pblm:

	Ex1	Ex2	Ex3	Ex4
Feature (x)	4	8	13	7
(y)	11	4	5	14

No. of features $n = 2$

" Samples $N = 4$

Step1 : Mean of variables.

$$\bar{x} = \frac{4+8+13+7}{4} = \frac{32}{4} = 8$$

$$\bar{y} = \frac{11+4+5+14}{4} = \frac{34}{4} = 8.5$$

Step 2 : Covariance matrix [S]

$\cancel{n \Rightarrow n^2 = 2^2 = 4.}$

(x, y)

Ordered pairs $= n^2 = 2^2 = 4$.

Ordered pairs $= (x, x)\ (x, y)\ (y, x), (y, y)$

Covariance matrix $\begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$

$$Cov(x, x) = \frac{1}{N-1} \sum_{k=1} (x_k - \bar{x})^2$$

$$= \frac{1}{4-1} \sum_{k=1}^{4} (4-8)^2 + (8-8)^2 + (13-8)^2 + (7-8)^2$$

$$= \frac{1}{3}(16+0+25+1)$$

$$= \frac{1}{3}(42) = 14.$$

$$Cov(x,x) = 14.$$

$$Cov(x,y) = \frac{1}{N-1}\sum_{k=1}^{N}(x_k-\bar{x})(y_k-\bar{y})$$

$$= \frac{1}{3}[(4-8)^2, (8-8)^2, (13-8)^2, (7-8)^2]$$

$$(11-8.5)^2 (4-8.5)^2$$

$$= \frac{1}{3}((4-8)^2(11-8.5)^2 + (8-8)^2(4-8.5)^2 +$$

$$(13-8)^2(5-8.5)^2 + (7-8)^2(14-8.5)$$

$$= \frac{1}{3}[(16)(6.25) + 0 + (25)(12.25) +$$

$$(1)(30.25)]$$

$$= \frac{1}{3}[100 + 306.25 +$$

$$= \frac{1}{3}((-4)(2.5) + 0 + 5(-3.5) + (1)(5.5))$$

$$= \frac{1}{3}(-10 + 0 - 17.5 + 5.5)$$

$= \frac{1}{3} (-33)$

$\boxed{cov(x,y) = -11}$ Both are Same.

$\boxed{cov(y,x) = -11}$

$$cov(y,y) = \frac{1}{N-1} \sum_{k=1}^{N} (y_k - \bar{y})^2$$

$$= \frac{1}{3} \left[(11-8.5)^2 + (4-8.5)^2 + (5-8.5)^2 + (14-8.5)^2 \right]$$

$$= 1/3 \left[6.25 + 20.25 + 12.25 + 30.25 \right]$$

$$= 1/3 \left[69 \right]$$

$\boxed{cov(y,y) = 23}$

$$S = \begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$$

Covariance matrix $S = \begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix}$

Step 3: Eigen value, Eigen vector & Normalized Eigen vector.

Step 3

3) Eigen value of the Covariance matrix:

$$\det(S - \lambda I) = 0$$

S ↓ Covariance matrix, λ ↓ Eigen value, I → Identity matrix.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{bmatrix} = 0$$

$$(14-\lambda)(23-\lambda) - (-11)(-11) = 0$$

$$322 - 14\lambda - 23\lambda + \lambda^2 - 121 = 0$$

$$\lambda^2 - 37\lambda + 201 = 0$$

Quadratic Equ. $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1, b = -37, c = 201$

$$= \frac{-(-37) \pm \sqrt{(37)^2 - 4(1)(201)}}{2(1)}$$

$$= \frac{37 \pm \sqrt{1369 - 804}}{2}$$

$$= \frac{37 \pm 23.7697}{2}$$

$$\lambda = \frac{60.7697}{2}, \lambda_2 = \frac{13.2303}{2}$$

$$\boxed{\begin{aligned} \lambda_1 &= 30.3849 \\ \lambda_2 &= 6.6151 \end{aligned}}$$

highest value

Step 9 Computation of

ii) Eigen Vector ~~of~~ λ_1

$$(S - \lambda_1 I) U_1 = 0.$$

$$\begin{bmatrix} 14-\lambda_1 & -11 \\ -11 & 23-\lambda_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0.$$

$$\begin{bmatrix} (14-\lambda_1)u_1 - 11u_2 \\ -11u_1 + (23-\lambda_1)u_2 \end{bmatrix} = 0$$

$(14-\lambda_1)u_1 - 11u_2 = 0$ — ①

$-11u_1 + (23-\lambda)u_2 = 0$ — ②.

take ①.

$(14-\lambda_1)u_1 - 11u_2 = 0.$

$u_1 = \frac{11u_2}{(14-\lambda_1)}$

$(14-\lambda_1)u_1 = 11u_2.$

$\frac{u_1}{11} = \frac{u_2}{14-\lambda_1} = t.$

$\boxed{t = 1 \text{ Constant}}$

$\frac{u_1}{11} = 1$ $\quad$ $\frac{u_2}{14-\lambda_1} = 1$

$\boxed{u_1 = 11}$ $\quad$ $\boxed{u_2 = 14-\lambda_1}$

Eigen vector of U_1 of λ_1.

$$U_1 = \begin{bmatrix} 11 \\ 14-\lambda_1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ 14-30.3849 \end{bmatrix}$$

$$U_1 = \begin{bmatrix} 11 \\ -16.3849 \end{bmatrix}$$

iii) Normalized Eigen Vector U_1

$$e_1 = \begin{bmatrix} 11 \\ -16.3849 \end{bmatrix}$$

$$e_1 = \frac{u_1}{|u_1|} = \begin{bmatrix} \dfrac{11}{\sqrt{(11^2 + (-16.3849)^2)}} \\ \dfrac{-16.3849}{\sqrt{(11^2 + (-16.3849)^2)}} \end{bmatrix}$$

Calculate the distance separately and apply $|u_1|$ in matrix.

$$= \begin{bmatrix} \dfrac{11}{\sqrt{121 + 268.4649}} \\ \dfrac{-16.3849}{\sqrt{121 + 268.4649}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{11}{19.7349} \\ \dfrac{-16.3849}{19.7349} \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 0.5574 \\ -0.8303 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0.8303 \\ 0.5574 \end{bmatrix}$$

Step ~~5~~ 2: Principal Component

		EX1	EX2	EX3	EX4
First principal Component	PC_1	P_{11}	P_{12}	P_{13}	P_{14}
		-4.3052	3.7361	5.6928	-5.1238

$$P_{11} = e_1^T \begin{bmatrix} x_{1k} - \bar{x} \\ x_{3k} - \bar{y} \end{bmatrix}$$

y_{1k}

$$= \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} x_{11} - \bar{x} \\ y_{11} - \bar{y} \end{bmatrix}$$

$$= \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 4 - 8 \\ 11 - 8.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} -4 \\ 2.5 \end{bmatrix}$$

$$= 0.5574 \times -4 - 0.8303 \times 2.5$$

$$= -\cancel{22.296}\ {-2.2296} - 2.07575$$

$$P_{11} = -4.3052$$

$$P_{12} = \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 8 - 8 \\ 4 - 8.5 \end{bmatrix}$$

$$= 3.7361$$

$$P_{13} = \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 13 - 8 \\ 5 - 8.5 \end{bmatrix}$$

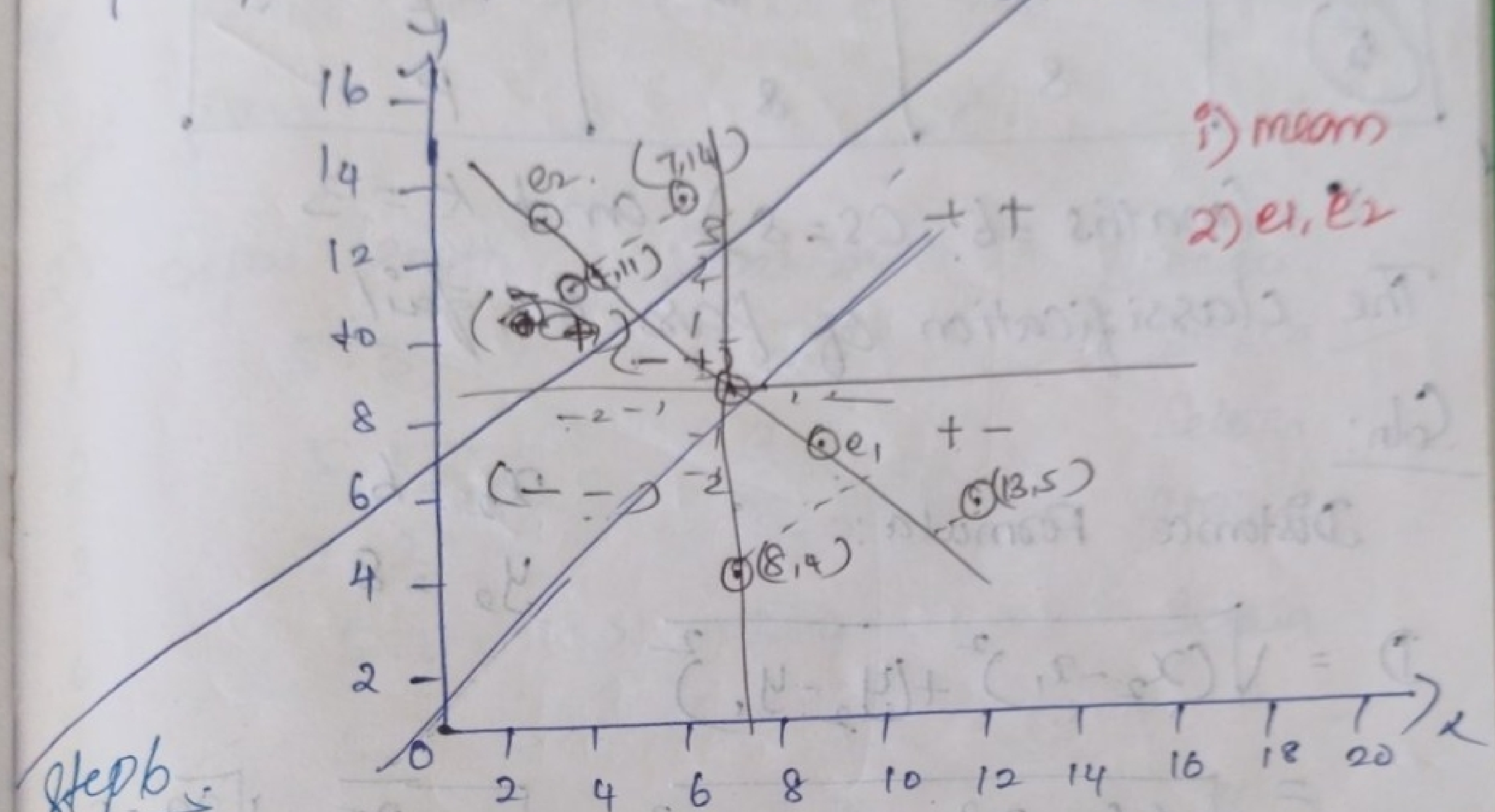
$$P_{13} = 5.6928$$

$P_{14} = \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 7-8 \\ 14-8.5 \end{bmatrix}$

$\boxed{P_{14} = -5.1238}$

Step 5 :

Find the Coordinate System for principal Component plot (x, y) From table..



Step 6 :

Geometric meaning of First principal Components.