Note: The goal of this document is to provide some guidance regarding what a complete solution to an assignment question might look like, as well as providing an example of a .tex file for those of you who have decided to type your assignments in LaTeX. I will post both the .tex source file as well as the compiled .pdf file for you to look at.

1. Consider the set $W = \{f \in F(-\infty, \infty)\,|f(x) = f(-x) \text{ for all } x\}$ with the usual scalar multiplication and vector addition from $F(-\infty, \infty)$. Is W a vector space?

Answer: Yes, W is a vector space.

Proof. Since we know that $F(-\infty, \infty)$ (with the usual operations) is a vector space, and since W is a subset of $F(-\infty, \infty)$ (with the same operations), it suffices to prove that W is a subspace of $F(-\infty, \infty)$. To this end we must show three things:

(a) That W is non-empty.

(b) That W is closed under addition.

(c) That W is closed under scalar multiplication.

Consider the function f defined by $f(x) =$ for all x. Then clearly $f(x) = f(-x)$ for all x (as both sides are equal to 0), so this f is in W, and W is non-empty. (Note: this f is the zero vector from $F(-\infty, \infty)$).

Now suppose f and g are two functions in W. We must show that $f + g$ is in W. Now, for all x we have

$$(f + g)(x) = f(x) + g(x) = f(-x) + g(-x) = (f + g)(-x)$$

so $f + g$ is in W (Note: the second equality above holds because f and g are in W, while the first and third equality are the definition of addition in $F(-\infty, \infty)$). Alternatively here we could have written:

$$\begin{aligned} (f + g)(x) &= f(x) + g(x) && \text{(definition of addition of functions)} \\ &= f(-x) + g(-x) && \text{(as } f \text{ and } g \text{ are in } W) \\ &= (f + g)(-x) && \text{(definition of addition of functions)} \end{aligned}$$

(**Note:** Normally you would not include both of the above in your solution, I have done it here as a further example of some of the things you can do in LaTeX.)

Finally, to show that W is closed under scalar multiplication, suppose f is in W and k is a scalar, then

$$(kf)(x) = kf(x) = kf(-x) = (kf(-x),$$

so (kf) is in W and W is closed under scalar multiplication.

Therefore W is a subspace of $F(-\infty, \infty)$ and hence is a vector space. □

2. Let V be a vector space and $\vec{v}$ be a vector in V. Prove that the negative of $\vec{v}$ is unique.

Proof. We must show that there is only one vector, $\vec{u}$, with the property that $\vec{u} + \vec{v} = \vec{v} + \vec{u} = \vec{0}$. To this end, we will assume that there are 2 such vectors and then show that the two vectors must in fact be equal to each other (i.e. the same vector).

Suppose $\vec{u_1}$ and $\vec{u_2}$ are both negatives (also known as additive inverses) of $\vec{v}$. Then

$$\begin{aligned}
\vec{u_2} &= \vec{0} + \vec{u_2} && \text{(vector space axiom 4)} \\
&= (\vec{u_1} + \vec{v}) + \vec{u_2} && \text{(since } \vec{u_1} \text{ is a negative of } \vec{v}) \\
&= \vec{u_1} + (\vec{v} + \vec{u_2}) && \text{(vector space axiom 3)} \\
&= \vec{u_1} + \vec{0} && \text{(since } \vec{u_2} \text{ is a negative of } \vec{v}) \\
&= \vec{u_1} && \text{(vector space axiom 4).}
\end{aligned}$$

Since we have shown that any two negatives must be equal to each other, we can conclude that the negative of $\vec{v}$ is unique. □