

Question 1

Find the transfer function $H(z)$ for the difference equation

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{4}y(n-2) + \frac{1}{3}x(n)$$

Solution

$$H(z) = \frac{\frac{1}{3}}{\left[1 - \frac{1}{4}z^{-1} - \frac{1}{4}z^{-2}\right]}$$

Proof

$$\begin{aligned} Y(z) &= \frac{1}{4}z^{-1}Y(z) + \frac{1}{4}z^{-2}Y(z) + \frac{1}{3}X(z) \\ Y(z) - \frac{1}{4}z^{-1}Y(z) - \frac{1}{4}z^{-2}Y(z) &= \frac{1}{3}X(z) \\ Y(z)\left[1 - \frac{1}{4}z^{-1} - \frac{1}{4}z^{-2}\right] &= \frac{1}{3}X(z) \\ \frac{Y(z)}{X(z)} &= H(z) = \frac{\frac{1}{3}}{\left[1 - \frac{1}{4}z^{-1} - \frac{1}{4}z^{-2}\right]} \quad \square \end{aligned}$$

Question 2

Find the transfer function $H(z)$ for the difference equation

$$y(n) = \frac{1}{4}y(n-2) + \frac{1}{4}y(n-3) + \frac{1}{3}x(n) + \frac{1}{6}x(n-2)$$

Solution

$$H(z) = \frac{\left[\frac{1}{3} + \frac{1}{6}z^{-2}\right]}{\left[1 - \frac{1}{4}z^{-2} - \frac{1}{4}z^{-3}\right]}$$

Proof

$$\begin{aligned} Y(z) &= \frac{1}{4}z^{-2}Y(z) + \frac{1}{4}z^{-3}Y(z) + \frac{1}{3}X(z) + \frac{1}{6}z^{-2}X(z) \\ Y(z) - \frac{1}{4}z^{-2}Y(z) - \frac{1}{4}z^{-3}Y(z) &= \frac{1}{3}X(z) + \frac{1}{6}z^{-2}X(z) \\ Y(z)\left[1 - \frac{1}{4}z^{-2} - \frac{1}{4}z^{-3}\right] &= X(z)\left[\frac{1}{3} + \frac{1}{6}z^{-2}\right] \\ \frac{Y(z)}{X(z)} &= H(z) = \frac{\left[\frac{1}{3} + \frac{1}{6}z^{-2}\right]}{\left[1 - \frac{1}{4}z^{-2} - \frac{1}{4}z^{-3}\right]} \quad \square \end{aligned}$$