

Recurrence Relation \rightarrow Recursive

$$T(n) = \begin{cases} 1 & n=1 \rightarrow \text{Base case condition} \\ \underline{T(n-1)} + \frac{1}{n} & n > 1 \end{cases}$$

Substitution

Method

$$\underline{T(n)} = \underline{T(n-1)} + \frac{1}{n}$$

\downarrow
Recursive Term

$$\begin{aligned} \underline{T(n-1)} &= T(n-1-1) + \frac{1}{n-1} \\ &= T(n-2) + \frac{1}{n-1} \end{aligned}$$

$$T(n) = \underline{T(n-2)} + \frac{1}{n-1} + \frac{1}{n}$$

$n-2+1$

$$T(n) = T(n-3) + \frac{1}{n-2} + \frac{1}{n-1} + \frac{1}{n}$$

$n-3+1$

$$\underline{T(1) = 1}$$

$\left\{ \begin{array}{l} \underline{k \text{ times}} = (n-1) \underline{\text{times}} \end{array} \right.$

$$n-k=1$$

$$\begin{aligned} \textcircled{n-1=k} \quad T(n) &= T(n-k) + \frac{1}{n-k+1} + \frac{1}{n-k+2} + \dots \\ &\quad + \frac{1}{n-1} + \frac{1}{n} \end{aligned}$$

$$\begin{aligned} \Rightarrow T(n-(n-1)) &+ \frac{1}{n-(n-1)+1} + \frac{1}{n-(n-1)+2} \\ &+ \dots + \frac{1}{n-1} + \frac{1}{n} \end{aligned}$$

$$\Rightarrow \frac{T(1)}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n-1} + \frac{1}{n}$$

$$\Rightarrow 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n-1} + \frac{1}{n}$$

$$\Rightarrow \underline{\underline{O(\log n)}}$$