This is just a compilation of results of calculations. In addition to the usual conventions I assume summation over repeated indices regardless of their position.

Definition 0.0.1. *For one particle operators* $A_1, \ldots, A_c, B_1, \ldots B_p$ *and* $c, p \in \mathbb{N}$ *define:*

$$L(A_1, \ldots, A_c; B_1, \ldots, B_m) := \prod_{l=1}^{p} a(\varphi_{-k_l}) \prod_{l=1}^{c} a^*(A_l \varphi_{n_l}) \prod_{l=1}^{p} a^*(B_l \varphi_{-k_l}) \prod_{l=1}^{c} a(\varphi_{n_l})$$

0.1   1 times 1

$$L(A_1;)L(B_1;) = -L(A_1, B_1;) + L(A_1 P_+ B_1;) \tag{1}$$

$$L(A_1;)L(; B_1) = -L(A_1, B_1) + L(B_1 P_- A_1;) + L(; A_1 P_+ B_1) - \operatorname{tr}(P_- A_1 P_+ B_1) \tag{2}$$

$$L(; A_1)L(B_1;) = -L(B_1; A_1) \tag{3}$$

$$L(; A_1)L(; B_1) = -L(A_1, B_1) + L(; B_1 P_- A_1) \tag{4}$$

0.2   2 times 1

$$L(A_1, A_2;)L(B_1;) = L(A_1, A_2, B_1;) + L(A_1, A_2 P_+ B_1;) + L(A_1 P_+ B_1, A_2;) \tag{5}$$

$$L(A_1, A_2;)L(; B_1) = L(A_1, A_2; B_1) + L(A_1; A_2 P_+ B_1)$$
$$+ L(A_2; A_1 P_+ B_1) + L(A_1, B_1 P_- A_2;) + L(A_2, B_1 P_- A_1;)$$
$$- L(A_2 P_+ B_1 P_- A_1;) - L(A_1 P_+ B_1 P_- A_2;)$$
$$+ L(A_1;) \operatorname{tr}(P_- A_2 P_+ B_1) + L(A_2;) \operatorname{tr}(P_- A_1 P_+ B_1) \quad (6)$$

$$L(A_1; A_2)L(B_1;) \quad = \quad L(A_1, B_1; A_2) \quad + \quad L(A_1 P_+ B_1; A_2) \quad (7)$$

$$L(A_1; A_2)L(; B_1) = L(A_1; A_2, B_1)$$
$$+ L(A_1; B_1 P_- A_2) + L(B_1 P_- A_1; A_2) + L(; A_1 P_+ B_1, A_2)$$
$$- L(; A_1 P_+ B_1 P_- A_2) + L(; A_2) \operatorname{tr}(P_- A_1 P_+ B_1) \quad (8)$$

$$L(; A_1, A_2)L(B_1;) \quad = \quad L(B_1; A_1, A_2) \quad (9)$$

$$L(; A_1, A_2)L(; B_1) = L(; A_1, A_2, B_1) + L(; A_1, B_1 P_- A_2) + L(; A_2, B_1 P_- A_1) \quad (10)$$

0.3   1 times 2

$$L(A_1;)L(B_1, B_2;) = L(A_1, B_1, B_2;) + L(A_1 P_+ B_1, B_2;) + L(A_1 P_+ B_2, B_1;) \quad (11)$$

$$L(A_1;)L(B_1;B_2) = L(A_1,B_1;B_2)$$
$$+ L(A_1P_+B_1;B_2) + L(B_1;A_1P_+B_2) + L(B_2P_-A_1,B_1;)$$
$$- L(B_2P_-A_1P_+B_1;) + L(B_1;)\operatorname{tr}(P_-A_1P_+B_2) \quad (12)$$

$$L(A_1;)L(;B_1,B_2) = L(A_1;B_1,B_2) + L(B_1P_-A_1;B_2)$$
$$+ L(B_1P_-A_1;B_1) + L(;B_1,A_1P_+B_2) + L(;A_1P_+B_1,B_2)$$
$$- L(;B_1P_-A_1P_+B_2) - L(;B_2P_-A_1P_+B_1)$$
$$+ L(;B_1)\operatorname{tr}(P_-A_1P_+B_2) + L(;B_2)\operatorname{tr}(P_-A_1P_+B_1) \quad (13)$$

$$L(;A_1)L(B_1,B_2;) \qquad = \qquad L(B_1,B_2;A_1) \quad (14)$$

$$L(;A_1)L(B_1;B_2) \quad = \quad L(B_1;A_1,B_2) \quad + \quad L(B_1;B_2P_-A_1) \quad (15)$$

$$L(;A_1)L(;B_1,B_2) = L(;A_1,B_1,B_2) + L(;B_1,B_2P_-A_1)$$
$$+ L(;B_1P_-A_1,B_2) \quad (16)$$

0.4   3 times 1

$$L(;A_1,A_2,A_3)L(B;) \qquad = \qquad -L(B;A_1,A_2,A_3) \quad (17)$$

$$L(; A_1, A_2, A_3)L(; B) = -L(; A_1, A_2, A_3, B) + L(; A_1, A_2, BP_-A_3)$$
$$+ L(; A_1, BP_-A_2, A_3) + L(; BP_-A_1, A_2, A_3) \quad (18)$$

$$L(A_1; A_2, A_3)L(B) = L(A_1, B; A_2, A_3) + L(A_1P_+B; A_2, A_3) \quad (19)$$

$$L(A_1; A_2, A_3)L(; B_1) = L(A_1; A_2, A_3, B) + L(A_1; A_2, BP_-A_3)$$
$$+ L(A_1; B_1P_-A_2, A_3) + L(BP_-A_1; A_2, A_3)$$
$$+L(; A_1P_+BP_-A_3)+L(; A_1P_+BP_-A_2, A_3)-L(; A_2, A_3)\operatorname{tr}(P_-A_1P_+B)$$
$$\quad (20)$$

$$L(A_1, A_2; A_3)L(B;) = -L(A_1, A_2, B; A_3)$$
$$+ L(A_1, A_2P_+B; A_3) + L(A_1P_+B, A_2; A_3) \quad (21)$$

$$L(A_1, A_2; A_3)L(; B) = -L(A_1, A_2; A_3, B)$$
$$+ L(BP_-A_1, A_2; A_3) + L(A_1, BP_-A_2; A_3) + L(A_1, A_2; BP_-A_3)$$
$$+ L(A_2; A_1P_+B, A_3) + L(A_1; A_2P_+B, A_3)$$
$$+ L(A_1P_+BP_-A_2; A_3) + L(A_2; A_1P_+BP_-A_3) + L(A_2P_+BP_-A_1; A_3)$$
$$+ L(A_1; A_2P_+BP_-A_3)$$
$$- L(A_1; A_3)\operatorname{tr}(P_-A_2P_+B) - L(A_2; A_3)\operatorname{tr}(P_-A_1P_+B) \quad (22)$$

$$L(A_1, A_2, A_3;)L(B;) = -L(A_1, A_2, A_3, B;) + L(A_1, A_2, A_3P_+B;)$$
$$+ L(A_1, A_2P_+B, A_3;) + L(A_1P_+B, A_2, A_3) \quad (23)$$

$$L(A_1, A_2, A_3;)L(; B) = -L(A_1, A_2, A_3; B)$$
$$+ L(BP_-A_1, A_2, A_3;) + L(A_1, BP_-A_2, A_3;) + L(A_1, A_2, BP_-A_3;)$$
$$+ L(A_2, A_3; A_1P_+B) + L(A_1, A_3; A_2P_+B) + L(A_1, A_2; A_3P_+B)$$
$$+L(A_1P_+BP_-A_2, A_3;)+L(A_1P_+BP_-A_3, A_2;)+L(A_2P_+BP_-A_1, A_3;)$$
$$+L(A_1, A_2P_+BP_-A_3;)+L(A_3P_+B_1P_-A_1, A2;)+L(A_1, A_3P_+BP_-A_2;)$$
$$-L(A_1, A_2;)\,\mathrm{tr}(P_-A_3P_+B)-L(A_1, A_3;)\,\mathrm{tr}(P_-A_2P_+B)-L(A_2, A_3;)\,\mathrm{tr}(P_-A_1P_+B)$$
$$(24)$$

0.5 2 times 2

$$L(A_1, A_2;)L(B_1, B_2;) = L(A_1, A_2, B_1, B_2;) - L(A_1P_+B_1, A_2, B_2;)$$
$$- L(A_1P_+B_2, A_2, B_1;) - L(A_1, B_1, A_2P_+B_2;) - L(A_1, A_2P_+B_1, B_2;)$$
$$- L(A_1P_+B_1, A_2P_+B_2;) - L(A_2P_+B_1, A_1P_+B_2;)\quad (25)$$

$$L(A_1, A_2;)L(B_1, B_2;) = L(A_1, A_2, B_1, B_2;) - L(A_1, B_1, A_2P_+B_2)$$
$$- L(A_1, B_2, A_2P_+B_1;) - L(A_2, B_1, A_1P_+B_2;) - L(A_2, B_2, A_1P_+B_1;)$$
$$- L(A_2P_+B_1, A_1P_+B_2;) - L(A_1P_+B_1, A_2P + B_2;)\quad (26)$$

$$L(A_1, A_2;)L(B_1; B_2) = L(A_1, A_2, B_1; B_2) - L(A_1, B_1, B_2P_-A_2;)$$
$$- L(B_2P_-A_1, A_2, B_1;) - L(A_1P_+B_1, A_2; B_2) - L(A_1, B_1; A_2P_+B_2)$$
$$- L(A_1, A_2P_+B_1; B_2) - L(A_2, B_1; A_1P_+B_2)$$
$$+ L(A_1, B_1)\,\mathrm{tr}(P_-A_2P_+B_2) + L(A_2, B_1;)\,\mathrm{tr}(P_-A_1P_+B_2)$$
$$-L(A_1, B_2P_-A_2P_+B_1;)-L(A_2P_+B_1, B_2P_-A_1;)-L(A_2P_+B_2P_-A_1, B_1;)$$

$$-L(A_1P_+B_1, B_2P_-A_2;\,)-L(A_2, B_2P_-A_1P_+B_1;\,)-L(A_1P_+B_2P_-A_2, B_1;\,)\blacksquare$$
$$- L(A_2P_+B_1; A_1P_+B_2) - L(A_1P_+B_1; A_2P_+B_2)$$
$$- L(A_2P_+B_1;\,)\,\mathrm{tr}(P_-A_1P_+B_2) - L(A_1P_+B_1;\,)\,\mathrm{tr}(P_-A_2P_+B_2)$$
$$+ L(A_1P_+B_2P_-A_2P_+B_1;\,) + L(A_2P_+B_2P_-A_1P_+B_1;\,) \quad (27)$$

$$L(A_1, A_2;\,)L(;B_1, B_2) = L(A_1, A_2; B_1, B_2) - L(A_1, B_2P_-A_2; B_1)$$
$$- L(B_1P_-A_1, A_2; B_2) - L(A_1, B_1P_-A_2; B_2) - L(B_1P_-A_1, A_2; B_1)$$
$$- L(A_2; B_1, A_1P_+B_2) - L(A_2; A_1P_+B_1, B_2) - L(A_1; B_2, A_2P_+B_2)$$
$$- L(A_1; A_2P_+B_1, B_2)$$
$$-L(B_1P_-A_1, B_2P_-A_2;\,)-L(B_2P_-A_1, B_1P_-A_2;\,)-L(A_2; B_2P_-A_1P_+B_1)\blacksquare$$
$$- L(A_2P_+B_2P_-A_1; B_1) - L(B_2P_-A_1; A_2P_+B_1) - L(A_1P_+B_2P_-A_2; B_1)$$
$$- L(B_2P_-A_2; A_1P_+B_1) - L(A_1; B_2P_-A_2P_+B_1) - L(A_2; B_1P_-A_1P_+B_2)$$
$$- L(B_1P_-A_2; A_1P_+B_2) - L(A_1P_+B_1P_-A_2; B_2) - L(A_1; B_1P_-A_2P_+B_2)$$
$$- L(B_1P_-A_1; A_2P_+B_2) - L(A_2P_+B_1P_-A_1; B_2)$$
$$- L(;A_1P_+B_1, A_2P_+B_2) - L(;A_1P_+B_2, A_2P_+B_1)$$
$$+ L(A_2; B_1)\,\mathrm{tr}(P_+B_2P_-A_1) + L(A_1; B_1)\,\mathrm{tr}(P_+B_2P_-A_2)$$
$$+ L(A_2; B_2)\,\mathrm{tr}(P_+B_1P_-A_1) + L(A_1; B_2)\,\mathrm{tr}(P_+B_1P_-A_2)$$
$$+ L(B_1P_-A_1P_+B_2P_-A_2;\,) + L(B_2P_-A_2P_+B_1P_-A_1;\,)$$
$$+ L(B_2P_-A_1P_+B_1P_-A_2;\,) + L(B_1P_-A_2P_+B_2P_-A_1;\,)$$
$$+ L(;A_2P_+B_2P_-A_1P_+B_1) + L(;A_1P_+B_2P_-A_2P_+B_1)$$
$$+ L(;A_2P_+B_1P_-A_1P_+B_2) + L(;A_1P_+B_1P_-A_2P_+B_2)$$
$$- L(B_2P_-A_2;\,)\,\mathrm{tr}(P_+B_1P_-A_1) - L(B_1P_-A_1;\,)\,\mathrm{tr}(P_+B_2P_-A_2)$$
$$- L(B_1P_-A_1;\,)\,\mathrm{tr}(P_+B_1P_-A_2) - L(B_1P_-A_2;\,)\,\mathrm{tr}(P_+B_2P_-A_1)$$
$$- L(;A_2P_+B_1)\,\mathrm{tr}(P_+B_2P_-A_1) - L(;A_1P_+B_1)\,\mathrm{tr}(P_+B_2P_-A_2)$$
$$- L(;A_2P_+B_2)\,\mathrm{tr}(P_+B_1P_-A_1) - L(;A_1P_+B_2)\,\mathrm{tr}(P_+B_1P_-A_2)$$
$$- \mathrm{tr}(P_+B_2P_-A_2P_+B_1P_-A_1) - \mathrm{tr}(P_+B_1P_-A_2P_+B_2P_-A_1)$$

$$+ \operatorname{tr}(P_+ B_1 P_- A_1) \operatorname{tr}(P_+ B_2 P_- A_2) + \operatorname{tr}(P_+ B_1 P_- A_2) \operatorname{tr}(P_+ B_2 P_- A_1) \tag{28}$$

$$L(A_1; A_2)L(B_1, B_2;) = L(A_1, B_1, B_2; A_2) - L(A_1 P_+ B_1, B_2; A_2)$$
$$- L(A_1 P_+ B_2, B_1; A_2) \tag{29}$$

$$L(A_1; A_2)L(B_1; B_2) = L(A_1, B_1; A_2, B_2) - L(B_2 P_- A_1, B_1; A_2)$$
$$- L(B_1; A_1 P_+ B_2, A_2) - L(A_1 P_+ B_1; A_2, B_2) - L(A_1, B_1; B_2 P_- A_2)$$
$$- L(B_1; A_1 P_+ B_2 P_- A_2) - L(B_2 P_- A_1 P_+ B_1; A_2) - L(A_1 P_+ B_1; B_2 P_- A_2)$$
$$+ L(B_1; A_2) \operatorname{tr}(P_- A_1 P_+ B_2) \tag{30}$$

$$L(A_1; A_2)L(; B_1, B_2) = L(A_1; A_2, B_1, B_2)$$
$$- L(A_1; B_1 P_- A_2, B_2) - L(B_2 P_- A_1; A_2, B_1) - L(A_1; B_1, B_2 P_- A_2)$$
$$- L(B_1 P_- A_1; A_2, B_2)$$
$$- L(B_1 P_- A_1; B_2 P_- A_2) - L(B_2 P_- A_1; B_1 P_- A_2) - L(; A_1 P_+ B_2 P_- A_2, B_1)$$
$$- L(; A_1 P_+ B_2, B_1 P_- A_2) - L(; A_2, B_2 P_- A_1 P_+ B_1) - L(; A_1 P_+ B_1, B_2 P_- A_2) \blacksquare$$
$$- L(; A_1 P_+ B_1 P_- A_2, B_2) - L(; A_2, B_1 P_- A_1 P_+ B_2)$$
$$+ L(; A_2, B_1) \operatorname{tr}(P_- A_1 P_+ B_2) + L(; A_2, B_2) \operatorname{tr}(P_- A_1 P_+ B_1)$$
$$+ L(B_1 P_- A_1 P_+ B_2 P_- A_2) + L(; B_2 P_- A_1 P_+ B_1 P_- A_2)$$
$$- L(; B_1 P_- A_2) \operatorname{tr}(P_- A_1 P_+ B_2) - L(; B_2 P_- A_2) \operatorname{tr}(P_- A_1 P_+ B_1) \tag{31}$$

$$L(; A_1, A_2)L(B_1, B_2;) \qquad = \qquad L(B_1, B_2; A_1, A_2) \tag{32}$$

$$L(; A_1, A_2)L(B_1; B_2) = L(B_1; A_1, A_2, B_2)$$
$$- L(B_1; B_2 P_- A_1, A_2) - L(B_1; A_1, B_2 P_- A_2) \quad (33)$$

$$L(; A_1, A_2)L(; B_1, B_2) = L(; A_1, A_2, B_1, B_2)$$
$$- L(; A_1, B_2 P_- A_2, B_1) - L(; A_1, B_1 P_- A_2, B_2)$$
$$- L(; B_2 P_- A_1, A_2, B_1) - L(; B_1 P_- A_1, A_2, B_2)$$
$$- L(; B_1 P_- A_1, B_2 P_- A_2) - L(; B_1 P_- A_2, B_2 P_- A_1) \quad (34)$$