This is just a compilation of results of calculations. In addition to the usual conventions I assume summation over repeated lower indices.

**Definition 0.0.1.** *For one particle operators $A_1, \ldots, A_c, B_1, \ldots B_p$ and $c, p \in \mathbb{N}$ define:*

$$L(A_1, \ldots, A_c; B_1, \ldots, B_m) := \prod_{l=1}^{p} a(\varphi_{-k_l}) \prod_{l=1}^{c} a^*(A_l \varphi_{n_l}) \prod_{l=1}^{p} a^*(B_l \varphi_{-k_l}) \prod_{l=1}^{c} a(\varphi_{n_l}) \quad (1)$$

# 0.1 N times 1

**Lemma 0.1.1.** *For any $a, b, \in \mathbb{N}_0$ and apropriate one particle operators $A_k, B_l, C$ for $1 \leq k \leq a$, $1 \leq l \leq b$ we have the following equality*

$$L\Big(\bigcup_{l=1}^{a}\{A_l\}; \bigcup_{l=1}^{b}\{B_l\}\Big) G(C) =$$

$$(-1)^{a+b}\Big[L\Big(\bigcup_{l=1}^{a}\{A_l\} \cup \{C\}; \bigcup_{l=1}^{b}\{B_l\}\Big) - L\Big(\bigcup_{l=1}^{a}\{A_l\}; \bigcup_{l=1}^{b}\{B_l\} \cup \{C\}\Big)\Big]$$

$$+ \sum_{f=1}^{a} \Big[L\Big(\bigcup_{\substack{l=1 \\ l \neq f}}^{a}\{A_l\} \cup \{A_f P_+ C\}; \bigcup_{l=1}^{b}\{B_l\}\Big) - L\Big(\bigcup_{\substack{l=1 \\ f \neq l}}^{a}\{A_l\} \cup \{C P_- A_f\}; \bigcup_{l=1}^{b}\{B_l\}\Big)$$

$$- L\Big(\bigcup_{\substack{l=1 \\ f \neq l}}^{a}\{A_l\}; \bigcup_{l=1}^{b}\{B_l\} \cup \{A_f P_+ C\}\Big)\Big]$$

$$- \sum_{f=1}^{b} L\Big(\bigcup_{l=1}^{a}\{A_l\}; \bigcup_{\substack{l=1 \\ l \neq f}}^{b}\{B_l\} \cup \{C P_- B_f\}\Big)$$

$$
\begin{aligned}
&+ (-1)^{a+b+1} \sum_{f=1}^{a} \operatorname{tr}\Big(P_+ C P_- A_f\Big) L\Big(\bigcup_{\substack{l=1\\ l\neq f}}^{a}\{A_l\}; \bigcup_{l=1}^{b}\{B_l\}\Big) \\
&+ (-1)^{a+b} \sum_{\substack{f_1,f_2=1\\ f_1\neq f_2}}^{a} L\Big( \bigcup_{\substack{l=1\\ l\neq f_1,f_2}}^{a} \{A_l\} \cup \{A_{f_2} P_+ C P_- A_{f_1}\}; \bigcup_{l=1}^{b}\{B_l\}\Big) \\
&+ (-1)^{a+b} \sum_{f=1}^{b}\sum_{g=1}^{a} L\Big(\bigcup_{\substack{l=1\\ l\neq g}}^{a}\{A_l\}; \bigcup_{\substack{l=1\\ l\neq f}}^{b}\{B_l\} \cup \{A_g P_+ C P_- B_f\}\Big).
\end{aligned}
$$

∎

**Proof:** The proof of this equality is a rather long calculation, where (1) is used repeatedly. We break up the calculation into several parts. Let us start with

$$
\begin{aligned}
&L\left(\bigcup_{l=1}^{a}\{A_l\}; \bigcup_{l=1}^{b}\{B_l\}\right) L(C;) = \\
&\prod_{l=1}^{b} a(\varphi_{-k_l}) \prod_{l=1}^{a} a^*(A_l \varphi_{n_l}) \prod_{l=1}^{b} a^*(B_l \varphi_{-k_l}) \prod_{l=1}^{a} a(\varphi_{n_l}) a^*(C\varphi_m) a(\varphi_m).
\end{aligned} \tag{2}
$$

We (anti)commute the creation operator involving $C$ to its place at the end of the second product, after that the term will be normally ordered and can be rephrased in terms of $L$s. During the commutation the creation operator in question can be picked up by any of the annihilation operators in the rightmost product. For each term where that happens we can perform the sum over the basis of $\mathcal{H}^-$ related to the annihilation operator whose anticommutator triggered. After this sum the corresponding term is also normally ordered and can be rephrased in terms of an $L$ after some reshuffling which may only produce signs. So performing these steps we get

$$
\begin{aligned}
L&\left(\bigcup_{l=1}^{a}\{A_l\};\bigcup_{l=1}^{b}\{B_l\}\right)L(C;)=\\
&\sum_{f=a}^{1}(-1)^{a-f}\prod_{l=1}^{b}a(\varphi_{-k_l})\prod_{l=1}^{f-1}a^*(A_l\varphi_{n_l})a^*(A_fP_+C\varphi_m)\\
&\prod_{l=f+1}^{a}a^*(A_l\varphi_{n_l})\prod_{l=1}^{b}a^*(B_l\varphi_{-k_l})\prod_{\substack{l=1\\l\neq f}}^{a}a(\varphi_{n_l})a(\varphi_m)\\
&\qquad+L\left(\bigcup_{l=1}^{a}\{A_l\}\cup\{C\};\bigcup_{l=1}^{b}\{B_l\}\right)\\
&=\sum_{f=1}^{a}+L\left(\bigcup_{\substack{l=1\\l\neq f}}^{a}\{A_l\}\cup\{A_fP_+C\};\bigcup_{l=1}^{b}\{B_l\}\right)\\
&\qquad+L\left(\bigcup_{l=1}^{a}\{A_l\}\cup\{C\};\bigcup_{l=1}^{b}\{B_l\}\right).
\end{aligned}
\tag{3}
$$

Now the remaining case is more laborious, that is why we will split off and treat some of the appearing terms separately. We start off analogous to before

$$
\begin{aligned}
L&\left(\bigcup_{l=1}^{a}\{A_l\};\bigcup_{l=1}^{b}\{B_l\}\right)L(;C)=\\
&\prod_{l=1}^{b}a(\varphi_{-k_l})\prod_{l=1}^{a}a^*(A_l\varphi_{n_l})\prod_{l=1}^{b}a^*(B_l\varphi_{-k_l})\prod_{l=1}^{a}a(\varphi_{n_l})a(\varphi_{-m})a^*(C\varphi_{-m}).
\end{aligned}
\tag{4}
$$

This time we need to (anti)commute the rightmost annihilation operator all the way to the end of the first product and the creation operator to the end of the second but last product. So there will be several qualitatively different terms. From the first step alone we get

$$L\left(\bigcup_{l=1}^{a}\{A_l\};\bigcup_{l=1}^{b}\{B_l\}\right)L(;C) =$$
$$(-1)^a \sum_{f=b}^{1}(-1)^{b-f}\prod_{l=1}^{b}a(\varphi_{-k_l})\prod_{l=1}^{a}a^*(A_l\varphi_{n_l})\prod_{\substack{l=1\\l\neq f}}^{b}a^*(B_l\varphi_{-k_l})$$
$$\prod_{l=1}^{a}a(\varphi_{n_l})a^*(CP_-B_f\varphi_{-k_f}) \tag{5}$$
$$+(-1)^{a+b}\sum_{f=a}^{1}(-1)^{b-f}\prod_{l=1}^{b}a(\varphi_{-k_l})\prod_{\substack{l=1\\l\neq f}}^{a}a^*(A_l\varphi_{n_l})$$
$$\prod_{l=1}^{b}a^*(B_l\varphi_{-k_l})\prod_{l=1}^{a}a(\varphi_{n_l})a^*(CP_-\varphi_{n_f}) \tag{6}$$
$$+(-1)^b\prod_{l=1}^{b}a(\varphi_{-k_l})a(\varphi_{-m})\prod_{l=1}^{a}a^*(A_l\varphi_{n_l})$$
$$\prod_{l=1}^{b}a^*(B_l\varphi_{-k_l})\prod_{l=1}^{a}a(\varphi_{n_l})a^*(C\varphi_{-m}). \tag{7}$$

We will discuss terms (5), (6) and (7) separately. In Term (5) we need to commute the last creation operator into its place in the third product, it can be picked up by one of the annihilation operators of the last product, but after performing the sum over the corresponding basis the resulting term can be rephrased in terms of an $L$ operator by commuting only creation operators of the second and third product. Performing these steps yields the identity

$$
\begin{aligned}
(5) = \sum_{f=1}^{b} L \left( \bigcup_{l=1}^{a} \{A_l\}; \bigcup_{\substack{l=1 \\ l\neq f}}^{b} \{B_l\} \cup \{CP_- B_f\} \right) \\
+ (-1)^{a+b+1} \sum_{f=1}^{b} \sum_{g=1}^{a} L \left( \bigcup_{\substack{l=1 \\ l\neq g}}^{a} \{A_l\}; \bigcup_{\substack{l=1 \\ l\neq f}}^{b} \{B_l\} \{A_g P_+ CP_- B_f\} \right) . \qquad (8)
\end{aligned}
$$

For (6) the last creation operator needs to be commuted to the end of the second product. It can be picked up by one of the annihilation operators of the last product, but here we have to distinguish between two cases. If the index of this annihilation operator equals $f$ the resulting commutator will be $\operatorname{tr} P_+ CP_- A_f$ otherwise one can again perform the sum over the corresponding index and express the whole Product in terms of an $L$ operator. All this results in

$$
\begin{aligned}
(6) = \sum_{f=1}^{a} L \left( \bigcup_{\substack{l=1 \\ l\neq f}}^{a} \{A_l\} \cup \{CP_- A_f\}; \bigcup_{l=1}^{b} \{B_l\} \right) \\
+ (-1)^{a+b} \sum_{f=1}^{a} L \left( \bigcup_{\substack{l=1 \\ l\neq f}}^{a} \{A_l\}; \bigcup_{l=1}^{b} \{B_l\} \right) \operatorname{tr}(P_+ CP_- A_f) \\
+ (-1)^{a+b+1} \sum_{\substack{f_1, f_2 = 1 \\ f_1 \neq f_2}}^{a} L \left( \bigcup_{\substack{l=1 \\ l\neq f_1, f_2}}^{a} \{A_l\} \cup \{A_{f_2} P_+ CP_- A_{f_1}\}; \bigcup_{l=1}^{b} \{B_l\} \right) . \qquad (9)
\end{aligned}
$$

For (7) the procedure is basically the same as for (5), it results in

$$
\begin{aligned}
(7) = (-1)^{a+b} L \left( \bigcup_{l=1}^{a} \{A_l\}; \bigcup_{l=1}^{b} \{B_l\} \right) \\
+ \sum_{f=1}^{a} L \left( \bigcup_{\substack{l=1 \\ l \neq f}}^{a} \{A_l\} \cup \{C P_- A_f\}; \bigcup_{l=1}^{b} \{B_l\} \cup \{A_f P_+ C\} \right) . \qquad (10)
\end{aligned}
$$

Putting the results of the calculation together results in the claimed equation □

## 0.2 1 times 1

$$L(A_1;)L(B_1;) = -L(A_1, B_1;) + L(A_1 P_+ B_1;) \qquad (11)$$

$$L(A_1;)L(;B_1) = -L(A_1;B_1) + L(B_1 P_- A_1;) + L(;A_1 P_+ B_1) - \mathrm{tr}(P_- A_1 P_+ B_1) \qquad (12)$$

$$L(;A_1)L(B_1;) = -L(B_1;A_1) \qquad (13)$$

$$L(;A_1)L(;B_1) = -L(;A_1, B_1) + L(;B_1 P_- A_1) \qquad (14)$$

## 0.3 2 times 1

$$L(A_1, A_2;)L(B_1;) = L(A_1, A_2, B_1;) + L(A_1, A_2 P_+ B_1;) + L(A_1 P_+ B_1, A_2;) \qquad (15)$$

$$L(A_1, A_2;\,)L(;B_1) = L(A_1, A_2; B_1) + L(A_1; A_2P_+B_1) + L(A_2; A_1P_+B_1) + L(A_1, B_1P_-A_2;\,) + L(A_2, B_1P_-A_1;\,) - L(A_2P_+B_1P_-A_1;\,) - L(A_1P_+B_1P_-A_2;\,) + L(A_1;\,)\,\mathrm{tr}(P_-A_2P_+B_1) + L(A_2;\,)\,\mathrm{tr}(P_-A_1P_+B_1) \quad (16)$$

$$L(A_1; A_2)L(B_1;\,) = L(A_1, B_1; A_2) + L(A_1P_+B_1; A_2) \quad (17)$$

$$L(A_1; A_2)L(;B_1) = L(A_1; A_2, B_1) + L(A_1; B_1P_-A_2) + L(B_1P_-A_1; A_2) + L(;A_1P_+B_1, A_2) - L(;A_1P_+B_1P_-A_2) + L(;A_2)\,\mathrm{tr}(P_-A_1P_+B_1) \quad (18)$$

$$L(;A_1, A_2)L(B_1;\,) = L(B_1; A_1, A_2) \quad (19)$$

$$L(;A_1, A_2)L(;B_1) = L(;A_1, A_2, B_1) + L(;A_1, B_1P_-A_2) + L(;A_2, B_1P_-A_1) \quad (20)$$

## 0.4 1 times 2

$$L(A_1;\,)L(B_1, B_2;\,) = L(A_1, B_1, B_2;\,) + L(A_1P_+B_1, B_2;\,) + L(A_1P_+B_2, B_1;\,) \quad (21)$$

$$\begin{aligned} L(A_1;)L(B_1;B_2) &= L(A_1,B_1;B_2) \\ &\quad + L(A_1P_+B_1;B_2) + L(B_1;A_1P_+B_2) + L(B_2P_-A_1,B_1;) \\ &\quad - L(B_2P_-A_1P_+B_1;) + L(B_1;)\,\mathrm{tr}(P_-A_1P_+B_2) \end{aligned} \tag{22}$$

$$\begin{aligned} L(A_1;)L(;B_1,B_2) &= L(A_1;B_1,B_2) + L(B_1P_-A_1;B_2) \\ &\quad + L(B_1P_-A_1;B_1) + L(;B_1,A_1P_+B_2) + L(;A_1P_+B_1,B_2) \\ &\quad - L(;B_1P_-A_1P_+B_2) - L(;B_2P_-A_1P_+B_1) \\ &\quad + L(;B_1)\,\mathrm{tr}(P_-A_1P_+B_2) + L(;B_2)\,\mathrm{tr}(P_-A_1P_+B_1) \end{aligned} \tag{23}$$

$$L(;A_1)L(B_1,B_2;) = L(B_1,B_2;A_1) \tag{24}$$

$$L(;A_1)L(B_1;B_2) = L(B_1;A_1,B_2) + L(B_1;B_2P_-A_1) \tag{25}$$

$$\begin{aligned} L(;A_1)L(;B_1,B_2) &= L(;A_1,B_1,B_2) + L(;B_1,B_2P_-A_1) \\ &\quad + L(;B_1P_-A_1,B_2) \end{aligned} \tag{26}$$

## 0.5 3 times 1

$$L(;A_1,A_2,A_3)L(B;) = -L(B;A_1,A_2,A_3) \tag{27}$$

$$L(;A_1,A_2,A_3)L(;B) = -L(;A_1,A_2,A_3,B) + L(;A_1,A_2,BP_-A_3) \\ + L(;A_1,BP_-A_2,A_3) + L(;BP_-A_1,A_2,A_3) \quad (28)$$

$$L(A_1;A_2,A_3)L(B;) = -L(A_1,B;A_2,A_3) + L(A_1P_+B;A_2,A_3) \quad (29)$$

$$L(A_1;A_2,A_3)L(;B_1) = L(A_1;A_2,A_3,B) + L(A_1;A_2,BP_-A_3) \\ + L(A_1;B_1P_-A_2,A_3) + L(BP_-A_1;A_2,A_3) \\ +L(;A_1P_+BP_-A_3)+L(;A_1P_+BP_-A_2,A_3)-L(;A_2,A_3)\,\mathrm{tr}(P_-A_1P_+B) \\ (30)$$

$$L(A_1,A_2;A_3)L(B;) = -L(A_1,A_2,B;A_3) \\ + L(A_1,A_2P_+B;A_3) + L(A_1P_+B,A_2;A_3) \quad (31)$$

$$L(A_1,A_2;A_3)L(;B) = -L(A_1,A_2;A_3,B) \\ + L(BP_-A_1,A_2;A_3) + L(A_1,BP_-A_2;A_3) + L(A_1,A_2;BP_-A_3) \\ + L(A_2;A_1P_+B,A_3) + L(A_1;A_2P_+B,A_3) \\ + L(A_1P_+BP_-A_2;A_3) + L(A_2;A_1P_+BP_-A_3) + L(A_2P_+BP_-A_1;A_3) \\ + L(A_1;A_2P_+BP_-A_3) \\ - L(A_1;A_3)\,\mathrm{tr}(P_-A_2P_+B) - L(A_2;A_3)\,\mathrm{tr}(P_-A_1P_+B) \quad (32)$$

$$L(A_1,A_2,A_3;)L(B;) = -L(A_1,A_2,A_3,B;) + L(A_1,A_2,A_3P_+B;) \\ + L(A_1,A_2P_+B,A_3;) + L(A_1P_+B,A_2,A_3) \quad (33)$$

$$
\begin{gathered}
L(A_1, A_2, A_3;)L(;B) = -L(A_1, A_2, A_3; B) \\
+ L(BP_-A_1, A_2, A_3;) + L(A_1, BP_-A_2, A_3;) + L(A_1, A_2, BP_-A_3;) \\
+ L(A_2, A_3; A_1P_+B) + L(A_1, A_3; A_2P_+B) + L(A_1, A_2; A_3P_+B) \\
+L(A_1P_+BP_-A_2, A_3;)+L(A_1P_+BP_-A_3, A_2;)+L(A_2P_+BP_-A_1, A_3;) \\
+L(A_1, A_2P_+BP_-A_3;)+L(A_3P_+B_1P_-A_1, A2;)+L(A_1, A_3P_+BP_-A_2;) \\
-L(A_1, A_2;)\,\mathrm{tr}(P_-A_3P_+B)-L(A_1, A_3;)\,\mathrm{tr}(P_-A_2P_+B)-L(A_2, A_3;)\,\mathrm{tr}(P_-A_1P_+B)
\end{gathered}
\tag{34}
$$

## 0.6 2 times 2

$$
\begin{gathered}
L(A_1, A_2;)L(B_1, B_2;) = L(A_1, A_2, B_1, B_2;) - L(A_1P_+B_1, A_2, B_2;) \\
- L(A_1P_+B_2, A_2, B_1;) - L(A_1, B_1, A_2P_+B_2;) - L(A_1, A_2P_+B_1, B_2;) \\
- L(A_1P_+B_1, A_2P_+B_2;) - L(A_2P_+B_1, A_1P_+B_2;)
\end{gathered}
\tag{35}
$$

$$
\begin{gathered}
L(A_1, A_2;)L(B_1, B_2;) = L(A_1, A_2, B_1, B_2;) - L(A_1, B_1, A_2P_+B_2) \\
- L(A_1, B_2, A_2P_+B_1;) - L(A_2, B_1, A_1P_+B_2;) - L(A_2, B_2, A_1P_+B_1;) \\
- L(A_2P_+B_1, A_1P_+B_2;) - L(A_1P_+B_1, A_2P + B_2;)
\end{gathered}
\tag{36}
$$

$$
\begin{gathered}
L(A_1, A_2;)L(B_1; B_2) = L(A_1, A_2, B_1; B_2) - L(A_1, B_1, B_2P_-A_2;) \\
- L(B_2P_-A_1, A_2, B_1;) - L(A_1P_+B_1, A_2; B_2) - L(A_1, B_1; A_2P_+B_2) \\
- L(A_1, A_2P_+B_1; B_2) - L(A_2, B_1; A_1P_+B_2) \\
+ L(A_1, B_1)\,\mathrm{tr}(P_-A_2P_+B_2) + L(A_2, B_1;)\,\mathrm{tr}(P_-A_1P_+B_2) \\
-L(A_1, B_2P_-A_2P_+B_1;)-L(A_2P_+B_1, B_2P_-A_1;)-L(A_2P_+B_2P_-A_1, B_1;)
\end{gathered}
$$

$$
\begin{aligned}
&-L(A_1P_+B_1, B_2P_-A_2;\,)-L(A_2, B_2P_-A_1P_+B_1;\,)-L(A_1P_+B_2P_-A_2, B_1;\,)\\
&\quad - L(A_2P_+B_1; A_1P_+B_2) - L(A_1P_+B_1; A_2P_+B_2)\\
&\quad - L(A_2P_+B_1;\,)\,\mathrm{tr}(P_-A_1P_+B_2) - L(A_1P_+B_1;\,)\,\mathrm{tr}(P_-A_2P_+B_2)\\
&\quad + L(A_1P_+B_2P_-A_2P_+B_1;\,) + L(A_2P_+B_2P_-A_1P_+B_1;\,)
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
&L(A_1, A_2;\,)L(; B_1, B_2) = L(A_1, A_2; B_1, B_2) - L(A_1, B_2P_-A_2; B_1)\\
&\quad - L(B_1P_-A_1, A_2; B_2) - L(A_1, B_1P_-A_2; B_2) - L(B_1P_-A_1, A_2; B_1)\\
&\quad - L(A_2; B_1, A_1P_+B_2) - L(A_2; A_1P_+B_1, B_2) - L(A_1; B_2, A_2P_+B_2)\\
&\quad - L(A_1; A_2P_+B_1, B_2)\\
&-L(B_1P_-A_1, B_2P_-A_2;\,)-L(B_2P_-A_1, B_1P_-A_2;\,)-L(A_2; B_2P_-A_1P_+B_1)\\
&- L(A_2P_+B_2P_-A_1; B_1) - L(B_2P_-A_1; A_2P_+B_1) - L(A_1P_+B_2P_-A_2; B_1)\\
&- L(B_2P_-A_2; A_1P_+B_1) - L(A_1; B_2P_-A_2P_+B_1) - L(A_2; B_1P_-A_1P_+B_2)\\
&- L(B_1P_-A_2; A_1P_+B_2) - L(A_1P_+B_1P_-A_2; B_2) - L(A_1; B_1P_-A_2P_+B_2)\\
&\quad - L(B_1P_-A_1; A_2P_+B_2) - L(A_2P_+B_1P_-A_1; B_2)\\
&\quad - L(; A_1P_+B_1, A_2P_+B_2) - L(; A_1P_+B_2, A_2P_+B_1)\\
&\quad + L(A_2; B_1)\,\mathrm{tr}(P_+B_2P_-A_1) + L(A_1; B_1)\,\mathrm{tr}(P_+B_2P_-A_2)\\
&\quad + L(A_2; B_2)\,\mathrm{tr}(P_+B_1P_-A_1) + L(A_1; B_2)\,\mathrm{tr}(P_+B_1P_-A_2)\\
&\quad + L(B_1P_-A_1P_+B_2P_-A_2;\,) + L(B_2P_-A_2P_+B_1P_-A_1;\,)\\
&\quad + L(B_2P_-A_1P_+B_1P_-A_2;\,) + L(B_1P_-A_2P_+B_2P_-A_1;\,)\\
&\quad + L(; A_2P_+B_2P_-A_1P_+B_1) + L(; A_1P_+B_2P_-A_2P_+B_1)\\
&\quad + L(; A_2P_+B_1P_-A_1P_+B_2) + L(; A_1P_+B_1P_-A_2P_+B_2)\\
&\quad - L(B_2P_-A_2;\,)\,\mathrm{tr}(P_+B_1P_-A_1) - L(B_1P_-A_1;\,)\,\mathrm{tr}(P_+B_2P_-A_2)\\
&\quad - L(B_1P_-A_1;\,)\,\mathrm{tr}(P_+B_1P_-A_2) - L(B_1P_-A_2;\,)\,\mathrm{tr}(P_+B_2P_-A_1)\\
&\quad - L(; A_2P_+B_1)\,\mathrm{tr}(P_+B_2P_-A_1) - L(; A_1P_+B_1)\,\mathrm{tr}(P_+B_2P_-A_2)\\
&\quad - L(; A_2P_+B_2)\,\mathrm{tr}(P_+B_1P_-A_1) - L(; A_1P_+B_2)\,\mathrm{tr}(P_+B_1P_-A_2)\\
&\quad - \mathrm{tr}(P_+B_2P_-A_2P_+B_1P_-A_1) - \mathrm{tr}(P_+B_1P_-A_2P_+B_2P_-A_1)
\end{aligned}
$$

$$+\operatorname{tr}(P_+B_1P_-A_1)\operatorname{tr}(P_+B_2P_-A_2)+\operatorname{tr}(P_+B_1P_-A_2)\operatorname{tr}(P_+B_2P_-A_1) \tag{38}$$

$$\begin{aligned} L(A_1;A_2)L(B_1,B_2;) = L(A_1,B_1,B_2;A_2) - L(A_1P_+B_1,B_2;A_2) \\ - L(A_1P_+B_2,B_1;A_2) \end{aligned} \tag{39}$$

$$\begin{aligned} &L(A_1;A_2)L(B_1;B_2) = L(A_1,B_1;A_2,B_2) - L(B_2P_-A_1,B_1;A_2) \\ &- L(B_1;A_1P_+B_2,A_2) - L(A_1P_+B_1;A_2,B_2) - L(A_1,B_1;B_2P_-A_2) \\ &- L(B_1;A_1P_+B_2P_-A_2) - L(B_2P_-A_1P_+B_1;A_2) - L(A_1P_+B_1;B_2P_-A_2) \\ &+ L(B_1;A_2)\operatorname{tr}(P_-A_1P_+B_2) \end{aligned} \tag{40}$$

$$\begin{aligned} &L(A_1;A_2)L(;B_1,B_2) = L(A_1;A_2,B_1,B_2) \\ &- L(A_1;B_1P_-A_2,B_2) - L(B_2P_-A_1;A_2,B_1) - L(A_1;B_1,B_2P_-A_2) \\ &- L(B_1P_-A_1;A_2,B_2) \\ &- L(B_1P_-A_1;B_2P_-A_2) - L(B_2P_-A_1;B_1P_-A_2) - L(;A_1P_+B_2P_-A_2,B_1) \\ &- L(;A_1P_+B_2,B_1P_-A_2) - L(;A_2,B_2P_-A_1P_+B_1) - L(;A_1P_+B_1,B_2P_-A_2) \\ &- L(;A_1P_+B_1P_-A_2,B_2) - L(;A_2,B_1P_-A_1P_+B_2) \\ &+ L(;A_2,B_1)\operatorname{tr}(P_-A_1P_+B_2) + L(;A_2,B_2)\operatorname{tr}(P_-A_1P_+B_1) \\ &+ L(B_1P_-A_1P_+B_2P_-A_2) + L(;B_2P_-A_1P_+B_1P_-A_2) \\ &- L(;B_1P_-A_2)\operatorname{tr}(P_-A_1P_+B_2) - L(;B_2P_-A_2)\operatorname{tr}(P_-A_1P_+B_1) \end{aligned} \tag{41}$$

$$L(;A_1,A_2)L(B_1,B_2;) = L(B_1,B_2;A_1,A_2) \tag{42}$$

$$
\begin{aligned}
L(;A_1,A_2)L(B_1;B_2) = {} & L(B_1;A_1,A_2,B_2) \\
& - L(B_1;B_2P_-A_1,A_2) - L(B_1;A_1,B_2P_-A_2)
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
L(;A_1,A_2)L(;B_1,B_2) = {} & L(;A_1,A_2,B_1,B_2) \\
& - L(;A_1,B_2P_-A_2,B_1) - L(;A_1,B_1P_-A_2,B_2) \\
& - L(;B_2P_-A_1,A_2,B_1) - L(;B_1P_-A_1,A_2,B_2) \\
& - L(;B_1P_-A_1,B_2P_-A_2) - L(;B_1P_-A_2,B_2P_-A_1)
\end{aligned} \tag{44}
$$