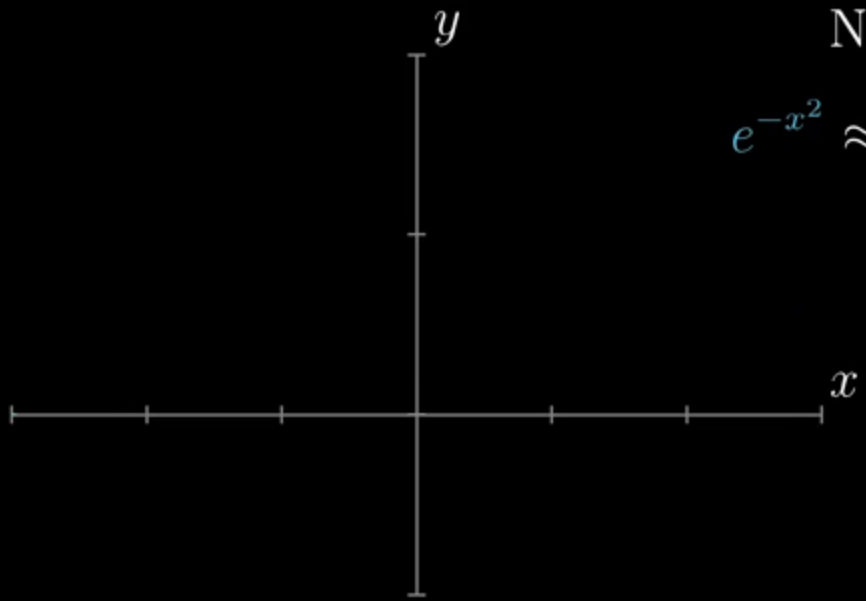
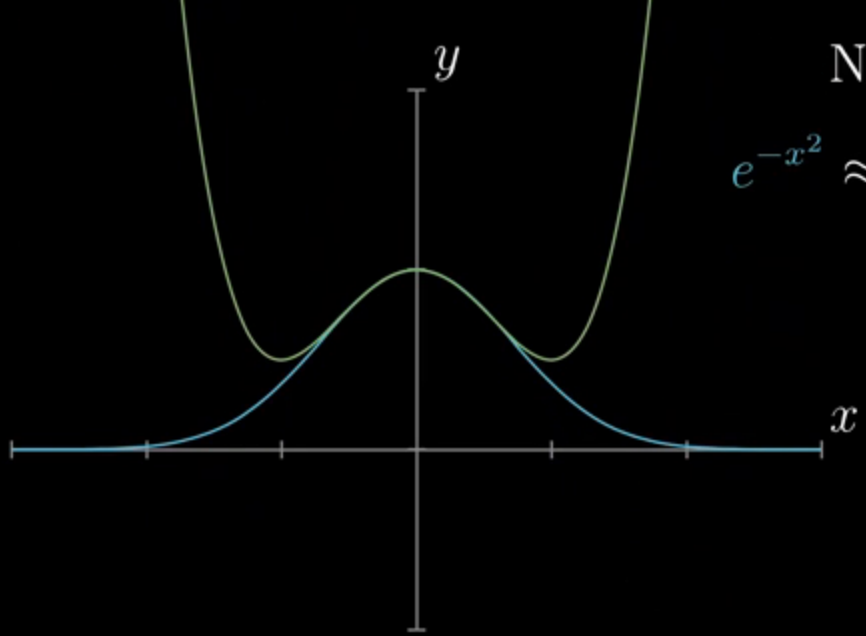
“For me, mathematics is a



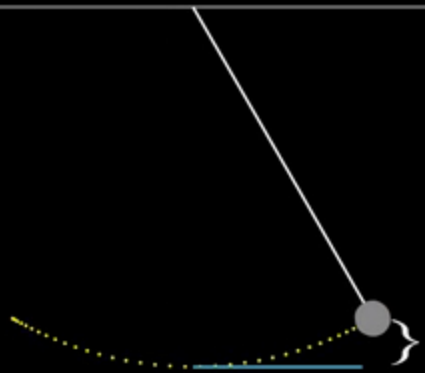
Near $x = 0$,

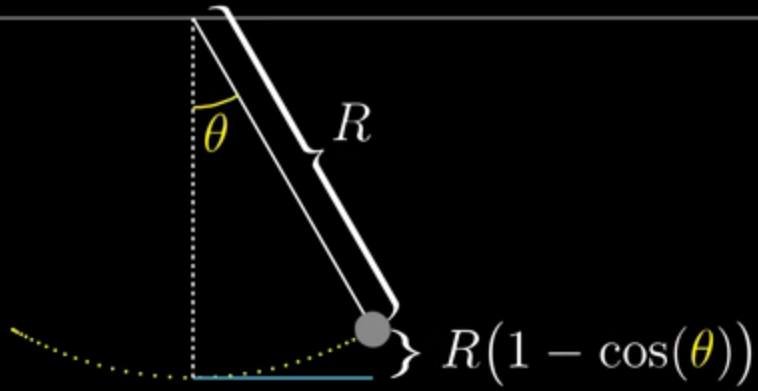
$$e^{-x^2} \approx$$

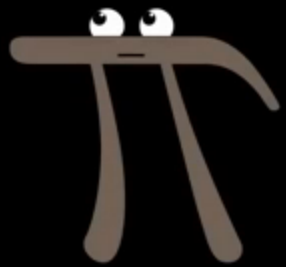


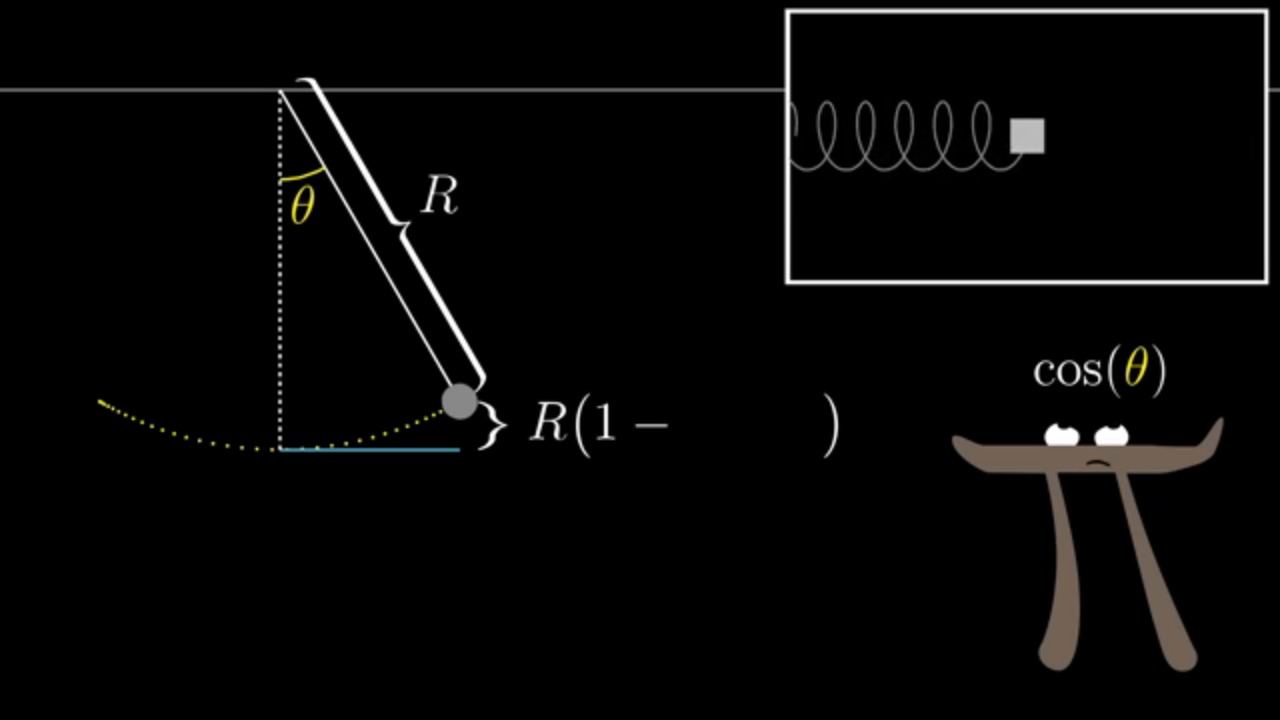
Near $x = 0$,

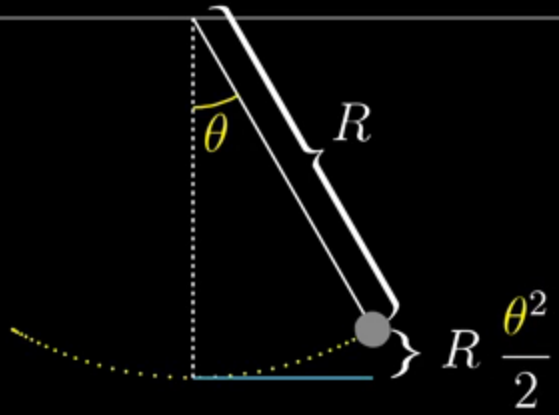
$$e^{-x^2} \approx 1 - x^2 + \frac{1}{2}x^4$$




θ
R
$R(1-\cos(\theta))$

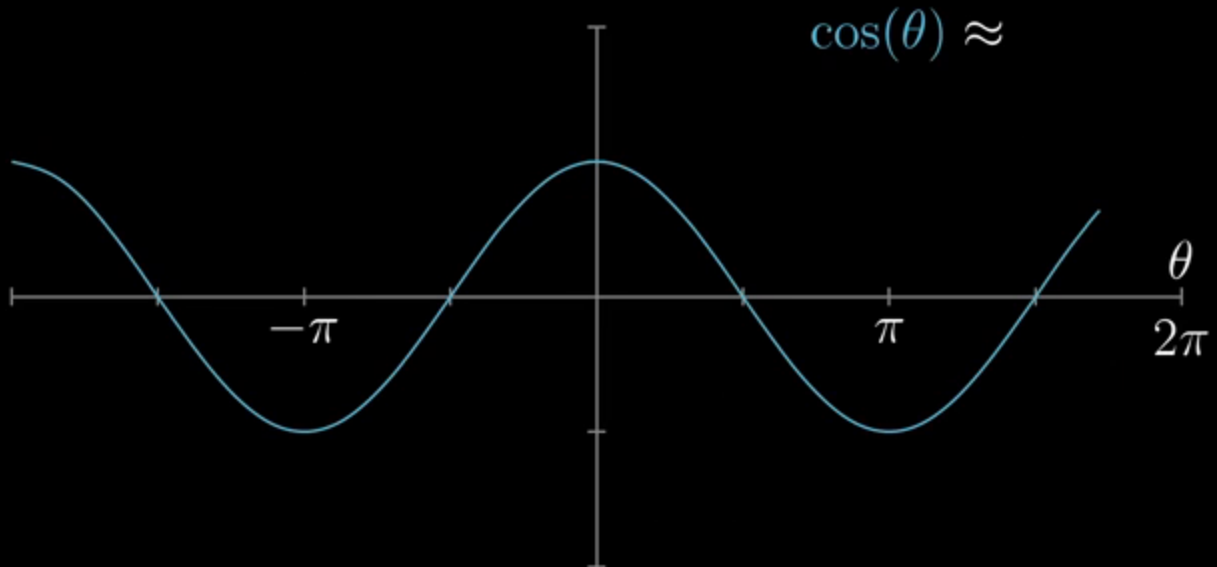


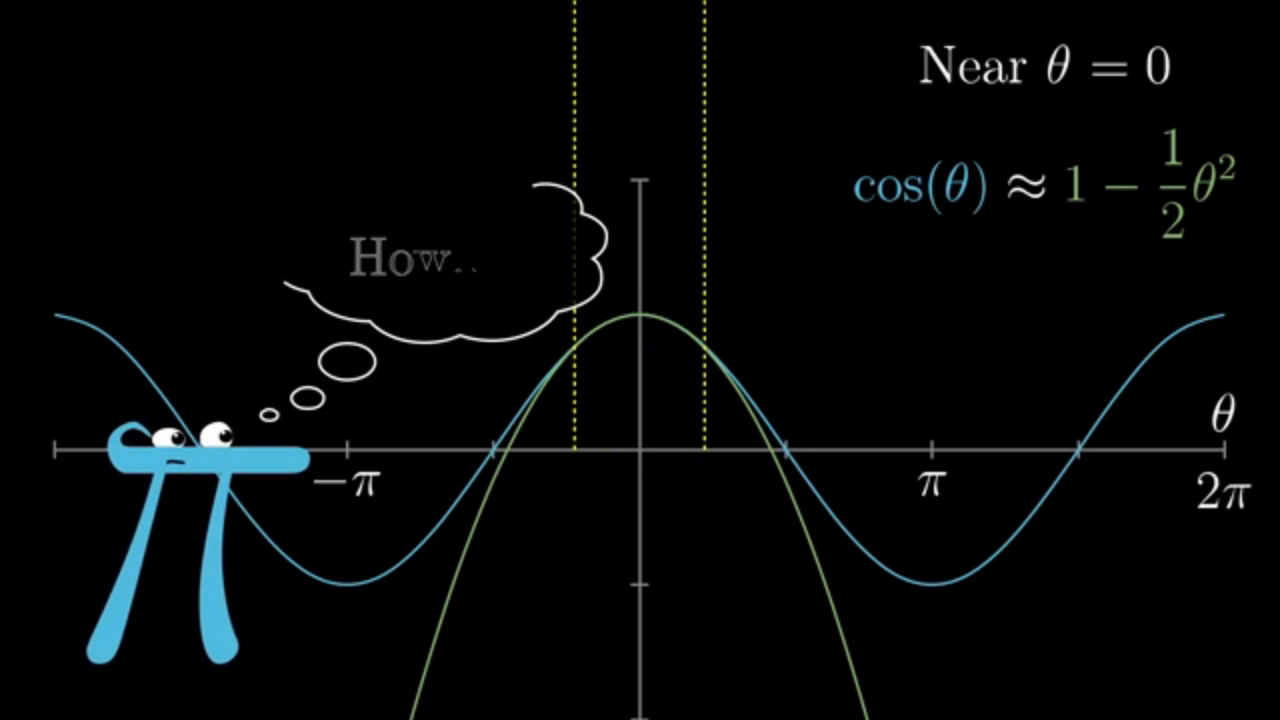

θ
R
$R(1 -$ $)$
$\cos(\theta)$

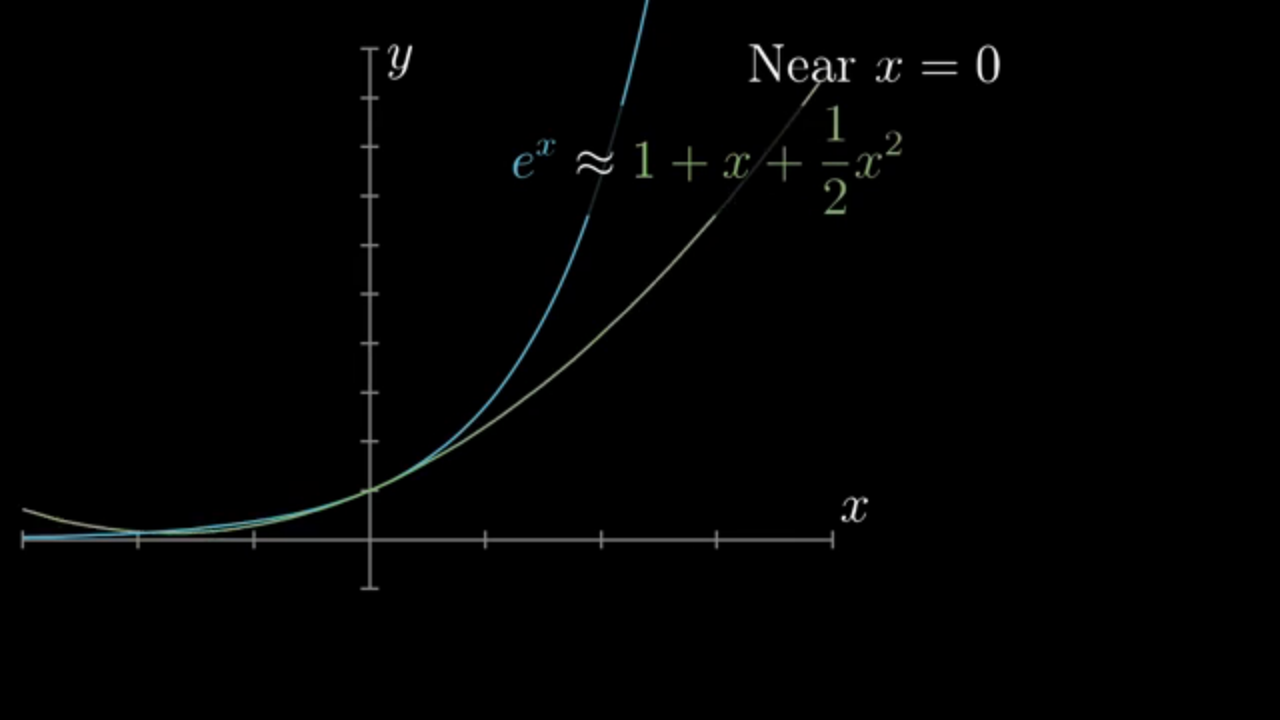


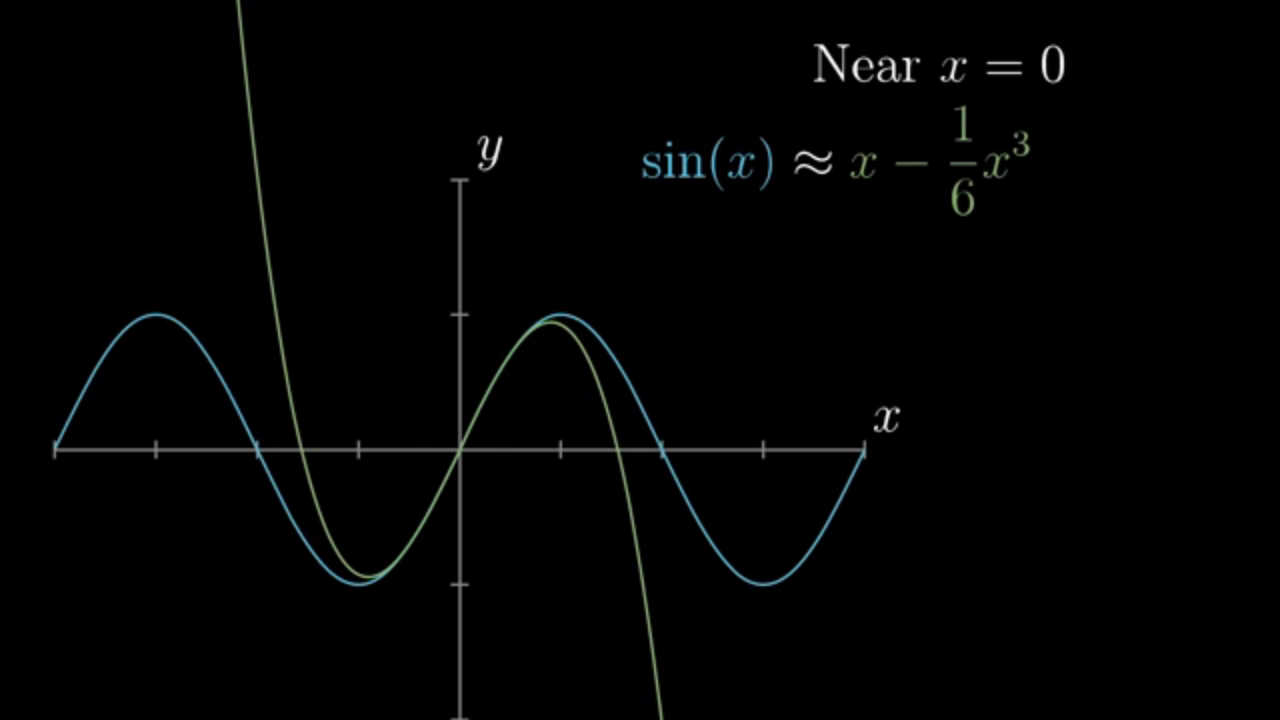
$$\cos(\theta) \approx 1 - \frac{\theta^2}{2}$$

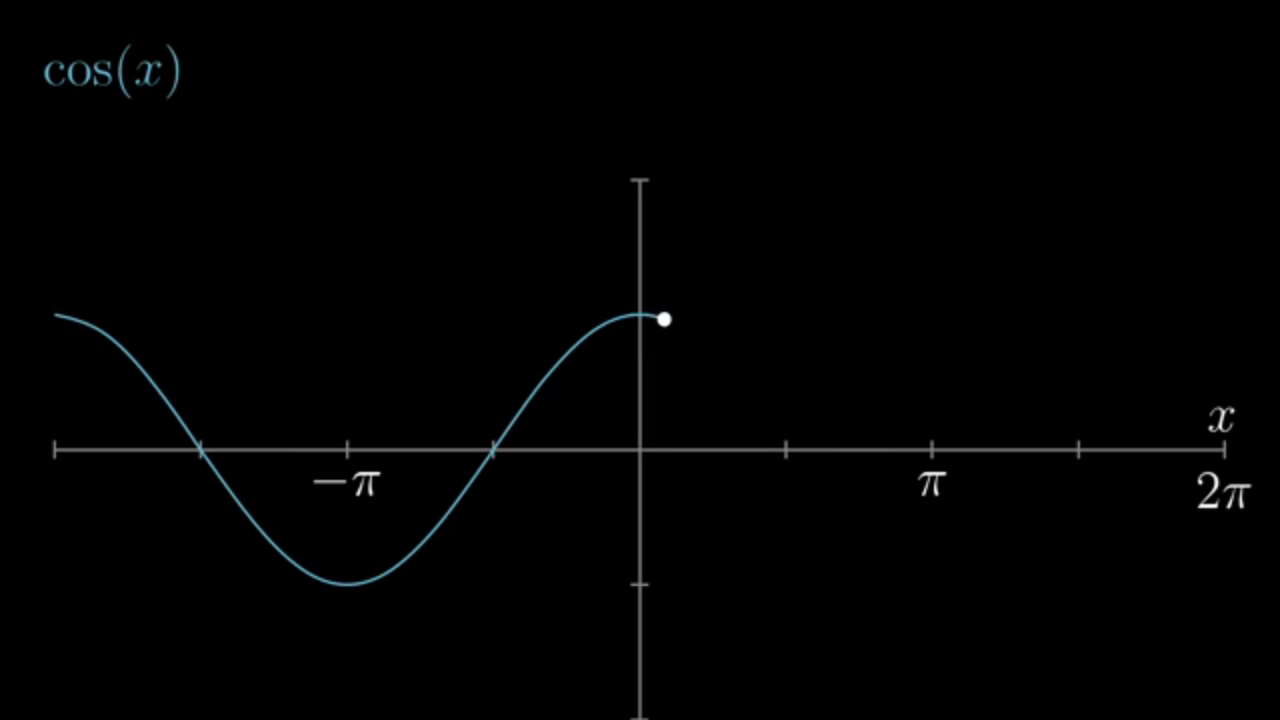
Near $\theta = 0$

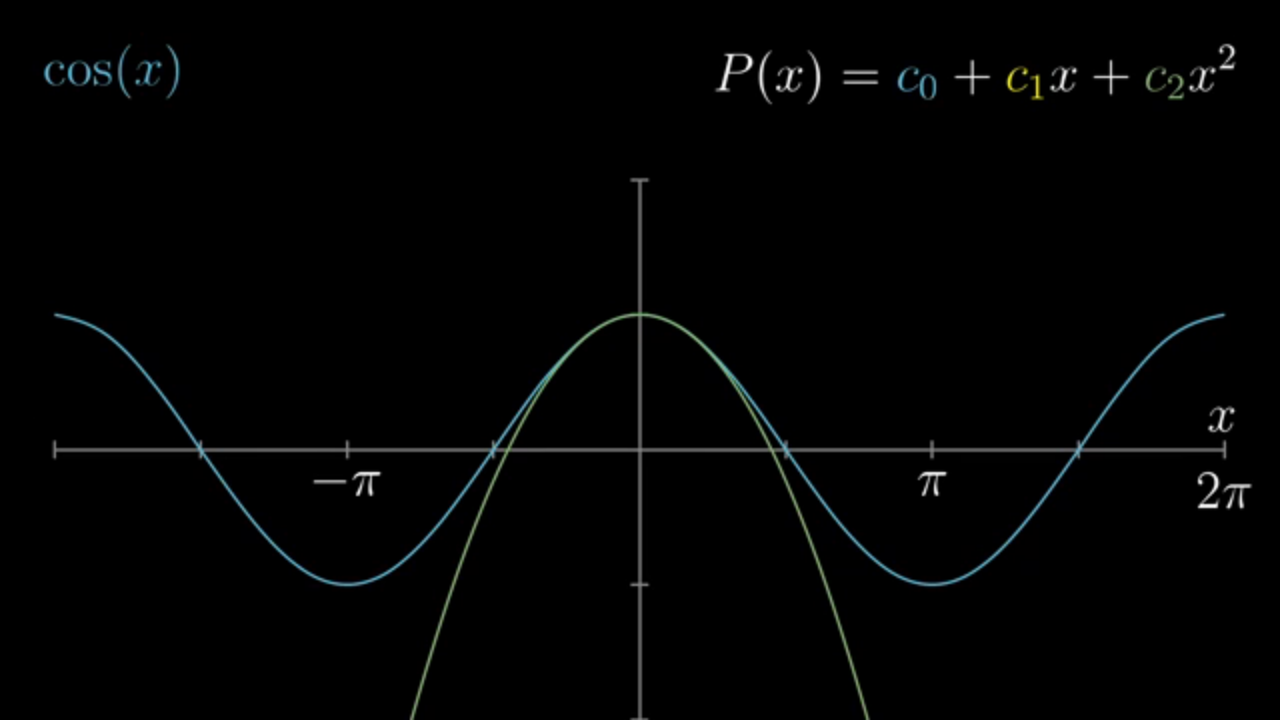


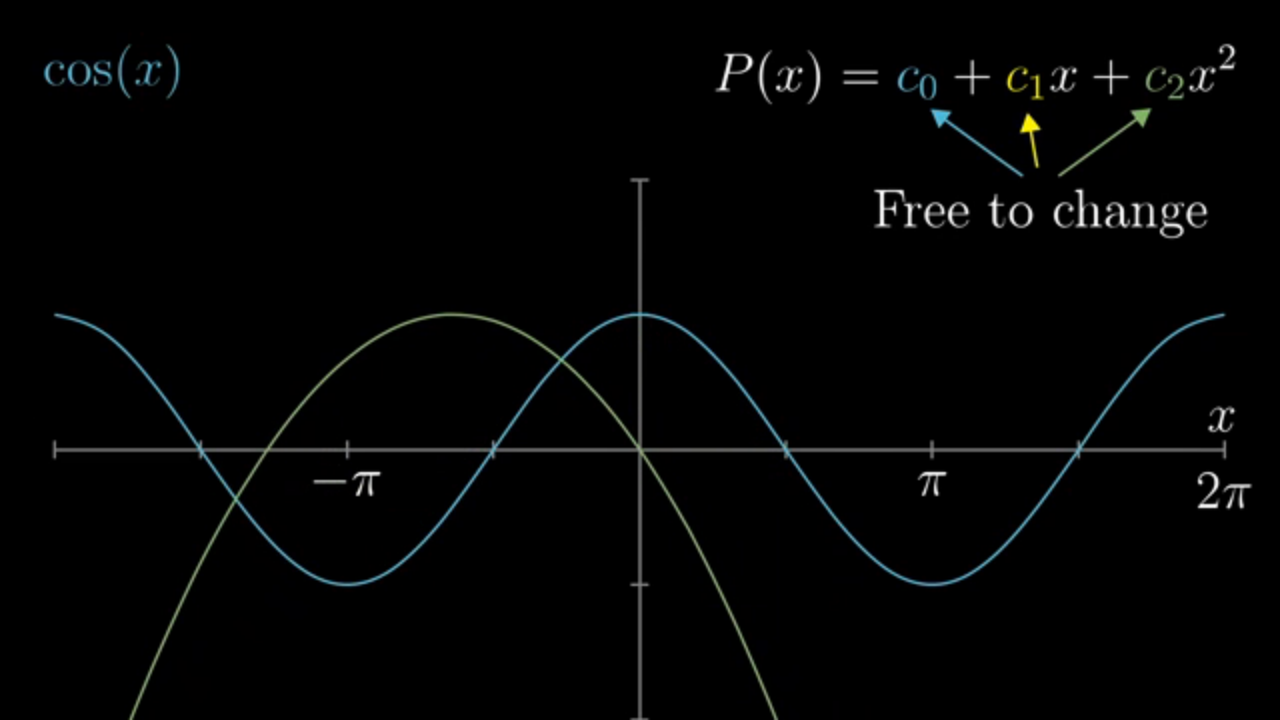

Near $\theta = 0$
$\cos(\theta) \approx 1 - \frac{1}{2}\theta^2$
How..
θ
$-\pi$
π
2π

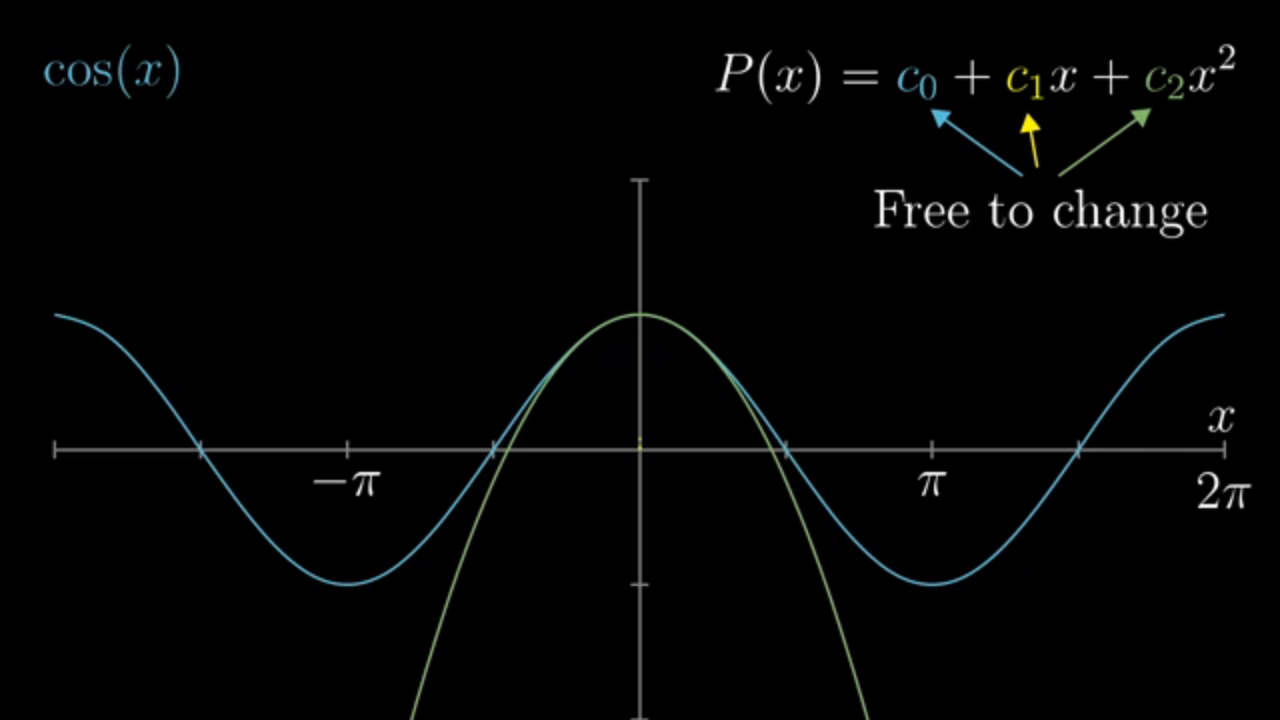

Near $x = 0$
$e^x \approx 1 + x + \frac{1}{2}x^2$
y
x


Near $x = 0$
$\sin(x) \approx x - \frac{1}{6}x^3$
y
x


$\cos(x)$
x
$-\pi$
π
2π


$\cos(x)$
$P(x) = c_0 + c_1 x + c_2 x^2$
x
$-\pi$
π
2π

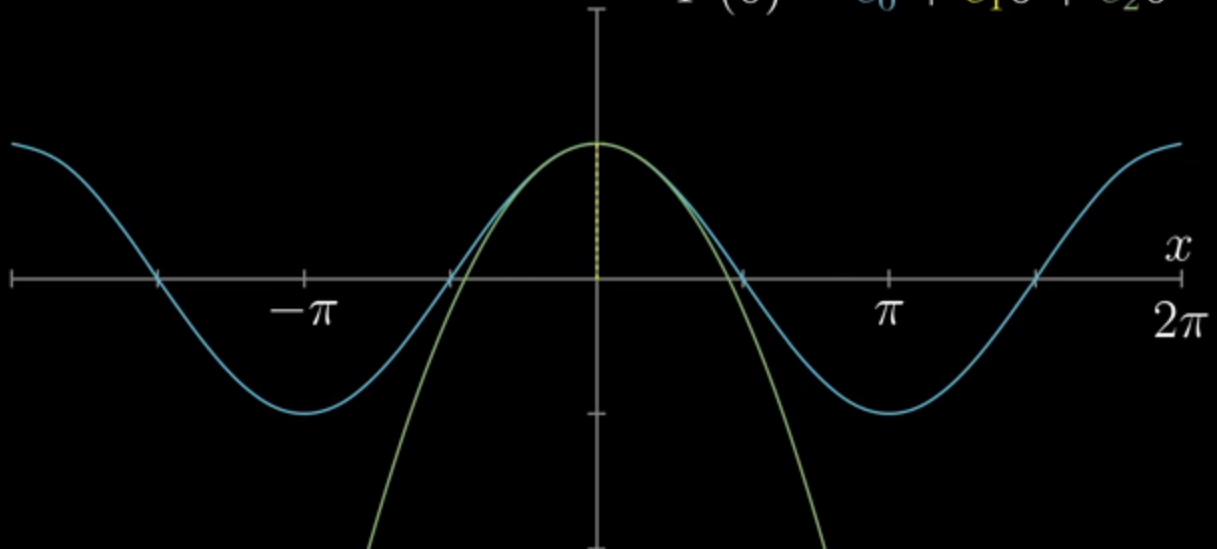

$\cos(x)$
$P(x) = c_0 + c_1 x + c_2 x^2$
Free to change
x
$-\pi$
π
2π


$\cos(x)$
$P(x) = c_0 + c_1 x + c_2 x^2$
Free to change
x
$-\pi$
π
2π

$$\cos(x) \xrightarrow{x=0} 1$$

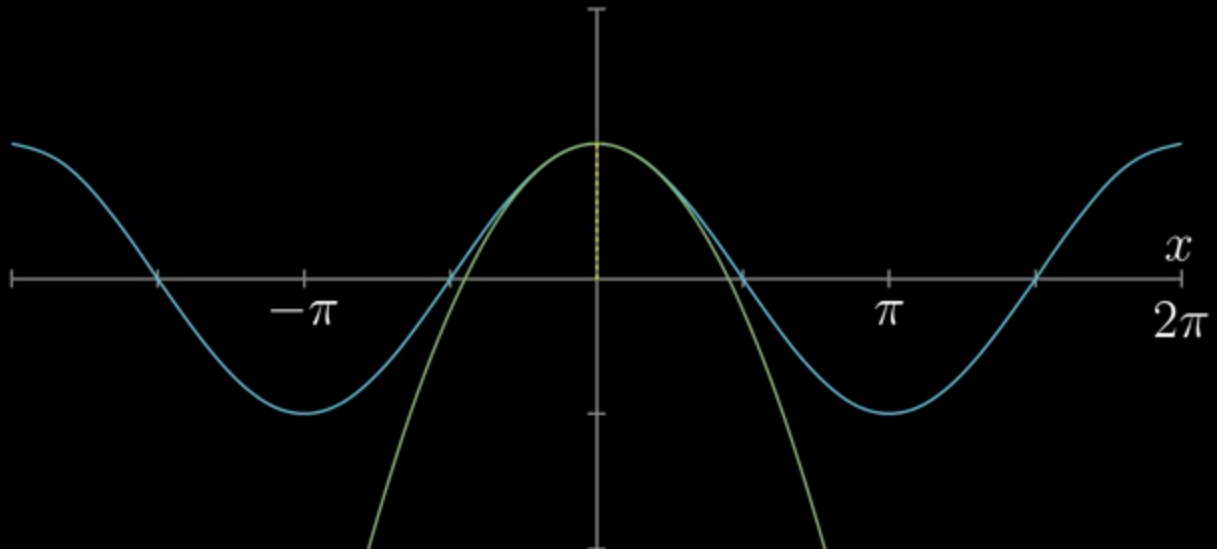
$$P(x) = c_0 + c_1 x + c_2 x^2$$

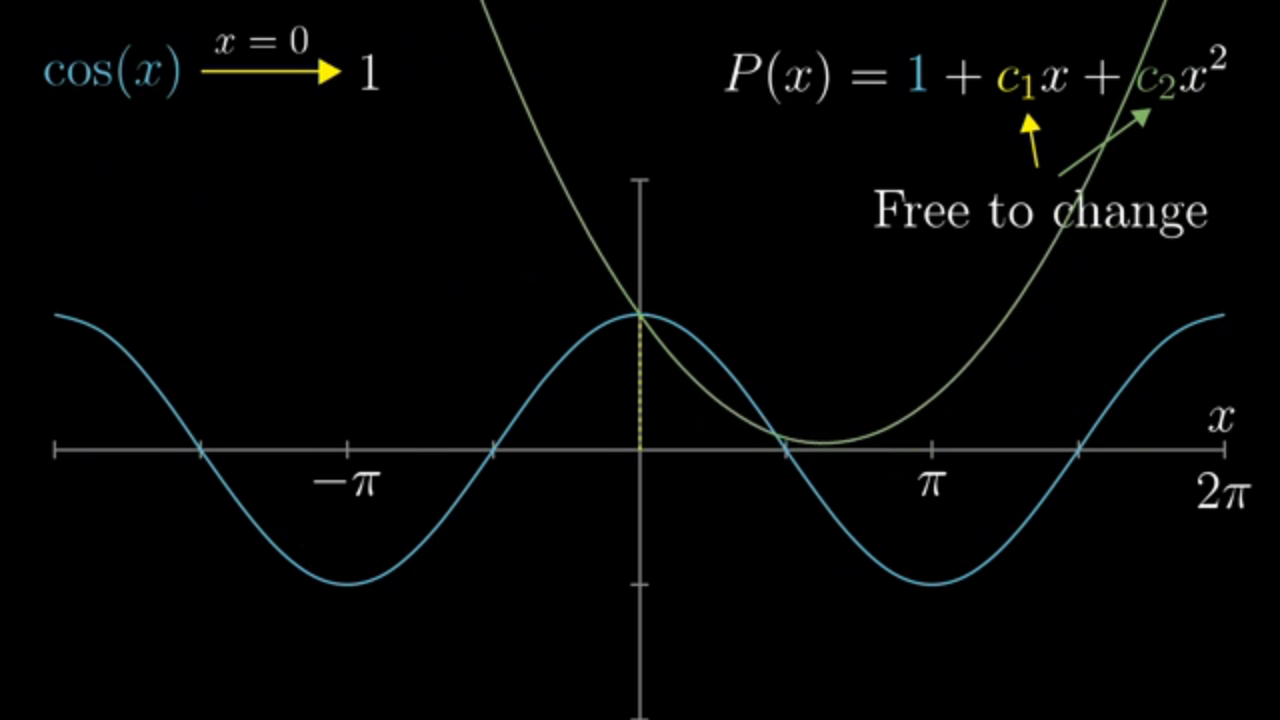
$$P(0) = c_0 + c_1 0 + c_2 0^2$$

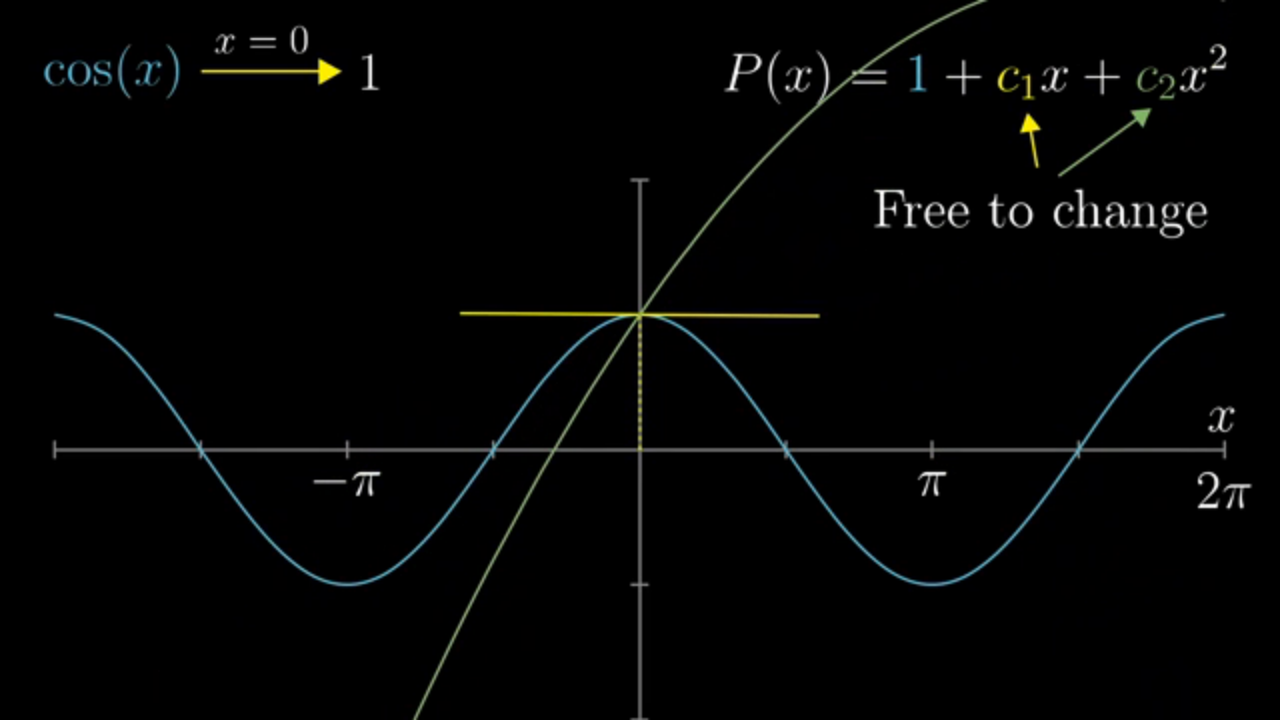


$\cos(x) \xrightarrow{x=0} 1$

$P(x) = 1 + c_1 x + c_2 x^2$




$\cos(x) \xrightarrow{x=0} 1$
$P(x) = 1 + c_1 x + c_2 x^2$
Free to change
x
$-\pi$
π
2π

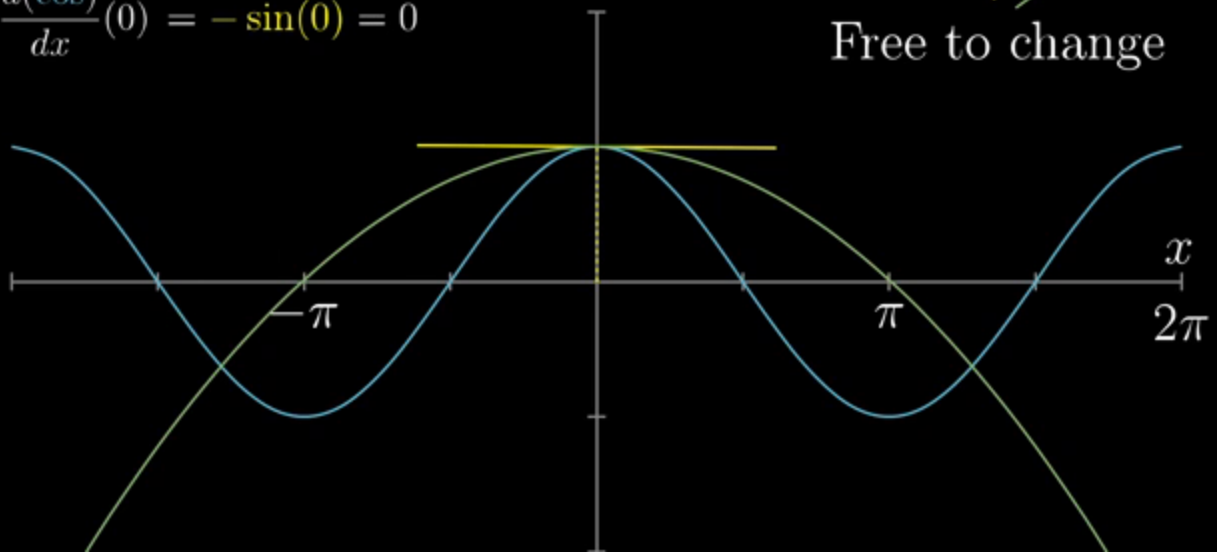

$\cos(x) \xrightarrow{x=0} 1$
$P(x) = 1 + c_1 x + c_2 x^2$
Free to change
x
$-\pi$
π
2π

$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + c_1 x + c_2 x^2$$

Free to change

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

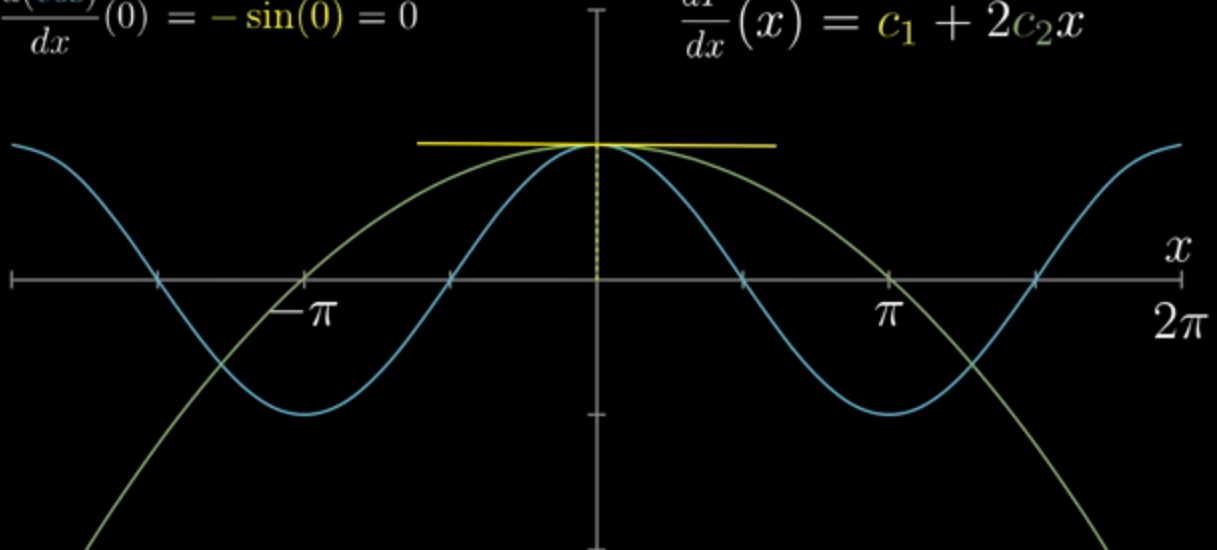


$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + c_1 x + c_2 x^2$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{dP}{dx}(x) = c_1 + 2c_2 x$$

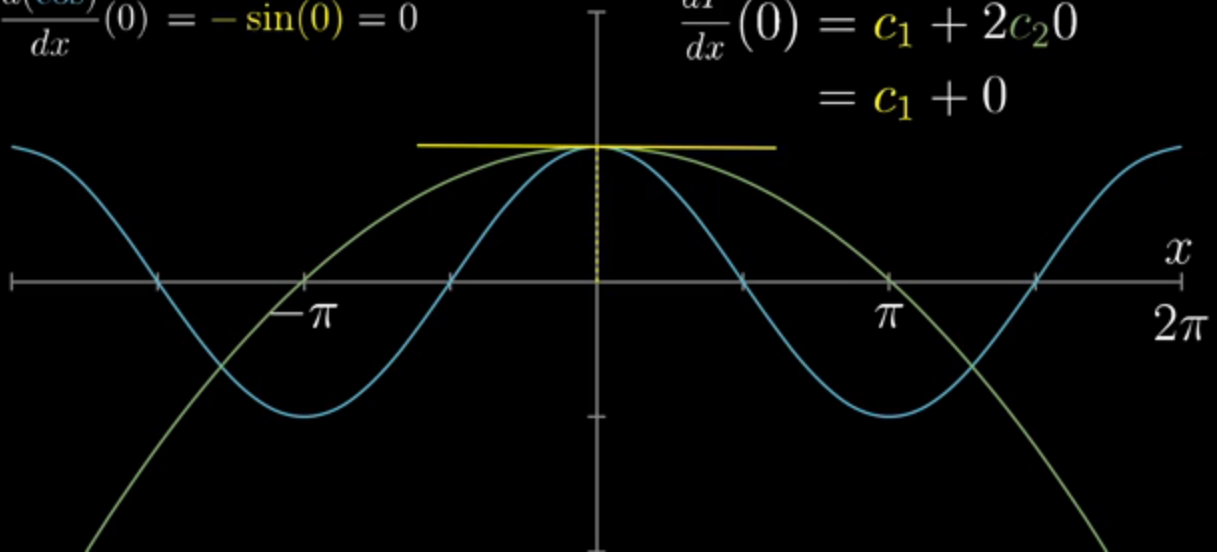


$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + c_1 x + c_2 x^2$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{dP}{dx}(0) = c_1 + 2c_2 0$$
$$= c_1 + 0$$

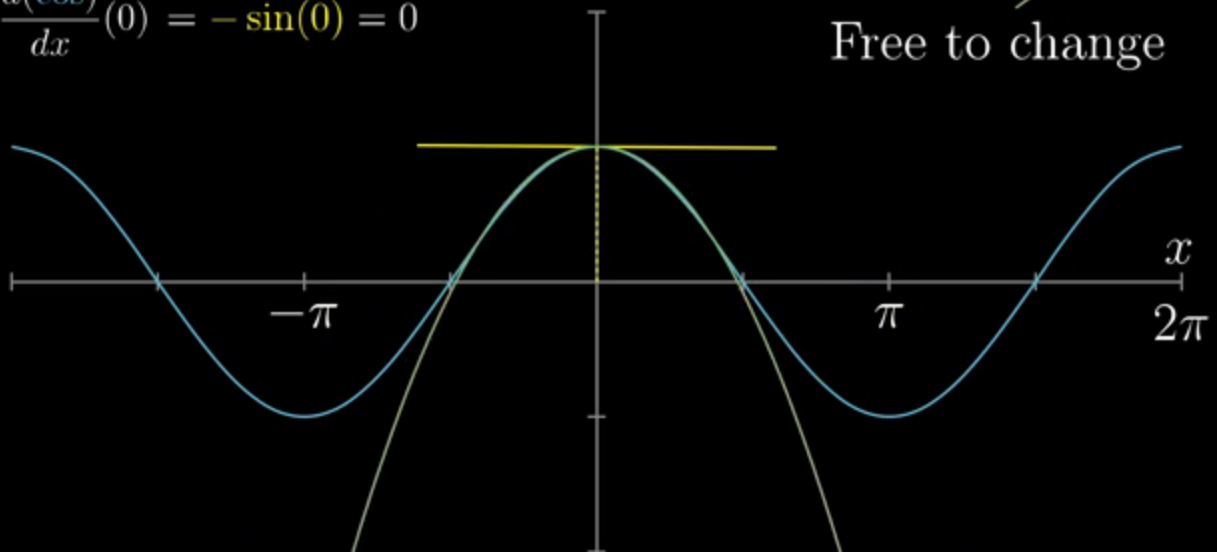


$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + 0x + c_2 x^2$$

Free to change

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$



$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + 0x + c_2x^2$$

Free to change

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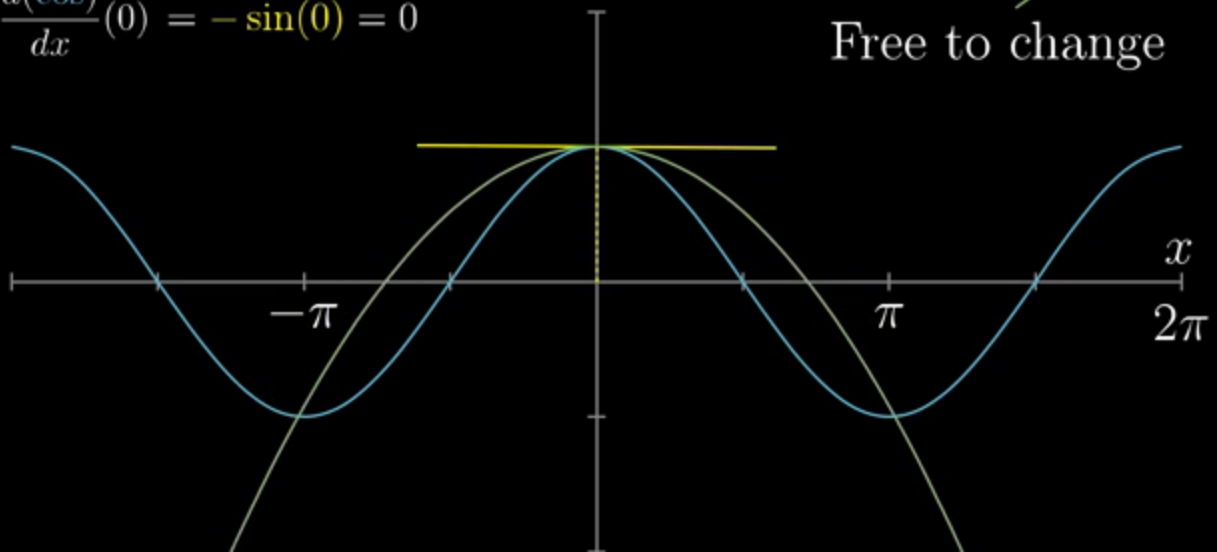
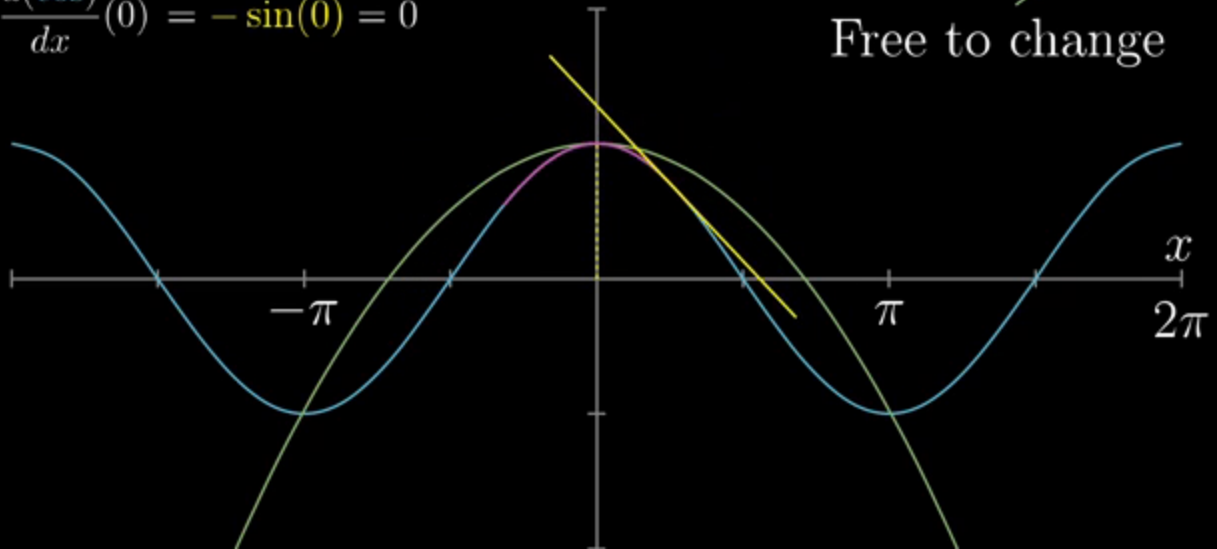


$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + 0x + c_2x^2$$

Free to change

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$



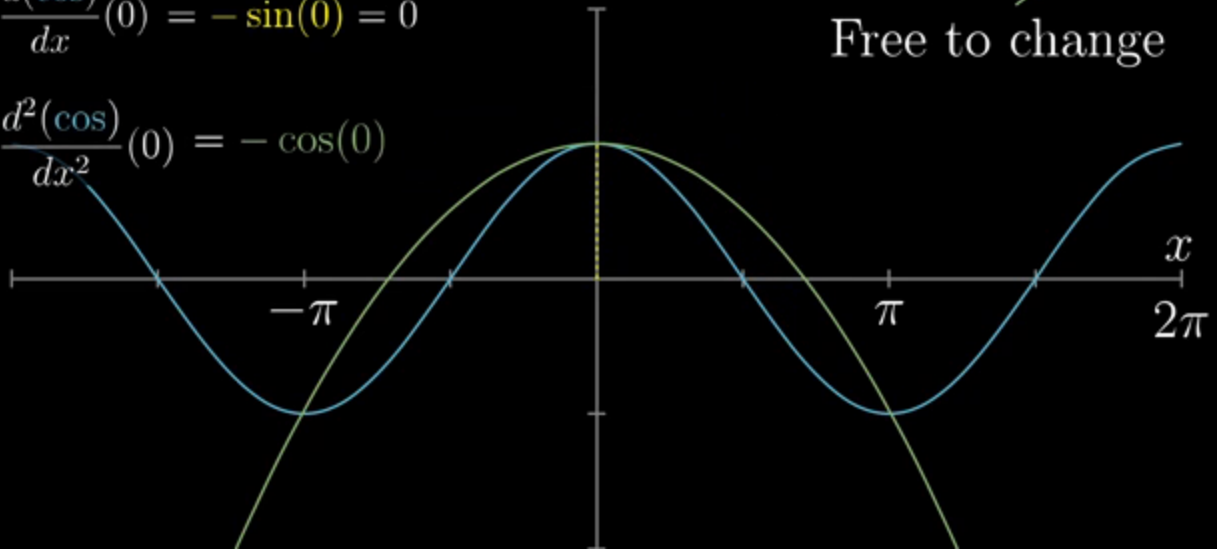
$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + 0x + c_2x^2$$

Free to change

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0)$$



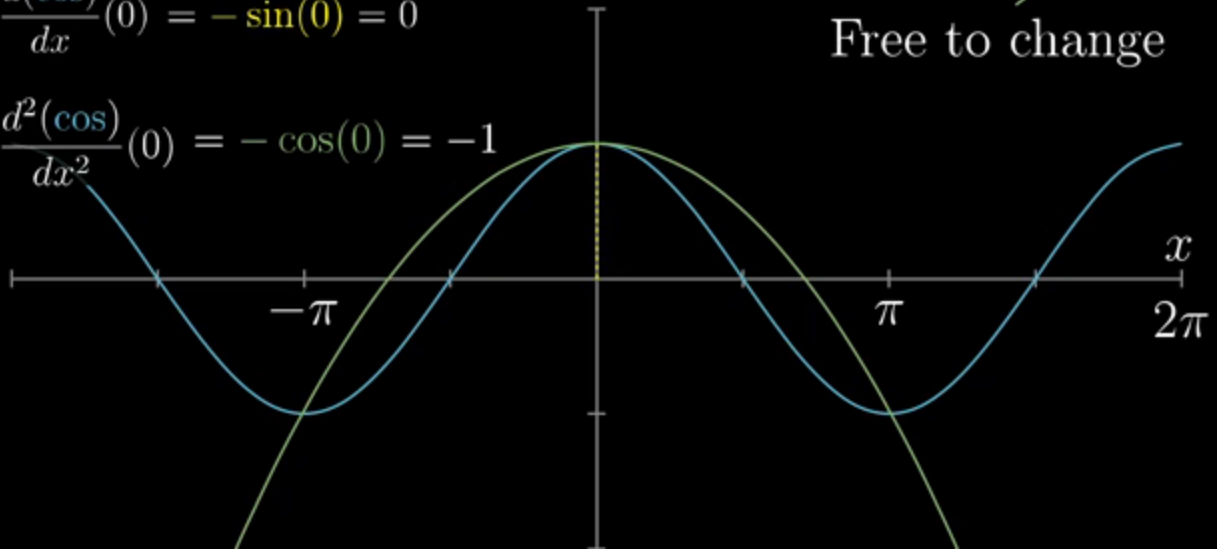
$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + 0x + c_2x^2$$

Free to change

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$



$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + 0x + c_2x^2$$

Free to change

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

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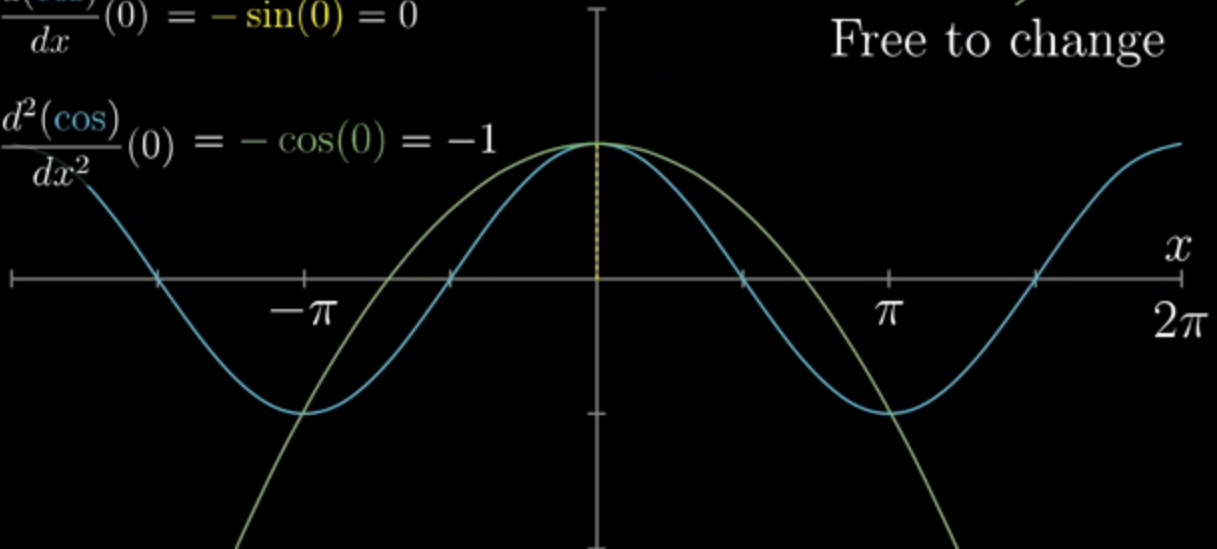


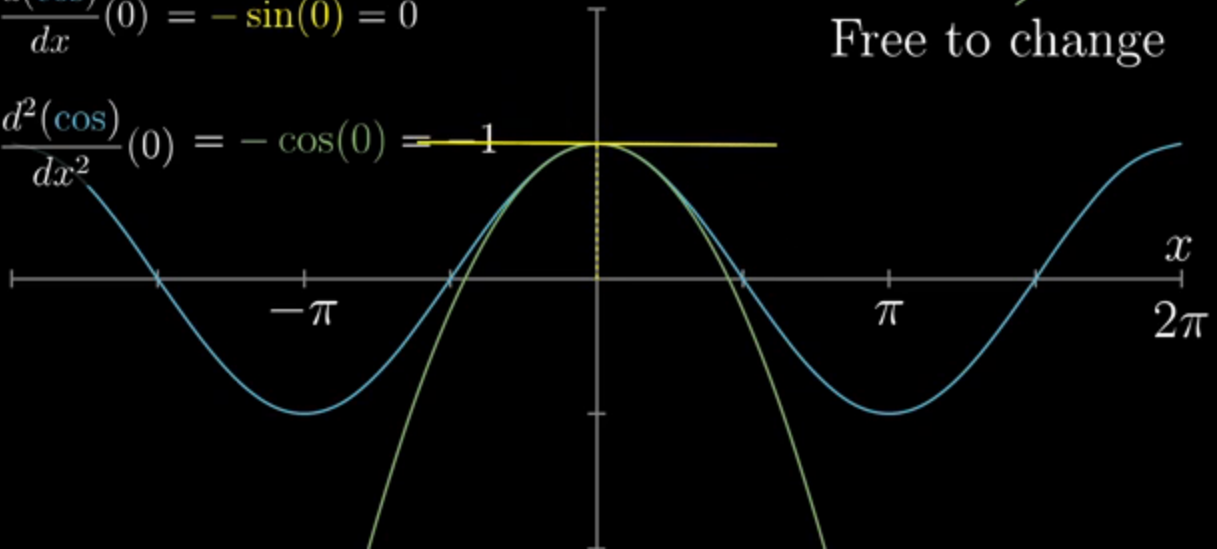
$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + 0x + c_2 x^2$$

Free to change

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$



$$\cos(x) \xrightarrow{x=0} 1$$

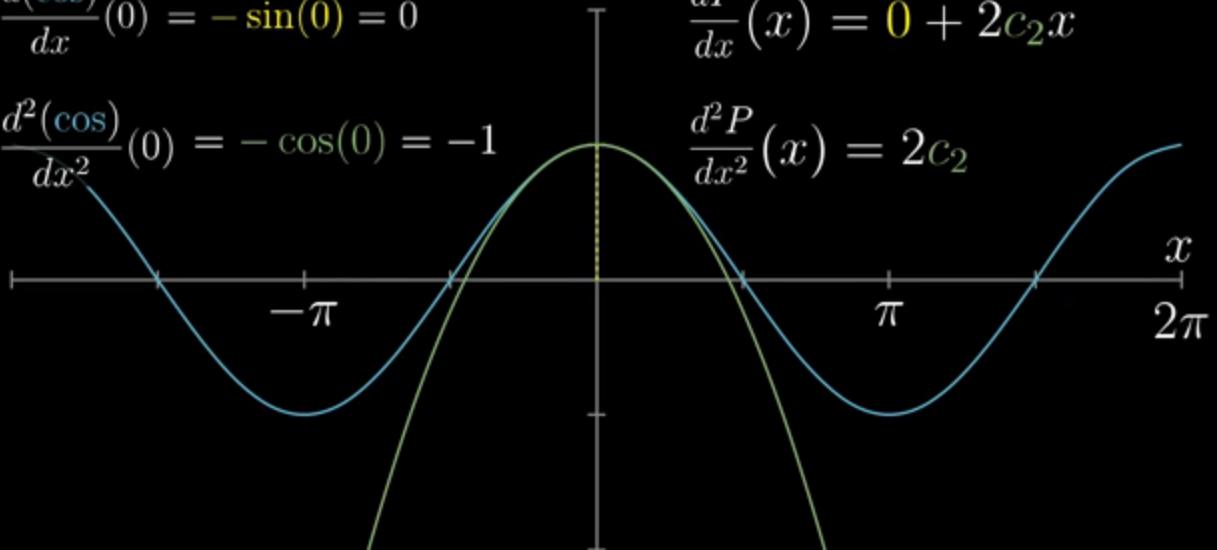
$$P(x) = 1 + 0x + c_2x^2$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{dP}{dx}(x) = 0 + 2c_2x$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$\frac{d^2P}{dx^2}(x) = 2c_2$$



$$\cos(x) \xrightarrow{x=0} 1$$

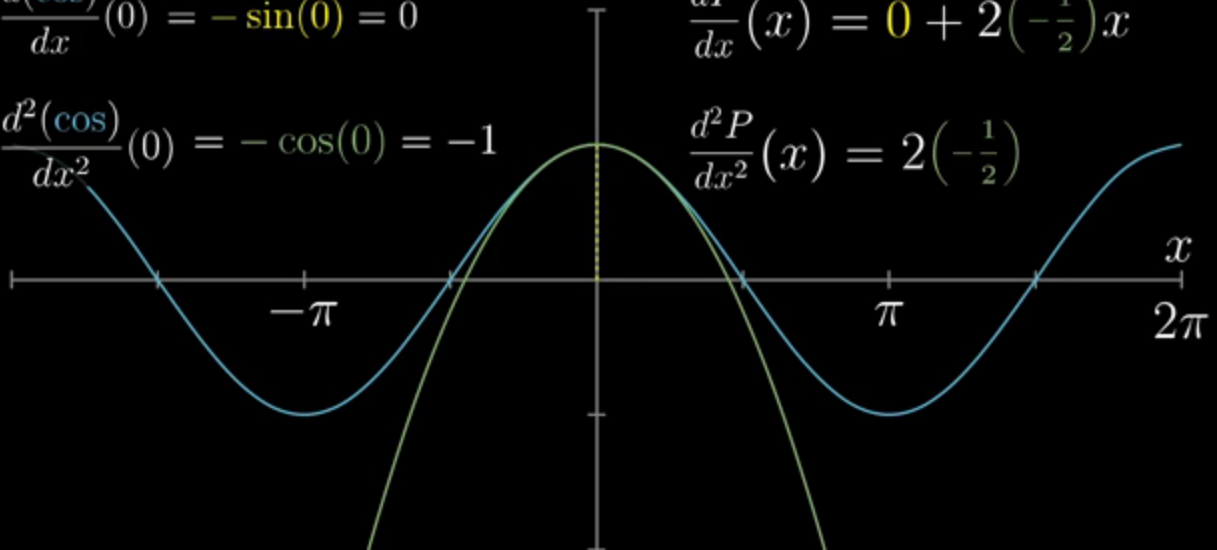
$$P(x) = 1 + 0x + \left(-\tfrac{1}{2}\right)x^2$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

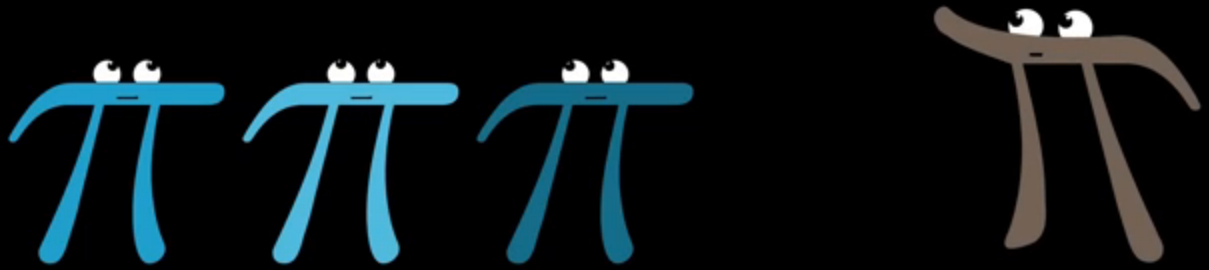
$$\frac{dP}{dx}(x) = 0 + 2\left(-\tfrac{1}{2}\right)x$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$\frac{d^2P}{dx^2}(x) = 2\left(-\tfrac{1}{2}\right)$$

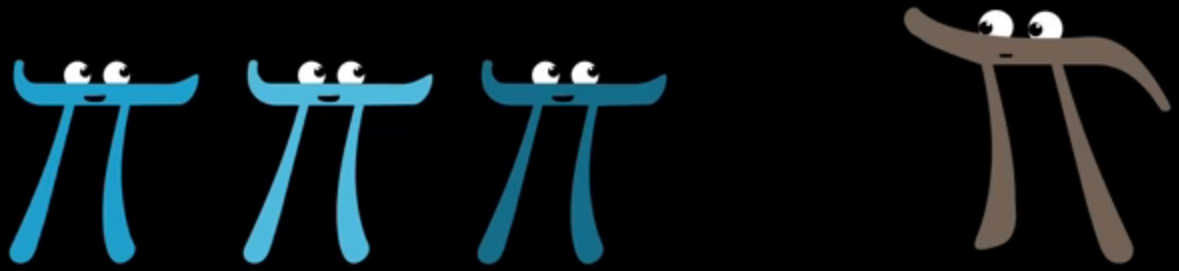


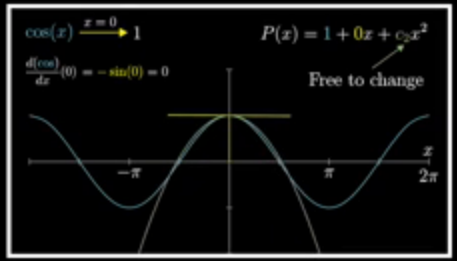
$$\cos(x) \approx 1 - \frac{1}{2}(x)^2$$



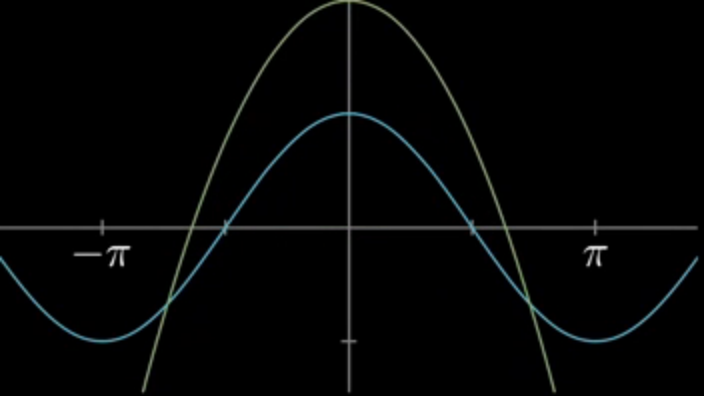
$$\cos(0.1) \approx 1 - \frac{1}{2}(0.1)^2 = 0.995$$

$$\cos(0.1) = 0.9950042\ldots$$



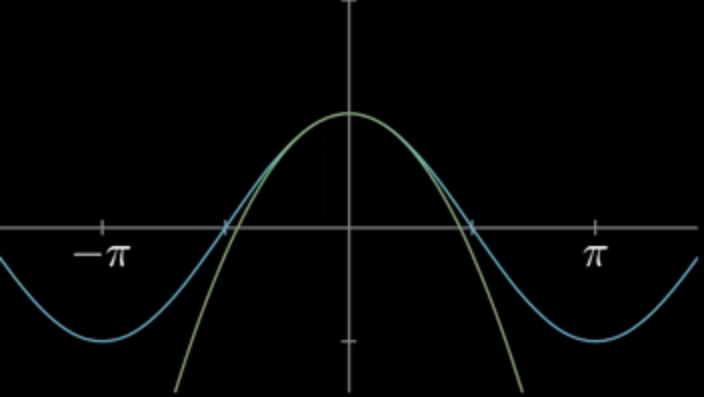


$$P(x) = c_0 + c_1x + c_2x^2$$

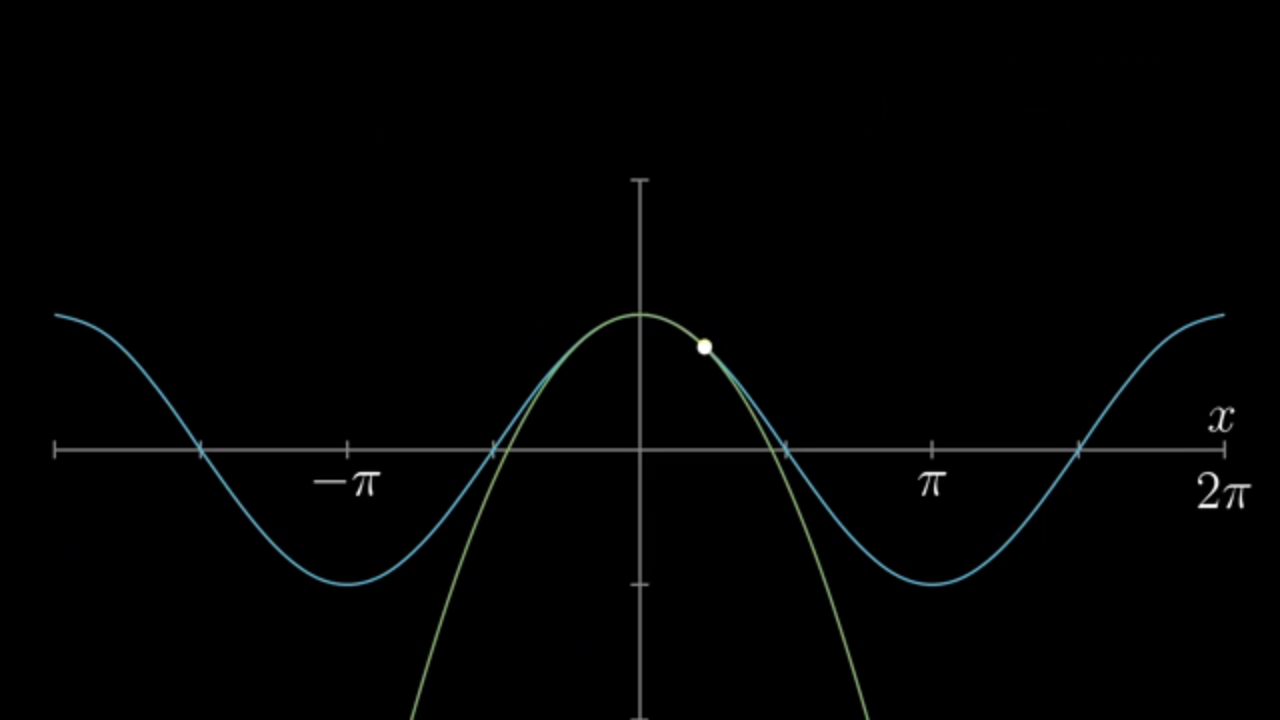


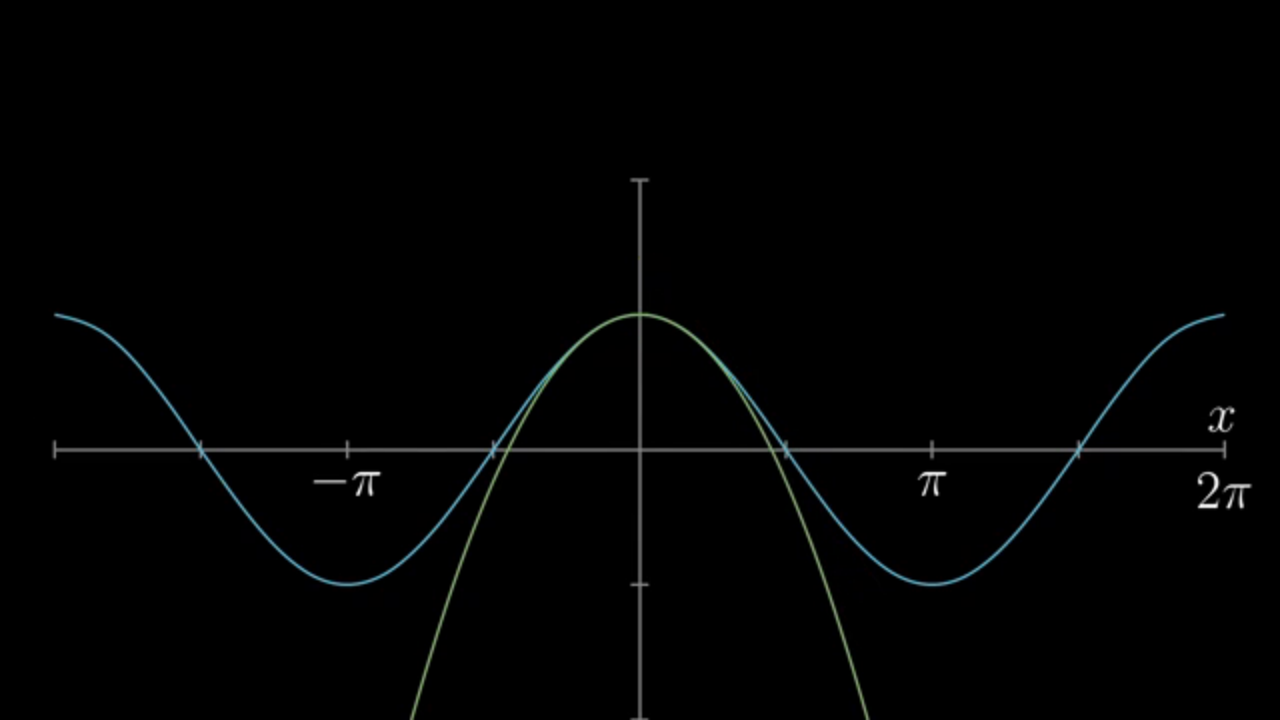
$$P(0) = \cos(0)$$

$$P(x) = c_0 + c_1 x + c_2 x^2$$



$$P(x) = 1 + 0x + c_2 x^2$$


x
$-\pi$
π
2π

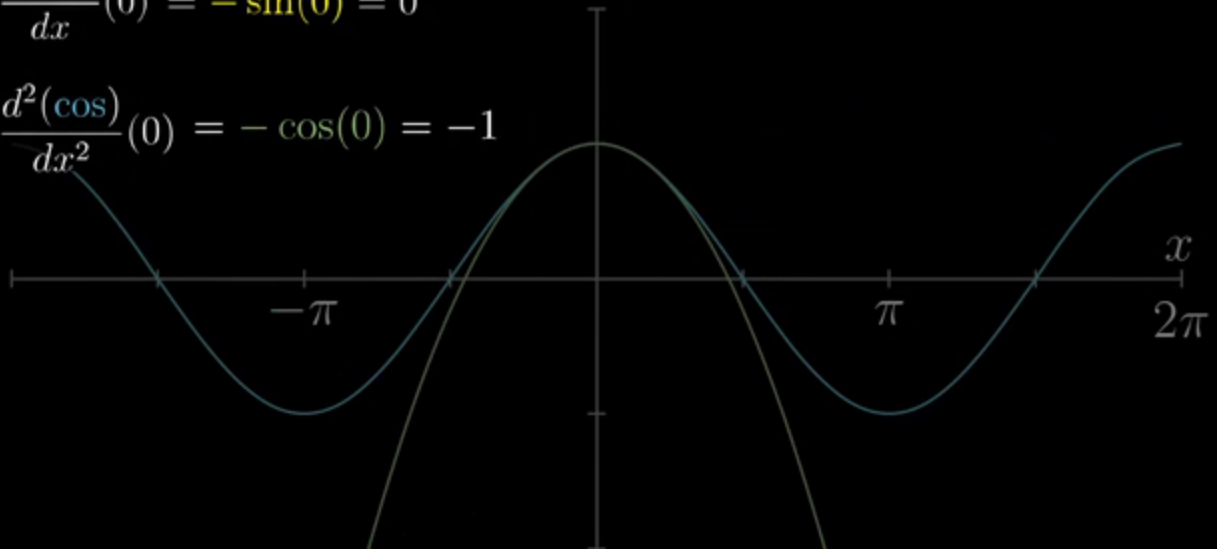

x
$-\pi$
π
2π

$$\cos(0) = 1$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$P(x) = 1 - \frac{1}{2}x^2 + c_3x^3$$



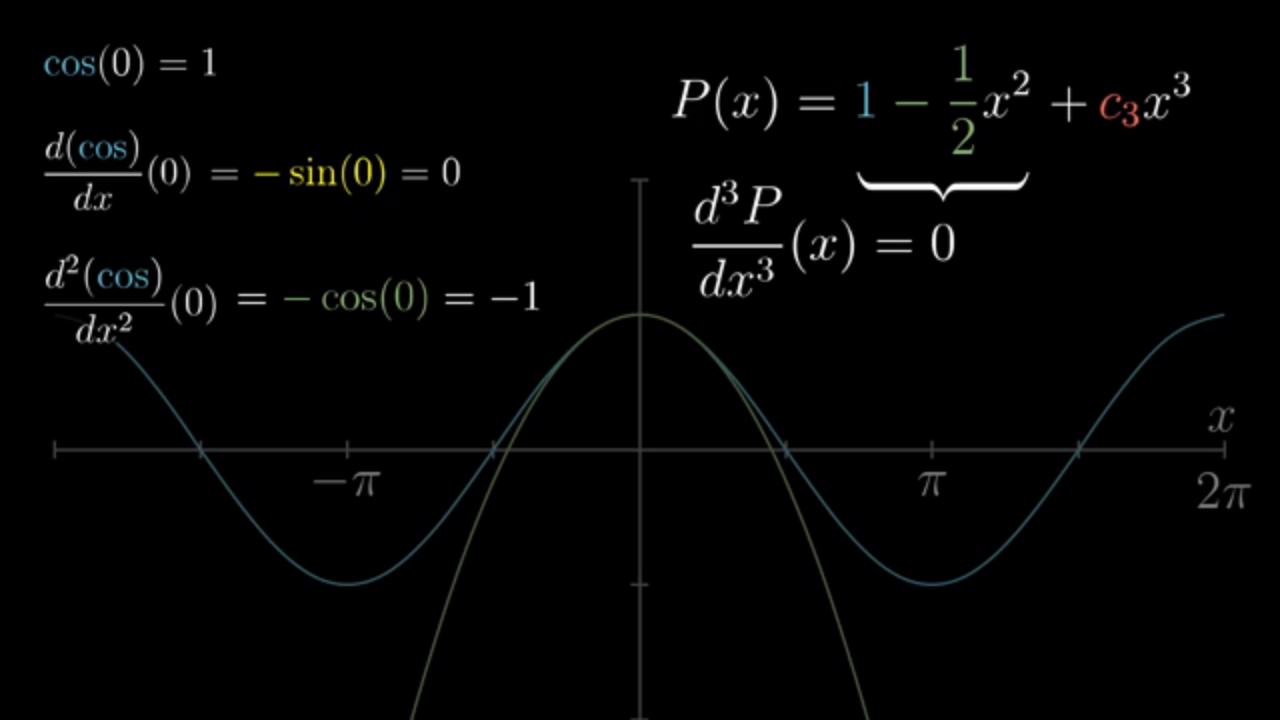
$$\cos(0) = 1$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$P(x) = \underbrace{1 - \frac{1}{2}x^2}_{} + c_3 x^3$$

$$\frac{d^3 P}{dx^3}(x) = 0$$



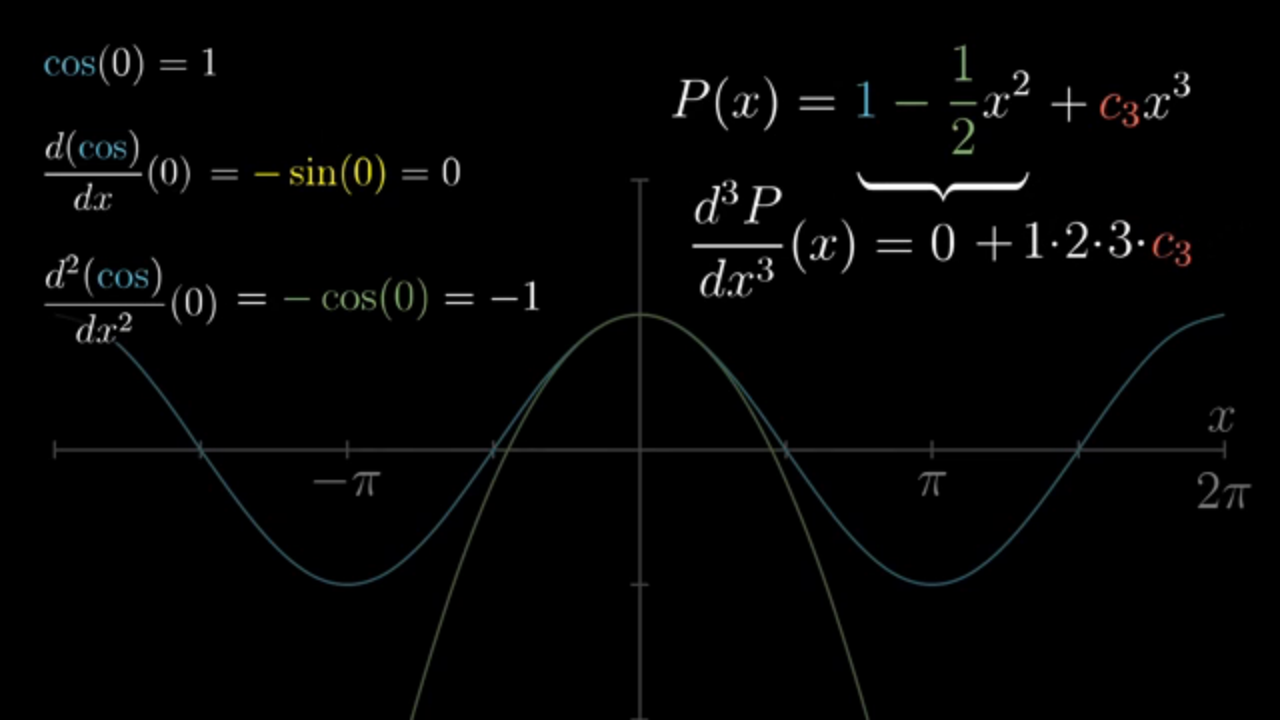
$$\cos(0) = 1$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$P(x) = \underbrace{1 - \frac{1}{2}x^2}_{} + c_3 x^3$$

$$\frac{d^3 P}{dx^3}(x) = 0 + 1 \cdot 2 \cdot 3 \cdot c_3$$



$$\cos(0) = 1$$

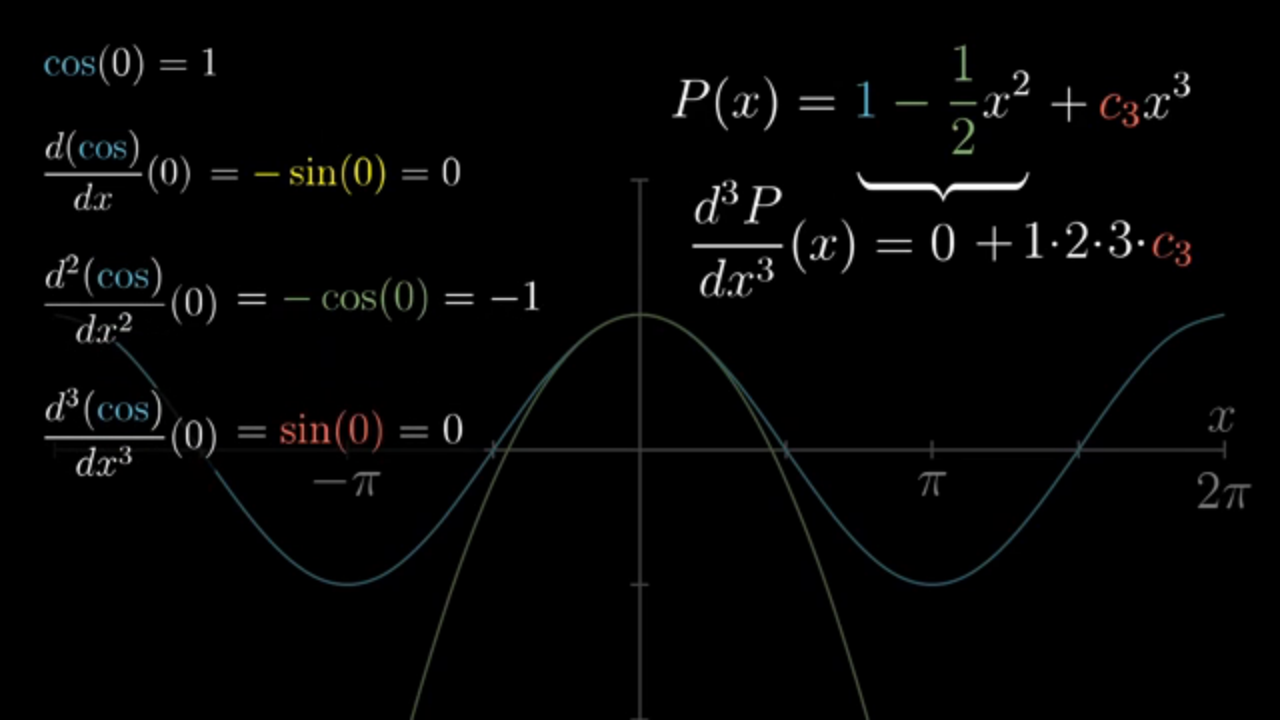
$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$\frac{d^3(\cos)}{dx^3}(0) = \sin(0) = 0$$

$$P(x) = \underbrace{1 - \frac{1}{2}x^2}_{} + c_3 x^3$$

$$\frac{d^3 P}{dx^3}(x) = 0 + 1 \cdot 2 \cdot 3 \cdot c_3$$



$$\cos(0) = 1$$

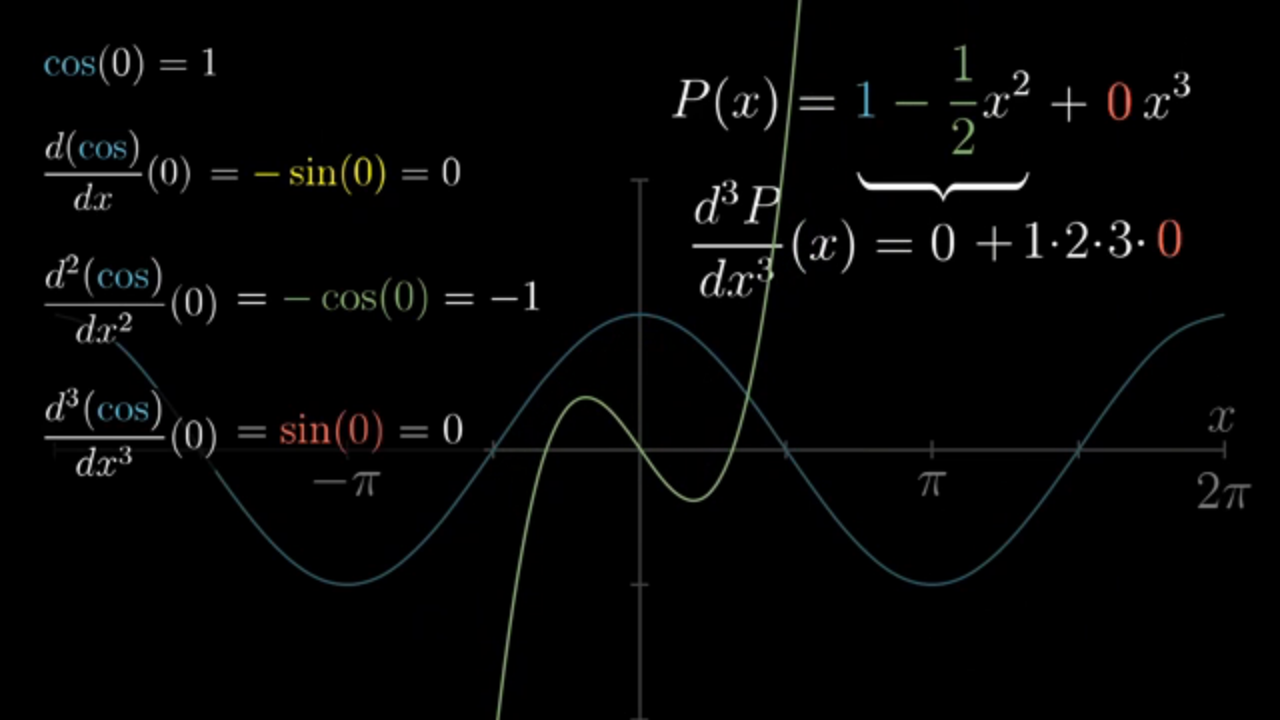
$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$\frac{d^3(\cos)}{dx^3}(0) = \sin(0) = 0$$

$$P(x) = \underbrace{1 - \frac{1}{2}x^2}_{} + 0\,x^3$$

$$\frac{d^3P}{dx^3}(x) = 0 + 1\cdot 2\cdot 3\cdot 0$$



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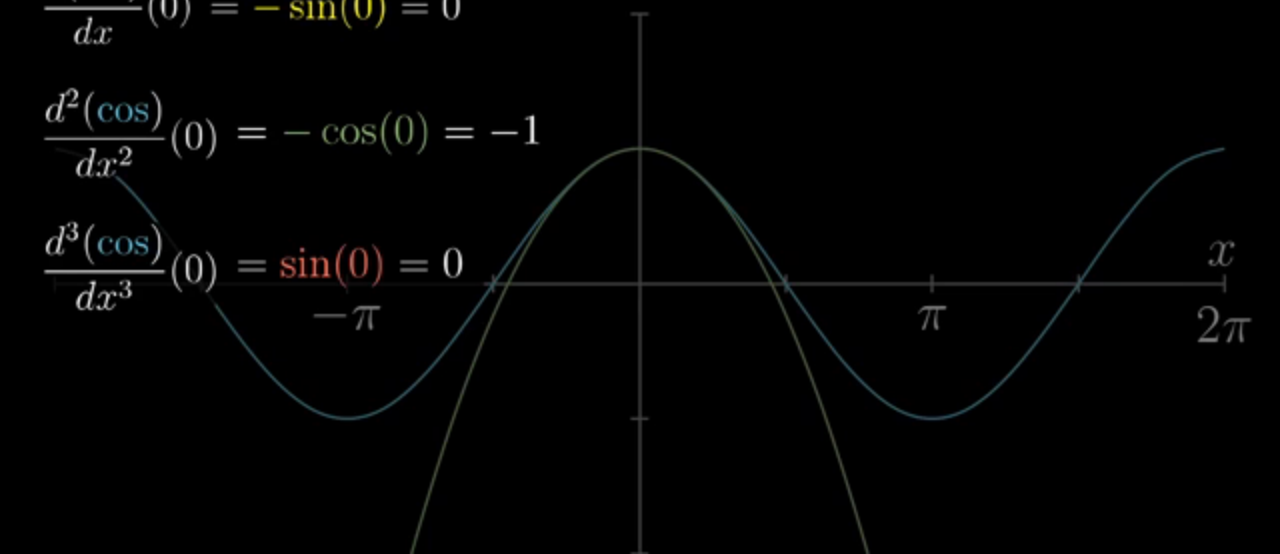
$$P(x) = 1 - \frac{1}{2}x^2 + c_4 x^4$$



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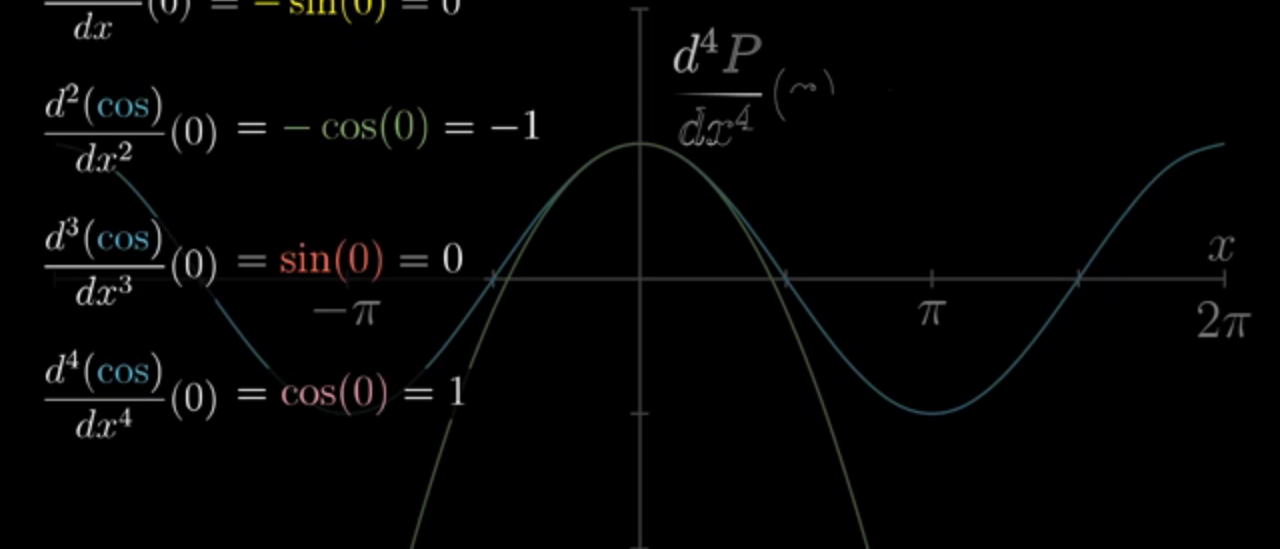
$$\frac{d^4(\cos)}{dx^4}(0) = \cos(0) = 1$$

$$P(x) = 1 - \frac{1}{2}x^2 + c_4 x^4$$

$$\frac{d^4 P}{dx^4}(0)$$



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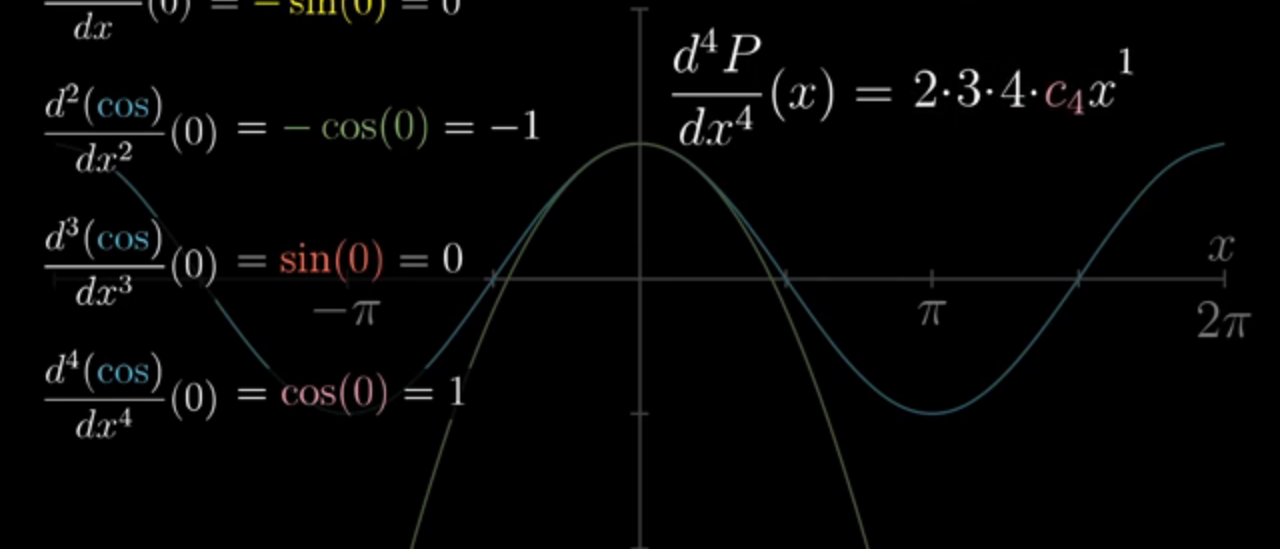
$$P(x) = 1 - \frac{1}{2}x^2 + c_4 x^4$$

$$\frac{d^4 P}{dx^4}(x) = 2 \cdot 3 \cdot 4 \cdot c_4 x^1$$



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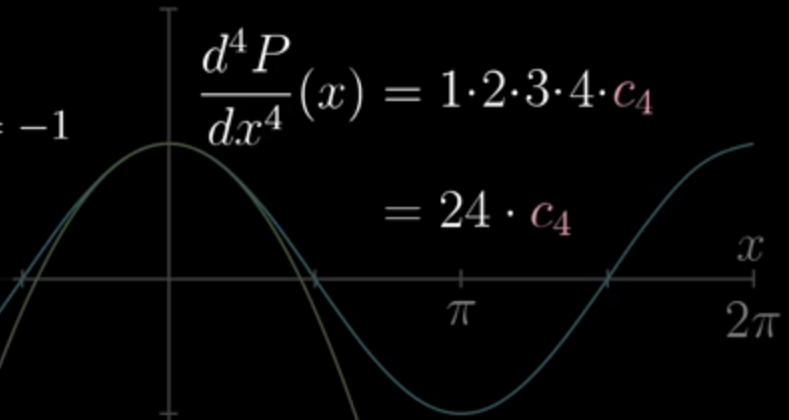
$$P(x) = 1 - \frac{1}{2}x^2 + c_4 x^4$$

$$\frac{d^4 P}{dx^4}(x) = 1 \cdot 2 \cdot 3 \cdot 4 \cdot c_4$$

$$= 24 \cdot c_4$$



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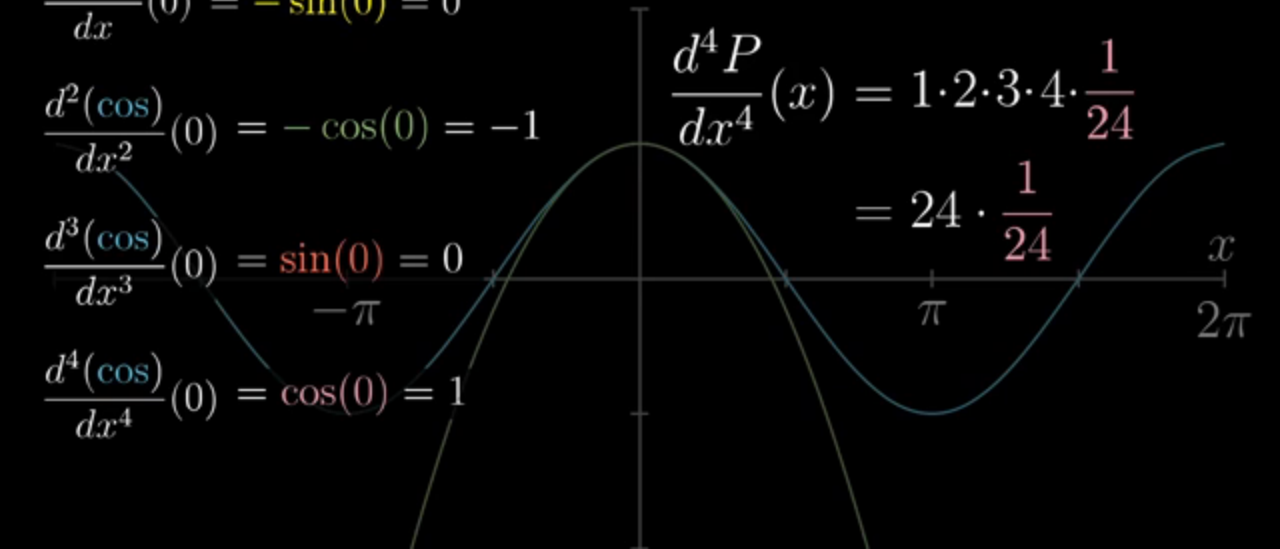
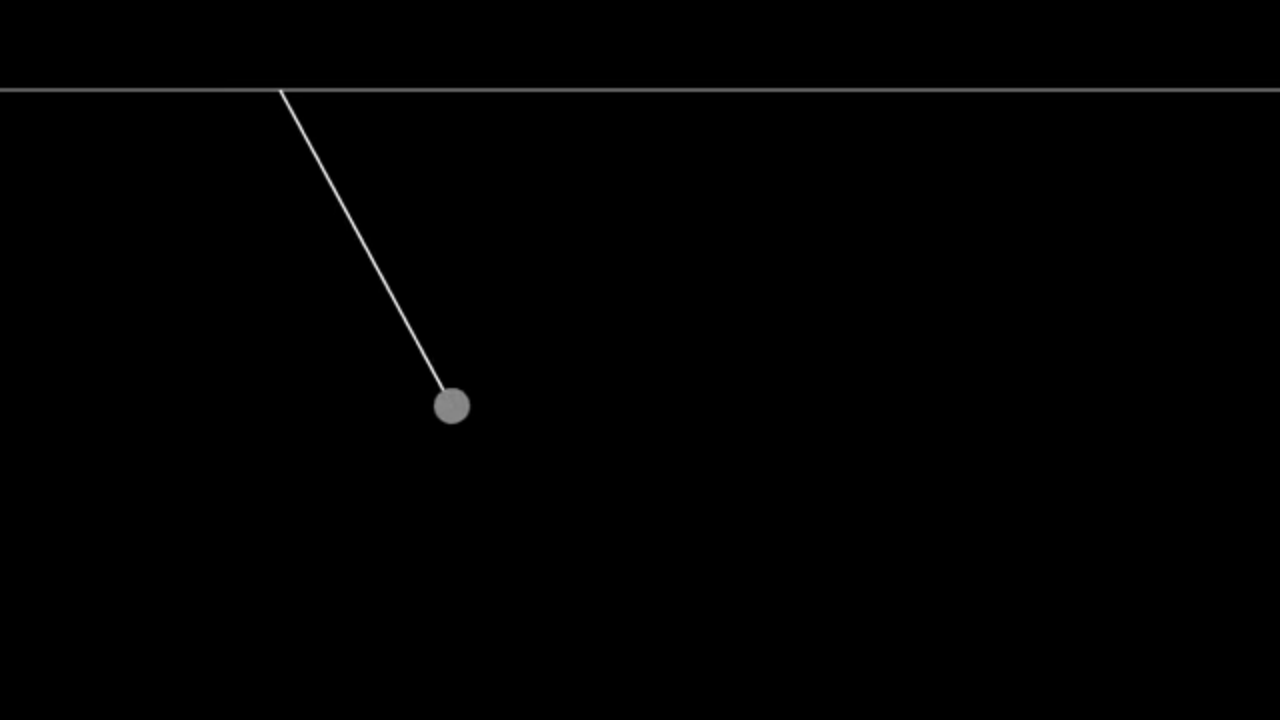
$$\frac{d^4(\cos)}{dx^4}(0) = \cos(0) = 1$$

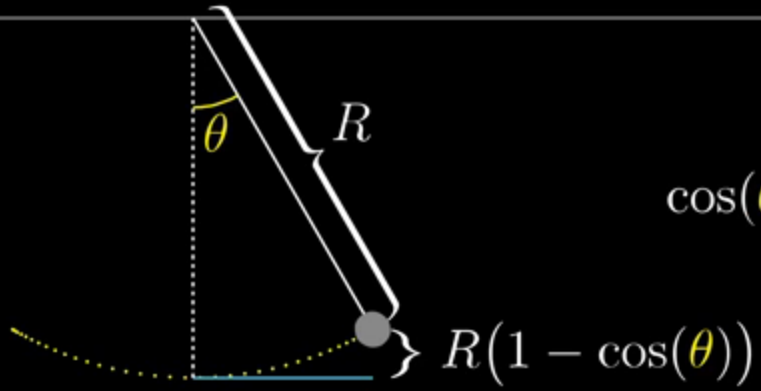
$$P(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$$

$$\frac{d^4P}{dx^4}(x) = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \frac{1}{24}$$

$$= 24 \cdot \frac{1}{24}$$







$$\cos(\theta) \approx 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24}$$

$$\frac{d^2}{dx^2}(c_8 x^8) = 7 \cdot 8 \cdot c_8 x^6$$

$$\frac{d^7}{dx^7}(c_8 x^8) = 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot c_8 x^1$$

$$\text{Set } c_8 = \frac{\text{Desired derivative value}}{8!}$$

$$\frac{d^8}{dx^8}(c_8 x^8) = \underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}_{8!} \cdot c_8$$

$$\cos(0) = 1$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

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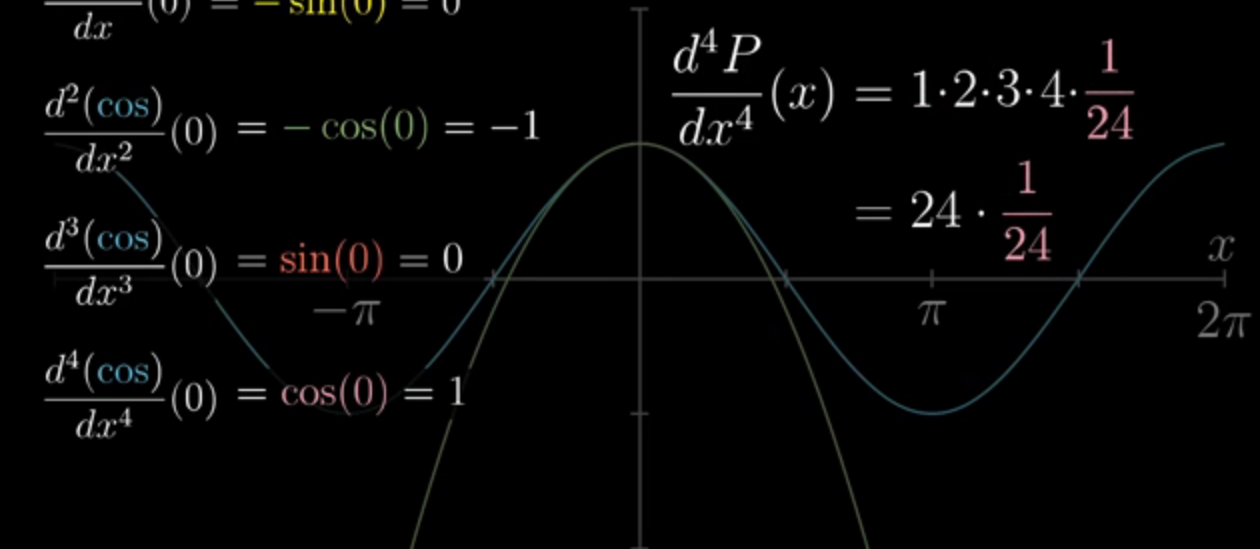
$$P(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$$

$$\frac{d^4P}{dx^4}(x) = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \frac{1}{24}$$

$$= 24 \cdot \frac{1}{24}$$



Doesn't affect previous terms

$$P(x) = 1 + \left(-\frac{1}{2}\right)x^2 + c_4 x^4$$

Doesn't affect previous terms

$$P(x) = 1 + \left(-\frac{1}{2}\right)x^2 + c_4 x^4$$

$$\frac{d^2P}{dx^2}(0) = 2\left(-\frac{1}{2}\right) + 3 \cdot 4c_4(0)^2$$

Doesn't affect previous terms

$$P(x) = 1 + \left(-\frac{1}{2}\right)x^2 + c_4 x^4$$

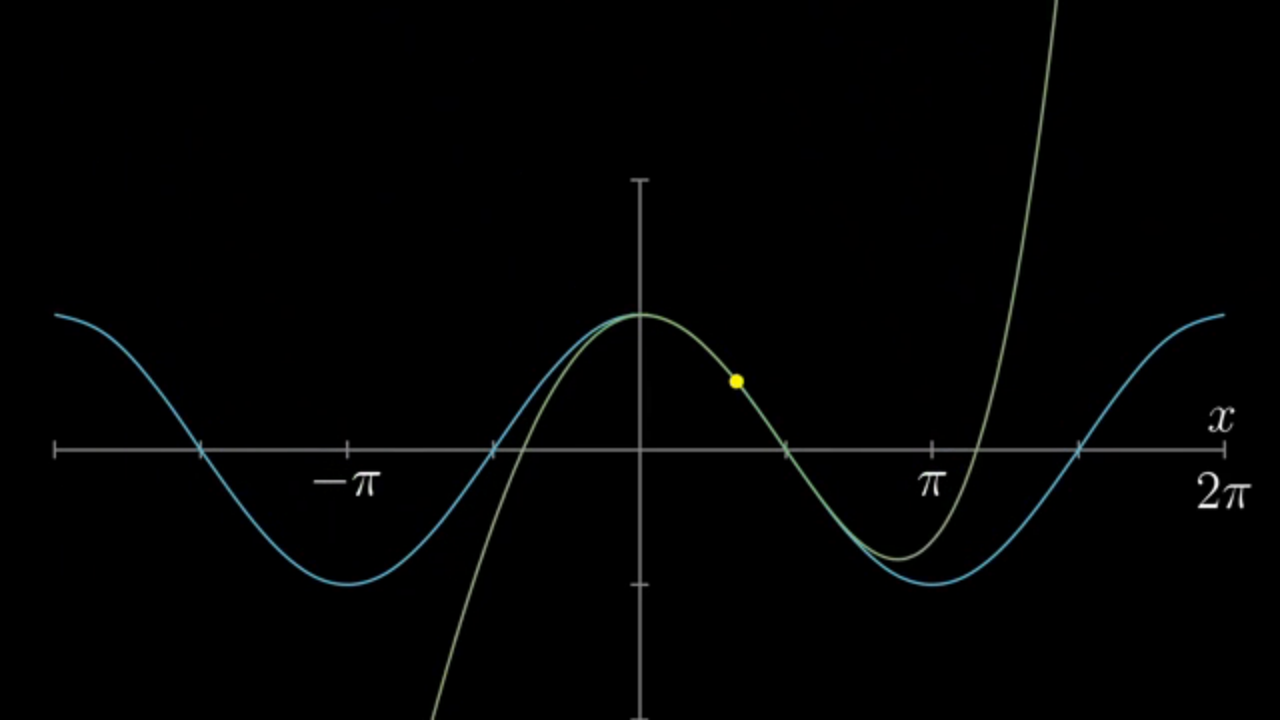
$$\frac{d^2P}{dx^2}(0) = 2\left(-\frac{1}{2}\right) + \underbrace{3 \cdot 4c_4(0)^2}_{=0}$$

Controls $P(0)$

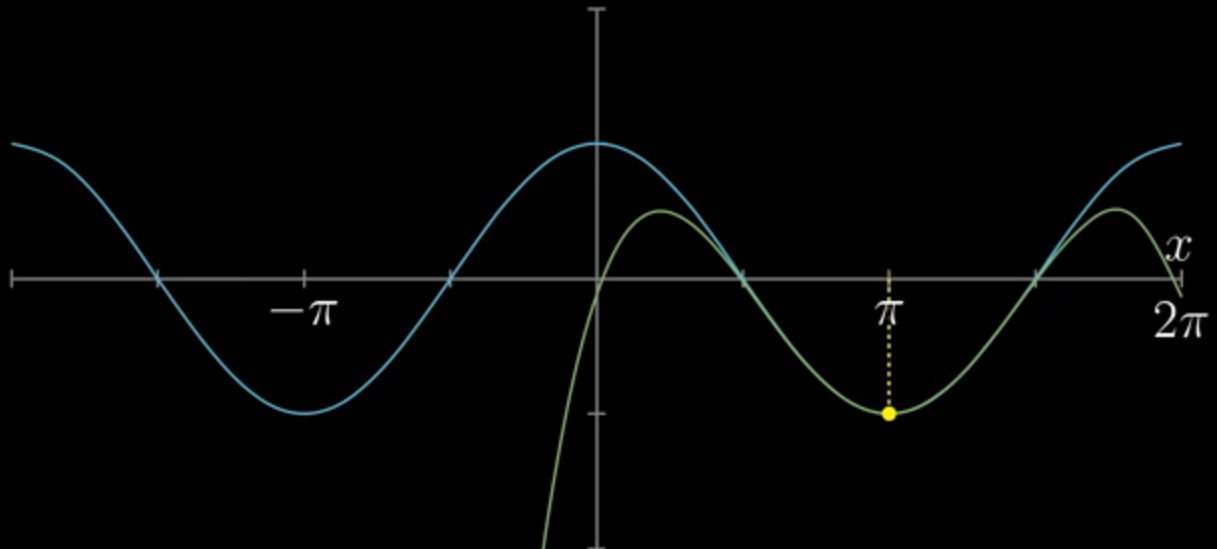
Controls $\frac{dP}{dx}(0)$

Controls $\frac{d^2P}{dx^2}(0)$

$$P(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4$$

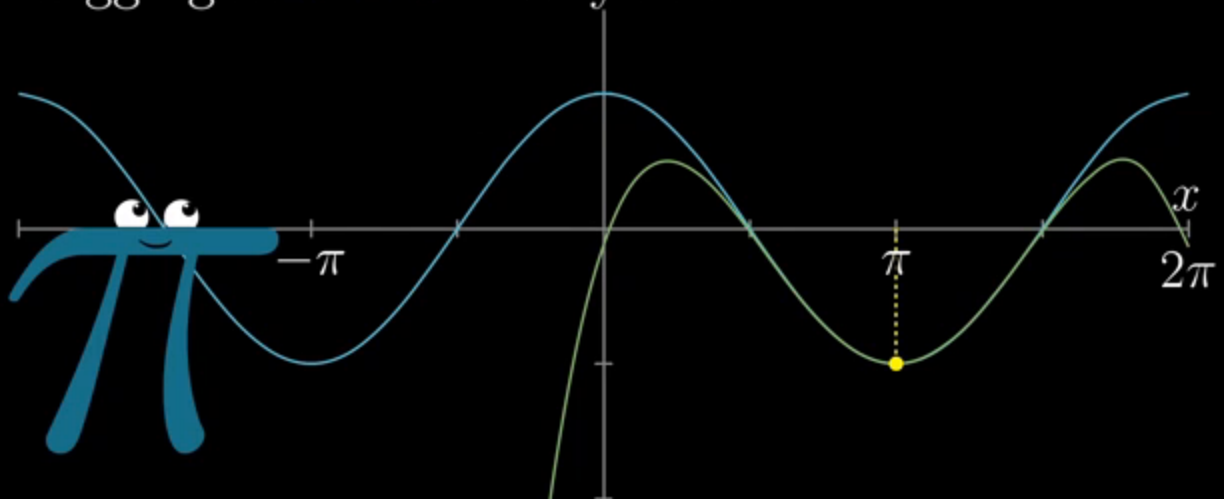

x
$-\pi$
π
2π

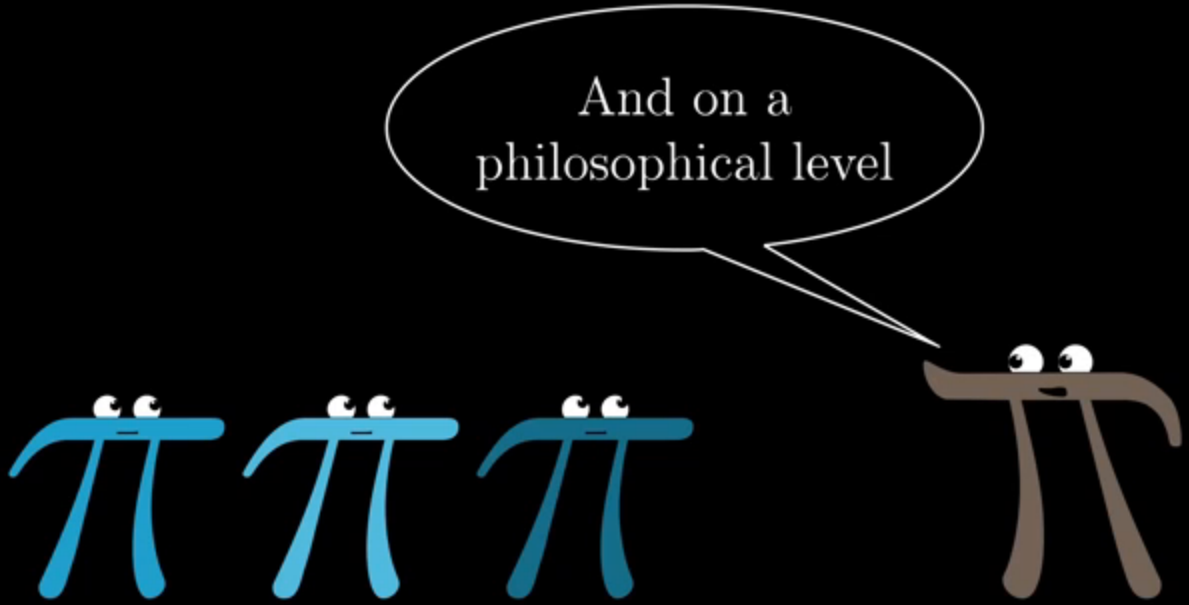
$$P_\pi(x) = c_0 + c_1(x-\pi)^1 + c_2(x-\pi)^2 + c_3(x-\pi)^3 + c_4(x-\pi)^4$$

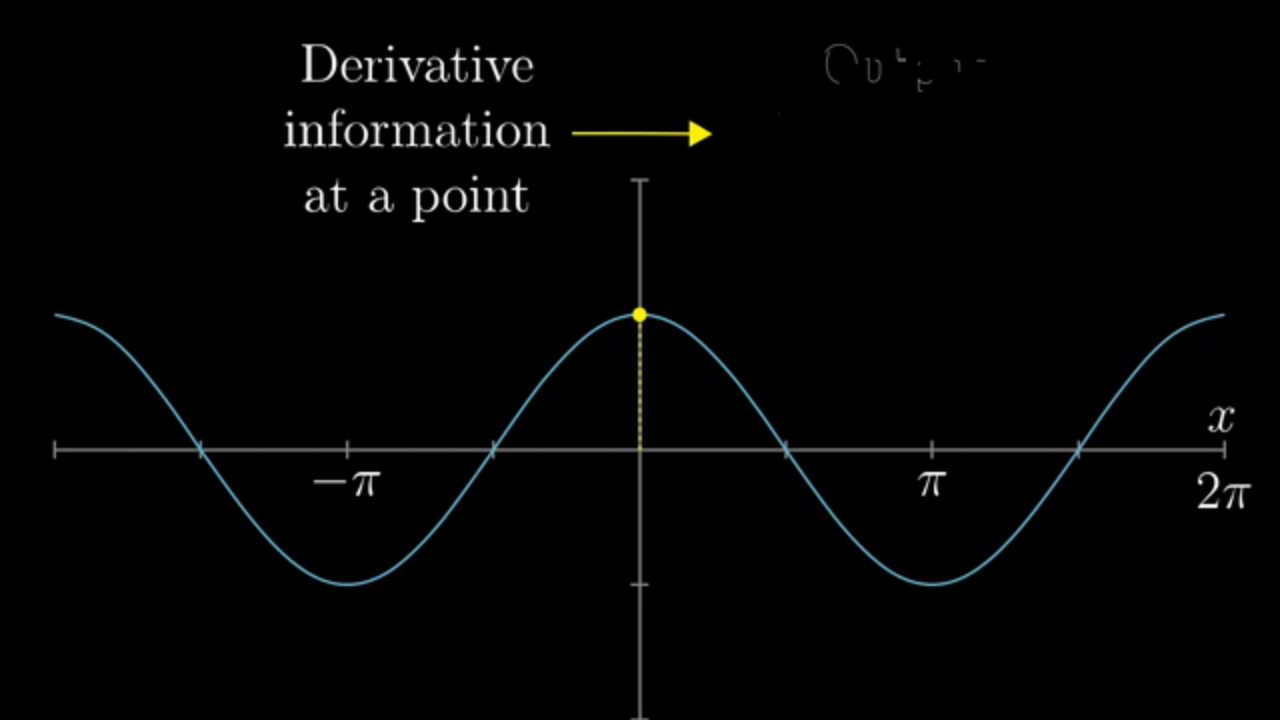


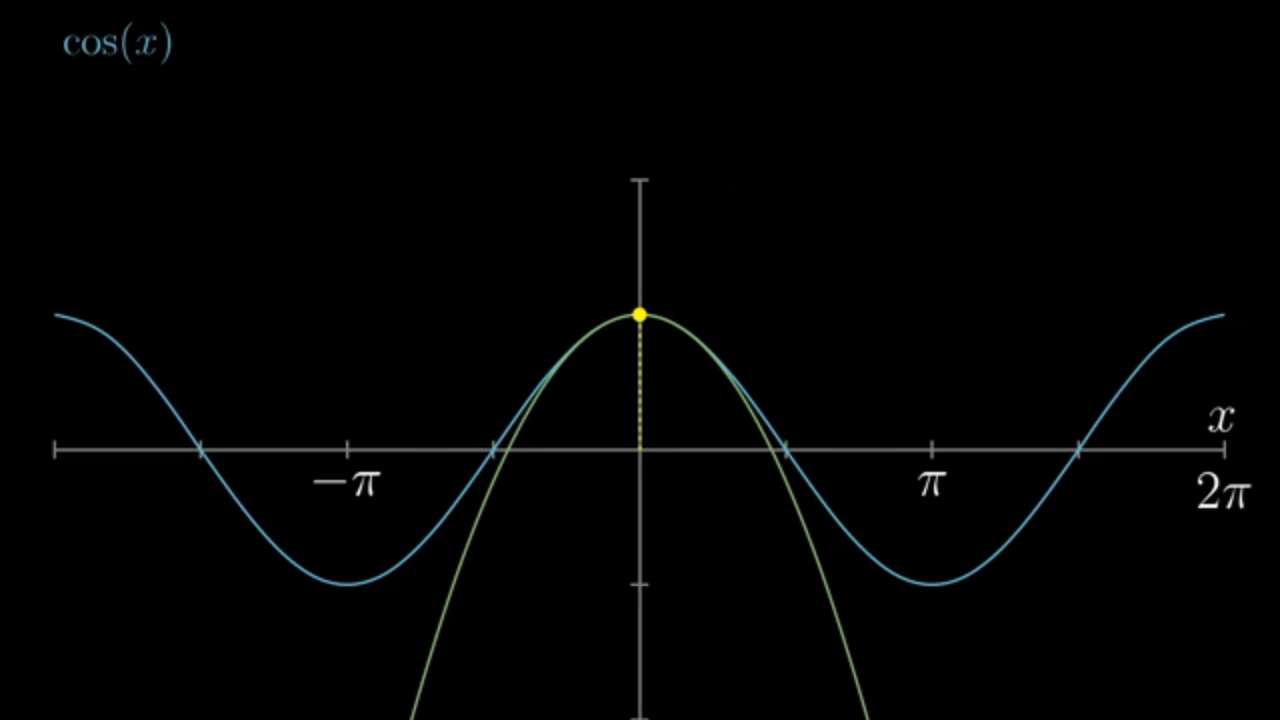
$$P_\pi(x) = c_0 + c_1(x-\pi)^1 + c_2(x-\pi)^2 + c_3(x-\pi)^3 + c_4(x-\pi)^4$$

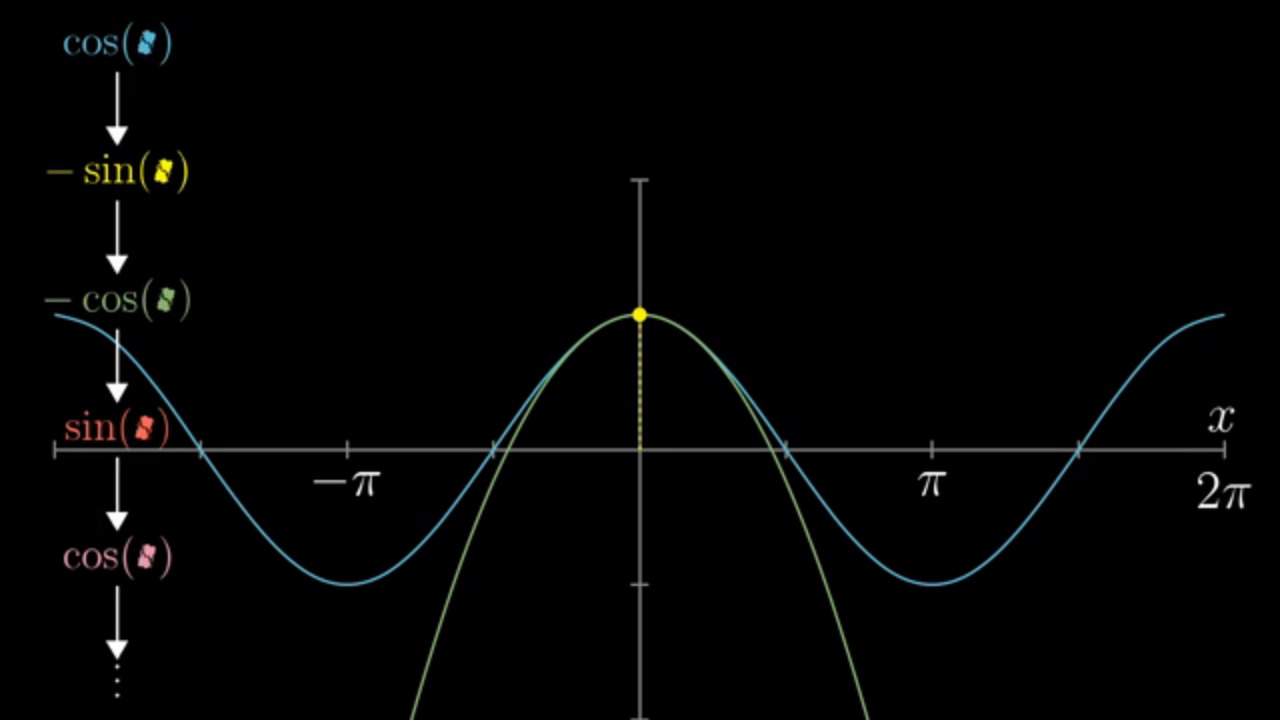
Plugging in $x = \pi$ is very nice.

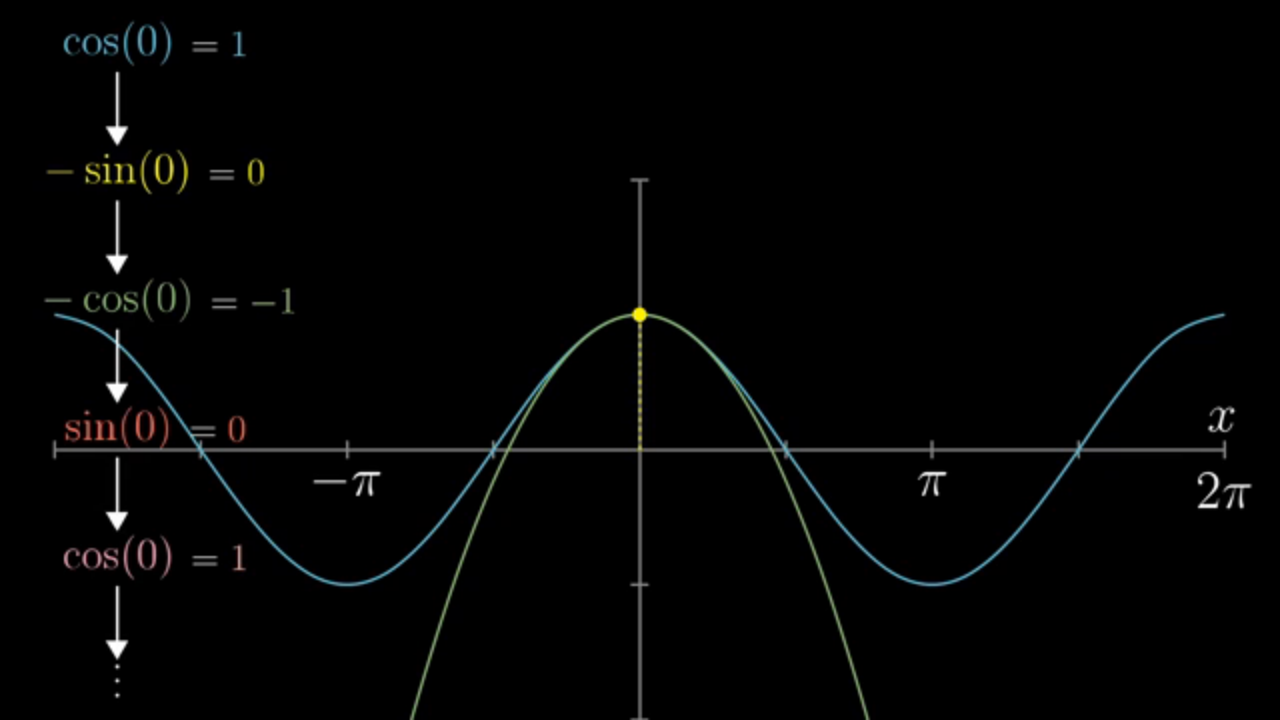


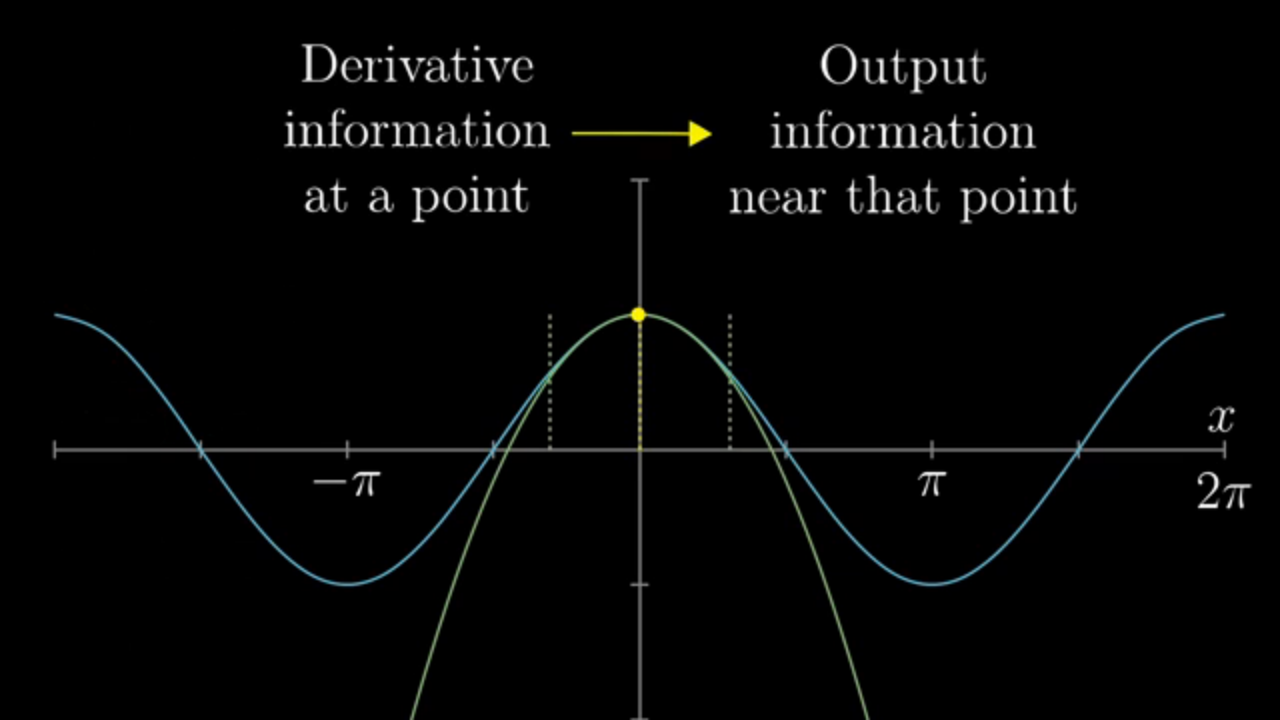

And on a philosophical level

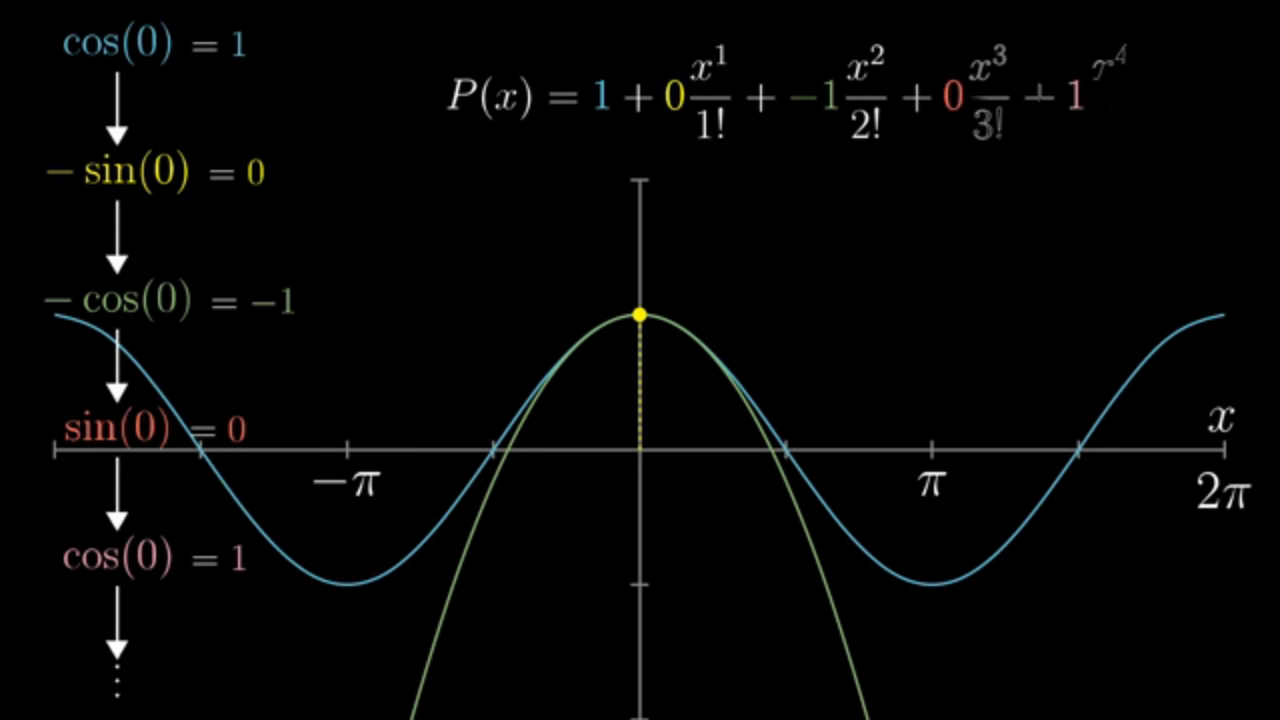

Derivative
information
at a point
x
$-\pi$
π
2π


$\cos(x)$
x
$-\pi$
π
2π


cos(x)
− sin(x)
− cos(x)
sin(x)
cos(x)
x
−π
π
2π


$\cos(0) = 1$
$-\sin(0) = 0$
$-\cos(0) = -1$
$\sin(0) = 0$
$\cos(0) = 1$
x
$-\pi$
π
2π


Derivative information at a point
Output information near that point
x
$-\pi$
π
2π


$\cos(0) = 1$
$-\sin(0) = 0$
$-\cos(0) = -1$
$\sin(0) = 0$
$\cos(0) = 1$
$P(x) = 1 + 0\frac{x^1}{1!} + -1\frac{x^2}{2!} + 0\frac{x^3}{3!} + 1\frac{x^4}{4!}$
x
$-\pi$
π
2π

$$\cos(0) = 1$$

$$-\sin(0) = 0$$

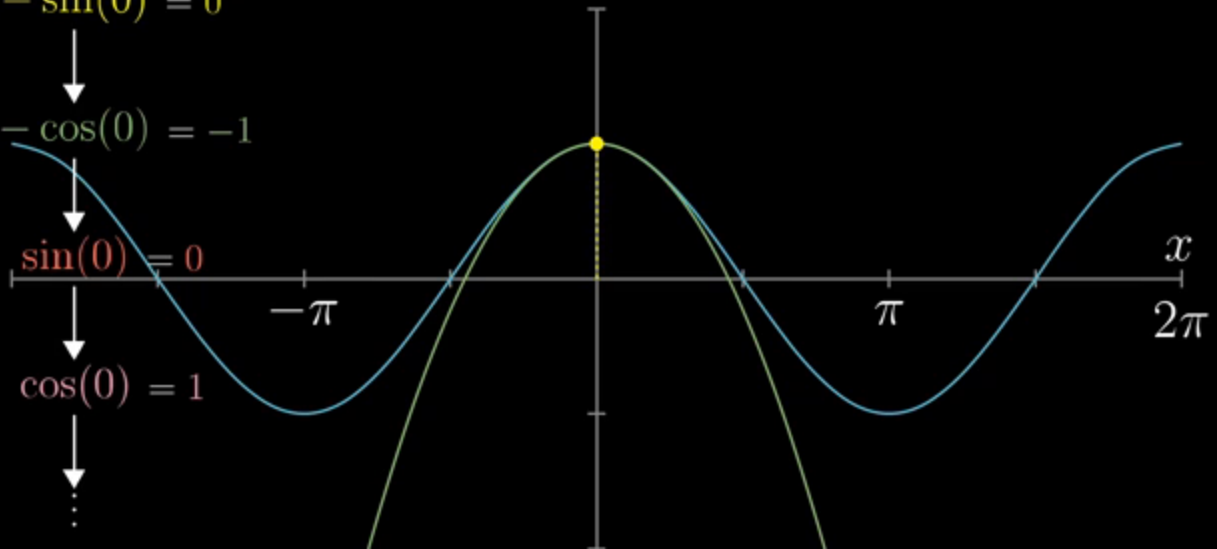
$$-\cos(0) = -1$$

$$\sin(0) = 0$$

$$\cos(0) = 1$$

$$\vdots$$

$$P(x) = 1 + 0\frac{x^1}{1!} + -1\frac{x^2}{2!} + 0\frac{x^3}{3!} + 1\frac{x^4}{4!} + \cdots$$



$$\cos(0) = 1$$

$$\downarrow$$

$$-\sin(0) = 0$$

$$\downarrow$$

$$-\cos(0) = -1$$

$$\downarrow$$

$$\sin(0) = 0$$

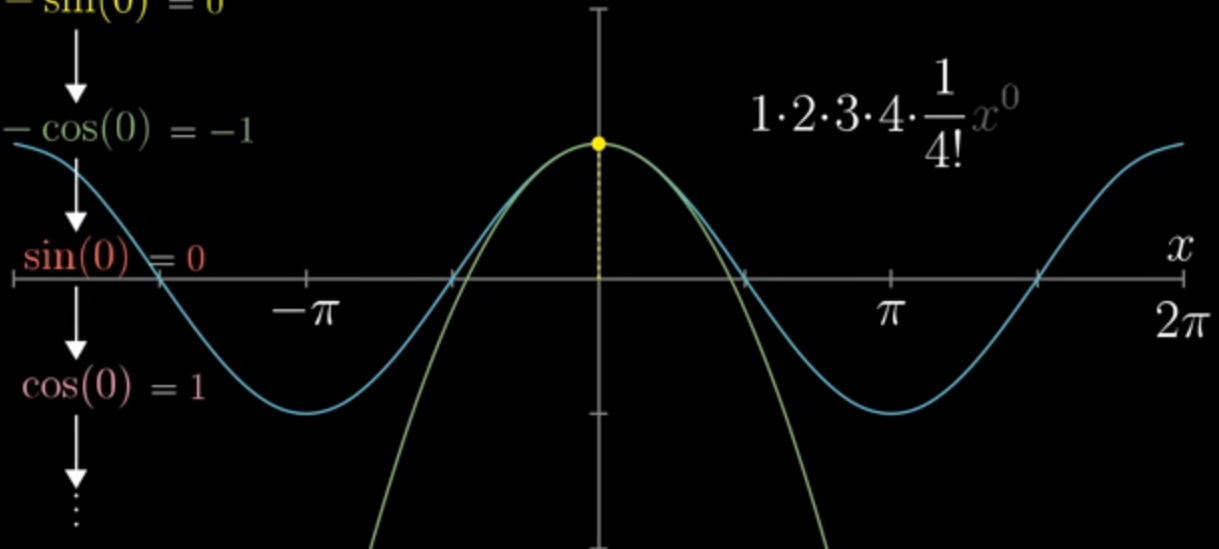
$$\downarrow$$

$$\cos(0) = 1$$

$$\downarrow$$

$$\vdots$$

$$P(x) = 1 + 0\frac{x^1}{1!} + -1\frac{x^2}{2!} + 0\frac{x^3}{3!} + 1\frac{x^4}{4!} + \cdots$$



$$\cos(0) = 1$$

$$-\sin(0) = 0$$

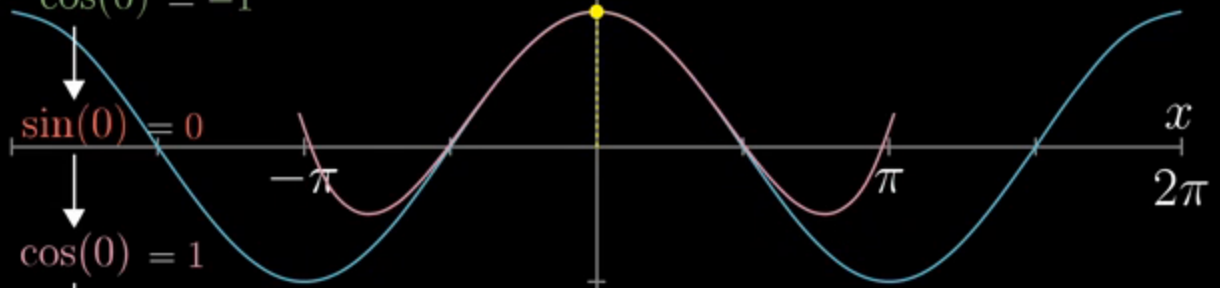
$$-\cos(0) = -1$$

$$\sin(0) = 0$$

$$\cos(0) = 1$$

$$\vdots$$

$$P(x) = \underbrace{1 + 0\frac{x^1}{1!} + -1\frac{x^2}{2!} + 0\frac{x^3}{3!} + 1\frac{x^4}{4!}}_{\text{“Taylor polynomial”}} + \cdots$$



$f(x)$

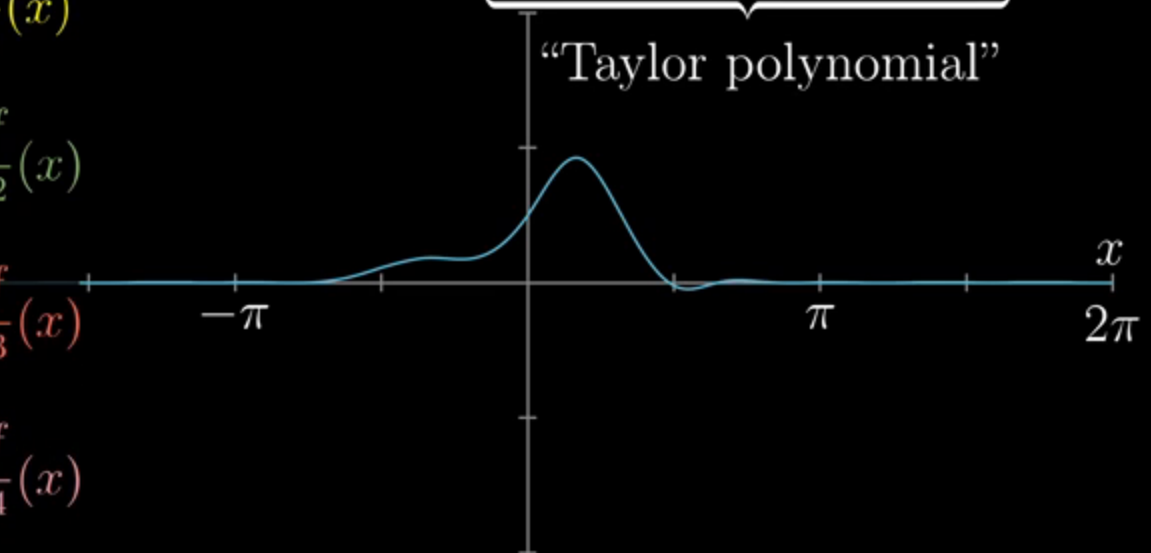
$\frac{df}{dx}(x)$

$\frac{d^2 f}{dx^2}(x)$

$\frac{d^3 f}{dx^3}(x)$

$\frac{d^4 f}{dx^4}(x)$

$$P(x) = \underbrace{1 + 0\frac{x^1}{1!} + -1\frac{x^2}{2!} + 0\frac{x^3}{3!} + 1\frac{x^4}{4!}}_{\text{“Taylor polynomial”}} + \cdots$$



$$P(x) = \underbrace{1 + 0\frac{x^1}{1!} + -1\frac{x^2}{2!} + 0\frac{x^3}{3!} + 1\frac{x^4}{4!}}_{\text{“Taylor polynomial”}} + \cdots$$

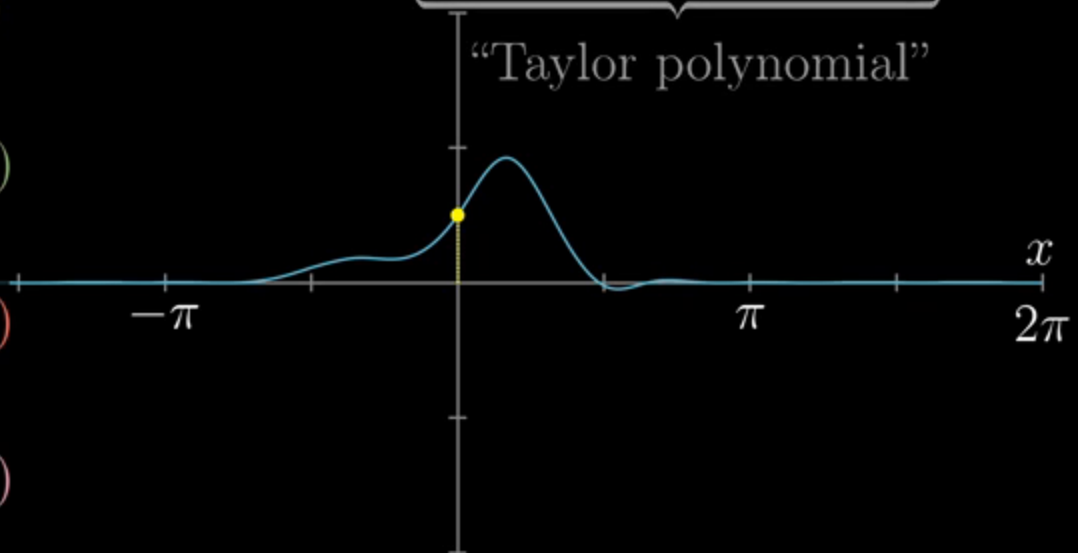
$f(0)$

$\frac{df}{dx}(0)$

$\frac{d^2 f}{dx^2}(0)$

$\frac{d^3 f}{dx^3}(0)$

$\frac{d^4 f}{dx^4}(0)$



$$f(0)$$

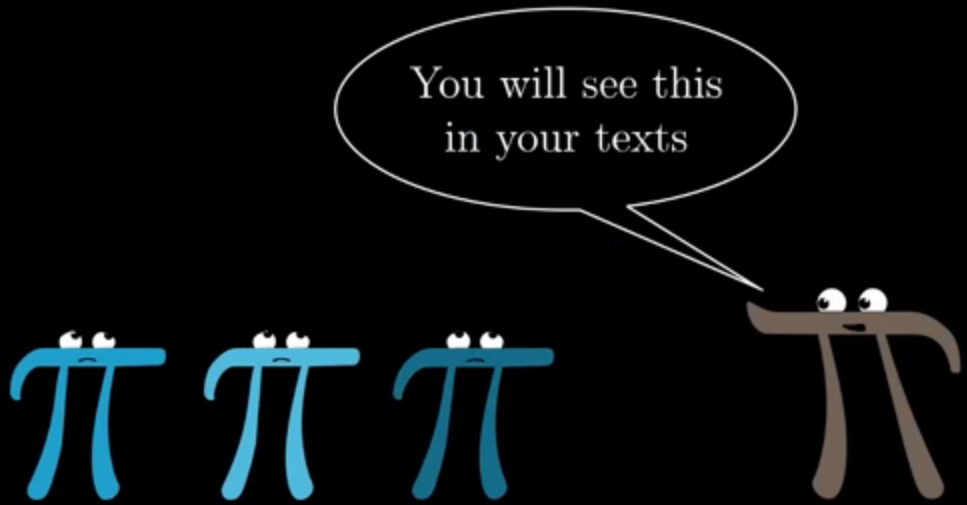
$$\frac{df}{dx}(0)$$

$$\frac{d^2 f}{dx^2}(0)$$

$$\frac{d^3 f}{dx^3}(0)$$

$$\frac{d^4 f}{dx^4}(0)$$

$$P(x) = f(0) + \frac{df}{dx}(0)\frac{x^1}{1!} + \frac{d^2 f}{dx^2}(0)\frac{x^2}{2!} + \frac{d^3 f}{dx^3}(0)\frac{x^3}{3!} + \cdots$$



$$P(x) = \underbrace{f(0)} + \frac{df}{dx}(0)\frac{x^1}{1!} + \frac{d^2 f}{dx^2}(0)\frac{x^2}{2!} + \frac{d^3 f}{dx^3}(0)\frac{x^3}{3!} + \cdots$$

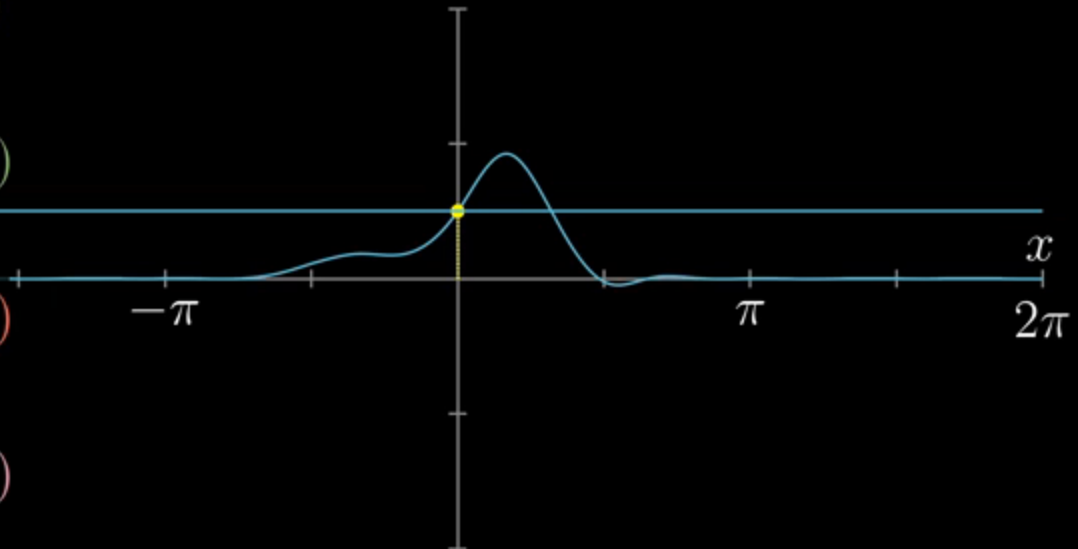
$f(0)$

$\frac{df}{dx}(0)$

$\frac{d^2 f}{dx^2}(0)$

$\frac{d^3 f}{dx^3}(0)$

$\frac{d^4 f}{dx^4}(0)$



$$P(x) = \underbrace{f(0) + \frac{df}{dx}(0)\frac{x^1}{1!}} + \frac{d^2 f}{dx^2}(0)\frac{x^2}{2!} + \frac{d^3 f}{dx^3}(0)\frac{x^3}{3!} + \cdots$$

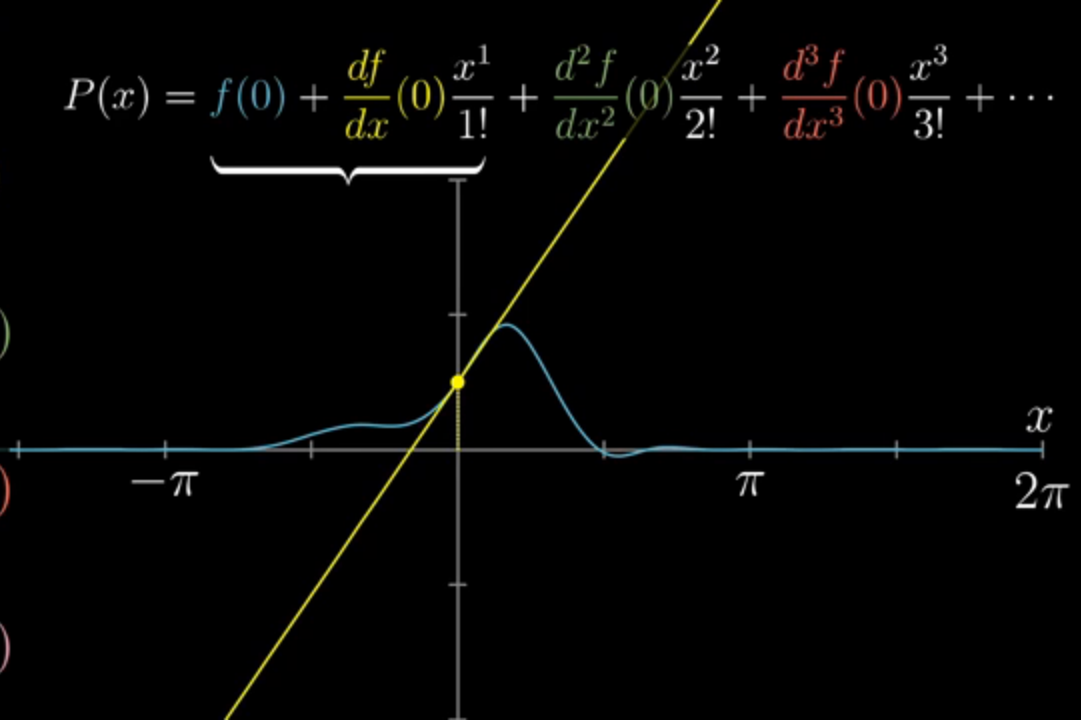
$f(0)$

$\frac{df}{dx}(0)$

$\frac{d^2 f}{dx^2}(0)$

$\frac{d^3 f}{dx^3}(0)$

$\frac{d^4 f}{dx^4}(0)$



$$P(x) = \underbrace{f(0) + \frac{df}{dx}(0)\frac{x^1}{1!} + \frac{d^2 f}{dx^2}(0)\frac{x^2}{2!} + \frac{d^3 f}{dx^3}(0)\frac{x^3}{3!} + \cdots}$$

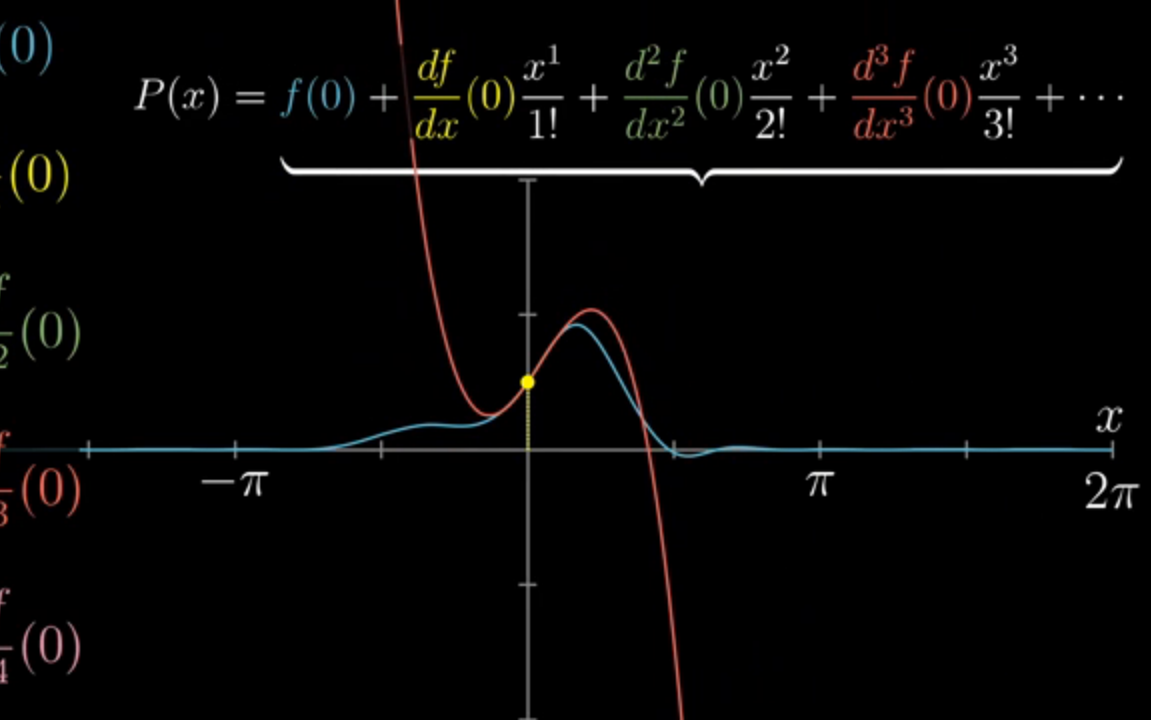
$f(0)$

$\frac{df}{dx}(0)$

$\frac{d^2 f}{dx^2}(0)$

$\frac{d^3 f}{dx^3}(0)$

$\frac{d^4 f}{dx^4}(0)$



$$P(x) = \underbrace{f(0) + \frac{df}{dx}(0)\frac{x^1}{1!} + \frac{d^2f}{dx^2}(0)\frac{x^2}{2!} + \frac{d^3f}{dx^3}(0)\frac{x^3}{3!} + \cdots}$$

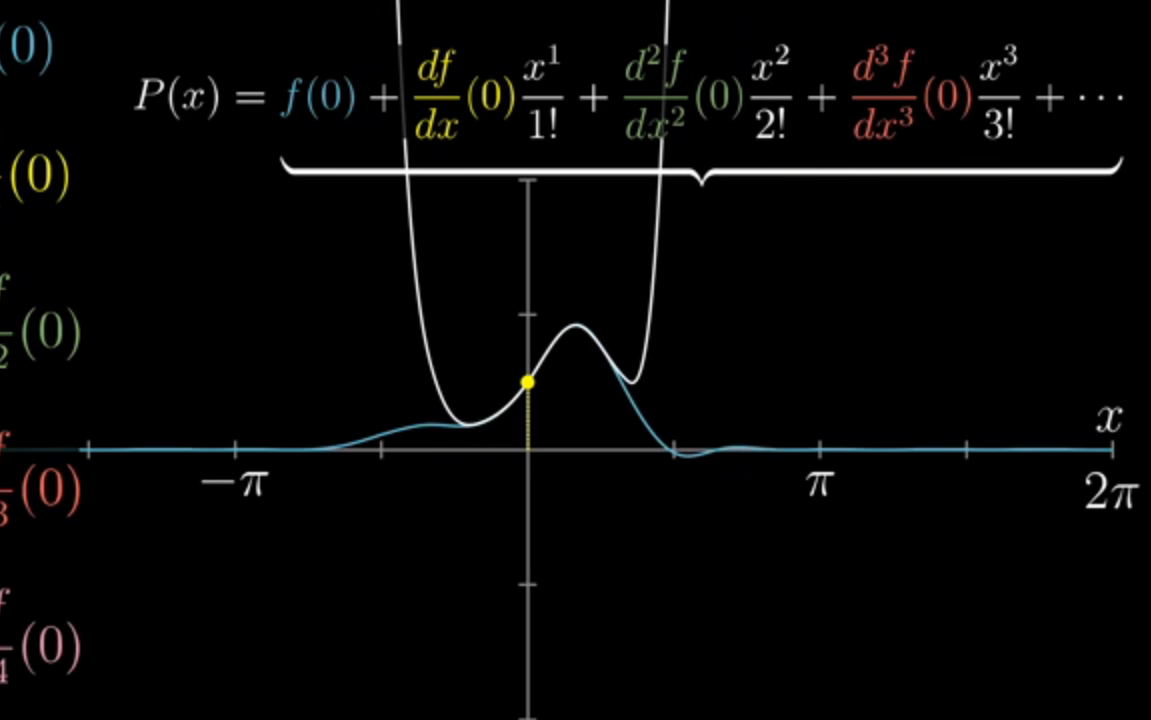
$f(0)$

$\frac{df}{dx}(0)$

$\frac{d^2f}{dx^2}(0)$

$\frac{d^3f}{dx^3}(0)$

$\frac{d^4f}{dx^4}(0)$



$$P(x) = f(a) + \frac{df}{dx}(a)\frac{(x-a)^1}{1!} + \frac{d^2f}{dx^2}(a)\frac{(x-a)^2}{2!} + \cdots$$

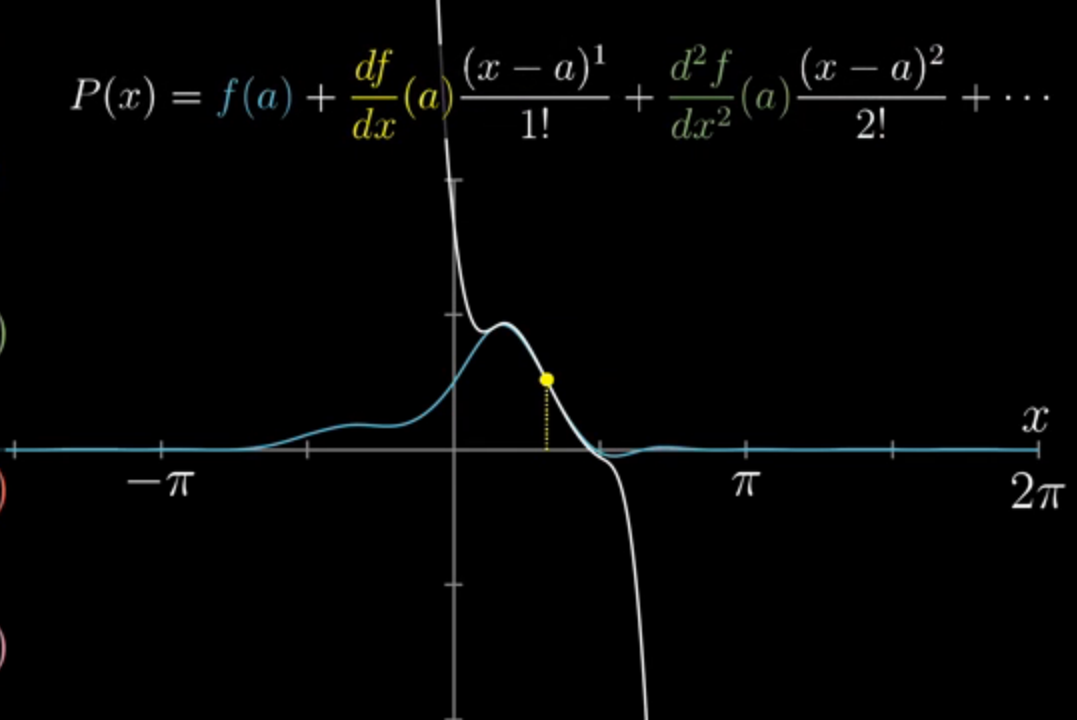
$f(a)$

$\frac{df}{dx}(a)$

$\frac{d^2f}{dx^2}(a)$

$\frac{d^3f}{dx^3}(a)$

$\frac{d^4f}{dx^4}(a)$



$$P(x) = f(a) + \frac{df}{dx}(a)\frac{(x-a)^1}{1!} + \frac{d^2f}{dx^2}(a)\frac{(x-a)^2}{2!} + \cdots$$

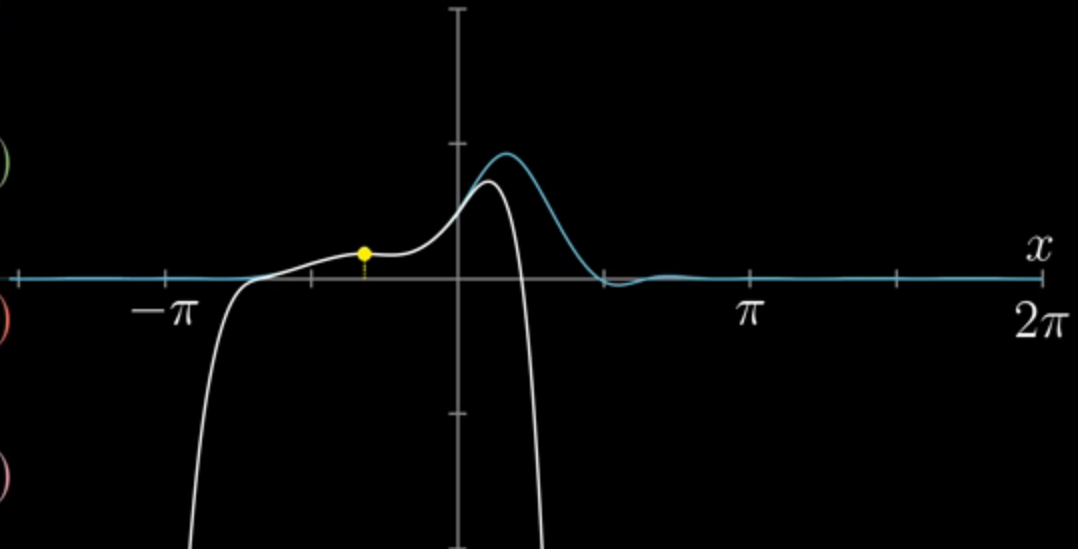
$f(a)$

$\frac{df}{dx}(a)$

$\frac{d^2f}{dx^2}(a)$

$\frac{d^3f}{dx^3}(a)$

$\frac{d^4f}{dx^4}(a)$



$$P(x) = f(a) + \frac{df}{dx}(a)\frac{(x-a)^1}{1!} + \frac{d^2f}{dx^2}(a)\frac{(x-a)^2}{2!} + \cdots$$

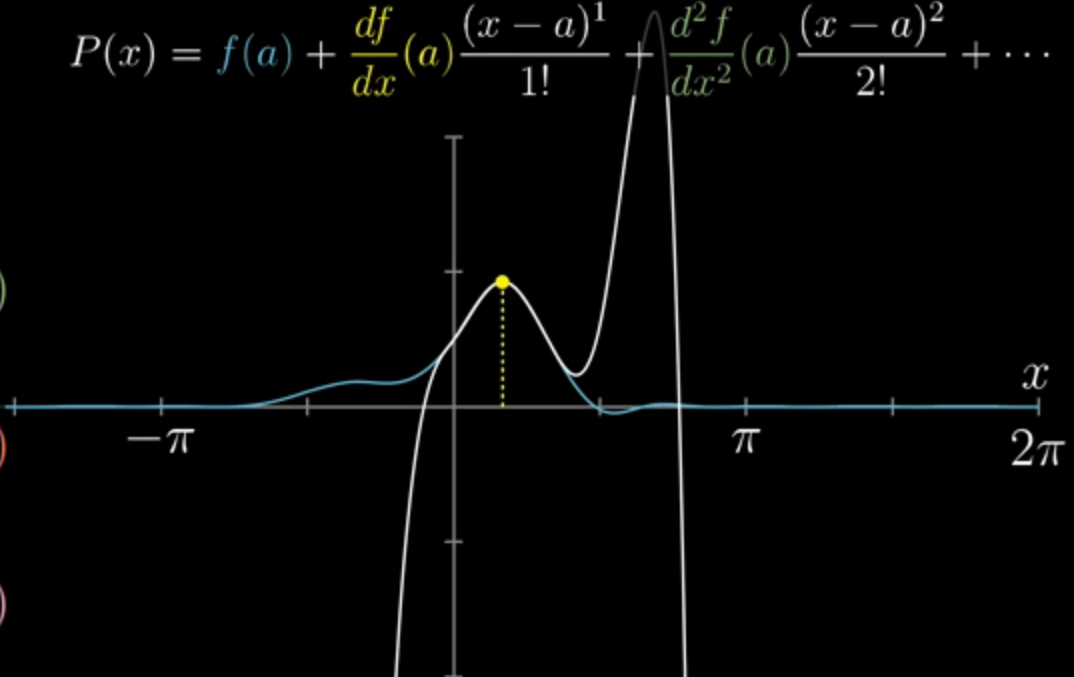
$f(a)$

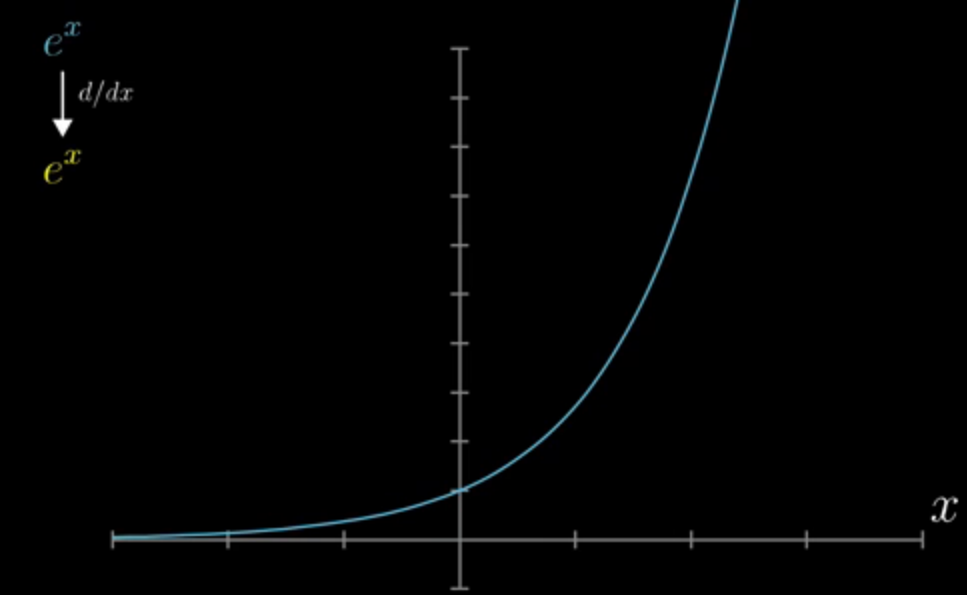
$\frac{df}{dx}(a)$

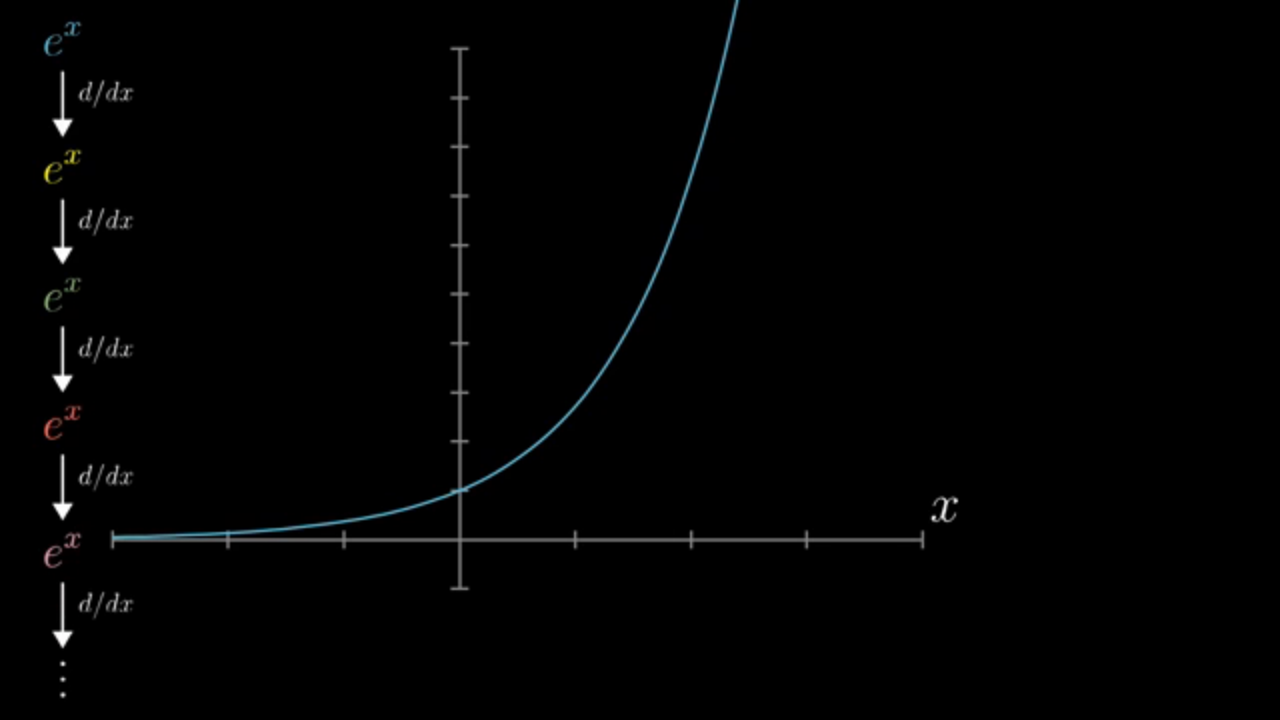
$\frac{d^2f}{dx^2}(a)$

$\frac{d^3f}{dx^3}(a)$

$\frac{d^4f}{dx^4}(a)$




e^x
d/dx
e^x
x


e^x
d/dx
e^x
d/dx
e^x
d/dx
e^x
d/dx
e^x
d/dx
x

$$e^0 = 1$$

$$\downarrow d/dx$$

$$e^0 = 1$$

$$\downarrow d/dx$$

$$e^0 = 1$$

$$\downarrow d/dx$$

$$e^0 = 1$$

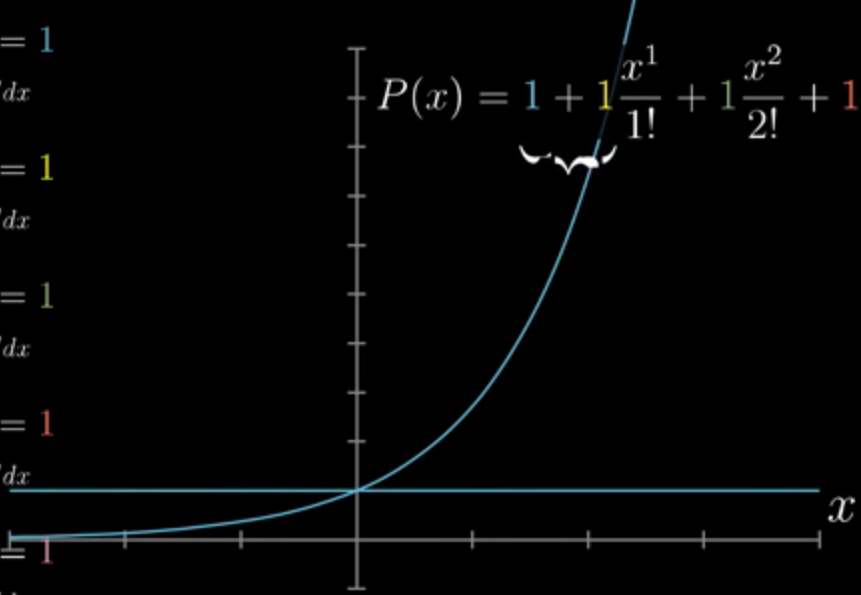
$$\downarrow d/dx$$

$$e^0 = 1$$

$$\downarrow d/dx$$

$$\vdots$$

$$P(x) = 1 + 1\frac{x^1}{1!} + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + 1\frac{x^4}{4!} + \cdots$$



$$e^0 = 1$$

$$\downarrow d/dx$$

$$e^0 = 1$$

$$\downarrow d/dx$$

$$e^0 = 1$$

$$\downarrow d/dx$$

$$e^0 = 1$$

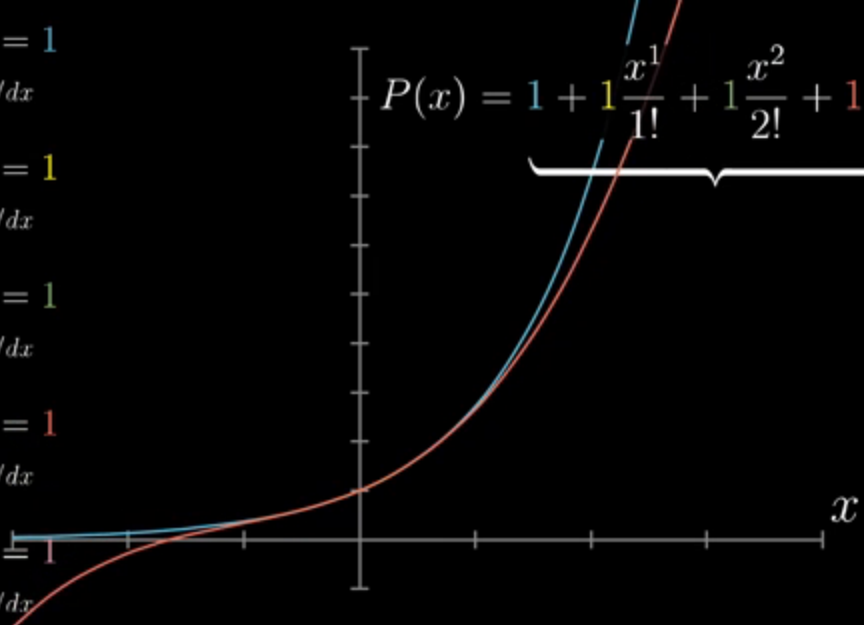
$$\downarrow d/dx$$

$$e^0 = 1$$

$$\downarrow d/dx$$

$$\vdots$$

$$P(x) = \underbrace{1 + 1\frac{x^1}{1!} + 1\frac{x^2}{2!} + 1\frac{x^3}{3!}} + 1\frac{x^4}{4!} + \cdots$$



$$e^0 = 1$$

$$\Big\downarrow d/dx$$

$$e^0 = 1$$

$$\Big\downarrow d/dx$$

$$e^0 = 1$$

$$\Big\downarrow d/dx$$

$$e^0 = 1$$

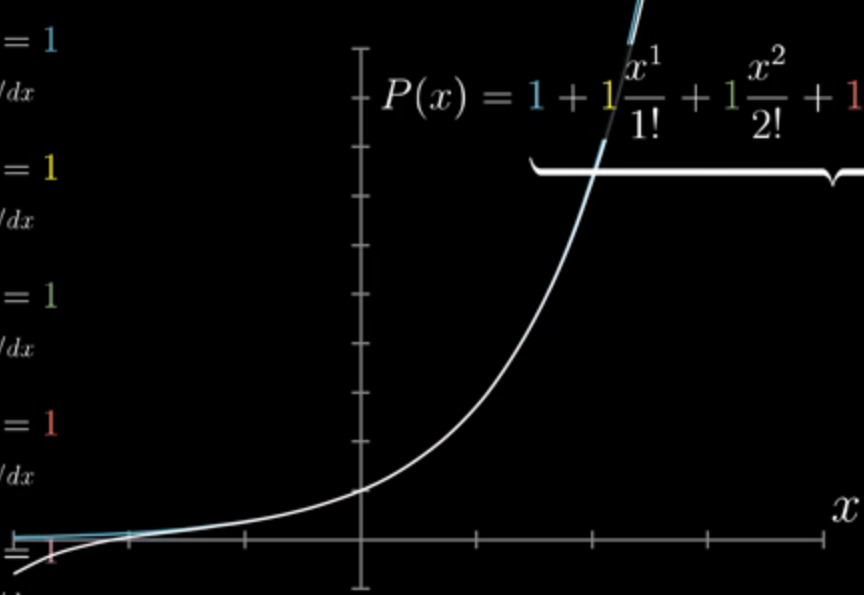
$$\Big\downarrow d/dx$$

$$e^0 = 1$$

$$\Big\downarrow d/dx$$

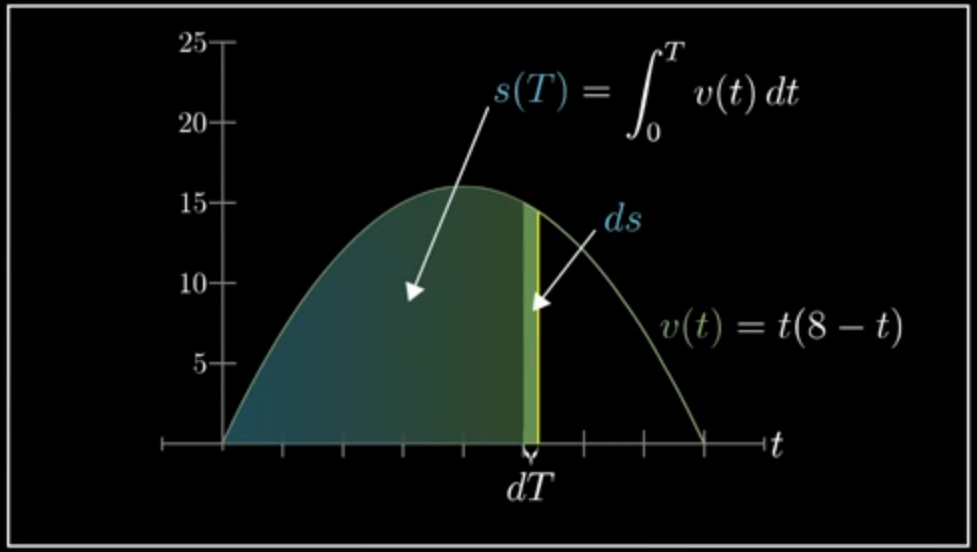
$$\vdots$$

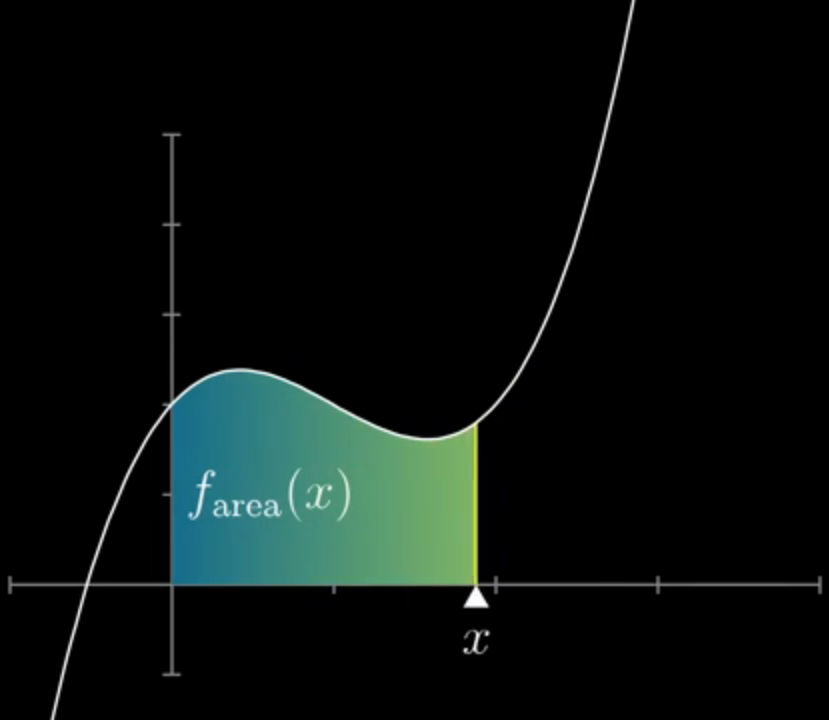
$$P(x) = \underbrace{1 + 1\frac{x^1}{1!} + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + 1\frac{x^4}{4!} + \cdots}$$

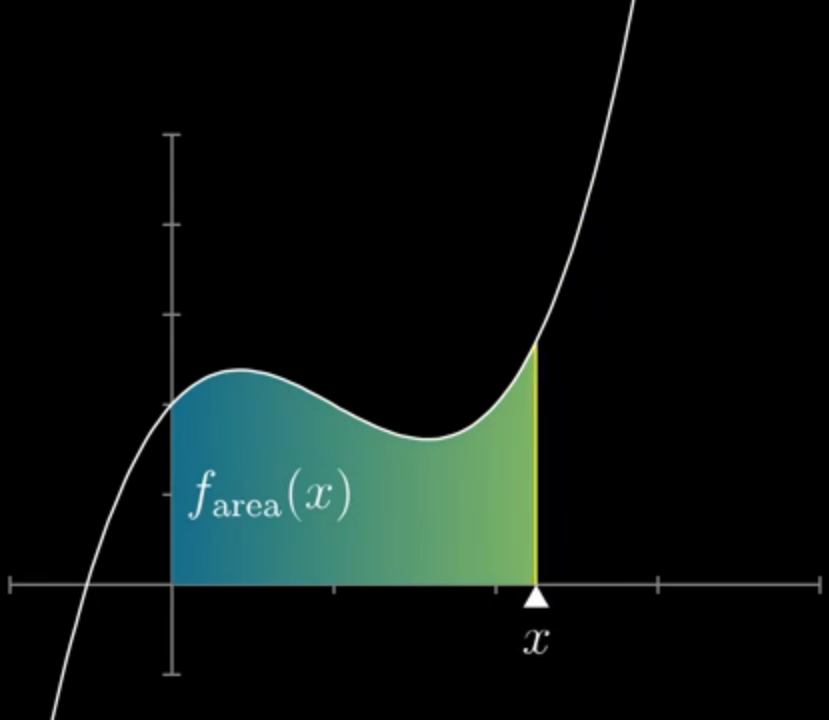


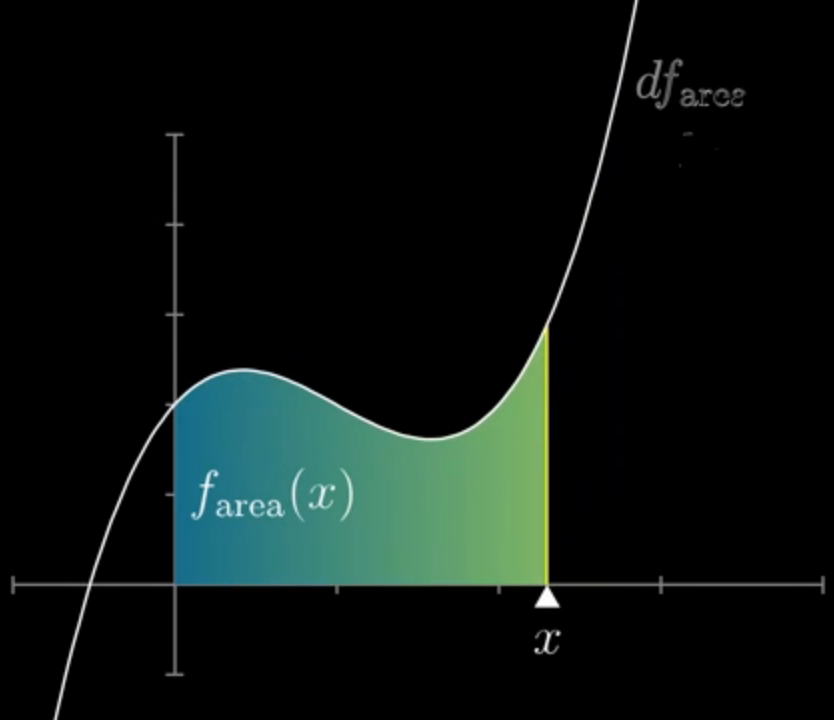
$$f(a) + \frac{df}{dx}(a)(x-a) + \frac{d^2 f}{dx^2}(a)(x-a)^2$$

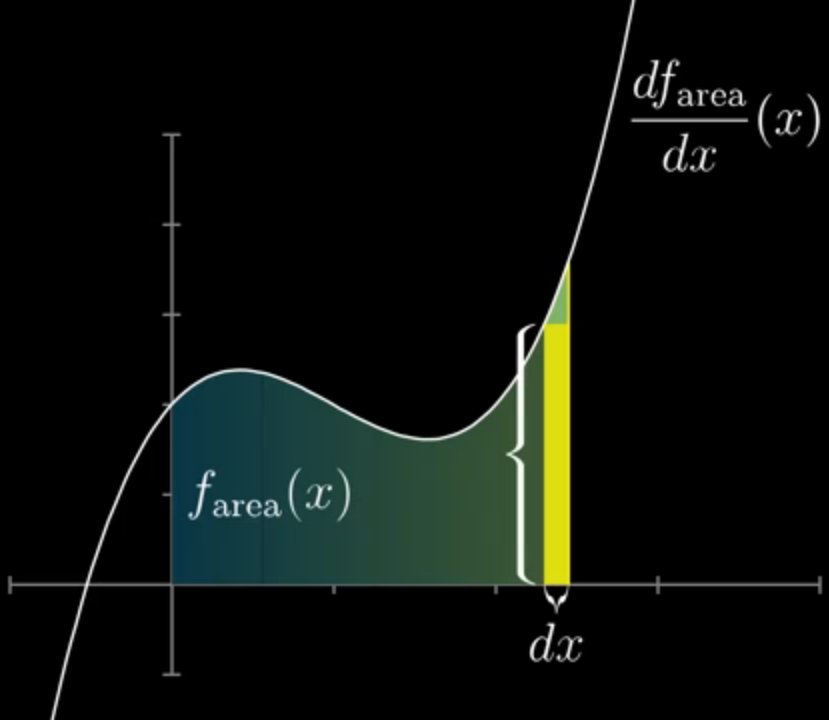
Chapter 8: Integrals

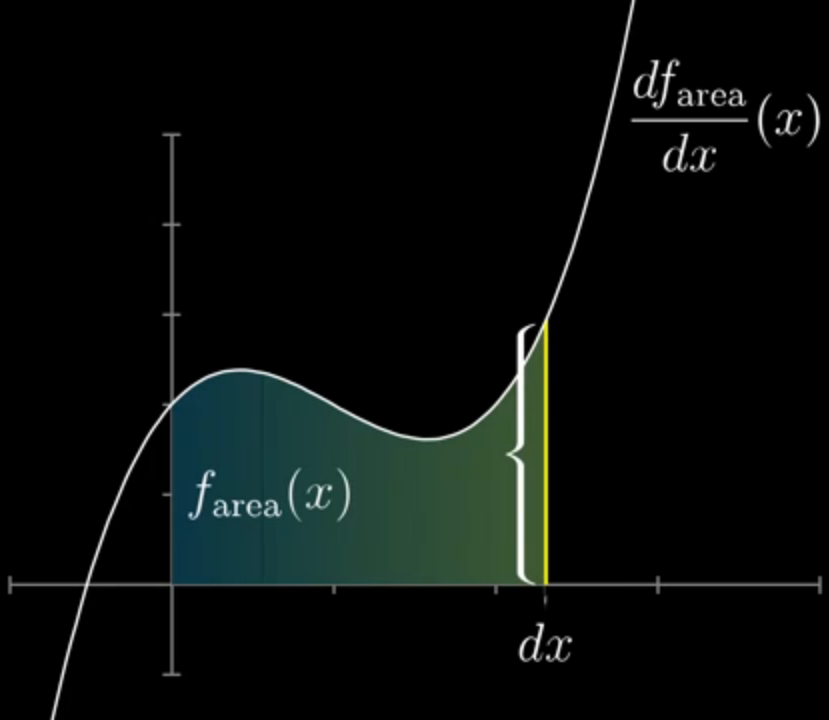


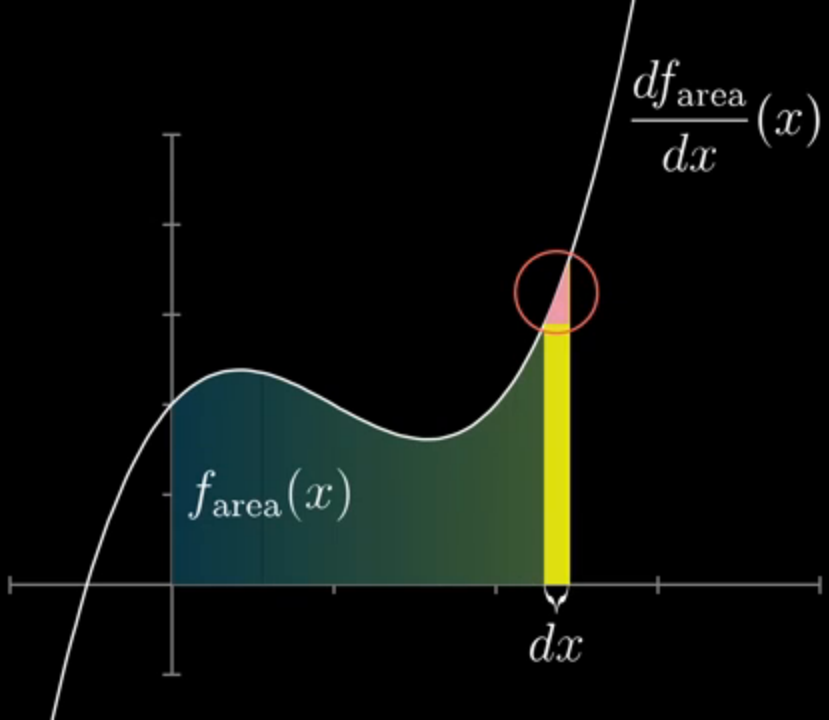

$f_{\text{area}}(x)$
x

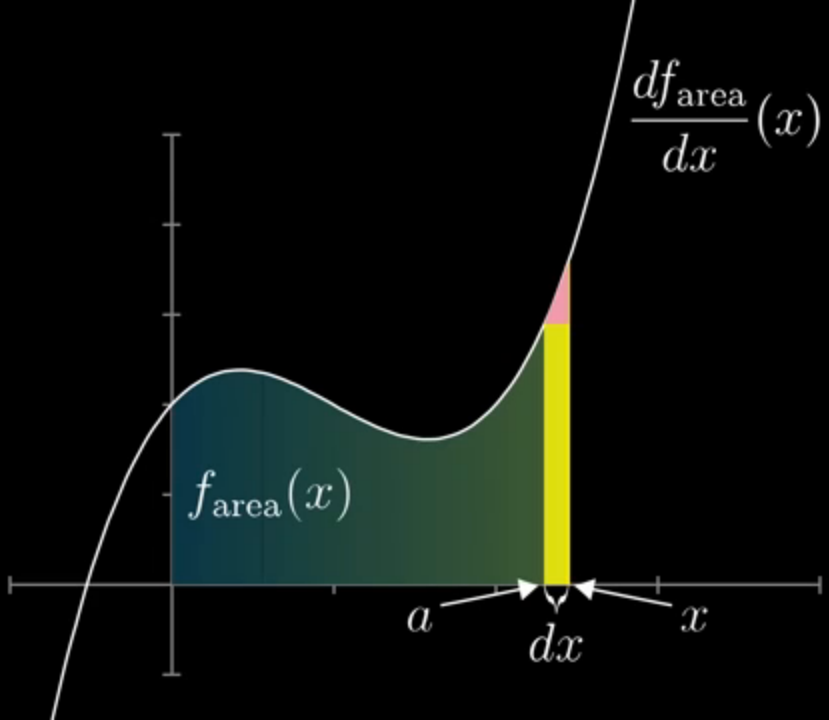

$f_{\text{area}}(x)$
x

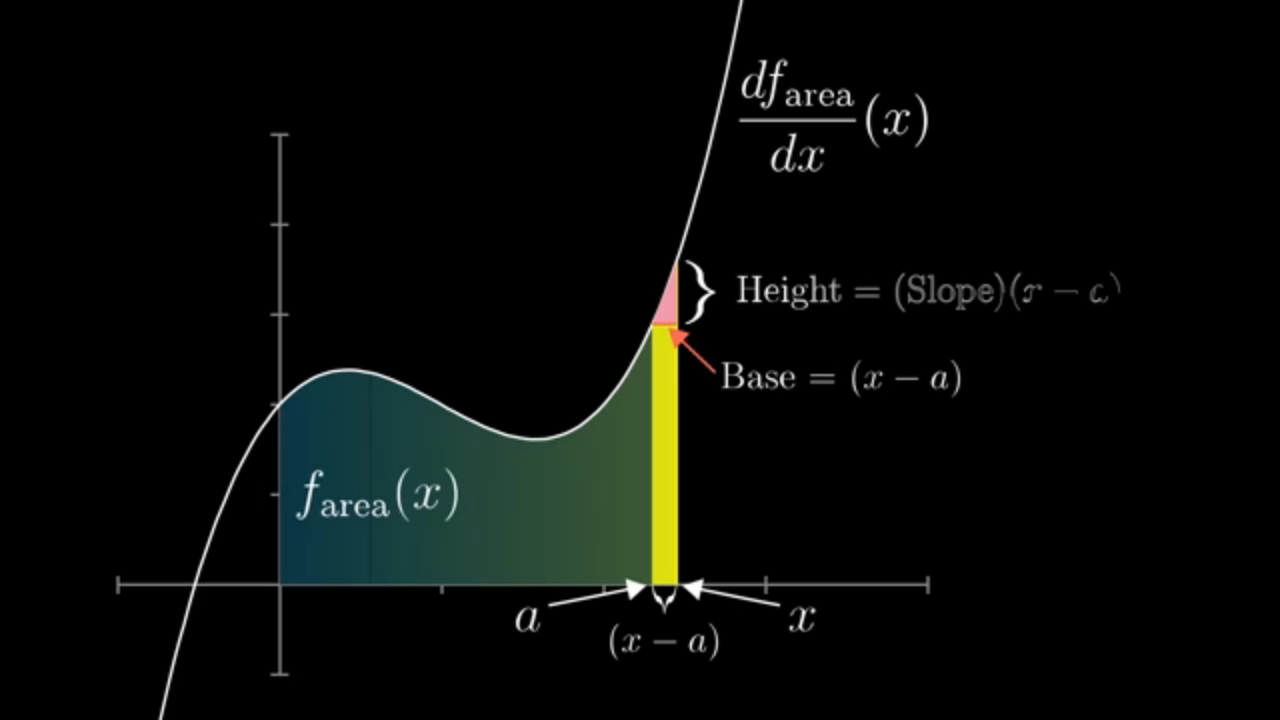

df_{area}
$f_{\text{area}}(x)$
x

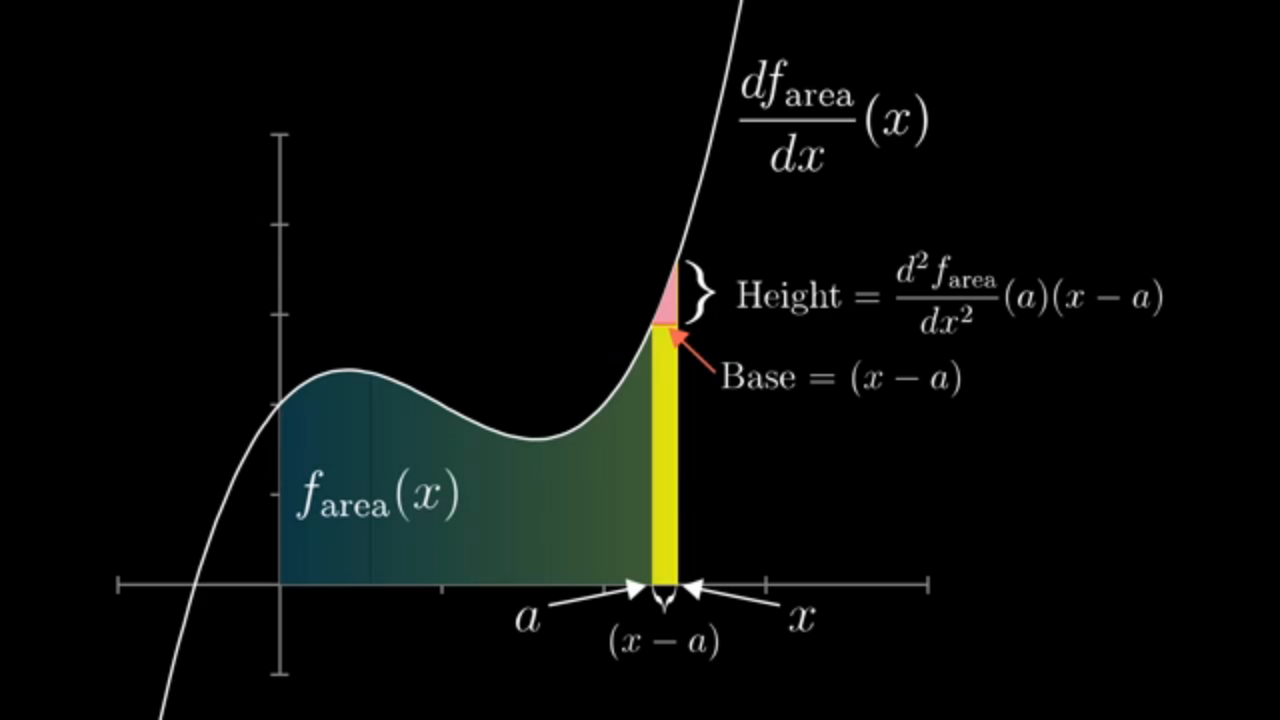

$\frac{df_{\text{area}}}{dx}(x)$
$f_{\text{area}}(x)$
dx

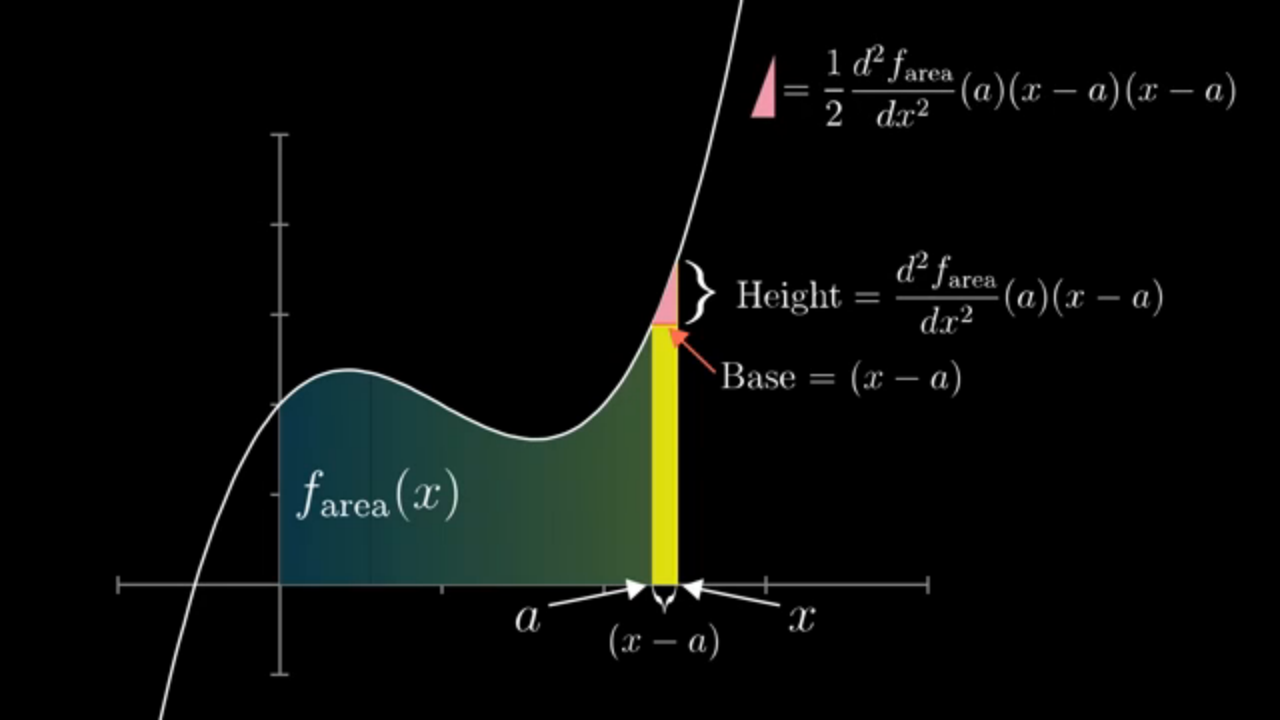

$\frac{df_{\text{area}}}{dx}(x)$
$f_{\text{area}}(x)$
dx

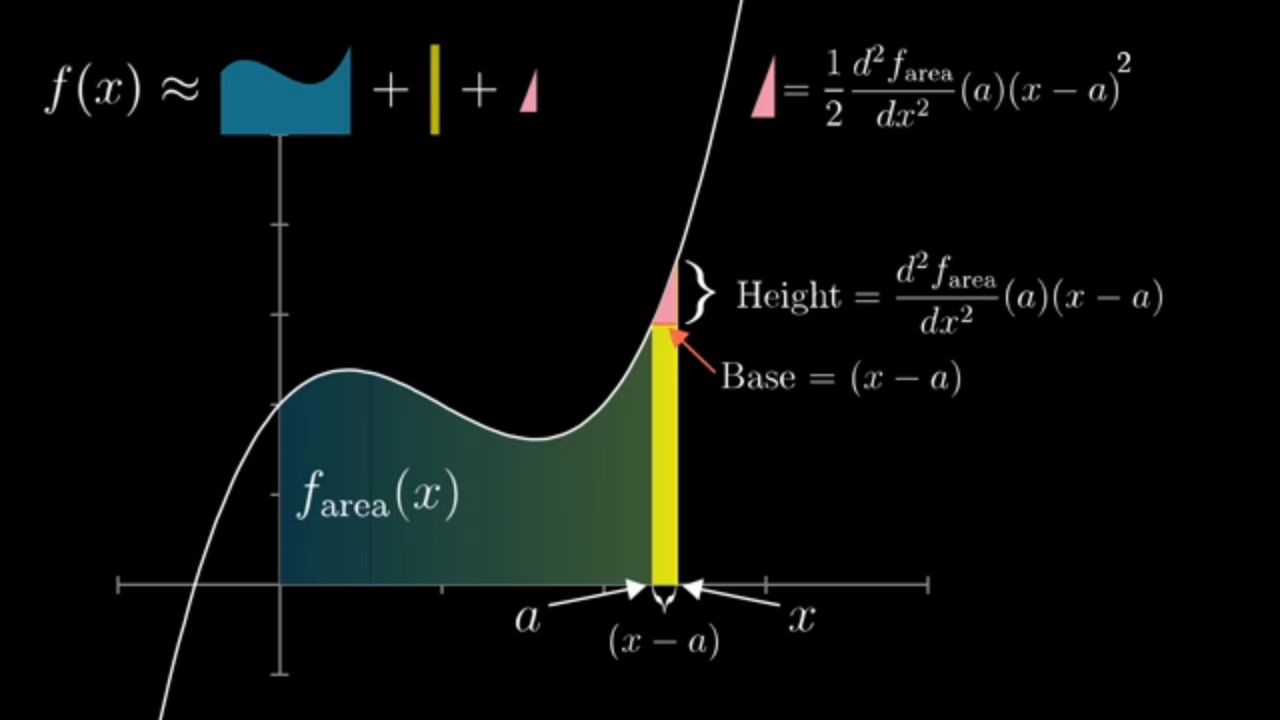

$\frac{df_{\text{area}}}{dx}(x)$
$f_{\text{area}}(x)$
dx

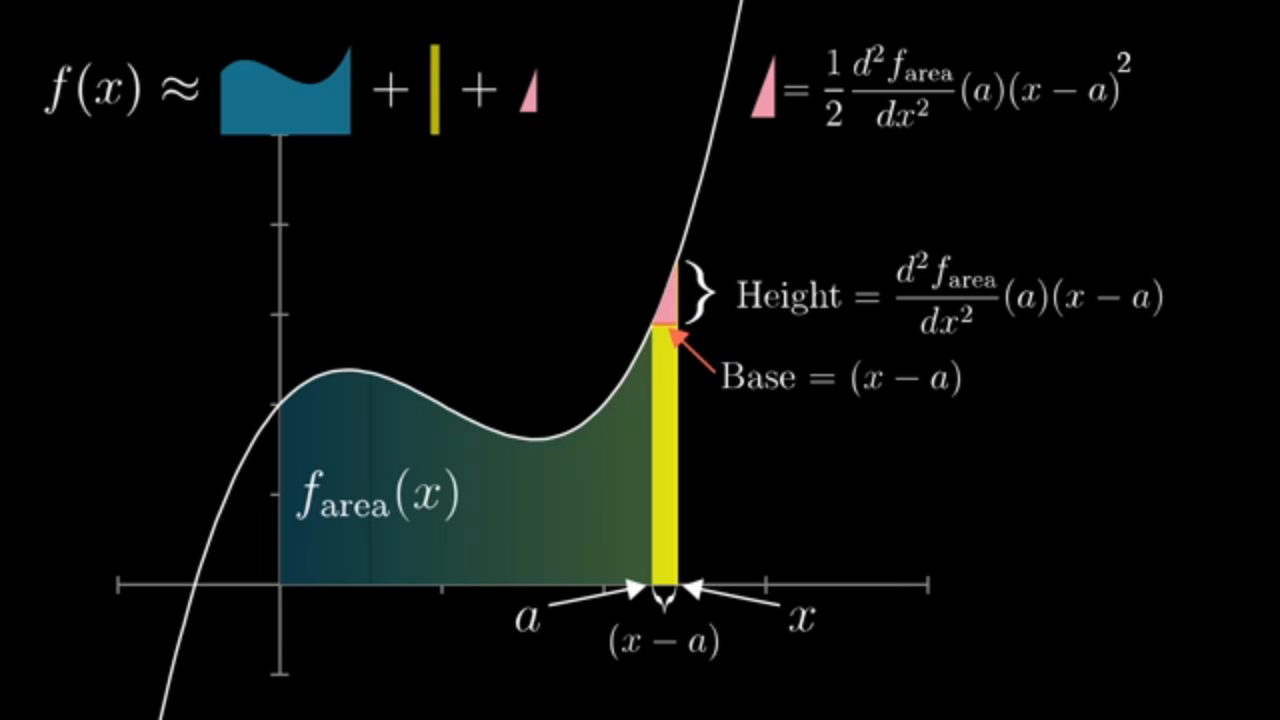

$\frac{df_{\text{area}}}{dx}(x)$
$f_{\text{area}}(x)$
a
dx
x

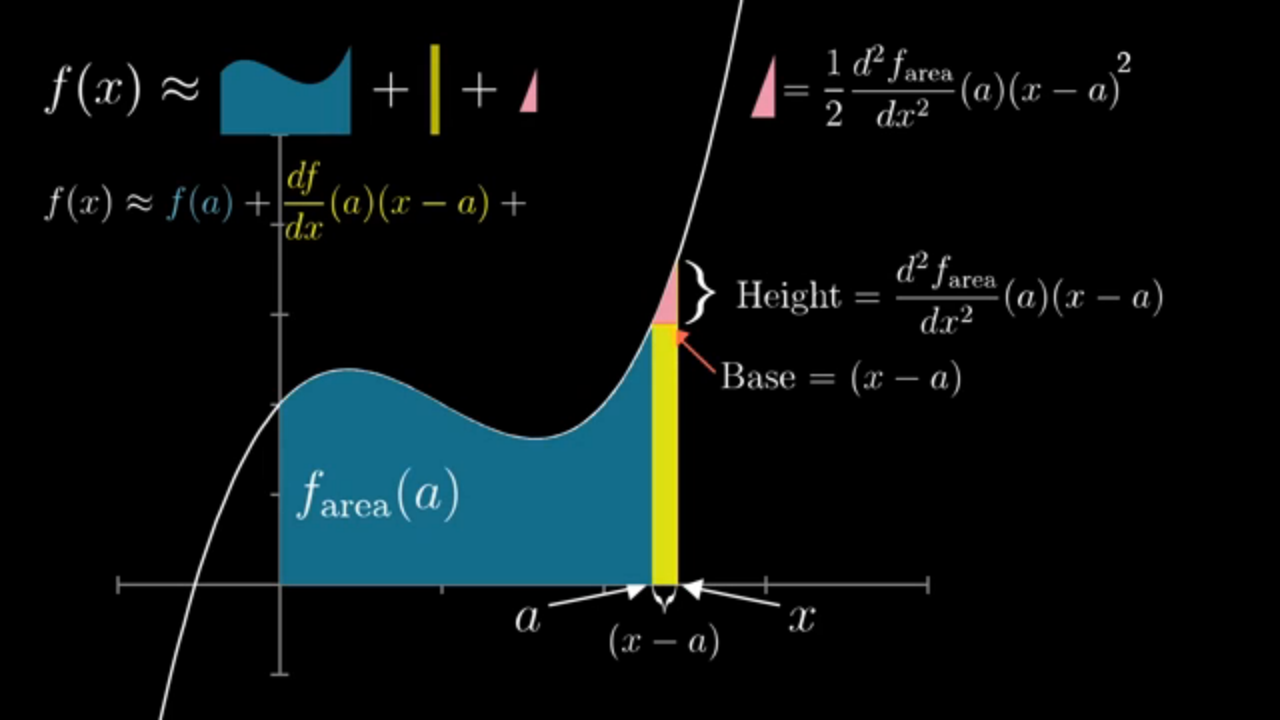

$\frac{df_{\text{area}}}{dx}(x)$
Height = (Slope)$(x - a)$
Base = $(x - a)$
$f_{\text{area}}(x)$
a
$(x - a)$
x


$\frac{df_{\text{area}}}{dx}(x)$
Height $= \frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)$
Base $= (x-a)$
$f_{\text{area}}(x)$
a
$(x-a)$
x


$= \frac{1}{2}\frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)(x-a)$
Height $= \frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)$
Base $= (x-a)$
$f_{\text{area}}(x)$
a
$(x-a)$
x


$f(x) \approx$ + +
$= \frac{1}{2}\frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)^2$
Height $= \frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)$
Base $= (x-a)$
$f_{\text{area}}(x)$
a
$(x-a)$
x

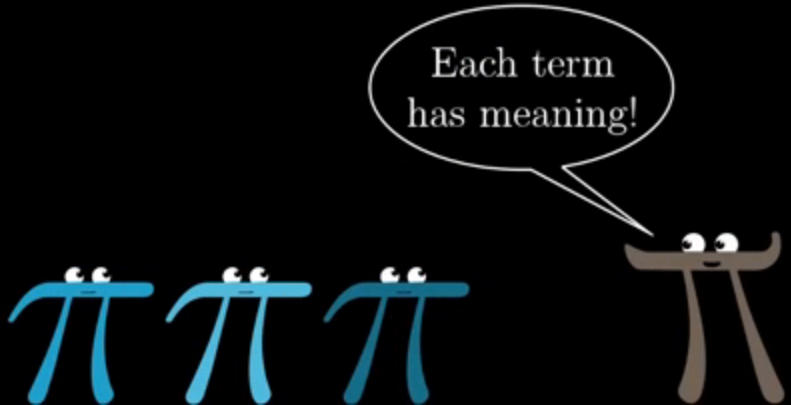

$f(x) \approx$ + +
$= \frac{1}{2}\frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)^2$
Height $= \frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)$
Base $= (x-a)$
$f_{\text{area}}(x)$
a
$(x-a)$
x

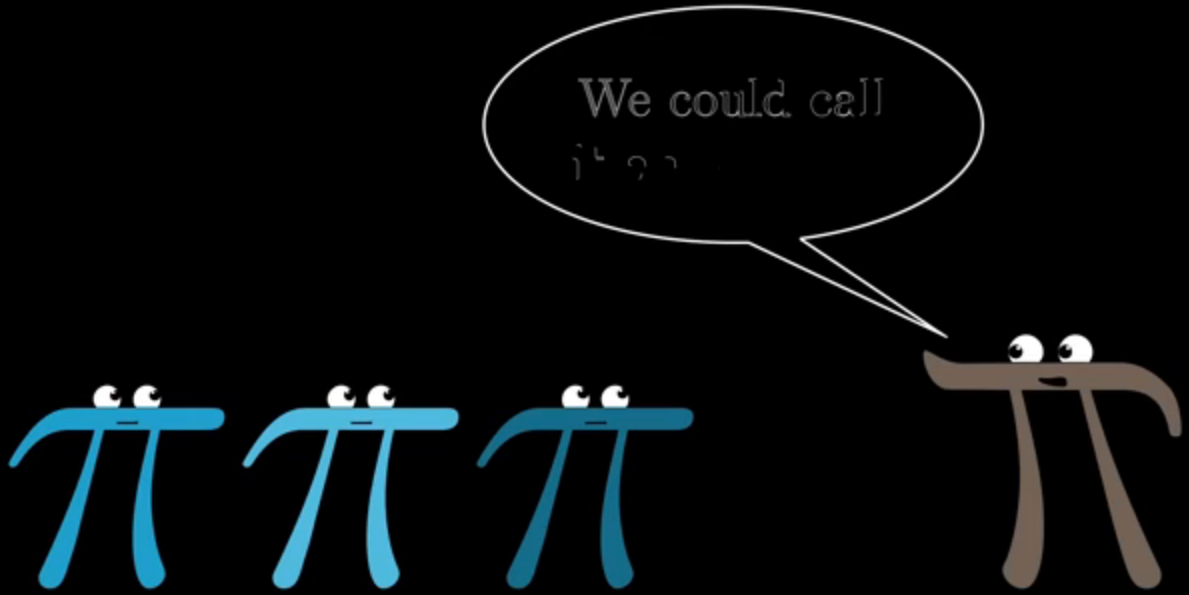

$f(x) \approx$ + +
$= \frac{1}{2}\frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)^2$
$f(x) \approx f(a) + \frac{df}{dx}(a)(x-a) +$
Height $= \frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)$
Base $= (x-a)$
$f_{\text{area}}(a)$
a
$(x-a)$
x

$f(x) \approx$ $+$ $+$

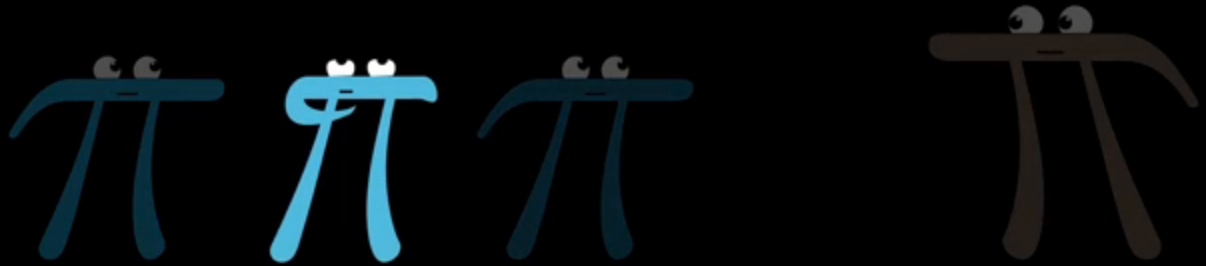
$= \frac{1}{2}\frac{d^2 f_{\text{area}}}{dx^2}(a)(x-a)^2$

$$f(x) \approx f(a) + \frac{df}{dx}(a)(x-a) + \frac{1}{2}\frac{d^2 f}{dx^2}(a)(x-a)^2$$




We could call

$$\cos(x) \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$



Infinite sum $\Leftrightarrow$ series

$$\cos(x) \approx \underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!}}_{\text{Taylor polynomial}} - \frac{x^6}{6!} + \cdots$$

Infinite sum $\Leftrightarrow$ series

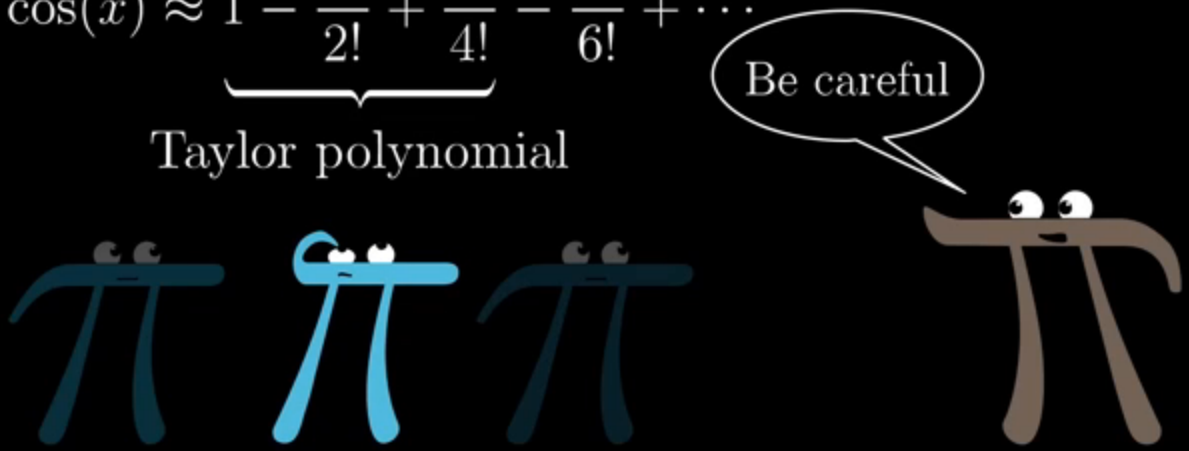
Taylor series

$$\cos(x) \approx \underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!}}_{\text{Taylor polynomial}} - \frac{x^6}{6!} + \cdots$$

Infinite sum $\Leftrightarrow$ series

Taylor series

$$\cos(x) \approx \underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!}}_{\text{Taylor polynomial}} - \frac{x^6}{6!} + \cdots$$



$$\underbrace{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}}_{0.4979424}+\cdots$$

"Converges" to $\frac{1}{2}$

$$\underbrace{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}+\cdots}_{0.4999915}$$

$$e^x \rightarrow 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

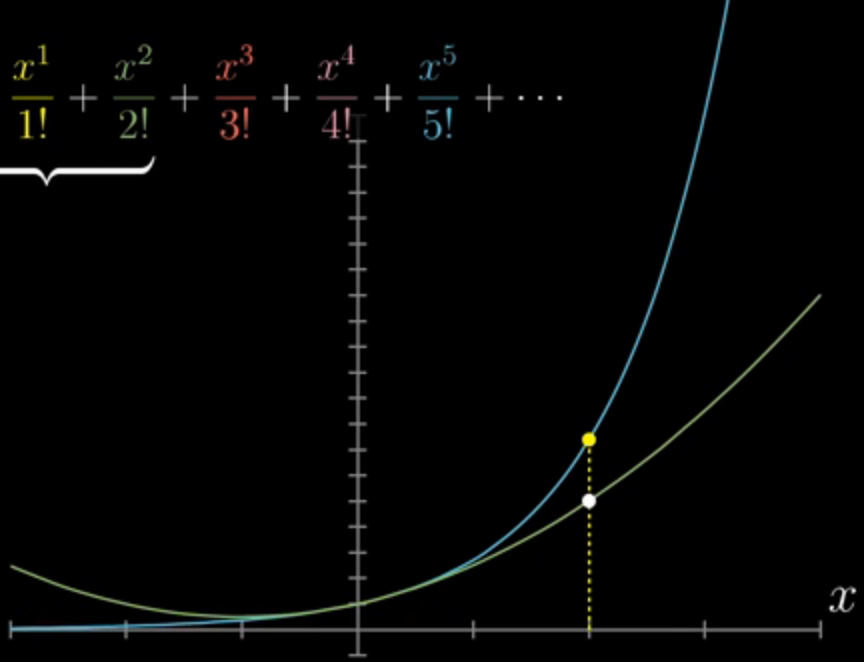
$$e^x \rightarrow 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

$$e^1 \rightarrow \underbrace{1 + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!}}_{2.7083333} + \frac{1^5}{5!} + \cdots$$

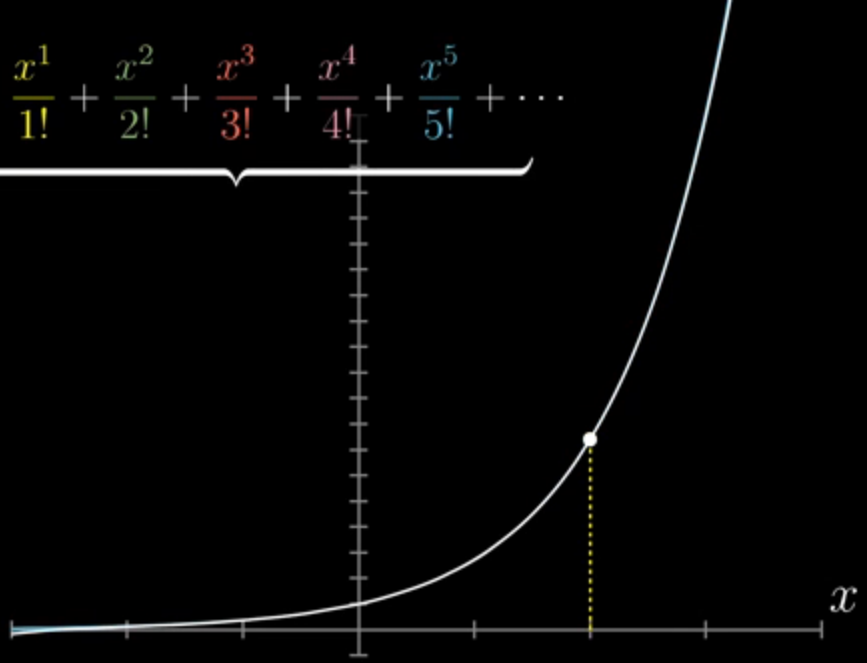
$$e^x \rightarrow 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

$$e^1 \rightarrow \underbrace{1 + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \frac{1^5}{5!} + \cdots}_{2.7182818}$$

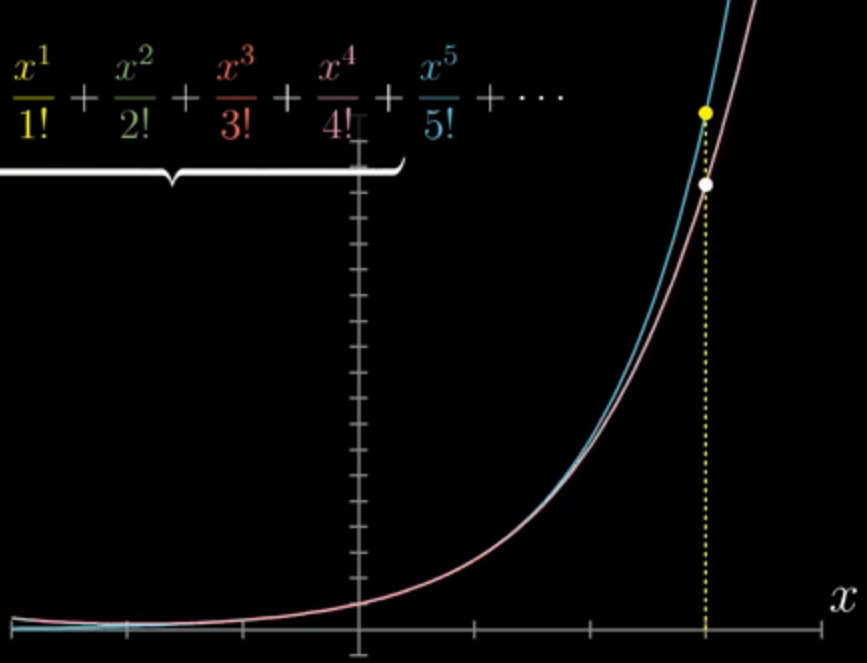
$$e^x \to \underbrace{1 + \frac{x^1}{1!} + \frac{x^2}{2!}} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$



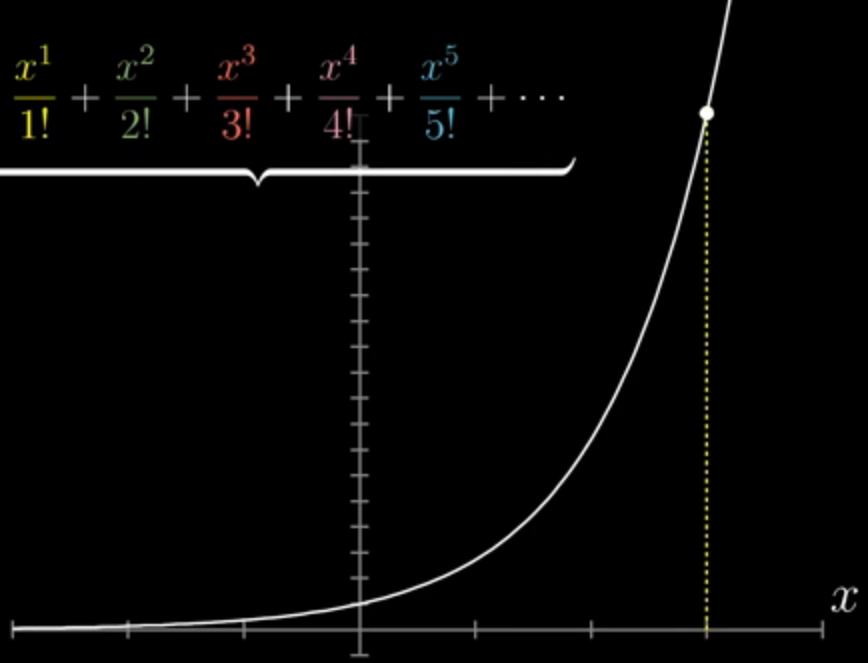
$$e^x \to \underbrace{1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots}$$

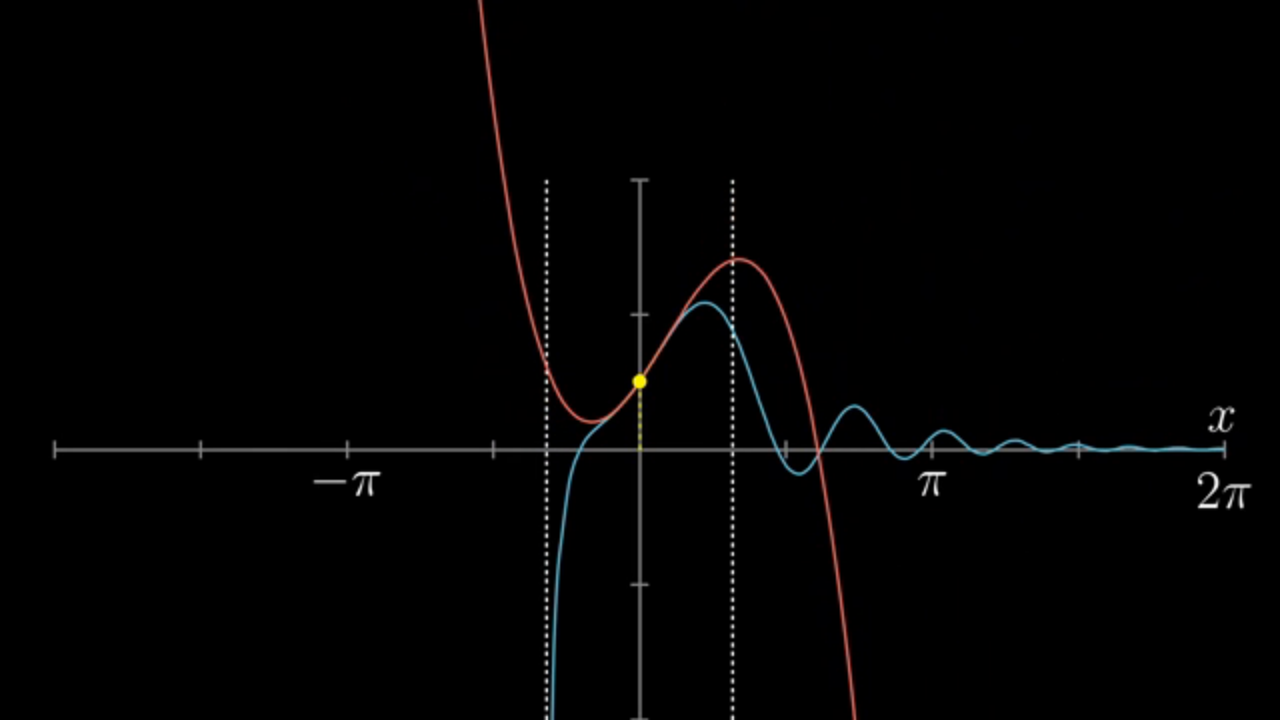


$$e^x \to \underbrace{1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}} + \frac{x^5}{5!} + \cdots$$

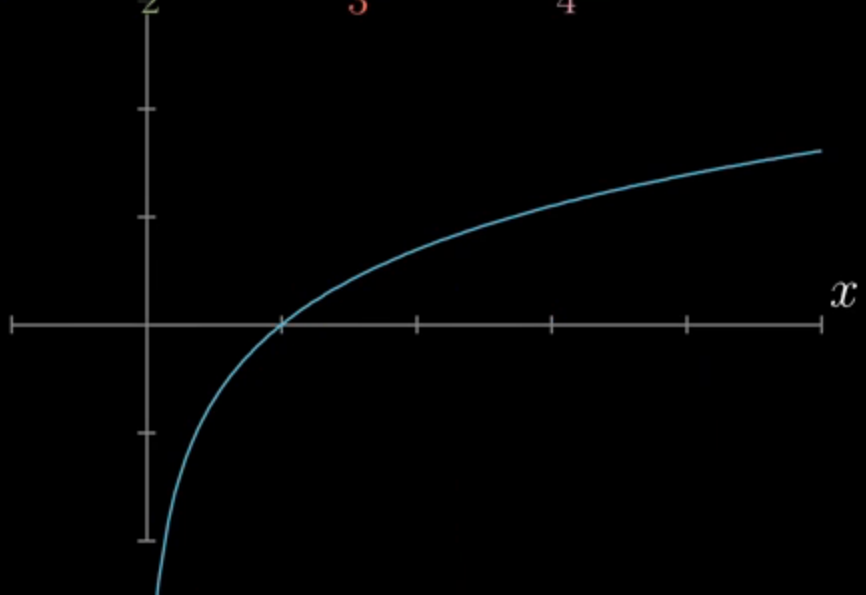


$$e^x \to \underbrace{1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots}$$

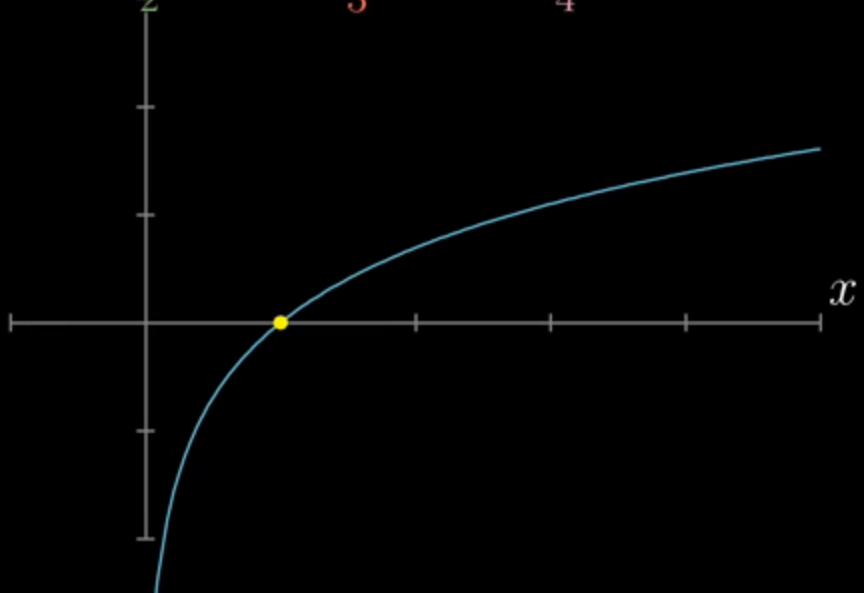



x
$-\pi$
π
2π

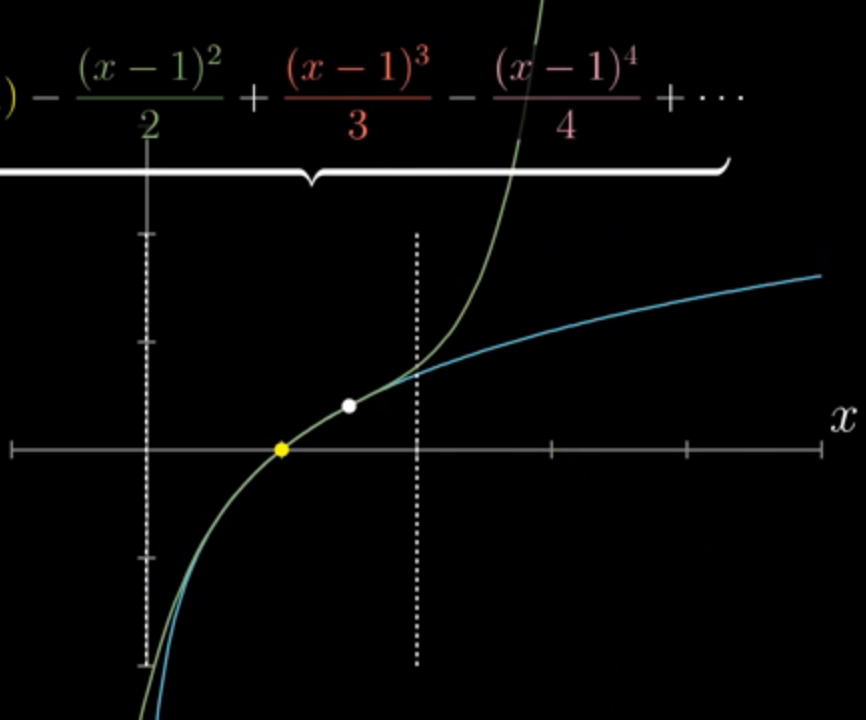
$$\ln(x) \to (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots$$



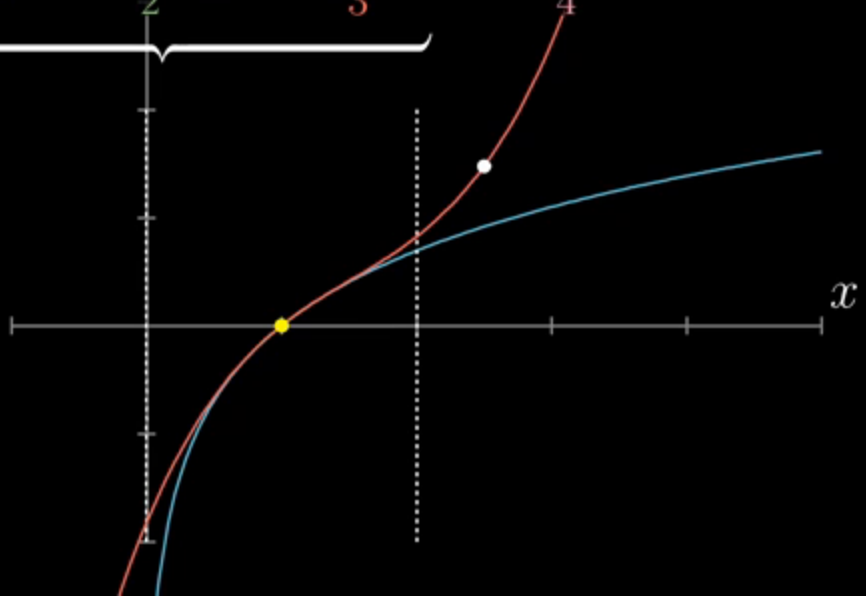
$$\ln(x) \to (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots$$



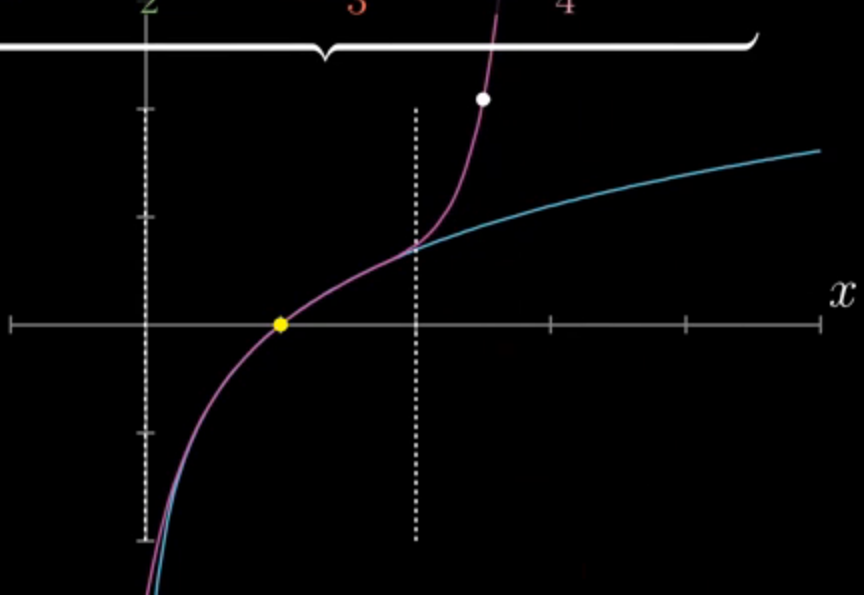
$$\ln(x) \to \underbrace{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots}$$



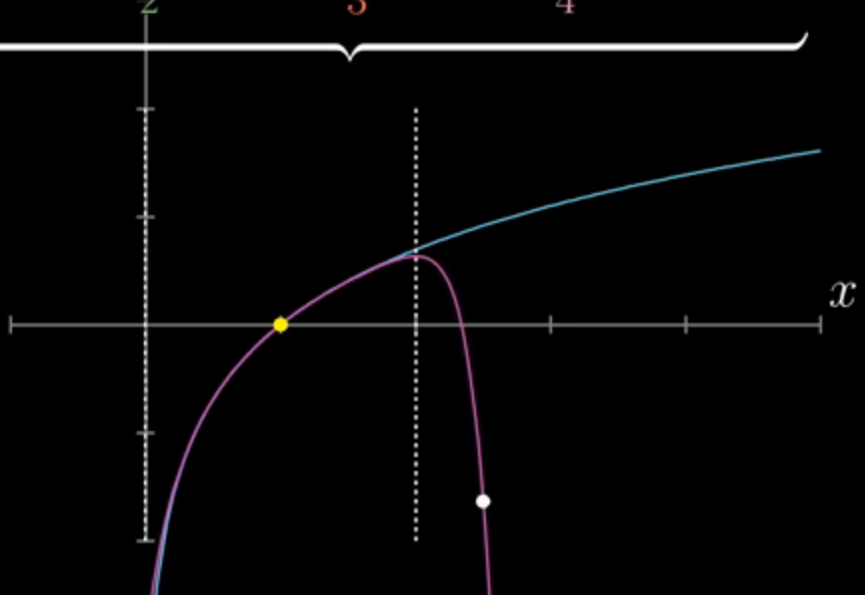
$$\ln(x) \to \underbrace{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}} - \frac{(x-1)^4}{4} + \cdots$$



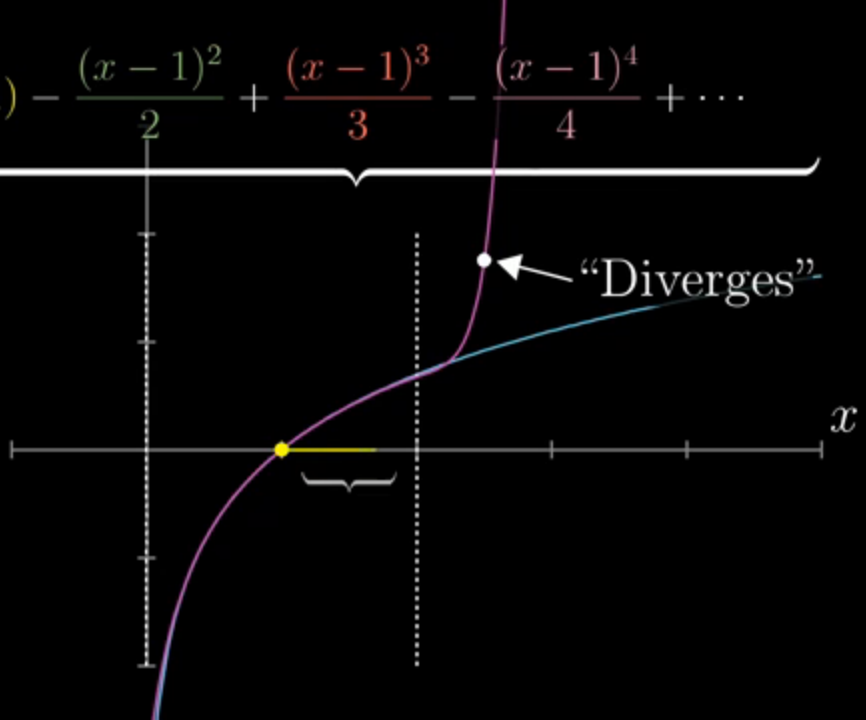
$$\ln(x) \to \underbrace{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots}$$



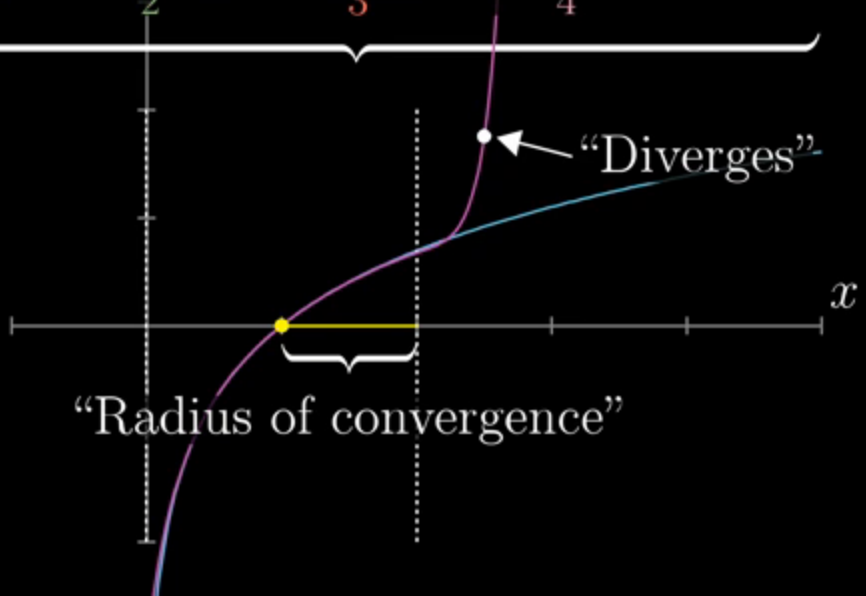
$$\ln(x) \to \underbrace{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots}$$



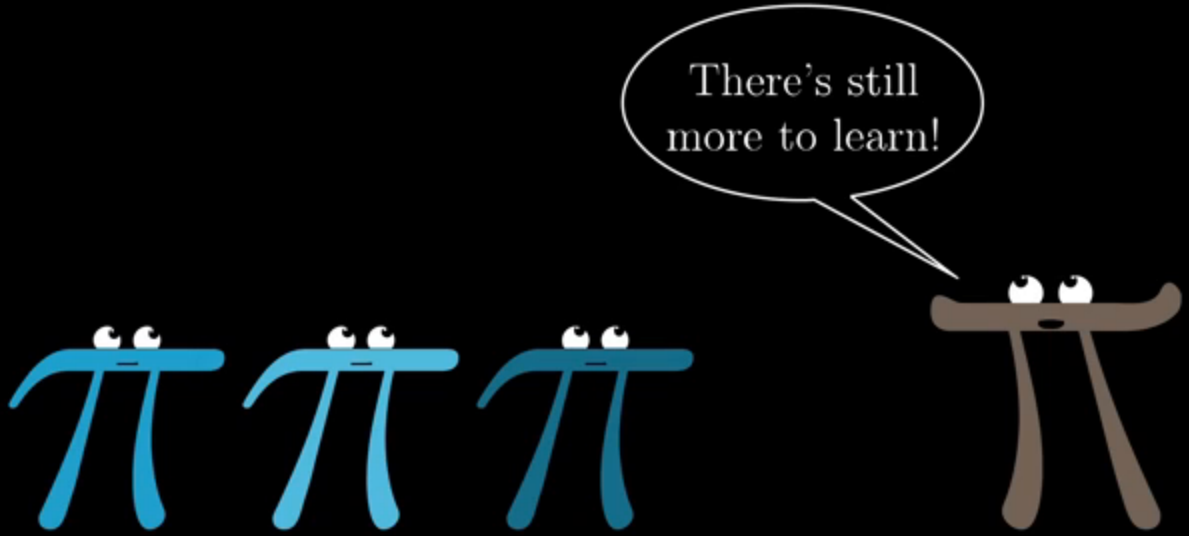
$$\ln(x) \to \underbrace{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots}$$



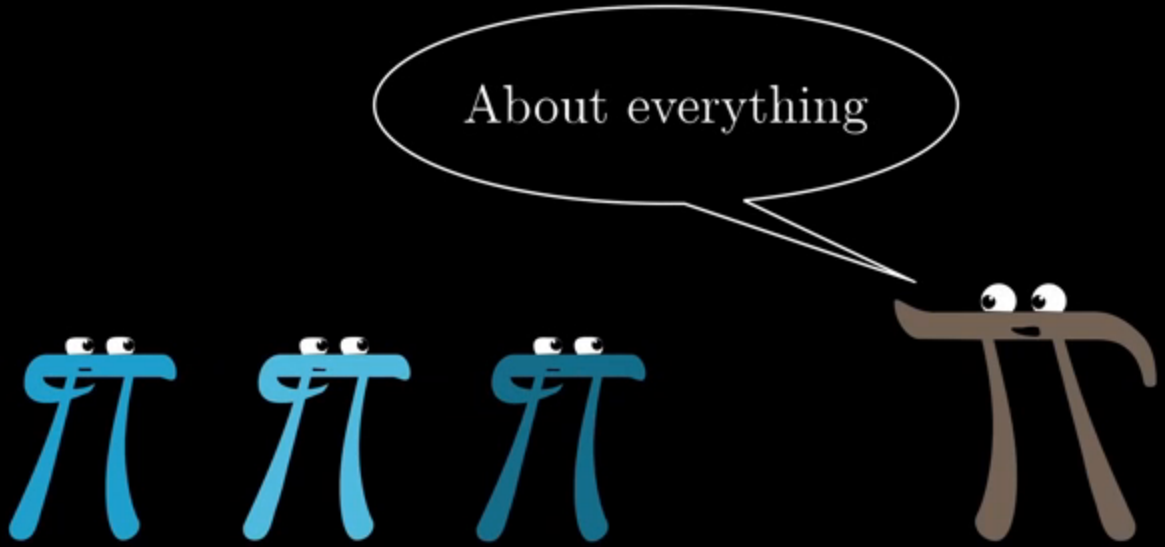
$$\ln(x) \to \underbrace{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots}$$

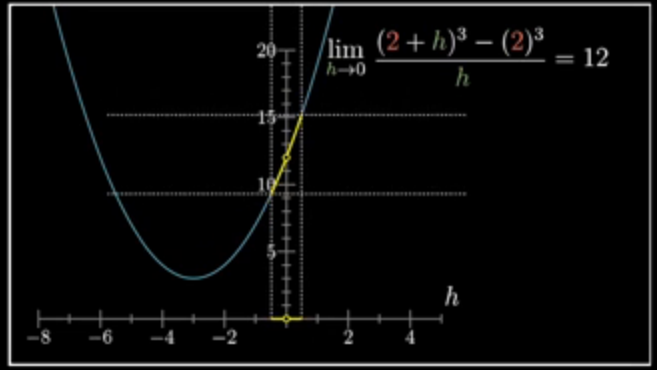


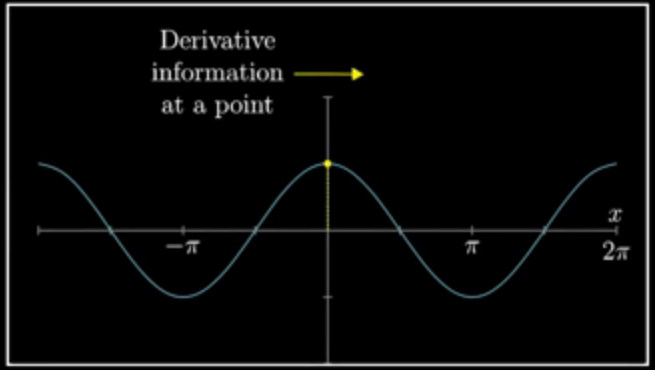
Lagrange error bounds, convergence tests,

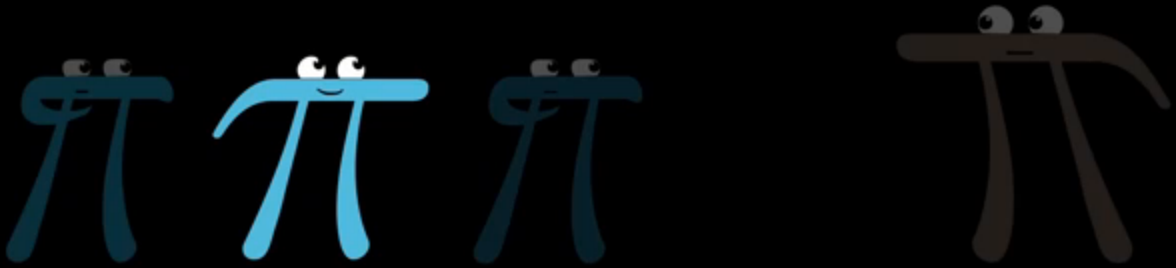


Lagrange error bounds, convergence tests, . . .




$\lim_{h \to 0} \frac{(2+h)^3 - (2)^3}{h} = 12$
20
15
10
5
h
−8
−6
−4
−2
2
4


Derivative
information
at a point
x
$-\pi$
π
2π



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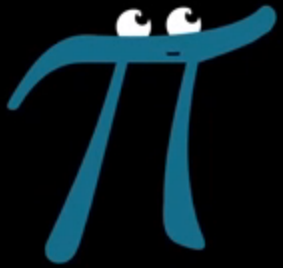
Derek Dai
Luc Ritchie
Ahmad Bamieh
Mark Govea
Zac Wentzell
Robert Teed
Jason Hise
Meshal Alshammari

Clicky stuffs

Clicky stuffs