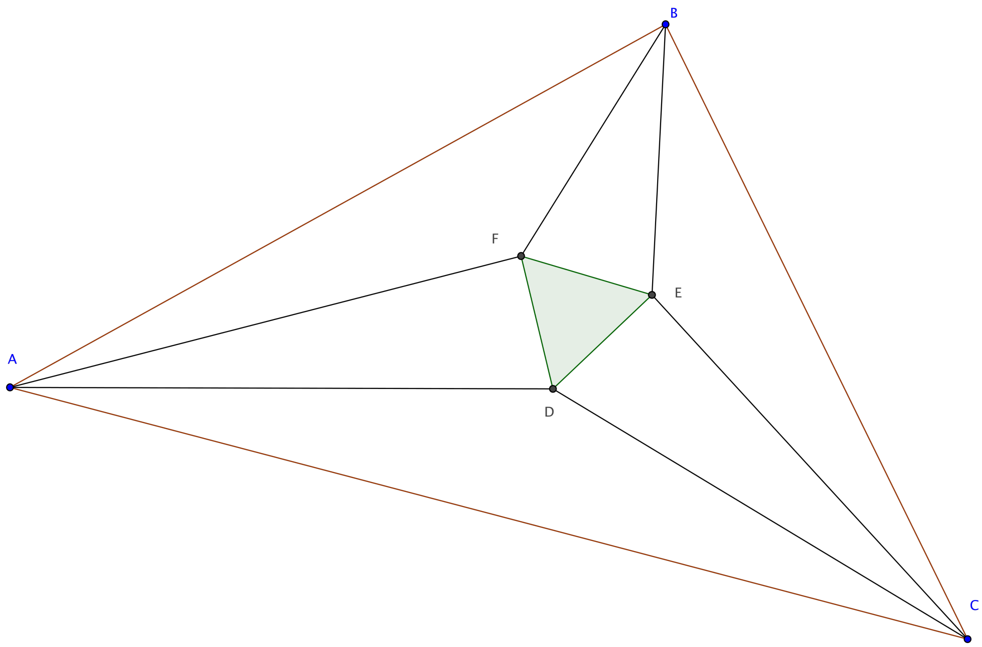
Transactions

in

Euclidean Geometry



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The Rhombus

Cameron Amos and Steven Flesch

September 22, 2017

Communicated by: Grant Kilburg

Abstract

We prove Theorem 1.1a using a direct proof with Postulate 1 and Euclid I.8 (SSS).

Theorem 1.1a. Let $ABCD$ be a rhombus. Then angle ABC is congruent to angle ADC.

Proof. Let $ABCD$ be a rhombus. Since $ABCD$ is a rhombus, then $ABCD$ is a quadrilateral and has vertices A, B, C, and D. Since $ABCD$ is a quadrilateral, then the segments AB, BC, CD, and DA are sides. Since $ABCD$ is a rhombus, then the sides are mutually congruent. By Postulate 1, draw a straight line segment, AC, from vertex A to vertex C. Since AB, BC, CD, DA are mutually congruent and AC is the base of both triangle ABC and triangle ADC, then triangle ABC and triangle ADC are congruent by Euclid I.8 (SSS). Since triangle ABC and triangle ADC are congruent, then angle ABC is congruent to angle ADC. $\square$

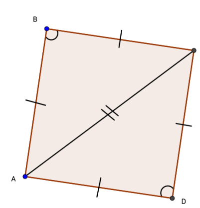

Figure 1: This is a picture of rhombus $ABCD$ representing angle ABC is congruent to angle ADC.

The Rhombus is a Parallelogram

Steven Flesch, Grant Kilburg, and Ashlyn Thompson

September 22, 2017

Communicated by: Cameron Amos

Definition 1. A parallelogram is a quadrilateral that has two sets of parallel sides that are opposite from each other.

Theorem 1.6. If $ABCD$ is a rhombus, then $ABCD$ is a parallelogram.

Proof. Let $ABCD$ be a rhombus. Since $ABCD$ is a rhombus, then its sides are mutually congruent. By the Amos-Flesch Theorem, angle ABC is congruent to angle ADC. We draw segment AC by Postulate 1.1. Triangle ABC is congruent to triangle CDA by Euclid I.4. Since triangle ABC is congruent to triangle CDA, angle BCA is congruent to angle CAD. Similarly, angle DCA is congruent to angle BAC. Line segment BC is parallel to line segment AD by Euclid I.27. Similarly, line segment AB is parallel to line segment CD. Thus, figure $ABCD$ is a parallelogram. $\square$

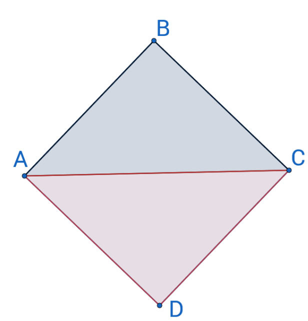

Figure 1: This is a picture of rhombus $ABCD$ showing Euclid I.4.

Diagonals of a Rhombus

Grant Kilburg

September 22, 2017

Communicated by: Ashlyn Thompson.

The following theorem proves that the diagonals of a rhombus will always bisect each other at right angles.

Definition 1. Let ABCD be a quadrilateral. We say ABCD is a parallelogram if its pairs of opposite sides AB, CD and BC, AD are both parallel.

Theorem 1.7. Let ABCD be a rhombus. Suppose that the diagonals AC and BD meet at a point X. The angle AXB is a right angle.

Proof. Let ABCD be a rhombus with diagonals AC and BD meeting at a point X. Since ABCD is a rhombus, then ABCD is a quadrilateral with vertices A, B, C, and D. By definition of a quadrilateral, ABCD also possesses four sides, which include segments AB, BC, CD, and DA. Since ABCD is a rhombus, sides AB, BC, CD, and DA are mutually congruent. Further, since ABCD is a rhombus, ABCD is a parallelogram by Theorem 1.6. Since ABCD is a parallelogram, opposite sides and angles are congruent to one another by Euclid I.34. Therefore, AD is parallel to BC by definition of a parallelogram.

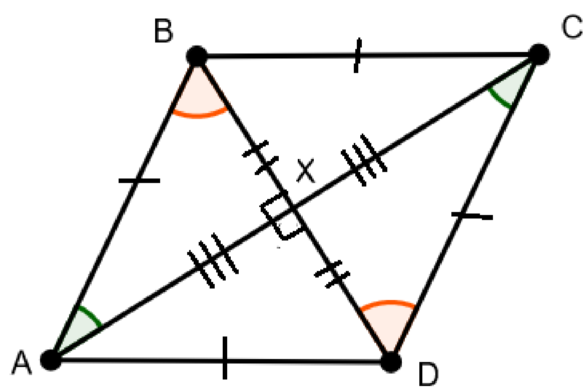

Figure 1: The rhombus ABCD is split into four congruent triangles via its diagonals.

Since AB is parallel to BC, and AC has fallen upon them, the alternate angles BAX and DCX are equal to one another by Euclid I.29. Similarly, since BC is parallel to AD, and BD has fallen upon them, the alternate angles ABX and CDX are equal to one another by

Euclid I.29. Thus, angle BAX is congruent to angle DCX, angle ABX is congruent to angle CDX, and sides AB and CD are congruent. Hence, triangle AXB is congruent to triangle CXD by Euclid I.26 (ASA).
Since triangle AXB is congruent to triangle CXD, segments AX and CX are congruent. Thus, X is the midpoint of the diagonal AC. Therefore, line segment BD bisects AC. Similarly, since triangle AXB is congruent to triangle CXD, segments BX and DX are congruent. Thus, X is the midpoint of the diagonal BD. Hence, line segment AC bisects BD.

Since ABCD is a rhombus with AC bisecting BD and BD bisecting AC, triangle AXB is congruent to triangle AXD by Euclid I.8 (SSS). Therefore, angle AXB is congruent to angle AXD. Furthermore, since line segment AX is a straight line set up on BD, angle AXB plus angle AXD will make right angles or angles equal to two right angles by Euclid I.13. Further, since angle AXB is congruent to angle AXD, both angle AXB and angle AXD must be right angles. Hence, angle AXB is a right angle. □

Diagonals of a Rhombus, II

Grant Kilburg, Kayla Schafbuch, Ashlyn Thompson

September 22, 2017

Communicated by: Emily Carstens

Theorem 1.2. The diagonals of a rhombus must cross.

Proof. Let $ABCD$ be a rhombus. By Euclid Postulate 2, let segments AB,BC,CD, and DA be extended into straight, continuous lines. Let point E be chosen such that it falls on line AB and follows points A and B. Let point F be chosen such that it falls on line DC and follows points D and C. By Mr. Flesch's Theorem 1.6, rhombus $ABCD$ is a parallelogram. By Euclid I.29, angle BCF is congruent to angle ABC and angle EBC is congruent to angle DCB. By Euclid I.13, when angle BCF and angle DCB are taken together, they are equal to two right angles. Since angle BCF is congruent to angle ABC, then by Euclid Common Notion 2, when angle ABC and angle DCB are taken together, they are equal to two right angles. By Mr. Hertzler's Conjecture E, diagonal AC bisects angle DCB, and diagonal BD bisects angle ABC. Since angle DCB and angle ABC when taken together are equal to two right angles, when bisected the angles taken together must be less than two right angles. Therefore, by Euclid Postulate 5, diagonals AC and BD must cross.

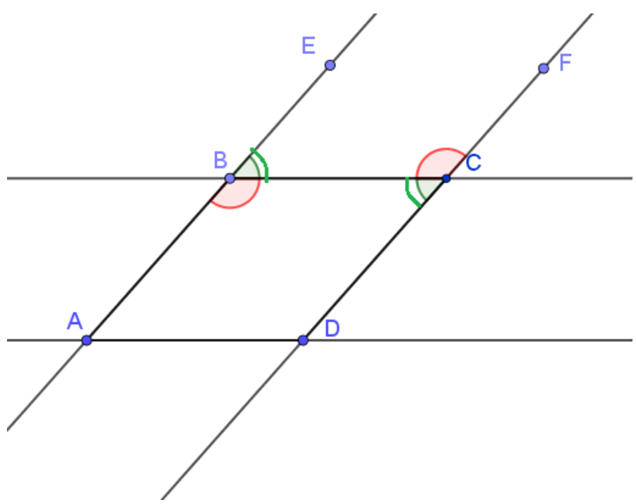

Figure 1: This is a diagram showing congruent angles in the rhombus as determined by Euclid I.29.

□

A Rhombus Which Is Not A Square

Emily Carstens

September 22, 2017

Communicated by: Kayla Schafbuch.

Theorem D. It is possible to construct a rhombus which is not a square.

Proof. The construction of a rhombus begins with constructing two equilateral triangles.

1. Let B and D be points, and draw the line BD using postulate 1.
2. Construct an equilateral triangle using proposition 1 on line BD. Label this triangle BAD.
3. Construct an equilateral triangle using proposition 1 and line BD such that angle BCD is opposite angle BAD. Label this triangle BCD.

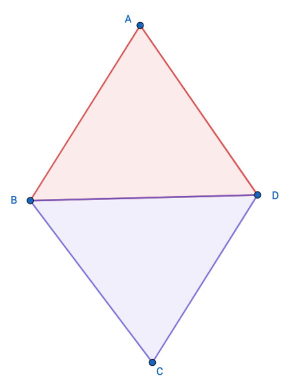

Figure 1: This is a picture of the two equilateral triangles BAD and BCD

By Euclid's Definition I.20, segment AB is congruent to segment AD is congruent to segment BD. By Euclid's Definition I.20, segment BD is congruent to segment CD is congruent to segment BC. Therefore, by Euclid's Common Notion I.1, segment AB is congruent to segment AD is congruent to segment CD is congruent to segment BC, then ABCD is a rhombus by the definition of a rhombus.

Since segment AB is congruent to segment AD, then by Euclid's Proposition I.5 angle ABD is congruent to angle ADB. Since segment AB is congruent to segment BD, then by Euclid's Proposition I.5 angle ADB is congruent to angle BAD. Therefore, angle ABD is congruent to angle ADB is congruent to angle BAD by Euclid's Common Notion I.1.

Since segment BC is congruent to segment CD, then by Euclid's Proposition I.5, angle DBC is congruent to angle BDC. Since segment BD is congruent to segment DC then by Euclid's Proposition I.5, angle BCD is congruent to angle DBC. Therefore, angle DBC is congruent to angle BDC is congruent to angle BCD by Euclid's Common Notion I.1.

By Euclid's Proposition I.32, all three angles of triangle BAD and all three angles of triangle BCD add up to two right angles. Since all three angles are congruent in each triangle, then using division it can be said that angle ABD, angle ADB, angle BAD, angle DBC, angle BDC, and angle BCD are congruent to 2/3 of a right angle.

By Theorem E, segment BD bisects angle ABC and angle ADC. Therefore angle ABC and angle ADC are congruent to 4/3 of a right angle using addition.

Angle BAD and angle BCD are each congruent to 2/3 of a right angle, which is less than a whole right angle. Angle ABC and angle ADC are congruent to 4/3 of a right angle, which is greater than a right angle. Therefore by Euclid's Definition I.22, ABCD is not a square. Therefore there exists a rhombus which is not a square.

□

Angles of a Kite

Grant Kilburg

September 22, 2017

Communicated by: Connor Coyle.

While working with kites and the angles formed by their sides, we stumbled upon the following property. The proof below illustrates our findings.

Theorem I. If ABCD is a kite with AB congruent to AD and DC congruent to BC, then line segment AC bisects angle DAB and angle DCB.

Proof. Let ABCD be a kite. Since ABCD is a kite, then segment AB is congruent to segment AD and segment DC is congruent to segment BC. By Postulate 1.1, construct line segment AC. Since AB is congruent to AD, DC is congruent to BC, and AC is shared, then triangle ADC is congruent to triangle ABC by Euclid I.8 (SSS). Since triangle ADC is congruent to triangle ABC, then angle DAC is congruent to angle BAC. Similarly, angle DCA is congruent to angle BCA. Hence, line segment AC bisects angle DAB and angle DCB. □

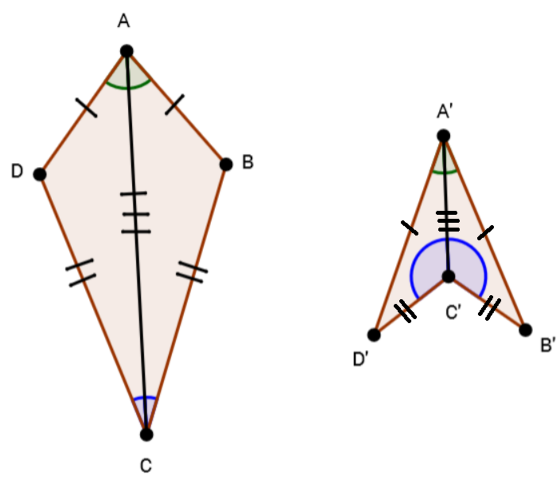

Figure 1: By Euclid I.8, triangle ADC is congruent to triangle ABC. The same holds true for Ms. Thompson's Star Trek kite which is shown above as ABCD'.

Diagonals of a Kite

Ashlyn Thompson

September 21, 2017

Communicated by: Grant Kilburg

Theorem 2.5. If the diagonals of a kite meet, then they meet at right angles.

Proof. Let $ABCD$ be a kite such that AB is congruent to BC and AD is congruent to CD. Let AC and BD be diagonals of kite $ABCD$, and let point E be the point of intersection of the diagonals. Since AB is congruent to BC, AD is congruent to CD, and triangle ABD and triangle CBD share side BD, by Euclid I.8, triangle ABD is congruent to triangle CBD. Therefore, angle ABD is congruent to angle CBD. Since AB is congruent to BC, angle ABD is congruent to angle CBD, and triangle ABE and triangle CBE share side BE, by Euclid I.4, triangle ABE is congruent to triangle CBE. Therefore angle AEB is congruent to angle CEB. By Euclid I.13, angle AEB and angle CEB taken together make two right angles. Since angle AEB is congruent to angle CEB, and when taken together make two right angles, then angle AEB and angle CEB must be right angles. By Mr. Hertzler's Lemma, angle DEC and angle DEA are also right angles. Therefore, diagonals BC and AD meet at right angles.

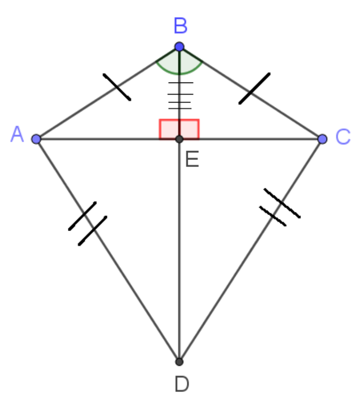

Figure 1: Kite $ABCD$ with congruent triangles ABD and CBD by Euclid I.8, and congruent triangles ABE and CBE by Euclid I.4.

□