finding of Power of an Element

$a \rightarrow 2$

$n \rightarrow 10$

Output $= a^n = 2^{10} = 1024$

↳ $n \geq 1$ ←

$n < 1$ ←

$\begin{cases} 2^1 = 2 \\ 4^1 = 4 \end{cases}$ $5^1 = 5$

Divide & conquer

Small problem ⟶ $n = 1$

↳ return solution ↳ return a

big Problem ⟶ $n > 1$

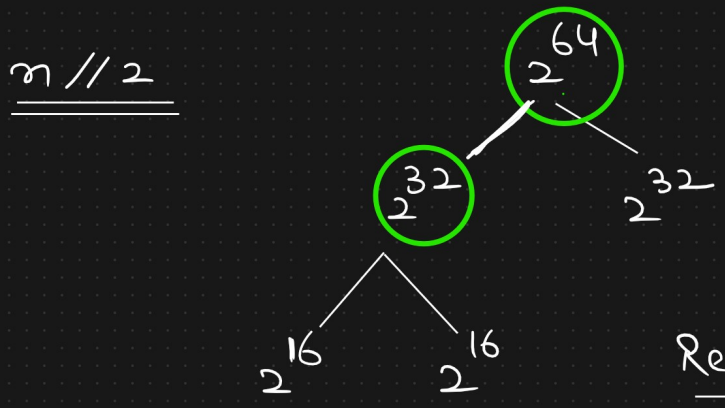
↳ Divide & conquer $a^n = a^{n/2} * a^{n/2}$

$n = 64$ $a = 2$

$n // 2$

$b = 2^{32}$

↳ b * b



Recursive Tree

$n \rightarrow$ even

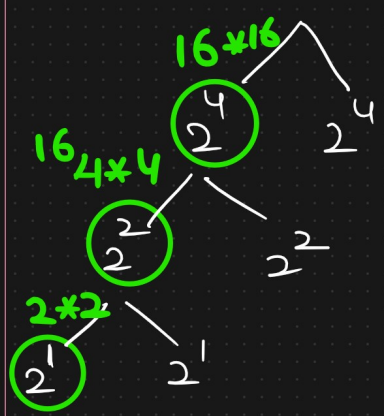
↳ $2^{10} = 1024$

256

16*16

16 4*4

2*2



$n \rightarrow$ odd $\rightarrow 2^{11}$

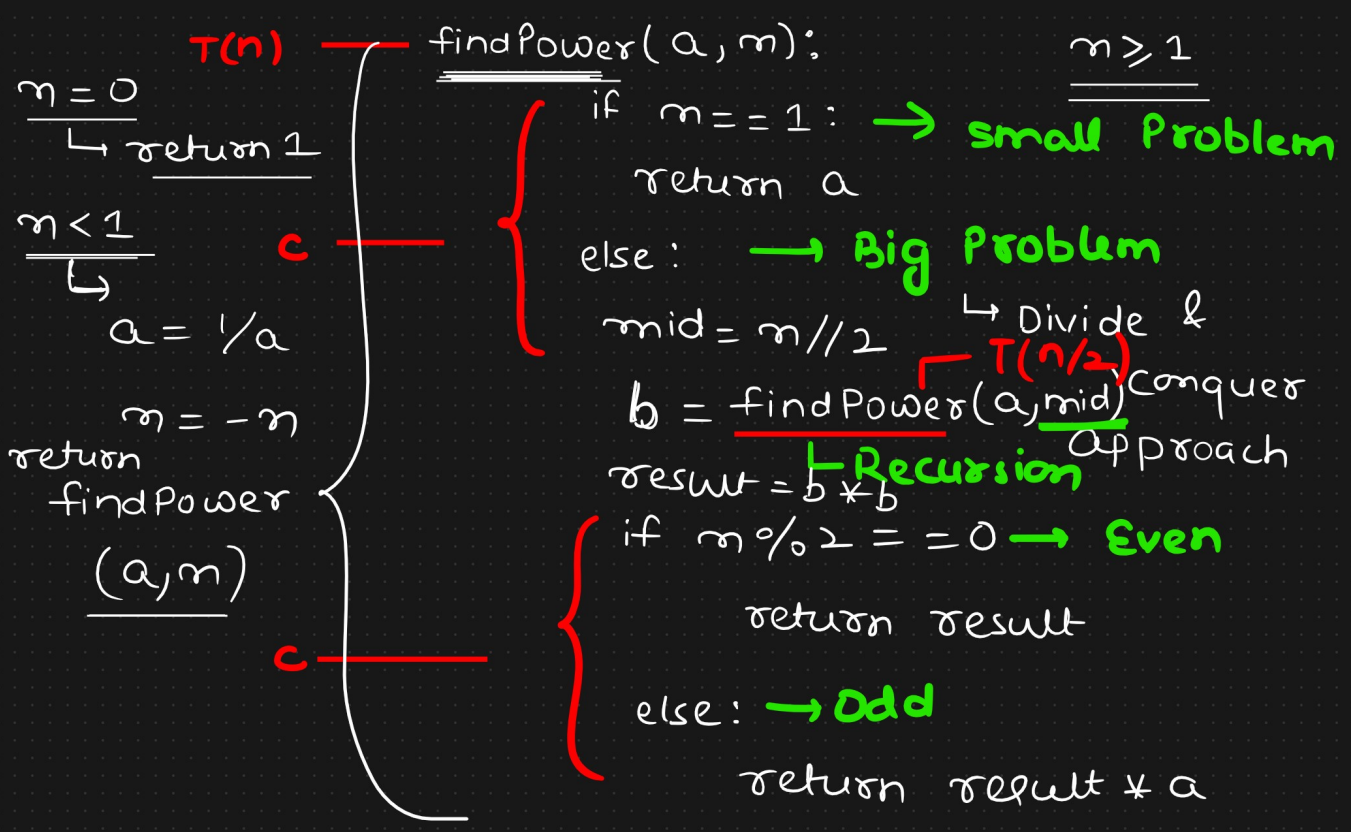
$2^{10} * \frac{2}{a}$

$a^n = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

$a^0 = 1$

$n = -4$

$a = 2$



Recurrence Relation

↳ $T(n) = T(n/2) + c$

↳ Master's Theorem or Substitution Method

Master's Theorem

$a = 1 \quad k = 0$

$b = 2 \quad p = 0$

$$\log_b^a = \log_2^1 = 0$$

$\log_b^a = k = 0$ — case 2

$p > -1 \Rightarrow p = 0$

$\Theta(\log n)$

Substitution Method → Recursive Term

$$T(n) = T(n/2) + c$$

$$= T(n/2^2) + c + c$$

$$= T\left(\frac{n}{2^3}\right) + c + c + c$$

$\frac{n}{2^k} = 1$ ↓ k times

$k = \log_2 n$

$$= T\left(\frac{n}{2^k}\right) + k \cdot c$$

$$= T\left(\frac{n}{2^{\log_2 n}}\right) + k \cdot c$$

$$= T\left(\frac{n}{n^{\log_2 2}}\right) + k \cdot c$$

$$= T(1) + k \cdot c$$

$$= 1 + \log_2 n \cdot c$$

$\Rightarrow O(\log_2 n)$