estimation of spectral norm of the repeated factor that disrupts the convolution structure of the kth term in the expansion of the S matrix

$$\begin{aligned}\left\|\frac{/p+m}{p^2-m^2}\right\|_{\text{spec}} &= \left\|\frac{\gamma^\alpha p_\alpha+m}{p^2-m^2}\right\|_{\text{spec}} = \left\|\frac{\gamma^\alpha\lambda_\alpha-i\epsilon\gamma^0+m}{\lambda^2-m^2-2i\epsilon\lambda^0-\epsilon^2}\right\|_{\text{spec}} \\ &= \frac{\|\gamma^\alpha\lambda_\alpha-i\epsilon\gamma^0+m\|_{\text{spec}}}{\sqrt{(\lambda^2-m^2-\epsilon^2)^2+4\epsilon^2\left(\lambda^0\right)^2}} = \frac{\epsilon\left\|\gamma^\alpha\frac{\lambda_\alpha}{\epsilon}-i\gamma^0+\frac{m}{\epsilon}\right\|_{\text{spec}}}{\epsilon^2\sqrt{\left(\left(\frac{\lambda}{\epsilon}\right)^2-\left(\frac{m}{\epsilon}\right)^2-1\right)^2+4\left(\frac{\lambda^0}{\epsilon}\right)^2}} \\ &\to_{\substack{\epsilon\to\infty \\ \frac{\lambda}{\epsilon}=\text{const}}} \text{overtext...} = \frac{\|\gamma^\alpha z_\alpha-i\gamma^0\|_{\text{spec}}}{\epsilon\sqrt{\left(z^2-1\right)^2+4\left(z^0\right)^2}} \end{aligned} \quad (1)$$

now continue to estimate the resulting expression maximizing over z

$$\begin{aligned}\frac{\|\gamma^\alpha z_\alpha-i\gamma^0\|_{\text{spec}}}{\sqrt{\left(z^2-1\right)^2+4\left(z^0\right)^2}} &\le \frac{\|\gamma^\alpha z_\alpha\|_{\text{spec}}+\|\gamma^0\|_{\text{spec}}}{\sqrt{\left(z^2-1\right)^2+4\left(z^0\right)^2}} \le \frac{4\|z\|+1}{\sqrt{\left(z^2-1\right)^2+4\left(z^0\right)^2}} \\ &= \frac{4\|z\|+1}{\sqrt{\left(z^2\right)^2+1+2\left\|z\right\|^2}} \le \frac{4\|z\|+1}{\sqrt{1+2\left\|z\right\|^2}} \le 2\sqrt{2} \end{aligned} \quad (2)$$

,where the last estimation follows from analyzing the differentiation of the second but last expression with respect to $\|z\|$: