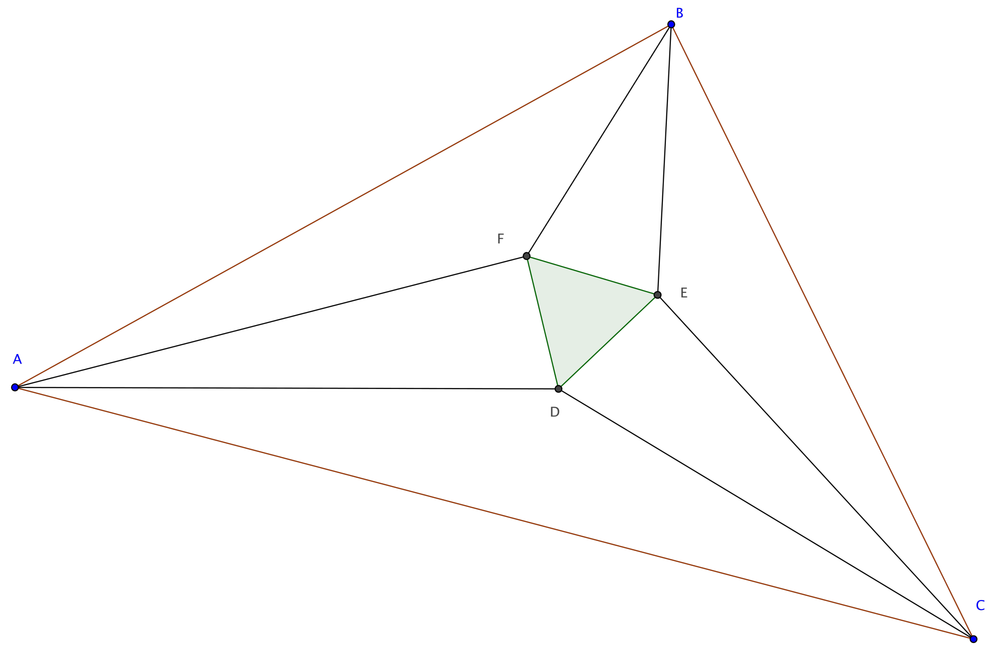
Transactions

in

Euclidean Geometry



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Special Case O

Katherine Bertacini and Kaelyn Koontz

November 20, 2017

Communicated by: Mr.Otterbein

Theorem O. Let ABCD be a quadrilateral. It is not possible to divide a non-simple quadrilateral into two triangles using the diagonals.

Proof. First, we will use a counter example to show that you cannot split a non-simple quadrilateral ABCD into two triangles using diagonals. By Challenge M, construct the hour glass quadrilateral ABCD. By Euclid's Postulate 1.1, draw the diagonals AC and BD. Notice the two diagonals are outside of quadrilateral ABCD. This shows that it is not possible to split quadrilateral ABCD into two triangles using the diagonals, since they are outside the figure, therefore the diagonals can not divide the quadrilateral ABCD. □

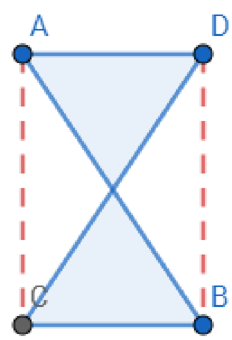

Figure 1: This is a picture of the Hour Glass with the diagonals.

A Rectangle Has Two Sets of Opposite Congruent Sides

Steven Flesch

November 20, 2017

Communicated by: Grant Kilburg

Theorem 3.1. Let R be a rectangle. Then R is a parallelogram.

Theorem 3.2. Let R be a rectangle. Then each pair of opposite sides of R is a pair of congruent sides.

Proof. Since R is a rectangle, angles ADC, DAB, ABC, and BCD are all right angles. By Postulate 1.1, draw segment AC. By Ms. Carstens's proof of Theorem 3.1, rectangle R is a parallelogram. By. Euclid I.34, AB is congruent to CD and AD is congruent to BC. Therefore, rectangle R has two pairs of opposite congruent sides. $\square$

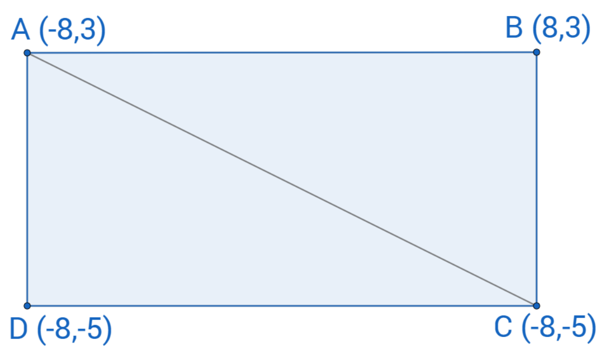

Figure 1: This proves that rectangle R has two pairs of opposite congruent sides by I.34.

Inside and Outside a Simple Polygon

Emily Carstens

November 20, 2017

Communicated by: Ms. Schafbuch.

Definition Inside and Outside a Simple Polygon.

Let m be a line. If line m does not intersect a polygon, then all the points of m are said to lie *outside* the polygon. If line m intersects a segment or vertex of a polygon, all points on line m after this point of intersection and until the next point of intersection with the polygon, will lie *inside* the polygon. Each time line m intersects the polygon, the points switch from inside to outside or vice versa.

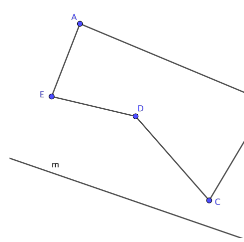

Figure 1: This is an example of a line m where all points of the line lie outside of the polygon ABCDE.

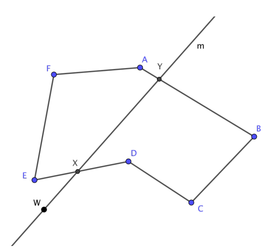

Figure 2: This is an example of a line m where the line intersects our polygon, ABCDE at points X and Y. Starting at point W and moving toward point X, these points, not including point X, lie outside of ABCDE until line m intersects ABCDE. Starting at point X moving toward point Y, not including point Y, these points lie inside ABCDE until line m intersects ABCDE. All following points of line m will lied outside ABCDE.

Diagonals and Bisectors

Mr. Hertzler

November 20, 2017

Communicated by: Ms. Carstens

Conjecture e. The diagonals of a rhombus bisect their respective angles.

Proof. Let ABCD be a rhombus. Join BD by Euclid postulate I.1. By Euclid's definition I.22, a rhombus is a quadrilateral that is equilateral but not right angled. Thus AB is congruent to BC is congruent to CD is congruent to DA. Triangles ABD and CBD have the same base BD. Triangles ABD and CBD also have two sides that are congruent with one another. Since triangles ABD and CBD have three congruent sides, then they are congruent triangles by Euclid I.8. Since triangles ABD and CBD are congruent, then angles ABD and angles CBD are congruent. Angles ADB and CDB make up angle ADC. Since the two angles are congruent and make up a larger angle, then BD bisects the larger angle ADC. Similarly, angles ABD and CBD are congruent and make up angle ABC. Thus, BD also bisects angle ABC.
Similarly, diagonal AC bisects angles DAB and DCB. □

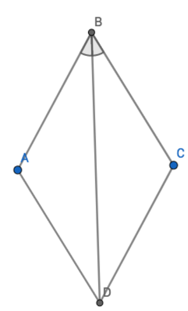

Figure 1: Above is a rhombus

5.3

Katherine Bertacini

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Communicated by: Ms. Feldmann

Like Theorem 5.2 this theorem will only work for simple polygons.

Theorem 5.3. The sum of the exterior angles of a hexagon is equal to four right angles, this is the same for any simple polygons as well.

Proof. Apply theorem 5.2 to a hexagon, where we use Postulate 1.2 to extend all lines of the polygon out one side, to get the sum of the interior and exterior right angles. Then use Postulate 1.1 to make the triangles in the polygon, to get the sum of the interior right angles.

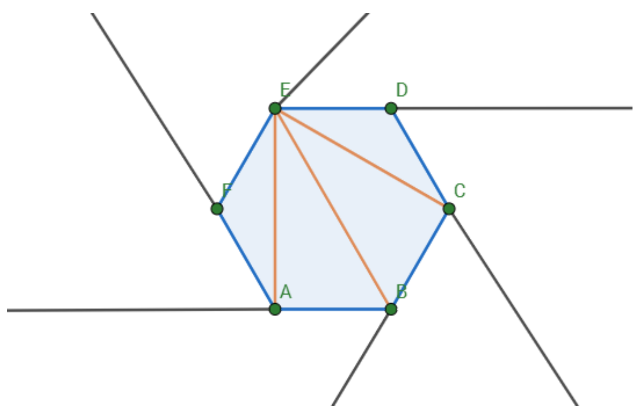

Figure 1: This is a hexagon with four triangles but the diagonals.

Using the example that a hexagon has six sides, we are going to apply two equations for the interior and exterior right angles. First we have n equals the number of sides, so n times 2 equals the sum of the interior and exterior right angles. For the hexagon example we have six times two is 12. Then (n-2) times 2 equals the sum of the interior right angles, so (6-2) times 2 is 8. Eight is the number of interior right angles of a hexagon. Using the first equation and subtracting the second, four exterior angles will always be produced. □

Angles of a Regular Pentagon

Grant Kilburg

November 20, 2017

Communicated by: Steven Flesch

In observing the angles of regular pentagon ABCDE, formed between angles CAD and ACD, it seemed as though the base angles of isosceles triangle ACD were equal to twice the measure of the third. If this is the case, it would be extremely noteworthy, as it would be the only isosceles triangle with such properties. The following theorem proves that this relationship exists.

Theorem 6.6. Let ABCDE be a regular pentagon. State the relationship between the angles CAD and ACD that shows how special the triangle is. Prove your observation.

Proof. Let ABCDE be a regular pentagon. Since ABCDE is a regular pentagon, it has five right angles, all of which are congruent. By Theorem 5.2, which was proved by Ms. Koontz, we know that the angles of a pentagon must be equal to six right angles. Thus, each angle of ABCDE must be equal to 6/5 of a right angle.

By Postulate 1.1, construct line segments AC and AD. By Mr. Amos's proof of Theorem 6.5, ACD is an isosceles triangle with AD congruent to AC. Since ACD is an isosceles triangle with sides AC and AD congruent, angle ADC is congruent to angle ACD by Euclid I.5. Since ABCDE is equilateral, AB is congruent to BC by definition. Similarly, AE is congruent to ED. Thus, angle BAC is congruent to angle BCA and angle EAD is congruent to angle EDA by Euclid I.5. By Euclid I.32, the sum of angles ABC, BAC, and BCA must be equal to two right angles. Since angle ABC is equal to 6/5 of a right angle, angle BAC plus angle BCA equals 4/5 of a right angle. Since angle BAC is congruent to angle BCA, and angle BAC plus angle BCA is 4/5 of a right angle, angles BAC and BCA must both be equal to 2/5 of a right angle.

By Euclid I.32, the sum of angles AED, EAD, and EDA must also be equal to two right angles. Since angle AED is equal to 6/5 of a right angle, angle EAD plus angle EDA equals 4/5 of a right angle. Since angle EAD is congruent to angle EDA and the sum of EAD and EDA is 4/5 of a right angle, angles EAD and EDA must both be equal to 2/5 of a right angle. Since angle EDA is 2/5 of a right angle, and angle EDA plus angle ADC equals 6/5 of a right angle, angle ADC equals 4/5 of a right angle. Similarly, since angle BCA is 2/5 of a right angle, and angle BCA plus angle ACD equals 6/5 of a right angle, angle ACD equals 4/5 of a right angle. Finally, since angle BAC plus angle CAD plus angle EAD equals 6/5 of a right angle and angles BAC and EAD are both equal to 2/5 of a right angle, angle DAC equals 2/5 of a right angle. Thus, angle DAC is half the as large angles ADC and ACD. Hence, triangle ACD is an isosceles triangle whose base angles are twice as large as the other angle.

□

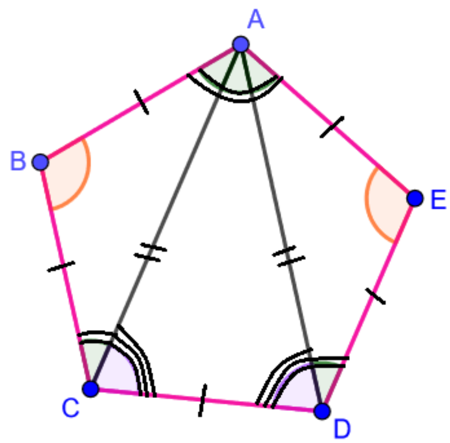

Figure 1: Since triangle DAC trisects angle EAB, angles ACD and ADC are twice as large as angle DAC. This is the only isosceles triangle in which the base angles are twice as large as the third angle.

Archimedes' Theorem of the Broken Chord

Ashlyn Thompson

November 20, 2017

Communicated by: Cameron Amos

Theorem 10.7. Let AB and BC be two chords of a circle Z, where BC is greater than AB. Let M be the midpoint of arc ABC and F the foot of the perpendicular from M to chord BC. Then F is the midpoint of the broken chord.

Proof. By Euclid I.2 construct a circle centered at C with radius length AB. Let E be the point of intersection of the circle and segment BC. By this construction, segment EC is congruent to segment AB.

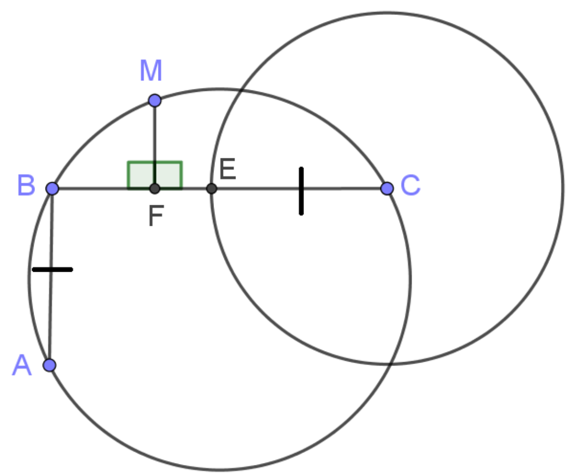

Figure 1: This is a picture of the construction for congruent segments AB and EC.

By Euclid's Postulate 1, construct segments AM, BM, EM and CM. Since M is the midpoint of arc ABC, then arc ABM is congruent to arc MC. By Euclid III.29, chord AM is congruent to chord CM since they cut off congruent arcs. Since angle BAM and angle BCM stand upon the circumference BM, by Euclid III.27 angle BAM and angle BCM are congruent angles.

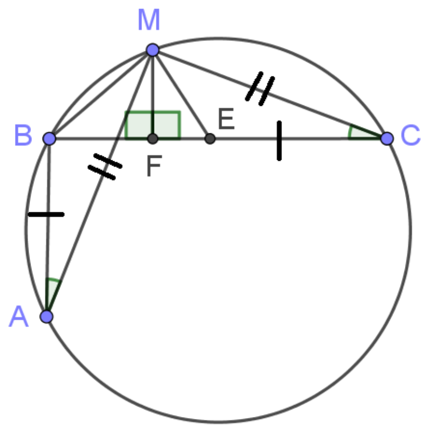

Figure 2: This picture shows congruent chords AM and CM by Euclid III.29 and congruent angles BAM and BCM by Euclid III.29.

Since chord AB is congruent to chord EC, chord AM is congruent to chord CM, and angle BAM is congruent to angle BCM, by Euclid I.4 triangle ABM is congruent to triangle CEM. It follows that segment BM is congruent to segment EM. Therefore, triangle BME is an isosceles triangle. By Euclid I.5, angle FBM is congruent to angle FEM. Therefore, by Euclid I.26, triangle BMF is congruent to triangle EMF. It follows that segment BF is congruent to segment FE.

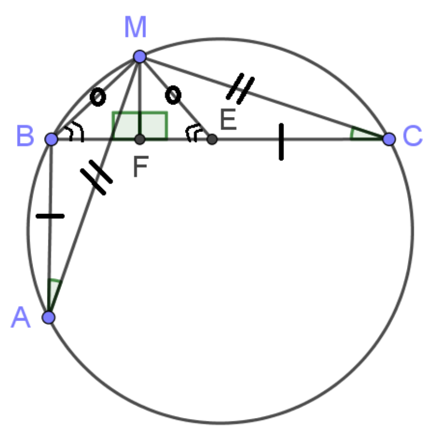

Figure 3: This diagram shows congruent triangles ABM and CEM by Euclid I.4 and congruent triangles FBM and FEM within the isosceles triangle BME.

Since segment AB is congruent to EC, and BF is congruent to FE, then AB and BF taken together is equal to FE and EC taken together by Common Notions 1 and 2. Therefore, F must be the midpoint of the two broken chords AB and BC.

□