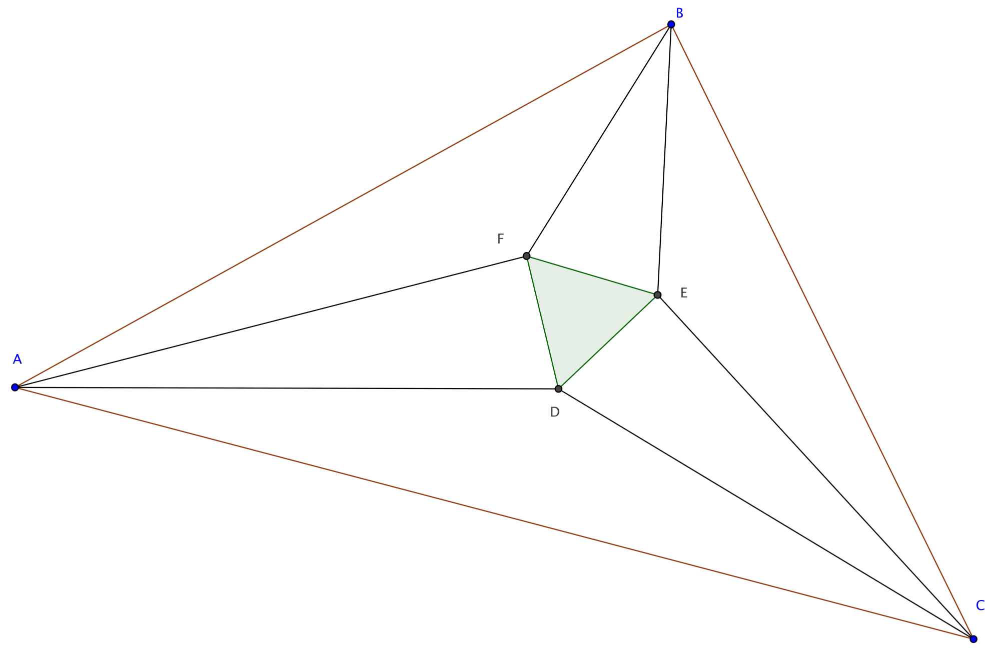
Transactions

in

Euclidean Geometry



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Constructing the Center of a Circle

Rachelle Feldmann

December 11, 2017

Communicated by: Ms. Koontz.

Theorem 11.7. Given the circumference of a circle, find the center of the circle.

Proof. We are given a circumference.

1. Create chord AC.
2. Draw circle AB.
3. Draw circle BA
4. Draw a line through the intersection points C and D. Then label the points where the line crosses the circle, E and F. Using the corollary from Euclid III.1, a line ,CD, that cuts another line ,AB, in half at right angles contains the center of the circle.
5. Draw circle EF.
6. Draw circle FE.
7. Connect the intersection points G and H, and label the points where the line crosses the circle, I and J. Label the intersection X.

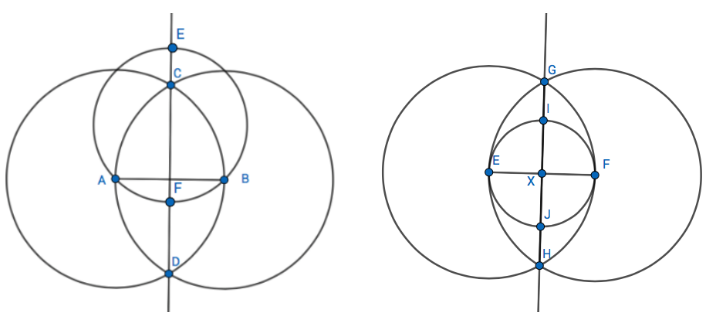

Figure 1: This picture shows the construction of the center of a circle.

The center of the circle is found by the intersection of lines, EF and IJ. Using Koontz 11.2, line EF has a midpoint at X by line IJ. Since line EF contains the center of the circle then center is the midpoint of the line. Hence X is the center of the circle. □

Theorem Y

Steven Flesch and Cameron Hertzler

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Communicated by: Dr. Hitchman

Theorem Y is based off of Mr. Amo's Thm 13.6 Proof. Theorem Y discusses the congruency of two rectangles using equal content.

Theorem Y. Suppose rectangles R and S have equal content and one pair of congruent sides. Then R and S are congruent.

Proof. From rectangles R and S, AB is congruent to CD is congruent to EF is congruent to GH. Then, create AC and EG by Euclid Postulate 1. Then, $ABCD$ and $EFGH$ are parallelograms by Thm 3.1. Also, BC is congruent to AD, and FG is congruent to EH by Ms. Carstens Thm 3.2 Proof. Triangle ABC is congruent to triangle CDA and triangle FGE is congruent to triangle GEH by Euclid I.8. AC bisects R and EG bisects S by Euclid I.34. Therefore, the area of triangle ABC plus the area of triangle CDA equals the area of rectangle R, and the area of triangle EFG plus the area of triangle GHE equals the area of rectangle S. We are given from our hypothesis that the area of rectangle R equals the area of rectangle S. Thus, the area of triangle ABC plus the area of triangle CDA equals the area of triangle EFG plus the area of triangle GHE.

Triangles ABC and FEG have one pair of congruent sides and a right angle. Triangles ABC and FEG also have the same area. By figure 3, triangles that share a side with a right angle on that side that also have the same area must be congruent. Let point D be taken at random on the extended line AC past C. Let the point E be taken at random on the segment AC. Then in order for a triangle to have a congruent side and right angle on that side as well as have the same area as triangle ABC, it must be congruent since a triangle with a side greater than BC will have greater area than triangle ABC. Also, a triangle with a side less than BC will result in a triangle with less area than ABC. Thus two triangles that share a side with a right angle on that side with equal area must be congruent. $\square$

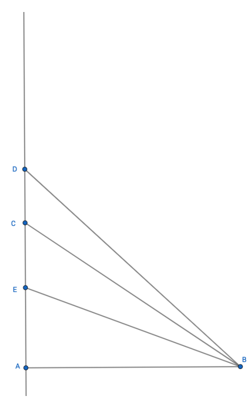

Figure 1:

Theorem 8.1

Micah Otterbein

December 11, 2017

Communicated by: Dr. Hitchman.

Theorem 8.1. Let ABC be a triangle, with rays r and s the angle bisectors at A and B, respectively. Suppose that r and s meet at point J which lies inside the triangle. Draw lines l and m through J that are perpendicular to AC and BC respectively. If l meets AC at point X and m meets BC at Y, then triangle JXC is congruent to triangle JYC.

Proof. Let ABC be a triangle. By Euclid I.9, construct the perpendicular bisectors for angle ABC and angle CAB. Let the point the two bisectors meet at be point J. By Euclid Postulate 1, draw the line JC. By Euclid I.12, construct the perpendicular lines from J to AC, from J to AB, and from J to BC, let the points where the lines meet be called X, Z, and Y respectively. By Euclid I.26, triangle XAJ is congruent to triangle ZAJ. Therefore, XJ is congruent to ZJ. By Euclid I.26, triangle ZBJ is congruent to triangle YBJ. Therefore, YJ is congruent to ZJ. Thus, YJ in congruent to XJ. By Hitchman's 7.2, triangle JXC is congruent to triangle JYC.

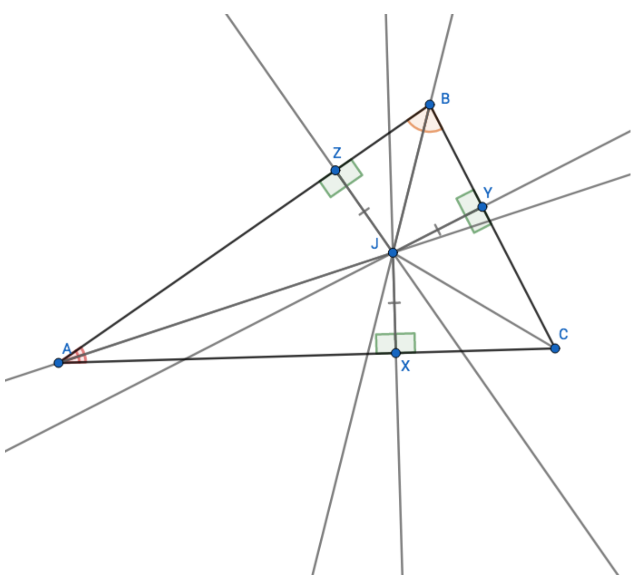

Figure 1: Triangle ABC.

□

Theorem 8.2

Micah Otterbein

December 11, 2017

Communicated by: Dr. Hitchman.

Theorem 8.2. The three angle bisectors of a triangle are concurrent.

Proof. Let ABC be a triangle. By Euclid I.9, construct the perpendicular bisectors for angle ABC and angle CAB. Let the point the two bisectors meet at be point J. By Euclid Postulate 1, draw the line JC. By Euclid I.12, construct the perpendicular lines from J to AC and from J to BC, let the points where the two lines meet be called X and Y respectively. By Otterbein 8.1, triangle JXC is congruent to triangle JYC. Thus, angle XCJ is congruent to angle CYJ. Therefore, JC is the angle bisector of angle BCA. Thus, the angle bisectors AJ, BJ, and CJ meet at point J and are thus concurrent.

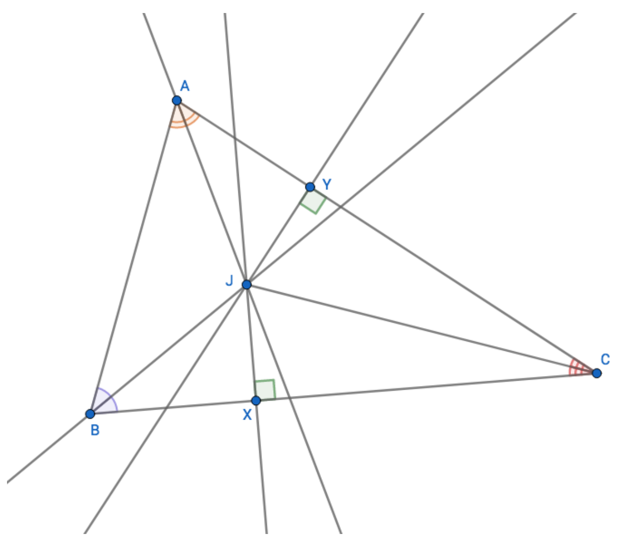

Figure 1: Triangle ABC.

□

Theorem 8.3

Micah Otterbein

December 11, 2017

Communicated by: Dr. Hitchman.

Theorem 8.3. Let T be a triangle. For any pair of sides of T, the perpendicular bisectors of those sides meet. (That is, they are not parallel.)

Proof. By Way of Contradiction: Assume that the perpendicular bisectors of triangle T are parallel. Let triangle T be the triangle ABC with perpendicular bisectors on AC and BC, at points E and F respectively.
Enter Cases.
Case 1: Angle ACB is right or obtuse. By definition and because angle ACB is right or obtuse, angle CAB and angle ABC are acute. By Euclid Postulate 5, the perpendicular bisector from E and from F both meet AB at a point, let the points they meet be G and H respectively. Let J be a point on the line EG above the line AB. By definition and because angle AEG and angle BFH are right, angle AGE and angle BHF are acute. By Euclid I.15, angle AGE is congruent to angle BGJ. Thus, angle BGJ is acute. By Euclid I.13, angle BHF and angle AHF, when taken together, are congruent to two right angels. It follows that, since angle BHF is acute, then angle AHF is obtuse. By Euclid I.29 and because EG is parallel to FH, angle BGJ is congruent to angle AHF. Angle BGJ is acute and angle AHF is obtuse, therefore they can not be congruent. Thus, angle BGJ is both congruent and not congruent to angle AHF, creating a contradiction.
Case 2: Angle ACB is acute. By Euclid Proposition 5, the perpendicular bisector from E meets BC at a point and F meets AC at a point, let the points they meet be G and H respectively. Let J be a point on the line EG. By definition and because angle CEG is right, angle EGC is acute. By Euclid I.15, angle EGC is congruent to angle BGJ. By Euclid I.29 and because EG is parallel to FH, angle BGJ is congruent to angle CFH. Angle BGJ is acute and angle CFH is right, therefore they can not be congruent. Thus, angle BGJ is both congruent and not congruent to angle CFH, creating a contradiction.
Exit Cases.
Therefore, Theorem 8.3 is true.

□

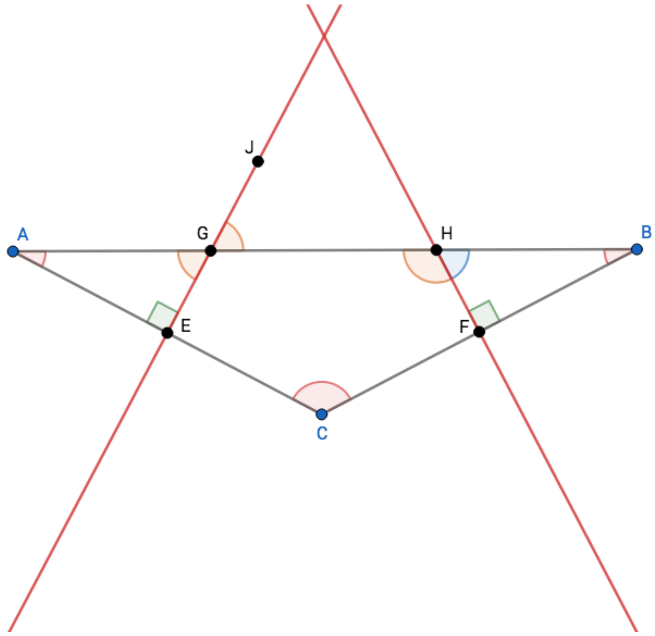

Figure 1: Obtuse Triangle ABC.

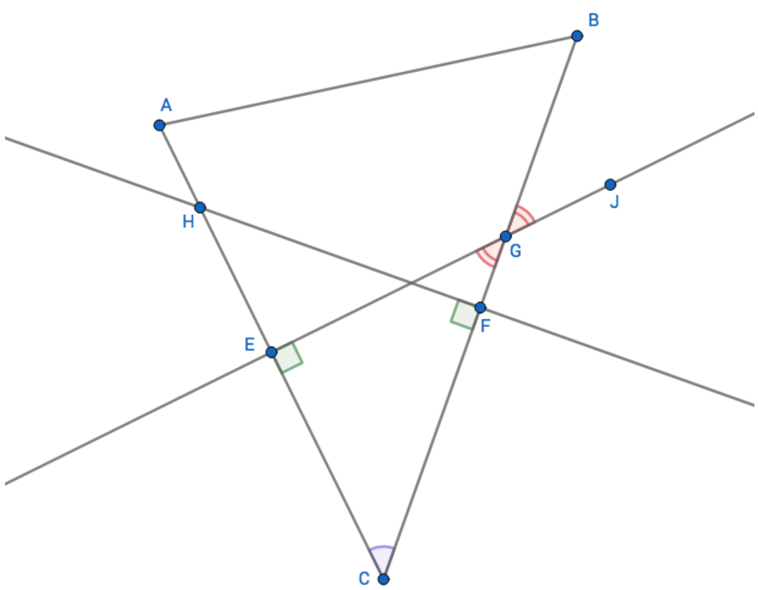

Figure 2: Acute Triangle ABC.

Parallel Lines in Tangent Circles

Micah Otterbein and Emily Carstens

December 11, 2017

Communicated by: Emily Carstens and Micah Otterbein

Theorem 9.5. Let two circles be tangent at a point A. If two lines are drawn through A meeting one circle at further points B and C and meeting the other circle at points D and E, then BC is parallel to DE.

Proof. There are three cases.

Case 1: One line is the diameter of the circles.

Let B, C, D and E be the points where the two lines intersect the two circles, where B and C lie on the same circle and D and E lie on the same circle. Form segments BC and DE using Euclid Postulate I.1.
By Euclid Proposition I.31 angle CBA and angle EDA are right angles. Thus by Euclid Proposition I.27 segment BC and segment DE are parallel.

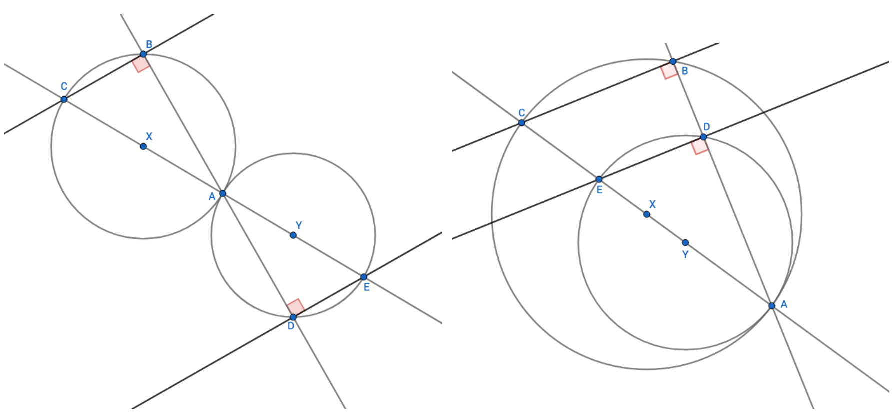

Figure 1:

Case 2: The two lines split the diameter on opposite sides.

Let B, C, D and E be the points where the two lines intersect the two circles, where B and C lie on the same circle and D and E lie on the same circle. Form segments BC and DE using Euclid Postulate I.1.

Using Euclid Proposition III.1 find the centers of the two circles. Label them X and Y, such that we have circles XB and circle YD. Form segments XY. By Euclid III.12 XY passes through point A.

By Euclid Postulate I.1 form segments CX, AX, AY and EY. By Euclid Proposition III.14 segment CX is congruent to segment AX, and segment AY is congruent to segment EY. By Euclid Definition I.20 triangle CXA and triangle AYE are isosceles. By Euclid Proposition I.5 angle XCA is congruent to angle XAC, and angle YAE is congruent to angle EAY. By Euclid Proposition I.15 angle XAC is congruent to angle EAY. Thus angle XCA is congruent to angle XAC is congruent to angle EAY is congruent to angle YAE by Euclid Common Notion I.1.

By Euclid Proposition I.32 angle XCA plus angle CXA plus angle XAC are equal to two right angles and angle YAE plus angle EAY plus angle AYE are equal to two right angles. By Euclid Common Notion I.1 angle XCA plus angle CXA plus angle XAC are equal to angle YAE plus angle EAY plus angle AYE. Thus angle CXA is congruent to angle AYE.

By Euclid Proposition I.20 angle CXA is double angle CBA, and angle AYE is double angle ADE. Since angle CXA is congruent to angle AYE, then angle CBA is congruent to angle ADE. Thus segment BC is parallel to segment DE by Euclid Proposition I.27.

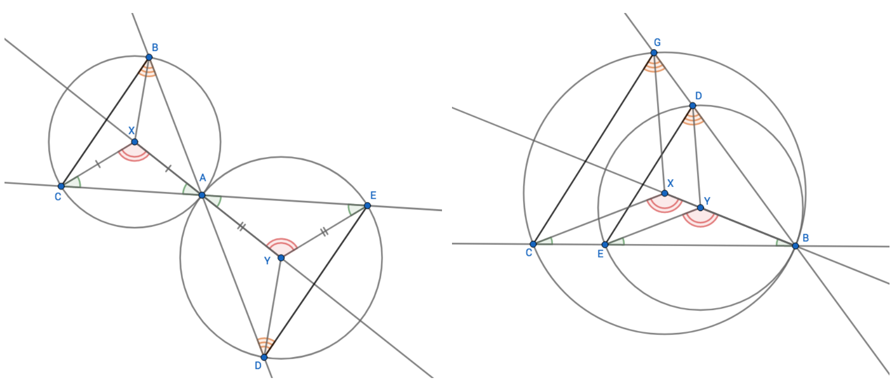

Figure 2:

Case 3: The two lines split the diameter on the same side.

Let B, C, D and E be the points where the two lines intersect the two circles, where B and C lie on the same circle and D and E lie on the same circle. Form segments BC and DE using Euclid Postulate I.1.

Using Euclid Proposition III.1 find the centers of the two circles. Label them X and Y, such that we have circles XB and circle YD. Form segments XY. By Euclid III.12 XY passes through point A. Extend segment XY using Euclid Postulate I.2. Create point F where line XY further intersects circle XB, and create point G where line XY further intersects circle

YD. By Euclid Postulate I.1 create segments FC and GE.

By Theorem 7.4 angle ACF an angle AEG are right angles. By Euclid Proposition I.15 angle DAE is congruent to angle BAC, and angle GAE is congruent to angle FAC.

By Euclid Proposition III.22 angle DAE plus angle GAE plus angle AEG plus angle AED is equal to two right angles, and angle BAC plus angle FAC plus angle ACF plus angle ACB is equal to two right angles. Thus angle DAE plus angle GAE plus angle AEG plus angle AED is equal to angle BAC plus angle FAC plus angle ACF plus angle ACB by Euclid Common Notion I.1. Then angle DAE plus angle GAE plus angle AEG plus angle AED is equal to angle DAE plus angle GAE plus angle ACF plus angle ACB by Euclid Common Notion I.1. Then angle AEG plus angle AED is equal angle ACF plus angle ACB. Since angle AEG and angle ACP are right angles then angle ACB is congruent to angle AED.

Thus by Euclid Proposition I.27 segment BC is parallel to segment DE.

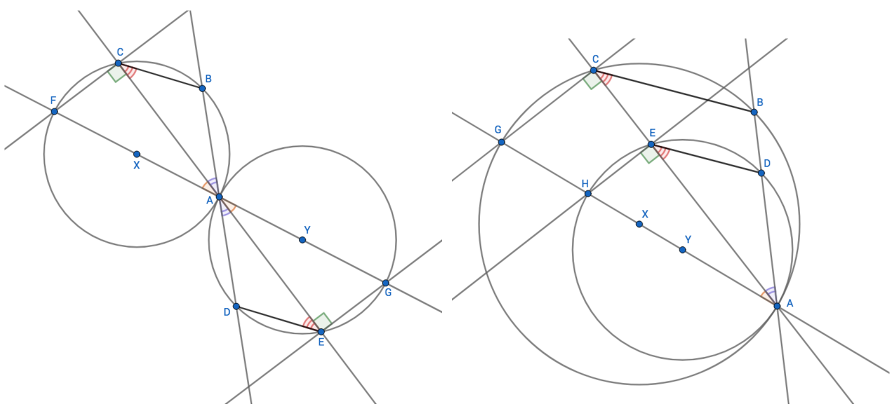

Figure 3:

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