Masters Theorem → Recurrence Relation

$$\boxed{T(n) = a\,T\left(\frac{n}{b}\right) + f(n)}$$

a & b → positive constants; $a \geq 1$, $b > 1$

$f(n) = \theta(n^k \log^p n)$

$f(n)$ → +ve function $\quad \theta(n^k \log^p n)$

case 1 if $\log_b a > k$

$\Rightarrow \theta(n^{\log_b a})$

1) $\log_b a$ $\quad f(n)$ $k=0$, $p=0$

$2T\left(\frac{n}{2}\right) + 1 = T(n)$

$a=2$, $b=2$ $\quad \log_2 2 = 1$

Case 2 if $\log_b a = k$

- $p > -1$: $\theta(n^k \log^{p+1} n)$
- $p = -1$: $\theta(n^k \log\log n)$
- $p < -1$: $\theta(n^k)$

2) compare $\log_b a$ & k

$k \rightarrow f(n)$

case 3 $\log_b a < k$

$p \geq 0 \quad \theta(n^k \log^p n)$

$p < 0 \quad \theta(n^k)$

Example 1 $T(n) = 2T(n/2) + 1$ $\quad$ case 1 $\log_b a > k$

$\log_b a = 1$, $k = 0$

↳ case 1 → $O(n^{\log_b a})$

$\Rightarrow O(n)$

Example 2

$T(n) = 4T\left(\frac{n}{2}\right) + n \rightarrow \theta(n^k \log^p n)$

$k = 1$, $p = 0$

$a = 4$, $b = 2$

$\log_2 4 = \log_2 2^2 = 2\cancel{\log_2 2} = 2$

$$\underset{(2)}{\log_b a} > \underset{(1)}{k} \longrightarrow \underline{\text{case 1}}$$

$$\Rightarrow O(n^2)$$

Example 3

$$T(n) = 8T\left(\frac{n}{2}\right) + n^2 \longrightarrow n^k \log^p n$$

$k = 2, \; p = 0$

$$a = 8, \; b = 2$$

$$\log_b a = \log_2 8 =$$

$$\log_2 2^3 = 3 \cancel{\log_2 2}^{\,1}$$

$$= 3$$

$$\underset{(3)}{\log_b a} > \underset{(2)}{k} \longrightarrow O(n^3)$$