Unit - II
2.9 DC Transient Analysis

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Syllabus

UNIT – II **14 Periods**

DC Circuit Analysis: Voltage source and current sources, ideal and practical, Kirchhoff's laws and applications to network solutions using mesh analysis, - Simplifications of networks using series- parallel, Star/Delta transformation, DC circuits-Current-voltage relations of electric network by mathematical equations to analyse the network (Superposition theorem, Thevenin's theorem, Maximum Power Transfer theorem), Transient analysis of R-L, R-C and R-L-C Circuits.

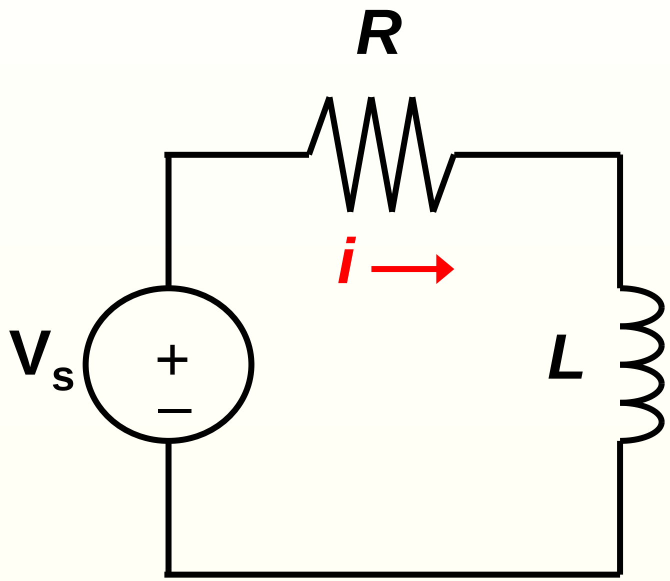
AC Steady-state Analysis: AC waveform definitions - Form factor - Peak factor - study of R-L - R-C -RLC series circuit - R-L-C parallel circuit - phasor representation in polar and rectangular form - concept of impedance - admittance - active - reactive - apparent and complex power - power factor, Resonance in R-L-C circuits - 3 phase balanced AC Circuits

OBJECTIVES

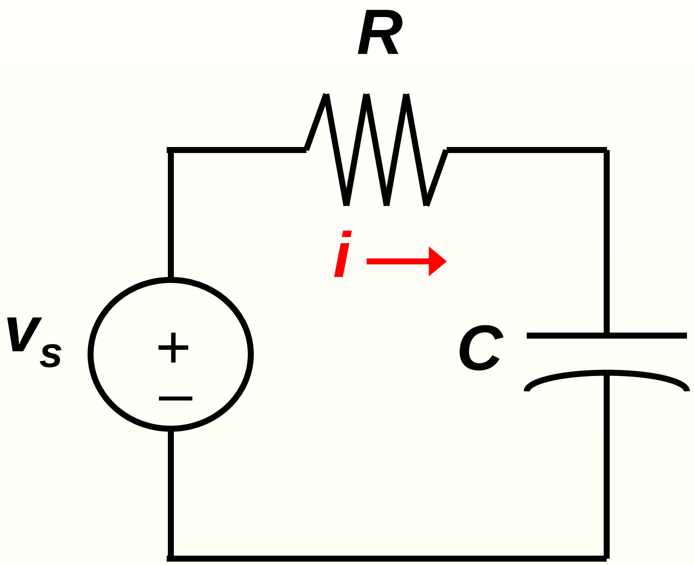
- To investigate the behavior of currents and voltages when energy is either released or acquired by inductors and capacitors when there is an abrupt change in dc current or voltage source.
- To do an analysis of natural response and step response of RL and RC circuit.

FIRST – ORDER CIRCUITS

- A circuit that contains only sources, resistor and inductor is called and RL circuit.
- A circuit that contains only sources, resistor and capacitor is called an RC circuit.
- RL and RC circuits are called first – order circuits because their voltages and currents are describe by first order differential equations.



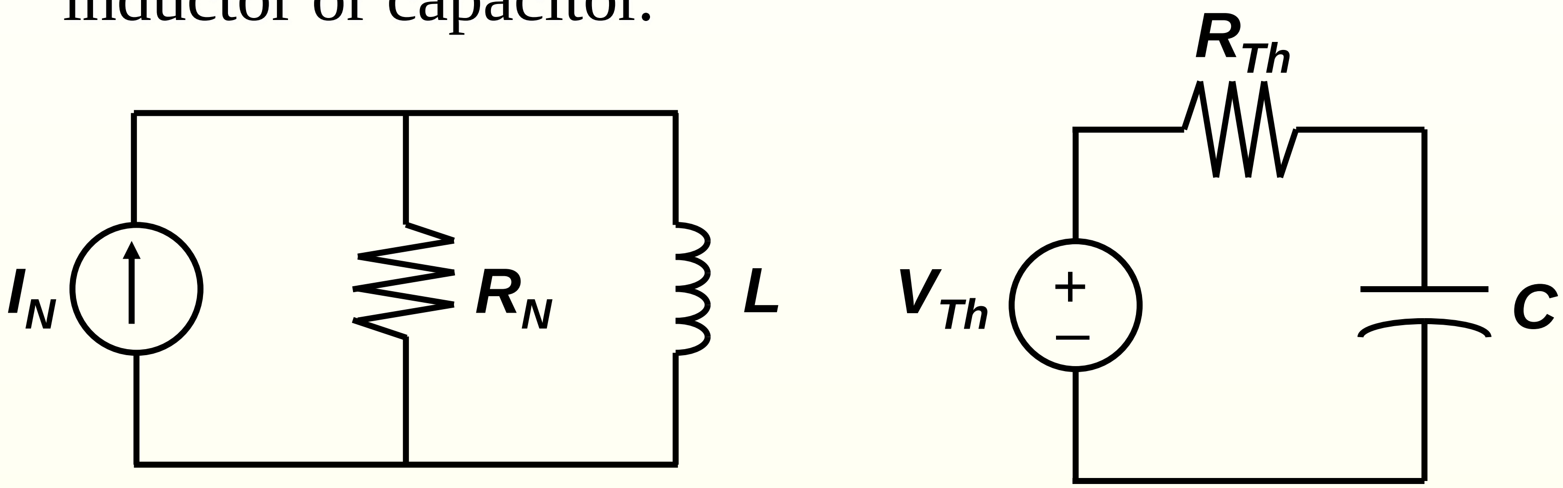
An RL circuit



An RC circuit

Review (conceptual)

- Any first – order circuit can be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.

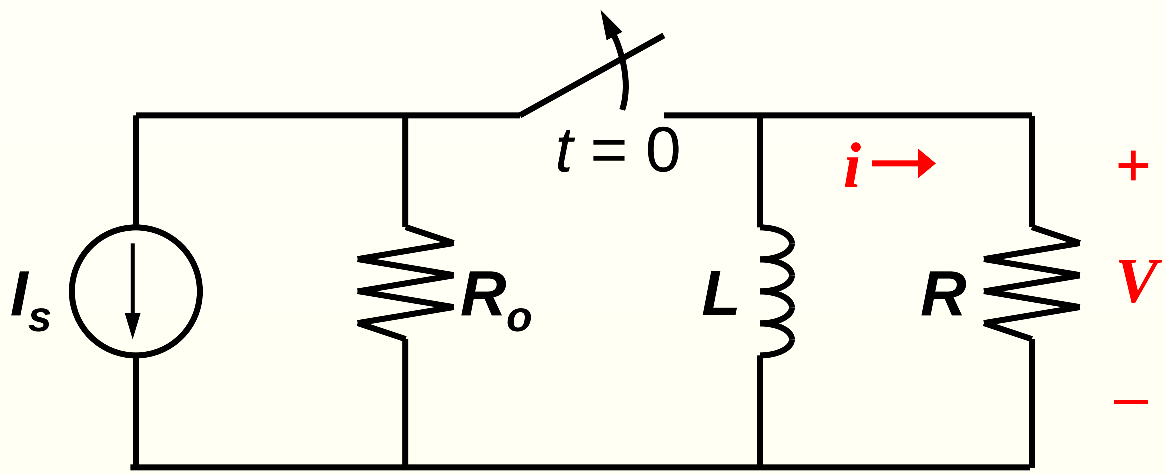


- In steady state, an inductor behave like a short circuit.
- In steady state, a capacitor behaves like an open circuit.

- The natural response of an RL and RC circuit is its behavior (i.e., current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources)
- The steps response of an RL and RC circuits is its behavior when a voltage or current source step is applied to the circuit, or immediately after a switch state is changed.

NATURAL RESPONSE OF AN RL CIRCUIT

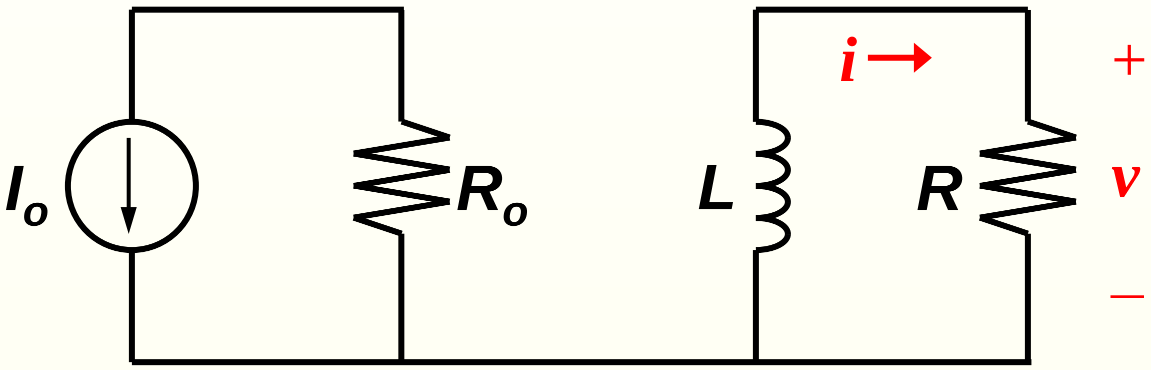
- Consider the following circuit, for which the switch is closed for $t<0$, and then opened at $t = 0$:



- The dc voltage V, has been supplying the RL circuit with constant current for a long time

Solving for the circuit

- For $t \leq 0$, $i(t) = I_o$
- For $t \geq 0$, the circuit reduce to



<u>Notation:</u>

- 0^- is used to denote the time just prior to switching.
- 0^+ is used to denote the time immediately after switching.

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- Applying KVL to the circuit:

$$v(t) + Ri(t) = 0 \qquad \text{———————} \quad \textbf{(1)}$$

$$L\frac{di(t)}{dt} + Ri(t) = 0 \qquad \text{———————} \quad \textbf{(2)}$$

$$L\frac{di(t)}{dt} = -Ri(t) \qquad \text{———————} \quad \textbf{(3)}$$

$$\frac{di(t)}{i(t)} = -\frac{R}{L}dt \qquad \text{———————} \quad \textbf{(4)}$$

Continue

- From equation (4), let say;

$$\frac{du}{u} = -\frac{R}{L}dv \quad \text{———————} \quad \textbf{(5)}$$

- Integrate both sides of equation (5);

$$\int_{i(t_o)}^{i(t)} \frac{du}{u} = -\frac{R}{L}\int_{t_o}^{t} dv \quad \text{———————} \quad \textbf{(6)}$$

- Where:

❖ *i(to)* is the current corresponding to time *to*

❖ *i(t)* ia the current corresponding to time *t*

Continue

- Therefore,

$$\ln \frac{i(t)}{i(0)} = -\frac{R}{L}t \quad \text{———————} \quad \textbf{(7)}$$

- hence, the current is

$$\boxed{i(t) = i(0)e^{-(R/L)t} = I_0 e^{-(R/L)t}}$$

Continue

- From the Ohm's law, the voltage across the resistor R is:

$$\boxed{v(t) = i(t)R = I_0\,\mathrm{R}e^{-(R/L)t}}$$

- And the power dissipated in the resistor is:

$$\boxed{p = v_R i(t) = I_0^2\,\mathrm{R}e^{-2(R/L)t}}$$

Continue

- Energy absorb by the resistor is:

$$\boxed{w = \frac{1}{2} L I_0^2 \left(1 - e^{-2(R/L)t}\right)}$$

Time Constant, τ

- Time constant, τ determines the rate at which the current or voltage approaches zero.

- Time constant, $\boxed{\tau = \dfrac{L}{R}}$ (sec)

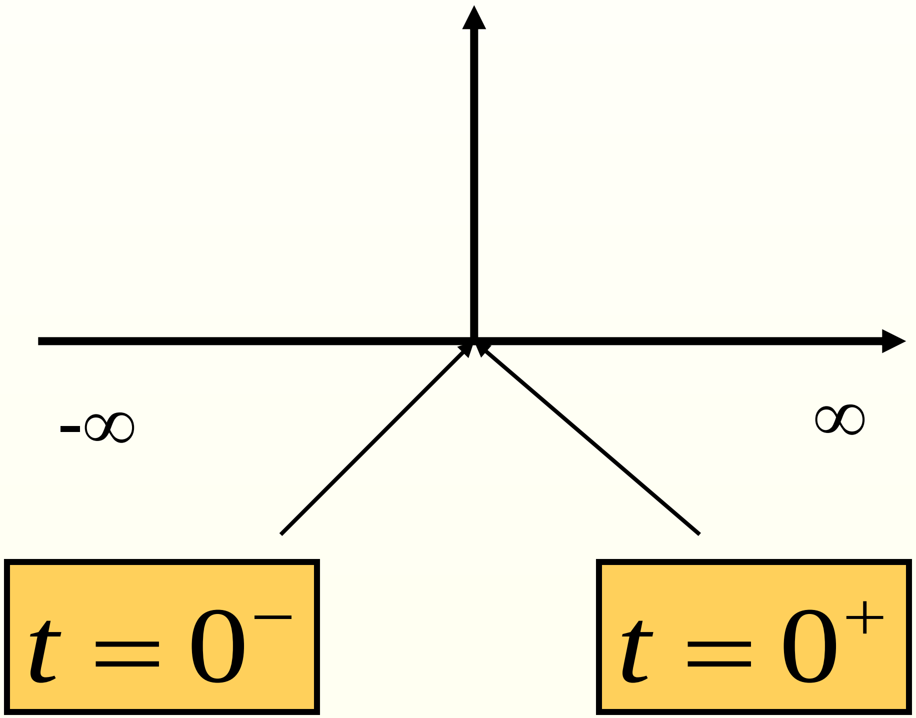
- The expressions for current, voltage, power and energy using time constant concept:

$$\boxed{\begin{aligned} i(t) &= I_0 e^{-t/\tau} \\ v(t) &= I_0 \mathrm{R} e^{-t/\tau} \\ p &= I_0^2 \mathrm{R} e^{-2t/\tau} \\ w &= \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}) \end{aligned}}$$

Switching time

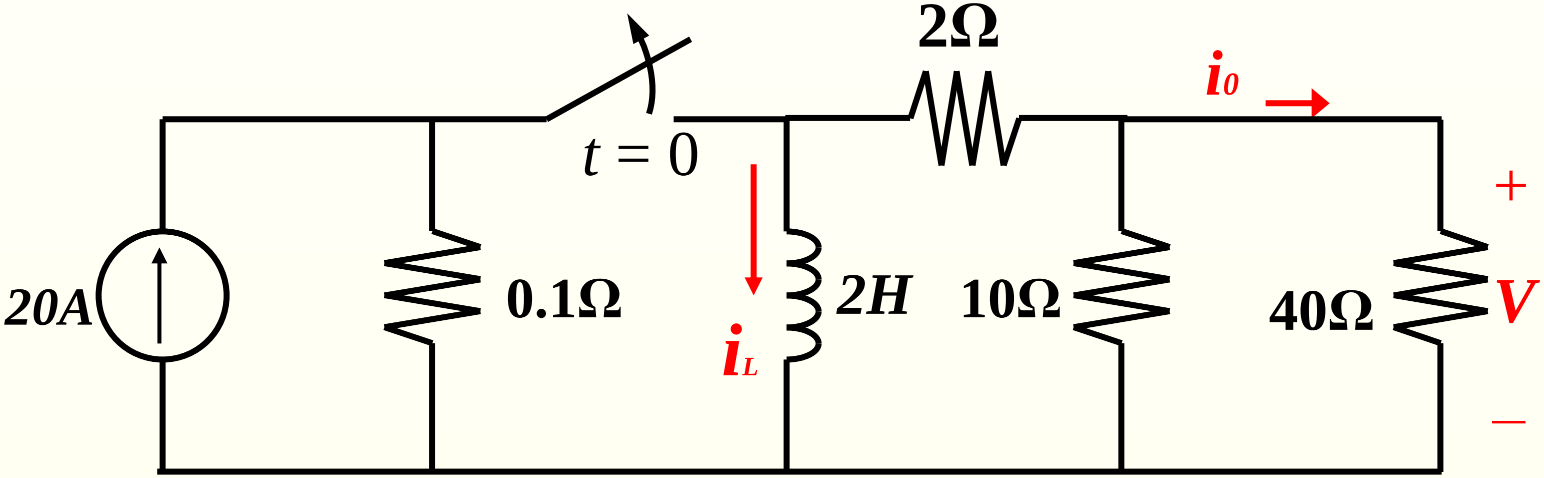
- For all transient cases, the following instants of switching times are considered.
- ✓ $t = 0^-$, this is the time of switching between -∞ to 0 or time before.
- ✓ $t = 0^+$, this is the time of switching at the instant just after time t = 0s (taken as initial value)
- ✓ $t = \infty$, this is the time of switching between $t = 0^+$ to ∞ (taken as final value for step response)

- The illustration of the different instance of switching times is:



Example

- For the circuit below, find the expression of $i_o(t)$ and $V_o(t)$. The switch was closed for a long time, and at $t = 0$, the switch was opened.



Solution :

Step 1:

Find τ for *t* > *0*. Draw the equivalent circuit. The switch is opened.

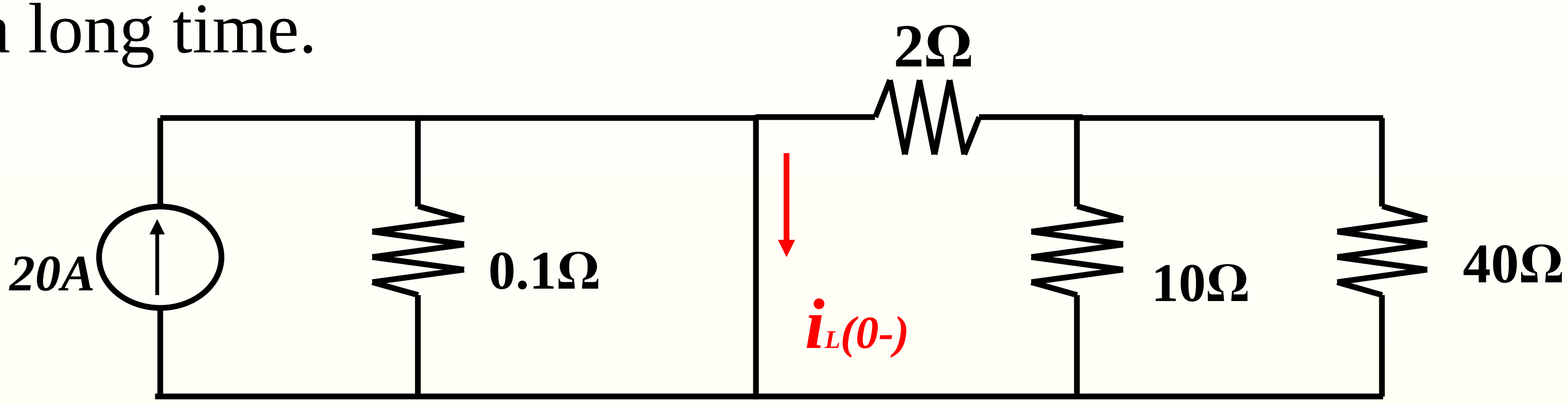
$$\boxed{R_T = (2 + 10 // 40) = 10\Omega}$$

So;

$$\boxed{\tau = \frac{L}{R_T} = \frac{2}{10} = 0.2} \text{ sec}$$

Step 2:

At $t = 0^-$, time from $-\infty$ to 0^-, the switch was closed for a long time.

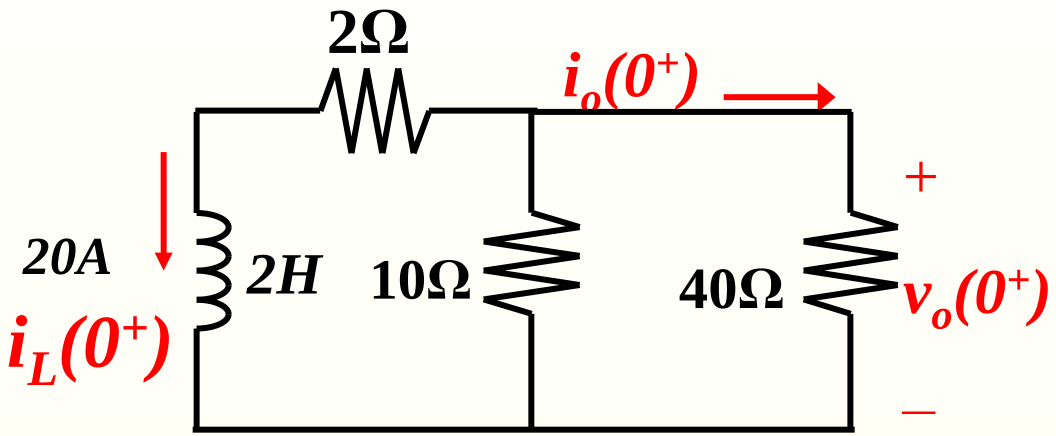


The inductor behave like a short circuit as it being supplied for a long time by a dc current source. Current 20A thus flows through the short circuit until the switch is opened.

Therefore; $i_L(0^-) = 20A$

Step 3:

At the instant when the switch is opened, the time $t = 0^+$,



The current through the inductor remains the same (continuous).

Thus, $\boxed{i_L(0^+) = i_L(0^-) = 20\,A}$ which is the initial current.

Only at this particular instant the value of the current through the inductor is the same.

Since, there is no other supply in the circuit after the switch is opened, this is the natural response case.

By using current division, the current in the 40Ω resistor is:

$$i_o = -i_L \frac{10}{10+40} = -4A$$

So,

$$\boxed{i_o(t) = -4e^{-5t} A}$$

Using Ohm's Law, the V_o is:

$$V_o(t) = -4 \times 40 = -160$$

So,

$$\boxed{V_0(t) = -160e^{-5t}}$$

Summary

- Transient RC/RL – Natural response