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Scattering of Closed Strings from Many D-branes

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Abstract

We develop an operator formalism to compute scattering amplitudes of arbitrary bosonic string states in the background of many D-branes. Specifically, we construct a suitable boundary state which we use to saturate the multi-Reggeon vertex in order to obtain the generator of multi-membrane scattering amplitudes. We explicitly show that the amplitudes with h parallel D-branes are similar to amplitudes with $h-1$ open string loops.

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1 Introduction

Dirichlet membranes, or simply D-branes, have acquired an increasing importance in the study of non-perturbative effects in string theory. As shown in a remarkable paper by Polchinski [1], they provide an exact and simple conformal field theory description of the extended solitons carrying one unit of Ramond-Ramond charge in type II string theories. Since there exist duality transformations [2] which relate pairs of apparently different string theories by exchanging the fundamental strings of a model with the solitons of its dual, it is of some importance to study the perturbative behavior of a string theory in the background of D-branes. In fact, from such a perturbative knowledge one can extract, in principle, some non-perturbative information on the dual theory.

In this paper we develop an operator formalism that allows to compute scattering amplitudes of arbitrary string states in the presence of many D-branes. Starting from the results already achieved in the old days of string theory [3], we first construct a state which inserts a boundary on the world-sheet and enforces Dirichlet or Neumann boundary conditions on the string coordinates. Then, we take the multi-Reggeon vertex of the operator formalism [4, 5], which describes the emission of many closed strings from a sphere, and saturate it with an appropriate number of boundary states to compute arbitrary multi-membrane scattering amplitudes. Our boundary state is different from those that have appeared in the literature [6, 7, 8, 9] since it contains not only the identity operator that identifies the left and right sectors of the closed string but also an open string propagator. It is precisely because of this feature that our boundary states can be directly used to saturate the multi-Reggeon vertex of the closed strings to obtain the multi-membrane amplitudes.

We would like to stress that the structure of an amplitude with h D-branes is formally similar to an open string amplitude at $h-1$ loops. While this analogy is quite obvious from a geometrical point of view, it is not so straightforward to describe explicitly the interactions of closed strings on surfaces with many boundaries. However, the operator formalism allows to obtain explicit results in a closed form. Even if these calculations are relevant and interesting in the case of superstrings, where duality is realized, in this paper we concentrate on the bosonic string in $D = 26$ where it is easier to understand and display the general structure of the formalism.

To illustrate our procedure, we begin by reviewing in Section 2 the operator formalism for the interaction among open and closed strings, and extend the old results to the case of Dirichlet boundary conditions. In Section 3, we factorize the amplitude for the emission of closed strings from an open string to obtain the boundary operator, and, by sewing two of these, we reproduce Polchinski's result for the interaction of two parallel D-branes [10]. Finally in Section 4 we reformulate the boundary operator as a boundary state, and saturate with it the multi-Reggeon vertex to compute the generator of the multi-brane amplitudes.

2 Mixed closed and open string amplitudes

The interactions among closed and open strings were extensively studied already in the early days of string theory [11, 3]. In particular, in Ref. [3] Ademollo et al. constructed vertex operators for the emission of a closed string out of an open string, and computed the scattering amplitudes among M closed and N open strings at tree level. The topology of the string world-sheet corresponding to these amplitudes is that of a disk emitting N open strings from its boundary and M closed strings from its interior. As customary in those days, only Neumann boundary conditions were imposed on the disk, and no target-space compactification was considered. However, we find useful to recall here the results of Ref. [3] because the introduction of Dirichlet boundary conditions and of compact directions is very simple in that approach.

By a conformal transformation a disk can be mapped onto the upper half complex z-plane ($z = \mathrm{e}^{\tau + i\sigma}$ with τ and σ being the timelike and spatial coordinates of the world sheet) and its boundary onto the real axis ($x = \pm \mathrm{e}^{\tau}$). Then, the emission from the disk boundary of an open string state with momentum p and internal quantum numbers α, is described by a vertex operator $\mathcal{V}_\alpha(x;p)$, while the emission from the disk interior of a closed string state with momentum k, and left and right quantum numbers β_L and β_R, is described by a vertex operator $\mathcal{W}_{\beta_L,\beta_R}(z,\bar{z};k)$. The presence of a boundary on the world sheet imposes a relation between the left and right parts of $\mathcal{W}_{\beta_L,\beta_R}$ which are not independent of each other. In fact, if one splits the (Neumann) open string coordinates into left and right components

$$X^\mu_N(\tau,\sigma) = \frac{1}{2}\left[X^\mu(z) + X^\mu(\bar{z})\right] \quad , \tag{2.1}$$

with

$$X^\mu(z) = q^\mu - \mathrm{i}\,(2\alpha')\,a_0^\mu \ln z + \mathrm{i}\,\sqrt{2\alpha'}\sum_{n\neq 0}\frac{a_n^\mu}{\sqrt{|n|}} z^{-n} \quad , \tag{2.2}$$

it is possible to write [3]

$$\mathcal{W}_{\beta_L,\beta_R}(z,\bar{z};k) = \mathcal{V}_{\beta_L}(z;k_L)\;\mathcal{V}_{\beta_R}(\bar{z};k_R) \quad , \tag{2.3}$$

where $k_L = k_R = k/2$. We would like to stress that the vertex operator $\mathcal{W}$ depends on a single set of oscillators (*i.e.* those of the open string), and that each factor in Eq. (2.3) is separately normal ordered. This is to be contrasted with the vertex operators describing the emission of a closed string out of a closed string. In fact, in this case there are two distinct sets of oscillators for the left and right sectors which only share the zero-mode.

Using the operators $\mathcal{V}$ and $\mathcal{W}$, the tree-level scattering amplitude among N open and M closed strings is given by

$$A(N,M) \quad = \quad \mathcal{C}_0\,\mathcal{N}^N\,\widehat{\mathcal{N}}^M \int \frac{1}{dV_{abc}}\prod_{i=1}^{N}\left[dx_i\;\theta(x_{i+1}-x_i)\right]\;\prod_{j=1}^{M} d^2 z_j \tag{2.4}$$

$$\langle 0| \, \mathrm{T} \left(\prod_{i=1}^{N} \mathcal{V}_{\alpha_i}(x_i; p_i) \, \prod_{j=1}^{M} \mathcal{W}_{\beta_{jL},\beta_{jR}}(z_j, \bar{z}_j; k_j) \right) |0\rangle \quad ,$$

where dV_{abc} is the volume of the projective group $SL(2, \mathbf{R})$, T denotes the time (radial) ordering prescription, and $\mathcal{C}_0$, $\mathcal{N}$ and $\widehat{\mathcal{N}}$ are respectively the normalizations of the disk, of the open and of the closed vertex operators respectively (see *e.g.* Ref. [12]). In Eq. (2.4) the variables x_i's are integrated on the real axis while the complex variables z_j's are integrated on the upper half plane. Because of Eq. (2.3) it is clear that $A(N, M)$ is formally similar to a pure open string amplitude with $N+2M$ external states provided suitable identifications of momenta are made. This has been recently re-proposed in [13].

It is interesting to note that amplitude (2.4) is ill defined if $N = 0$. In fact, as we will see, $A(N, M)$ can be written as a pure closed string diagram with $M + 1$ legs one of which sewn to a disk with N external open strings. The propagator sewing the closed string amplitude to the disk must carry a momentum k equal to the sum of the N open string momenta p_i's; if $N = 0$, this sum vanishes and Eq. (2.4) becomes ill defined, since the closed string propagator has a pole when $k = 0$.

The situation is different if some target space directions (labeled by an index I) are compactified on a circle of radius R. In this case, the left and right parts of the closed string momenta contain a Kaluza-Klein term proportional to $1/R$ and a winding term proportional to R:

$$k_L^I = \frac{1}{2} \left(\frac{n^I}{R} + \frac{w^I R}{\alpha'} \right) \quad , \quad k_R^I = \frac{1}{2} \left(\frac{n^I}{R} - \frac{w^I R}{\alpha'} \right) \quad . \tag{2.5}$$

In the compactified theory Eq. (2.4) is well defined even if $N = 0$. In fact, it is still true that the amplitude (2.4) can be written as a pure closed string diagram sewn to a disk, but now momentum conservation [1] constrains only the Kaluza-Klein part of the sewing propagator, leaving its winding number arbitrary. Thus, the singularity is avoided.

Notice that while in a pure closed string amplitude both the Kaluza-Klein and the winding numbers are separately conserved, when the closed strings interact with a disk, only the conservation of the Kaluza-Klein part of the momentum seems to hold. However, contrarily to what happens to the external open strings, the world-sheet boundary of a virtual open string can wind along the compact directions, and since

$$\oint d\sigma^\alpha \, \frac{\partial_\alpha X_N^I}{2\pi\alpha'} = \sum_{j=1}^{M} \frac{w_j^I R}{\alpha'} \quad , \tag{2.6}$$

[1] For $N = 0$ it is enforced by a factor $\langle 0| \, \exp \left[\mathrm{i} \, q_I \sum_{j=1}^{M} \left(k_{jL}^I + k_{jR}^I \right) \right] |0\rangle = 2\pi R \; \delta_{\sum_j n_j^I \, , 0}$ for any compactified direction.

where the contour integral is over the disk boundary, the conservation of the winding number can be recovered if one also considers the boundary of the world sheet. Eq. (2.6) suggests that it is necessary to add to the string action of the compactified Neumann theory the following topological term [8]

$$\mathrm{i}\, Y_I \oint d\sigma^\alpha\ \frac{\partial_\alpha X_N^I}{2\pi\alpha'} \quad , \tag{2.7}$$

where the parameters Y_I, conjugate to the winding number, are constant $U(1)$ gauge potentials that couple to the boundary of the disk. Obviously, the term (2.7) does not change the equations of motion of the string coordinates X^I and their mode expansion, but it couples to the winding number of the external closed strings so that the amplitude (2.4) must be multiplied by the factor $\exp\left(\mathrm{i}\, Y_I \sum_{j=1}^{M} w_j^I\, R/\alpha'\right)$. Such a factor can be automatically generated by shifting the left and right parts of the open string compact coordinates X_N^I according to

$$X^I(z) \to X^I(z) + Y^I \quad , \quad X^I(\bar{z}) \to X^I(\bar{z}) - Y^I \quad . \tag{2.8}$$

Using this approach, it is now rather simple to introduce Dirichlet boundary conditions along compact directions and study the scattering amplitudes of closed strings with a D-brane. To do this, let us first recall that the mode expansion of a compact string coordinate X_D^I with Dirichlet boundary conditions $X_D^I(\tau, \sigma = 0) = X_D^I(\tau, \sigma = \pi) = Y^I \bmod 2\pi R$, is

$$X_D^I(\tau, \sigma) = Y^I + 2w^I R\, \sigma + \mathrm{i}\sqrt{2\alpha'} \sum_{n\neq 0} \frac{a_n^\mu}{\sqrt{n}} (z^n - \bar{z}^{-n}) \quad . \tag{2.9}$$

As in the Neumann case, also in the Dirichlet case one can divide the string coordinates into left and right components, and then follow the same steps as before [8]. In fact, using the expansion (2.2), we have

$$X_D^I(\tau, \sigma) \equiv \frac{1}{2}\Big[X^I(z) - X^I(\bar{z})\Big] \quad . \tag{2.10}$$

Notice that the eigenvalues of a_0^I in a compactified Neumann coordinate X_N^I are n^I/R, while in a Dirichlet coordinate X_D^I they are $w^I R/\alpha'$ (with $n^I, w^I \in \mathbf{Z}$). Thus, the transformation $R \to \alpha'/R$, $X^I(\bar{z}) \to -X^I(\bar{z})$ and $n^I \to w^I$ changes Neumann into Dirichlet boundary conditions in the open strings. Under this duality transformation the parameters Y^I of the Neumann theory become the coordinates of the D-brane in the Dirichlet theory, while the closed string sector remains unchanged.

We are now in the position of writing the amplitude of closed strings interacting with a D-brane of dimension p. For notational simplicity we suppose that all 26 coordinates are compactified with scale R and that the first $p+1$ have Neumann boundary conditions, while the remaining $25-p$ have Dirichlet boundary conditions.

Then, the scattering amplitude of M closed string states with a Dp-brane is given by

$$
\begin{aligned}
A(M) &= V\,\mathcal{C}_0\,\widehat{\mathcal{N}}^M\,\mathrm{e}^{\mathrm{i}\,Y\cdot\sum\limits_{j=1}^{M}\left(k_{jL}-Sk_{jR}\right)}\int\frac{1}{dV_{abc}}\prod_{j=1}^{M}d^2z_j \\
&\times\langle 0|\ \mathrm{T}\left(\prod_{j=1}^{M}\mathcal{V}_{\beta_j}(z_j;k_{jL})\,\mathcal{V}_{S\beta_{jR}}(\bar{z}_j;Sk_{jR})\right)|0\rangle \quad ,
\end{aligned}
\tag{2.11}
$$

where S is a diagonal matrix with eigenvalues $+1$ (-1) for the Neumann (Dirichlet) directions, and $\mathcal{V}_{S\beta_R}(\bar{z};Sk_R)$ stands for the antiholomorphic part of the vertex (2.3) in which $X(\bar{z})$ has been replaced by $SX(\bar{z})$. Note that in Eq. (2.11), differently from Eq. (2.4), the vacuum $|0\rangle$ has been normalized to one. This explains the overall appearance of the volume V, which, according to footnote 1, is given in this case by

$$
V = (2\pi R)^{p+1}\left(\frac{2\pi\alpha'}{R}\right)^{25-p} \quad . \tag{2.12}
$$

The specific expression of the vertices $\mathcal{V}$ in Eq. (2.11) depends on the emitted states, but it always contains a factor proportional to $:\exp\left(\mathrm{i}\,k_{jL}\cdot X(z_j)\right):$ $:\exp\left(\mathrm{i}\,Sk_{jR}\cdot X(\bar{z}_j)\right):$. Therefore, the expectation value over the zero-modes in Eq. (2.11) yields the momentum conservation

$$
\sum_{j=1}^{M}\left(k_{jL}+Sk_{jR}\right)=0 \tag{2.13}
$$

i.e. $\sum\limits_j n_j^I = 0$ along the Neumann directions, and $\sum\limits_j w_j^J = 0$ along the Dirichlet directions. We conclude by mentioning that using this formalism in the decompactification limit $R\to\infty$ one can easily reproduce the results of Refs. [14, 13] for the amplitudes of massless states from a D-brane.

3 Factorization and boundary operator

We now show that the amplitude $A(M)$ of Eq. (2.11) can be factorized as a $M+1$ closed string diagram in which one leg is saturated with a boundary operator B that encodes the presence of the D-brane. To do this, following Ref. [3], we first exploit the $SL(2,\mathbf{R})$ invariance to fix $z_1=\mathrm{i}$, and then make the change of variables

$$
z\to z' = -\frac{z-\mathrm{i}}{z+\mathrm{i}} \quad , \tag{3.1}
$$

so that the upper half z-plane is mapped into the circle of unit radius. After this transformation, the variables z' and $(\bar{z})'$ are no longer complex conjugate of each other since $(\bar{z})' = 1/\overline{z'}$. Therefore, radial ordering forces to split the vertices $\mathcal{W}$ into their constituent factors, and put all the holomorphic parts on the right and all the antiholomorphic ones on the left. Then, Eq. (2.11) becomes

$$\begin{aligned} A(M) \;&=\; \frac{V\,\mathcal{C}_0\,\widehat{\mathcal{N}}^M}{2\pi}\;\mathrm{e}^{\,\mathrm{i}Y\cdot\sum\limits_{j=1}^{M}\left(k_{jL}-Sk_{jR}\right)}\int\prod_{j=2}^{M}\frac{d^2z'_j}{\overline{z'_j}^{\,2}}\;\langle Sk_{1R};S\beta_{1R}| \\ &\times \mathrm{T}\Bigg(\prod_{j=2}^{M}\mathcal{V}_{S\beta_{jR}}\Big(1/\overline{z'_j};Sk_{jR}\Big)\Bigg)\,\mathrm{T}\Bigg(\prod_{j=2}^{M}\mathcal{V}_{\beta_{jL}}(z'_j;k_{jL})\Bigg)|k_{1L};\beta_{1L}\rangle \quad , \end{aligned} \tag{3.2}$$

where the states $|k_{1L};\beta_{1L}\rangle$ and $\langle Sk_{1R};S\beta_{1R}|$ correspond, respectively, to the vertices $\mathcal{V}_{\beta_{1L}}(z'_1;k_{1L})$ and $\mathcal{V}_{S\beta_{jR}}\Big(1/\overline{z'_1};Sk_{1R}\Big)$ in the limit $z'_1\to 0$. Notice that the overall factor of $1/2\pi$ in Eq. (3.2) is what remains of $1/dV_{abc}$ after fixing $z_1=\mathrm{i}$, *i.e.* the inverse volume of the translations. To simplify the notation, from now on we will suppress the primes on the z-variables.

Then, using the relation $\mathcal{V}_{\beta_j}(z_j) = z_M^{L_0-1}\mathcal{V}_{\beta_j}(z_j/z_M)z_M^{-L_0}$, and inserting a complete set of open string states $|q;\lambda\rangle$ twice, Eq. (3.2) becomes

$$\begin{aligned} A(M) \;&=\; \frac{V\,\mathcal{C}_0\,\widehat{\mathcal{N}}^M}{2\pi}\;\mathrm{e}^{\,\mathrm{i}Y\cdot\sum\limits_{j=1}^{M}\left(k_{jL}-Sk_{jR}\right)}\int\prod_{j=2}^{M}\frac{d^2z_j}{\bar{z}_j^2}\left(\frac{\bar{z}_M}{z_M}\right)^{M-2}\frac{1}{z_M^2} \\ &\times\sum_{\{q;\lambda\},\{q';\lambda'\}}\langle Sk_{1R};S\beta_{1R}|\,\mathrm{T}\Bigg(\prod_{j=2}^{M}\mathcal{V}_{S\beta_{jR}}\left(\bar{z}_M/\bar{z}_j;Sk_{jR}\right)\Bigg)|q;\lambda\rangle \\ &\times\,\langle -q;\lambda|\;|z_M|^{2L_0}\,|q';\lambda'\rangle\,\langle -q';\lambda'|\,\mathrm{T}\Bigg(\prod_{j=2}^{M}\mathcal{V}_{\beta_{jL}}\left(z_j/z_M;k_{jL}\right)\Bigg)|k_{1L};\beta_{1L}\rangle \quad . \end{aligned} \tag{3.3}$$

Using the invariance of the second line of Eq. (3.3) under the transformation $X(\bar{z})\to SX(\bar{z})$, that is $(Sk_{jR},S\beta_{jR},q,\lambda)\to(k_{jR},\beta_{jR},Sq,S\lambda)$, the transposition property $[\mathcal{V}_{\beta_R}(1/\bar{z})]^{\mathrm{T}} = \bar{z}^2\mathcal{V}_{\beta_R}(\bar{z})$, and the conservation law $q' = -\sum\limits_j k_{jL} = S\sum\limits_j k_{jR}$, we can rewrite the amplitude $A(M)$ as follows

$$A(M) = \sum_{\{q;\lambda\},\{q';\lambda'\}}\langle -q';\lambda'|\;\langle Sq;S\lambda|\,W\;\times\;\langle -q;\lambda|\,B\,|q';\lambda'\rangle \tag{3.4}$$

where

$$\begin{aligned} W \;&=\; \Phi\,\widehat{\mathcal{C}_0}\,\widehat{\mathcal{N}}^{M+1}\int\prod_{j=2}^{M-1}d^2\xi_j\;\mathrm{T}\Bigg(\prod_{j=2}^{M}\mathcal{V}_{\beta_{jR}}\left(\bar{\xi}_j;k_{jR}\right)\Bigg)|k_{1R};\beta_{1R}\rangle \\ &\times\mathrm{T}\Bigg(\prod_{j=2}^{M}\mathcal{V}_{\beta_{jL}}\left(\xi_j;k_{jL}\right)\Bigg)|k_{1L};\beta_{1L}\rangle \quad , \end{aligned} \tag{3.5}$$

with $\xi_j = z_j/z_M$, and

$$B = \frac{V\,\mathcal{C}_0}{\Phi\,\widehat{\mathcal{C}_0}\,\widehat{\mathcal{N}}}\;\mathrm{e}^{-2\mathrm{i}Y\cdot q'}\int_{|z_M|\le 1}\frac{d^2z_M}{2\pi}\;|z_M|^{2L_0-4} \quad . \tag{3.6}$$

For reasons that will be clear in a moment, we have introduced the coefficients $\widehat{\mathcal{C}_0}$ and Φ which are, respectively, the normalization of the sphere (see *e.g.* [12]) and the self-dual "volume" factor (see *e.g.* [15]) which normalize all closed string scattering amplitudes at tree level.

Eqs. (3.5) and (3.6) can be given a simple interpretation. In fact, we can use two independent sets of oscillators (say a_n and $\tilde{a}_n$) for the holomorphic and the antiholomorphic parts of W, and relate Sq and q' to the right and left momenta, q_R and q_L, of a closed string, and $S\lambda$ and λ' to its right and left quantum numbers λ_R and λ_L, according to

$$Sq = q_R \quad , \quad q' = -q_L \quad , \quad S\lambda = \lambda_R \quad , \quad \lambda' = \lambda_L \quad . \tag{3.7}$$

Then, the first factor in Eq. (3.4), namely

$${}_{a}\langle q_L; \lambda_L| \; {}_{\tilde{a}}\langle q_R; \lambda_R| \, W^{a,\tilde{a}} \tag{3.8}$$

can be read as the correctly normalized amplitude among $M+1$ closed strings in which the first M ones are on shell, and the last one is on an arbitrary excited state. In this amplitude, the $SL(2, \mathbf{C})$ invariance has been fixed by the conditions $\xi_1 = 0$, $\xi_M = 1$ and $\xi_{M+1} = \infty$; moreover as is clear from Eq. (3.4), the last excited string is sewn to the operator B which encodes the presence of the boundary.

Since the states $|q; \lambda\rangle$ are eigenstates of L_0, $\lambda = \lambda'$ and $q = q'$. Therefore, Eq. (3.7) implies that the closed string states exchanged between W and B satisfy the conditions $\lambda_L = S\lambda_R$ and $q_L = -Sq_R$, that is $n = 0$ ($w = 0$) along the Neumann (Dirichlet) directions. Thus, the explicit form of the boundary operator is [2]

$$B = \frac{V\,\mathcal{C}_0 \widehat{\mathcal{N}} \alpha'}{4\pi\Phi} \, \frac{1}{2L_0 - 2} \, \prod_{I=0}^{p} \left(\mathrm{e}^{-\mathrm{i}\, Y_I w^I R/\alpha'} \, \delta_{n^I,0} \right) \prod_{J=p+1}^{25} \left(\mathrm{e}^{-\mathrm{i}\, Y_J n^J / R} \, \delta_{w^J,0} \right) \Bigg|_{\lambda_L = S\lambda_R} . \tag{3.9}$$

It is important to realize that the operator B contains the information about the presence of a Dp-brane with coordinates Y_J, as well as a closed string propagator with the level matching condition enforced. This fact is also showed by the normalization of Eq. (3.9): in fact, $V\mathcal{C}_0$ is the factor common to all amplitudes having a disk with no holes as world-sheet, $\widehat{\mathcal{N}}$ signals the emission of a closed string state, and α'/Φ gives the right dimension to the closed string propagator attached to the disk. As we will see in the next section, it is the presence of the closed string propagator in B that makes it easy to write scattering amplitudes with many D-branes in the operator formalism.

The geometrical meaning of the operator B is quite clear. For example, if we sew B to a graph with one closed and N open strings as in Eq. (2.4), we obtain a N-point open string amplitude at one loop. Thus, as expected, B inserts a boundary on the world sheet, transforming the disk into an annulus [6]. The latter turns out to be

[2] From closed string unitarity, one can derive the relation $\widehat{\mathcal{C}_0} \widehat{\mathcal{N}}^2 \alpha' = 4\pi$; thus the prefactor of Eq. (3.6) can be written as in Eq. (3.9).

described in the "crossed channel", where the role of τ and σ is exchanged and the modulus t measures essentially the distance between the two boundaries. Making the modular transformation $t \to 2\pi/t$, one can obtain the one-loop amplitude in the usual configuration as it results when one computes the trace of open string vertex operators (see chapter VIII of Ref. [16]). Furthermore, by carefully comparing the normalization coefficients in the two descriptions, one finds that

$$\widehat{\mathcal{N}} = \mathcal{N}\sqrt{\frac{8\pi\Phi\,\mathcal{C}_1}{V\,\mathcal{C}_0}} \quad , \tag{3.10}$$

where $\mathcal{C}_1 = (2\pi)^{-26}(2\alpha')^{-13}$ is the normalization of the annulus [12].

As a check, let us now consider two boundary operators B_1 and B_2 and sew them together to calculate the interaction between two parallel Dp-branes that exchange closed string states. Since both operators include a propagator, their sewing must be done using the inverse propagator, namely

$$\mathcal{A}|_R = \mathrm{Tr}'\left(B_1^\dagger\ \left(\frac{2\Phi}{\alpha'}(L_0 + \tilde{L}_0 - 2)\right)\ B_2\right) \quad . \tag{3.11}$$

where Tr' means trace over the physical states; this amounts to change the space-time dimension D into $D-2$ in the trace over the non-zero modes. Thus, with this prescription, one obtains

$$\begin{aligned} \mathcal{A}|_R \;=\; & \frac{V\,\mathcal{C}_1}{2\pi}\int_0^\infty dt\ \mathrm{e}^t\ \prod_{n=1}^{\infty}\left(1-\mathrm{e}^{-nt}\right)^{-24} \\ & \times \prod_{I=0}^{p}\sum_{w^I}\exp\left[-t\alpha'\left(\frac{w^I R}{2\alpha'}\right)^2 + 2\mathrm{i}(Y_1-Y_2)_I\left(\frac{w^I R}{2\alpha'}\right)\right] \\ & \times \prod_{J=p+1}^{25}\sum_{n^J}\exp\left[-t\alpha'\left(\frac{n^J}{2R}\right)^2 + 2\mathrm{i}(Y_1-Y_2)_J\left(\frac{n^J}{2R}\right)\right] \quad . \end{aligned} \tag{3.12}$$

It is now easy to take the decompactification limit $R \to \infty$ of Eq. (3.12): in this case the sum over w simply picks up the value $w=0$, while the sum over n becomes a gaussian integral, so that using Eq. (3.10), one gets

$$\mathcal{A} = \lim_{R\to\infty}\frac{V\,\mathcal{C}_1}{2\pi}\int_0^\infty dt\ \left(\frac{4\pi R^2}{\alpha' t}\right)^{(25-p)/2}\left(f_1(\mathrm{e}^{-t/2})\right)^{-24}\exp\left[-\frac{\Delta Y^2}{\alpha' t}\right] \quad , \tag{3.13}$$

where $f_1(q) \equiv q^{1/12}\prod_{n=1}^{\infty}(1-q^{2n})$, and $\Delta Y^2 \equiv \sum\limits_{J=p+1}^{25}(Y_1-Y_2)_J\,(Y_1-Y_2)^J$ is the square of the distance between the two D-branes.

By making the modular transformation $t \to 2\pi/t$, this same amplitude can be reinterpreted as the one-loop free energy of an open string whose ends are fixed on two parallel D-branes. In fact, using the relation $\left(f_1(\mathrm{e}^{-\pi/t})\right)^{-24} = t^{-12}\left(f_1(\mathrm{e}^{-\pi t})\right)^{-24}$ and the explicit expression of $\mathcal{C}_1$, Eq. (3.13) becomes

$$\mathcal{A} = V_{p+1}\int_0^\infty \frac{dt}{t}\,(8\pi^2\alpha' t)^{-(p+1)/2}\left(f_1(\mathrm{e}^{-\pi t})\right)^{-24}\exp\left[-\frac{\Delta Y^2}{2\pi\alpha'}t\right] \quad , \tag{3.14}$$

where $V_{p+1} = (2\pi R)^{p+1}$ is the world volume of the Dp-branes. Eq. (3.14) agrees completely, including the overall normalization, with the result of Ref. [10]. By analyzing the poles of $\mathcal{A}$ due to graviton and dilaton exchanges, one can obtain the tension of the D-brane which is the normalization of its world-volume action.

The same calculation can be done for two non-parallel D-branes, even of different dimensionality; the main change in Eqs. (3.12)-(3.13) is that, in the directions parallel to a D-brane and ortogonal to the other, $f_1(q)$ is replaced by $f_2(q) = q^{1/12}\prod_{n=1}^\infty(1+q^{2n})$ and the zero modes of the exchanged closed string are forced to be zero.

4 Scattering amplitudes with many D-branes

The purpose of this section is to compute the interaction among M closed strings in the presence of h parallel Dp-branes. From the geometrical point of view, this process is associated to a world-sheet $\Sigma_{M;h}$ with M punctures and h boundaries on which $25-p$ string coordinates have Dirichlet boundary conditions. The surface $\Sigma_{M;h}$ can be obtained from a sphere with $M+h$ punctures in which h holes are cut around h punctures. This operation has an explicit realization in the operator formalism used in Refs. [4, 5] for the calculation of multiloop string amplitudes. In this formalism the fundamental object is the multi-Reggeon vertex that generates all scattering amplitudes among arbitrary string states, and encodes all geometric information about the corresponding world-sheet. In particular, to a sphere with $M+h$ punctures one associates a tree-level multi-Reggeon vertex with $M+h$ closed strings, which we denote by $\mathbf{V}_{M+h}$. The second fundamental ingredient that is necessary for our purpose is the boundary state $|B\rangle$ that inserts a hole around a given puncture and enforces the appropriate boundary conditions on the string coordinates. Once $|B\rangle$ is given, the scattering amplitude of M closed strings in the background of h D-branes can be obtained by saturating $\mathbf{V}_{M+h}$ with M on-shell closed string states and h boundary states. The generator of all such amplitudes is then the vertex operator

$$\mathbf{V}_{M;h} \equiv \mathbf{V}_{M+h}\prod_{\mu=0}^{h-1}|B_\mu\rangle \tag{4.1}$$

whose corresponding world-sheet is $\Sigma_{M;h}$. Since this surface is conformally equivalent to a disk with $h-1$ holes and M punctures, one can guess that $\mathbf{V}_{M;h}$ will be similar in its structure to the Reggeon vertex of the open string at $h-1$ loops [4]. We remark that one can construct $\mathbf{V}_{M;h}$ for any M and h, since all legs of $\mathbf{V}_{M+h}$

are off shell. Futhermore, as emphasized in Ref. [5], one of the distinctive features of the operator formalism is that no knowledge of $\Sigma_{M;h}$ is a priori necessary; in fact all geometrical objects of the world-sheet are the result of the sewing precedure and are given explicitly in the Schottky representation.

Let us now give some details. The tree-level multi-Reggeon vertex for the compactified closed string is [5]

$$
\begin{aligned}
\mathbf{V}_M &= \Phi\widehat{\mathcal{C}_0}\widehat{\mathcal{N}}^M \, \mathbf{V}_M^{(gh)} \prod_{i=1}^{M}\left[\sum_{n^i,w^i}\langle n^i,w^i;0|\right]\int\prod_{i=1}^{M}\left(\frac{d^2 z_i}{|z_{i+1}-z_i|^2}\right)\frac{1}{d\widehat{V}_{abc}} \\
&\times\,\delta\Big(\sum_{i=1}^{M}a_0^i\Big)\,\delta\Big(\sum_{j=1}^{M}\tilde{a}_0^j\Big)\;\exp\!\left[-\sum_{i<j=1}^{M}\sum_{k,l=0}^{\infty}a_k^i D_{kl}(\Gamma V_i^{-1}V_j)\cdot a_l^j\right] \\
&\times\;\exp\!\left[-\sum_{i<j=1}^{M}\sum_{k,l=0}^{\infty}\tilde{a}_k^i D_{kl}(\Gamma \bar{V}_i^{-1}\bar{V}_j)\cdot \tilde{a}_l^j\right]\,. \qquad (4.2)
\end{aligned}
$$

In this expression, V_i is a projective transformation related to the choice of the local coordinates around the puncture z_i such that $V_i(0)=z_i$, Γ is the transformation $z\to 1/z$, the matrices D_{nm} are the infinite dimensional representation of the projective group of weight zero, and $|n,w;0\rangle$ is an eigenstate of a_0 and $\tilde{a}_0$ with eigenvalues k_L and k_R as in Eq. (2.5). Finally, the variables z_i are integrated over the whole complex plane and $d\widehat{V}_{abc}$ is the volume of the projective group $SL(2,\mathbf{C})$. For sake of simplicity we do not write the ghost vertex $\mathbf{V}_M^{(gh)}$ which can be found in Ref. [17]. Note that Eq. (4.2) exhibits a complete left-right factorization and its structure is formally similar to the product of two independent open string vertices.

Let us now turn to the boundary state $|B\rangle$. To obtain it, we start from the boundary operator of the previous section which, however, must be modified for two reasons. In fact, the operator B of Eq. (3.9) geometrically represents a closed string propagator attached to a disk, and is formally composed by two parts: the first one identifies the left and the right sectors of a closed string, and the second is an open string propagator that sews them together [3]. In Section 3, the identification was performed by transforming one of the two bras of Eq. (3.8) into a ket, and then the sewing was realized by taking the trace on the open string propagator. Now the situation is slightly different: in fact, since the state attached to the boundary operator is not necessarily fixed at $z=0$ or $z=\infty$, it is represented by a vertex $\mathcal{W}$, and not simply by a bra or a ket. Thus, to identify the two sectors of the closed string we have first to take the adjoint of, say, the right part of $\mathcal{W}$. This operation contains a twist since the transformation property of the vertices is $\mathcal{V}^\dagger_{\beta_R}(1/\bar{z}) = z^2(-1)^{\ell_R}\mathcal{V}_{\beta_R}(z)$, where ℓ_R is the level of the state. If one wants that the closed string ends on a membrane (and not on a crosscap), this twist has to be removed. This can be done by sewing the two sectors of the closed string by means of a *twisted* propagator.

[3]This explains why the operator B brings only *one* real modulus, even if it contains a closed string propagator.

The second modification of B stems out of the presence of unphysical states. In Section 3 they have been projected out by hand, but in the background of three or more D-branes this is not possible, and thus one has to include ghosts and work with BRST invariant objects. Since the boundary conditions of the string coordinates do not influence the local geometry on the world-sheet, the reparametrization ghosts do not feel the boundary conditions, and are the same as in the standard Neumann open string theory. Therefore, the propagator $1/(L_0 - 1)$ in Eq. (3.9) must be replaced by a BRST invariant twisted open string propagator T. Its explicit form depends on the local coordinates V_i around the punctures that are sewn together; for example, if Lovelace coordinates are used, we have [17]

$$T = (b_0 - b_1) \int_0^1 \frac{dx}{x(1-x)} P(x) \tag{4.3}$$

where $P(x)$ is the operator that realizes the transformation $z \to (xz-x)/(xz-1)$ [4], and b_0 and b_1 are the antighost zero-modes. (For the ghost fields, here and in the following we adopt the notations of Ref. [18]). By attaching this propagator to the operator that identifies the left and right sectors of the closed string [6, 7, 8, 9] one gets the BRST invariant state

$$|B\rangle = \sqrt{\frac{V\,\mathcal{C}_1 \alpha'}{2\pi\Phi}} \int_0^1 \frac{dx}{x(1-x)} |B(x)\rangle_X |B(x)\rangle_{gh} \quad , \tag{4.4}$$

where

$$\begin{aligned} |B(x)\rangle_X \;&=\; \exp\left[- \sum_{n,m=0}^{\infty} a_n^\dagger D_{nm}(P(x)) \cdot S \tilde{a}_m^\dagger \right] \\ &\quad \times \, \mathrm{e}^{-iY\cdot(a_0 - S\tilde{a}_0)} \prod_{I=0}^{p} \left(\sum_{w^I} |0, w^I; 0\rangle \right) \prod_{J=p+1}^{25} \left(\sum_{n^J} |n^J, 0; 0\rangle \right) \quad , \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} |B(x)\rangle_{gh} \;&=\; (b_0 - b_1) \exp\left[\sum_{k,l=-1}^{\infty} c_k^\dagger E_{kl}(P(x)) \tilde{b}_l^\dagger + \sum_{m,n=2}^{\infty} \tilde{c}_m^\dagger E_{mn}(P(x)) b_n^\dagger \right] \\ &\quad \times \exp\left[- \sum_{n=2}^{\infty} \sum_{r,s=-1}^{1} \tilde{c}_n^\dagger E_{nr}(P(x)) E_{rs}(\Gamma P(x)) \tilde{b}_s^\dagger \right] |q=0\rangle |\tilde{q}=3\rangle \quad , \end{aligned} \tag{4.6}$$

with E_{nm} being the infinite dimensional representation of the projective group of weight -1 [17]. Notice that this structure of the boundary state $|B\rangle$ is completely general; if different local coordinates are used, the only thing that changes is the

[4] Note that this transformation contains a twist since its determinant is negative.

explicit form of the twisted propagator T. The operator $|B(x)\rangle_X$ satisfies the equations

$$\left(a_n + \sum_{m=0}^{\infty} D_{nm}(P(x)) S\, \tilde{a}_m^\dagger\right) |B(x)\rangle_X = 0 \tag{4.7}$$

with $n > 0$, and

$$(a_0 + S\, \tilde{a}_0)\, |B(x)\rangle_X = 0 \quad . \tag{4.8}$$

Analogous equations hold for $|B(x)\rangle_{gh}$. We remark that the boundary states considered in the literature [6, 7, 8, 9] satisfy Eqs. (4.7) and (4.8) with D_{nm} replaced by δ_{nm}.

It is easy to check that by saturating one leg of $\mathbf{V}_{m+1}$ of Eq. (4.2) with $|B\rangle$, one obtains an operator

$$\mathbf{V}_{M;1} = \mathbf{V}_{M+1}\, |B\rangle \quad , \tag{4.9}$$

which generates the amplitudes (2.11) when furtherly saturated with M physical closed string states. Notice that the insertion of the boundary soaks up three real parameters of $SL(2, \mathbf{C})$ leaving a residual $SL(2, \mathbf{R})$ invariance.

We have now all ingredients to compute $\mathbf{V}_{M;h}$; it is only a matter of saturating $\mathbf{V}_{M+h-1;1}$ with further $h-1$ boundary states, and of using the oscillator algebra to calculate the expectation values in Eq. (4.1). In the ghost sector, this task is simplified by saturating also the remaining M legs with the ghost part of the external physical states, which, due to BRST invariance, can always be written as $c_1\, \tilde{c}_1 |q = \tilde{q} = 0\rangle$. As we have already stressed, the calculation over the orbital part gives the geometrical objects of the surface in the Schottky representation, while the ghost sector simply modifies the measure of integration. Note that the presence of Dirichlet or Neumann boundary conditions does not alter the result of the traces on the non-zero modes. In fact, after the insertion of the first $|B\rangle$ (Eq. (4.9)) the right part of all external states appears multiplied by the factor S, see Eq. (2.11). When further boundaries are introduced, one identifies the right and the left part of the string field by means of a propagator that contains another factor S, see for example Eq. (4.5). Thus, only $S^2 = 1$ appears and the calculation of the traces is the same as in the standard open string theory (see Ref. [5] for details). Evidence of this fact already appeared in Section 3: indeed, the trace in Eq. (3.12) yielded the usual partition function $f_1(q)$ characterizing the one-loop amplitudes of the open string for all 24 transverse directions independently of the boundary conditions. Thus, as long as the matrix S is equal for all the boundaries, *i.e.* the D-branes are parallel, a sphere with h boundaries yields the same geometrical objects that describe an amplitude of open string with $h-1$ loops.

We now concentrate on the decompactification limit $R \to \infty$ and write explicitly $\mathbf{V}_{M;h}$ in terms of the prime form $E(z_i, z_j)$, the first abelian differential $\omega(z)$ and the period matrix $\tau_{\mu\nu}$ of the surface $\Sigma_{M;h}$. As we have already seen in the previous section, in this limit the only possible value for the winding number is $w = 0$ and thus the zero modes a_0 and $\tilde{a}_0$ have to be identified, while the structure of the orbital vacua in Eq. (4.5) becomes

$$\prod_{I=0}^{p}\left(\sum_{w^I}|0,w^I;0\rangle\right)\prod_{J=p+1}^{25}\left(\sum_{n^J}|n^J,0;0\rangle\right)\rightarrow\prod_{I=0}^{p}|0;0\rangle\prod_{J=p+1}^{25}\left(\int|q^J;0\rangle dq^J\right)\quad . \tag{4.10}$$

In order to write $\mathbf{V}_{M;h}$ in a more compact form, we fix $2\alpha'=1$, neglect the normalization factors and introduce a new convention for the oscillators

$$\alpha_n^i=\begin{cases}\sqrt{n}\,a_n^i\\ \\ \sqrt{n}\,S\,\tilde{a}_n^{2M-i+1}\end{cases}\quad,\quad \alpha_0^i=\begin{cases}a_0^i & \text{for } i=1,\ldots,M\,,\\ \\ Sa_0^{2M-i+1} & \text{for } i=M+1,\ldots,2M\,.\end{cases} \tag{4.11}$$

Then, after integrating over the loop momenta q^J along the Dirichlet directions, and setting $Y_0=0$, the Reggeon vertex $\mathbf{V}_{M;h}$ for the orbital degrees of freedom can be written as

$$\begin{aligned}\mathbf{V}_{M;h} \;=\; & \prod_{i=1}^{M}\left(\int dp_i\langle p_i;0|\right)\,\delta^{p+1}\left(\sum_{i=1}^{M}\alpha_0^i\right)\int dm_h\;(\det\mathrm{Im}\,\tau)^{\;(p-25)/2}\\ & \times\exp\left(\sum_{i<j=1}^{2M}\sum_{n,m=0}^{\infty}\frac{\alpha_n^i}{n!}\partial_z^n\partial_y^m\log E(V_i(z),V_j(y))\Big|_{z=y=0}\frac{\alpha_m^j}{m!}\right)\\ & \times\exp\left(\frac{1}{2}\sum_{i=1}^{2M}\sum_{n,m=0}^{\infty}\frac{\alpha_n^i}{n!}\partial_z^n\partial_y^m\log\frac{E(V_i(z),V_i(y))}{V_i(z)-V_i(y)}\Bigg|_{z=y=0}\frac{\alpha_m^i}{m!}\right)\\ & \times\prod_{J=p+1}^{25}\exp\Bigg\{\frac{1}{2}\sum_{\mu,\nu=1}^{h-1}\left[\left(\sum_{i=1}^{2M}\sum_{n=0}^{\infty}\frac{\alpha_n^i}{n!}\partial_z^n\int_{z_0}^{V_i(z)}\omega^\mu\right)\Bigg|_{z=0}-2\mathrm{i}Y^\mu\right]_J\\ & \times(2\pi\mathrm{Im}\tau)^{-1}_{\mu\nu}\left[\left(\sum_{j=1}^{2M}\sum_{m=0}^{\infty}\frac{\alpha_m^j}{m!}\partial_y^m\int_{z_0}^{V_j(y)}\omega^\nu\right)\Bigg|_{y=0}-2\mathrm{i}Y^\nu\right]_J\Bigg\}\quad ,\end{aligned} \tag{4.12}$$

where the local coordinates V_i of the external states have been chosen as

$$V_i(z)=\begin{cases}z+z_i & \text{for } i=1,\ldots,M\,,\\ \\ z+\bar{z}_{2M-i+1} & \text{for } i=M+1,\ldots,2M\,,\end{cases} \tag{4.13}$$

and the measure dm_h, that already takes into account the ghost contributions [4], is

$$dm_h=\frac{\prod_{i=1}^{M}d^2z_i}{dV_{abc}}\prod_{\mu=1}^{h-1}\left(\frac{dk_\mu d^2\eta_\mu(1-k_\mu)^2}{k_\mu^2(\eta_\mu-\bar{\eta}_\mu)^2}\right)\prod_\alpha{}'\left(\prod_{n=1}^{\infty}(1-k_\alpha^n)^{-D}\prod_{n=2}^{\infty}(1-k_\alpha^n)^2\right)\;. \tag{4.14}$$

The parameters in Eq. (4.14) are the moduli of the surface $\Sigma_{M;h}$ in the Schottky representation, namely a real multiplier k_μ and two complex conjugate fixed points η_μ and $\bar{\eta}_\mu$ for each of the $h-1$ generators of the Schottky group. The explicit expressions of the prime form, abelian differentials and period matrix in terms of the Schottky parameters can be found in Ref. [5]. The vertex $\mathbf{V}_{M;h}$ depends on $2M + 3(h-1) - 3$ moduli, that is exactly what one expects, because of the close analogy between multi-brane processes and multi-loop open string amplitudes. Note that for $M = 0$ and $h = 2$ Eq. (4.12) gives the same result for the interaction of two D-branes found in Section 3. It is interesting to note that, along the Dirichlet directions, the bilinear part of Eq. (4.12) with $i \neq j$ can be rewritten in terms of the Dirichlet Green function used in the functional approach. If $i = j$, where the Green function is singular and the functional approach requires a suitable regularization [19], our operator formalism already gives a well-defined expression because the closed string emission vertex is separately normal ordered in the left and the right parts.

We end by commenting that our formalism is suitable for several extensions and applications; in particular it is possible to take into account the emission of open strings from the boundaries, consider the case of many non parallel D-branes or boost the boundary states. We plan to extend this formalism to the superstring and compute the field theory limit of the multi-membrane scattering amplitudes. Recently some of these issues have been addressed in Ref. [20].

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