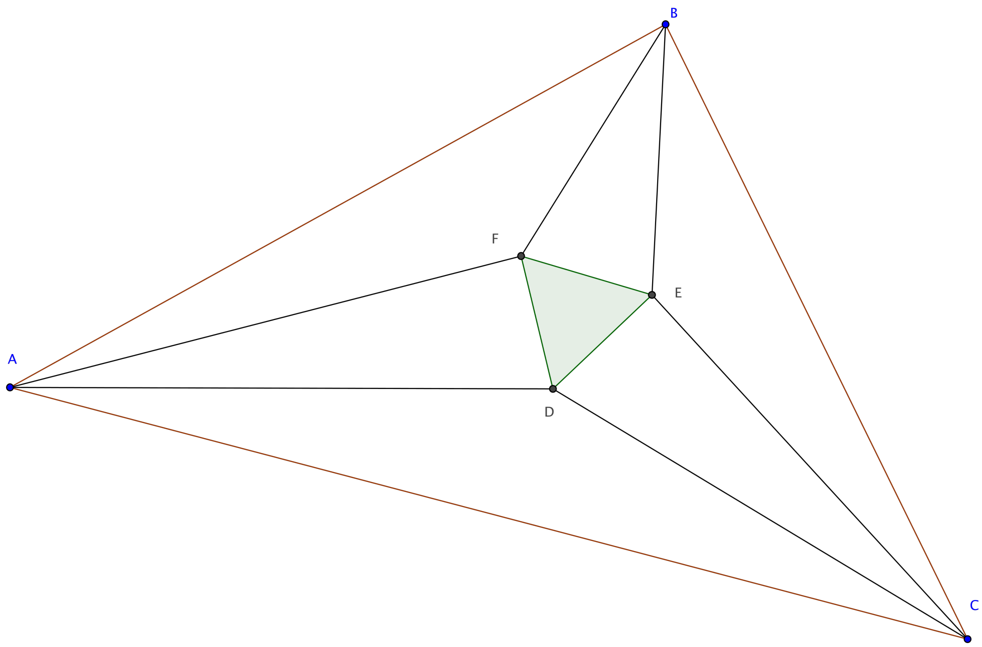
Transactions
in
Euclidean Geometry



Volume 2017F Issue # 2

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Construction of a Rhombus

Micah Otterbein

September 30, 2017

Communicated by: Cameron Amos.

This is one way to construction a rhombus. This is a very rigid construction.

Theorem 1.4. Given a segment AB, we can find a compass and straightedge construction of a rhombus ABCD.

Proof. Given segment AB. Let AB be a finite straight line. By Postulate 3, construct the circle AB and the circle BA to get point D. By Postulate 1, construct the lines AD and BD. By Proposition 1 we know that ABD is an equilateral triangle, therefore AB is congruent to both AD and BD. Then, by Postulate 3, construct circle DB and circle BD to get point C. By Postulate 1, construct the lines DC and BC. By Proposition 1 we know that BCD is an equilateral triangle, therefore BD is congruent to both DC and BC. Since AB is congruent to BD, then AB is congruent to BC, CD, and AD. Therefore ABCD is a Rhombus.

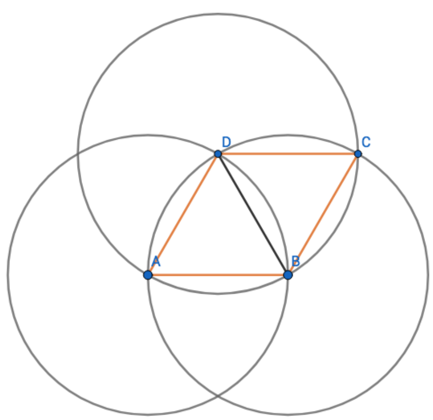

Figure 1: Rhombus ABCD

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Kite Construction

Emily Carstens

September 30, 2017

Communicated by: Katherine Bertacini.

Theorem 2.3. A general construction of a kite.

Proof. First, we will construct two circles that will be important for our kite constructions.

1. Let A, B and C be points, such that point C is not collinear to points A and B.
2. Construct circle AC using Euclid's Postulate I.3.
3. Construct circle BC using Euclid's Postulate I.3.

Notice point C is a point of intersection between circle AC and circle BC. There exists a point E in the interior of both circle AC and circle BC. There exists a point F in the interior of circle AC and on the exterior of circle BC. Thus by the circle-circle intersection property, circle AC and BC meet at two points. Let point D be the second point of intersection. Construct segment AC, segment BC, segment BD and segment AD by Euclid's Postulate I.1.

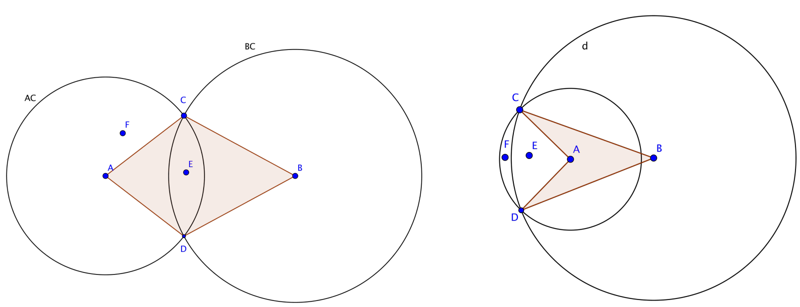

Figure 1: This is a picture of the two types of kites ACBD.

Segment AC is congruent to segment AD because they are both the radius of circle AC. Segment AC is adjacent to segment AD because they share the same vertex, angle DAC. Segment BC is congruent to segment BD because they are both the radius of circle BC. Segment BC is adjacent to segment BD because they share the same vertex, angle CBD. Since there are two pairs of congruent adjacent sides, then ACBD is a kite.

□

Constructing Kites

Grant Kilburg

September 30, 2017

Communicated by: Kaelyn Koontz

Our quest to solving this problem began with Ms. Carstens proving that a kite could be constructed using two circles, circle AC and circle BC. Let Ms. Carstens' kite be denoted as ACBD in the figure below. The construction of our figure ACED begins from here. Draw circle AB. By Postulate 1.1, extend line segment AB so that it intersects circle AB. Label this point, E. By Postulate 1.1, construct line segments EC and ED to get the quadrilateral ACED. It remains to show what type of figure ACED is.

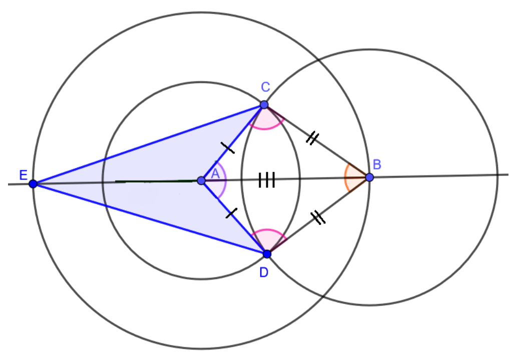

Figure 1: This figure shows the construction described above.

Theorem K. ACED is a kite.

Proof. Since ACBD is a kite, line segments AC and AD are congruent to one another. By Theorem I, segment AB bisects angle DAC. Therefore, angle CAB and angle DAB are congruent. Since EB is a straight line with CA falling upon it, angles EAC and CAB will add to two right angles by Euclid I.13. Because DA is a straight line which falls upon EB, angles EAD and DAB will also form two right angles by Euclid I.13. Since angles CAB and DAB are congruent, angles EAC and EAD must also be congruent by Common Notion 3. Since EA is shared, it follows by Euclid I.4 (SAS), that triangle EAC is congruent to triangle EAD. Thus, EC is congruent to ED. Hence, ACED is a kite. □

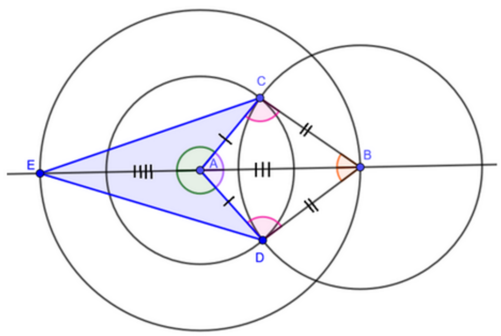

Figure 2: By Euclid I.4, triangles EAC and EAD are congruent. Thus, it follows that ACED is a kite.

Star Trek Kite

Cameron Amos and Connor Coyle

September 28, 2017

Communicated by: Mr. Otterbein

Theorem J. The Star Trek figure AEBC is a kite.

Proof. To construct the Star Trek kite AEBC, first draw line AB. Draw a circle AB and BA using postulate 3. Label point C and point D at the intersection of circles AB and BA. By using Euclid I.1, create the equilateral triangle ABC. Draw a circle CD using postulate 3. Using postulate 1, create the line CD. Using Euclid I.10, the line CD bisects triangle ABC, and where the line CD bisects line AB, create the point F. Using postulate 2, extend the line CD up to the circle CD and create the point E on circle CD. Using postulate 1, create lines AE and BE. Using postulate 2, extend the lines AC and BC. On extended line AC, create the point X. On extended line BC, create the point Y. By Euclid I.13, the angle ACX is the sum of two right angles. By Euclid I.13, the angle BCY is the sum of two right angles. Since AF is congruent to BF because ED bisects AB, and AC is congruent to BC because they are part of the equilateral triangle ABC, and the triangle AFC and BFC share the same base, then by Euclid I.8 the triangles AFC and BFC are congruent. By using Euclid I.15, the vertical angles ACB is congruent to YCX. Since line DC bisects angle ACB, and DC is extended to point E, then angle YCX is also bisected by DE. By Euclid I.4, triangle ACE is congruent to BCE because AC is congruent to BC and AE is congruent to BE. Thus AEBC is a kite. ☐

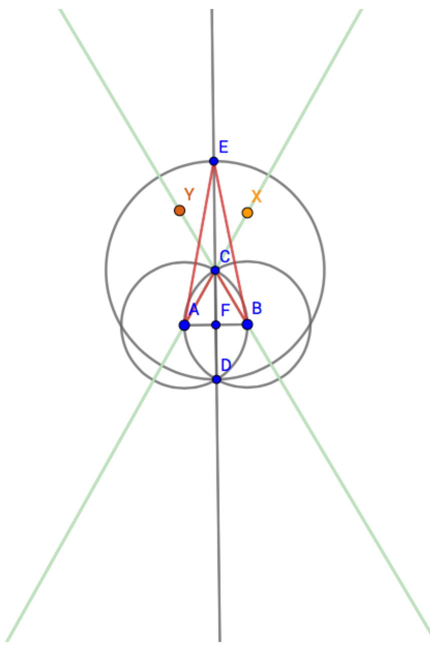

Figure 1: This is a picture of the kite AEBC

Congruent Kites

Cameron Amos

September 30, 2017

Communicated by: Connor Coyle

Theorem L. Suppose $ABCD$ is a kite with AB congruent to AD and CB congruent to CD and suppose $LMNO$ is a kite such that LM is congruent to LO and NM is congruent to NO. If AB is congruent to LM and CB is congruent to NM and angle ABC is congruent to angle LMN. Then kite $ABCD$ is congruent to kite $LMNO$.

Proof. Let $ABCD$ be a kite with AB congruent to AD and CB congruent to CD. Let $LMNO$ be a kite with LM congruent to LO and NM congruent to NO. Let AB be congruent to LM and CB be congruent to NM. Let angle ABC be congruent to angle LMN. By Postulate 1, draw straight lines AC and LN. Then, triangle ABC is congruent to triangle LMN by Euclid I.4. Since AD is congruent to AB and AB is congruent to LM, then AD is congruent to LM. Since LM is congruent to LO, then AD is congruent to LO. Since CB is congruent to CD and CB is congruent to NM, then CD is congruent to NM. Since NM is congruent to NO, then CD is congruent to NO. Since triangle ABC is congruent to triangle LMN, then AC is congruent to LN. By Euclid I.8, then triangle ADC is congruent to triangle LON. Since triangle ADC is congruent to triangle LON and triangle ABC is congruent to triangle LMN, then kite $ABCD$ is congruent to kite $LMNO$. □

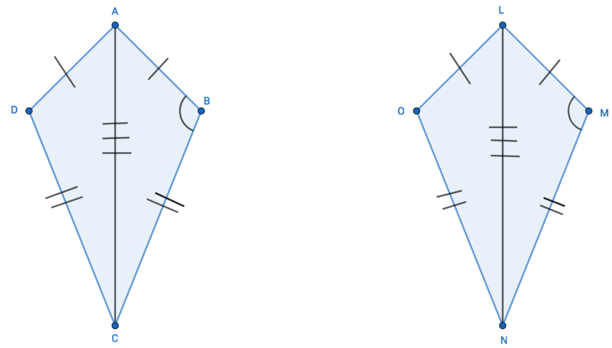

Figure 1: This is a picture representing kite $ABCD$ is congruent to kite $LMNO$.

The Congruent Nature of Opposite Angles in a Kite

Kayla Schafbuch

September 30, 2017

Communicated by: Emily Carstens

Theorem 2.1. Let ABCD be a kite with segment AB congruent to segment BC and segment DA congruent to segment DC. Then angle BAD is congruent to angle BCD.

Proof. Let ABCD be a kite. By Euclid's Postulate 1, construct segment BD. Now we have triangle BAD and triangle BCD. By Euclid I.8, triangle ABD is congruent to triangle CBD, angle CBD is congruent to angle ABD, and angle CDB is congruent to ADB. Also by Euclid I.8, angle BAD is congruent to angle BCD. □

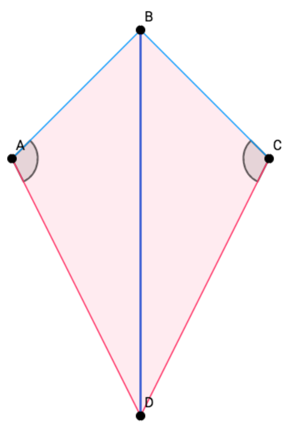

Figure 1: Kite ABCD with triangles BAD and BCD.

Schafbuch's Conjecture on the Angles of a Kite

Theron J Hitchman

September 29, 2017

Communicated by: The Editor

The quadrilaterals known as *kites* are interesting because of their symmetry. In particular, they seem to have less structure than rhombuses, but still have more structure than a random quadrilateral. The goal of our work is to settle a conjecture of Schafbuch about the nature of the pairs of opposite angles of a kite.

Amos and Flesch prove that each pair of opposite angles of a rhombus makes a pair of congruent angles. It is natural to ask if this symmetry is also true for kites. Schafbuch found a partial result, that each kite has at least one congruent pair of opposite angles.

Theorem 2.1. [Schafbuch] Let ABCD be a kite with AB congruent to BC and AD congruent to DC. Then angle DAB is congruent to angle DCB.

There are several constructions which are either known or suspected to give kites, including work of Amos-Coyle, Carstens, and Kilburg. Schafbuch notes that some of these generate examples of kites which are not parallelograms, especially following the work of Flesch, who proves that a kite which is not a rhombus is certainly not a parallelogram. She makes the following conjecture, which we intend to settle.

Conjecture G. [Schafbuch] If a kite is not a parallelogram, then it has only one pair of congruent opposite angles.

Our approach is to prove the contrapositive statement. Specifically, we shall prove the following.

Theorem G. Let ABCD be a kite with AB congruent to BC and AD congruent to DC. If ABCD has two pairs of opposite angles congruent, then it is a parallelogram.

Note that we need only consider the case of two pairs of congruent opposite angles, since by Schafbuch's theorem it is impossible to have no pairs of congruent opposite angles.

Proof of Theorem G. By assumption, our kite has two pairs of opposite angles congruent. Thus angle ABC is congruent to angle ADC and angle BAD congruent to angle BCD.

Using Euclid Postulate 1, construct the diagonal segment AC. By a theorem of Kilburg, this diagonal bisects the angles DAB and DCB. Hence, the angles BAC and DAC are congruent, and each is half of angle BAD. Similarly, the angles BCA and DCA are congruent

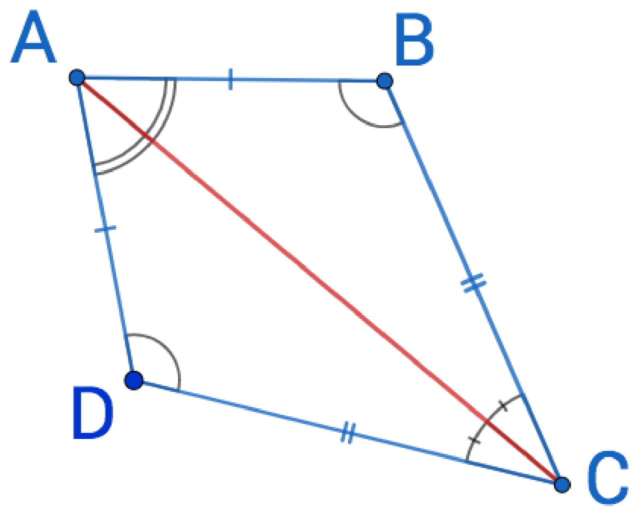

Figure 1: Our kite, with congruent sides marked.

and half of angle BCD. Halves of congruent angles must also be congruent, so we deduce that the four angles BAC, DAC, BCA, and DCA are all mutually congruent.

Consider the triangle ABC. We have that angle BAC is congruent to angle BCA, so by Euclid I.6 we know that segment AB is congruent to segment BC. Applying the same logic to triangle DAC, we deduce that AD is congruent DC. By our hypothesis, AB is congruent to AD and BC is congruent to CD. So by Euclid's Common Notion 1, we deduce that all four of the segments AB, BC, CD, and DA are mutually congruent.

So, by definition, ABCD is a rhombus. By a theorem of Flesch-Kilburg-Thompson, a rhombus is a parallelogram. So we conclude that our kite ABCD must be a parallelogram. This completes the argument. □

Remark: Note that along the way we have proved something a bit easier to remember, namely that a kite which has two pairs of congruent opposite angles, rather than just one, must be a rhombus.

A Rectangle is a Parallelogram

Emily Carstens

September 30, 2017

Communicated by: Rachelle Feldmann.

Theorem 3.1. Let R be a rectangle. Then R is a parallelogram.

Proof. Let ABCD be a rectangle. Since ABCD is a rectangle then angle ABC, angle BCD, angle CDA and angle DAB are right angles by the definition of a rectangle.

Extend segment BC, segment AD and segment AB using Euclid's Postulate I.2.

Let E be a point on line AB such that point E follows point B on the line. By Euclid's Proposition I.13, angle EBC is a right angle. Angle EBC is congruent to angle DCB by Euclid's Postulate I.4, then by Euclid's Proposition I.27 segment AB is parallel to segment DC.

Let F be a point on line BC such that point F comes before point B on the line. By Euclid's Proposition I.13, angle ABF is a right angle. Angle ABF is congruent to angle DAB by Euclid's Postulate I.4, then segment AD is parallel to segment BC by Euclid's Proposition I.27.

Since opposite sides of the ABCD are parallel, then ABCD is a parallelogram.

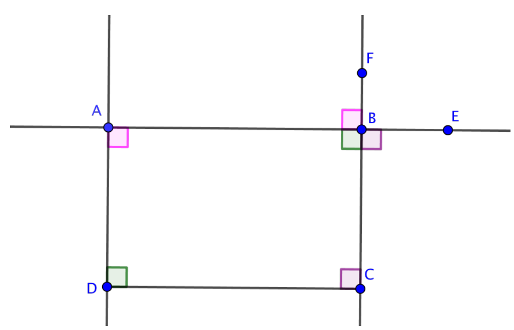

Figure 1: The rectangle ABCD.

□

Diagonals of a Rectangle

Ashlyn Thompson

September 30, 2017

Communicated by: Micah Otterbein

Theorem P. The diagonals of a rectangle must cross.

Proof. Let $ABCD$ be a rectangle with diagonals BD and CA. Since $ABCD$ is a rectangle, then angle ABC and angel DCB taken together make two right angles. Since and ABC and angle DCB taken together make two right angles, then angle CBD and angle BCA must be less than two right angles. Therefore, by Euclid Postulate 5, diagonals BD and CA must cross.

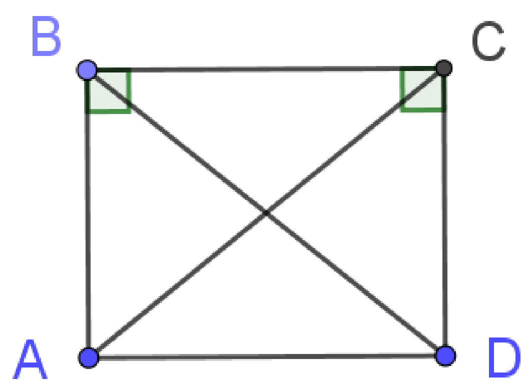

Figure 1: This is a diagram showing angle ABC and DCB taken together make two right angles. Therefore, angle CBD and BCA taken together must be less than two right angles.

□

Diagonals of a Rectangle

Grant Kilburg

September 30, 2017

Communicated by: Katherine Bertacini

The following theorem is proven using Theorem P, which states that the diagonals of a rectangle must cross.

Theorem 3.3. The two diagonals of a rectangle are congruent and bisect each other.

Proof. Let ACBD be a rectangle. Since ABCD is a rectangle, then AB is congruent to DC and AD is congruent to BC, by Mr. Flesch's Theorem. Since ABCD is a rectangle, all four interior angles are right triangles, by definition of a rectangle. By Postulate 1.1, construct line segments AC and BD. By Theorem P, AC and BD must cross. Let X be the point at which they intersect. Since AB is congruent to DC and BC is congruent to AD, and angle DCB is congruent to angle CDA, triangle ADC is congruent to triangle BCD by Euclid I.4 (SAS). Thus AC is congruent BD. It remains to show that the diagonals AC and BD bisect each other.

Since ABCD is a parallelogram, AB is parallel to DC and AD is parallel to BC. Since AD is parallel to BC, and AC has fallen upon them, angle BAX is congruent to angle DCX by Euclid I.29. Similarly, since AB is parallel to DC and DB has fallen upon them, angle ABX is congruent to angle CDX by Euclid I.29. By Euclid I.26 (ASA), triangle ABX is congruent to triangle CDX. Thus BX is congruent to DX and AX is congruent to CX. Thus X is the midpoint of BD and X is the midpoint of AC. Hence, AC and BD bisect each other. □

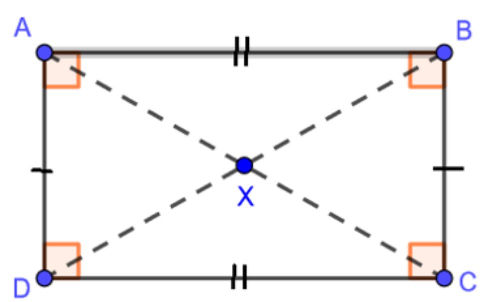

Figure 1: Rectangle ABCD with diagonals AC and BD meeting at X.

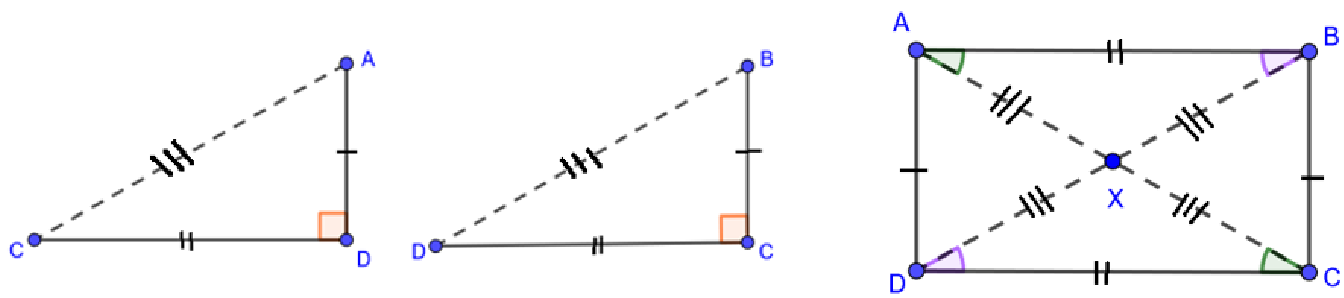

Figure 2: Since triangles CDA and DCB are congruent, AC is congruent to BD. Similarly, since triangles AXB and CXD are congruent, AX and CX are congruent as are DX and BX.