Masters Theorem → Recurrence Relation

$$\Rightarrow \boxed{T(n) = aT\left(\frac{n}{b}\right) + f(n)}$$

a & b → Positive constants ; $a \geq 1$, $b > 1$

$f(n) = \theta(n^k \log^p n)$

$f(n)$ → +ve function $\theta(n^k \log^p n)$

case 1 if $\log_b a > k$

$\Rightarrow \theta(n^{\log_b a})$

Case 2 if $\log_b a = k$

$p > -1$ → $\theta(n^k \log^{p+1} n)$

$p = -1$ → $\theta(n^k \log\log n)$

$p < -1$ → $\theta(n^k)$

case 3 $\log_b a < k$

$p \geq 0$ $\theta(n^k \log^p n)$

$p < 0$ $\theta(n^k)$

1) $\log_b a$ $f(n)$ $k = 0$, $p = 0$

$$2T\left(\frac{n}{2}\right) + 1 = T(n)$$

$a = 2$ $\log_2 2 = 1$

$b = 2$

2) compare $\log_b a$ & k

k ↓ $f(n)$

case 1

$$T(n) = 9T\left(\frac{n}{3}\right) + 1$$

$O(n^k \log^p n)$

$k = 0$

$p = 0$

$a = 9$

$b = 3$

$$\log_b a = \log_3 9 = \log_3 3^2 = 2$$

$2 > 0$

↳ $O(n^2)$

Example 1

$$T(n) = 2T(n/2) + n \rightarrow n^k \log^p n$$

$a = 2$

$b = 2$ $\qquad k = 1,\ p = 0$

$$\log_b a = \log_2 2 = 1$$

$$\log_b a = k = 1$$

$p = 0 \quad p > -1$

$$\Theta(n^k \log^{p+1} n)$$

$$\Theta(n \log n)$$

Example 2

$$T(n) = 8T\left(\frac{n}{2}\right) + n^3 \longrightarrow k = 3,\ p = 0$$

$\log_b a = 3$ $\qquad O(n^k \log^{p+1} n)$

$$\Rightarrow O(n^3 \log n) \leftrightarrow O(n^3 \log n)$$

Example 3

$n \log^{-1} n$

$\downarrow\ n^k \log^p n$

$$T(n) = 2T(n/2) + \frac{n}{\log n}$$

$\log_b a = 1 \quad k = 1 \quad p = -1$

$$O(n \log \log n)$$

Example 4

$n \log^{-2} n$

$\downarrow$

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log^2 n}$$

$\log_b a = 1,\ k = 1$

$p = -2$

$p < -1$

$$\underline{\Theta(n)} \Leftarrow \Theta(n^k)$$