PERMUTATION & COMBINATION

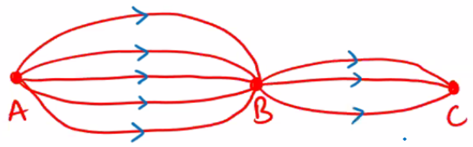
Selection & Arrangement   Selection

- KOUSTAV

PRINCIPLE OF COUNTING

OR => +

AND => X



$5 \times 3 = 15$

$(1+1+1+1+1) \times (1+1+1) = 15$

CONCEPT

1. $n! = n(n-1)(n-2)(n-3)(\ldots\ldots) \times 3 \times 2 \times 1$
   Note: 0! (factorial of zero) is 1.
2. ${}^{n}C_{r} = n! / [r! \times (n-r)!]$
3. ${}^{n}P_{r} = n! / (n-r)!$
4. Number of arrangements of n different objects in a line = n!
5. Number of arrangements of n different objects in a circle = (n − 1)!
6. Number of arrangements of n different objects in a circle where clockwise and anti-clockwise are same = (n − 1)! / 2
7. Number of arrangements of n objects in a line where p objects are alike of one kind, q are alike of another kind, r are alike of still another kind = n!p!q!r!
8. Number of arrangements when some objects are together = (Arrangement of all objects considering box as single object) × (Arrangement within box)

Formulae 7 and 8 are mostly applicable for rearrangement of letters of a word.

NOTE: For digit problems,

1. Draw the dashes equal to number of digits.
2. Fill the dashes from left to right, except when some condition is given. For e.g. the condition could be that the units digitis prime or even.
3. If Zero is among the given digits, then it cannot occupy leftmost place unless otherwise clarified

1. You have 3 pens and 2 markers. In how many ways:

i. Can you pick an item (pen or marker)?

Ans: $3+2=5$

ii. Can you pick a pen and a marker?

Ans: $3\times2=6$

2. If you have 3 shirts and 2 pairs of pants, in how many different ways can you dress up?

Ans: ________

$3\times2=6$

J_1

3. A shopping mall has 3 distinct glass doors and 2 distinct metal doors for entry, and has 5 distinct glass doors and a wooden door for exit.

i. In how many ways can you enter and exit the mall? Ans: $5\times6=30$

ii. In how many ways can you enter and exit the mall using only glass doors? Ans: $3\times5=15$

iii. In how many ways can you enter and exit the mall without using glass doors? Ans: $2\times1=2$

ENTRY	EXIT
G_1	G_1
G_2	G_2
G_3	G_3
M_1	G_4
M_2	G_5
	W_1

COMBINATION

- Selection
- Order does not matter

$n \rightarrow$ Total No.

$r \rightarrow$ Req. No.

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^nP_r = {}^nC_r \text{ \& Arr}$$

$$= \frac{n!}{\cancel{r!}(n-r)!} \times \cancel{r!} = \frac{n!}{(n-r)!}$$

PERMUTATION

- Selection & Arrangement
- Order matters

4. How many three-digit numbers can be formed using the digits 7, 8 and 9?

i. When repetition is allowed. Ans: _______

ii. When repetition is not allowed. Ans: _______

i) $\rightarrow \underset{(7,8,9)}{\underline{3}} \times \underline{3} \times \underline{3} = 27$

ii) $\rightarrow \underset{(7,8,9)}{\underline{3}} \times \underline{2} \times \underline{1} = 6$

5. How many three-digit numbers can be formed using the digits 0, 7, 8 and 9?

i. When repetition is allowed. Ans: _______

ii. When repetition is not allowed. Ans: _______

i) $\rightarrow \underset{(7,8,9)}{\underline{3}} \times \underset{(0,7,8,9)}{\underline{4}} \times \underline{4} = 48$

ii) $\rightarrow \underset{0 \times}{\underline{3}} \times \underset{0 \checkmark}{\underline{3}} \times \underline{2} = 18$

6. How many four-digit numbers can be formed using the first 6 natural numbers?

i. When repetition is allowed. Ans: _______

ii. When repetition is not allowed. Ans: _______

i) → $\underset{(1-6)}{\underline{6}}\ \underline{6}\ \underline{6}\ \underline{6} = 6^4$

ii) → $\underset{(1-6)}{\underline{6}} \times \underline{5} \times \underline{4} \times \underline{3} = 360$

$^6P_4 = \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360$

7. How many three-digit numbers can be formed so that the unit's place of the number is prime?

i. When repetition is allowed. Ans: _______

ii. When repetition is not allowed. Ans: _______

i) → $\underset{0 \times}{\underline{9}} \times \underset{0 \checkmark}{\underline{10}} \times \underset{(2,3,5,7)}{\underline{4}} = 360$

ii) → $\underset{0 \times}{\underline{8}} \times \underset{0 \checkmark}{\underline{8}} \times \underset{(2,3,5,7)}{\underline{4}} = 256$

8. How many four-digit even numbers can be formed using the first 7 whole numbers?

(0-6)

i. When repetition is allowed. Ans: _______

ii. When repetition is not allowed. Ans: _______

i) → $\underset{(1-6)}{\underline{6}} \times \underset{(0-6)}{\underline{7}} \times \underset{(0-6)}{\underline{7}} \times \underset{(0,2,4,6)}{\underline{4}} = 1176$

ii) → $\underset{0 \times}{\underline{5}} \times \underset{0 \checkmark}{\underline{5}} \times \underline{4} \times \underset{(2,4,6)}{\underline{3}}$ OR → $\underset{(1-6)}{\underline{6}} \times \underline{5} \times \underline{4} \times \underset{(\text{"0"})}{\underline{1}}$

300 + 120

= 420

9. In how many different ways can the letters of the word "LEADING" be arranged?

i. When repetition is not allowed. Ans: _______

ii. Such that all the vowels are together. Ans: _______

iii. Such that the vowels are together and the consonants are together. Ans: _______

iv. All the vowels are not together. Ans: _______

i) $\underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 7! = 5040$

ii) $\boxed{E, A, I}, L, D, N, G \rightarrow 5! \times 3! = 120 \times 6 = 720$

iii) $\boxed{E, A, I}, \boxed{L, D, N, G} \rightarrow 2! \times 3! \times 4! = 2 \times 6 \times 24 = 288$

iv) n(vowels NOT together) = n(Total) − n(vowels ARE together)

$= 5040 - 720 = 4320$

10. How many 11-letter words can be formed using all the letters of the word "MATHEMATICS"?

i. (No condition.) Ans: _______

ii. Such that all the vowels are together. Ans: _______

MATHEMATICS

i) $\dfrac{11!}{2! \times 2! \times 2!}$

ii) $\boxed{A, E, A, I}, M, T, H, M, T, C, S \rightarrow \dfrac{8!}{2! \times 2!} \times \dfrac{4!}{2!}$

$= \dfrac{\cancel{8} \times 7! \times 4!}{\cancel{2 \times 2 \times 2}} = 7! \times 4!$

12. A committee of 10 people needs to be seated on 10 chairs in a straight line. In how many different ways can they be seated if 8 particular people never sit together?

Ans: _______

$n(\text{Total}) = 10!$

$n(\text{Together}) = \boxed{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8}\ 9\ 10 = 3! \times 8!$

$n(\text{NOT tog}) = 10! - 3! \times 8!$

$= 8!(90 - 6)$

$= 84 \times 8!$

13. In how many ways can we select a team of 4 men and 2 women from a group of 8 men and 5 women?

Ans: _______

$^8C_4 \times {}^5C_2$

$\frac{8!}{4! \times 4!} \times \frac{5!}{2! \times 3!}$

$= \frac{\cancel{8} \times 7 \times \cancel{6} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2}} \times \frac{5 \times 4}{\cancel{2}} = 700$

14. In how many ways can a team of 11 be selected from 5 men and 11 women such that the team must comprise of not more than 3 men?

A) 1565 B) 2256 C) 2456 D) 1243

3M & 8W or 2M & 9W or 1M & 10W or 11W

$^5C_3 \times {}^{11}C_8 + {}^5C_2 \times {}^{11}C_9 + {}^5C_1 \times {}^{11}C_{10} + {}^{11}C_{11}$

$\frac{5\times\cancel{4}}{\cancel{2}} \times \frac{11\times10\times\cancel{9}^{3}}{\cancel{3\times2}} + \frac{5\times\cancel{4}}{\cancel{2}} \times \frac{11\times10}{\cancel{2}} + 5\times11 + 1$

$1650 + 550 + 55 + 1$

$= 2256$

15. In how many ways can a team of 8 be selected from 10 men and 6 women such that the number of women is less than that of men and there is at least one woman?

A) 8190 B) 8910 C) 9810 D) 9180

1W & 7M or 2W & 6M or 3W & 5M

$^6C_1 \times {}^{10}C_7 + {}^6C_2 \times {}^{10}C_6 + {}^6C_3 \times {}^{10}C_5$

$\cancel{6} \times \frac{10\times9\times8}{\cancel{3\times2}} + \frac{\cancel{6}\times5}{\cancel{2}} \times \frac{10\times9\times\cancel{8}\times7}{\cancel{4\times3\times2}} + \frac{\cancel{6\times5\times4}}{\cancel{3\times2}} \times \frac{10\times9\times8\times7\times\cancel{6}}{\cancel{5\times4\times3\times2}}$

$720 + 3150 + 5040$

$= 8910$

16. There are 10 people in a meeting. Everybody shakes hand with everybody. What is the minimum number of handshakes possible?

Ans: _______

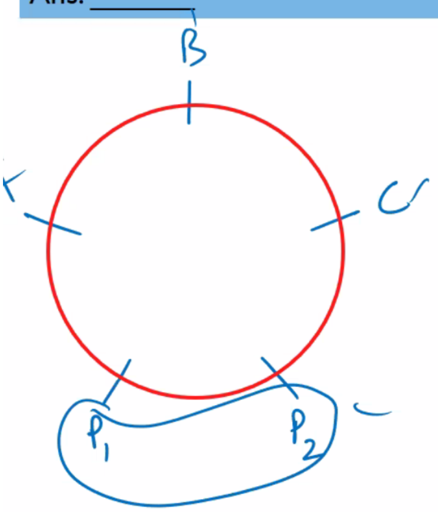
$^{10}C_2 = \frac{10!}{2!\times8!} = \frac{10\times9}{2} = 45$

17. A circle has 10 points on its circumference. What is the ratio of the number of quadrilaterals to the number of hexagons that can be formed using these 10 points?
Ans: _______

$$\frac{\text{QUAD} = {}^{10}C_4}{\text{HEXA} = {}^{10}C_6} = \frac{\frac{10!}{4! \times 6!}}{\frac{10!}{6! \times 4!}} = 1:1$$

$${}^{n}C_{r} = {}^{n}C_{n-r}$$

18. In how many ways can 5 family members sit around a circle so that the parents always sit together?
Ans: _______

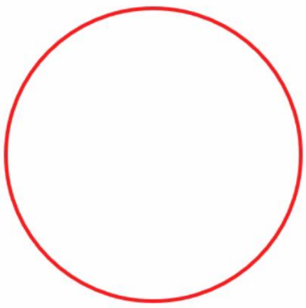


$(4-1)! \times 2!$

$3! \times 2!$

$6 \times 2 = 12$

19. How many necklaces/garlands can be formed with 7 different beads/flowers?
Ans: _______



$$\frac{(n-1)!}{2} = \frac{(7-1)!}{2}$$

$$\frac{6!}{2} = \frac{720}{2} = 360.$$

20. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

A) 24400 B) 21300 C) 210 D) 25200

SELECTION $= {}^7C_3 \times {}^4C_2 = \dfrac{7!}{3! \times \cancel{4!}} \times \dfrac{\cancel{4!}}{2! \times 2!}$

$= \dfrac{7 \times 6 \times 5 \times 4}{\cancel{2 \times 2}} = 210$

SELECTION & ARRANGEMENT $= 210 \times 5!$

$= 210 \times 120$

$= 25200$