Math 2311 - Assignment 2

Due: Wednesday, Feb. 9th, 2022.

1. Show that $S = \{1 + 2x + x^2, 2 + 9x, 3 + 3x + 4x^2\}$ is a basis for P_2 and write $p = 2 + 17x - 3x^2$ as a linear combination of vectors in S. Finally, write $[p]_S$.

2. Recall the standard basis of $\mathbb{R}^3$, $\vec{e_1} = [1\ 0\ 0]^T$, $\vec{e_2} = [0\ 1\ 0]^T$, $\vec{e_3} = [0\ 0\ 1]^T$

   (a) Consider the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 3 & 2 \\ 0 & 3 & 2 \end{bmatrix}$. Does the set of vectors $S_1 = \{A\vec{e_1}, A\vec{e_2}, A\vec{e_3}\}$ form a basis for $\mathbb{R}^3$?

   (b) Consider the matrix $B = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 3 & 0 \\ 0 & 3 & -2 \end{bmatrix}$. Does the set of vectors $S_2 = \{B\vec{e_1}, B\vec{e_2}, B\vec{e_3}\}$ form a basis for $\mathbb{R}^3$?

   (c) Make a conjecture of the form "$S = \{A\vec{e_1}, A\vec{e_2}, A\vec{e_3}\}$ forms a basis for $\mathbb{R}^3$ if and only if A *(insert appropriate property of A here)*".
   Bonus: Prove your conjecture.

3. For each of the following subspaces of M_{33} find a basis and state the dimension. You should check for yourself that it is in fact a basis, but this verification does not need to be a part of your submitted answer.

   (a) $W_1 = \{A \in M_{33} | A \text{ is a diagonal matrix}\}$
   (b) $W_2 = \{A \in M_{33} | A = A^T\}$ (the symmetric matrices)
   (c) $W_1 = \{A \in M_{33} | A = -A^T\}$ (the anti-symmetric matrices)

4. Find a basis for the subspace of P_3 spanned by the following polynomials (vectors):

   $p_1 = 1 + x + 3x^2 + 4x^3,\ p_2 = 1 + 2x^2 + 3x^3,\ p_3 = x + x^2 + 2x^3,\ p_4 = 1 + x + 3x^2 + 5x^3$

5. Let $\vec{x} = [1\,2\,3]^T$, $\mathcal{B} = \{[1\,0\,0]^T,\ [1\,1\,0]^T,\ [1\,1\,1]^T\}$, and $\mathcal{C} = \{[1\,1\,0]^T,\ [0\,1\,1]^T,\ [1\,0\,1]^T\}$.

   (a) Find $[\vec{x}]_{\mathcal{B}}$
   (b) Find $[\vec{x}]_{\mathcal{C}}$
   (c) Find $P_{\mathcal{C}\leftarrow\mathcal{B}}$ and compute $P_{\mathcal{C}\leftarrow\mathcal{B}}[\vec{x}]_{\mathcal{B}}$
   (d) Find $P_{\mathcal{B}\leftarrow\mathcal{C}}$ and compute $P_{\mathcal{B}\leftarrow\mathcal{C}}[\vec{x}]_{\mathcal{C}}$

6. Let $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

   (a) Find a basis for each of null(A), row(A), col(A), and state the dimension of each of these subspaces.

   (b) Is the vector $\vec{b} = \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix}$ in the column space of A? If so, write $\vec{b}$ as a linear combination of the columns of A.