3. What is the remainder when 2^{25} is divided by 3?

A. 2 B. 1 C. 0 D. 3

$$2^{1} \quad 2^{2} \quad 2^{3} \quad 2^{4} \quad 2^{5}$$

$$\div 3 \xrightarrow{R} 2 \quad 1 \quad 2 \quad 1 \quad 2$$

OR

3. What is the remainder when 2^{25} is divided by 3?

A. 2 B. 1 C. 0 D. 3

$$\frac{2^{25}}{3} \xrightarrow{R} \frac{2^{25}}{3} \xrightarrow{R} (-1)^{25} = -1$$

$$\Rightarrow 3-1 = 2$$

$$3)\,2\,(0 \qquad \text{or} \qquad 3)\,2\,(1$$
$$\underline{-0} \qquad\qquad \underline{-3}$$
$$2 \iff -1$$

$$\frac{5^{44}}{3} \xrightarrow{R} \frac{2^{44}}{3} \xrightarrow{R} (-1)^{44} = 1$$

$$\frac{34^{34}}{7} \xrightarrow{R} \frac{6^{34}}{7} \xrightarrow{R} (-1)^{34} = 1$$

$$\frac{19^{19}}{5} \xrightarrow{R} \frac{4^{19}}{5} \xrightarrow{R} (-1)^{19} = -1$$

$$\Rightarrow 5-1 = 4$$

$$\frac{19^{19}}{9} \xrightarrow{R} 1^{19} = 1$$

4. What is the remainder when $(1^1 + 2^2 + 3^3 + \ldots + 100^{100})$ is divided by 4?

A. 3 B. 1 C. 2 D. 0

$$\frac{1^1 + 2^2 + 3^3 + 4^4}{4}$$

$$\xrightarrow{R} 1 + 0 + (-1)^3 + 0^4$$

$$= 1 + 0 - 1 + 0$$

$$= 0$$

$$\frac{5^5 + 6^6 + 7^7 + 8^8}{4}$$

$$\xrightarrow{R} \frac{1^5 + 2^6 + (-1)^7 + 0^8}{4}$$

$$\xrightarrow{R} 1 + 0 - 1 + 0$$

$$= 0$$

5. Find the remainder when 53^{12} is divided by 17.

A. 8 B. 0 C. 1 D. 16

$\frac{53^{12}}{17} \xrightarrow{R} \frac{2^{12}}{17} = \frac{(2^4)^3}{17}$

Power of 2, nearest to 17

$(a^x)^y = a^{xy}$

$= \frac{16^3}{17} \xrightarrow{R} (-1)^3 = -1 \Rightarrow 17-1 = 16$

$\frac{32^{32}}{15} \xrightarrow{R} \frac{2^{32}}{15} = \frac{(2^4)^8}{15} = \frac{16^8}{15} \xrightarrow{R} 1^8 = 1$

$\frac{16^{24}}{9} \xrightarrow{R} (-2)^{24} = \frac{2^{24}}{9} = \frac{(2^3)^8}{9} = \frac{8^8}{9} \xrightarrow{R} (-1)^8 = 1$

$\frac{20^{21}}{9} \xrightarrow{R} \frac{2^{21}}{9} = \frac{(2^3)^7}{9} = \frac{8^7}{9} \xrightarrow{R} (-1)^7 = -1$

$\Rightarrow 9-1 = 8$

$\frac{27^{22}}{8} \xrightarrow{R} \frac{3^{22}}{8} = \frac{(3^2)^{11}}{8} = \frac{9^{11}}{8} \xrightarrow{R} 1^{11} = 1$

$\frac{32^{33}}{15} \xrightarrow{R} \frac{2^{33}}{15} = \frac{(2^4)^8 \times 2^1}{15} = \frac{16^8 \times 2}{15} \xrightarrow{R} 1^8 \times 2 = 2$

$\frac{20^{23}}{9} \xrightarrow{R} \frac{2^{23}}{9} = \frac{(2^3)^7 \times 2^2}{9} = \frac{8^7 \times 4}{9} \xrightarrow{R} (-1)^7 \times 4 = -4$

$\Rightarrow 9-4 = 5$

6. The remainder when $(7^{21}+7^{22}+7^{23}+7^{24})$ is divided by 25:

A. 1 B. 24 C. 0 D. 12

$7^{21} + 7^{22} + 7^{23} + 7^{24}$

$$\frac{(7^2)^{10} \times 7 + (7^2)^{11} + (7^2)^{11} \times 7 + (7^2)^{12}}{25}$$

$\frac{7^2}{25} \xrightarrow{R} -1$

$\xrightarrow{R} (-1)^{10} \times 7 + (-1)^{11} + (-1)^{11} \times 7 + (-1)^{12}$

$= 1 \times 7 - 1 + (-7) + 1$

$= 7 - 1 - 7 + 1$

$= 0$

7. $P = (1!)^2 + (2!)^2 + (3!)^2 + \ldots + (100!)^2$.
The remainder when 5^{2P} is divided by 13 is:

A. 1 B. 12 C. 0 D. 2

$$\frac{5^{2P}}{13} = \frac{25^P}{13} \xrightarrow{R} (-1)^P$$

If P is odd; ans = -1 or 12
If P is even; ans = 1

$$P = (1!)^2 + \underbrace{(2!)^2 + (3!)^2 + \ldots + (100!)^2}_{\text{even}}$$

$(1!)^2 \rightarrow$ odd + even

$n \geq 2$; $n! =$ even

P = odd

$$\text{Ans} = (-1)^{\text{odd}} = -1 \Rightarrow 13 - 1 = 12$$