1 For each of the following subspaces of M_{33} find a basis and state the dimension.

1.a $W_1 = \{A \in M_{33} | A$ is a diagonal matrix$\}$

$\boxed{\text{Solution: } \dim(W_1) = 3}$

Proof.

$$D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore D = aA_1 + bA_2 + cA_3$$

The matricies A_1, A_2, A_3 form a basis for W_1 consequently, the dimension of W_1 is 3. □

1.b $W_2 = \{A \in M_{33} | A = A^T\}$ (the symmetric matrices)

$\boxed{\text{Solution: } dim(W_2) = 6}$

Proof.

$$S = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, A_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore S = aA_1 + bA_2 + cA_3 + dA_4 + eA_5 + fA_6$$

The matricies $A_1, A_2, A_3, A_4, A_5, A_6$ form a basis for W_2 consequently, the dimension of W_2 is 6. □

1.c $W_1 = \{A \in M_{33} | A = -A^T\}$ (the anti-symmetric matrices)

$\boxed{\text{Solution: } dim(W_3) = 3}$

Proof.

$$S = \begin{bmatrix} 0 & b & c \\ -b & 0 & d \\ -c & -d & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\therefore S = aA_1 + bA_2 + cA_3$$

The matricies A_1, A_2, A_3 form a basis for W_3 consequently, the dimension of W_3 is 3. □