Basic Feasible solution :- (BFS)

A feasible solution to a $(m \times n)$ transportation problem that contains no more than $(m+n-1)$ non-negative allocations is called basic feasible solution to the transportation pbm.

NORTH - WEST CORNER RULE:-

Determine basic feasible solution to the following transportion pbm using NWCR :-

		A	B	C	D	E	Supply
origin	P	2	11	10	3	7	4
	Q	1	4	7	2	1	8
	R	3	9	4	8	12	9
Demand		3	3	4	5	6	21

Sol:- the given transportation pbm is balance

Since $\Sigma a_i = \Sigma b_j$

$\Sigma a_i = 21$ and $\Sigma b_j = 21$

Step 1:-

	A	B	C	D	E	Supply
P	2	11	10	3	7	4 1
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
Demand	~~3~~	3	4	5	6	

Step : 2

	B	C	D	E	Supply
P	~~11~~	~~10~~	~~3~~	~~7~~	~~1~~
Q	~~4~~	7	2	1	8
R	~~9~~	4	8	12	9
Demand	~~3~~ 2	4	5	6	

Step 3 :-

	B	C	D	E	Supply.
Q	~~4~~	7	2	1	~~8~~ 6
R	~~9~~	4	8	12	9
Demand :	~~2~~	4	5	6	

Step 4 :-

	C	D	E	supply
Q	7	2	1	~~6~~ 2
R	4	8	12	9
Demand :	~~4~~	5	6	

Step 5:

	D	E	Supply
Q	2	1	$\cancel{2}$
R	8	12	$\cancel{9}$
Demand:	$\cancel{5}$ 3	6	

Step 6:

	D	E	Supply
R	8	12	$\cancel{9}$ $\cancel{6}$
Demand	$\cancel{3}$	6	

Step 7:

	E	Supply
R	12	$\cancel{6}$
Demand	$\cancel{6}$	

2)

	A	B	C	D	E
P	$\boxed{3}$ 2	$\boxed{1}$ 11	10	3	7
Q	1	$\boxed{2}$ 4	$\boxed{4}$ 7	$\boxed{2}$ 2	1
R	3	9	4	$\boxed{3}$ 8	$\boxed{6}$ 12

$$Z = 3\times2 + 1\times11 + 2\times4 + 4\times7 + 2\times2 + 3\times8 + 6\times12$$

$Z =$ Rs. 153

Least cost Method (or) Matrix Minimum method:-

Pbm: Obtain an initial basic feasible solution to the following T.P using the matrix minimum method.

	D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

Where O_i and D_j denote i^{th} origin and j^{th} destination respectively.

$\Sigma a_i = 24$, $\Sigma b_j = 24$

So. Given pbm is balanced.

Therefore given pbm has feasible solution.

Step 1 :-

	D1	D2	D3	D4	Capacity
O1	1	2	3	4	6
O2	4	3	2	0	8
O3	$\boxed{4}$ 0	2	2	1	~~10~~ 6
Demand	~~4~~	6	8	6	

Step 2 :-

	D2	D3	D4	Capacity
O1	2	3	4	6
O2	3	2	$\boxed{6}$ 0	~~8~~ 2
O3	2	2	1	6
Demand	6	8	~~6~~	

Step 3 :-

	D2	D3	C
O1	2	3	6
O2	3	$\boxed{2}_2$	$\cancel{2}$
O3	2	2	6
	6	$\cancel{8}$ 6	

Step 4 :

	D2	D3	C
O1	$\boxed{6}_2$	3	$\cancel{6}$
O3	2	2	6
	$\cancel{6}$	6	

Step 5 :-

	D2	D3	C
O3	2	$\boxed{6}_2$	$\cancel{6}$
	6	$\cancel{6}$	

	D1	D2	D3	D4
O1	1	$\boxed{6}$ 2	3	4
O2	4	3	$\boxed{2}$ 2	$\boxed{6}$ 0
O3	$\boxed{4}$ 0	2	$\boxed{6}$ 2	1

$Z = 4\times0 + 6\times2 + 2\times2 + 6\times2 + 6\times0.$

$Z =$ Rs. 28.

Vogel's Approximation Method (VAM)

pbm: Use vogel's approximation method to obtain initial basic feasible solution of the transportation prbm.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

$\Sigma a_i = 950$, $\Sigma b_j = 950$

Step 1

200 11	13	17	14	~~250~~ 50	(2) ← 13-11
16	18	14	10	300	(4) ← 14-10
21	24	13	10	400	(3) ← 13-10
~~200~~	225	275	250		
(5)	(5)	(1)	(0)		

2)

Step 2 :

$\boxed{50}$ 13	17	14	~~50~~ 1	(1)
18	14	10	300	(4)
24	13	10	400	(3)
175 ~~225~~	275	250		
(5)	(1)	(0)		

Step 3 .

$\boxed{175}$ 18	14	10	125 ~~300~~	(4)
24	13	10	400	(3)
~~175~~	275	250		
(6)	(1)	(0)		

Step 4 :-

14	10 $\boxed{125}$	~~125~~ 1	(4)
13	10	400	(3)
275	~~250~~ 125		
(1)	(0)		

Step 5 :-

$\boxed{275}$ 13	$\boxed{125}$ 10	~~400~~ 125	(3)
~~275~~	125		
~~(13)~~	(10)		

$\boxed{275}$ 13	$\boxed{125}$ 10	~~400~~ 275	(3)
275	~~125~~		
(0)	(0)		

Step 6

$\boxed{275}$ 13	275
275	

$\boxed{200}$ 11	$\boxed{50}$ 13	17	14
16	$\boxed{175}$ 18	14	$\boxed{125}$ 10
21	24	$\boxed{275}$ 13	$\boxed{125}$ 10

$$Z = 200\times11 + 50\times13 + 175\times18 + 125\times10 + 275\times13 + 125\times10.$$

$$Z = 12075.$$

2) MODI METHOD (Modified distribution method)

Test for optimal solution

Pbm :-

Solve the following pbm using MODI Method

	1	2	3	4	Supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	$\boxed{43}$

$\Sigma a_i = \Sigma b_j$

First we should solve this pbm by Vogel's approximation method or N-W corner rule

	1	2	3	4		
I	21	16	25	13	~~11~~	(3)
II	17	18	14	23	13	(3)
III	32	27	18	41	19	(9)
	6	10	12	~~15~~ 4		
	(4)	(2)	(4)	(10)		

	1	2	3	4		
II	17	18	14	23	$\cancel{18}$ 9	(3)
III	32	27	18	41	19	(9)
	6	10	12	$\cancel{4}$		
	(15)	(9)	(4)	(18)		

	1	2	3		
II	17	18	14	$\cancel{9}$3	(3)
III	32	27	18	19	(9)
	$\cancel{6}$	10	12		
	(15)	(9)	(4)		

	2	3		
II	18	14	$\cancel{3}$	(4)
III	27	18	19	(9)
	$\cancel{10}$7	12		
	(9)	(4)		

	2	3		
III	27	18	$\cancel{19}$ 12	(9)
	$\cancel{0}$ $\cancel{4}$ $\cancel{7}$	$\cancel{0}$ 12		
	0	0		

	3	
III	18	$\cancel{12}$
	$\cancel{12}$	

2)

	1	2	3	4	v_j
I	21	16	25	(11) 13	$\to u_1$
II	(6) 17	(3) 18	14	(4) 23	u_2
III	32	(7) 27	(12) 18	41	u_3
v_j	v_1				

From this table, we see that the no. of non negative independent allocation is $(m+n-1)=6$.

Hence the solution is non degenerate basic feasible.

$\therefore$ The initial transportation cost is 796/-

To find the optimal solution by modi method.

Consider the above transportation table.

since $m+n-1=6$, we can apply modi method.

Now we determine the set of values u_i and v_j for each occupied cell (i,j) by using the relation $c_{ij}=u_i+v_j$

since the second row contains the maximum no. of allocations, then we choose $u_2=0$.

$c_{21} = u_2+v_1 \Rightarrow 17=0+v_1 = v_1=17$

$c_{22} = u_2+v_2 \Rightarrow 18=0+v_2 = v_2=18$

$c_{24} = u_2+v_4 \Rightarrow 23=0+v_4 = v_4=23$

$C_{14} = u_1 + v_4 \Rightarrow 13 = u_1 + 23 \Rightarrow u_1 = -10.$

$C_{32} = u_3 + v_2 \Rightarrow 27 = u_3 + 18 \Rightarrow u_3 = 9.$

$C_{33} = u_3 + v_3 \Rightarrow 18 = 9 + v_3 \Rightarrow v_3 = 9.$

	1	2	3	4	
I	$\boxed{u_i+v_j}$ 21 +	16	25 +	$\boxed{11}$ 13	$u_1 = -10$
II	$\boxed{6}$ 17.	$\boxed{3}$ 18	14 +	$\boxed{4}$ 23	$u_2 = 0$
III	32 +	$\boxed{7}$ 27	$\boxed{12}$ 18	41 +	$u_3 = 9$
	$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$	

Now we find $u_i + v_j$ for each unoccupied cell (i, j) and enter at the corresponding unoccupied cell. (i, j) then we find the cell evaluation $d_{ij} = C_{ij} - (u_i + v_j)$ for each unoccupied cell (i, j) and enter the corresponding unoccupied cell (i, j).

Thus we get the following table.

	1	2	3	4	
I	$\boxed{7}$ $\boxed{14}$ 21 ×	$\boxed{8}$ $\boxed{8}$ 16 ×	$\boxed{-1}$ $\boxed{26}$ 25 ×	$\boxed{11}$ 13	$u_1 = -10$
II	$\boxed{6}$ 17	$\boxed{3}$ 18	$\boxed{9}$ $\boxed{5}$ 14 ×	$\boxed{4}$ 23	$u_2 = 0$
III	$\boxed{26}$ $\boxed{6}$ 32 ×	$\boxed{7}$ 27	$\boxed{12}$ 18	$\boxed{32}$ $\boxed{9}$ 41 ×	$u_3 = 9$
	$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$	

2) $d_{11} = C_{11} - (U_1 + V_1) = 21 - 7 = 14$

Since all $d_{ij} > 0$, the solution under the test is optimal and unique

The optimum allocation schedule is given by

$x_{14} = 11$, $x_{21} = 6$, $x_{22} = 3$, $x_{24} = 4$, $x_{32} = 7$,

$x_{33} = 12$ and the optimum (minimum)

transportation cost $Z = Rs. 796/-$

Pbm:-

Find the optimal transportation cost of the following matrix using least cost method for finding the critical solution.

	A	B	C	D	E	Available
P	4	1	2	6	9	100
Q	6	4	3	5	7	120
R	5	2	6	4	8	120
Demand	40	50	70	90	90	340

$\Sigma a_i = \Sigma b_j$

	A	B	C	D	E		
P	4	1	2	6	9	100	(1)
Q	6	4	3	5	7	120	(1)
R	5	2	6	4	8	~~120~~ 80	(2)
	~~40~~	~~50~~	70	90	90		
	(1)	(1)	(1)	(1)	(1)		

	B	C	D	E		
P	1	2	6	9	100	(1)
Q	4	3	5	7	120	(1)
R	2	6	4	8	~~80~~ 30	(2)
	~~50~~	70	90	90		
	(1)	(1)	(1)	(1)		

	C	D	E		
P	2	6	9	100	(4)
Q	~~4~~ 3	5	7	120	(2)
R	6	4	8	30	(2)
	70	90	90		

2)

	A	B	C	D	E	
P	4	1	2	6	9	~~100~~ 50
Q	6	4	3	5	7	120
R	5	2	6	4	8	120
	40	~~80~~	70	90	90.	

	A	C	D	E	
P	4	2	6	9	~~50~~
Q	6	3	5	7	120
R	5	6	4	8	120
	40	~~70~~ 20.	90	90.	

	A	C	D	E	
Q	6	3	5	7	~~120~~ 100.
R	5	6	4	8	120.
	40	~~20~~	90	90.	

	A	D	E	
Q	6	5	7	100
R	5	4	8	~~120~~ 80.
	40	~~90~~	90.	

	A	E	
Q	6	7	100
R	5	8	~~30~~
	~~40~~ 10	90	

	A	E	
Q	6	7	~~100~~ 90
	~~10~~	90	

	E	
Q	7	90
	90	

	A	B	C	D	E	
P	4	$\boxed{50}$ 1	$\boxed{50}$ 2	6	9	u_1
Q	$\boxed{10}$ 6	4	$\boxed{20}$ 3	5	$\boxed{90}$ 7	u_2
R	$\boxed{30}$ 5	2	6	$\boxed{90}$ 4	8	u_3
	v_1	v_2	v_3	v_4	v_5	

$Z = 1410/-$

2) For optimality since the no. of non-negative indepe allocations is 7 which is equal to m+n-1, then we can apply MODI method.

$u_2 = 0$

$u_2 + v_1 = 6 \Rightarrow v_1 = 6$

$u_2 + v_3 = 3 \Rightarrow v_3 = 3$

$u_2 + v_5 = 7 \Rightarrow v_5 = 7$

$v_3 + u_1 = 2 \Rightarrow 3 + u_1 = 2 \Rightarrow u_1 = -1$

$v_2 + u_1 = 1 \Rightarrow v_2 - 1 = 1 \Rightarrow v_2 = 2.$

$v_1 + u_3 = 5 \Rightarrow 6 + u_3 = 5 \Rightarrow u_3 = -1$

$v_4 + u_3 = 4 \Rightarrow v_4 - 1 = 4 \Rightarrow v_4 = 5.$

	A	B	C	D	E	
P	5 4 -1 $+\theta$	50 1	50 2 $-\theta$	4 6 2 ×	6 9 3 ×	$u_1 = -1$
Q	10 6 $-\theta$	2 4 2 ×	20 3 $+\theta$	5 5 0 ×	90 7	$u_2 = 0$
R	30 5	1 2 1 ×	2 6 4 ×	90 4	6 8 2 ×	$u_3 = -1$
	$v_1 = 6$	$v_2 = 2$	$v_3 = 3$	$v_4 = 5$	$v_5 = 7$	

Since $d_{11} = -1 < 0$, the current solution is not optimal. Now let us form a new basic feasible solution by given maximum allocation to the cell (i, j) for which d_{ij} is most negative by making an unoccupied cell empty. Here the cell $(1, 1)$ having the negative value $d_{11} = -1$. We draw a closed loop consisting of horizontal and vertical lines beginning and ending at the cell $(1, 1)$ and having other corners at some occupied cells. Along the closed loop indicate $+\theta$ and $-\theta$ alternatively at the corners, we have.

	A	B	C	D	E
P	$+\theta$ 4	[50] 1	[50] $-\theta$ 2	6	9
Q	[10] $-\theta$ 6	4	[20] $+\theta$ 3	5	[90] 7
R	[30] 5	2	6	[90] 4	8

From the two cell $(1, 3)$ and $(2, 1)$ having $-\theta$, we find that the minimum of the allocations that is $\min\{10, 50\} = 10$. Add this 10 to the cells with $+\theta$ and subtract this 10 to the cells θ.

Hence the new basic feasible solution is displayed in the following table.

[10] 4	[50] 1	[40] 2	6	9
6	4	[30] 3	5	[90] 7
[30] 5	2	6	[90] 4	8

2)

$u_1 = 0$

$u_1 + v_1 = 4 \Rightarrow v_1 = 4$

$u_1 + v_2 = 1 \Rightarrow v_2 = 1$

$u_1 + v_3 = 2 \Rightarrow v_3 = 2$

$u_2 + v_3 = 3 \Rightarrow u_2 + 2 = 3 \Rightarrow u_2 = 1$

$u_2 + v_5 = 7 \Rightarrow 1 + v_5 = 7 \Rightarrow v_5 = 6.$

$u_3 + v_1 = 5 \Rightarrow u_3 + 4 = 5 \Rightarrow u_3 = 1$

$u_3 + v_4 = \cancel{3}\ 4 \Rightarrow 1 + v_4 = 4 \Rightarrow v_4 = 3.$

[10] 4	[50] 1	[40] 2	[3] 6 [3] ×	[6] 9 [3] ×	$u_1 = 0$
[5] 6 [1] ×	[2] 4 [2] ×	[30] 3	[4] 5 [1] ×	[90] 7 ×	$u_2 = 1$
[30] 5	[2] 2 [0] ×	[3] 6 [3] ×	[90] 4	[7] 8 [1] ×	$u_3 = 1$
$v_1 = 4$	$v_2 = 1$	$v_3 = 2$	$v_4 = 3$	$v_5 = 6.$	

All $d_{ij} > 0$ the solution under the test is optimal & unique.

$Z = 4 \times 10 + 1 \times 50 + 2 \times 40 + 3 \times 30 + 7 \times 90 + 30 \times 5 + 4 \times 90.$

$Z =$ Rs. 1400/-

$z = 1160$

Pbm :-

				Avaiable unit	
6	1	9	3	$\cancel{70}$	(2)
11	5	2	8	55	(3)
10	12	4	7	90	(2)
Required units					
$\cancel{85}$ 15	35	$\cancel{50}$	45		
(4)	(4)	$\cancel{(2)}$	(4)		

11	5	2	8	$\cancel{55}$ 20	(3)
10	12	4	7	90	(3)
15	$\cancel{35}$	50	45		
(1)	(7)	(2)	(1)		

11	2	8	$\cancel{20}$	(6)
10	4	7	90	(3)
15	$\cancel{50}$ 30	45		
(1)	(2)	(1)		

10	4	7	60 $\cancel{90}$	(3)
15	$\cancel{30}$	45		
(0)	(0)	(0)		

2)

10	7	~~60~~ 15	(3)
15	~~45~~		
(0)	(0)		

10	15
15	

[70] 6	1	9	3
11	[35] 5	[20] 2	8
[15] 10	12	[30] 4	[45] 7

$m+n-1 \geq 6$

$Z = 70\times6 + 35\times5 + 20\times20 + 15\times10 + 30\times4 + 45\times7$

Z = Rs - 1580/-

$U_3 = 0$

$V_1 + U_3 = 10 \Rightarrow V_1 = 10$

$V_3 + U_3 = 4 \Rightarrow V_3 = 4$

$V_4 + U_3 = 7 \Rightarrow V_4 = 7$

$U_2 + V_2 = 5 \Rightarrow -2 + V_2 = 5 \Rightarrow V_2 = 7$

$U_2 + V_3 = 2 \Rightarrow U_2 + 4 = 2 \Rightarrow U_2 = -2$

PM Mithun

$U_1 + V_1 = 6 \Rightarrow U_1 + 10 = 6 \Rightarrow U_1 = -4$

$\boxed{70}$ 6 $-\theta$	$\boxed{3}$ 1 $\boxed{-2}$ $+\theta$	$\boxed{0}$ 9 $\boxed{9}$	$\boxed{3}$ 3 $\boxed{0}$	$U_1 = -4$
$\boxed{8}$ 11 $\boxed{3}$	$\boxed{35}$ 5 $-\theta$	$\boxed{20}$ 2 $+\theta$	$\boxed{5}$ 8 $\boxed{3}$	$U_2 = -2$
$\boxed{15}$ 10 $+\theta$	$\boxed{7}$ 12 $\boxed{5}$	$\boxed{30}$ 4 $-\theta$	$\boxed{45}$ 7	$U_3 = 0$
$V_1 = 10$	$V_2 = 7$	$V_3 = 4$	$V_4 = 7$	

cell $(2,2)$, $(3,3)$, $(1,1)$

$\min\{70, 35, 30\} = 30$

$\boxed{70}$ 60	$+\theta$ 40 1	9	3
11	$\boxed{35}$ $\cancel{0}$ 5	$\boxed{20}$ $\cancel{0}$ 2	8
$\boxed{15}$ 10 $\cancel{0}$	12	$\boxed{30}$ 4 0	$\boxed{45}$ 7

40

$\boxed{40}$ 6	$\boxed{30}$ 1	9	3	$U_1 = -4$
11	$\boxed{05}$ 5	$\boxed{50}$ 2	8	$U_2 = -2$
$\boxed{45}$ 10	12	$\boxed{0}$ 4	$\boxed{45}$ 7	$U_3 = 0$
$V_1 = 10$	$V_2 = 7$	$V_3 = 4$	$V_4 = 7$	

$U_3 = 0$

$U_3 + V_4 = 7 \Rightarrow V_4 = 7$

$U_3 + V_1 = 10 \Rightarrow V_1 = 10$

$U_3 + V_3 = 4 \Rightarrow V_3 = 4$

2)

$U_2 + V_2 = 5 \Rightarrow -2 + V_2 = 5 \Rightarrow V_2 = 7$

$U_2 + V_3 = 2 \Rightarrow U_2 + 4 = 2 \Rightarrow U_2 = -2$

$U_1 + V_1 = 6 \Rightarrow U_1 + 10 = 6 \Rightarrow U_1 = -4$

~~$U_1 + V_2 = 11 \Rightarrow$~~

6 [40]	[30] 1	[0] 9 [9] ×	[3] 3 [0] ×	$U_1 = -4$
[8] 11 [3] ×	[5] 5	[50] 2	[5] 8 [3] ×	$U_2 = -2$
[45] 10	[7] 12 [5] ×	[4] 4 [0]	[45] 7	$U_3 = 0$
$V_1 = 10$	$V_2 = 7$	$V_3 = 4$	$V_4 = 7$	

$Z = 6 \times 40 + 30 \times 1 + 5 \times 5 + 50 \times 2 + 45 \times 10 + 45 \times 7$

$Z = $ Rs - 1160/-

Degeneracy in Transportation pbm :

In a tran pbm, whenever the no. of non-negative independent allocation is less the $m+n-1$, the Tp is said to be degenerate. Degeneracy may occur either at the initial stage (or) at an intermediate stage at some subsequent iteration.

To resolve degeneracy, we allocate an extremely small amount (which is close to zero) to one (or) more empty cells of the T table (generally minimum cost cells if possible), so that the total number of occupied cells becomes $(m+n-1)$ at independent positions. We denote this small amount by ε (epsilon) satisfying the following condition

i) $0 < \varepsilon < x_{ij}$ for all $x_{ij} > 0$

ii) $x_{ij} \pm \varepsilon = x_{ij}$ for all $x_{ij} > 0$.

pbm :-

NWC rule

				Supply
10	20	5	7	~~10~~
13	9	12	8	20
4	5	7	9	30
14	7	1	0	40
3	12	5	19	50.
Demand ~~60~~ 50.	60	20	10	

13	9	12	8	~~20~~
4	5	7	9	30
14	7	1	0	40
3	12	5	19	50.
~~50~~ 30	60	20	10	

2)

4	5	7	9	$\cancel{30}$
14	7	1	0	40
3	12	5	19	50
$\cancel{30}$	60	20	10	

14	7	1	0	$\cancel{40}$
3	12	5	19	50
$\cancel{0}$	60	20	10	

7	1	0	$\cancel{40}$
12	5	19	50
$\cancel{60}$ 20	20	10	

12	5	19	$\cancel{50}$ 30
$\cancel{20}$	20	10	

5	19	$\cancel{30}$ 10
$\cancel{20}$	10	

19	10

10

~~path~~

[10] 10	[19] 20 [1]	[12] 5 [7]	[26] 7 [19]	$u_1 = 7$
[20] 13	[22] 9 [-13]	[15] 12 [-3]	[29] 8 [-19]	$u_2 = 10$
[30] 4	[13] 5 [-8]	[6] 7 [1]	[20] 9 [-11]	$u_3 = 1$
[-2] 4 [16] $+\theta$	[40] 7 $-\theta$	[0] 1 [1]	[14] 8 [-14]	$u_4 = -5$
[ε] 3 $-\theta$	[20] 12 $+\theta$	[20] 5	[10] 19	$u_5 = 0$
$v_1 = 3$	$v_2 = 12$	$v_3 = 5$	$v_4 = 19$	

6

$\varepsilon \to$ min value

$Z = 1230 + 3 \times \varepsilon$

$\varepsilon = 0$

min $Z = 1230$.

$u_5 = 0$

$u_5 + v_1 = 3 \Rightarrow v_1 = 3$

$u_5 + v_2 = 12 \Rightarrow v_2 = 12$

$u_5 + v_3 = 5 \Rightarrow v_3 = 5$

$u_5 + v_4 = 19 \Rightarrow v_4 = 19$.

$u_4 + v_2 = 7 \Rightarrow u_4 + 12 = 7 \Rightarrow u_4 = -5$

$u_3 + v_1 = 4 \Rightarrow u_3 + 3 = 4 \Rightarrow u_3 = 1$

$u_2 + v_1 = 13 \Rightarrow u_2 + 3 = 13 \Rightarrow u_2 = 10$.

$u_1 + v_1 = 10 \Rightarrow u_1 + 3 = 10 \Rightarrow u_1 = 7$

min $\{(4,2)\ (5,1)\} = \{7, 3\} = 3$.

2)

$\boxed{10}$ 10	20	5	7
$\boxed{20}$ 13	9	12	8
$\boxed{30}$ 4	5	7	9
14	$\boxed{40}$ 7	1	0
$\boxed{\varepsilon}$ 30	$\boxed{20}$ 12	$\boxed{20}$ 5	$\boxed{10}$ 19

CB					
M	$\boxed{10}$ 10	20	5	7	$U_1 = 10$
0	$\boxed{20}$ 13	9	12	8	$U_2 = 13$
-M	$\boxed{30}$ 4	5	7	9	$U_3 = 4$
	$\boxed{3}$ 14	$\boxed{37}$ 7	1	0	$U_4 = 14$
	$\boxed{3-\varepsilon}$ 30	$\boxed{23}$ 12	$\boxed{20}$ 5	$\boxed{10}$ 19	$U_5 = 30$
	$V_1 = 0$	$V_2 = -7$	$V_3 = -25$	$V_4 = -11$	

$V_1 = 0$

$V_1 + U_1 = 10 \Rightarrow U_1 = 10$

$V_1 + U_2 = 13 \Rightarrow U_2 = 13$

$V_1 + U_3 = 4 \Rightarrow U_3 = 4$

$V_1 + U_4 = 14 \Rightarrow U_4 = 14$

$V_1 + U_5 = 30 \Rightarrow U_5 = 30$

$v_2 + u_4 = 7 \Rightarrow v_2 + 14 = 7 \Rightarrow v_2 = -7$

$v_3 + u_5 = 5 \Rightarrow v_3 + 30 = 5 \Rightarrow v_3 = -25$

$v_4 + u_5 = 19 \Rightarrow v_4 + 30 = 19 \Rightarrow v_4 = -11$

$Z = 10 \times 10 + 20 \times 13 + 30 \times 4 + 3 \times 14 + 37 \times 7$

$+ (3 - \varepsilon) \times 30 + 23 \times 12 + 20 \times 5 + 10 \times 19$

$Z = 1347 + 90 - 30\varepsilon$

$\varepsilon = 0$

$Z = 1347 + 90 \Rightarrow$ Rs 1437/-

2) Vogel's

				Supply	
10	20	5	7	10	(2)
13	9	12	8	20	(1)
4	5	7	9	30	(1)
14	7	1	0	~~40~~ 30	(1)
3	12	5	19	50	(2)
Demand 60	60	~~20~~ 20	~~10~~		
(1)	(2)	(4)	(7)		

2)

10	20	~~4~~5	10	(5)
13	9	~~8~~12	20	(3)
4	5	~~9~~7	30	(1)
14	7	~~0~~1	~~30~~10	(6)
3	12	~~10~~ 5	50	(2)
60	60	~~20~~		
(1)	(2)	(4)		

CE

M

O

10	20	~~5~~ ~~10~~	(10)
13	9	~~10~~ 20	(4)
4	5	30	(1)
14	7	10	(7)
3	12	50	(9)
~~60~~ 50	60		
(1)	(2)		

13	9	20	(4)
4	5	30	(1)
14	7	10	(7)
50 3	12	~~50~~	(9)
~~50~~	60		
(1)	(2)		

9	20
5	~~30~~
7	10
12	0

~~60~~ 30

9	20
~~7~~	~~10~~
12	0

~~30~~ 20

$\boxed{20}$ 9	~~20~~
12	0

~~20~~

$\boxed{10}$ 10	20	5	7
13	9	12	8
4	$\boxed{30}$ 5	$\boxed{10}$ 7	$\boxed{20}$ 9
14	7	$\boxed{20}$ 1	$\boxed{10}$ 0
$\boxed{80}$ 3	12	5	12

Z = Rs. 670/-

2) 3) Least count:-

10	20	5	7	10
13	9	12	8	20
4	5	7	9	30
14	7	1	0	~~40~~ 30 20
3	12	5	19	50
60	60	20	~~10~~	

10	20	5	10
13	9	12	20
4	5	7	30
14	7	1	~~30~~ 10
3	12	5	50
60	60	~~20~~	

10	20	10
13	9	20
4	5	30
14	7	10
3	12	~~50~~
~~60~~ 10	60	

10	20	10
15	9	20
4	5	~~30~~ 20
14	7	10
~~10~~	60	

20	10
9	20
5	~~20~~
7	10
~~60~~ 40	

20	10
9	20
7	~~10~~
~~40~~ 30 20	

20	10
9	~~20~~
~~30~~ ~~20~~ 10	

2)

$\boxed{20}$ $\cancel{10}$

$\cancel{10}$

10	[10] 20	5	7
13	[20] 9	12	8
[10] 4	[20] 5	7	[2] 9
14	[10] 7	[20] 1	[10] 0
[50] 3	12	5	19

Assignment Method (or) Hungarian Method.

Pbm:- A department has five emp with 5 jobs to be performed the time (in hours) each man will take to perform each job is given in the effectiveness matrix.

emp

Jobs	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

Select minimum in row and sub

Step 1: Row wise subtraction

5	$\boxed{0}$	8	10	11
$\boxed{0}$	6	15	10	3
8	5	0	$\cancel{0}$	0
$\cancel{0}$	4	2	$\boxed{0}$	5
3	5	6	$\cancel{0}$	8

Step 2 : Column wise

5	$\boxed{0}$	8	10	11
$\boxed{0}$	6	15	10	3
8	5	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
$\cancel{0}$	4	2	$\boxed{0}$	5
3	5	6	$\cancel{0}$	8

first we have to see row wise zero, and mark it as □ and mark X in correspondi.. 0 after see column wise zero same procedu..

sub min value of the table

5	$\boxed{0}$	3	10	~~8~~ 6
$\boxed{0}$	6	15	10	3
8	0	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
$\cancel{0}$	4	2	$\boxed{0}$	5
3	5	6	$\cancel{0}$	8

Enthe row the assigned 0 "□" illaiyo verum X mattum irundha anthe row ah ✓ tick pannikanum next col la □ and X enga iruko anga tick pannikanu.. tick agirukura col la enga □ iruko anga anthe row la tick pannanum step 2 :- Column wise.

$$\begin{array}{c}\begin{bmatrix} 5 & \boxed{0} & 8 & 10 & 11 \\ \boxed{0} & 6 & 15 & 10 & 3 \\ 8 & 5 & \boxed{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & 4 & 2 & \boxed{0} & 5 \\ 3 & 5 & 6 & \cancel{0} & 8 \end{bmatrix} \begin{array}{c} \\ \checkmark \\ \\ \checkmark \\ \checkmark \end{array} \\ \begin{array}{ccccc} \checkmark & \quad & \quad & \checkmark & \quad \end{array}\end{array}$$

unmarked rows and marked column.

Sub min value of the table.

$$\begin{matrix} 5 & \boxed{0} & 8 & 10 & 11 \\ \boxed{0} & 6 & 15 & 10 & 3 \\ 8 & 5 & \boxed{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & 4 & 2 & \boxed{0} & 5 \\ 3 & 5 & 6 & \cancel{0} & 8 \end{matrix}$$

intersection point

+

↓

add min

2)

	I	II	III	IV	V
A	7	$\boxed{0}$	8	12	11
B	$\boxed{0}$	4	15	10	1
C	10	5	$\cancel{0}$	2	$\boxed{0}$
D	$\cancel{0}$	2	$\boxed{0}$	$\cancel{0}$	3
E	3	3	4	$\boxed{0}$	6

A → II　　C → V　　E ⇒ IV

B → I　　D → III

$Z = 5 + 3 + 2 + 9 + 4 \Rightarrow 23$ hours.

Maximization case in the Assignment problem :-

	1	2	3	4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

Select max no and sub it with all value in table

	1	2	3	4
A	0	6	2	5
B	2	5	1	1
C	1	1	3	4
D	3	4	2	1

Step 1 & 2 :-

	1	2	3	4	
A	$\boxed{0}$	6	2	5	✓
B	1	4	$\boxed{0}$	$\cancel{0}$	
C	$\cancel{0}$	$\boxed{0}$	2	3	
D	2	3	1	$\boxed{0}$	✓
				✓	

2)

	1	2	3	4
A	$\boxed{0}$	5	1	5
B	1	4	$\boxed{0}$	1
C	1	$\boxed{0}$	2	4
D	2	2	$\cancel{0}$	$\boxed{0}$

A → 1 B → 3 C → 2 D → 4

Rs:- 16+15+15+15 = 61.

---

2)

	1	2	3	4
A	60	50	40	30
B	40	30	20	15
C	40	20	35	10
D	30	30	25	20

	1	2	3	4
A	0	10	20	30
B	20	30	40	45
C	20	40	25	50
D	30	30	35	40

	1	2	3	4
A	0	10	20	30
B	0	10	20	25
C	0	20	15	30
D	10	10	15	20

	1	2	3	4
A	0	0	5	10
B	0	0	5	5
C	~~0~~	10	$\boxed{0}$	10
D	10	~~0~~	~~0~~	$\boxed{0}$

	1	2	3	4
A	$\boxed{0}$	0	15	10
B	0	$\boxed{0}$	15	5
C	~~0~~	10	$\boxed{0}$	10
D	10	~~0~~	10	$\boxed{0}$

	1	2	3	4
A	0	10	20	30
B	0	10	20	25
C	0	20	5	30
D	0	0	5	10

	1	2	3	4	
A	$\boxed{0}$	10	15	20	✓
B	~~0~~	10	15	15	✓
C	~~0~~	20	$\boxed{0}$	20	✓
D	~~0~~	$\boxed{0}$	~~0~~	~~0~~	
	✓		✓		

	1	2	3	4	
A	$\boxed{0}$	10	15	20	✓
B	~~0~~	10	15	15	✓
C	~~0~~	20	$\boxed{0}$	20	✓
D	~~0~~	$\boxed{0}$	~~0~~	~~0~~	
	✓		✓		

		1	2	3	4
	A	$\boxed{\cancel{0}}$	$\boxed{\cancel{0}}$	15	10
✓	B	$\boxed{\cancel{0}}$	$\boxed{\cancel{0}}$	15	5
✓	C	~~0~~	10	$\boxed{0}$	10
✓	D	10	~~0~~	10	$\boxed{0}$

5	**0**	8	10	11	
0	6	15	10	3	✓
0	5	**0**	~~0~~	~~0~~	
~~0~~	4	2	**0**	5	✓
3	5	6	~~0~~	8	✓
✓			✓		

	I	II	III	IV	V
A	7	**0**	8	12	11
B	**0**	4	13	10	1
C	10	5	~~0~~	2	**0**
D	~~0~~	2	**0**	~~0~~	3
E	3	3	4	**0**	6

A → II   B → I   C → V   D → III

E → IV

$5+3+2+9+4 = 23$