Row Vector

* Use space or comma to separate elements
* E.g.

```
>> A=[1 2 3]

A =

     1     2     3

>> B=[4,5,6]

B =

     4     5     6
```

Column Vector

* Use semicolon to separate elements
* E.g.

```
>> C=[7;8;9]

C =

     7
     8
     9
```

Matrix

* Matrix can be viewed as a column vector of row vectors
* Use semicolon to separate rows, and use space or comma to separate elements in a row
* E.g.

```
>> D=[1 2 3;4,5,6]

D =

     1     2     3
     4     5     6
```

Vector and Matrix

* Row vector is a special case of matrix: only one row
* Column vector is a special case of matrix: only one column
* Scalar is also a special case of matrix: only one row and only one column

Element of Matrix

* If A is a matrix, then A(i,j) is the element in the i^{th} row and j^{th} column
* E.g.

```
>> A=[1 2 3 4;5 6 7 8;1 3 5 7]

A =

     1     2     3     4
     5     6     7     8
     1     3     5     7

>> A(3,2)

ans =

     3
```

Submatrix

* If A is a matrix, then A(i:j,m:n) is the submatrix of A, which is the intersection of i^{th} row to j^{th} row of A and m^{th} column to n^{th} column of A (colon means "to")
* E.g.

```
>> A=[1 2 3 4;5 6 7 8;1 3 5 7]

A =

     1     2     3     4
     5     6     7     8
     1     3     5     7

>> A(2:3,1:2)

ans =

     5     6
     1     3
```

Basic Operations of Matrix

* Addition: A+B
* Subtraction: A-B
* Scalar multiplication: a*A
* Element by element multiplication: A.*B
* Matrix multiplication: A*B
* Power of matrix: A^n
* Determinant of matrix: det(A)

Example

* Use Matlab to verify that AB=BA is not necessarily true
* Observe the difference between A.*B and A*B
* E.g. Use

```
A =
      1      2
      3      4

B =
      4      3
      2      1
```

Example

```
>> A=[1 2;3 4]
A =
     1     2
     3     4

>> B=[4 3;2 1]

B =
     4     3
     2     1
```

```
>> A*B

ans =
     8     5
    20    13

>> B*A

ans =
    13    20
     5     8

>> A.*B

ans =
     4     6
     6     4
```

Transpose

* Transpose of A: A’
* Transpose of a row vector is a column vector, vice versa

```
>> A=[1 2;3 4;5 6]

A =

     1     2
     3     4
     5     6

>> A'

ans =

     1     3     5
     2     4     6
```

```
>> u=[1 2 3]

u =

     1     2     3

>> u'

ans =

     1
     2
     3
```

Norm and Size

* norm() computes the magnitude of a vector
* size() gives the dimensionality of a matrix
* E.g.

```
>> u=[1;1]

u =

     1
     1

>> norm(u)

ans =

    1.4142
```

```
>> A=[1 2 3;4 5 6]

A =

     1     2     3
     4     5     6

>> size(u)

ans =

     2     1
```

```
>> size(A)

ans =

     2     3
```

Dot (Inner) Product of Vectors

* dot(u,v)
* If both u and v are column vectors, we can also use matrix multiplication: u'*v

```
>> u=[1;2;3]

u =

     1
     2
     3

>> v=[3;2;1]

v =

     3
     2
     1
```

```
>> dot(u,v)

ans =

    10

>> u'*v

ans =

    10
```

Example: Angle and Projection

* u=(2,0,0), v=(3,3,3)
* Use Matlab to compute

1. The angle between u and v in degrees
2. The projection of v on u

$$\cos\ \theta = \frac{\boldsymbol{u} \cdot \boldsymbol{v}}{\|\boldsymbol{u}\|\,\|\boldsymbol{v}\|}$$

$$proj_{\boldsymbol{u}}\boldsymbol{v} = \frac{\boldsymbol{u} \cdot \boldsymbol{v}}{\|\boldsymbol{u}\|^2}\boldsymbol{u}$$

Example: Angle and Projection

```
>> u=[2;0;0]

u =

     2
     0
     0

>> v=[3;3;3]

v =

     3
     3
     3
```

```
>> angle=acosd(dot(u,v)/(norm(u)*norm(v)))

angle =

   54.7356

>> proj_v_on_u=dot(u,v)/norm(u)^2*u

proj_v_on_u =

     3
     0
     0
```

Cross Product and Moment

* cross(u,v) computes the cross product of u and v

* E.g.
* F=(3,1,1) is a force on point B, the vector from B to point A is (2,0,0), what is the moment of F about point A?

Cross Product and Moment

* F=(3,1,1) is a force on point B, the vector from B to point A is (2,0,0), what is the moment of F about point A?

```
>> F=[3,1,1]

F =

     3     1     1

>> B_A=[2,0,0]

B_A =

     2     0     0

>> moment=cross(F,B_A)

moment =

     0     2    -2
```

Cross Product and Moment

* P=(2,3,1), Q=(4,0,3), R=(6,1,0), what is P×(Q×R)?

```
>> P=[2 3 1]

P =

     2     3     1

>> Q=[4 0 3]

Q =

     4     0     3

>> R=[6 1 0]

R =

     6     1     0

>> cross(P,cross(Q,R))

ans =

    -6   -11    45
```

Linear Equations

- $x_1+3x_2=9$
- $2x_1+x_2=8$

- First method:
- $A=[1\ 3;\ 2\ 1]$, $x=[x_1;\ x_2]$, $b=[9;\ 8]$
- Equations: $Ax=b$
- Solution: $x=A^{-1}b$

```
>> A=[1 3;2 1]

A =

     1     3
     2     1

>> b=[9;8]

b =

     9
     8

>> x=inv(A)*b

x =

     3
     2

>> x=A^(-1)*b

x =

     3
     2
```

Linear Equations

* $x_1+3x_2=9$
* $2x_1+x_2=8$

* Second method:
* rref(): reduced row echelon form
* B=[1 3 9; 2 1 8], $x=[x_1; x_2]$
* B is the augmented matrix

```
>> B=[1 3 9;2 1 8]

B =

     1     3     9
     2     1     8

>> B=rref(B)

B =

     1     0     3
     0     1     2

>> x=B(:,end)

x =

     3
     2
```

Exercise

* Solve:
* $2x_1+x_2-4x_3=5$
* $3x_1+8x_3+10x_4=7$
* $x_1-5x_2+9x_3+3x_4=8$
* $5x_1+3x_3+6x_4=-15$

Key to Exercise

```
>> A=[2 1 -4 0;3 0 8 10;1 -5
9 3;5 0 3 6]

A =

     2     1    -4     0
     3     0     8    10
     1    -5     9     3
     5     0     3     6

>> b=[5;7;8;-15]

b =

     5
     7
     8
   -15

>> x=inv(A)*b

x =

  -12.6680
  -17.0810
  -11.8543
   13.9838
```