Q-(i) - Discrete Cosine transform on (FFT).

$$DCT(x) = e^{\frac{i\omega k}{2}} \overline{F_{2N}(x)} + e^{\frac{i\omega k}{2}} F_2 N(x).$$

$$F_{2N} = \sum_{n=0}^{N-1} x(n) e^{\frac{2\pi i k n}{2N}}$$

for $K = \{0, 1, —, N-1\}$

Discrete Cosine Transform is special case of Fast Fourier Transform.

$$F_{2N} = C_K = \sum_{n=0}^{N-1} x_n e^{\left(-i\frac{2\pi kn}{N}\right)}$$

$$= \sum_{n=0}^{\frac{1}{2}N} x_n e^{\left(-i\frac{2\pi kn}{N}\right)} + \sum_{n=\frac{1}{2}N+1}^{N-1} x_n e^{\left(-i\frac{2\pi kn}{N}\right)}$$

$$= \sum_{n=0}^{\frac{1}{2}N} x_n e^{\left(-i\frac{2\pi kn}{N}\right)} + \sum_{n=\frac{1}{2}N+1}^{N-1} x_{N-n} e^{i\frac{2\pi K(N-n)}{N}}$$

function is symmetric $x_0 = x_N$, $x_1 = x_{N-1}$ — and

$e^{i2\pi K} = 1 \quad \forall K \in \{0, 1, 2, —, N-1\}$.

change $(N-n) \rightarrow n$ in right hand expr

$$C_K = \sum_{n=0}^{\frac{1}{2}N} x_n e^{-i\left(\frac{2\pi kn}{N}\right)} + \sum_{n=1}^{\frac{1}{2}N-1} x_n e^{\left(i\frac{2\pi kn}{N}\right)}$$

$$= x_0 + x_{N/2} \cos\left(\frac{2\pi K(N/2)}{N}\right) + 2\sum_{n=1}^{\frac{1}{2}N-1} x_n \cos\left(\frac{2\pi kn}{N}\right)$$

Cosine transform is applied to real samples, which implies that coefficients C_K, will all be real as the sum of all real terms.

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As x_n and c_K are real

$$C_{N-\nu} = C_\nu^* = C_\nu$$

inverse function

$$x_n = \frac{1}{N}\sum_{K=0}^{N-1} c_K\, e^{i\left(\frac{2\pi K n}{N}\right)}$$

$$x_n = \frac{1}{N}\left\{\sum_{K=0}^{\frac{1}{2}N} c_K\, e^{i\left(\frac{2\pi K n}{N}\right)} + \sum_{K=\frac{1}{2}N+1}^{N-1} c_K\, e^{\left(\frac{i 2 K\pi n}{N}\right)}\right\}$$

for each c_K one has to N complex multiplication and $(N-1)$ addition i.e, i.e, $(2N-1)$ complex operations.

Since N c_K's so the total number of operations is $O(N^2)$

Fast fourier transform algrithm is simple to understand when no. of sample $N = 2^m$ divide the group into two as even and odd groups.

$$\equiv E_K + e^{-i\frac{2\pi K}{N}} O_K$$

and $$C_K = \begin{cases} \frac{1}{\sqrt{2}} & K=0 \\ 1 & \text{otherwise} \end{cases}$$

so

$$x_n = \frac{1}{N}\sum_{K=0}^{N-1} \frac{1}{\sqrt{2}}\, e^{i\left(\frac{2\pi K n}{N}\right)}$$

$$x_n = \frac{1}{N}\sum_{K=0}^{N-1} \frac{1}{\sqrt{2}}\, e^{i(n\omega_K)} \qquad \omega_K = \left(\frac{2\pi}{N}K\right)$$

$$= \frac{1}{\sqrt{2}N}\sum_{K=0}^{N-1} e^{i(n\omega_K)}$$

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Q-NO-4:-

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row vector of size $1 \times N$

$$C_N^K(n) = e^{\frac{\pi i n (n-N)K}{N}}$$

$j, K \in (0 \longrightarrow N-1)$

$$V_{j,K} = \frac{1}{N} e^{\frac{2\pi i j n}{N}} C_N^K(n) = \frac{1}{N} e^{\frac{\pi i n}{N}(2j + (n-K)K}$$

Form $(N \times N^2)$ matrix V Columns are $\{V_{j,K}\}_{j,K=0}^{N-1}$

row Vectors

$$N=1, \quad C_1^K(n) = e^{\frac{\pi(1-N)K}{(1)}}$$

$$C_1^K(n) = e^{\pi(n-1)K}$$

$$C_2^K(n) = e^{\frac{\pi(n-2)K}{2}}$$

$$V_{j,K} = \frac{1}{N} e^{\frac{2\pi i j n}{N}}$$

$j = 0, 1, 2, \ldots, N-1$

$K = 0, 1, 2, \ldots, N-1$

$$V_{(0,0)} = \frac{1}{N} e^0 = \frac{1}{N}$$

$$V(j,K) = \frac{1}{N} e^{\frac{2\pi i j n}{N}} C_N^K(n) = \frac{1}{N} e^{\frac{\pi i n}{N}(2j + (n-N)K)}$$

$j=0, \; K=0$

$$V_{(0,0)} = \frac{1}{N} e^{\frac{\pi i n}{N}(0)} = \frac{1}{N} e^0 = \frac{1}{N}$$

$$V(1,1) = \frac{1}{N} e^{\frac{\pi i n}{N}(2+n-N)}$$

$$V(2,2) = \frac{1}{N} e^{\frac{\pi i n}{N}(4+2n-2K)}$$

$$V(N-1, N-1) = \frac{1}{N-1} e^{\frac{\pi i n}{N}(2N-2+(n-N)(N-1))}$$

$$2N-2+nN-n-N^2+N$$

$$(3N+nN-N^2-n-2)$$

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Date: P-4

$$V_{j,K} = \frac{1}{N}\begin{bmatrix} e^{\frac{\pi 2 i n}{N}}\,(n+2-N) \\ e^{\frac{2\pi n}{N}}\,(2n+4-2K) \end{bmatrix}$$

$$V_{j,K} = \frac{1}{N} e^{\frac{2\pi n}{N}} \begin{bmatrix} 1 & (n+2-N) & (2n+4-2K) \text{---} \\ 2 & (n+4-N) & (2n+8-4K) \text{---} \\ \vdots & & \\ n & \text{---}\ (n+2N) \text{---} & (2n+2N-2K) \end{bmatrix}$$

$\forall \quad j = 0, 1, 2, \text{---}, N-1$

$K = 0, 1, 2, \text{---}, N-1$

$$C^{K}_{N-1}(n) = e^{\frac{\pi 2 i n}{N-1}}\left(\frac{n-(N-1)}{N-1}\right)^{K}$$

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