Viererintegral durch Residuensatz

Sei m,$\epsilon > 0$ beliebig:

$$\int_{\mathcal{M}} \frac{\not{p}+m}{2m^2} f(p) i_p(\mathrm{d}^4 p) = m^2 \int_{\mathbb{R}^3} \frac{\not{p}+m}{2m^2} f(p) \frac{\mathrm{d}^3 p}{p^0}\bigg|_{p^0=\sqrt{\vec{p}^2+m^2}} + m^2 \int_{\mathbb{R}^3} \frac{\not{p}+m}{2m^2} f(p) \frac{\mathrm{d}^3 p}{p^0}\bigg|_{p^0=-\sqrt{\vec{p}^2+m^2}}$$

$$= \frac{2}{2\pi i} m^2 \left(\oint_{\left|p^0-\sqrt{\vec{p}^2+m^2}\right|=\epsilon} \mathrm{d}p^0 + \oint_{\left|p^0+\sqrt{\vec{p}^2+m^2}\right|=\epsilon} \mathrm{d}p^0 \right) \int_{\mathbb{R}^3} \frac{\not{p}+m}{2m^2} f(p) \frac{\mathrm{d}^3 p}{(p^0+\sqrt{\vec{p}^2+m^2})(p^0-\sqrt{\vec{p}^2+m^2})}$$

$$\overset{\text{Umformen der Integralkontur}}{=} \frac{2m^2}{2\pi i} \left(\int_{\mathbb{R}-i\epsilon} \mathrm{d}p^0 - \int_{\mathbb{R}+i\epsilon} \mathrm{d}p^0 \right) \int_{\mathbb{R}^3} \frac{\not{p}+m}{2m^2} f(p) \frac{\mathrm{d}^3 p}{p^2-m^2}$$

$$= \frac{1}{2\pi i} \left(\int_{\mathbb{R}^4 - i\epsilon e_0} - \int_{\mathbb{R}^4 + i\epsilon e_0} \right) \frac{\not{p}+m}{p^2-m^2} f(p) \mathrm{d}^4 p$$

$$= \frac{1}{2\pi i} \left(\int_{\mathbb{R}^4 - i\epsilon e_0} - \int_{\mathbb{R}^4 + i\epsilon e_0} \right) (\not{p}-m)^{-1} f(p) \mathrm{d}^4 p \tag{1}$$

also

$$\int_{\mathcal{M}} \frac{\not{p}+m}{2m^2} f(p) i_p(\mathrm{d}^4 p) = \frac{1}{2\pi i} \left(\int_{\mathbb{R}^4 - i\epsilon e_0} - \int_{\mathbb{R}^4 + i\epsilon e_0} \right) (\not{p}-m)^{-1} f(p) \mathrm{d}^4 p \tag{2}$$