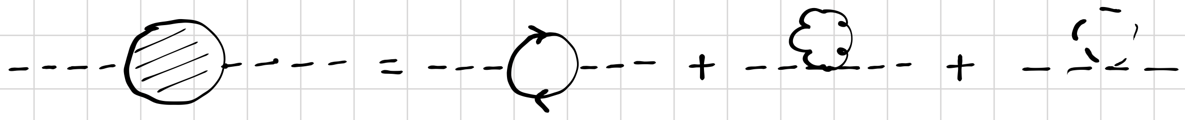
BSM models

- Hierarchy problem in SM:

$$V = -\underbrace{\mu^2}_{\text{mass param.}}|H|^2 + \lambda |H|^4$$

mass param. →



$$\delta\mu^2 = \frac{\Lambda^2}{32\pi^2}\left\{ -y_t^2 + \frac{1}{4}\left(9g^2 + 3g'^2\right) + 6\lambda \right\}$$

↳ quadratically div

We know:

$$v = 246 \text{ GeV}$$
$$m_H \simeq 125 \text{ GeV} = \sqrt{2\lambda}\, v \qquad \left(\lambda \sim \tfrac{1}{8}\right)$$

⇒ What is Λ ?

$\Lambda \gg \mu \Rightarrow$ very large corrections: $\Lambda \sim M_P = 10^{19}$ GeV (big hierarchy)

$\Lambda \sim 10$ TeV (little hierarchy)

⇒ What really is Λ ? (artifact of calc or deeper meaning?)

Λ: physical cutoff scale of effective theory (SM) → mass scale of new particles BSM

[theoretical reasons]

SM is not the FINAL th.! $\left\{\begin{array}{l} - U(1)_Y \text{ is not asymptotically free} \\ - \text{Gravity } [@ M_{Pl} \sim 10^{19} \text{ GeV}] \end{array}\right.$

→ We expect new thresholds to show up! ⇒ new particles! [@ Λ mass scale!]

↳ H is <u>quadratically</u> <u>sensitive to any new mass scale</u>

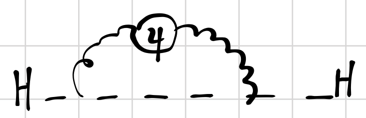
[NB: in DIMENSIONAL REGULARIZATION there are no quad. divergence!

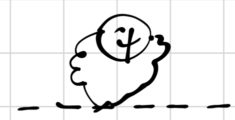
↳ introduce new particles: $\lambda_s |H|^2 |S|^2$ (new superheavy particle)

 $\propto \frac{\lambda_s}{16\pi^2}\left[\cancel{\Lambda_{UV}^2} - 2m_s^2 \ln\left(\frac{\Lambda}{m_s}\right) + O(m_s^2)\right]$

even if I forget Λ_{UV}, I will find sth $\propto m_s^2$ ⟶ NEW PHYSICS SCALE!]

If we were to intro. sth NOT directly coupled to H → new massive fermions (with SM quantum no.) ⇒ 2 LOOPS CONTRIBUTIONS:





$$\downarrow$$

$$\frac{m_\psi^2}{(16\pi^2)^2}$$

⇒ anytime we introduce (super) heavy particles we find sth like

$$\delta m_H^2 \propto \Lambda^2$$

but then why didn't this new physics push m_H much higher?

$$\Downarrow$$

HIERARCHY PROBLEM : $\mu^2 |H|^2$ relevant operator in SM expected to grow towards the IR!

→ If I start in UV with arbitrary b.c.'s for most trajectories I don't expect a light Higgs mass

NB Hierarchy problem is specific to elementary scalars!

↳ fermion masses do not have this problem ⇒ m_ψ are protected by chiral symmetry (when $m_\psi \to 0$)

$$\Rightarrow \quad \bar\psi(i\partial\!\!\!/ - m)\psi = \chi_L^\dagger i\bar\sigma^\mu \partial_\mu \chi_L + \psi_R^\dagger i\sigma^\mu \partial_\mu \psi_R - m(\chi_L^\dagger \psi_R + \psi_R^\dagger \chi_L)$$

$m \to 0$: $U(1)_L \times U(1)_R$ chiral symmetry

↳ mass correction must be proportional to the symm. breaking term

$$\Delta m_\psi \propto m_\psi \ln\left(\frac{\Lambda}{m_e}\right)$$

The same applies to spin 1 ⇒ $M_W \to 0$: gauge symmetry is restored $\left[\Delta M_W^2 \propto M_W^2 \ln\left(\frac{\Lambda}{M_W}\right)\right]$

One possible way to avoid hierarchy problem:

$$\delta m_H^2 = \frac{1}{32\pi^2}\left[-y_t^2\Lambda^2 + \frac{1}{4}\left(9g^2+3g'^2\right)\Lambda^2 + \dots\right]$$

maybe they cancel!

$\Rightarrow$ not the right way!

- Λ^2 for t loops might be $\neq$ from vector bos.

$$\rightarrow -y_t^2\Lambda_T^2 + \frac{1}{4}\left(9g^2+3g'^2\right)\Lambda_g^2$$

- even if $\Lambda_T = \Lambda_g$, H self interactions do not lead to 125 GeV

These are some motivations to go BSM $\longrightarrow$ what is the way out?

- Λ is actually not that high: new physics early on

$\delta m_H^2 \propto \frac{\Lambda^2}{32\pi^2} \longrightarrow \boxed{\Lambda \sim 1\,\text{TeV}} \longrightarrow$ new particles s.t. UV completion is SPECIAL

$\boxed{\text{NO NEW CONTRIBUTIONS TO QUADR. SENSITIVITY IN THE MORE COMPLETE TH!}}$

* Supersymmetry: new symm. relating fermions and bosons (Higgs $\longleftrightarrow$ Higgsino)

$\Rightarrow$ SUSY + chiral sym for ferm. are sufficient to protect Higgs mass

(NB: the higher we push the limit of superpartners the less we solve the hierarchy problem)

* Goldstone theorem: spontaneously broken global symmetry
* Composite Higgs models: no fundamental scalars (bound state of ferm) [still needs Goldstone thm]

AdS/CFT

* Warped extra-dimensions:
* Gauge-Higgs unification: extra-dimens $\longrightarrow$ 5 gauge fields $\longrightarrow$ A_5 looks like a scalar!

$\Downarrow$

5D Gauge symm. protects Higgs mass $\longrightarrow$ NO HIERARCHY PROBL

More radical idea (excluded): TECHNICOLOR

+ new strong interactions (no elem. scalars)

↳ condensate directly broke EW symm. (NO Higgs)

* Large extra-dimensions: Weak scale IS the fundamental scale!
* Relaxion: μ^2 could be very large, but it's cosmologically scanned [i.e. "time dep"]
  ↳ EW symm. breaking will trigger the stopping of scanning.
* Anthropic consideration: ok with hierarchy problem → multiverse: each has diff. m_H^2
  [m_H^2 large: no chemistry, no life]

Goldstone theorem:

spontaneously broken GLOBAL symmetry → MASSLESS THEORY

↳ If minimum breaks symmetry → there has to be a valley of minima

[minimum → apply symm → new minimum ⇒ continuous valley of minima]

↳ GOLDSTONE BOSON := motion along the valley of diff. vacua

- acting with BROKEN generator on a VEV will move along the valley.
- acting with UNBROKEN will not change anything

e.g.:
- th of scalar.
- strongly interacting (QCD-like): $G = SU(3) \times SU(3)$ → $H = SU(3)_L$

$\to e^{i\varepsilon^a T^a} U_0$

$\varepsilon^a(x) := \frac{\pi^a(x)}{f}$; T^a: broken direction ; U_0: vacuum

⇓

Goldstone boson field: $U(x) = e^{i\pi^a T^a/f}\, U_0$. → non-linearly realised field

⇒ EFT for Goldstone's → ∃Λ cutoff: Some new particles have to show up here

QCD and chiral Lagrangian

→ Consider 3 flavours (u, d, s)

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \sum \bar{q}_i (i\not{D} - m_q) q_i$$

→ if $m_q \to 0$: $SU(3)_L \times SU(3)_R$ global symmetry.

$$\Psi = \begin{pmatrix} \chi \\ \bar{\psi} \end{pmatrix} \longrightarrow \text{use } \chi, \psi \text{ LH in } 3, \bar{3}$$

$$\Rightarrow i\chi^\dagger \bar{\sigma}^\mu D_\mu \chi + i\psi^\dagger \bar{\sigma}^\mu D_\mu \psi + m(\psi^\alpha \chi_\alpha + h.c.)$$

In flavour space:

$$\begin{aligned} \psi &\to U_R \psi \\ \chi &\to U_L \chi \end{aligned} \quad \Rightarrow \quad SU(3)_L \times SU(3)_R \text{ global symm. of QCD}$$

- naïve expectation → should be realised in the bound state spectrum $\boxed{SU(3) \text{ of Gell-Mann}}$

→ it's actually smaller than that! (smaller than what expected from $\mathcal{L}_{QCD}$)
↳ WAY OUT: QCD vacuum is NON TRIVIAL

$$\langle \bar{q}_i q_j \rangle = \langle \bar{q}_{Li} q_{Rj} + h.c. \rangle \propto \delta_{ij} \Lambda^3_{QCD}$$

quark condensate (colour indices are contracted: $SU(3)_c$ is unbroken)

$U_L = U_R$: $SU(3)_V$ → unbroken Gell-Mann's $SU(3)$

$U_L = U_R^\dagger$: $SU(3)_A$ → spont. broken in QCD

We expect 8 Goldstone bosons → they are not massless because the symm. is not exact

$$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta \qquad \text{(PSEUDO-GOLDSTONE bosons)}$$

Where is EW symm.?

expectation: SM $SU(2)_L$ is embedded in $SU(3)_L$ $\left[\boxed{\begin{pmatrix} u \\ d \end{pmatrix} \} SU(2)_L} \text{ with heavy } c \right]$ $\begin{matrix} s \\ (c) \end{matrix} \} SU(2)$

Where is the EW hypercharge?

$$Y = (T_R)_3 + \frac{1}{2} B$$

$\Rightarrow$ EFT

Act with broken generator on VEV:

$\langle \bar{q} q \rangle$ is in bi-fundamental of $SU(3)_L \times SU(3)_R$

$$\Rightarrow \quad U_0 = \underset{\substack{\uparrow \\ \text{(scale)}}}{f} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$\Rightarrow U(x) = \underbrace{U_L U_0 U_R^\dagger}_{\text{general transf. of a bi-fundamental}}$

$U_L = U_R$: unbroken

$U_L = U_R^\dagger$: broken $\Rightarrow U_L = e^{i \frac{\pi^a}{f} T^a} = U_R^\dagger$

$$\Downarrow$$

$$U(x) = f\, e^{2i\pi^a(x)\frac{T^a}{f}}$$

$\rightsquigarrow$ how does this transforms? $U(x) \to U_L U(x) U_R^\dagger$

- unbroken symm.: $U(x) \to U_V U(x) U_V^\dagger$ [regular linear transf of adj]
  $\hookrightarrow \pi \to U_V \pi U_V^\dagger$

- broken symm.: $U(x) \to U_A U(x) U_A$

$$\Rightarrow e^{2i\pi'/f} = e^{2ic^a T^a} e^{2i\frac{\pi}{f}} e^{2ic^a T^a}$$

$$\to \mathbb{I} + 2i\frac{\pi'}{f} = (\mathbb{I} + 2ic^a T^a)(\mathbb{I} + 2i\frac{\pi}{f})(\mathbb{I} + 2ic^a T^a) =$$

$$\sim \mathbb{I} + \dots$$

$\to$ leading term linear in c^a's doesn't cancel!

$\hookrightarrow \pi'^a T^a = \pi^a T^a + f c^a T^a + O(\pi^2, c^2)$

$\Rightarrow \pi^a \longrightarrow \pi^a + c^a$ SHIFT SYMMETRY

Now I want to write a Lagrangian!

Just write all terms that are invariant:

<u>NO DERIV</u> • $\mathrm{Tr}\left[U^\dagger U\right] = \text{const}$

<u>2 DERIV.</u> • $\frac{f^2}{4}\mathrm{Tr}\left[(\partial_\mu U^\dagger)(\partial^\mu U)\right]$ ⟶ at lowest level this is the term that matters

$$\frac{f^2}{4}\mathrm{Tr}\left[(\partial_\mu U^\dagger)(\partial^\mu U)\right] = \frac{1}{2}\partial_\mu\pi\cdot\partial^\mu\pi + \underbrace{\frac{1}{f^2}(\partial_\mu\pi)(\partial^\mu\pi)\pi^2 + \dots}_{\text{INTERACTIONS}}$$

How to write interactions between $\pi^a(x)$ and EW sector (W, Z, γ)?

$$SU(2)_L \times U(1)_Y \subset SU(3)_L \times SU(3)_R \times U(1)_B$$

• $\partial_\mu \to D_\mu$

$$(\tau^a)^i_{\ k} = \left(\begin{array}{c|c} \frac{1}{2}\tau^a & \\ \hline & \end{array}\right) \subset SU(3)_L$$

Then:

$$D_\mu U^i_{\ j} = \partial_\mu U^i_{\ j} - ig W^a_\mu \left(\tfrac{1}{2}(\tau^a)^i_{\ k}\right) U^k_{\ j} + ig' B_\mu \left(\tfrac{1}{2}(\tau_{3R})^k_{\ j}\right) U^i_{\ k}$$

$\Rightarrow \frac{f^2}{4}\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] \Rightarrow$ GIVES A MASS TO W and Z!

(because we embedded $SU(2)_L \subset SU(3)_L$: QCD vacuum breaks EW symmetry!)

↳ W, Z get part of their masses from QCD! (NB without Higgs!)

$$\Rightarrow \frac{g^2 f^2}{4} W^+_\mu W^{\mu -}, \quad \frac{g^2+g'^2}{4} f^2 \, \frac{1}{2} Z_\mu Z^\mu$$

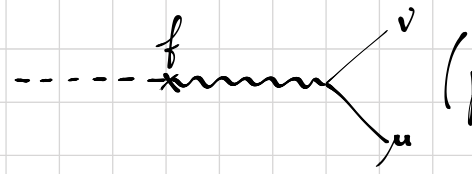
same structure as in SM from Higgs

How do I fix f?

$$\frac{g}{2} f \, W^+_\mu \partial^\mu \pi^-(x) + h.c.$$

[NB Higgs has larger contribution than π] ⟶ < GeV

↳ (pion decay) ⟶ $f_\pi \simeq 130$ MeV



How do I take care of EXPLICIT BREAKING (and Spurion analysis)?

$\boxed{\bar{q}\, m_q\, q}$ $\longrightarrow$ explicit breaking term $\longrightarrow$ how to include the effect in U?

→ what does it take to make this symm.?

- if m_q was actually a <u>field</u> then I could make it symm. (bifundamental)

  ↳ VERY HEAVY field with VEV ⇒ I do not see it in the EFT because they're integrated out!

  I know the breaking term $(\bar{q} m_q q)$ so that the analysis makes sense!

Now I have U, m_q both <u>bi-fundamental</u>:

- write invariants: $\mu^2 \, \mathrm{Tr}\left[m_q U\right] + h.c. =$

$$= \mu^2 \, \mathrm{Tr}\left[\begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} e^{2i\pi^a \frac{T^a}{f}}\right] + h.c.$$

$$\Rightarrow \quad \Delta m_\pi^2 \propto \frac{m_q}{f}\mu^2$$

Consider $m_u \sim m_d \longrightarrow$ ex: show $m_\eta^2 + m_\pi^2 = 4 m_K^2$ (Gell-Mann Okubo mass formula)

ex: QED case → mass contribution:

$U(1)_{EM} \subset SU(3)_V \Rightarrow Q = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}$ (treat this as spurion)

$\Delta f \sim \mathrm{Tr}\left[Q U^\dagger Q U\right]$

CUTOFF and NDA for chiral Lagrangian

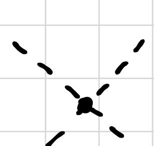
- Has built-in cutoff:
  - where do new particles show up?

⇒ Estimate for cutoff (naive dimensional analysis - NDA):

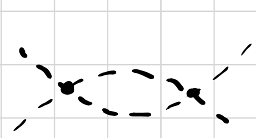
Λ: Scale at which pert. theory ceases to be meaningful (inside EFT)

⇒ require 1-loop contrib. < tree level

$$\frac{f^2}{4}\mathrm{Tr}\left[(\partial_\mu U)(\partial^\mu U^\dagger)\right] \Rightarrow$$



$$\frac{1}{f^2}(\partial_\mu \pi)^2 \pi^2 \longrightarrow \frac{p^2}{f^2}$$



$$\frac{1}{f^4}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2)^2}\, p^2 k^2 \sim \frac{p^2}{f^4}\cdot\frac{1}{16\pi^2}\Lambda^2$$

$1/2\sqrt{2}$

$m_H = \sqrt{\lambda}\, v$ shows up at $\sqrt{\lambda} f < 4\pi f$ ⟸ weakly interacting particles can show up earlier!

Then:

AT MOST

$$\frac{p^2}{f^4} \cdot \frac{1}{16\pi^2} \Lambda^2 \overset{!}{=} p^2/f^2 \implies \Lambda = 4\pi f$$

[AT THE LATEST: at scale $4\pi f$ some new particles have to show up!]

UPPER BOUND! (they can show up earlier)

How to use these for CH models?

- simplest idea: scale up QCD → $\Lambda_{QCD} \sim$ TeV → TECHNICOLOR $\left(SU(2)_L \times SU(2)_R\right)$

  proved wrong! $\langle \bar{q}_{TC} q_{TC} \rangle \sim \Lambda^3_{QCD}$

- Higgs composite:

  - expect in generic theory: $\Lambda = \begin{cases} 100 \text{ GeV} \longrightarrow \text{NO (LEP, Tevatron, ...)} \\ 1 \text{ TeV} \longrightarrow \text{Higgs is a GOLDSTONE boson } (m_H \sim 125 \text{ GeV}) \end{cases}$

Generic theory of $\Lambda = 1$ TeV problematic because of EWPT

→ $\Lambda = 10$ TeV ⟶ safe from generic EW PT

↳ 125 GeV ⟶ $\frac{\Lambda^2}{16\pi^2}$ ⟶ "little hierarchy problem"

How can we make the Higgs a Goldstone?

- light particle ⇝ $\Lambda \sim 10$ TeV, $m_H = 125$ GeV (hierarchy)
- no derivative inter. ⟶ pseudo - GB

⇒ **COLLECTIVE SYMM. BREAKING**

$G = SU(3)$

$H = SU(2)$

$G \to H$ [8 - 3 = 5 broken directions] $\left(\begin{array}{cc|c} & & \times \\ & & \times \\ \hline \times & \times & \end{array}\right)$

$$\Rightarrow \quad \Pi = \left(\begin{array}{cc|c} \eta/\sqrt{6} & & \\ & \eta/\sqrt{6} & H \\ \hline & H^\dagger & -\frac{2\eta}{\sqrt{6}} \end{array}\right)$$

H: SU(2) doublet

η: "the 5th GB needed"

* Naive attempt 1 $\longrightarrow$ gauge SU(2) group $\Rightarrow$ EXPLICIT BREAKING OF SU(3)

[explicit non-derivative interactions]

$\Rightarrow$ [as in $U(x) = e^{i\pi^a \frac{T^a}{f}} U_0$]

$$\phi = e^{i\Pi/f}\begin{pmatrix}0\\0\\f\end{pmatrix} \sim \begin{pmatrix} iH \\ f - \frac{H^\dagger H}{2f}\end{pmatrix}$$ + higher order in H's (and η term)

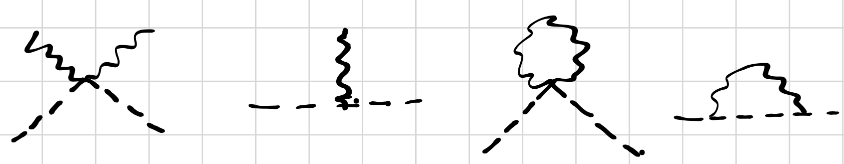
↳ $(D_\mu\phi)^\dagger(D^\mu\phi) \longrightarrow (\partial_\mu\phi)^\dagger(\partial^\mu\phi) = |\partial_\mu H|^2 + |\partial_\mu H|^2 H^\dagger H$

$\left[D_\mu = \partial_\mu - igW^a_\mu \begin{pmatrix} \tau^a/2 & \\ & \end{pmatrix}\right]$

↳ gauged SU(2)

$\Rightarrow (D_\mu H)^\dagger(D^\mu H) = |D_\mu H|^2 + |D_\mu H|^2 \frac{H^\dagger H}{f}$ $\longrightarrow$ breakes shift symm.

↳ we reintroduced $|gWH|^2$



$\Rightarrow$ quadratic divergences! $\rightarrow$ cannot respect hierarchy

The problem is in:

$$\left| g \begin{pmatrix} W_\mu & \\ & 0\end{pmatrix}\phi\right|^2$$

$\rightarrow$ COLEMAN-WEINBERG potential:

$$\frac{\Lambda}{16\pi^2}\mathrm{Tr}(M^\dagger M) + \frac{3}{64\pi^2}\mathrm{Tr}(M^\dagger M)^2 \ln\left(\frac{M^\dagger M}{\Lambda^2}\right)$$ $\rightarrow$ M: field dep. mass for GB

"spurion"

$$M = \begin{pmatrix}1&&\\&1&\\&&0\end{pmatrix}\phi$$

$\Rightarrow$ sub. in CW pot:

$$\frac{\Lambda^2}{16\pi^2}\phi^\dagger M^\dagger M\phi \longrightarrow \frac{g^2\Lambda^2}{16\pi^2}(H^\dagger H)$$

* Naive attempt no. 2 $\longrightarrow$ gauge the entire $SU(3)$

$$M \to \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$\to$ no quad. div. but also no GB (no Higgs)

* FINAL ATTEMPT: two copies of $\phi \Rightarrow \phi_1, \phi_2$

$$\begin{array}{ccc} & SU(3) \times SU(3) & \text{(global starting symm.)} \\ H_1 \swarrow & \downarrow \quad\quad \downarrow & \searrow H_2 \\ & SU(2) \times SU(2) & \end{array} \Rightarrow \text{gauge } \begin{array}{c} SU(3)_D \text{ (DIAGONAL)} \\ \downarrow \\ SU(2)_D \end{array}$$

$\to$ one Higgs disappears and a combination remains! $\to$ "best of both worlds"

$\rightsquigarrow f_1 = f_2 = f$ VEV (aligned for simplicity)

$$\phi_1 = e^{i\pi_1/f}\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \qquad \underline{\pi_1 + \pi_2 \text{ eaten}}$$

$$\phi_2 = e^{i\pi_2/f}\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \qquad \underline{\pi_1 - \pi_2 \text{ physical}}$$

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 \longrightarrow \text{quad div } \frac{\Lambda}{16\pi^2} g^2 \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2\right) \Rightarrow \text{NO QUAD DIVERG.}$$

$\downarrow$

collective symm. breaking

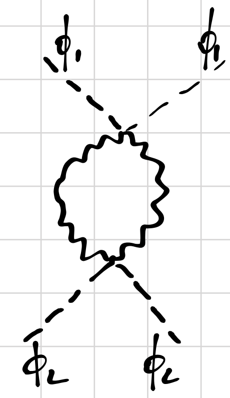
Special structure of explicit breaking:

* several explicit breaking term $\rightsquigarrow$ more than one turned on to truly break the global symm.

Suppose: $|g A_\mu \phi_1|^2 + |g A_\mu \phi_2|^2 \longrightarrow$ if I switch off one of them then $SU(3)_1 \times SU(3)_2$ is unbroken!

Only if both of them are on then we can have a symm breaking!

↳ potential:



$\Rightarrow$ NOT quad. divergent

$\Rightarrow$ SIMPLEST LITTLE HIGGS (Schmaltz)

$$\underbrace{|\phi_1^\dagger \phi_2|^2 \frac{g^2}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right)} \longrightarrow \phi_1^\dagger \phi_2 = f^2 - \frac{2H^\dagger H}{f} + \dots$$

$$\Rightarrow \text{MASS CORR.}: \quad \frac{g^4}{16\pi^2} f^2 \ln\left(\frac{\Lambda^2}{\mu^2}\right)(H^\dagger H) \longrightarrow \delta m = \frac{f}{4\pi} \sim 100\ \text{GeV}$$

Who cancelled the quad div.? → the additional SU(3) GB!

$$\begin{pmatrix} & W^+ & X^+ \\ W^- & & Y_1^0 + iY_2^0 \\ X^- & Y_1^0 - iY_2^0 & \end{pmatrix} + \quad A_3, A_8 \text{ in diagonals}$$

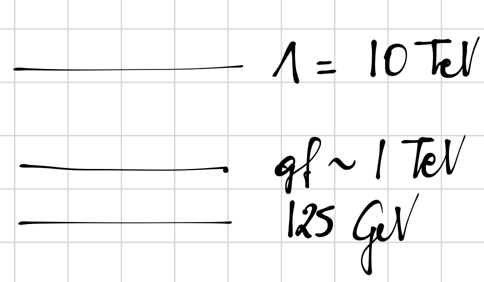
$\boxed{W_\mu^\pm \quad A_\mu^3}$ ~ W, Z → no mass

$X_\mu^\pm$, $Y_{1,2}$, A_μ^8 ⇒ Mass

$$\hookrightarrow |(\partial_\mu - ig A_\mu^a T^a)\phi_{1,2}|^2$$

$$\Rightarrow \frac{g^2}{4} H^\dagger H \left[2W_\mu^+ W_\mu^- + A_\mu^3 A^{\mu 3} - X_\mu^+ X^{\mu -} - \frac{1}{2}\left(Y_{1\mu}^0 Y_1^{0\mu} + Y_{2\mu}^0 Y_2^{\mu 0}\right) - A_\mu^8 A^{\mu 8}\right]$$

→ mass of spin 1 particle: gf



→ FERMIONS?

Intro: • $SU(3) \times SU(3)$ global $\Rightarrow \begin{pmatrix} t \\ b \end{pmatrix}_L \longrightarrow \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L = \Psi$

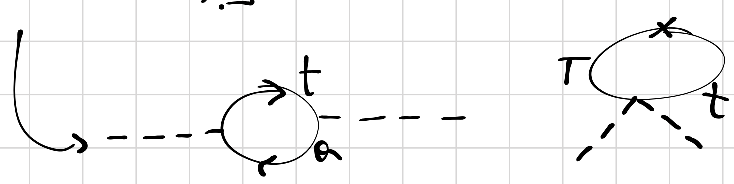
• t_1^c, t_2^c (2 right handed tops)

$$\Rightarrow \lambda_1 \phi_1^\dagger \Psi t_1^c + \lambda_2 \phi_2^\dagger \Psi t_2^c \text{ — trivially collective symm}$$

$\nearrow \pi_1 + \pi_2 = 0$

$$\Rightarrow \mathcal{L} = \frac{\lambda}{\sqrt{2}} \left\{ \left[-i H_1^\dagger f - \frac{H_1^\dagger H}{2} \right] \begin{pmatrix} Q \\ T \end{pmatrix} t_1^c + \left(i H_1^\dagger f - \frac{H_1^\dagger H}{f} \right) \begin{pmatrix} Q \\ T \end{pmatrix} t_2^c \right] \Big\} = \frac{\lambda}{\sqrt{2}} \left[f\, T \left(t_1^c + t_2^c \right) \left(1 - \frac{H^\dagger H}{2f^2} \right) \right.$$

$$\left. + i H^\dagger Q \left(t_2^c - t_1^c \right) \right]$$



$\Longrightarrow$ 1-loop structure is fixed by symm. breaking term.

$\Rightarrow$ LITTLEST HIGGS

$SU(5) \longrightarrow SO(5)$

24 generators $-$ 10 generators $\Rightarrow$ 14 broken generators

$$\left(\begin{array}{c|c|c} & H & \phi \\ \hline H^+ & & H^T \\ \hline & \bar{H} & \end{array} \right)$$ $\leftarrow$ complex triplet

$\left[SU(2) \times U(1) \right]^2$ gauged

$\hookrightarrow$ MAIN NEW INGREDIENT: tree level sized quartic self interaction:

- Coleman-Weinberg: $a(g_1^2+g_2^2) f^2 \left| \phi_{ij} + \frac{i}{4f^2}\left(H_i H_j + H_j H_i\right)\right|^2$

$\Rightarrow$ it has quad. diverg. and cubic/quartic self-interactions

$\Rightarrow$ Little Higgs $\longrightarrow$ tree level collective quartic

collective Higgs $\longrightarrow$ quartic induced at loop level

$$\lambda = a \frac{(g_1^2+g_2^2)(g_1'^2+g_2'^2)}{g_1^2+g_1'^2+g_2^2+g'^2} = O(1) \quad \text{tree level coupling}$$

$$\rightarrow V_H = - \frac{g_{sm}^2 m_T^2}{16\pi^2} |H|^2 + g_{sm}^2 |H|^4 \rightsquigarrow \text{no need to tune anything}$$

BUT $\quad \lambda \sim O(1) \Rightarrow m_H \gtrsim 300\,\text{GeV}$

OBSERVED: $\lambda \sim 1/8 \quad (m_H \simeq 125\,\text{GeV})$

MINIMAL COMPOSITE HIGGS MODEL [MCHM]

→ EW precision observables: S, T parameter

* strongest from T param.

⇒ T-parity : $\mathbb{Z}_2$ symm.

→ No tree level effects! ⇒ $\underbrace{\frac{1}{16\pi^2 m_T^2}}$ suppression scale $4\pi m_T$

⇒ CUSTODIAL SYMMETRY [forbid just T-param]

- additional symmetry of SM Higgs sector :

$$-\mu^2 |H|^2 + \lambda |H|^4 = \qquad H = \begin{pmatrix} h_2 + i h_3 \\ h_1 + i h_4 \end{pmatrix}$$

$$= -\mu^2 \left(h_1^2 + h_2^2 + h_3^2 + h_4^2 \right) + \dots$$

↳ full SO(4) global symmetry : $SO(4) = SU(2)_L \times SU(2)_R$ — gauged $SU(2)_L$

$\downarrow$

$SO(3) \simeq SU(2)_c$ — SU(2) DIAGONAL

⇝ Custodial symm. is approximate ⇒ $U(1)_Y$, Yukawa coupl break it!!!

⇒ MCHM → custodial symm. is explicitly incorporated

$$G = SO(5) \longrightarrow SO(4) = H$$

→ 10 gen of SO(5) − 6 gen of SO(4) = 4 GOLDSTONE BOSONS

⇒ Adj of SO(5)

$$\left(\begin{array}{c|c} & \times \\ & \times \\ & \times \\ & \times \\ \hline & \end{array} \right) = \left(\begin{array}{c|c} SO(4) & \begin{matrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{matrix} \\ \hline -h_1, -h_2 \dots & 0 \end{array} \right) = \Pi(x)$$

broken generators

$$\Rightarrow \Sigma = e^{i\pi/f}\underset{\begin{pmatrix}0\\0\\0\\0\\f\end{pmatrix}}{\Sigma_0} = \frac{\sin(\pi/f)}{h}\begin{pmatrix}h_1\\h_2\\h_3\\h_4\\h\cot(\frac{h}{f})\end{pmatrix} \qquad h = \sqrt{|H|^2} \qquad H = \begin{pmatrix}h_1 - ih_2\\h_3 - ih_4\end{pmatrix}$$

corrections to SM interactions

$$\to \underset{\text{w.r.t. SM fields.}}{(D^\mu\Sigma)^\dagger(D_\mu\Sigma)} \to \mathcal{L}_{INT} = \frac{1}{8}f^2\sin^2\left(\frac{h}{f}\right)\left(B_\mu B_\nu + W^3_\mu W^3_\nu - 2W^3_\mu B_\nu + 2W^+_\mu W^-_\nu\right)\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)$$

$$\downarrow$$

$$h = \langle h\rangle + \tilde{h}$$

$\Rightarrow$ MASS TERM: $\quad f^2\sin^2\left(\frac{\langle h\rangle}{f}\right) \equiv v^2 \simeq (246\text{ GeV})^2$

$+$ corrections: $\quad \xi = \frac{v^2}{f^2} = \sin^2\left(\frac{\langle h\rangle}{f}\right) \Rightarrow \left(v^2 + 2v(1-\xi)h + (1-2\xi)h^2 + \dots\right)$

$$g_{VVh} = g_{VVh}^{SM}(1-\xi)$$
$$g_{VVhh} = g_{VVhh}^{SM}(1-2\xi)$$

$\to$ part num. values for coeff of ξ depends on group.

$\to$ MCH has no additional GB $\to$ quartics are loop induced

$$V_{MCH} = \frac{1}{16\pi^2}g_{SM}^2 g_*^2 f^2|h|^2 + \frac{g_{SM}^2 g_*^2}{16\pi^2}|h|^4$$

$\rightsquigarrow$ roughly the right magnitude (quartic) $\to$ Higgs VEV at scale f:

$$\frac{v}{f} \sim 1 \quad \left(\text{vs } \frac{v}{f} \sim \frac{1}{4\pi} \text{ in LH}\right)$$

$\to$ now $\frac{v^2}{f^2}$ tuning in the H potential.

PARTIAL COMPOSITENESS for fermion masses

* Technicolour models: 4 fermion operators $\longrightarrow \psi\psi\langle\psi_{TC}\psi_{TC}\rangle$
  $\hookrightarrow$ it has FCNC

* Partial compositeness: Source of fermion masses from mixing of SM ferm. with heavy composite fermions

SM fermions are ext to strong interactions: ψ_L, ψ_R

$$\underbrace{\lambda_L \bar{\psi}_L \mathcal{O}_R + \lambda_R \bar{\psi}_R \mathcal{O}_L}$$

SM fields (elementary)

Composite fermions $\rightarrow \mathcal{O}_L, \mathcal{O}_R \rightarrow \phi_L, \phi_R$ lowest excitations

$$\Downarrow$$

"totally another fields"

$$\lambda_L f\, \bar{\psi}_L \phi_L + \lambda_R f\, \bar{\psi}_R \phi_R + \underbrace{m_L \bar{\phi}_L \phi_L + m_R \bar{\phi}_R \phi_R}_{\text{HEAVY COMPOSITE FERMIONS!}}$$

Higgs is composite $\rightarrow$ interacts only with composite fermions:

Yukawa couplings: $\underbrace{Y \bar{\phi}_L \phi_R H} + \text{h.c.}$

the mass correction is small for $\phi_L, \phi_R, \dots$

but generate Yukawa's for ψ

$\Rightarrow \phi_L \left(\lambda_L f \bar{\psi}_L + m_L \bar{\phi}_L \right) \longrightarrow$ We know by chirality that there is a massless boson

$$s_L = \frac{\lambda_L f}{\sqrt{(\lambda_L f)^2 + m_L^2}} \qquad c_L = \dots$$

$\Rightarrow$ MASSLESS FIELD: $f_L = \frac{\lambda_L f}{m_L} \phi_L$ $\qquad \psi_L$ light field

Effective Yukawa coupling

$$y_u = f_q Y_u f_u$$

$$y_d = f_q Y_d f_d$$

$\Rightarrow$ nice ways to make $f \ll 1$ by dynamics [anomalous dim large]

ANARCHY MODELS [3x3 flavour]

$$y_u = f_q Y_u f_u \quad O(1)$$

$$= L_u \cdot y_u^{diag} R_u^\dagger$$

ex: $L_u \sim \min\left(\frac{f_{q_i}}{f_{q_j}}, \frac{f_q^j}{q^i}\right)$

$y_u^{diag} \sim f_q^i f_u^i$

$R_u^{ij} = \min\left(\frac{f_u^i}{f_u^j}, \frac{f_u^j}{f_u^i}\right)$ — $V_{CKM} = L_u L_d^\dagger$

Comments:

1) t mass should be reasonably large $\left(\frac{\lambda_{qR}}{\sqrt{\lambda_{qR}^2 + g_*^2}} \rightsquigarrow g_*^{t_R} \sim O(1)\right)$

2) in partial comp. we produce FCNC's in from composite sector!

Natural suppression!

$\hookrightarrow$ RS-GIM

EXTRA DIMENSIONS

5D theory $\Rightarrow$ 5^{th} dim. compact $\longrightarrow$ circle of radius R

$$\int d^5x \, \frac{1}{2} \partial_M \phi(x,y) \, \partial^M \phi(x,y) = \int d^5x \, \frac{1}{2} \left[(\partial_\mu \phi)^2 - (\partial_y \phi)^2 \right]$$

$$\phi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum \phi_n(x) \, e^{i \frac{n}{R} y} \qquad \left(\phi_n^* = \phi_{-n} \right)$$

$\hookrightarrow$ $S_{eff} = \int d^4x \left\{ \sum_{n>0} (\partial_\mu \phi^{(n)})^* (\partial^\mu \phi^{(n)}) - \left(\frac{n}{R}\right)^2 |\phi^{(n)}|^2 \right\}$ $\Rightarrow$ 5D field = Kaluza-Klein tower of massive 4D fields

The same applies to gauge fields:

$$A_M \longrightarrow \begin{pmatrix} A_\mu \\ A_5 \end{pmatrix}$$

A_μ $\longrightarrow$ 4D gauge field

A_5 $\longrightarrow$ scalar

NB: $m = 0 \longrightarrow 2$ dofs

$m > 0 \longrightarrow 3$ dofs

$\Rightarrow$ $D=5$ massless $\longrightarrow$ 3 physical dofs

$\rightsquigarrow$ the massive tower from A_5 gets eaten by the other polarization

$$\Rightarrow \quad F_{MN} F^{MN} \longrightarrow (\partial_\mu A_5 - \partial_5 A_\mu)(\partial^\mu A_5 - \partial^5 A^\mu)$$

$\rightarrow$ If we have $4+n$ - dimensional gauge theory:

$$\begin{pmatrix} A_\mu \\ A_5 \\ A_6 \\ \vdots \end{pmatrix}$$

$A_\mu \longrightarrow$ one tower of massive gauge fields

$\Rightarrow$ $(n-1)$ tower of scalars + one single scalar mode

GRAVITON

$$G_{MN} = \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline & \varphi \end{array} \right) \quad \text{in} \quad (4+n)\text{-dim} \Rightarrow$$

MASSLESS

4D graviton

n massless gauge fields

scalars

MASSIVE

massive graviton (5 dof)

eat one KK tower + 1 scalar tower

(n-1) massive gauge towers

$$\frac{n(n+1)}{2} - 1 - (n-1) = \frac{n(n-1)}{2}$$

massive scalars

MATCHING of COUPLING

$$\int d^{4+n}x \; (\partial\phi)^2$$

$$\hookrightarrow 2 + 2[\phi] = 4+n \Rightarrow [\phi] = 1 + \frac{n}{2} \text{ (mass dimensions)}$$

$$\left(\text{NB: } [\psi] = \frac{3}{2} + \frac{n}{2} \right)$$

→ couplings?

$$D_M = \partial_M - i \underset{\text{dim } -\frac{1}{2}}{g_5} \overset{\text{dim } 3/2}{A_M}$$

→ What's the effective 4D gauge coupling?

$$\Rightarrow A_M \longrightarrow A_\mu = \frac{A_\mu^{(0)}(x)}{\sqrt{2\pi R}}$$

$$D_M \longrightarrow \partial_\mu - i \frac{g_5}{\sqrt{2\pi R}} A_\mu^{(1)} + \dots$$

then: $g_4 = \dfrac{g_5}{\sqrt{2\pi R}}$

In general:

$$g_4^2 = \frac{g_{4+n}^2}{Vol_n}$$

Similar for grav. coupling: $S_{4+n} = -M_{4+n}^{2+n} \int d^{4+n}x \sqrt{G} \, R_{4+n}$ ← (4+n)-dim Planck scale

Matching: $M_{pl}^2 = M_{4+n}^{2+n} \, V_{(n)}$ for flat extra-dim

Theory with single scale

$$M_{4+n} = M_*$$
$$g_{4+n} \sim M_*^{-n/2}$$

From gauge coupling $\rightarrow$ $Vol \sim \dfrac{1}{M_*^n}$

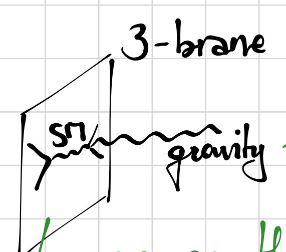
From the gravitational matching $\longrightarrow$ $M_* \sim M_{Pl}$

BRANES

$\rightarrow$ can change naïve scaling of the size of dimensions

↳ LOWER DIMENSIONAL OBJECTS

Appear in:



gravity must be matched

gauge th. is naturally 4D

1) LARGE EXTRA-DIMENSIONS

$\rightarrow$ I can start increasing V_n

$\Downarrow$

decrease M_* [what if $M_* \sim M_{EW} \sim 1$ TeV? $M_{Pl} \sim M_{EW} \Rightarrow$ only ONE fundam scale!]

↳ There's also the $R \sim \frac{1}{1 \text{ TeV}}$ $\Rightarrow$ why is R so large? $\rightarrow$ Hierarchy problem!

2) WARPED EXTRA DIMENSIONS

$\rightarrow$ True solution to hierarchy problem

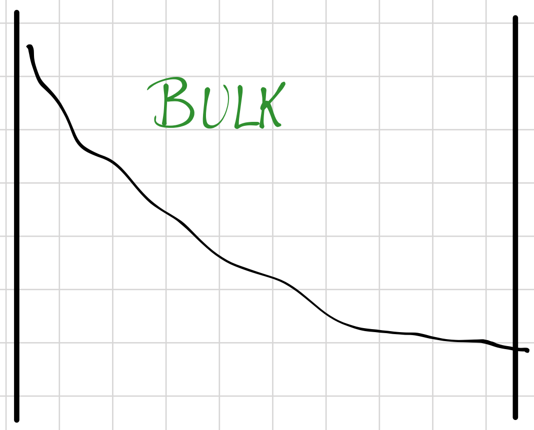
$\rightarrow$ related to pGB composite Higgs through AdS/CFT

$\rightarrow$ single extra dimension $\rightarrow$ <u>non trivial metric</u> $[AdS_5]$

$$ds^2 = \underbrace{\left(\frac{R}{z}\right)^2}_{\text{WARP FACTOR}} (dx^2 - dz^2) \qquad \text{"conformally flat" form}$$

In addition

(Planck brane) UV brane — (TeV brane) IR brane



$\Rightarrow$ mass scales are NOT const!

$$M(z) = \frac{R}{z}\, m(R)$$

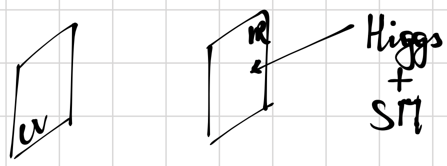
$z = R$ — $z = R'$ (exponentially large)

$\downarrow$

R : AdS curvature

$\Rightarrow$ RS1 model

- Assume the Higgs stuck to IR brane



$$S = \int d^4x \sqrt{g^{ind}} \left[\partial_\mu H \partial_\nu H\, g^{\mu\nu}_{(ind)} - \lambda \left(|H|^2 - v^2 \right)^2 \right] =$$

induced metric

$$= \int d^4x \left(\frac{R}{R'}\right)^4 \left\{ (\partial_\mu H)^2 \left(\frac{R'}{R}\right)^2 - \lambda \dots \right\}$$

$\tilde{H} = \frac{R}{R'} H \rightsquigarrow$ the effect is the rescaling of v

Higgs VEV : $v \frac{R}{R'}$ $\leftarrow$ "Goldberger - Wise" stabilization mechanism

if $\frac{R}{R'}$ is exponentially small :

$$R \sim \frac{1}{M_{pl}} \qquad R' \sim \frac{1}{1\,\text{TeV}}$$

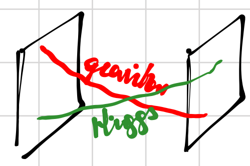
$\Rightarrow$ Matching of gravity:

$$M_*^3 \int dz \int d^4x \sqrt{g^5}\, R_{(5)} \longrightarrow M_{pl}^2 = M_*^3 R \left(1 - \frac{R^2}{R'^2}\right)$$ barely depends on R'

(even if $R' \to \infty \Rightarrow M_{pl} < \infty$)

- RS2 model $\Rightarrow$ $R' \to \infty$

(has hierarchy problem (: no IR brane))



graviton 0-mode $\rightarrow$ remains normalizable as $R' \to \infty$

$$\longrightarrow \quad M_* \frac{R}{R'} \sim \text{TeV} \quad \ll M_{pl} \sim M_*$$

$\Rightarrow$ KK decomp.

$$S = \int_R^{R'} dz \int d^4x \sqrt{g} \left\{ \partial_M \phi^* \partial_N \phi \, g^{MN} - m^2 |\phi|^2 \right\}$$

$\hookrightarrow$ e.o.m : $\phi = \frac{1}{\sqrt{R}} \sum_n \phi^{(n)}(x) f^{(n)}(z)$

$$\longrightarrow \left[\partial_z^2 - \frac{3}{z} \partial_z + \left[m_n^2 - \left(\frac{R}{z}\right)^2 m^2 \right] \right] f^{(n)}(z) = 0$$

$\Downarrow$

Bessel functions!

ALL peaked towards IR!

(0-modes are just powers : $f^{(0)}(z) = z^{2 \pm \sqrt{4 + m^2 R^2}}$)

$\Rightarrow$ KK tower in $\frac{n}{R'}$ — steps given by TeV scale

AdS / CFT correspondence

Maldacena : $\quad$ IIB $AdS_5 \times S_5 \longleftrightarrow \mathcal{N}=4$ SYM (SCFT)

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2) \xrightarrow{z \to e^{\alpha} z}$$ compensate by rescaling $\underbrace{x \longrightarrow e^{\alpha} x}$

equivalent to rescaling D=4 theory

→ Motion along z ⟷ 4D scale transf. [HOLOGRAPHIC RG]

→ no UV/IR branes ⟷ CFT

→ UV brane ⟷ cutoff for the CFT at $\Lambda \sim 1/R$

→ IR brane ⟷ CFT now spontaneously broken

(KK like a confining gauge theory with a tower of hadrons)

DILATON ("radion")
GB for scale inv. SSB

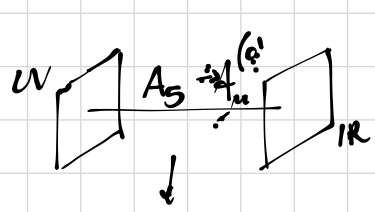
MASS SCALE (~QCD) for hadronic excitations!

UV localized → "elementary fields"

IR localized → "composite modes"

AdS/CFT global symm. → MALDACENA: gauge field in the bulk!

- with UV and IR brane this 0-mode can be flat



↓ FLAT:

- $A_\mu^{(0)}$ flat on both branes → weakly gauge global
- $A_\mu^{(0)}$ has D b.c. on UV → just global symm.
- $A_\mu^{(0)}$ has D b.c. on IR → broken global symm. → A_5 has 0-mode.
  N " → unbroken global cond

MCH from warped extra dimensions

Spin 1 partners are KK modes