

## Math 2311 - Assignment 1

Due: Wed., Jan. 26th

**Note:** You may work alone or in pairs on the assignment. If you work in a pair, hand in one assignment with both names on it. If you work in a pair, you should still be able to explain your answer to every question on the assignment.

1. Determine if each of the following sets is a vector space. If it is, write a convincing argument to show that it is (verify all of the axioms, or briefly explain why you only need to verify a small number of them); if it is not a vector space demonstrate at least one of the axioms that fails.
  - (a)  $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid x \geq y \right\}$  with the usual scalar multiplication and vector addition from  $\mathbb{R}^2$
  - (b)  $\{f \in F(-\infty, \infty) \mid f(1) = 0\}$  with the usual scalar multiplication and vector addition from  $F(-\infty, \infty)$
2. Let  $V$  be a vector space.
  - (a) If  $k$  is any scalar, prove that  $k\vec{0} = \vec{0}$ .
  - (b) Prove that the zero vector in  $V$  is unique. (Hint: assume that there are two different “zero vectors” and show that they must be equal to each other.)
3. Determine if each of the following are subspaces of  $M_{nn}$ 
  - (a)  $\{A \in M_{nn} \mid \det(A) = 0\}$
  - (b)  $\{A \in M_{nn} \mid \text{tr}(A) = 0\}$  Note:  $\text{tr}(A)$  is the *trace* of  $A$  and is equal to the sum of the diagonal entries of  $A$ , see section 1.3 of the text. In particular, you may use the results of questions 35 and TF question (j) from section 1.3 (11th or 12th edition).
  - (c)  $\{A \in M_{nn} \mid A^T = A\}$  In other words, the symmetric matrices. Theorem 1.4.8 from section 1.4 may be of use.
4. Consider the following vectors in  $P_2$ :  $p_1 = 2 + x + 4x^2$ ,  $p_2 = 1 - x + 3x^2$ ,  $p_3 = 3 + 2x + 5x^2$ .
  - (a) Express the vector  $g = 6 + 11x + 6x^2$  as a linear combination of  $p_1, p_2, p_3$ .
  - (b) Does  $\{p_1, p_2, p_3\}$  span  $P_2$ ?
  - (c) Is  $\{p_1, p_2, p_3\}$  linearly independent?
5. Consider the following planes in  $\mathbb{R}^3$ .  $P_1 : 2x + 3y - z = 0$  and  $P_2 : x + 2y - 2z = 0$ 
  - (a) Find a set of vectors that spans  $P_1$ .
  - (b) Find a set of vectors that spans  $P_2$ .
  - (c) Find a set of vectors that spans the intersection of  $P_1$  and  $P_2$ . (Recall that we showed the intersection of two subspaces is a subspace).