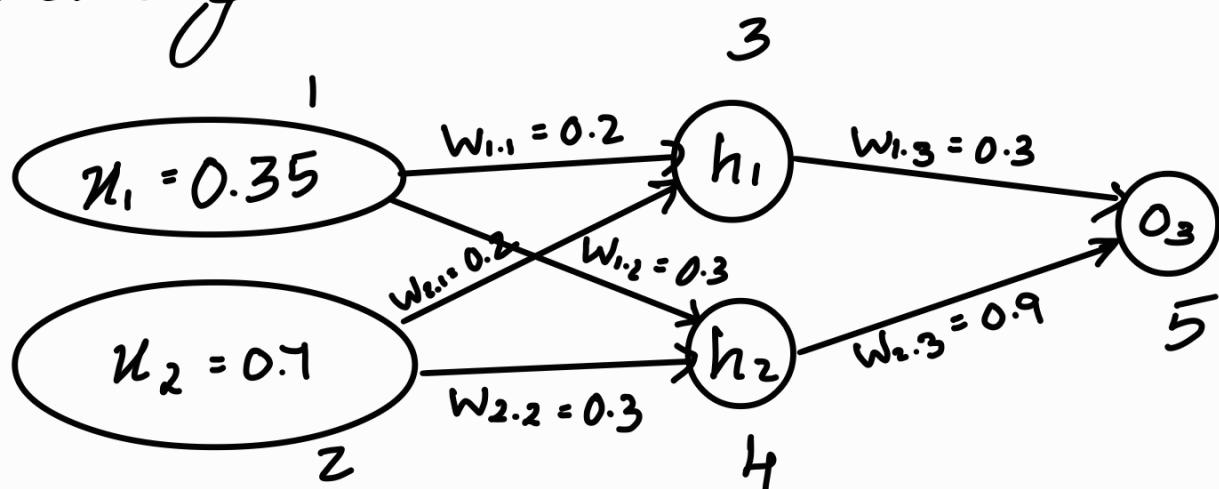


Q. Assume the neurons use the sigmoid activation func. for the forward and backward pass. The target output is 0.5 and the learning rate is 1.



Soln:- Forward Pass :-

$$h_1 = x_1 w_{1,1} + x_2 w_{2,1} = 0.35 \times 0.2 + 0.7 \times 0.2 = 0.21$$

$$y_1 = \frac{1}{1 + e^{-0.21}} = 0.552$$

$$h_2 = x_1 w_{1,2} + x_2 w_{2,2} = 0.35 \times 0.3 + 0.7 \times 0.3 = 0.315$$

$$y_2 = \frac{1}{1 + e^{-0.315}} = 0.518$$

$$O_3 = Y_1 \omega_{13} + Y_2 \omega_{23} = 0.552 \times 0.3 + \\ 0.578 \times 0.9 \\ = 0.6858$$

$$O_3 = \frac{1}{1+e^{-0.685}} = 0.665$$

$$t = 0.5$$

$$\Sigma = \frac{1}{2} [t - O_3]^2 = \frac{1}{2} (0.5 - 0.66)^2 \\ = [0.0132]$$

Backward Pass :-

$$\delta_5 = 0.665 (1 - 0.665) (0.5 - 0.665) = -0.036$$

$$\delta_3 = 0.552 (1 - 0.552) 0.3 \times (-0.036) \\ = -0.00267$$

$$\delta_4 = 0.578 (1 - 0.578) W_{23} \delta_3 = 0.578 (1 - 0.578) \\ 0.9 \times -0.036 \\ = -0.0079$$

$$\begin{aligned}\Delta W_{13} &= \alpha \times \delta_5 \times Y_1 \\ &= 1 \times -0.036 \times 0.552 \\ &= -0.0198\end{aligned}$$

$$W_{13}(\text{new}) = 0.3 - 0.0198 = 0.28$$

$$\begin{aligned}\Delta W_{23} &= \alpha \times \delta_5 \times Y_2 \\ &= 1 \times -0.036 \times 0.578 \\ &= -0.0208\end{aligned}$$

$$W_{23}(\text{new}) = 0.9 - 0.0208 = 0.879$$

$$\begin{aligned}\Delta W_{11} &= \alpha \times \delta_3 \times X_1 \\ &= 1 \times -0.0026 \times 0.35 = -0.00093\end{aligned}$$

$$W_{11}(\text{new}) = 0.2 - 0.00093 = 0.199$$

$$\begin{aligned}\Delta W_{12} &= \alpha \times \delta_4 \times X_2 \\ &= -0.0079 \times 0.35 = -0.0027\end{aligned}$$

$$W_{12}(\text{new}) = 0.3 - 0.0027 = 0.297$$

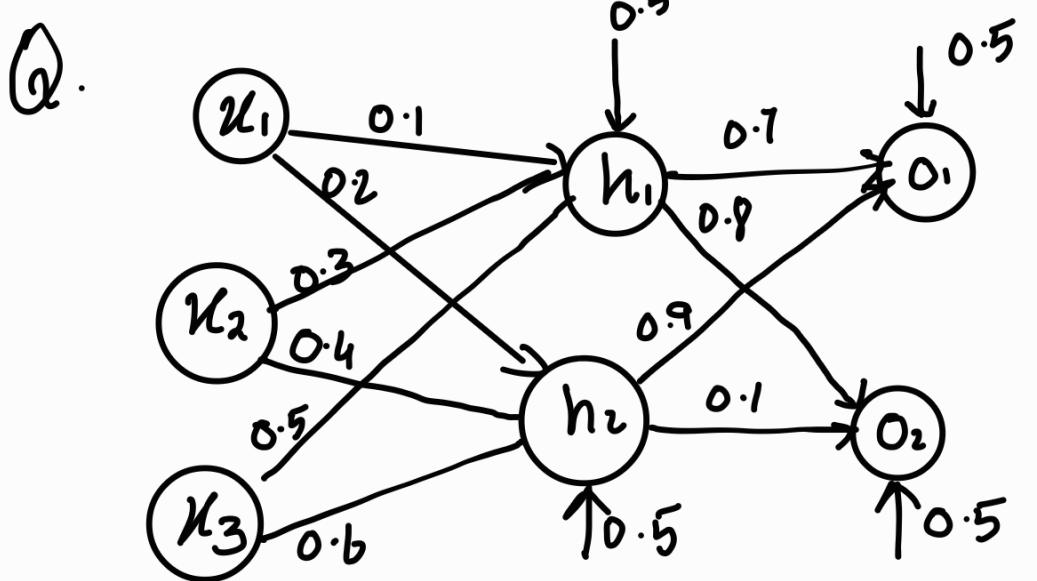
$$\begin{aligned}\Delta W_{21} &= 1 \times \delta_3 \times X_2 \\ &= -0.00267 \times 0.7 = -0.00186\end{aligned}$$

$$W_{\alpha 1(\text{new})} = 0.2 - 0.00065 = 0.199$$

$$\begin{aligned}\Delta W_{22} &= 1 \times \delta_4 \times \kappa_2 \\ &= -0.0079 \times 0.7 \\ &= -0.00553\end{aligned}$$

$$W_{22(\text{new})} = 0.294$$

x ————— x ————— x



Now.

$$\begin{aligned}Z_{h_1} &= 0.1 \times 1 + 0.2 \times 4 + 0.3 \times 5 + 0.5 \\ &= 2.9\end{aligned}$$

$$\begin{aligned}Z_{h_2} &= 0.4 \times 1 + 0.5 \times 4 + 0.6 \times 5 + 0.5 \\ &= 5.9\end{aligned}$$

$$h_1 = \frac{1}{1 + e^{-2.9}} = 0.947$$

$$h_2 = \frac{1}{1 + e^{-5.9}} = 0.997$$

$$\begin{aligned}Z_{O_1} &= 0.1 \times 0.947 + 0.8 \times 0.99 + 0.5 \\&= 1.9613\end{aligned}$$

$$\begin{aligned}Z_{O_2} &= 0.9 \times 0.947 + 0.1 \times 0.99 + 0.5 \\&\approx 1.452\end{aligned}$$

$$O_1 = 0.88$$

$$O_2 = 0.81$$

$$\begin{aligned}\text{Loss} &= \frac{1}{2} \left((0.1 - 0.8766)^2 + (0.05 - 0.810)^2 \right) \\&= 0.5907\end{aligned}$$

Backward pass :-

$$\delta_{O_1} = (O_1 - t_1) \cdot O_1 (1-O_1) = \frac{(0.876 - 0.1)}{0.87 (1 - 0.876)} \\ \approx 0.0839$$

$$\delta_{O_2} = (O_2 - t_2) \cdot O_2 (1-O_2) \approx 0.11682$$

$$\Delta W_{h_1, O_1} = \delta_{O_1} \times h_1 = 0.083 \times 0.947 = 0.0795$$

$$\Delta W_{h_2, O_1} = 0.0839 \times 0.99 = 0.0837$$

$$\Delta W_{h_1, O_2} = \delta_{O_2} \times h_1 = 0.1107$$

$$\Delta W_{h_2, O_2} = \delta_{O_2} \times h_2 = 0.1165$$

$$\Delta b_{O_2} = \delta_{O_2} = 0.1168$$

$$\delta_{h_1} = (\delta_{O_1} W_{h_1, O_1} + \delta_{O_2} W_{h_1, O_2}) \times h_1 (1-h_1) \\ = (0.083 \times 0.7 + 0.11 \times 0.9) \cdot 0.947 (1 - 0.947) \\ \approx 0.00810$$

$$\delta_{h_2} = (\delta_{O_1} W_{h_2, O_1} + \delta_{O_2} W_{h_2, O_2}) \cdot h_2 (1-h_2) \\ = 0.000214$$

$$\Delta W_{h_1 h_1} = \delta_{h_1} \chi_1 = 0.0081 \times 1 = 0.0081$$

$$\Delta W_{h_2 h_1} = \delta_{h_1} \chi_2 = 0.0081 \times 4 = 0.0324$$

$$\Delta W_{h_3 h_1} = \delta_{h_1} \chi_3 = 0.0081 \times 5 = 0.0405$$

$$\Delta b_{h_1} = \delta_{h_1} = 0.0081$$

$$\Delta W_{h_1 h_2} = \delta_{h_2} \chi_1 = 0.000214 \times 1$$

$$\Delta W_{h_2 h_2} = \delta_{h_2} \chi_2 = 0.00085$$

$$\Delta W_{h_3 h_2} = \delta_{h_2} \chi_3 = 0.00107$$

$$\Delta b_{h_2} = 0.000214$$

$$W_{h_1,0,(\text{new})} = 0.7 - 0.1 \times 0.079 = 0.692$$

$$W_{h_2,0,(\text{new})} = 0.8 - 0.1 \times 0.083 = 0.7916$$

$$b_{0,1}^{(\text{new})} = 0.5 - 0.1 \times 0.083 = 0.49$$

$$W_{h_1,h_1,(\text{new})} = 0.1 - 0.1 \times 0.08$$

$$W_{h_2,h_1,(\text{new})} = 0.1964$$

$$W_{h_3,h_1,(\text{new})} = 0.295$$

$$b_{h_1,(\text{new})} = 0.4991$$

