# Recurrence Relation

**Substitution Method**

$$T(n) = \begin{cases} 1 & n = 1 \rightarrow \text{Base case condition} \\ T(n/2) + n & n > 1 \end{cases}$$

↑ Recursive Term

$T(n) = T(n/2) + n$ — 1st time

$T(n/2) = T\left(n/2^2\right) + \frac{n}{2}$

**Pattern**

$$\boxed{T(n) = T\left(\frac{n}{2^2}\right) + \frac{n}{2} + n}$$ — 2nd time

$T(1) = 1$

$T(n) = T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} + \frac{n}{2} + n$ — 3rd time

$k \rightarrow$ ??

$\frac{n}{2^k} = 1 \rightarrow$ base case condition

$n = 2^k$

$k = \log_2 n$

$T(n) = T\left(\frac{n}{2^k}\right) + \frac{n}{2^{k-1}} + \frac{n}{2^{k-2}} + \dots + \frac{n}{2} + n$

$2^{\log_2 n} \Rightarrow n^{\cancel{\log_2 2}} = n$

← Reverse Order

$T(n) = T\left(\frac{n}{2^{\log_2 n}}\right) + \left\{\frac{n}{2^{\log_2 n - 1}} + \frac{n}{2^{\log_2 n - 2}} + \dots + n/2 + n\right\}$

Mathematical Series (Basic)

$= T(1) + n\left(\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \dots + \left(\frac{1}{2}\right)^{\log_2 n - 1}\right)$

↳ GP Series

$r = 1/2$

$a = \left(\frac{1}{2}\right)^0 = 1$

$T(1) = 1$

$r < 1$, $\frac{a(1 - r^n)}{1 - r}$

$$\Rightarrow 1 + \left\{ \frac{\left(1 - \frac{1}{2}^{\log_2 n}\right)}{1 - \frac{1}{2}} \right\} * n$$

$$\Rightarrow 1 + \underline{(1)\, n} \rightarrow \text{Higher term}$$

$$\Rightarrow O(n)$$