Masters Theorem → Recurrence Relation

$$\boxed{T(n) = aT\left(\frac{n}{b}\right) + f(n)}$$

a & b → positive constants; $a \geq 1$, $b > 1$

$f(n) = \theta(n^k \log^p n)$

$f(n)$ → +ve function $\quad \theta(n^k \log^p n)$

case 1 $\quad$ if $\log_b^a > k$

$\Rightarrow \theta(n^{\log_b^a})$

1) $\log_b^a$ $\quad$ $f(n)$ $k=0$, $p=0$

$2T\left(\frac{n}{2}\right) + 1 = T(n)$

$a = 2$ $\quad \log_2 2 = 1$

$b = 2$

Case 2 $\quad$ if $\log_b^a = k$

- $p > -1$: $\theta(n^k \log^{p+1} n)$
- $p = -1$: $\theta(n^k \log\log n)$
- $p < -1$: $\theta(n^k)$

2) compare $\log_b^a$ & k

k ↓ $f(n)$

case 3 $\quad \log_b^a < k$ ←

$p \geq 0 \qquad \theta(n^k \log^p n)$

$p < 0 \qquad \theta(n^k)$

$T(n) = 2T\left(\frac{n}{2}\right) + n^2 \rightarrow k = 2, \; p = 0$

$\log_b^a = \log_2^2 = 1 < 2$

$\theta(n^2)$

$\log_b^a < k \rightarrow \theta(f(n))$

$T(n) = 2T(n/2) + n^2 \log n$

$\log_b^a = 1 < 2$

$\theta(n^2 \log n)$

$$T(n) = 4T\left(\frac{n}{2}\right) + \frac{n^3}{\log n}$$

$a = 4$ $\quad \log_b a = \log_2 4 = 2$

$b = 2$

$k = 3$

$\log_b a < k \rightarrow p = -1 < 0$

$\underline{\Theta(n^3)}$