Assignment 1 MATH 4199

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```
function [V, D] = ft(n,j,k)
% Code for Assignment 1 MATH 4199

W = exp(sym(1))^(2*pi*1i/n);
V = zeros(n:n);
D = zeros(n:n);
l = 0;
m = 0;
for a = 0:j
    for b = 0:k
        l = l + 1;
        V(l) = W^(a*b);
        if(a == b)
            D(l) = W^(-m);
            m = m + 1;
        end
    end
end
end
```

1.

8. Exercises

1. Consider the mask $\mathbf{c} = \frac{1}{10}(6, 4, 2, 0, -2)$ and the corresponding circulant matrix C.

   (a) Calculate the eigenvalues of C.

   (b) Find the singular value decomposition of C.

Figure 1: Problem 1

Solution:

```
c = 1/10*[6, 4, 2, 0, -2]' % mask c
C = gallery('circul', c)' % circulant matrix C
[V, D] = ft(5, 4, 4) % ft function for Assignment 1
(V*c)' % Calculate eigenvalues of C.
(eig(C))' % verify eigenvalues of C.
[U,S,V] = svd(C,"vector") % Find the singular value decomposition of C.
```

```
>> c = 1/10*[6, 4, 2, 0, -2]' % mask c
c =
    0.6000
    0.4000
    0.2000
         0
   -0.2000
>> C = gallery('circul', c)' % circulant matrix C
C =
    0.6000   -0.2000         0    0.2000    0.4000
    0.4000    0.6000   -0.2000         0    0.2000
    0.2000    0.4000    0.6000   -0.2000         0
         0    0.2000    0.4000    0.6000   -0.2000
   -0.2000         0    0.2000    0.4000    0.6000
>> [V, D] = ft(5, 4, 4) % ft function for Assignment 1
V =
   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i
   1.0000 + 0.0000i   0.3090 + 0.9511i  -0.8090 + 0.5878i  -0.8090 - 0.5878i   0.3090 - 0.9511i
   1.0000 + 0.0000i  -0.8090 + 0.5878i   0.3090 - 0.9511i   0.3090 + 0.9511i  -0.8090 - 0.5878i
   1.0000 + 0.0000i  -0.8090 - 0.5878i   0.3090 + 0.9511i   0.3090 - 0.9511i  -0.8090 + 0.5878i
   1.0000 + 0.0000i   0.3090 - 0.9511i  -0.8090 - 0.5878i  -0.8090 + 0.5878i   0.3090 + 0.9511i
D =
   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.3090 - 0.9511i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.8090 - 0.5878i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.8090 + 0.5878i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.3090 + 0.9511i
>> (V*c)' % Calculate eigenvalues of C.
ans =
   1.0000 + 0.0000i   0.5000 - 0.6882i   0.5000 - 0.1625i   0.5000 + 0.1625i   0.5000 + 0.6882i
>> (eig(C))' % verify eigenvalues of C.
ans =
   0.5000 - 0.6882i   0.5000 + 0.6882i   1.0000 + 0.0000i   0.5000 - 0.1625i   0.5000 + 0.1625i
>> [U,S,V] = svd(C,"vector") % Find the singular value decomposition of C.
U =
   -0.4472    0.3717    0.5117   -0.1954   -0.6015
   -0.4472    0.6015   -0.1954    0.5117    0.3717
   -0.4472    0.0000   -0.6325   -0.6325    0.0000
   -0.4472   -0.6015   -0.1954    0.5117   -0.3717
   -0.4472   -0.3717    0.5117   -0.1954    0.6015
S =
    1.0000
    0.8507
    0.8507
    0.5257
    0.5257
V =
   -0.4472    0.6325         0         0   -0.6325
   -0.4472    0.1954   -0.6015    0.3717    0.5117
   -0.4472   -0.5117   -0.3717   -0.6015   -0.1954
   -0.4472   -0.5117    0.3717    0.6015   -0.1954
   -0.4472    0.1954    0.6015   -0.3717    0.5117
>>
```

2.

2. Consider the mask $\mathbf{c} = \frac{1}{12}(1, 1, 1, 1, 2, 1, 1, 1, 1)^T$ and the corresponding circulant matrix C.

(a) Calculate the eigenvalues of C.

(b) Find the singular value decomposition of C.

Figure 2: Problem 2

Solution:

```
c = 1/12*[1, 1, 1, 1, 2, 1, 1, 1, 1]' % mask c
C = gallery('circul', c)' % circulant matrix C
[V, D] = ft(9, 8, 8); % ft function for Assignment 1
(V*c)' % Calculate eigenvalues of C.
(eig(C))' % verify eigenvalues of C.
[U,S,V] = svd(C,"vector") % Find the singular value decomposition of C.
```

```
>> c = 1/12*[1, 1, 1, 1, 2, 1, 1, 1, 1]' % mask c
c =
    0.0833
    0.0833
    0.0833
    0.0833
    0.1667
    0.0833
    0.0833
    0.0833
    0.0833
>> C = gallery('circul', c)' % circulant matrix C
C =
    0.0833    0.0833    0.0833    0.0833    0.0833    0.1667    0.0833    0.0833    0.0833
    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.1667    0.0833    0.0833
    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.1667    0.0833
    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.1667
    0.1667    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833
    0.0833    0.1667    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833
    0.0833    0.0833    0.1667    0.0833    0.0833    0.0833    0.0833    0.0833    0.0833
    0.0833    0.0833    0.0833    0.1667    0.0833    0.0833    0.0833    0.0833    0.0833
    0.0833    0.0833    0.0833    0.0833    0.1667    0.0833    0.0833    0.0833    0.0833
>> [V, D] = ft(9, 8, 8); % ft function for Assignment 1
>> (V*c)' % Calculate eigenvalues of C.
ans =
   0.8333 + 0.0000i  -0.0783 - 0.0285i   0.0638 + 0.0536i  -0.0417 - 0.0722i   0.0145 + 0.0821i   0.0145 - 0.0821i  -0.0417 + 0.0722i   0.0638 - 0.0536i  -0.0783 + 0.0285i
>> (eig(C))' % verify eigenvalues of C.
ans =
   0.8333 + 0.0000i   0.0638 - 0.0536i   0.0638 + 0.0536i   0.0145 - 0.0821i   0.0145 + 0.0821i  -0.0783 - 0.0285i  -0.0783 + 0.0285i  -0.0417 - 0.0722i  -0.0417 + 0.0722i
>> [U,S,V] = svd(C,"vector") % Find the singular value decomposition of C.
U =
   -0.3333   -0.0571    0.3531   -0.0301    0.7870   -0.1800   -0.2057    0.2281   -0.1179
   -0.3333   -0.1485    0.1869   -0.0840   -0.5512   -0.4672   -0.1441    0.5178   -0.1179
   -0.3333    0.2332    0.1645    0.3915   -0.0646    0.5311    0.4661    0.3698   -0.1179
   -0.3333   -0.2206   -0.4004    0.1236   -0.0646    0.4555   -0.6624    0.0178   -0.1179
   -0.3333   -0.0000    0.0000   -0.0000    0.0000   -0.0000   -0.0000   -0.0000    0.9428
   -0.3333    0.0787    0.6041   -0.0727   -0.2473    0.1211   -0.1260   -0.6379   -0.1179
   -0.3333   -0.1272   -0.3308    0.6049    0.0571   -0.4579    0.2140   -0.3532   -0.1179
   -0.3333   -0.5196   -0.2117   -0.5581    0.0571    0.1318    0.4681   -0.0947   -0.1179
   -0.3333    0.7611   -0.3658   -0.3752    0.0266   -0.1345   -0.0100   -0.0477   -0.1179
S =
    0.8333
    0.0833
    0.0833
    0.0833
    0.0833
    0.0833
    0.0833
    0.0833
    0.0833
V =
   -0.3333         0         0         0         0         0         0         0    0.9428
   -0.3333    0.0787    0.6041   -0.0727   -0.2473    0.1211   -0.1260   -0.6379   -0.1179
   -0.3333   -0.1272   -0.3308    0.6049    0.0571   -0.4579    0.2140   -0.3532   -0.1179
   -0.3333   -0.5196   -0.2117   -0.5581    0.0571    0.1318    0.4681   -0.0947   -0.1179
   -0.3333    0.7611   -0.3658   -0.3752    0.0266   -0.1345   -0.0100   -0.0477   -0.1179
   -0.3333   -0.0571    0.3531   -0.0301    0.7870   -0.1800   -0.2057    0.2281   -0.1179
   -0.3333   -0.1485    0.1869   -0.0840   -0.5512   -0.4672   -0.1441    0.5178   -0.1179
   -0.3333    0.2332    0.1645    0.3915   -0.0646    0.5311    0.4661    0.3698   -0.1179
   -0.3333   -0.2206   -0.4004    0.1236   -0.0646    0.4555   -0.6624    0.0178   -0.1179
>>
```

3.

3. Consider the frequency response $\mathbf{d} = (-1, 1+2i, 1-i, i, 3, -i, 1+i, 1-2i)^T$. Find the circulant matrix that would implement this frequency response.

Figure 3: Problem 3

Solution:

```
d = [-1, 1+2i, 1-i, i, 3, -i, 1+i, 1-2i] % frequency response.
c=ifft(d) % get the mask
C=gallery('circul', c)' % mask is first column the rest is now shifted.

% test
% 0th frequency is -1 so the sign of the average should be negative
here.
mean(C*[1, 2, 3, 4, 5, 6, 7, 8]')

% confirm test
mean([1, 2, 3, 4, 5, 6, 7, 8])
```

```
>> d = [-1, 1+2i, 1-i, i, 3, -i, 1+i, 1-2i] % frequency response.
d =
  -1.0000 + 0.0000i   1.0000 + 2.0000i   1.0000 - 1.0000i   0.0000 + 1.0000i   3.0000 + 0.0000i   0.0000 - 1.0000i   1.0000 + 1.0000i   1.0000 - 2.0000i
>> c=ifft(d) % get the mask
c =
    0.7500   -0.6036   -0.2500   -1.4571    0.2500    0.1036    0.2500   -0.0429
>> C=gallery('circul', c)' % mask is first column the rest is now shifted.
C =
    0.7500   -0.0429    0.2500    0.1036    0.2500   -1.4571   -0.2500   -0.6036
   -0.6036    0.7500   -0.0429    0.2500    0.1036    0.2500   -1.4571   -0.2500
   -0.2500   -0.6036    0.7500   -0.0429    0.2500    0.1036    0.2500   -1.4571
   -1.4571   -0.2500   -0.6036    0.7500   -0.0429    0.2500    0.1036    0.2500
    0.2500   -1.4571   -0.2500   -0.6036    0.7500   -0.0429    0.2500    0.1036
    0.1036    0.2500   -1.4571   -0.2500   -0.6036    0.7500   -0.0429    0.2500
    0.2500    0.1036    0.2500   -1.4571   -0.2500   -0.6036    0.7500   -0.0429
   -0.0429    0.2500    0.1036    0.2500   -1.4571   -0.2500   -0.6036    0.7500
>> mean(C*[1, 2, 3, 4, 5, 6, 7, 8]') % 0th frequency is -1 so the sign of the average should be negative
ans =
   -4.5000
>> mean([1, 2, 3, 4, 5, 6, 7, 8]) % confirm test
ans =
    4.5000
>>
```