

Backward propagation:

target output is 0.5

learning rate is 1.

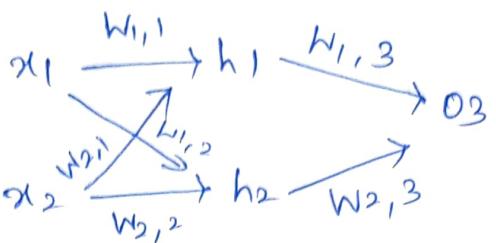
$$x_1 = 0.35 \quad x_2 = 0.7$$

$$w_{1,1} = 0.2 \quad w_{2,2} = 0.3$$

$$w_{2,1} = 0.2 \quad w_{2,2} = 0.3$$

$$w_{1,3} = 0.3 \quad (h_1 \rightarrow o_3)$$

$$w_{2,3} = 0.9 \quad (h_2 \rightarrow o_3)$$



1) forward pass:

$$\begin{aligned} \text{net } h_1 &= x_1 w_{1,1} + x_2 w_{2,1} \\ &= (0.35)(0.2) + (0.7)(0.2) \Rightarrow 0.07 + 0.14 = 0.21 \end{aligned}$$

$$h_1 = \sigma(\text{net } h_1) = \sigma(0.21) = \frac{1}{1+e^{-0.21}} \approx 0.5523079$$

$$\begin{aligned} \text{net } h_2 &= x_1 w_{1,2} + x_2 w_{2,2} \\ &= (0.35)(0.3) + (0.7)(0.3) = 0.105 + 0.21 = 0.315 \end{aligned}$$

$$h_2 = \sigma(\text{net } h_2) = \sigma(0.315) = \frac{1}{1+e^{-0.315}} \approx 0.5781052$$

$$\text{net } o_3 = h_1 w_{1,3} + h_2 w_{2,3}$$

$$= 0.5523079(0.3) + 0.5781052(0.9)$$

$$= 0.685987$$

$$o_3 = \sigma(\text{net } o_3) = \frac{1}{1+e^{-0.685987}} \approx 0.66507364$$

2) Error calculation

$$E = \frac{1}{2} (y - \hat{y})^2$$

$$E = \frac{1}{2} (0.5 - 0.66507364)^2 \approx 0.01362465$$

3) Backward pass

$$\frac{\partial E}{\partial \hat{y}} = (\hat{y} - y)$$

$$\delta_0 = (\hat{y} - y) \cdot \sigma'(\text{net}_{03}) = (\hat{y} - y) \cdot \hat{y}(1 - \hat{y})$$

$$\hat{y} - y = 0.66507364 - 0.5 = 0.16507364$$

$$\sigma'(\text{net}_{03}) = \hat{y}(1 - \hat{y})$$

$$= 0.66507364 (1 - 0.5) \Rightarrow 0.2230116$$

$$\delta_0 = (0.16507364)(0.2230116)$$

$$= 0.03677027$$

$$\frac{\partial E}{\partial w_{1,3}} = \delta_0 \cdot h_1 , \quad \frac{\partial E}{\partial w_{2,3}} = \delta_0 \cdot h_2$$

$$\frac{\partial E}{\partial w_{1,3}} = (0.03677027)(0.5523079) \approx 0.02030851$$

$$\frac{\partial E}{\partial w_{2,3}} = (0.03677027)(0.5523079) \approx 0.02125708$$