. I also think it is interesting because the general solution to the differential equation is

$$x(t) = c_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}t\right) \tag{2}$$

where c_1 and c_2 are constants. You can see that the solution is sinusoidal which is expected if you have ever hung a mass from a spring. This is the solution to an ideal spring system. This means that there are no other forces acting on the object. Things like gravity and air resistance are ignored. This solution also only works if the spring is an ideal spring. This means there are no flaws in the spring and the displacement linearly relates to the force on the object.

I think equation 1 is also interesting because it can be rearranged to the form

$$m\frac{d^2x(t)}{dt^2} + kx(t) = 0. \tag{3}$$

Equation 1 and 3 also contain the position function as well as its second derivative. It is missing the first derivative which could also lead to some interesting characteristics (admittedly, I have probably known what these characteristics mean, but I can definitely use a refresher). It would also be interesting to see how variations in the mass or the spring constant would alter the c constants in the solution.

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Differential Equations
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$$\frac{dm}{dr} = 4\pi r^2 \rho(r)/c^2$$

And

$$\frac{dP}{dr} = -\frac{Gm(r)}{r^2}\rho(r)/c^2$$

These ordinary differential equations yield the total mass of a neutron star, m, and the pressure of the star, P, as a function of ρ, where ρ is the mass-energy density of the star.

This ODE is interesting to me personally because I've always thought astronomy was very interesting. It's useful for Astronomers to determine the total mass of a neutron star, and when coupled with the ODE for the pressure of the star, they can learn a lot about the star, and use this information to determine things about the distance the star is from us, its luminosity, and perhaps properties of the star that died to give rise to it.

The symbols represent physical quantities about the universe and about the individual star itself; r is the radius of the star, ρ is the mass-energy density of the star, G is the universal gravitational constant, and c is the speed of light. ρ is the parameter for both these equations, and if changed, would change the output of each. Presumably, were ρ to grow, the total mass of the star and the pressure at any given point within it would also grow.

These equations can be solved numerically, and have the initial boundary conditions which are dictated by the mass of the star being zero at its center, m (r = 0) = 0, and that the pressure at the surface of the star must also be zero. These are non-linear first order ordinary differential equations.

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Example Differential Equation

The barometric equation describes how the pressure of a gas P changes with height z as a function of pressure:

$$\frac{dP}{dz} = -\frac{mg}{kT}P \ .$$

This first order linear ordinary differential equation applies to the atmosphere near the surface of the Earth where m is the average mass of air molecules, g is the acceleration due to gravity, k is Boltzmann's constant, and T is temperature.

The parameters in the equation (m and T) could be varied to scale up or scale down the rate at which the pressure changes with height. For example, for higher temperatures, air pressure decreases less rapidly as you increase in altitude. For a more dense atmosphere (larger values of m), the air pressure decreases more rapidly for the same pressures as you increase in altitude.

This equation can be "experienced" in nature especially on a cold day when driving up or down large hills or when taking off or landing in an airplane. You can feel the pressure change when the air pressure builds up inside or outside your ear drum. If the pressure builds up enough, you may hear a pop.

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Differential Equations in Weather

Ever since I was a young child, weather has always fascinated me. While today I am a mathematics major, sometimes I love to look up some new things that is happening in weather. So, when trying to find a differential equation for something I care about, I knew I would look to see if there are any such equations in meteorology. In fact, I found that there is something called Numerical Weather Prediction (or NWP for short). NWP takes current weather data and runs that data through computer models in order to predict the weather in the future[1]. One part of the weather is moisture. Moisture would be interesting to know about because you will be able to predict when it would rain or not. When the air is warm and moist, if the air is cooled, the moisture in the air condenses to liquid, which causes rain. One equation that has been used to predict this is:

$$\frac{\partial q}{\partial t} = -\overline{\mathrm{U}}\frac{\partial q}{\partial x}. \tag{1}$$

In this equation, q represents the moisture gradient across land west to east, t represents time, and $\overline{\mathrm{U}}$ represents the average wind between the west and the east. A more conceptual version of this equation is:

$$q^{forecast} = q^{now} - \overline{\mathrm{U}}\frac{\Delta t}{2\Delta x}(q_{east} - q_{west})^{now}. \tag{2}$$

In this equation, q^{now} represents the current moisture value when the forecast was taken, $q_{east} - q_{west}$ represent the moisture gradient across land west to east, $\overline{\mathrm{U}}$ represents the average wind between q_{east} and q_{west}, Δt represents the change in time, and Δx represents the distance between two land points.[2]

The two defining parameters in this equation is moisture and the average wind speed. If you varied moisture, it would change your forecast for the moisture significantly. For example, in equation 2, if your current moisture value is a large number and the product on the right side of the subtraction is small number, then a higher moisture content would be forecasted. Though if it was switched, then a

[1] I found this information here: https://www.ncdc.noaa.gov/data-access/model-data/model-datasets/numerical-weather-prediction
[2] I found the equations here: http://rams.atmos.colostate.edu/at540/fall03/fall03Pt7.pdf. This PowerPoint also has a fantastic visual of how the equations are put in action.

lower moisture content would be forecasted. For the average wind speed, this can determine whether or not the product on the right side of equation 2 is a high number or a small number, which in turn will again determine what forecast will be presented for the moisture content for future dates. Although this is an ordinary differential equations class, many equations in meteorology have multiple different variables that need to be considered, so this type of equation is a partial differential equation. It is exciting to know that many different differential equations are being used in order to forecast and model weather patterns and predictions for the future, and just this example proves just how useful modeling would be so that weather can be accurately predicted.

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Differential Equations

Dr. Hitchman

August 24, 2016

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta) \longrightarrow \frac{d^2\theta}{dt^2} + \frac{g}{l}\sin(\theta) = 0$$

Being a physics major, I chose an equation related to physics for this assignment. The equation above represents the motion of a simple pendulum, where drag is ignored. Pendulums are used in a variety of ways, for example, in grandfather clocks, metronomes, or to show the Earth rotates, to name a few. Since they have multiple uses, it is important to understand how they function. However, most physics classes start out with the simple pendulum equation. It comes from setting the forces acting on the pendulum equal to one another and solving for angular acceleration. Since this equation ignores drag, I find it interesting because the mass of the object does not affect the motion of the pendulum. If θ is a small angle, then $\sin(\theta) = \theta$, therefore, this equation is homogenous. On the other hand, if drag is not ignored, this equation will be non-homogenous for any θ since drag is considered to be a forcing term.

In this equation g represents gravitational force, l is the length between the pivot point and the mass, θ is the angle of the pendulum, and $\frac{d^2\theta}{dt^2}$ is the angular acceleration. For small θ, the parameters are constant. If the parameters are varied, then this equation will be a non-homogeneous equation since $\frac{g}{l}$ will be a function of time. If this occurs, the angular acceleration would increase as the pendulum swings. This second order differential equation can be both linear and non-linear. It is linear when θ is a small angle because the coefficients remain constant, since $\sin(\theta) = \theta$. However, for large θ this equation is considered to be non-linear since the direction of the acceleration is dependent on the angle.

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Differential Equations

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Rate Law

Since my majors are in the chemistry field, I chose a very basic differential equation from chemistry: the rate law. The rate law of a chemical reaction shows how the amount of each reactant changes over time. Depending on the number of reactants and the proportion in which they are used to make products, the rate law has an order (typically zero, one, two, or three). The order of the rate law helps you to work out the equation for the system. A zero order rate law (a basic reaction with only one reactant) looks like: $[A]=[A]_0 -kt$. This is the "integrated rate law". [A] is the concentration at a given time t. $[A]_0$ is the initial concentration of the reactant. k is a constant specific to the system. $-kt$, as a whole, is the change in concentration from time zero to time t. (note, the "differential rate law" is $-d[A]/dt = k$)

This equation is of interest to me, because it is fairly basic concept within chemistry that I have already learned to use and manipulate. I am sure as I learn more in the chemistry field, I will learn more applications and uses for the rate law, as well as more of the math behind it.

Seeing as I am unsure of what is meant by parameters in this case. I don't really know how to answer part of this assignment. If parameter is meant to mean like constraints on the values of the variables, then t must be greater than or equal to zero, $[A]_0$ must be greater than zero, and [A] cannot go below zero.

This differential equation is an ordinary differential equation, because it has one independent variable to differentiate by. I also think it would be considered linear. (in "integrated rate law" form, it is a linear equation.)

Joseph Kosmicki

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section 01

Since I'm a physics major I decided to look for a common differential equation used in physics. The equation I chose is $\frac{dQ}{dt} = kA\frac{T_H - T_C}{L}$ and it is used to model conductive heat transfer. There are many real world heat transfer problems that this differential equation can be used to solve but I took a MIG welding class in high school and something that you can run into fairly quickly when welding is welding distortion. Welding distortion is the phenomenon where a weldment, or a welded assembly, warps. This can be a result of a lack of material around the weld for heat to transfer to. However, if there is more material for the heat to transfer to there is a smaller chance welding distortion will occur. It may be possible to use the differential equation I've chosen to determine an ideal length away from the weld and cross section of the metal necessary for the weld to cool to a certain temperature in a certain time, which may prevent welding distortion in that weldment.

In the equation for conductive heat transfer, $\frac{dQ}{dt} = kA\frac{T_H - T_C}{L}$, the variables are Q, t, T_H, and T_C. The variable Q is the amount of heat transferred from the position of greater temperature to the position of lesser temperature, the variable t is the time over which this heat transfer occurs, the variable T_H is the temperature at the position having a greater temperature, and the variable T_C is the temperature at the position having a lesser temperature. The parameters in this equation are A and L. A is the cross sectional area of the material the heat is flowing through and L is the length between the two points where the temperature is being measured. The term k is a constant that has a different value for different materials and it is called the thermal conductivity and so it also is a parameter for this equation.

An increase in the values of k, the thermal conductivity of the material the heat is flowing through, and A, the cross sectional area of the material, will result in an increase in the rate at which heat is transferred through the material, $\frac{dQ}{dt}$. Likewise, a decrease in the value of these two parameters will result in a decrease in the rate at which heat is transferred through the material, $\frac{dQ}{dt}$. An increase in the value of L, the length between the two points where temperature measurements are taken, will result in an decrease in the rate heat is transferred through the material, $\frac{dQ}{dt}$, and a decrease in the value of L would result in a decrease in $\frac{dQ}{dt}$.

The conductive heat transfer differential equation $\frac{dQ}{dt} = kA\frac{T_H - T_C}{L}$ is an ordinary differential equation since no partial derivatives are taken. It is a first order differential equation as the rate at which heat is transferred through the material, $\frac{dQ}{dt}$, is a first order derivative. It is a separable differential equation because dQ could be written as dT since Q, the amount of heat transferred, is analogous to a difference in temperature and so we could move all of the temperature variables to one side of the equation and differentiate the other side with respect to t, time. This equation is also linear as all of the terms are functions of time only.

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