In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import contour
```

In [ ]:
```
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

def loss_function(theta, X, y):
    y_hat = sigmoid(theta[0] + theta[1] * X)
    loss = -np.sum(y * np.log(y_hat) + (1 - y) * np.log(1 - y_hat))
    return loss

def gradient(theta, X, y):
    y_hat = sigmoid(theta[0] + theta[1] * X)
    gradient_0 = np.sum(y_hat - y)
    gradient_1 = np.sum((y_hat - y) * X)
    return np.array([gradient_0, gradient_1])
```

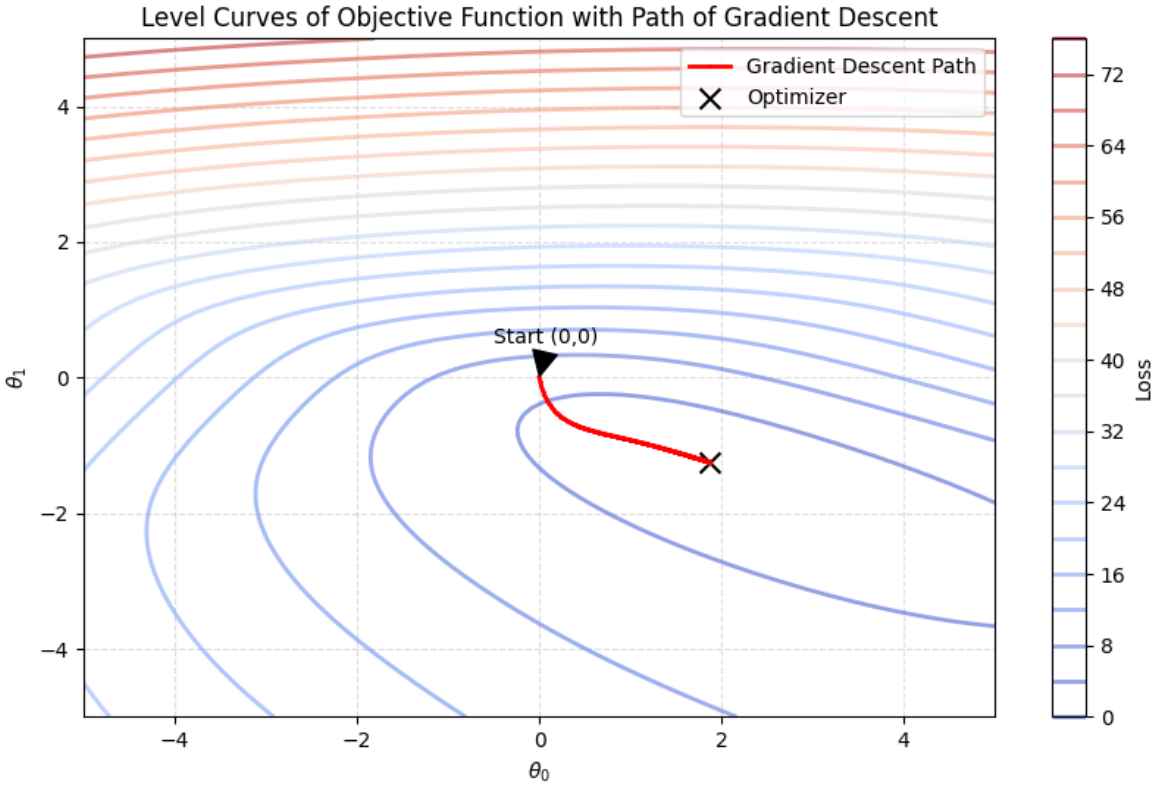
In [ ]:
```
def gradient_descent(X, y, learning_rate, num_iterations):

    theta = np.array([0.0, 0.0])  # Initialization
    losses = []
    theta_path = [theta.copy()]

    for _ in range(num_iterations):

        grad = gradient(theta, X, y)
        theta -= learning_rate * grad
        theta_path.append(theta.copy())
        losses.append(loss_function(theta, X, y))

    return theta, theta_path, losses
```

In [ ]:
```
X = np.array([-3, -2, -1, 0, 1, 2, 3, 4])
y = np.array([1, 1, 1, 1, 0, 1, 0, 0])

learning_rate = 0.01
num_iterations = 1000

theta_opt, theta_path, losses = gradient_descent(X, y, learning_rate, num

theta0_vals = np.linspace(-5, 5, 100)
theta1_vals = np.linspace(-5, 5, 100)
theta0_grid, theta1_grid = np.meshgrid(theta0_vals, theta1_vals)
loss_grid = np.zeros_like(theta0_grid)

for i in range(len(theta0_vals)):
    for j in range(len(theta1_vals)):
        loss_grid[i, j] = loss_function([theta0_grid[i, j], theta1_grid[i

print("Optimizer (theta_0, theta_1):", theta_opt)
```

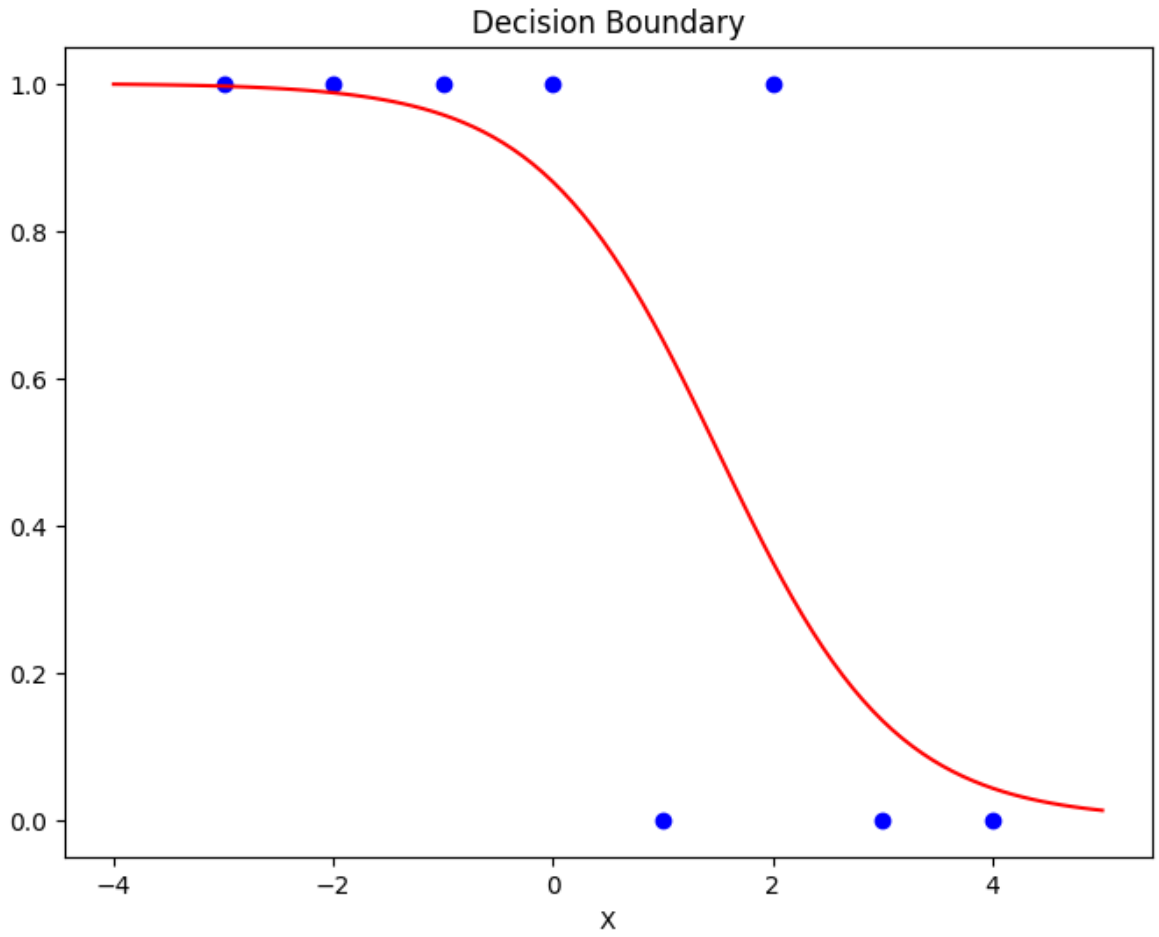
```
Optimizer (theta_0, theta_1): [ 1.86797788 -1.24319766]
```

In [ ]:
```
plt.figure(figsize=(10, 6))
plt.contour(theta0_grid, theta1_grid, loss_grid, levels=20, cmap='coolwar
plt.colorbar(label='Loss')
plt.xlabel(r'$\theta_0$')
```

```
plt.ylabel(r'$\theta_1$')

theta_path = np.array(theta_path)
plt.plot(theta_path[:, 0], theta_path[:, 1], color='red', marker='o', mar
plt.scatter(theta_opt[0], theta_opt[1], color='black', marker='x', s=100,
plt.title('Level Curves of Objective Function with Path of Gradient Desce
plt.legend()

plt.annotate('Start (0,0)', xy=(theta_path[0, 0], theta_path[0, 1]), xyte
             arrowprops=dict(facecolor='black', shrink=0.05), fontsize=10

#plt.annotate(f'End ({theta_opt})', xy=(theta_path[-1, 0], theta_path[-1,

plt.grid(True, linestyle='--', alpha=0.4)

plt.show()
```



In [ ]:
```
x_vals = np.linspace(-4,5,100)
logit = theta_opt[0] + theta_opt[1] * x_vals
y_vals = sigmoid(logit)

plt.figure(figsize=(8, 6))
plt.scatter(X, y, color='blue')
plt.plot(x_vals, y_vals, 'r')
plt.xlabel('X')
plt.title('Decision Boundary')

plt.show()
```


Decision Boundary
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4
X