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GAME THEORY AND NASH EQUILIBRIUM: A PERSPECTIVE

Sri Naren Chandra Boro*1

*1Assistant Professor, Department Of Mathematics Rabindranath Tagore University, Hojai, Assam, India.

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ABSTRACT

Game theory is the mathematical study of strategic decision making in various situations of conflict. In this theory a single interaction is defined as game, and those who involved in decision making are called players, who are assumed to act rationally and at the same time Nash equilibrium is another concept of game theory that has been widely used in various fields especially in economics and various social sciences. In this paper I will explore the basic idea of game theory including the ideas of Nash Equilibrium. John Von Neumann who discussed the zero sum game of two people while Nash discusses a broader range of games. Seeing the importance of Nash equilibrium in different field of social sciences like philosophy, political science, psychology, biology, economics and machine learning's it is becoming increasingly important to understand the concept of Nash equilibrium. In political science Nash equilibrium is applied to analyze international relations, game theory diplomacy etc., in psychology it is used to analyze cooperation and competition, decision making etc., in economics it is used to analyze market competition ,pricing strategies and industry organizations etc, with the development of machine learning and artificial intelligence technology Nash equilibrium has been applied to the design of adaptive intelligence agents networks in machine learning etc.

Keywords: Game, Zero-Sum, Payoff, Strategies, Equilibrium, Nash Equilibrium.

I. INTRODUCTION

A game is an abstract of a strategic situations involving interdependence. A simple form of game is defined by: players, strategies and payoffs. In a game theory players are the agents who are involved in the decision making process. Each player has a number of strategies to choose, these strategies of each player determine the outcome of the game, with each possible outcome the pay off of each player. In a game theory there may be two or more players. The game theory basically studies the decision making processes in the game which is not only a mathematical theory but as a way of thinking for solving decision taking problems. Game theory provides a new perspective and method for solving various decision making social problems by establishing models and analyzing strategies. The research result of a game theory have been applied in many fields such as political science ,psychology , sociology Biology, computer science and most importantly in economics.

In game theory another important concept is the Nash Equilibrium that is widely used in various fields. Mostly in economics it is used in analyzing market competition, pricing strategies and industry organizations. In psychology Nash equilibrium is used to analyze cooperation and competitions, decision making etc, in political science it is used to analyze international relations, game theory diplomacy etc.. Nash equilibrium refers to the state in which all the payers choose the best strategy for themselves in a certain game which is their best game strategy. The prisoner's dilemma is one of the most popular cases in game theory, which is widely used in research works in various fields. In this paper I have taken prisoners dilemma as an example, the concept and application methods of Nash equilibrium will be introduced in details. Besides I will discuss the limitations of Nash equilibrium and solutions as well as the applications and prospects of Nash equilibrium in various research fields.

I hope through this study readers can gain a deeper knowledge or understanding of game theory as well as the applications also the limitations of Nash equilibrium. I believe this would help rather better knowledge the theory and practice of game theory and provide new insights and new direction for further research. The research methodology of this paper basically focuses on literature review and theoretical analysis of the topic. Here I will first introduce the basic concept and application of prisoner's dilemma game and then explain the concept and application method of Nash equilibrium. Subsequently I will analyze the limitations of Nash equilibrium, especially in the applications of related games. Finally some methods and schemes to overcome the limitations of Nash equilibrium were proposed and the applications and prospects of Nash equilibrium in

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different fields were discussed. The final result of this paper shows that Nash equilibrium as a game strategy can be applied to decision making problems in various fields and has already been widely using in practice.

However Nash equilibrium has some limitations. To overcome these limitations scholars can adopt some methods and schemes such as introducing uncertainty and establishing trust mechanisms. In conclusion this study aims to explore the main concept and strategies of game theory, especially the application and limitations of Nash equilibrium. I hope this paper will provide a new perspective and methods for the research and application of game theory and also provides readers with a comprehensive reference of the game theory.

Basic concepts of Game:

Players: In a game player are the agents playing the game, they may be individual or firms. In the duopoly game for instance the players are taken two firms .Generally in n-person game players are numbered as 1,2,3,……n. etc.

Strategies: Strategies are the actions or the set of actions available to players through which they earn payoffs.

Payoff: Payoffs are the returns to the player at the end of the game. For instance payoffs are the profits in case of profit maximizing farms.

Zero sum and Non zero sum game: A zero sum game is one in which the gain of one player comes at the expense of the other player and is exactly equal to the loss of the other player. In other words the sum of the payoffs of the two players always adds to zero. On the other hand a non-zero sum game is where gain or loss doesn't come at the expense of the other player. As for example this might arise if increased advertisement leads to higher profits for the companies.

Pure strategy: A pure strategy provides a complete definition of how a player will play the game. In particular it determines the move a player will make for any situation they could face. A player's strategy set is the set of pure strategies available to that player.

Mixed strategy: A mixed strategy is an assignment of a probability to each pure strategy. This allows for a player to randomly select a pure strategy. Since the probabilities are continuous there are infinitely many mixed strategies available to player. Of course one can regard a pure strategy as a degenerate case of mixed strategy in which that particular pure strategy is related with probability 1 and every other strategy with probability 0.

Optimal strategy: One of the pair of mixed strategies carried by the two players of a matrix game when each player adjusts strategy so as to minimize the maximum loss that an opponent can inflict.

Minimax principle: The minimax principle for decision making by which, when presented with two various and conflicting strategies one should by the use of logic, determine and use the strategy that will minimize the maximum loss that could occur. This technique of financial and business strategy strives to attain result that will cause that least amount of loss should the strategy failure.

Maximin principle: This principle is for making choices when one is not sure of the outcome that will result from ones choice. The principle says to evaluate each point in terms of the worst possible outcome that could result from choosing the specific options and to pick that offers the best worst out come. Rational choice theory generally divides situations in which agents don't know for sure about the outcome.

Nash equilibrium: Nash equilibrium is a solution concept of a game theory involved in two or more players in which each payer is assumed to know the equilibrium strategies of each of the other player and no player has anything to gain by changing only his/her own strategy unilaterally. Named after the Nobel Laurent John Nash Jr., who proposed Nash equilibrium as a collective set of strategies for each player who takes decision taking into account the other player's decision. In other case each player is assumed to comprehend the equilibrium strategies of other players. Hence by changing only his/her strategies unilaterally no player has anything to gain. The players are known to be in Nash equilibrium if each one is making a strategic decision taking into account the decisions of other players. However Nash equilibrium does not always mean best collective pay off for all the players involved. Some strategic game may have single Nash equilibrium, some may possess no Nash equilibrium and others may have many Nash equilibrium. Nash first proposed this theory in his paper "The Bargaining problem" in 1950. He later discussed the same in his paper "Equilibrium points in n-person games "and in "Non co-operative games" in 1951 and "two person cooperative games" in 1953. He was

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awarded John Von Neumann theory prize in 1978, for his discovery of "Non co-operative equilibria", famously known as Nash equilibrium.

Case Analysis: Consider the case of involving two firms X and Y. both the firm provides the same type of products thus creating a direct competitive relationship between them. Each firm has a market share of 50% and both firms desire to acquire a greater market share by adjusting their pricing strategies. Specifically each firm can choose either a high pricing strategy or a low pricing strategy with their profit related to the pricing strategies they choose. If both firms adopt the same pricing strategy, their market share will remain unchanged. If on the other hand one firm adopts a high pricing strategy while the other adopts low pricing strategy the firm with low pricing strategy will acquire greater market share. If both firms adopt a low pricing strategy their market share will remain unchanged but their profits will be reduced. Nash equilibrium has many problems in reality. As for example in the price war in the business world, both sides are hurt, but "if the other side uses the means, I also have to use it, and who ever changes first will be unlucky". This is a Nash equilibrium Group conflicts, political disturbances and international disputes may all lead to such deadlock.

One popular example of game theory that is often cited is prisoner's dilemma. Two suspects in a crime are detained separately and kept in independent cells. Enough evidences is found to convict each one for a minor offence, but both or either can't be convicted for a major crime unless one of them converts as an informer and provide information about the other person. In a situation where both of them stay quiet they would be convicted of a minor crime and may spend less duration in prison the situation can be modeled as a strategic game by informing each one separately that if the other one provide information against him/her then he/she may be convicted for longer duration and as informer to other person may be freed or given lesser punishment. Both suspects 1 and suspects 2 would start wondering what to do? There are four options to them .if one of them confesses the crime, he will be freed but the other one will have to spend 4 years in prison, If both confess each will spend 3 years in prison, if both stay quiet and don't confess the crime cannot be probed, so they will get nominal punishment by spending only one year in prison. Thus each player has two possible strategies not confess (N) or confess(C) and they decide simultaneously. Here the situation where both the prisoners confess is called as a pure strategy Nash equilibrium as if one prisoner chooses to confess then it is better for the other prisoner too to confess rather than quiet and face longer punishment.

Issues Analysis: In the example cited above there are four pricing strategies adopted by two firms, namely High pricing by firm X, High pricing by firm Y; high pricing by firm X, low pricing by firm Y; low pricing by firm X, high pricing by firm Y, and low pricing by firm X, low pricing by firm Y. here one can represent the different pricing strategies and the corresponding profit using a game matrix as follows.

	High price	Low price
High price	50,50	30,70
Low price	70,30	60,60

An observer of the game matrix reveals that when both firms adopt the high price strategy their returns are both 50. If firm X adopts the high pricing while firm Y adopts the low pricing, firm X's returns are 30 while farm Y's return is 70.if firm X adopts the low pricing strategy while farm Y adopts the high pricing strategy firm X's return are 70 while firm Y's return is 30. When both the firm adopts the low price strategy their returns are both 60. In this scenario each firm has two different strategies to choose thus leading to four possible cases. When both firm adopt the High pricing strategy this is a Nash equilibrium state as under this situation neither firm can increase their return by choosing their strategies. Likewise when firm X adopts high pricing strategy while firm Y adopts low pricing strategy this is also a Nash equilibrium state, as neither firm can increase their returns by changing their strategies. Similarly when firm X adopt Low pricing strategy while firm Y adopts the high pricing strategy this is a Nash equilibrium state as well. However when both firms adopts the low pricing strategy this is not a Nash equilibrium state as firm X can increase its return by changing its strategy, leading to an unstable strategy state.

II. SUGGESTIONS

In above example if both firms adopt a high price strategy, their returns would be 50. Although this is a Nash equilibrium state the profits of both the firms are not maximized. Therefore firm X can adopt a low price

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strategy, which will increase its market share and thus increase its profits. At the same time firm Y can also adopt low price strategy which will reduce its market share but since the total market size remains unchanged firm Y's profit may be little bit reduced but still obtain decent profits. In this scenario both firms can achieve relatively maximum profits and this leading a Nash equilibrium state.

Here my suggestion is only for this specific case and may not be applicable to other similar situations. In practical application, participants need to select the optimal strategy according to different situations, and at the same time we need to consider other factors such as market demand, product quality, cost etc.

Besides I recommend that the firms engage in open and transparent communication to improve their information sharing this may include sharing information about their cost structure and product capacities which can help to reduce uncertainty and improve in decision making.

III. CONCLUSION

In this paper I have introduced the basic concept of game and the concept of Nash equilibrium. Also I have analyzed the significance and limitations in practical application through an illustrative example. At this stage this paper also explore the application of Nash equilibrium in different fields and looked forward to the prospects in the field of machine learning and artificial intelligence. The study specifies that the study of Nash equilibrium can help us to predict the ultimate outcome in game involving multiple participants, as well as to find optimal strategies, but it may not be necessarily a maximum payoff. Therefore in practical application we need to select the optimal strategies according to specific circumstances while taking the factors into account.

Nash equilibrium is an important and most frequently used game theoretic concept that is useable in various fields such as computer science, biology, psychology, political science and mostly in economics. Economic applications include oligopoly entry and exit, market equilibrium, search, location, bargaining product quality, auctions, insurance, principal-agent, higher education, discrimination, public goods etc. on the political front applications include voting, arms control and inspection, as well as most international political models. Biological applications all deals with forms of strategic equilibrium; they suggest an interpretation of equilibrium quite different from the usual overt rationalism. Although Nash equilibrium suffers from certain limitations in practical applications it remains an important concept in game theory that can help us to better understanding the social behavior. With the continuous development of machine earning and artificial intelligence technologies Nash equilibrium has also been applied to design adaptive intelligent and adversarial generative networks in machine learning which has a broad application prospect. One needs to choose the optimal strategy according to situation arise and to consider other factors. Thus the concept and the method of Nash equilibrium is very important for better understanding human behavior and social phenomena. At the same time this paper has some deficiencies in research method. This paper mainly focuses on the carding theory and the description of classical contents, but lacks the calculation of equivalence. Besides in the aspect of problem analysis due to limited data resources there is a lack of quantitative analysis of data in the entire process which can be extended further.

IV. REFERENCES

[1] Jenny Duffy, Game theory and Nash equilibrium.
[2] Roger B Myerson, Nash equilibrium and the history of economic theory.
[3] Madhavi Garikaparthi, Nash equilibrium- A strategy of game theory.
[4] John Nash, Non co-operative games.
[5] Leonard R 1994, Reading cournot reading Nash: The creation and stabilization of the Nash equilibrium. Economic Journal 140: 492-511
[6] ……1950. The Bargaining problem. Econometrica 18: 155-162
[7] Von Neumann, John and Oskar Morgenstern, 1944. The theory of games and economic behavior Princeton: Princeton university press.
[8] Yiting Xie, Application of Nash equilibrium. Taking the game behavior Enterprises.
[9] Von Damme E 1987: stability and perfection of Nash equilibria Berlin: springer verlag.
[10] Nash J.F. equilibrium points in n-person games. Proceedings of the National Academy of sciences, 1950.