HOMEWORK ON SEPARABLE AND HOMOGENEOUS EQUATIONS

MATH 3425, FALL 2015

Do these for Friday, September 11.

Task 1. Verify that each of the equations below is homogeneous. Solve the equations.

(1)

$$(x^2 - 2y^2) \, dx + xy \, dy = 0$$

(2)

$$x^2y' - 3xy - 2y^2 = 0$$

(3)

$$x^2y' = 3(x^2 + y^2)\arctan\left(\frac{y}{x}\right) + xy$$

(4)

$$x\sin\left(\frac{y}{x}\right)\frac{dy}{dx} = y\sin\left(\frac{y}{x}\right) + x$$

(5)

$$xy' = \sqrt{x^2 + y^2}$$

Task 2. By making a substitution of the style $z = y/x^n$ or $y = zx^n$ and choosing a convenient value of n, show that the following odes can be transformed into equations with separable variables and thereby solve them:

(1)

$$\frac{dy}{dx} = \frac{1 - xy^2}{2x^2y}$$

(2)

$$\frac{dy}{dx} = \frac{2 + 3xy^2}{4x^2y}$$

(3)

$$\frac{dy}{dx} = \frac{y - xy^2}{x + x^2y}$$