

Build Heap  $\rightarrow O(n)$

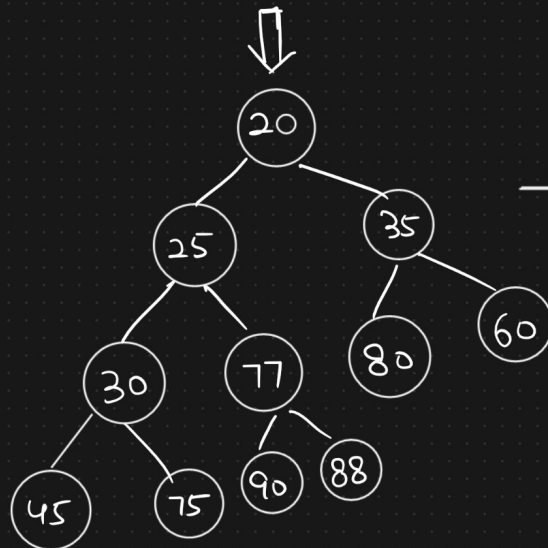
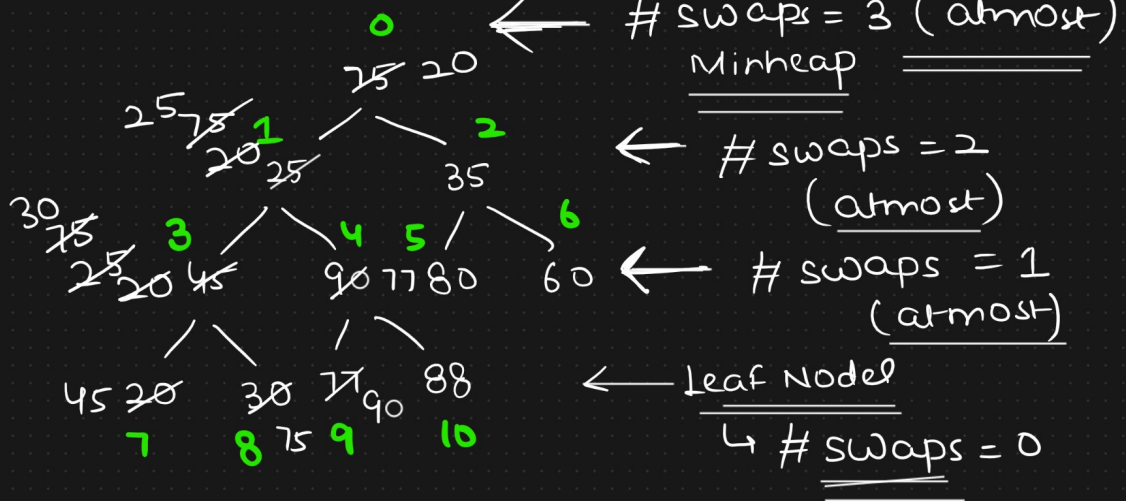
75, 25, 35, 45, 90, 80, 60, 20, 30, 77, 88

complete

Binary

Tree

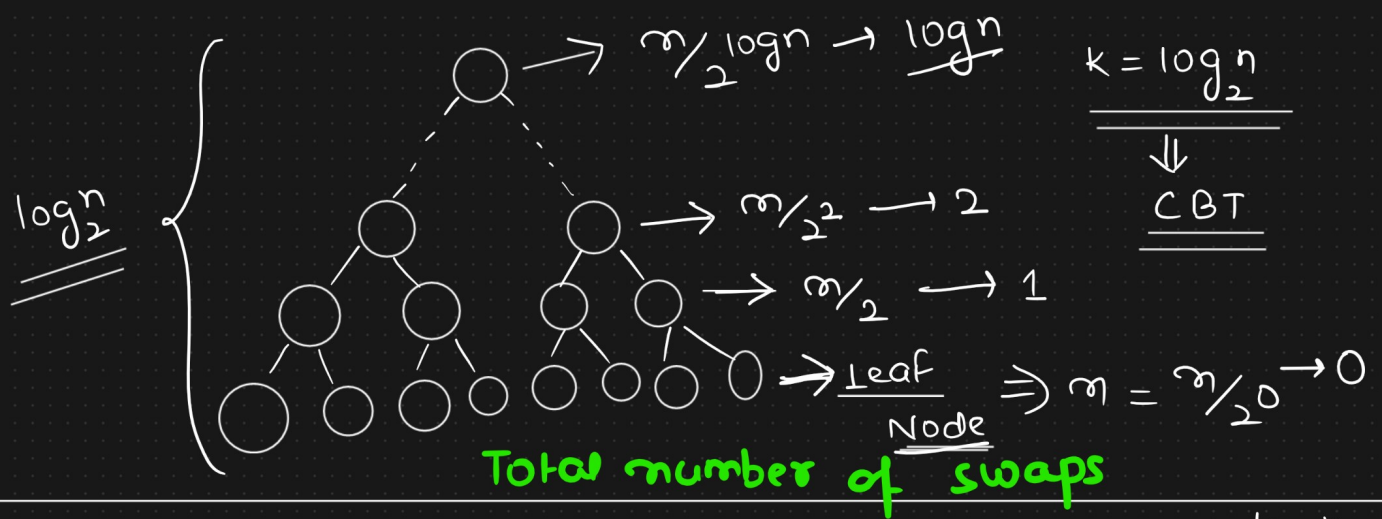
(CBT)



$\rightarrow$  valid Minheap

Heapify Method

Time complexity to build heap =  $O(n)$



$$S = \frac{n}{2^0} * 0 + \frac{n}{2^1} * 1 + \frac{n}{2^2} * 2 + \dots + \frac{n}{2^{\log n}} * \log n$$

$$S = n \left( \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{\log n}{2^{\log n}} \right) \text{--- ①}$$

Divide ① / 2

$\hookrightarrow$  GP series

$$\frac{S}{2} = n \left( \frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \dots + \frac{\log n - 1}{2^{\log n}} + \frac{\log n}{2^{\log n + 1}} \right) \text{--- ②}$$

Subtraction of ② from ①

$$\frac{S}{2} = n \left( \left( \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{\log n}} \right) - \frac{\log n}{2^{\log n + 1}} \right)$$

$\hookrightarrow$  **valid GP series**

$$a = \frac{1}{2}$$

$\downarrow$

first element

(sum of GP series)

$$r < 1$$

$$r = \frac{1/2^2}{1/2^1} = \frac{1}{2} < 1$$

$\downarrow$   
common ratio

$$\text{Sum of GP Series} = \frac{a(1-r^n)}{1-r} ; \underline{\underline{r < 1}}$$

$$\frac{S}{2} = n \left( \frac{\cancel{\frac{1}{2}} \left( 1 - \frac{1}{2} \log n \right)}{\cancel{1 - \frac{1}{2}}} - \frac{\log n}{2^{\log n + 1}} \right)$$

$$\frac{S}{2} = n \left( \frac{2^{\log n} - 1}{2^{\log n}} - \frac{\log n}{2^{\log n + 1}} \right)$$

$\Downarrow$   
 $2^{\log n} \cdot 2$

$$2^{\log_2 n} = n^{\frac{1}{\log_2 2}} = n$$

$$\frac{S}{2} = n \left( \left( \frac{n-1}{n} \right) - \frac{\log n}{n \times 2} \right)$$

$$\frac{S}{2} = \cancel{n} \frac{(n-1)}{\cancel{n}} - \cancel{n} \times \frac{\log n}{\cancel{n} \times 2}$$

$$\frac{S}{2} = n-1 - \frac{\log n}{2}$$

$$S = 2n - 2 - \log n$$

$$\underline{\underline{T(n) = O(n)}}$$

