1 Lifting Charge Conjugation

We will define the second quantised charge conjugation operator $\mathfrak{C}$ on all of Fockspace analogously to the way we are currently trying to define the second quantised S matrix operator. Fock space is defined as:

$$\mathcal{F} := \bigoplus_{m,p=0}^{\infty} \left(\mathcal{H}^+\right)^{\Lambda m} \otimes \left(\mathcal{H}^-\right)^{\Lambda p} \tag{1}$$

We will denote the fixed particles sectors of Fockspace by $\mathcal{F}_{m,p} := \left(\mathcal{H}^+\right)^{\otimes m} \otimes \left(\mathcal{H}^-\right)^{\otimes p}$ The following property called "Lift condition" will play a central role in our definition:

$$\begin{aligned} \forall \phi \in \mathcal{H} : \quad & a\left(C\phi\right) \circ \mathfrak{C} = \mathfrak{C} \circ a^*(\phi), \\ & a^*\left(C\phi\right) \circ \mathfrak{C} = \mathfrak{C} \circ a(\phi) \end{aligned} \tag{2}$$

This property together with the charge conjugation operator on the one particle Hilbertspace, linearity and the following property shall be sufficient to define $\mathfrak{C}$.

$$\mathfrak{C}\Omega = \Omega \tag{3}$$

Lemma 1. properties of $\mathfrak{C}$:

$$\mathfrak{C}\mathfrak{C} = \mathbb{1} \tag{4}$$

$$\mathfrak{C}^*\mathfrak{C} = \mathbb{1} \tag{5}$$

Proof: We will first proof (4): Let α be an arbitrary Fockspace element. Let further $(\varphi_n)_{n\in\mathbb{N}}$ be an orthonormal basis of $\mathcal{H}_+$ and $(f_n)_{n\in\mathbb{N}}$ be an orthonormal basis of $\mathcal{H}_-$.

$$\begin{aligned} \mathfrak{C}\mathfrak{C}\alpha = \mathfrak{C}\mathfrak{C} &\sum_{m,p\in\mathbb{N}_0} \sum_{a_1,\ldots a_m,b_1,\ldots b_p\in\mathbb{N}} \alpha_{a_1,\ldots a_m,b_1,\ldots b_p} \prod_{l=1}^{m} a^*(\varphi_{a_l}) \prod_{d=1}^{p} a(f_{b_d})\Omega \\ &= \mathfrak{C} \sum_{m,p\in\mathbb{N}_0} \sum_{a_1,\ldots a_m,b_1,\ldots b_p\in\mathbb{N}} \alpha_{a_1,\ldots a_m,b_1,\ldots b_p} \left(\prod_{l=1}^{m} a(C\varphi_{a_l})\right) \mathfrak{C} \prod_{d=1}^{p} a(f_{b_d})\Omega \\ &= \sum_{m,p\in\mathbb{N}_0} \sum_{a_1,\ldots a_m,b_1,\ldots b_p\in\mathbb{N}} \alpha_{a_1,\ldots a_m,b_1,\ldots b_p} \left(\prod_{l=1}^{m} a^*(CC\varphi_{a_l})\right) \mathfrak{C} \prod_{d=1}^{p} a^*(Cf_{b_d})\mathfrak{C}\Omega \\ &\overset{CC=(1)}{=} \sum_{m,p\in\mathbb{N}_0} \sum_{a_1,\ldots a_m,b_1,\ldots b_p\in\mathbb{N}} \alpha_{a_1,\ldots a_m,b_1,\ldots b_p} \left(\prod_{l=1}^{m} a^*(\varphi_{a_l})\right) \prod_{d=1}^{p} a(CCf_{b_d})\mathfrak{C}\mathfrak{C}\Omega \\ &\overset{CC=(1)}{=} \sum_{m,p\in\mathbb{N}_0} \sum_{a_1,\ldots a_m,b_1,\ldots b_p\in\mathbb{N}} \alpha_{a_1,\ldots a_m,b_1,\ldots b_p} \left(\prod_{l=1}^{m} a^*(\varphi_{a_l})\right) \prod_{d=1}^{p} a(f_{b_d})\Omega = \alpha, \end{aligned} \tag{6}$$

which proves the first claim. The calculation for the second claim is similar. Let $\alpha, \beta \in \mathfrak{F}$ be arbitrary elements and $(\varphi_n)_{n\in\mathbb{N}}$, $(f_n)_{n\in\mathbb{N}}$ as before.

$$\langle\beta,\mathfrak{C}^*\mathfrak{C}\alpha\rangle = \langle\mathfrak{C}\beta,\mathfrak{C}\alpha\rangle = \sum_{n,g,m,p\in\mathbb{N}_0}\sum_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g\in\mathbb{N}}\sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}}$$
$$\left\langle \mathfrak{C}\beta_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g}\prod_{l=1}^{n}a^*(\varphi_{\tilde{a}_l})\prod_{d=1}^{g}a(f_{\tilde{b}_d})\Omega, \mathfrak{C}\alpha_{a_1,\dots a_m,b_1,\dots b_p}\prod_{l=1}^{m}a^*(\varphi_{a_l})\prod_{d=1}^{p}a(f_{b_d})\Omega\right\rangle$$
$$= \sum_{n,g,m,p\in\mathbb{N}_0}\sum_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g\in\mathbb{N}}\sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}}\beta_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g}\alpha_{a_1,\dots a_m,b_1,\dots b_p}$$
$$\left\langle \mathfrak{C}\prod_{l=1}^{n}a^*(\varphi_{\tilde{a}_l})\prod_{d=1}^{g}a(f_{\tilde{b}_d})\Omega, \mathfrak{C}\prod_{l=1}^{m}a^*(\varphi_{a_l})\prod_{d=1}^{p}a(f_{b_d})\Omega\right\rangle$$
$$= \sum_{n,g,m,p\in\mathbb{N}_0}\sum_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g\in\mathbb{N}}\sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}}\beta_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g}\alpha_{\tilde{a}_1,\dots\tilde{a}_m,\tilde{b}_1,\dots\tilde{b}_p}$$
$$\left\langle \prod_{l=1}^{n}a(C\varphi_{\tilde{a}_l})\prod_{d=1}^{g}a^*(Cf_{\tilde{b}_d})\mathfrak{C}\Omega, \prod_{l=1}^{m}a(C\varphi_{a_l})\prod_{d=1}^{p}a^*(Cf_{b_d})\mathfrak{C}\Omega\right\rangle$$
$$= \sum_{n,g,m,p\in\mathbb{N}_0}\sum_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g\in\mathbb{N}}\sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}}\beta_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g}\alpha_{a_1,\dots a_m,b_1,\dots b_p}$$
$$\left\langle \prod_{l=1}^{n}a(C\varphi_{\tilde{a}_l})\prod_{d=1}^{g}a^*(Cf_{\tilde{b}_d})\Omega, \prod_{l=1}^{m}a(C\varphi_{a_l})\prod_{d=1}^{p}a^*(Cf_{b_d})\Omega\right\rangle$$
$$= \sum_{m,p\in\mathbb{N}_0}\sum_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g\in\mathbb{N}}\sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}}\beta_{a_1,\dots a_n,b_1,\dots b_g}\alpha_{a_1,\dots a_m,b_1,\dots b_p}$$
$$\sum_{\pi\in S_m,\sigma\in S_p}(-1)^\pi(-1)^\sigma\prod_{l=1}^{m}\left\langle C\varphi_{a_{\pi(l)}},C\varphi_{\tilde{a}_l}\right\rangle\prod_{d=1}^{p}\left\langle Cf_{\tilde{b}_d},Cf_{b_{\sigma(d)}}\right\rangle$$
$$= \sum_{m,p\in\mathbb{N}_0}\sum_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g\in\mathbb{N}}\sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}}\beta_{a_1,\dots a_n,b_1,\dots b_g}\alpha_{a_1,\dots a_m,b_1,\dots b_p}$$
$$\sum_{\pi\in S_m,\sigma\in S_p}(-1)^\pi(-1)^\sigma\prod_{l=1}^{m}\left\langle \varphi_{\tilde{a}_l},\varphi_{a_{\pi(l)}}\right\rangle\prod_{d=1}^{p}\left\langle f_{b_{\sigma(d)}},f_{\tilde{b}_d}\right\rangle$$
$$= \sum_{n,g,m,p\in\mathbb{N}_0}\sum_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g\in\mathbb{N}}\sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}}$$
$$\left\langle \beta_{\tilde{a}_1,\dots\tilde{a}_n,\tilde{b}_1,\dots\tilde{b}_g}\prod_{l=1}^{n}a^*(\varphi_{\tilde{a}_l})\prod_{d=1}^{g}a(f_{\tilde{b}_d})\Omega, \alpha_{a_1,\dots a_m,b_1,\dots b_p}\prod_{l=1}^{m}a^*(\varphi_{a_l})\prod_{d=1}^{p}a(f_{b_d})\Omega\right\rangle = \langle\beta,\alpha\rangle \quad (7)$$

$\square$

Theorem 1. $\mathfrak{C}S^A = S^{-A}\mathfrak{C}$

Proof: possible?

We can now use lemma 1 to show the following

Lemma 2. $\mathfrak{C}T_1(A)\mathfrak{C} = T_1(-A)$.

Proof: The analogous property holds for the one-particle operator: $CZ_1(A)C = Z_1(-A)$. We will make use of the image of an arbitrary Fockspace element of T_1 as was derived in "defining_T1.pdf". We will consider the the image part by part, separated by the fixed-particle subspace they belong to. Let $\alpha \in \mathcal{F}_{m,p}$ be arbitrary. $(\varphi_n)_{n\in\mathbb{N}}$, $(f_n)_{n\in\mathbb{N}}$ be as usual.

Case 1: $T_1|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m,p}}$

$$
\begin{aligned}
&\sum_{\tilde{m},\tilde{p}\in\mathbb{N}} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{\tilde{m},\tilde{p}}\to\mathcal{F}_{\tilde{m},\tilde{p}}}\, \mathfrak{C}\alpha = \\
&\sum_{m,p\in\mathbb{N}} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m,p}}\, \mathfrak{C} \prod_{l=1}^{m} a^*(\varphi_{a_l}) \prod_{d=1}^{p} a(f_{b_d})\Omega = \\
&\sum_{m,p\in\mathbb{N}} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m,p}} \prod_{l=1}^{m} a(C\varphi_{a_l}) \prod_{d=1}^{p} a^*(Cf_{b_d})\mathfrak{C}\Omega = \\
&(-1)^{mp} \sum_{m,p\in\mathbb{N}} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m,p}} \prod_{d=1}^{p} a^*(Cf_{b_d}) \prod_{l=1}^{m} a(C\varphi_{a_l})\Omega \overset{\text{defining\_T1.pdf}}{=} \\
&\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C} \\
&\Bigg[\sum_{c=1}^{p} \left[\prod_{d=1}^{c-1} a^*(Cf_{b_d})\right] a^*\left(Z_{1,++}(A)Cf_{b_c}\right) \prod_{k=c+1}^{p} a^*(Cf_{b_k}) \prod_{l=1}^{m} a(C\varphi_{a_l})\Omega + \\
&\sum_{e=1}^{m} \prod_{d=1}^{p} a^*(Cf_{b_d}) \left[\prod_{l=1}^{e-1} a(C\varphi_{a_l})\right] a\left(Z_{1,--}(A)C\varphi_{a_e}\right) \prod_{k=e+1}^{m} a(C\varphi_{a_k})\Omega\Bigg] = \\
&\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \\
&\Bigg[\sum_{c=1}^{p} \left[\prod_{d=1}^{c-1} a(f_{b_d})\right] a\left(CZ_{1,++}(A)Cf_{b_c}\right) \prod_{k=c+1}^{p} a(f_{b_k}) \prod_{l=1}^{m} a^*(\varphi_{a_l})\mathfrak{C}\Omega + \\
&\sum_{e=1}^{m} \prod_{d=1}^{p} a(f_{b_d}) \left[\prod_{l=1}^{e-1} a^*(\varphi_{a_l})\right] a^*\left(CZ_{1,--}(A)C\varphi_{a_e}\right) \prod_{k=e+1}^{m} a^*(\varphi_{a_k})\mathfrak{C}\Omega\Bigg] = \\
&\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \\
&\Bigg[(-1)^{mp} \sum_{c=1}^{p} \prod_{l=1}^{m} a^*(\varphi_{a_l}) \left[\prod_{d=1}^{c-1} a(f_{b_d})\right] a\left(Z_{1,--}(-A)f_{b_c}\right) \prod_{k=c+1}^{p} a(f_{b_k})\Omega + \\
&(-1)^{pm} \sum_{e=1}^{m} \left[\prod_{l=1}^{e-1} a^*(\varphi_{a_l})\right] a^*\left(Z_{1,++}(-A)\varphi_{a_e}\right) \prod_{k=e+1}^{m} a^*(\varphi_{a_k}) \prod_{d=1}^{p} a(f_{b_d})\Omega\Bigg] = \\
&\sum_{m,p\in\mathbb{N}} T_1(-A)|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m,p}}\, \alpha
\end{aligned} \tag{8}
$$

For case 2 we need to pick the bases of $\mathcal{H}^+, \mathcal{H}^-$ in such a way that $\forall n \in \mathbb{N}\ C\varphi_n = f_n$, which is always possible. We also make use of properties of the one-particle charge conjugation operator. $C : \mathcal{H}^- \to \bar{\mathcal{H}}^+$,linear.

$$\text{Case 2: } T_1|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m+1,p+1}}$$

$$\sum_{\tilde{m},\tilde{p}\in\mathbb{N}} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{\tilde{m},\tilde{p}}\to\mathcal{F}_{\tilde{m}+1,\tilde{p}+1}}\, \mathfrak{C}\alpha =$$

$$\sum_{m,p\in\mathbb{N}} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m+1,p+1}}\, \mathfrak{C} \prod_{l=1}^{m} a^*(\varphi_{a_l}) \prod_{d=1}^{p} a(f_{b_d})\Omega =$$

$$\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m+1,p+1}} \prod_{d=1}^{p} a^*(Cf_{b_d}) \prod_{l=1}^{m} a(C\varphi_{a_l})\mathfrak{C}\Omega \overset{\text{defining\_T1.pdf}}{=}$$

$$\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C}$$

$$(-1)^m \sum_{\substack{\varphi\in\mathrm{ONB}(\mathcal{H}^+)\\ f\in\mathrm{ONB}(\mathcal{H}^-)}} \langle Z_{1,-+}(A)\varphi|\, f\rangle \left[\prod_{d=1}^{p} a^*(Cf_{b_d})\right] a^*(\varphi) \left[\prod_{l=1}^{m} a(C\varphi_{a_l})\right] a(f)\Omega =$$

$$\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p}$$

$$(-1)^m \sum_{\substack{\varphi\in\mathrm{ONB}(\mathcal{H}^+)\\ f\in\mathrm{ONB}(\mathcal{H}^-)}} \langle Z_{1,-+}(A)\varphi|\, f\rangle \left[\prod_{d=1}^{p} a(f_{b_d})\right] a(C\varphi) \left[\prod_{l=1}^{m} a^*(\varphi_{a_l})\right] a^*(Cf)\Omega =$$

$$\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p}$$

$$(-1)^m(-1)^{m+1+mp+p} \sum_{\substack{\varphi\in\mathrm{ONB}(\mathcal{H}^+)\\ f\in\mathrm{ONB}(\mathcal{H}^-)}} \langle Z_{1,-+}(A)\varphi|\, f\rangle \left[\prod_{l=1}^{m} a^*(\varphi_{a_l})\right] a^*(Cf) \left[\prod_{d=1}^{p} a(f_{b_d})\right] a(C\varphi)\Omega =$$

$$\sum_{m,p\in\mathbb{N}} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p}$$

$$(-1)^{1+p} \sum_{\substack{\varphi\in\mathrm{ONB}(\mathcal{H}^+)\\ f\in\mathrm{ONB}(\mathcal{H}^-)}} \langle CCZ_{1,-+}(A)Cf|\, C\varphi\rangle \left[\prod_{l=1}^{m} a^*(\varphi_{a_l})\right] a^*(\varphi) \left[\prod_{d=1}^{p} a(f_{b_d})\right] a(f)\Omega =$$

$$\sum_{m,p\in\mathbb{N}} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p}$$

$$(-1)^{1+p} \sum_{\substack{\varphi\in\mathrm{ONB}(\mathcal{H}^+)\\ f\in\mathrm{ONB}(\mathcal{H}^-)}} \langle Z_{1,-+}(A)\varphi|\, f\rangle \left[\prod_{l=1}^{m} a^*(\varphi_{a_l})\right] a^*(\varphi) \left[\prod_{d=1}^{p} a(f_{b_d})\right] a(f)\Omega =$$

$$\sum_{\tilde{m},\tilde{p}\in\mathbb{N}} T_1(-A)|_{\mathcal{F}_{\tilde{m},\tilde{p}}\to\mathcal{F}_{\tilde{m}+1,\tilde{p}+1}}\, \alpha = \quad (9)$$

Where in the second but last equality we used that

$$\langle CCZ_{1,-+}(A)Cf|\, C\varphi\rangle_{\mathcal{H}} = \langle CZ_{1,+-}(-A)f|\, C\varphi\rangle_{\mathcal{H}} = \langle Z_{1,+-}(-A)f|\, \varphi\rangle_{\bar{\mathcal{H}}} = \\ \langle\varphi|\, Z_{1,+-}(-A)f\rangle_{\mathcal{H}} = \langle Z_{1,-+}(A)\varphi|\, f\rangle_{\mathcal{H}} \quad (10)$$

Which we will use again in case 3.

Case 3: $T_1|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m-1,p-1}}$

$$\sum_{\tilde{m},\tilde{p}\in\mathbb{N}} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{\tilde{m},\tilde{p}}\to\mathcal{F}_{\tilde{m}-1,\tilde{p}-1}}\, \mathfrak{C}\alpha =$$

$$\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C}\, T_1(A)|_{\mathcal{F}_{m,p}\to\mathcal{F}_{m-1,p-1}} \prod_{d=1}^{p} a^*(Cf_{b_d}) \prod_{l=1}^{m} a(C\varphi_{a_l})\Omega \overset{\text{defining\_T1.pdf}}{=}$$

$$\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p} \mathfrak{C}$$

$$\sum_{j=1}^{m}\sum_{c=1}^{p} (-1)^{p-c+j-1} \left\langle C\varphi_{a_j}\right|\, Z_{1,-+}(A)Cf_{b_c}\rangle \prod_{\substack{d=1\\ d\neq c}}^{p} a^*(Cf_{b_l}) \prod_{\substack{l=1\\ l\neq j}}^{m} a(C\varphi_{a_l})\Omega =$$

$$\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p}$$

$$\sum_{j=1}^{m}\sum_{c=1}^{p} (-1)^{p-c+j-1} \left\langle C\varphi_{a_j}\right|\, CCZ_{1,-+}(A)Cf_{b_c}\rangle \prod_{\substack{d=1\\ d\neq c}}^{p} a(f_{b_l}) \prod_{\substack{l=1\\ l\neq j}}^{m} a^*(\varphi_{a_l})\Omega =$$

$$\sum_{m,p\in\mathbb{N}} (-1)^{mp} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p}$$

$$\sum_{j=1}^{m}\sum_{c=1}^{p} (-1)^{p-c+j-1}(-1)^{mp-m-p+1} \left\langle f_{b_c}\right|\, Z_{1,-+}(A)\varphi_{a_j}\rangle \prod_{\substack{l=1\\ l\neq j}}^{m} a^*(\varphi_{a_l}) \prod_{\substack{d=1\\ d\neq c}}^{p} a(f_{b_l})\Omega =$$

$$\sum_{m,p\in\mathbb{N}} \sum_{a_1,\dots a_m,b_1,\dots b_p\in\mathbb{N}} \alpha_{a_1,\dots a_m,b_1,\dots b_p}$$

$$\sum_{j=1}^{m}\sum_{c=1}^{p} (-1)^{m-j+c-1}(-1) \left\langle f_{b_c}\right|\, Z_{1,-+}(A)\varphi_{a_j}\rangle \prod_{\substack{l=1\\ l\neq j}}^{m} a^*(\varphi_{a_l}) \prod_{\substack{d=1\\ d\neq c}}^{p} a(f_{b_l})\Omega =$$

$$\sum_{\tilde{m},\tilde{p}\in\mathbb{N}} T_1(-A)|_{\mathcal{F}_{\tilde{m},\tilde{p}}\to\mathcal{F}_{\tilde{m}-1,\tilde{p}-1}}\, \alpha \quad (11)$$

□