Substitution Method

Example 1

(Assumption → Already provided Recurrence Relation)

$$T(n) = \begin{cases} 1 & n=1 \\ T(n-1)+n & n>1 \end{cases}$$

$T(1) = 1$ — Base case condition

($n>1$ — Termination)

$$T(n) = T(n-1) + n \quad \text{---(2)}$$

(Substitute the Recursive term)

$n-1+1$

$$T(n-1) = T(n-2) + n-1 \quad \text{---(1)}$$

$n-2+1$

$$T(n) = T(n-2) + n-1 + n$$

$$= T(n-3) + n-2 + n-1 + n$$

$n-3+1$

k times $\Rightarrow$ $n-k = 1$

$n-1 = k$

$$= T(n-k) + (n-k+1) + (n-k+2) + \cdots + n-2 + n-1 + n$$

$$= T(n-(n-1)) + (n-(n-1)+1) + (n-(n-1)+2) + \cdots + n-2 + n-1 + n$$

$$= T(n-n+1) + (n-n+1+1) + (n-n+1+2) + \cdots + n-2 + n-1 + n$$

$$\Rightarrow T(1) + 2 + 3 + \cdots + n-2 + n-1 + n$$

$\Downarrow$ → Mathematical Series

$$1 + 2 + 3 + \cdots + n-2 + n-1 + n$$

(Sum of n natural numbers ≥ 1)

$$n(n+1)/2 = (n^2+n)/2$$

$\Rightarrow O(n^2)$

↳ AP, GP