

NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

MID - TERM EXAMINATION, 2022

SESSION: 2022 - 2023 (Autumn)

B.Tech./Int. M.Sc./M.Tech. Dual Degree - Ist Semester

Subject Code: MA 1001 Subject Name: Mathematics I Dept. Code: MA

No. of Pages: 1

Full Marks: 30

Duration: 2 Hours

Answer all the questions. Each question is of three marks.

- (1) Prove that every convergent sequence of real numbers is a Cauchy sequence.
- (2) Test the convergence of the sequence $(a_n)_{n \in \mathbb{N}}$ defined by $a_1 = 2$ and $a_{n+1} = \sqrt{2 + 2a_n}$, for $n \geq 1$. If it is convergent, then find its limit.
- (3) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
- (4) Find all the values of x for which the series $\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2}$ is (i) absolutely convergent (ii) conditionally convergent.
- (5) (i) If $\sum_{n=1}^{\infty} a_n$ is convergent, then prove that $\lim_{n \rightarrow \infty} a_n = 0$.
(ii) Examine the convergence of the series $\sum_{n=1}^{\infty} \cos(n\pi)$.
- (6) Prove that the function $f(x) = 2x + 3$ is Riemann integrable in the interval $[1, 3]$ by showing the equality of upper and lower Riemann integrals.
- (7) By using $\epsilon - \delta$ definition of limit show that $\lim_{x \rightarrow -3} \sqrt{1 - 5x} = 4$.
- (8) For every $x > 0$, prove that $\frac{x}{1+x} < \ln(1+x) < x$.
- (9) Find the number of distinct real roots of $e^{2x} + \cos x + x = 0$ by using Rolle's theorem.
- (10) Test the continuity of the function

$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

at $(0, 0)$. Also find $f_x(0, 0)$ and $f_y(0, 0)$.