Assignment 1 MATH 2200

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(Problem 1a)

Compute $\int csc(x)dx$.

Solution. $\boxed{\int csc(x) \cdot dx = ln|csc(x) - cot(x)| \ + \ C.}$

Start of 1a.

$$\begin{aligned}
\int csc(x) \cdot dx &= \int csc(x) \cdot \left(\frac{csc(x) - cot(x)}{csc(x) - cot(x)} \right) \cdot dx \\
&= \int \frac{csc^2(x) - csc(x) \cdot cot(x)}{csc(x) - cot(x)} \cdot dx \\
let\ u &= csc(x) - cot(x) \\
du &= \left(-csc(x) \cdot cot(x) \ + csc^2(x) \right) \cdot dx \\
&= \left(csc^2(x) - csc(x) \cdot cot(x) \right) \cdot dx \\
\int u^{-1} \cdot dx &= ln|u| + C.
\end{aligned}$$

End of 1a. $\int csc(x) \cdot dx = ln|csc(x) - cot(x)| \ + \ C.$ ■

(Problem 1b)

Find the arc length of the curve $y = ln(sin(x))$. *Over the interval* $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

Solution. $\boxed{L = ln\left(\sqrt{2}-1\right)^{-1}}$

Start of 1b.

Formula. $L = \int_a^b \sqrt{1+[f'(x)]^2} \cdot dx$

$f(x) = ln(sin(x))$

Derivative. $f'(x) = \frac{1}{sin(x)} \cdot cos(x)$

$= cot(x)$

$1 + [f'(x)]^2 = 1 + cot^2(x)$

$= csc^2(x)$

Interval. $let\ a = \frac{\pi}{4}$

$let\ b = \frac{\pi}{2}$

Integragte. $L = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\sqrt{csc^2(x)}\right] \cdot dx$

From (a). $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} csc(x) \cdot dx$

$= ln|csc(x) - cot(x)|\ ]_{x=pi/4}^{pi/2}$

$= ln(csc(pi/2) - cot(pi/2)) - ln(csc(pi/4) - cot(pi/4))$

$= ln(1) - ln\left(\sqrt{2}-1\right)$

$= -ln\left(\sqrt{2}-1\right)$

End of 1b. $= ln\left(\sqrt{2}-1\right)^{-1}$ ■

(Problem 2)

Find the volume of the solid that results when the region enclosed by $y = tan(x),\ y = 1\ and\ x = 0$ *is revolved around the* $x - axis$.

Solution. $\boxed{V = \dfrac{\pi(4-\pi)}{4}}$

Start of 2.

Compute bounds of integration.

$$\begin{aligned} tan(x) &= 1; \\ tan^{-1}(tan(x)) &= tan^{-1}(1), \\ \Longrightarrow x &= \frac{\pi}{4}, \\ \Longrightarrow tan^{-1}(0) &= 0. \\ let\ a &= 0, \\ b &= \frac{\pi}{4}. \end{aligned}$$

Approximate the volume of the Kth layer with the volume of a cylinder of width Δx *and radius* r.

Dimensions of cylinder.

$\mathbf{r} \ = f(x) = tan(x)$

$\mathbf{V} \ = \pi \cdot (\mathbf{r})^2 \cdot \Delta x$

$$\textbf{Formula}.\ V = \int_a^b \pi \cdot [f(x)]^2 \cdot dx$$

$$\textbf{Integrate}.\ V = \pi \cdot \int_0^{\frac{\pi}{4}} tan^2(x) \cdot dx$$

$$\textbf{Formula} \int tan^n x \cdot dx = \frac{tan^{n-1}x}{n-1} - \int tan^{n-2} x \cdot dx$$

$$\begin{aligned} \therefore V &= \pi \cdot (tanx\ - x)\Big|_{x=0}^{\pi/4} \\ &= \pi \cdot \left[\left(tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} \right) - (tan(0) - 0) \right] \end{aligned}$$

$$\textbf{End of 2}.\ = \pi\left(1 - \frac{\pi}{4}\right) = \frac{4\pi - \pi^2}{4} = \frac{\pi(4-\pi)}{4}\ \blacksquare$$

Page 1 of 4 Compute Integrand.
(Problem 3)

Find the surface area that results from revolving the curve $y = e^{x/2} + e^{-x/2}$ *on the interval* $[0,\ ln(4)]$ around the y-axis.

Solution. $\boxed{2\pi\int_0^{ln(4)}\left(x\cdot\sqrt{\frac{1}{4e^x}\left(e^x+1\right)^2}\right)\cdot dx = 2\pi\cdot(3\cdot ln(2)-1)\ .}$

Start of 3.

Formula. $A = 2\pi\int_a^b\left(x\cdot\sqrt{1+f'(x)^2}\right)\cdot dx$

$$f(x) = e^{x/2} + e^{-x/2}$$

$$f'(x) = \frac{d}{dx}\left[e^{x/2} + x^{-x/2}\right]$$

$$= e^{x/2}\cdot\left(\frac{1}{2}\right) + x^{-x/2}\cdot\left(-\frac{1}{2}\right)$$

$$= \frac{1}{2}\cdot\left[e^{x/2} - e^{-x/2}\right]$$

$$1 + [f'(x)]^2 = 1\ +\ \left(\frac{1}{2}\cdot\left[e^{x/2} - e^{-x/2}\right]\right)^2$$

$$= \frac{1}{4}\left(4\ +\cdot\ \left(e^{x/2} - e^{-x/2}\right)^2\right)$$

$$= \frac{1}{4}\left[4 + e^{x/2}\left(e^{x/2} - e^{-x/2}\right) - e^{-x/2}\left(e^{x/2} - e^{-x/2}\right)\right]$$

$$= \frac{1}{4}\left[4 + e^{(x/2+x/2)} - e^{(-x/2+x/2)} - e^{(-x/2+x/2)} + e^{(-x/2+-x/2)}\right]$$

$$= \frac{1}{4}\left[4 + e^x - e^0 - e^0 + e^{-x}\right] = \frac{1}{4}\left[4\ -\ 2\ + e^x + e^{-x}\right]$$

$$= \frac{1}{4}\left[2 + e^x + e^{-x}\right] = \frac{1}{4}\left[\frac{2\cdot e^x}{e^x} + \frac{\left(e^x\cdot e^x\right)}{e^x} + \frac{1}{e^x}\right]$$

$$= \frac{1}{4e^x}\left[\left(e^{2x} + 2\cdot e^x + 1\right)\right] = \frac{1}{4e^x}\left(e^x+1\right)^2$$

(Problem 3)

Find the surface area that results from revolving the curve $y = e^{x/2} + e^{-x/2}$ *on the interval* $[0,\ ln(4)]$ around the y-axis.

$$\textbf{Integrate.}\ A = 2\pi \int_0^{ln(4)} \left(x \cdot \sqrt{\frac{1}{4e^x} \left(e^x + 1\right)^2} \right) \cdot dx$$

$$= 2\pi \int_0^{ln(4)} (1/2) \cdot \left(x \cdot \left(e^x + 1\right) \cdot \left(e^{-\frac{x}{2}}\right) \right) \cdot dx$$

$$= \pi \int_0^{ln(4)} \left(x \cdot \left(e^x + 1\right) \cdot \left(e^{-\frac{x}{2}}\right) \right) \cdot dx$$

$$let\ U(x) = u = -\frac{x}{2}$$

$$-2u = x$$

$$-2du = dx$$

$$a = U(0) = 0$$

$$b = U(ln(4)) = -\left(\frac{ln(4)}{2}\right) = -ln(2)$$

$$\pi \int_0^{ln(4)} \left(x \cdot \left(e^x + 1\right) \cdot \left(e^{-\frac{x}{2}}\right) \right) \cdot dx = \pi \int_a^b \left[-2u \cdot \left(e^{-2u} + 1\right) \cdot e^u \right] \cdot (-2) \cdot du$$

$$= 4\pi \cdot \int_a^b \left[u \cdot \left(e^{-2u} + 1\right) \cdot e^u \right] \cdot du$$

$$= 4\pi \cdot \int_a^b \left[\left(u \cdot e^{-2u} + u\right) \cdot e^u \right] \cdot du$$

$$= 4\pi \cdot \int_a^b \left[u \cdot e^{(-2u+u)} + u \cdot e^u \right] \cdot du$$

$$= 4\pi \cdot \int_a^b \left(u \cdot e^{-u} + u \cdot e^u \right) \cdot du$$

$$= 4\pi \cdot \int_a^b \left(u \cdot e^{-u} \right) \cdot du + \pi \int_a^b \left(u \cdot e^u \right) \cdot du$$

Page 3 of 4 Integration by parts.
(Problem 3)

Find the surface area that results from revolving the curve $y = e^{x/2} + e^{-x/2}$ *on the interval* $[0,\ ln(4)]$ around the y-axis.

Tabular Method $4\pi \int_a^b \left(u \cdot e^{-u}\right) \cdot du$

+u	e^{-u}
-1	$-e^{-u}$
+0	e^{-u}

$$\Longrightarrow 4\pi \int_a^b \left(u \cdot e^{-u}\right) \cdot du = 4\pi \cdot \left[u \cdot \left(-e^{-u}\right) + (-1) \cdot e^{-u}\right]_{u=a}^{b}$$

Tabular Method $4\pi \int_a^b \left(u \cdot e^{u}\right) \cdot du$

+u	e^{u}
-1	e^{u}
+0	e^{u}

$$\Longrightarrow 4\pi \int_a^b \left(u \cdot e^{u}\right) \cdot du \ = \ 4\pi \cdot \left[u \cdot e^{u} - (1)e^{u}\right]_{u=a}^{b}$$

Page 4 of 4 Evaluation at bounds.
(Problem 3)

Find the surface area that results from revolving the curve $y = e^{x/2} + e^{-x/2}$ *on the interval* $[0,\ ln(4)]$ around the y-axis.

$$\textbf{Solve } 4\pi \cdot \int_a^b \left(u \cdot e^{-u}\right) \cdot du + 4\pi \cdot \int_a^b \left(u \cdot e^{u}\right) \cdot du$$

$$4\pi \cdot \left[u \cdot \left(-e^{-u}\right) + (-1) \cdot e^{-u}\right]_{u=a}^{b} + 4\pi \cdot \left[u \cdot e^{u} - (1)e^{u}\right]_{u=a}^{b}$$

$$= 4\pi \cdot \left[-u \cdot e^{-u} - e^{-u} + u \cdot e^{u} - e^{u}\right]_{u=a}^{b}$$

$$= 4\pi \cdot \left[e^{x} \cdot (x-1) - e^{-x} \cdot (x+1)\right]$$

$$\textbf{Substitute constants } a, b = 4\pi \cdot \{\left[e^{b} \cdot (b-1) - e^{-b} \cdot (b+1)\right]$$

$$Cont... \ - \left[e^{a} \cdot (a-1) - e^{-a} \cdot (a+1)\right]\}$$

$$\textbf{Right Hand Side}: \left[e^{b}(b-1) - e^{-b}(b+1)\right] = e^{-ln(2)} \cdot (-ln(2) - 1) - e^{ln(2)}(-ln(2) + 1)$$

$$= \left(\frac{3 \cdot ln(2) - 5}{2}\right)$$

$$\textbf{Left Hand Side}: \left[e^{a}(a-1) - e^{-a}(a+1)\right] = \left[e^{0}(0-1) - e^{-0}(0+1)\right]$$

$$= -2$$

$$\textbf{Right Hand Side} - \textbf{Left Hand Side}: = \left(\frac{3 \cdot ln(2) - 5}{2}\right) - (-2)$$

$$= \left(\frac{3 \cdot ln(2) - 5}{2}\right) + \frac{4}{2}$$

$$= \frac{(3 \cdot ln(2) - 5 + 4)}{2} = \frac{3 \cdot ln(2) - 1}{2}$$

$$4\pi \cdot \left[-ue^{-u} - e^{-u} + ue^{u} - e^{u}\right]_{u=a}^{b} = 4\pi \cdot \left(\frac{3 \cdot ln(2) - 1}{2}\right) = 2\pi \cdot (3 \cdot ln(2) - 1)$$

End of 3.

$$\therefore \textbf{ The solution to } 2\pi \int_0^{ln(4)} \left(x \cdot \sqrt{\frac{1}{4e^{x}}\left(e^{x} + 1\right)^{2}}\right) \cdot dx = 2\pi \cdot (3 \cdot ln(2) - 1) \ \blacksquare$$

(Problem 4a)

Consider the curve $x = 1 - cos(y)$ for values of y in $[0, \pi]$.
If this curve is revolved around the y-axis the result is a "bowl" with curved sides.
Suppose x and y are both measured in meters, and suppose that the "bowl" is filled with water to a depth of $\pi / 2$ meters. ***What is the volume of the water in the bowl****?*

Solution. The volume of water in the bowl when it is filled to a depth of $\frac{\pi}{2}$ *meters* is,

$$\int_0^{\frac{\pi}{2}} \pi \cdot (1 - cos(y))^2 \cdot dy = \frac{\pi}{4} \cdot [3\pi - 8] \text{ cubic meters.}$$

Start of 4a.

Approximate the volume of the Kth layer with the volume of a cylinder of height Δt and radius r.

Dimensions of cylinder.

$$\begin{aligned} let\ r &= g(y) = 1 - cos(y) \\ V &= \pi \cdot (r)^2 \cdot h \\ &= \pi \cdot (1 - cos(y))^2 \cdot dy \end{aligned}$$

Formula. $V = \int_0^{\frac{\pi}{2}} \pi \cdot [g(y)]^2 \cdot dy$

$$\begin{aligned} &= \pi \cdot \int_0^{\frac{\pi}{2}} (1 - cos(y))^2 \cdot dy \\ &= \pi \cdot \int_0^{\frac{\pi}{2}} \left[cos^2(y) - 2cos(y) + 1\right] \cdot dy \\ &= \pi \left\{ \int_0^{\frac{\pi}{2}} \left[cos^2(y)\right] \cdot dy - \int_0^{\frac{\pi}{2}} 2 \cdot cos(y) \cdot dy + \int_0^{\frac{\pi}{2}} dy \right\} \end{aligned}$$

Page 2 of 3 Evaluation at bounds.
(Problem 4a)

Consider the curve $x = 1 - cos(y)$ for values of y in $[0, \pi]$.
If this curve is revolved around the y-axis the result is a "bowl" with curved sides.
Suppose x and y are both measured in meters, and suppose that the "bowl" is filled with water to a depth of $\pi / 2$ meters. ***What is the volume of the water in the bowl****?*

$$\textbf{Use formula} \int cos^n(y) \cdot dy = \frac{1}{n} cos^{(n-1)}(y) \cdot sin(y) + \frac{n-1}{n} \int cos^{n-2}(y) \cdot dy$$

$$\Longrightarrow \int_0^{\frac{\pi}{2}} \left[cos^2(y)\right] \cdot dy = \left[\frac{1}{2} cos(y) \cdot sin(y)\right]_{y=0}^{\frac{\pi}{2}} + \frac{1}{2} \int_0^{\frac{\pi}{2}} cos^0(y) \cdot dy$$

$$= \left[\frac{1}{2} cos(y) \cdot sin(y) + \frac{y}{2}\right]_{y=0}^{\frac{\pi}{2}}$$

$$\Longrightarrow \int_0^{\frac{\pi}{2}} 2 \cdot cos(y) = [2 \cdot sin(y)]_{y=0}^{\frac{\pi}{2}}$$

$$\Longrightarrow \int_0^{\frac{\pi}{2}} dy = [t]_{t=0}^{\frac{\pi}{2}}$$

$$\textbf{let } b = \frac{\pi}{2}$$

(Problem 4a)

Consider the curve $x = 1 - cos(y)$ for values of y in $[0, \pi]$.
If this curve is revolved around the y-axis the result is a "bowl" with curved sides.
Suppose x and y are both measured in meters, and suppose that the "bowl" is filled with water to a depth of $\pi / 2$ meters. ***What is the volume of the water in the bowl****?*

$$\begin{aligned}
\int_0^b \pi\left[cos^2(y) - 2cos(y) + 1\right] \cdot dy &= \pi \cdot \left[\left(\frac{1}{2}cos(y) \cdot sin(y) + \frac{t}{2}\right) - 2 \cdot sin(y) + y\right]_{y=0}^{\frac{\pi}{2}} \\
&= \pi \cdot \left[\left(\frac{1}{2}cos(b) \cdot sin(b) + \frac{b}{2}\right) - 2 \cdot sin(b) + b\right] - 0 \\
&= \pi \cdot \left[\frac{(cos(b) \cdot sin(b) + b)}{2} - 2sin(b) + b\right] \\
&= \pi \cdot \left[\frac{(cos(b) \cdot sin(b) + b - 4sin(b) + 2b}{2}\right] \\
&= \frac{\pi}{2} \cdot [cos(b) \cdot sin(b) - 4sin(b) + 3b] \\
&= \frac{\pi}{2} \cdot \left[-4 + 3\frac{\pi}{2}\right] = \frac{\pi}{4} \cdot [3\pi - 8]
\end{aligned}$$

End of 4a.

$\therefore$ **The solution to** $\int_0^{\frac{\pi}{2}} \pi \cdot (1 - cos(y))^2 \cdot dy = \frac{\pi}{4} \cdot [3\pi - 8]$ *cubic meters.* ■

(Problem 4b)

Consider the curve $x = 1 - cos(y)$ *for values of* y *in* $[0, \pi]$.
If this curve is revolved around the y*-axis the result is a "bowl" with curved sides. Suppose x and y are both measured in meters, and suppose that the "bowl" is filled with water to a depth of* $\pi / 2$ *meters.* ***How much work must be done to pump all the water to the top of the bowl?***

Solution.
$$\boxed{\begin{array}{l} 9.8 \cdot 1000 \cdot \pi \int_0^{\frac{\pi}{2}} (\pi - y) \cdot (1 - cos(y))^2 \cdot dy \;= \\ \frac{9800}{16}\left(9\pi^3 - 16\pi^2 - 28\pi\right) \approx 20321.663060413 \; Joules \end{array}}$$

Start of 4b.

Dimensions of cylinder.

$$\begin{aligned}
let\ r &= g(y) = 1 - cos(y) \\
V &= \pi \cdot (r)^2 \cdot h \\
&= \pi \cdot (1 - cos(y))^2 \cdot dy
\end{aligned}$$

Variables.

$$\begin{aligned}
Force\ of\ gravity\ g &= 9.8 \frac{N}{kg} \\
Water\ density\ d &= 1000\ kg\ per\ cubic\ meter. \\
\textbf{Force.}\ F_k &= g \cdot d \cdot \pi[1 - cos(y)]^2 \cdot dy \\
Distance\ to\ top\ from\ Kth\ cylinder. &= (\pi - y) \\
Work\ to\ move\ Kth\ cylinder\ to\ top.\ W_k &= gd(\pi - y) \cdot \pi \cdot [1 - cos(y)]^2 \cdot dy \\
Bounds\ of\ integration\ a &= 0 \\
b &= \frac{\pi}{2} \\
\textbf{Work.}\ W &= gd \int_0^{\frac{\pi}{2}} (\pi - y) \cdot \pi \cdot [1 - cos(y)]^2 \cdot dy
\end{aligned}$$

Page 2 of 5 Integration by parts.
(Problem 4b)

Consider the curve $x = 1 - cos(y)$ for values of y in $[0, \pi]$.
If this curve is revolved around the y-axis the result is a "bowl" with curved sides.
Suppose x and y are both measured in meters, and suppose that the "bowl" is filled with water to a depth of $\pi / 2$ meters. ***How much work must be done to pump all the water to the top of the bowl?***

$$W = gd \int_0^{\frac{\pi}{2}} (\pi - y) \cdot \pi \cdot [1 - cos(y)]^2 \cdot dy$$

Tabular method of integration all work done on page 3 of 4 and in problem 4a.

$+(\pi - y)$	$\pi \cdot [1 - \cos(y)]^2$
$-(-1)$	**From 4a.** $\frac{\pi}{2} \cdot [cos(y) \cdot sin(y) - 4sin(y) + 3y]$
$+0$	**From Asside.** $\left[\frac{3\pi}{4} \cdot y^2 + 2\pi \cdot cos(y) - \frac{\pi}{8} \cdot cos(2y)\right]$

$$\therefore gd \int_0^{\frac{\pi}{2}} (\pi - y) \cdot \pi \cdot [1 - cos(y)]^2 \cdot dy$$

$$= gd \cdot \left[\frac{\pi}{4}(\pi - y) \cdot [sin(2y) - 8sin(y) + 6y] + \frac{\pi}{8} \cdot \left[6y^2 + 16cos(y) - cos(2y)\right] \right]_{y=0}^{\frac{\pi}{2}}$$

Page 3 of 5 Asside work.
(Problem 4b)

Consider the curve $x = 1 - cos(y)$ for values of y in $[0, \pi]$.
If this curve is revolved around the y-axis the result is a "bowl" with curved sides.
Suppose x and y are both measured in meters, and suppose that the "bowl" is filled with water to a depth of $\pi / 2$ meters. ***How much work must be done to pump all the water to the top of the bowl?***

Asside work for Page 2 of 4. (*Problem 4b*)

Integral of $\frac{\pi}{2}\int [cos(y) \cdot sin(y) - 4sin(y) + 3y] \cdot dy$

$$= \frac{\pi}{2}\int [cos(y) \cdot sin(y) - 4sin(y) + 3y] \cdot dy$$

$$= \frac{2}{2} \cdot \frac{\pi}{2}\int [cos(y) \cdot sin(y) - 4sin(y) + 3y] \cdot dy$$

$$= \frac{\pi}{4}\int [2 \cdot cos(y) \cdot sin(y) - 8sin(y) + 6y] \cdot dy$$

$$= \frac{\pi}{4}\int [sin(2y) - 8sin(y) + 6y] \cdot dy$$

$$= \frac{\pi}{4} \cdot \left[-\frac{cos(2y)}{2} + 8cos(y) + 3y^2\right] + C.$$

$$= \frac{\pi}{8} \cdot \left[6y^2 + 16cos(y) - cos(2y)\right] + C.$$

Sum of tabular method.

$$+(\pi - y) \cdot \frac{\pi}{4} \cdot [sin(2y) - 8sin(y) + 6y] + -(-1)\frac{\pi}{8} \cdot \left[6y^2 + 16cos(y) - cos(2y)\right]$$

$$= \frac{\pi}{4}(\pi - y) \cdot [sin(2y) - 8sin(y) + 6y] + \frac{\pi}{8} \cdot \left[6y^2 + 16cos(y) - cos(2y)\right]$$

Page 4 of 5 Evaluation at bounds.
(Problem 4b)

Consider the curve $x = 1 - cos(y)$ for values of y in $[0, \pi]$.
If this curve is revolved around the y-axis the result is a "bowl" with curved sides.
Suppose x and y are both measured in meters, and suppose that the "bowl" is filled with water to a depth of $\pi / 2$ meters. ***How much work must be done to pump all the water to the top of the bowl?***

$$gd \cdot \left[\frac{\pi}{4}(\pi - y) \cdot [sin(2y) - 8sin(y) + 6y] + \frac{\pi}{8} \cdot \left[6y^2 + 16cos(y) - cos(2y)\right] \right]_{y=0}^{\frac{\pi}{2}}$$

Simplify left hand side $y = \frac{\pi}{2}$, *without constants* $gd \cdot$

$$\frac{\pi}{4}\left(\pi - \frac{\pi}{2}\right) \cdot \left[sin\left(2 \cdot \frac{\pi}{2}\right) - 8 \cdot sin\left(\frac{\pi}{2}\right) + 6 \cdot \frac{\pi}{2}\right] + \frac{\pi}{8} \cdot \left[6\left(\frac{\pi}{2}\right)^2 + 16cos\left(\frac{\pi}{2}\right) - cos\left(2 \cdot \frac{\pi}{2}\right)\right]$$

$$= \frac{\pi^2}{8} \cdot [0 - 8 + 3\pi] + \frac{\pi}{8} \cdot \left[\frac{6\pi^2}{4} + 0 - (-1)\right]$$

$$= \frac{-16\pi^2 + 6\pi^3 + 3\pi^3 + 2\pi}{16}$$

$$= \frac{9\pi^3 - 16\pi^2 + 2\pi}{16}$$

Simplify right hand side $y = 0$, *without constants* $gd \cdot$

$$\frac{\pi}{4}(\pi - 0) \cdot [sin(2 \cdot 0) - 8sin(0) + 6 \cdot 0] + \frac{\pi}{8} \cdot \left[6 \cdot 0^2 + 16cos(0) - cos(2 \cdot 0)\right]$$

$$= \frac{\pi}{4}(\pi) \cdot [0 - 0 + 0] + \frac{\pi}{8} \cdot [0 + 16(1) - cos(1)]$$

$$= \frac{15\pi}{8}$$

Page 5 of 5 Evaluation at bounds.
(Problem 4b)

Consider the curve $x = 1 - cos(y)$ for values of y in $[0, \pi]$.
If this curve is revolved around the y-axis the result is a "bowl" with curved sides.
Suppose x and y are both measured in meters, and suppose that the "bowl" is filled with water to a depth of $\pi / 2$ meters. ***How much work must be done to pump all the water to the top of the bowl?***

Calculate $gd \cdot (left\ hand\ side\ -\ right\ hand\ side)$.

$$gd \cdot \left(\frac{9\pi^3 - 16\pi^2 + 2\pi}{16} - \frac{15\pi}{8} \right)$$
$$= 9.8 \cdot 1000 \left(\frac{9\pi^3 - 16\pi^2 + 2\pi}{16} - \frac{30\pi}{16} \right)$$
$$= 9.8 \cdot 1000 \left(\frac{9\pi^3 - 16\pi^2 - 28\pi}{16} \right)$$
$$= \frac{9800}{16} \left(9\pi^3 - 16\pi^2 - 28\pi \right) \approx 20321.663060413\ Joules$$

End of 4b.

$\therefore$ **The solution to** $9.8 \cdot 1000 \cdot \pi \int_0^{\frac{\pi}{2}} (\pi - y) \cdot (1 - cos(y))^2 \cdot dy$

$\approx 20321.663060413\ Joules$ ■