Computability and Complexity theory
Assignment # 5

April 18, 2023

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1. Which of the following statements are true, which are false? Give a short proof of your answers.
   (a) If L and L' are P-complete, so is $L \cup L'$.

   Ans:

   (b) If $\mathsf{L} = \mathsf{P}$, then every nontrivial language in L is P-complete.

   Ans:

   (c) $\mathsf{PolyL} \triangleq \bigcup_{i \in \mathbb{N}} \mathsf{DSpace}(\log^i n)$ has complete problems (under log space many-one reductions).

   Ans:

2. Show that the following problem is hard for NL under log-space many-one reductions.
   Input: A set A with an associative binary operation '$*$', a subset $S \subseteq A$, and an element $w \in A$.
   Output: YES if there is a set of elements $a_1, \ldots, a_k \in S$ such that $a_1 * a_2 * \ldots * a_k = w$.
   Assume that A is closed under $*$. Is the problem complete for NL? Prove your answer.

   Ans:

3. Let $G = (V, E)$ be a directed graph. Consider two persons playing the following game: The first player occupies some given vertex v_0. Then the second player chooses a non-occupied neighbor v_1 of v_0 (i.e. $(v_0, v_1) \in E$) and occupies it. Then the first player chooses some non-occupied neighbor v_2 of v_1 and occupies it (in addition to v_0). The game is continued in this way until one play is not able to extend the path $v_0, v_1, \ldots$ by a non-occupied neighbor of the current vertex. The player who played the last move wins the game.

   GEO is the following problem: Given a directed graph G and a vertex v_0 of G decide whether the first player has a winning strategy for the game above.

Prove that GEO is PSPACE-complete.

Ans:

4. Let $G = (V, E)$ be a directed graph. Two vertices u and v are said to be strongly connected if there is a directed path from u to v in G as well as there is a directed path from v to u in G. Let

$$\mathsf{StrConn} = \{\langle G, s, t\rangle \mid s \text{ and } t \text{ are strongly connected in } G\}.$$

Show that StrConn is NL-complete (under logarithmic space many-one reductions.)

Ans:

5. A language A is called *downward self-reducible*, if there is a polynomial time oracle deterministic Turing machine M that on input x, only queries oracle strings with length $< |x|$ such that $A = L(M^A)$.
   (a) Show that SAT is downward self reducible.
   (b) Show that if A is downward self reducible, then $A \in \mathsf{PSPACE}$.

Ans: