Unit 4

Probabilistic reasoning in Artificial intelligence

- **Uncertainty**
- **Causes of uncertainty:**
    - Information occurred from unreliable sources.
    - Experimental Errors
    - Equipment fault
    - Temperature variation
    - Climate change.

Probabilistic reasoning:

- way of knowledge representation where we apply the concept of probability to indicate the uncertainty in knowledge
- combine probability theory with logic to handle the uncertainty.
- the certainty of something is not confirmed, such as "It will rain today," "behavior of someone for some situations," "A match between two teams or two players."

- **Need of probabilistic reasoning in AI:**
  - When there are unpredictable outcomes.
  - When specifications or possibilities of predicates becomes too large to handle.
  - When an unknown error occurs during an experiment.
- In probabilistic reasoning, there are two ways to solve problems with uncertain knowledge:
  - **Bayes' rule**
  - **Bayesian Statistics**

- **Probability:** Probability can be defined as a chance that an uncertain event will occur. It is the numerical measure of the likelihood that an event will occur. The value of probability always remains between 0 and 1 that represent ideal uncertainties.
- $0 \leq P(A) \leq 1$, where $P(A)$ is the probability of an event A.
- $P(A) = 0$, indicates total uncertainty in an event A.
- $P(A) = 1$, indicates total certainty in an event A.
- We can find the probability of an uncertain event by using the below formula.

$$\textbf{Probability of occurrence} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

- $P(\neg A)$ = probability of a not happening event.
- $P(\neg A) + P(A) = 1$.
-

Event: Each possible outcome of a variable is called an event.

Sample space: The collection of all possible events is called sample space.

Random variables: Random variables are used to represent the events and objects in the real world.

Prior probability: The prior probability of an event is probability computed before observing new information.

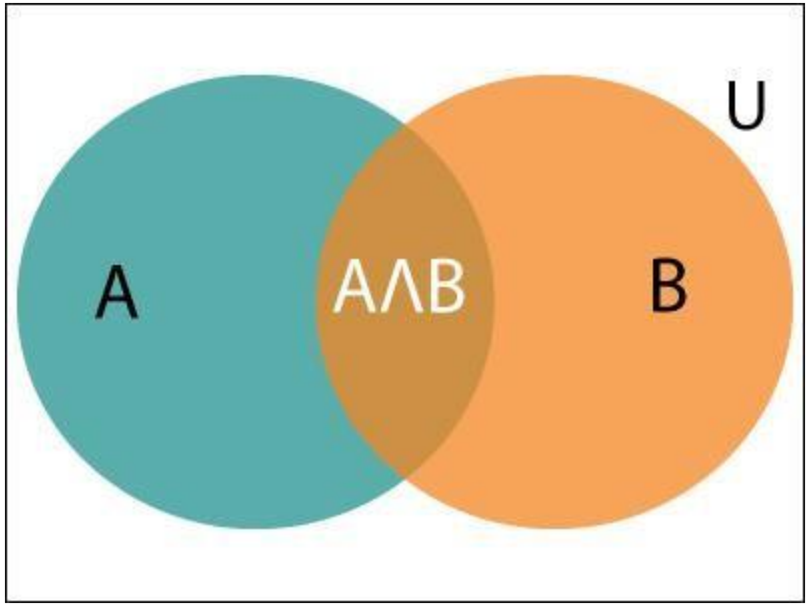
Posterior Probability: The probability that is calculated after all evidence or information has taken into account. It is a combination of prior probability and new information.

Conditional probability:

Conditional probability is a probability of occurring an event when another event has already happened.

Let's suppose, we want to calculate the event A when event B has already occurred, "the probability of A under the conditions of B", it can be written as:

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$



- **Example:**
- In a class, there are 70% of the students who like English and 40% of the students who likes English and mathematics, and then what is the percent of students those who like English also like mathematics?

Conditional probability:

Conditional probability is a probability of occurring an event when another event has already happened.

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$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

- **Solution:**
- Let, A is an event that a student likes Mathematics B is an event that a student likes English.

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} = \frac{0.4}{0.7} = 57\%$$

Hence, 57% are the students who like English also like Mathematics.

Bayes' theorem:

- Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.
- In probability theory, it relates the conditional probability and marginal probabilities of two random events.
- Bayes' theorem was named after the British mathematician **Thomas Bayes**. The **Bayesian inference** is an application of Bayes' theorem, which is fundamental to Bayesian statistics.

- It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).
- Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.
- **Example**: If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age

- Bayes' theorem can be derived using product rule

conditional probability of event A with known event B:

$$P(A \wedge B) = P(A|B)\,P(B) \text{ or}$$

Similarly, the probability of event B with known event A:

$$P(B \wedge A) = P(B|A)\,P(A)$$

Equating right hand side of both the equations, we will get:

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}} \quad \text{....(a)}$$

Bayes' rule or **Bayes' theorem**. This equation is basic of most modern AI systems for **probabilistic inference**.

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} \quad \text{....(a)}$$

- It shows the simple relationship between joint and conditional probabilities. Here,
- P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.
- P(B|A) is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.
- P(A) is called the **prior probability**, probability of hypothesis before considering the evidence
- P(B) is called **marginal probability**, pure probability of an evidence.

- **Applying Bayes' rule:**
- Bayes' rule allows us to compute the single term P(B|A) in terms of P(A|B), P(*B*), and P(A). This is very useful in cases where we have a good probability of these three terms and want to determine the fourth one. Suppose we want to perceive the effect of some unknown cause, and want to compute that cause, then the Bayes' rule becomes:

$$\mathbf{P(cause \mid effect)} = \frac{P(effect|cause)\, P(cause)}{P(effect)}$$

Example-1:

- **Question: what is the probability that a patient has diseases meningitis with a stiff neck?**
- **Given Data:**
- A doctor is aware that disease meningitis causes a patient to have a stiff neck, and it occurs 80% of the time. He is also aware of some more facts, which are given as follows:
- The Known probability that a patient has meningitis disease is 1/30,000.
- The Known probability that a patient has a stiff neck is 2%.

- Let a be the proposition that patient has stiff neck and b be the proposition that patient has meningitis. , so we can calculate the following as:
- P(a|b) = 0.8
- P(b) = 1/30000
- P(a)= .02

$$\mathbf{P(b|a)} = \frac{P(a|b)P(b)}{P(a)} = \frac{0.8*(\frac{1}{30000})}{0.02} = 0.001333333.$$

Hence, we can assume that 1 patient out of 750 patients has meningitis disease with a stiff neck.

Example-2:

- **Question: From a standard deck of playing cards, a single card is drawn. The probability that the card is king is 4/52, then calculate posterior probability P(King|Face), which means the drawn face card is a king card.**

- **Solution:**

$$P(king|face) = \frac{P(Face|king) * P(King)}{P(Face)} \quad ....(i)$$

P(king): probability that the card is King= 4/52= 1/13

P(face): probability that a card is a face card= 3/13

P(Face|King): probability of face card when we assume it is a king = 1

substituting all values in equation (i) we will get:

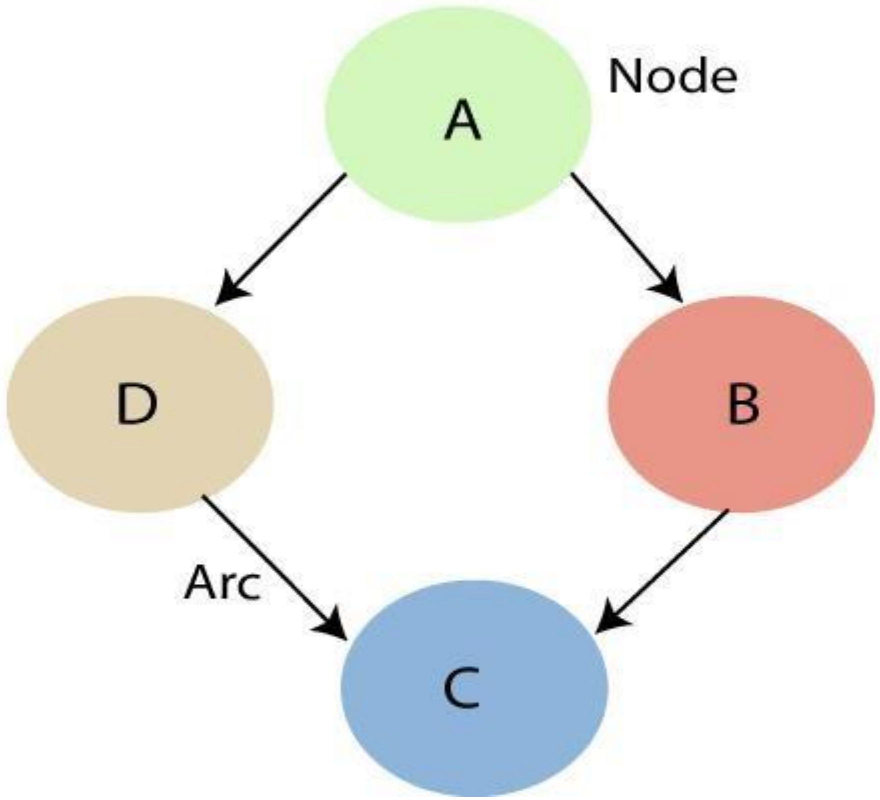
$$P(king|face) = \frac{1 * (\frac{1}{13})}{(\frac{3}{13})} = \mathbf{1/3,}$$ **it is a probability that a face card is a king card.**

Bayesian Belief Network in artificial intelligence

- "A Bayesian network is a probabilistic graphical model which represents a set of variables and their conditional dependencies using a directed acyclic graph."
- **Bayes network, belief network,**
- **decision network**, or **Bayesian model**.

- Real world applications are probabilistic in nature, and to represent the relationship between multiple events, we need a Bayesian network.
- It can also be used in various tasks including **prediction, anomaly detection, diagnostics, automated insight, reasoning, time series prediction**, and **decision making under uncertainty**.
- Bayesian Network can be used for building models from data and experts opinions, and it consists of two parts:
  - **Directed Acyclic Graph**
  - **Table of conditional probabilities.**

Influence diagram



- **The Bayesian network graph does not contain any cyclic graph. Hence, it is known as a** directed acyclic graph or DAG
- Each node in the Bayesian network has condition probability distribution **$P(X_i \mid Parent(X_i))$**, which determines the effect of the parent on that node.
- Bayesian network is based on Joint probability distribution and conditional probability.

Joint probability distribution:

- If we have variables x1, x2, x3,....., xn, then the probabilities of a different combination of x1, x2, x3.. xn, are known as Joint probability distribution.
- **$P[x_1, x_2, x_3, \ldots, x_n]$**, it can be written as the following way in terms of the joint probability distribution.
- **$= P[x_1 | x_2, x_3, \ldots, x_n] P[x_2, x_3, \ldots, x_n]$**
- **$= P[x_1 | x_2, x_3, \ldots, x_n] P[x_2 | x_3, \ldots, x_n] \ldots P[x_{n-1} | x_n] P[x_n]$.**
- In general for each variable Xi, we can write the equation as:
- $P(X_i | X_{i-1}, \ldots, X_1) = P(X_i | Parents(X_i))$

Illustration of Bayesian network:

- Harry installed a new burglar alarm at his home to detect burglary.
- The alarm reliably responds at detecting a burglary but also responds for minor earthquakes.
- Harry has two neighbors David and Sophia, who have taken a responsibility to inform Harry at work when they hear the alarm.
- David always calls Harry when he hears the alarm, but sometimes he got confused with the phone ringing and calls at that time too.
- On the other hand, Sophia likes to listen to high music, so sometimes she misses to hear the alarm.
- Here we would like to compute the probability of Burglary Alarm.

- **Problem:**
- **Calculate the probability that alarm has sounded, but there is neither a burglary, nor an earthquake occurred, and David and Sophia both called the Harry.**

- **Solution:**
- The Bayesian network for the above problem is given below. The network structure is showing that burglary and earthquake is the parent node of the alarm and directly affecting the probability of alarm's going off, but David and Sophia's calls depend on alarm probability.
- The network is representing that our assumptions do not directly perceive the burglary and also do not notice the minor earthquake, and they also not confer before calling.
- The conditional distributions for each node are given as conditional probabilities table or CPT.
- Each row in the CPT must be sum to 1 because all the entries in the table represent an exhaustive set of cases for the variable.
- In CPT, a boolean variable with k boolean parents contains 2^K probabilities. Hence, if there are two parents, then CPT will contain 4 probability values

List of all events occurring in this network:

- **Burglary (B)**
- **Earthquake(E)**
- **Alarm(A)**
- **David Calls(D)**
- **Sophia calls(S)**

P[D, S, A, B, E], can rewrite the above probability statement using joint probability distribution:

P[D, S, A, B, E]= P[D | S, A, B, E]. P[S, A, B, E]

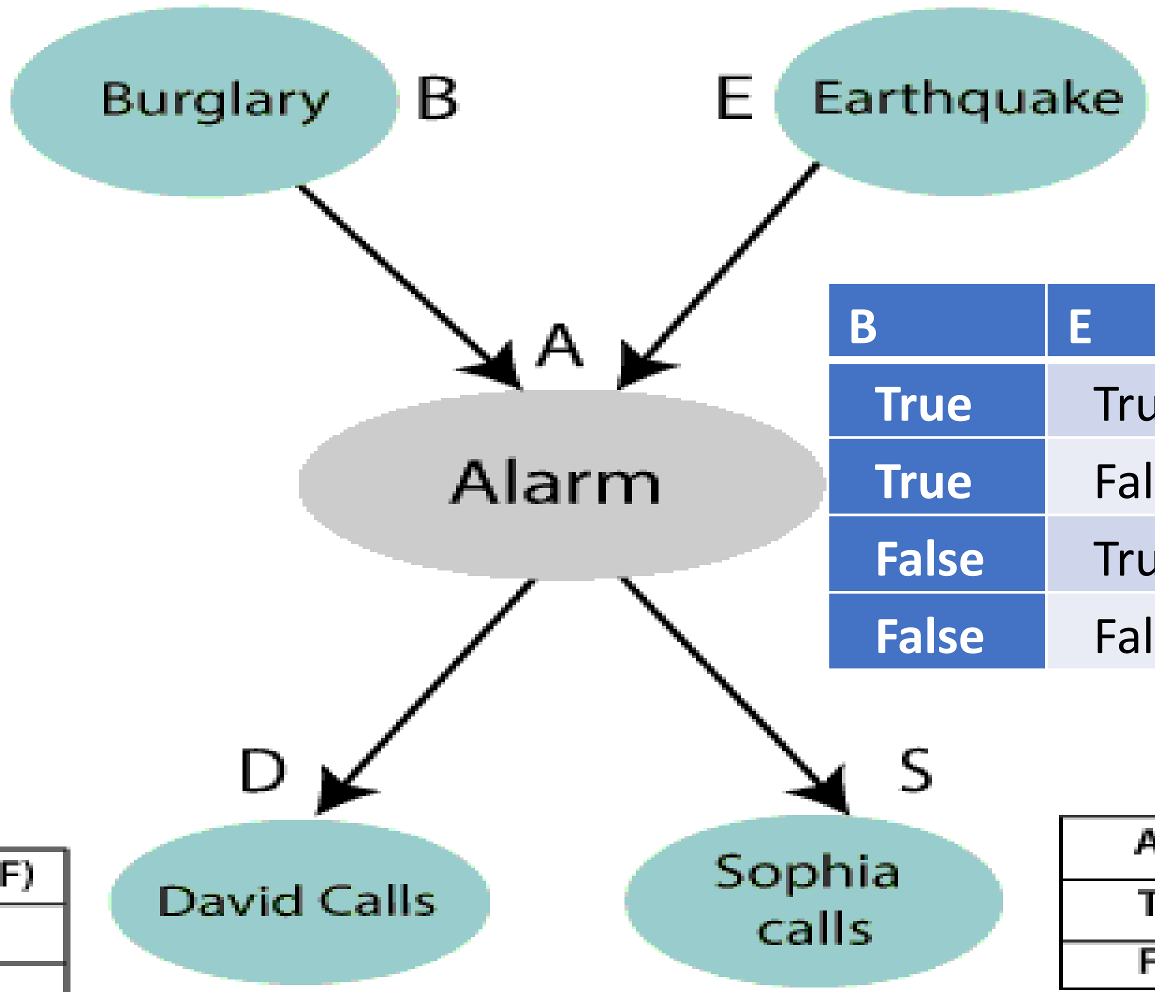
=P[D | S, A, B, E]. P[S | A, B, E]. P[A, B, E]

= P [D| A]. P [S| A, B, E]. P[A, B, E]

= P[D | A]. P[S | A]. P[A| B, E]. P[B, E]

= P[D | A]. P[S | A]. P[A| B, E]. P[B |E]. P[E]

T	0.002
F	0.998



T	0.001
F	0.999

B	E	P(A= True)	P(A= False)
True	True	0.94	0.06
True	False	0.95	0.04
False	True	0.31	0.69
False	False	0.001	0.999

A	P (D=T)	P (D=F)
T	0.91	0.09
F	0.05	0.95

A	P (S=T)	P (S=F)
T	0.75	0.25
F	0.02	0.98

- Let's take the observed probability for the Burglary and earthquake component:
- P(B= True) = 0.002, which is the probability of burglary.
- P(B= False)= 0.998, which is the probability of no burglary.
- P(E= True)= 0.001, which is the probability of a minor earthquake
- P(E= False)= 0.999, Which is the probability that an earthquake not occurred

Conditional probability table for Alarm A:

B	E	P(A= True)	P(A= False)
True	True	0.94	0.06
True	False	0.95	0.04
False	True	0.31	0.69
False	False	0.001	0.999

Conditional probability table for David Calls:

A	P(D= True)	P(D= False)
True	0.91	0.09
False	0.05	0.95

Conditional probability table for Sophia Calls:

A	P(S= True)	P(S= False)
True	0.75	0.25
False	0.02	0.98

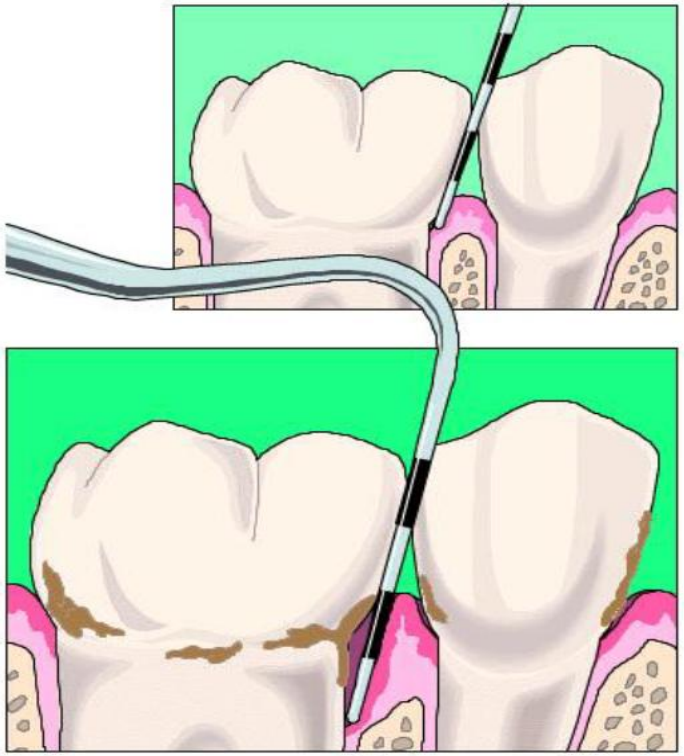
From the formula of joint distribution, we can write the problem statement in the form of probability distribution:

P(S, D, A, ¬B, ¬E) = P (S|A) *P (D|A)*P (A|¬B ^ ¬E) *P (¬B) *P (¬E).

= 0.75. * 0.91. * 0.001. * 0.998. *0.999

= 0.00068045.

Dilemma at the Dentist's



What is the probability of a cavity given a toothache?
What is the probability of a cavity given the probe catches?

Example 2

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the $Toothache$, $Cavity$, $Catch$ world.

13.8 Given the full joint distribution shown in Figure 13.3, calculate the following:

a. $\mathbf{P}(toothache)$.

b. $\mathbf{P}(Cavity)$.

c. $\mathbf{P}(Toothache \mid cavity)$.

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the *Toothache*, *Cavity*, *Catch* world.

a. P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

b. P(Cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2

c. P(Toothache | cavity) = toothache and cavity / cavity = (0.108 + 0.012) / 0.2 = 0.6

Start with the joint distribution:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	**.108**	**.012**	**.072**	**.008**
¬ cavity	**.016**	**.064**	**.144**	**.576**

Can also compute conditional probabilities:

$$P(\neg cavity | toothache) = \frac{P(\neg cavity \land toothache)}{P(toothache)}$$
$$= \frac{0.016 + 0.064}{0.108 + 0.012 + 0.016 + 0.064} = 0.4$$

- Hence, a Bayesian network can answer any query about the domain by using Joint distribution.
- The semantics of Bayesian Network:
- There are two ways to understand the semantics of the Bayesian network, which is given below:
- To understand the network as the representation of the Joint probability distribution.
- It is helpful to understand how to construct the network.
- To understand the network as an encoding of a collection of conditional independence statements.

Bayes Rule

posterior

$$P(x,y) = P(x \mid y)P(y) = P(y \mid x)P(x)$$

$$\boxed{P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}}$$

Useful for assessing diagnostic probability from causal probability:

$$P(Cause|Effect) = \frac{P(Effect|Cause)P(Cause)}{P(Effect)}$$