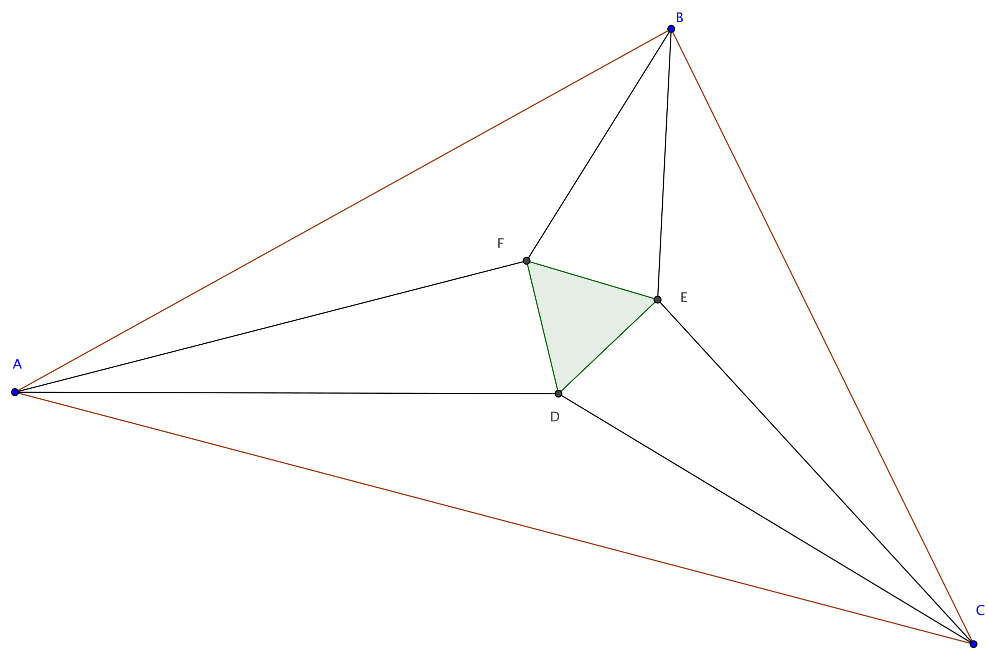
# Transactions
# in
# **Euclidean Geometry**

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# Rhombus Angle Equivalence

Alexa DeVore

August 24, 2018

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**Theorem 1.1a.** Let ABCD be a rhombus. Then angle DAB is congruent to angle BCD.

*Proof.* Let the points B and D be collinear. Now we have two triangles that we will call DAB and BCD. Based off the definition of a rhombus, we know that all four sides of the rhombus must be congruent so therefore segments AB, BC, CD, and DA must be congruent. Segment DB must be congruent to itself. Hence, our two triangles have segments DA and BC, AB and CD, and BD to BD respectively congruent to each other. Therefore our two triangles DAB and BCD have three pairs of congruent sides and we have two congruent triangles based off of Euclid 1.8. Since we know the triangles are congruent, we know that each pair of corresponding angles must be congruent from the definition of congruent triangles. Therefore, angle DAB is congruent to angle BCD. □

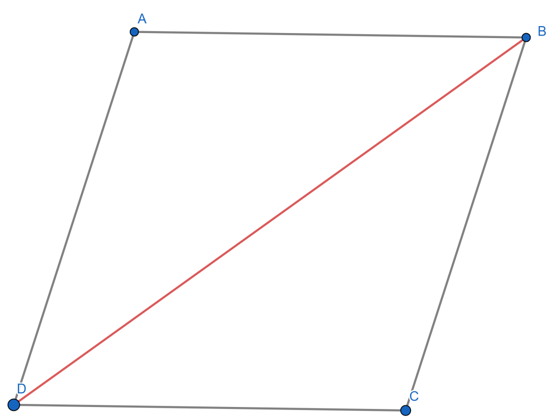


Figure 1: This is a picture of a rhombus.

# Diagonals of a Rhombus

Jaclyn Miller

August 29, 2018

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**Lemma 1.2a.** The diagonals of a rhombus bisect the angles of the rhombus.

*Proof.* Let ACDB be a rhombus. By the definition of a rhombus, sides AB, BD, DC, and AC are congruent. By Euclid Postulate I.1, one can construct line segment BC. According to the Miller, Stine, Warner Theorem 1.6, a rhombus is a parallelogram, thus line segments AB and DC are parallel. Therefore, the alternate interior angles BCA and CBD are congruent. Similarly, line segments BD and AC are parallel, thus alternate interior angles ABC and BCD are congruent.

Line segments AB and AC are congruent so triangle ABC is an isosceles triangle by Euclid Definition I.20. Therefore, by Euclid I.5, the base angles ABC and BCA are congruent. Similarly, line segments BD and DC are congruent, so triangle DCB is an isosceles triangle by Euclid Definition I.20. Therefore, by Euclid I.5, the base angles BCD and CBD are congruent.

But angle CBD was already found to be congruent with angle BCA. Thus, angle BCA is congruent to angle BCD because two angles congruent to the same angle are congruent. Thus, angle ACD is bisected by line segment BC.

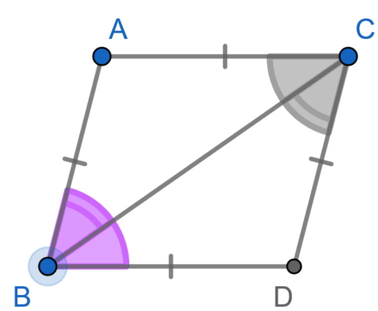


Figure 1: Rhombus ACDB with alternate angles congruent and bisected angles congruent.

Similarly, recall that alternate interior angles ABC and BCD are congruent. We also have base angles ABC and BCA are congruent. But, angle BCA was already found to be congruent with angle CBD. Therefore, angles ABC and CBD are congruent because two angles congruent to the same angle are congruent. Thus, angle ABD is bisected by line segment BC.

In the same way it can be shown that line segment AD bisects angles BAC and CDB.

□

**Theorem 1.2.** The diagonals of a rhombus must cross.

*Proof.* Let BCDA be a rhombus. By Euclid Postulate I.1, the line segment AB can be extended. Let arbitrary point F be placed on line AB and follow points A and B. Similarly, line segment DC can be extended. Let arbitrary point E be placed on line DC and follow points D and C. Similarly, line segments AD and BC can be extended.

Since a rhombus is a parallelogram by Miller, Stine, Warner Theorem 1.6, the sides AB and DC are parallel. Similarly sides BC and AD are parallel.

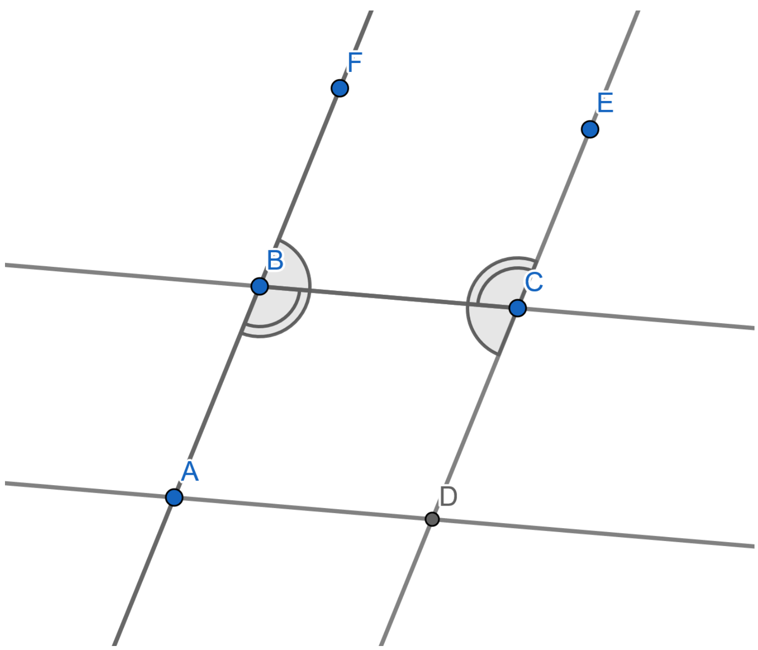


Figure 2: Rhombus with Euclid I.29 applied

By Euclid I.29, angle FBC is congruent to angle BCD. Similarly, angle ABC is congruent to angle ECB. Also by Euclid I.29, angles ABC and FBC taken together are congruent to two right angles. Similarly, angles ECB and BCD taken together are congruent to two right angles. Since angle ABC is congruent to angle ECB, it follows that angles ABC and BCD taken together are congruent to two right angles.

By Lemma 1.2a (above) line segment AC bisects angle BCD, and line segment BD bisects angle ABC. Thus angle CBD and angle BCA taken together would be less than two right angles. By Euclid Postulate I.5, the line segments AC and BD must cross.

$\square$

# Construction of a Rhombus

Jaclyn Miller

August 29, 2018

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**Theorem 1.4.** Given a line segment AB, it is possible to construct, with a compass and straightedge, a rhombus ABDC having sides congruent to AB.

*Proof.* Let AB be a line segment. It is required that a rhombus be constructed from line segment AB. By Euclid's Postulate I.3, draw a circle with center A and line segment AB as its radius. Similarly, draw a circle with center B and line segment AB as its radius.

Select an arbitrary point C on the edge of circle A. By Euclid's Postulate I.1, draw line segment AC. By Euclid's Postulate I.3, draw a circle with center C and line segment AC as its radius. By the Circle-Circle Intersection Property, circle C and circle B will intersect twice. Once is at point A. Place point D at the second intersection of circle C and circle B. By Euclid's Postulate I.1, draw the line segment CD. Similarly, draw line segment BD. Thus, we have quadrilateral ABDC.

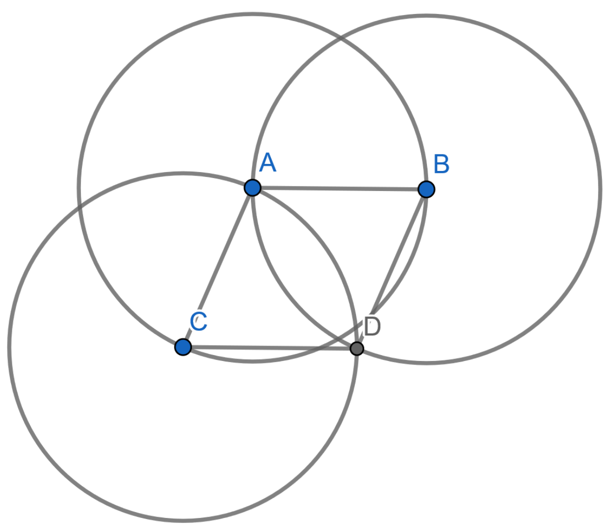


Figure 1: Rhombus ABDC inside circles A, B, and C.

Similar to Euclid I.1: since point A is the center of circle A, line segment AB is congruent to line segment AC. Since point B is the center of circle B, line segment AB is congruent to line segment BD. Similarly, since point C is the center of circle C, line segment AC is congruent to line segment CD.

But AC was also proved to be congruent to line segment AB, and line segment AB was also proved to be congruent to line segment BD. Thus the four line segments that comprise quadrilateral ABDC: AB, AC, BD, and CD, are congruent. Therefore, the quadrilateral ABDC is a rhombus.

$\square$

# A Rhombus is a Parallelogram

Jaclyn Miller, Jason Stine, Brad Warner

August 29, 2018

*Communicated by: The Editor*

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**Definition 1.** A parallelogram is a quadrilateral with two sets of opposite and parallel sides.

**Theorem 1.6.** If ADCB is a rhombus, then ADCB is a parallelogram.

*Proof.* Let ADCB be a rhombus. Since ADCB is a rhombus, it has four mutually congruent sides. By the DeVore Theorem 1.1A, angle BAD and angle DCB are congruent. By Euclid's Postulate I.1, one can construct line segment BD. Since line segments AD, DC, BC, and BA are congruent, and line segment BD is common, triangle BAD and triangle DCB are congruent by Euclid I.8. Since triangle BAD is congruent to triangle DCB, the corresponding components of each triangle are also congruent. Thus, angle ADB is congruent to angle DBC. Therefore, since the alternate interior angles ADB and DBC are congruent, by Euclid I.27, the opposite line segments AD and BC are parallel.

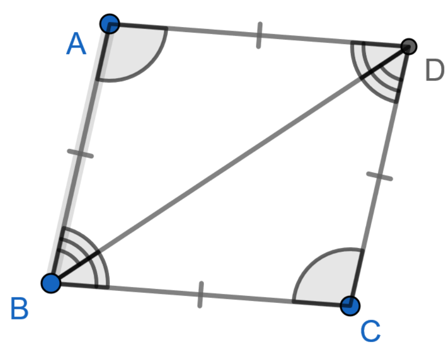


Figure 1: Rhombus ABDC

Similarly, angle ABD is congruent to angle BDC. Thus, since the alternate interior angles ABD and BDC are congruent, by Euclid I.27, the opposite line segments AB and DC are parallel. Therefore, a rhombus ADCB is a parallelogram because it is composed of two sets of opposite and parallel sides.

□

# Bisectors and Parallel Lines

Jaclyn Miller

September 4, 2018

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This theorem is based on a conjecture from Ms. Falck. Ms. Faulk proved that diagonal AC bisects diagonal BD and that diagonal BD bisects diagonal AC. With Ms. Falck's proof and the following, Theorem 1.6 (a rhombus is a parallelogram) is proved.

**Theorem B.** Let ABCD be a rhombus. Assume that the diagonals AC and BD bisect each other at a point X. Then AB and CD are parallel.

*Proof.* Let ABCD be a rhombus with diagonals AC and BD bisecting each other at point X. Line segments AB, BC, CD, AD are congruent by definition of a rhombus. Since AC and BD bisect each other, line segment AX is congruent to line segment XC. Similarly, line segment BX is congruent to line segment XD.

By Euclid Proposition I.8, triangle AXD is congruent to triangle BXC, therefore the components of these triangles are congruent. Thus, angle ADX is congruent to angle XBC.

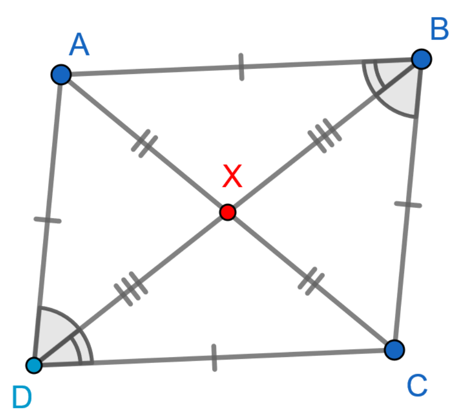


Figure 1: Rhombus ABCD

By Euclid Proposition I.8, triangle AXB is congruent to triangle DXC, which means the components of these triangles are congruent. Thus, angle XDC is congruent to angle XBA.

Therefore, since alternate interior angles ADX and XBC are congruent, and alternate interior angles XDC and XBA are congruent, line segment AB is parallel to line segment CD by Euclid Proposition I.27. In the same way, it can be shown that line segment AD is parallel to line segment BC. Thus, a rhombus is a parallelogram by definition.

$\square$