Math 2311 - Assignment 1
Due: Wed., Jan. 26th

Note: You may work alone or in pairs on the assignment. If you work in a pair, hand in one assignment with both names on it. If you work in a pair, you should still be able to explain your answer to every question on the assignment.

1. Determine if each of the following sets is a vector space. If it is, write a convincing argument to show that it is (verify all of the axioms, or briefly explain why you only need to verify a small number of them); if it is not a vector space demonstrate at least one of the axioms that fails.

   (a) $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid x \geq y \right\}$ with the usual scalar multiplication and vector addition from $\mathbb{R}^2$

   (b) $\{f \in F(-\infty, \infty) \mid f(1) = 0\}$ with the usual scalar multiplication and vector addition from $F(-\infty, \infty)$

2. Let V be a vector space.

   (a) If k is any scalar, prove that $k\vec{0} = \vec{0}$.

   (b) Prove that the zero vector in V is unique. (Hint: assume that there are two different "zero vectors" and show that they must be equal to each other.)

3. Determine if each of the following are subspaces of M_{nn}

   (a) $\{A \in M_{nn} \mid det(A) = 0\}$

   (b) $\{A \in M_{nn} \mid tr(A) = 0\}$ Note: $tr(A)$ is the *trace* of A and is equal to the sum of the diagonal entries of A, see section 1.3 of the text. In particular, you may use the results of questions 35 and TF question (j) from section 1.3 (11th or 12th edition).

   (c) $\{A \in M_{nn} \mid A^T = A\}$ In other words, the symmetric matrices. Theorem 1.4.8 from section 1.4 may be of use.

4. Consider the following vectors in P_2: $p_1 = 2+x+4x^2$, $p_2 = 1-x+3x^2$, $p_3 = 3+2x+5x^2$.

   (a) Express the vector $g = 6 + 11x + 6x^2$ as a linear combination of p_1, p_2, p_3.

   (b) Does $\{p_1, p_2, p_3\}$ span P_2?

   (c) Is $\{p_1, p_2, p_3\}$ linearly independent?

5. Consider the following planes in $\mathbb{R}^3$. $P_1 : 2x + 3y - z = 0$ and $P_2 : x + 2y - 2z = 0$

   (a) Find a set of vectors that spans P_1.

   (b) Find a set of vectors that spans P_2.

   (c) Find a set of vectors that spans the intersection of P_1 and P_2. (Recall that we showed the intersection of two subspaces is a subspace).

6. Show that the functions $f_1(x) = e^x$, $f_2(x) = xe^x$, and $f_3(x) = x^2e^x$ are linearly independent in $C^\infty(-\infty, \infty)$.