a) void f1 (int n) {

    int i = 2;

    while (i < n)

    {

      /* do action that takes $O(1)$ */

      i = i * i;

    }

}

$$i = 2^{2^k}$$

isolate k:

$$\log_2 \left(\log_2 \left(2^{2^k} \right) \right)$$

$$= \Theta \left(\log_2 \left(\log_2 (n) \right) \right)$$

k	i	new i
0	2	4
1	4	16
2	16	256
3	256	65,536

b)
```
void f2 (int n) {
    for (int i=1; i<=n; i++) {
        if ((i % (int) sqrt (n) == 0) {
            for (int k=0; k<pow(i,3); k++) {
                /* something that takes O(1) time */
```

$$\sum_{i=1}^{n} \left(\Theta \left(\sum_{i=1}^{\sqrt{n}} \sum_{k=0}^{(i\sqrt{n})^3 - 1} \Theta(1) \right) \right)$$

$$= \sum_{i=1}^{n} \left(\Theta \left(\sum_{i=1}^{\sqrt{n}} (i\sqrt{n})^3 - 1 \right) \right)$$

$$= \sum_{i=1}^{n} \Theta \left(\frac{\sqrt{n}(\sqrt{n}+1)}{2} \cdot (\sqrt{n^3}) - \sqrt{n} \right)$$

$$= \sum_{i=1}^{n} \Theta \left(\frac{\sqrt{n}}{2} \cdot \frac{\sqrt{n}+1}{2} \right) (n^{3/2} - \sqrt{n}),$$

essentially equals:
$$\sum_{i=1}^{n} (n) (n^{3/2}) = \sum_{i=1}^{n} (n^{5/2})$$

$$= \Theta(n^{7/2})$$

c)
```
for (int i=1; i<=n; i++) {
    for (int k=1; k<=n; k++) {
        if ( A[k]==i ) {
            for (int m=1; m<=n; m=m+m)
            /* action that takes O(1) time,
            assure contents of A[] unchanged */
```

assure always true

$m = 1, 2, 4, 8, 16, 32 \ldots 2^{m-1}$

$\log_2(m-1)$

if statement

i	k	m
i	1	1
i	1	2
i	1	4

$$\therefore \sum_{i=1}^{n} \sum_{k=1}^{n} (\Theta(1)) +$$

$$\sum_{i=1}^{n} \sum_{m=1}^{\log_2 n} \Theta(1)$$

$$= \Theta(n^2) + \Theta(n\log n)$$

$$= \Theta(n^2)$$

```
d) int f (int n) {
        int * a = new int [10];

        int size = 10;

        for (int i=0; i<n; i++) {          ∑_{i=0}^{n-1}
            if (i == size) {                              +
                int newsize = 3*size/2;
                int *b = new int [newsize];
                for (int j = 0; j<size; j++)
                    b[j] = a[j];

                delete [] a;

                a = b;

                size = newsize;

            }
            a[i] = i*i;

    }
```

i	newsize
1	3/2
2	3
3	9/2
4	6

$$\sum_{i=0}^{n-1} \Theta(1) + \sum_{i=0}^{\log_{3/2}(n/10)} \sum_{j=0}^{size-1} \Theta(1)$$

$$= \sum_{i=0}^{n-1} \Theta(1) + \sum_{j=0}^{k} 10\left(\frac{3}{2}\right)^{j} =$$

$$10 \cdot \sum_{j=0}^{\log_{3/2}(\frac{n}{10})} \left(\frac{3}{2}\right)^{j}$$

$$= \sum_{i=0}^{n-1} \Theta(1) + \sum_{i=0}^{n-1} 15 \log_{3/2}(n/10)$$

$$= \Theta(n) + \Theta(n) = \underline{\Theta(n)}$$