

## Recurrence Relation

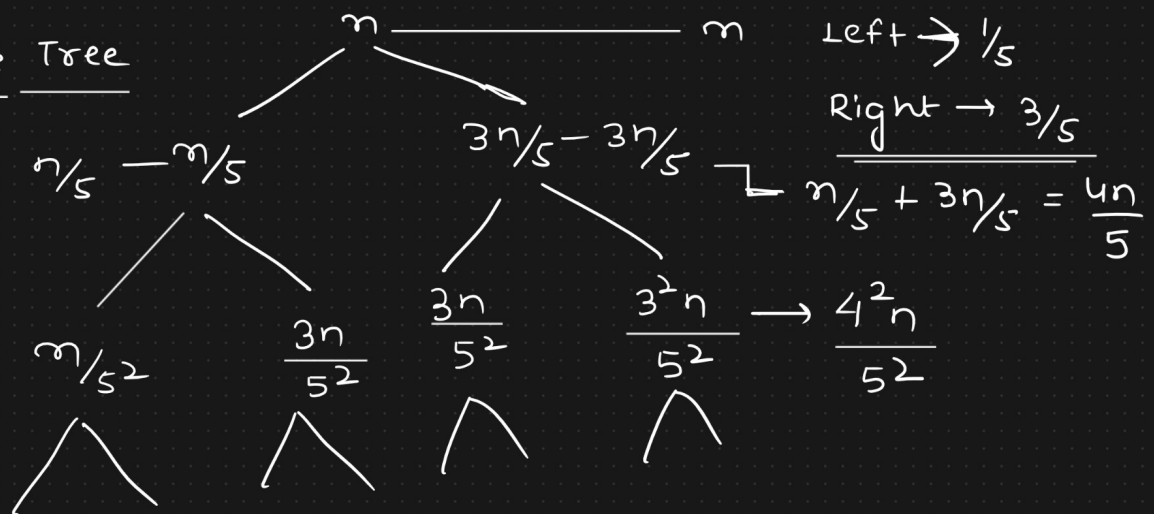
$$T(n) = \begin{cases} 1 & n=1 \\ T\left(\frac{n}{5}\right) + T\left(\frac{3n}{5}\right) + n & n>1 \end{cases}$$

## Recursive Tree

### Approach

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{3n}{5}\right) + \underline{n}$$

## Recursive Tree



$$n = 5^k$$

$$k = \log_5 n$$

①

$$\text{Right side} \rightarrow \frac{n}{\left(\frac{5}{3}\right)^k} = 1$$

High value

$$k = \log_{5/3} n$$

②

$$\left(\frac{4}{5}\right)^0 n + \left(\frac{4}{5}\right)^1 n + \left(\frac{4}{5}\right)^2 n + \dots + \left(\frac{4}{5}\right)^{\log_{5/3} n} n$$

$$= n \left( \left(\frac{4}{5}\right)^0 + \left(\frac{4}{5}\right)^1 + \left(\frac{4}{5}\right)^2 + \dots + \left(\frac{4}{5}\right)^{\log_{5/3} n} \right)$$

$$a = \left(\frac{4}{5}\right)^0 = 1 \quad \rightarrow \text{GP Series}$$

$$r = \frac{4}{5} \quad (r < 1)$$

$$S = \frac{a(1-r^n)}{1-r} \times n$$

$$\rightarrow O(1) \times n$$

$$\Rightarrow \underline{\underline{O(n)}}$$