D branes in string theories, II [1]

P. Di Vecchia[a] **and A. Liccardo**[b]

[a] *NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

[b] *Dipartimento di Fisica, Università di Napoli and I.N.F.N.,
Sezione di Napoli, Mostra d'Oltremare Pad. 19, I-80125 Napoli, Italy*

Abstract

In these lectures we review the properties of a boosted and rotated boundary state and of a boundary state with an abelian gauge field deriving from it the Dirac-Born-Infeld action and a newly constructed class of classical solutions. We also review the construction of the boundary state for the stable non-BPS state of type I theory corresponding to the perturbative state present at the first excited level of the $SO(32)$ heterotic string and transforming according to the spinor representation of $SO(32)$.

1   Introduction

Dp-branes are classical solutions of the eqs. of motion of the low-energy string effective action [2], charged under a $(p+1)$-form R-R field, that correspond to new non-perturbative BPS states of string theory, break 1/2 supersymmetry, and are required by T-duality in theories with open strings [3]. They are characterized by the fact that open strings have their end-points attached to them [3] and are conveniently described by a state of closed string theory called the boundary state. A review of their properties and of the origin and of the construction of the boundary state can be found in Ref. [4]. For the sake of completeness we rewrite here its

---

[1] Work partially supported by the European Commission TMR programme ERBFMRX-CT96-0045 and by 'Programma di breve mobilità per scambi internazionali dell'Università di Napoli "Federico II" '.

[2] See Ref. [1] and references therein.

[3] See Ref. [2] and references therein.

explicit form. It is given by

$$|B, \eta\rangle_{R,NS} = \frac{T_p}{2} |B_{mat}, \eta\rangle |B_g, \eta\rangle \quad ; \quad T_p = \sqrt{\pi} \left(2\pi\sqrt{\alpha'} \right)^{3-p} \tag{1.1}$$

where the boundary states for the matter part and for the ghost degrees of freedom are given by

$$|B_{mat}\rangle = |B_X\rangle |B_\psi, \eta\rangle \quad ; \quad |B_g\rangle = |B_{gh}\rangle |B_{sgh}, \eta\rangle \tag{1.2}$$

The ghost part of the boundary state can be found in eqs.(6.216) and (7.251)-(7.253) of Ref. [4]. Here we will only write the explicit form of the matter part of the boundary state. The part corresponding to the bosonic coordinate X is equal to

$$|B_X\rangle = \delta^{d-p-1}(\hat{q}^i - y^i) \left(\prod_{n=1}^{\infty} e^{-\frac{1}{n}\alpha_{-n} S \cdot \widetilde{\alpha}_{-n}} \right) |0\rangle_\alpha |0\rangle_{\widetilde{\alpha}} |p = 0\rangle , \tag{1.3}$$

where

$$S^{\mu\nu} = (\eta^{\alpha\beta}, -\delta^{ij}) \tag{1.4}$$

α, β are indices along the world volume of the Dp-brane, while i, j span the transverse directions of the brane.

The fermionic part of the matter boundary state is equal to

$$|B_\psi, \eta\rangle = -i \prod_{t=1/2}^{\infty} \left(e^{i\eta\psi_{-t} \cdot S \cdot \widetilde{\psi}_{-t}} \right) |0\rangle \tag{1.5}$$

in the NS-NS sector and to

$$|B_\psi, \eta\rangle = -\prod_{t=1}^{\infty} e^{i\eta\psi_{-t} \cdot S \cdot \widetilde{\psi}_{-t}} |B_\psi, \eta\rangle^{(0)} \tag{1.6}$$

in the R-R sector. The zero mode contribution $|B_\psi, \eta\rangle^{(0)}$ is given by

$$|B_\psi, \eta\rangle^{(0)} = \mathcal{M}_{AB} |A\rangle |\widetilde{B}\rangle \tag{1.7}$$

where

$$\mathcal{M}_{AB} = \left(C\Gamma^0 ... \Gamma^p \frac{1 + i\eta\Gamma^{11}}{1 + i\eta} \right)_{AB} \tag{1.8}$$

C is the charge conjugation matrix and Γ^μ are the Dirac Γ matrices in the 10-dimensional space. Their properties are summarized in the Appendix. The boundary state in eq.(1.1) depends on the two values of $\eta = \pm 1$. Actually we must take a combination of them corresponding to the GSO projection. The GSO-projected states are given by:

$$|B\rangle_{NS} = \frac{1}{2} \left(|B, +\rangle_{NS} - |B, -\rangle_{NS} \right) \quad ; \quad |B\rangle_R = \frac{1}{2} \left(|B, +\rangle_R + |B, -\rangle_R \right) \tag{1.9}$$

respectively for the NS and R sectors.

In Ref. [4] we have reviewed the properties of the Dp-branes, described by the boundary state discussed above, and considered as static and rigid objects to which open strings are attached. We have not discussed the fact that they can be boosted, rotated and that the excitations of the attached open strings provide dynamical degrees of freedom to them. In particular the massless excitations that have the property of not changing the energy of the brane, can be interpreted as collective coordinates of the Dp-branes. In these lectures we fill this gap by showing how to construct a boosted [5] [4] and rotated boundary state and a boundary state containing a constant abelian gauge field living in the world volume of the brane [7]. We then show that some of those boundary states are related by T-duality. We then use the boundary state with an external gauge field in order, on the one hand, to derive the Born-Infeld action and, on the other hand, to reconstruct, with a specific choice of the external field, newly found solutions [8, 9, 10] of the eqs. of motion of the low-energy string effective action. Finally we discuss the boundary state for a toroidally compactified space-time and we use it for describing the properties of stable non BPS states recently discussed in the literature [5].

These lectures, that are partially based on the Ph.D. thesis of Antonella Liccardo, are organized as follows. In sect. 2 we discuss the properties of the boosted and rotated boundary state. Sect. 3 is devoted to the boundary state with an abelian gauge field. In sect. 4 we show how some of the previously discussed boundary states are related by T-duality and we discuss the boundary state with transverse excitations. In sect. 5 we show that, by choosing a particular form of the gauge field, we reproduce the (F, Dp) bound states. Sect. 6 is devoted to the construction of the boundary state in the case of a compactified space-time and sect. 7 to the construction of the boundary state for the stable non-BPS particle of type I string theory. Finally in the Appendix we summarize the properties of the ten-dimensional Γ matrices.

2 Boosted and Rotated Boundary State

In the introduction we have considered the boundary state corresponding to a static Dp-brane. In this section we want to extend this construction to a boosted and a rotated one. It is easy to see that a boost along a longitudinal direction of a Dp-brane does not modify the boundary state explicitly written in the introduction as expected from the Poincaré invariance of the classical solution along the longitudinal directions of the brane. We can therefore concentrate us on a boost along one of the transverse directions and let us call this direction k. The way to construct a boosted boundary state is, as we have done in Ref. [4], to start from the boundary

---

[4]See also Ref. [6].
[5]For reviews on stable non BPS states see Ref. [11].

conditions for an open string attached to such a Dp-brane and then translate them into the language of the closed string channel through a conformal transformation and a conformal rescaling (see Ref. [4] for details).

The boundary conditions for an open string attached to a Dp-brane boosted with velocity v in the direction k are [12]

$$\partial_\sigma X^\alpha|_{\sigma=0} = 0 \qquad\qquad \alpha = 1, ..., p \tag{2.10}$$

$$\partial_\sigma \left(X^0 + vX^k \right)\Big|_{\sigma=0} = 0 \tag{2.11}$$

$$X^i|_{\sigma=0} = y^i \qquad\qquad i = p+1, ...D-1, \quad \text{and} \quad i \neq k \tag{2.12}$$

$$\left(\frac{X^k + vX^0}{\sqrt{1-v^2}} \right)\Bigg|_{\sigma=0} = \frac{y^k}{\sqrt{1-v^2}} \tag{2.13}$$

where $\vec{y}$ is a vector belonging to the space transverse to the Dp-brane and therefore has zero component along the time and the other world volume directions of the Dp-brane. In the closed channel the previous conditions translate into the following equations that characterize the boosted boundary state $|B, v, y\rangle$ in the bosonic string:

$$\partial_\tau X^\alpha|_{\tau=0} |B, v, y\rangle = 0 \qquad\qquad \alpha = 1, ..., p \tag{2.14}$$

$$\partial_\tau \left(X^0 + vX^k \right)\Big|_{\tau=0} |B, v, y\rangle = 0 \tag{2.15}$$

$$\left(X^i - y^i \right)\Big|_{\tau=0} |B, v, y\rangle = 0 \qquad\qquad i = p+1, ...D-1, \quad \text{and} \quad i \neq k \tag{2.16}$$

$$\left(\frac{(X^k - y^k) + vX^0}{\sqrt{1-v^2}} \right)\Bigg|_{\tau=0} |B, v, y\rangle = 0 \tag{2.17}$$

Instead of the last equation we can also have a less restrictive one

$$\partial_\sigma \left(X^k + vX^0 \right)\Big|_{\tau=0} |B, v\rangle = 0 \tag{2.18}$$

which corresponds to the case of a brane which is delocalized in the k direction.

The only overlap conditions that differ from those of the static case given in Ref. [4] are those in the directions of the boost, namely the time and the k directions and they are equal to

$$\left(\hat{p}^0 + v\hat{p}^k \right) |B, v, y\rangle = 0 \tag{2.19}$$

$$\left[\left(\alpha_n^0 + \tilde{\alpha}_{-n}^0 \right) + v \left(\alpha_n^k + \tilde{\alpha}_{-n}^k \right) \right] |B, v, y\rangle = 0 \qquad \forall n \neq 0 \tag{2.20}$$

$$\frac{\hat{q}^k + v\hat{q}^0}{\sqrt{1-v^2}} |B, v, y\rangle = \frac{y^k}{\sqrt{1-v^2}} |B, v, y\rangle \tag{2.21}$$

$$\left[\left(\alpha_n^k - \tilde{\alpha}_{-n}^k \right) + v \left(\alpha_n^0 - \tilde{\alpha}_{-n}^0 \right) \right] |B, v, y\rangle = 0 \qquad \forall n \neq 0 \tag{2.22}$$

4

Let us now determine the explicit expression of the state $|B, v, y\rangle$ which fulfills all the previous conditions. For the zero mode part eq.(2.21) tells us that the boundary state must contain a δ function of the type

$$\delta\left(\frac{q^k + vq^0 - y^k}{\sqrt{1-v^2}}\right) = \sqrt{1-v^2}\delta(q^k + vq^0 - y^k), \qquad (2.23)$$

Since the operator that acts on the boundary state in eq.(2.19) commutes with the δ-function in eq.(2.23), in order to satisfy eq.(2.19), it is sufficient to write the zero mode part as follows:

$$\sqrt{1-v^2}\delta\left(q^k + vq^0 - y^k\right)|p=0\rangle \qquad (2.24)$$

It is easy to check that it satisfies both zero mode eqs.(2.19) and (2.21). Let us consider now the non zero modes part of the overlap conditions. It is easy to see that, in order to satisfy eqs.(2.20) and (2.22), the non-zero mode part of the boundary state must have the following structure

$$\prod_{n=1}^{\infty}\left(e^{-\frac{1}{n}\alpha_{-n}\cdot M(v)\cdot\widetilde{\alpha}_{-n}}\right)|0\rangle_{\alpha}|0\rangle_{\widetilde{\alpha}} \qquad (2.25)$$

where the matrix M is obtained from the matrix S in eq.(1.4) by substituting its elements $(S_{00}, S_{0k}, S_{k0}, S_{kk})$ with the correspondent ones

$$M_{00} = M_{kk} = -\frac{1+v^2}{1-v^2} \quad ; \quad M_{0k} = M_{k0} = -\frac{2v}{1-v^2} \qquad (2.26)$$

Putting eqs.(2.24) and (2.25) together we get the final expression for the boosted boundary state:

$$|B, v, y\rangle = \frac{T_p}{2}\prod_{i=p+1, i\neq j}^{d-1}\left[\delta(\hat{q}^i - y^i)\right]\sqrt{1-v^2}\,\delta(q^k + v\,q^0 - y^k)$$

$$e^{-\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}\cdot M(v)\cdot\widetilde{\alpha}_{-n}}|0\rangle_{\alpha}|0\rangle_{\widetilde{\alpha}}|p=0\rangle\ . \qquad (2.27)$$

We have fixed the normalization factor to be $T_p/2$ as in the static case, but the overlap conditions alone do not allow to fix it uniquely. In general the boundary state in eq.(2.27) could include an arbitrary function $N(v)$ of the physical velocity v that can only be determined by requiring agreement between the calculation of the interaction between two D-branes in the closed and open string channel. In this case, however, we have an independent way of uniquely fixing its normalization by applying to the static boundary state $|B_X\rangle$ in eq.(1.3) an operator that performs a boost along the direction k transverse to the world volume of the D-brane

$$|B, y, w\rangle = e^{iw^k J^0{}_k}\,|B, {}^{(w)}y\rangle\ , \qquad (2.28)$$

where w is related to the physical velocity v through the relation

$$v = \text{tgh}w \tag{2.29}$$

$^{(w)}y^k = y^k \cosh w$ is the boosted position of the D-brane and the generator of the Lorentz transformation is equal to

$$J^{\mu\nu} = q^\mu p^\nu - q^\nu p^\mu - i \sum_{n=1}^{\infty} \left(a_n^{\dagger\mu} a_n^\nu - a_n^{\dagger\nu} a_n^\mu + \tilde{a}_n^{\dagger\mu} \tilde{a}_n^\nu - \tilde{a}_n^{\dagger\nu} \tilde{a}_n^\mu \right) \ . \tag{2.30}$$

with $a_n \sqrt{n} = \alpha_n$ and $a_n^\dagger \sqrt{n} = \alpha_{-n}$ with $n > 0$. After some algebra it can be seen that the boosted boundary state in eq.(2.28) can be written in the following form

$$|B, y, w(v)\rangle = \frac{T_p}{2} \prod_{i=p+1, i \neq k}^{d-1} \left[\delta(\hat{q}^i - y^i) \right] \frac{1}{\cosh w} \delta(q^k + \text{tgh}w \ q^0 - y^k)$$

$$e^{-\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \cdot M(w) \cdot \tilde{\alpha}_{-n}} |0\rangle_\alpha |0\rangle_{\tilde{\alpha}} |p = 0\rangle \ , \tag{2.31}$$

that exactly coincides with the one given in eq.(2.27), as it can be easily seen by making use of eq.(2.29). In this way we have shown that the overall normalization of the boosted boundary state in eq.(2.27) is correct.

The previous construction can be easily generalized to describe a rotated Dp-brane. Obviously the configuration of a Dp-brane embedded in a d-dimensional space-time is invariant under rotations in the longitudinal space as well as in the transverse space. This implies that the boundary state is invariant under rotations in the planes (α, β) or (i, j) $\forall \alpha, \beta \in \{0, ..., p\}$ and $\forall i, j \in \{p + 1, ..., d - 1\}$. This means that, in order to get a new boundary state, we must consider a Dp-brane which is rotated with an angle ω in one of the planes specified by the directions (α, k).

The open string attached to this brane at $\sigma = 0$ satisfies the boundary conditions

$$\partial_\sigma X^\beta|_{\sigma=0} = 0 \qquad \forall \beta \in \{0, ..., p\} \ \text{and} \ \beta \neq \alpha \tag{2.32}$$

$$\partial_\sigma \left(X^\alpha \cos \omega + X^k \sin \omega \right)\Big|_{\sigma=0} = 0 \tag{2.33}$$

$$X^i|_{\sigma=0} = y^i \qquad i = p + 1, ... D - 1, \quad \text{and} \quad i \neq k \tag{2.34}$$

$$\left(X^k \cos \omega - X^\alpha \sin \omega - y^k \cos \omega \right)\Big|_{\sigma=0} = 0 \tag{2.35}$$

Then the overlap conditions that the rotated boundary state must satisfy in the directions different from (α, k) are the same as in the unrotated case. On the other hand along the directions (α, k) we must impose the following conditions:

$$\partial_\tau \left(X^\alpha \cos \omega + X^k \sin \omega \right)\Big|_{\tau=0} |B, \omega, y\rangle = 0 \tag{2.36}$$

and

$$\left(X^k \cos \omega - X^\alpha \sin \omega - y^k \cos \omega \right)\Big|_{\tau=0} |B, \omega, y\rangle = 0 \tag{2.37}$$

that in terms of the oscillators become:

$$\left(\hat{p}^\alpha \cos\omega + \hat{p}^k \sin\omega\right)|B,\omega,y\rangle = 0 \tag{2.38}$$

$$\left[\left(\alpha_n^\alpha + \tilde{\alpha}_{-n}^\alpha\right) + \tan\omega \left(\alpha_n^k + \tilde{\alpha}_{-n}^k\right)\right]|B,\omega,y\rangle = 0 \quad \forall n \neq 0 \tag{2.39}$$

$$\left(\hat{q}^k \cos\omega - \hat{q}^\alpha \sin\omega\right)|B,\omega,y\rangle = y^k \cos\omega|B,\omega,y\rangle \tag{2.40}$$

$$\left[\left(\alpha_n^k - \tilde{\alpha}_{-n}^k\right) - \tan\omega \left(\alpha_n^\alpha - \tilde{\alpha}_{-n}^\alpha\right)\right]|B,\omega,y\rangle = 0 \quad \forall n \neq 0 \tag{2.41}$$

Proceeding as in the previous case it is easy to see that the rotated boundary state has the following form:

$$|B,\omega,y\rangle = \frac{T_p}{2} \prod_{i=p+1,i\neq k}^{d-1} \left[\delta(\hat{q}^i - y^i)\right] \delta(\cos\omega \; q^k - \sin\omega \; q^\alpha - y^k \cos\omega)$$

$$e^{-\sum_{n=1}^\infty \frac{1}{n}\alpha_{-n}\cdot M(\omega)\cdot\tilde{\alpha}_{-n}}|0\rangle_\alpha|0\rangle_{\tilde{\alpha}}|p=0\rangle \tag{2.42}$$

where in this case the matrix M is obtained from the matrix S appearing in eq.(1.4) by substituting its elements $(S_{\alpha\alpha}, S_{\alpha k}, S_{k\alpha}, S_{kk})$ with the correspondent elements

$$M_{(\alpha\alpha)} = -M_{(kk)} = \cos 2\omega \quad ; \quad M_{(0k)} = M_{(k0)} = \sin 2\omega \tag{2.43}$$

The previous boundary state for a rotated Dp-brane can again also be obtained by acting on the boundary state given in eq.(1.3) with the rotation operator

$$|B,y,\omega\rangle = e^{i\omega^k J^\alpha{}_k} |B, {}^{(\omega)}y\rangle \;, \tag{2.44}$$

where ${}^{(\omega)}y^k = y^k \cos\omega$ is the rotated position of the D-brane and $J^{\mu\nu}$ is defined in eq.(2.30). The previous considerations can be easily extended to the fermionic coordinate obtaining the boosted boundary state in the case of superstring. We will not present here its detailed derivation as in the bosonic case, but we write only its final form. We get

$$|B_\psi,\eta\rangle = -i \prod_{t=1/2}^\infty \left(e^{i\eta\psi_{-t}\cdot M\cdot\tilde{\psi}_{-t}}\right)|0\rangle \tag{2.45}$$

in the NS-NS sector and

$$|B_\psi,\eta\rangle = -\prod_{t=1}^\infty e^{i\eta\psi_{-t}\cdot M\cdot\tilde{\psi}_{-t}}|B_\psi,\eta\rangle^{(0)} \tag{2.46}$$

in the R-R sector. The matrix M is obtained from S as in eq.(2.26). The zero mode contribution $|B_\psi,\eta\rangle^{(0)}$ is given by

$$|B_\psi,\eta\rangle^{(0)} = \frac{1}{\sqrt{1-v^2}} \left(C[\Gamma^0 + v\Gamma^k]\Gamma^1...\Gamma^p\frac{1+i\eta\Gamma^{11}}{1+i\eta}\right)_{AB} |A\rangle|\tilde{B}\rangle \tag{2.47}$$

7

At the end of this section we write the interaction between two branes moving with velocity v relative to each other originally performed in Ref. [12] in the open string channel and then reproduced in Ref. [5] in the closed string channel. The total contribution of the NS-NS sector is given by:

$$A_{NS} = V_{NN-1}(8\pi^2\alpha')^{-NN/2} \, iv \int_0^\infty dt \left(\frac{1}{t}\right)^{DD/2} \int_{-\infty}^\infty d\tau e^{-(y^2+v^2\tau^2)/(2\pi\alpha't)} \times$$

$$\times \left[\frac{\Theta_3(\theta|it)}{\Theta_1(\theta|it)}\left(\frac{f_3}{f_1}\right)^{6-\nu}\left(\frac{f_4}{f_2}\right)^{\nu} - \frac{\Theta_4(\theta|it)}{\Theta_1(\theta|it)}\left(\frac{f_4}{f_1}\right)^{6-\nu}\left(\frac{f_3}{f_2}\right)^{\nu}\right] \qquad (2.48)$$

where $\Theta_i(\theta|it)$ is the Jacobi Θ-function and its argument is related to the velocity by:

$$\theta = \frac{1}{2\pi i}\log\frac{1-v}{1+v} \qquad (2.49)$$

ν is equal to the number of mixed N-D boundary conditions for the open strings with their endpoints on the two branes and the functions f_i can be found in eqs.(9.282) and (9.283) of Ref. [4]. For the sake of simplicity we give the R-R contribution in the case of equal branes. In this case we get

$$A_R = V_p(8\pi^2\alpha')^{-(p+1)/2}(-iv)\int_0^\infty dt\left(\frac{1}{t}\right)^{(9-p)/2}\int_{-\infty}^\infty d\tau e^{-(y^2+v^2\tau^2)/(2\pi\alpha't)} \times$$

$$\frac{\Theta_2(\theta|it)}{\Theta_1(\theta|it)}\left(\frac{f_2}{f_1}\right)^6 \qquad (2.50)$$

Summing the two contribution we get the following expression for equal D branes:

$$A = V_p(8\pi^2\alpha')^{-(p+1)/2}(-iv)\int_0^\infty dt\left(\frac{1}{t}\right)^{(9-p)/2}\int_{-\infty}^\infty d\tau e^{-(y^2+v^2\tau^2)/(2\pi\alpha't)} \times$$

$$\times \left\{\frac{\Theta_3(\theta|it)}{\Theta_1(\theta|it)}\left(\frac{f_3}{f_1}\right)^6 - \frac{\Theta_4(\theta|it)}{\Theta_1(\theta|it)}\left(\frac{f_4}{f_1}\right)^6 - \frac{\Theta_2(\theta|it)}{\Theta_1(\theta|it)}\left(\frac{f_2}{f_1}\right)^6\right\} \qquad (2.51)$$

Using known identities between Θ-functions one can rewrite the previous expression as:

$$A = V_p(8\pi^2\alpha')^{-(p+1)/2}(-iv)\int_0^\infty dt\left(\frac{1}{t}\right)^{(9-p)/2}\int_{-\infty}^\infty d\tau e^{-(y^2+v^2\tau^2)/(2\pi\alpha't)} \times$$

$$\times \frac{[\Theta_1(\theta/2|it)]^4}{\Theta_1(\theta|it)f_1^9} \qquad (2.52)$$

that is equal to the expression obtained in Ref. [13] using the light-cone boundary state. In the limit of small velocity we get

$$A \to V_{p+1}(2\pi\alpha')^{3-p}\frac{\Gamma(7/2-p/2)}{(4\pi)^{(p+1)/2}}\frac{v^4}{y^{7-p}} \qquad (2.53)$$

that agrees with the calculation using M(atrix) theory performed in Ref. [14] in the case of a D particle ($p=0$).

3   Boundary state with an external field

Until now we have considered the branes as static or rigidly moving objects, a sort of geometrical hyperplanes having open strings attached with their endpoints. We have completely neglected the dynamics of the open strings attached to the D branes. In this section we will consider those excitations. We are in particular interested to the zero mode (massless) excitations that do not change the energy of the brane and that correspond to its collective coordinates. In absence of Chan Paton factors the massless excitations of an open string are described by a $U(1)$ gauge field. In presence of a Dp-brane the ten-dimensional gauge vector field splits into a longitudinal vector field living in the world volume of the brane and a transverse part correponding to $d - p - 1$ scalar fields that have the physical interpretation of coordinates of the brane. In this section we will neglect the scalar fields corresponding to the transverse components of the gauge field and concentrate on the construction of the boundary state describing a Dp-brane with an abelian gauge field living on its world volume. Then we will use it for deriving the Dirac-Born-Infeld (DBI) action with the inclusion of the Wess-Zumino term, that is the action describing the low-energy dynamics of a Dp-brane.

In order to construct the boundary state with an abelian field on it we follow the same procedure that we have followed in the previous section. We start looking at the boundary conditions of an open string stretching between two Dp-branes with a gauge field on it and then we translate these conditions into those for the boundary state with a gauge field on it.

Let us start considering the bosonic string. An open string interacts with a gauge field through the pointlike charges located at its endpoints. Therefore the action for an open string interacting with an abelian gauge field contains besides the free string action also a term describing this interaction that occurs only at the endpoints $\sigma = 0$ and $\sigma = \pi$:

$$S = \int d\tau \int_0^\pi d\sigma \left\{ \frac{1}{4\pi\alpha'}[(\partial_\tau X)^2 - (\partial_\sigma X)^2] - [\delta(\sigma) - \delta(\sigma - \pi)]\dot{X}^\mu A_\mu(X) \right\} . \quad (3.54)$$

For the sake of simplicity we have taken the two charges located at the endpoints of the string to have the same absolute value and opposite sign. The eqs. of motion and the boundary conditions are obtained by varying the previous action. One gets:

$$\delta S = -\frac{1}{2\pi\alpha'} \int d\tau \int_0^\pi d\sigma \{ (\partial_\tau^2 X_\mu - \partial_\sigma^2 X_\mu)\delta X^\mu + \partial_\sigma(\partial_\sigma X_\mu \delta X^\mu) +$$

$$+ 2\pi\alpha'[\delta(\sigma) - \delta(\sigma - \pi)](-\delta X^\mu \partial_\tau A_\mu(X) + \partial_\tau X^\mu \delta A_\mu(X)) \} \quad (3.55)$$

Requiring this variation to be zero we obtain, together with the equations of motion that are of course unchanged with respect to the free case, also the following boundary constraint

$$\frac{1}{2\pi\alpha'} \int d\tau \left\{ \partial_\sigma X_\mu - \partial_\tau X^\rho \hat{F}_{\mu\rho} \right\} \delta X^\mu |_{\sigma=0}^{\sigma=\pi} = 0 \quad (3.56)$$

9

where we have defined $\hat{A}^\mu = 2\pi\alpha' A^\mu$ and $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$. We will now assume that the gauge field is non zero only in the directions of the world volume of the brane, while it is constant along the transverse ones: $A^i = const$. This means that eq.(3.56) is satisfied if we impose

$$(\partial_\sigma X_\alpha + \hat{F}_{\beta\alpha}(X)\partial_\tau X^\beta)|_{\sigma=0,\pi} = 0 \tag{3.57}$$

along the world volume of the brane, while the transverse coordinates still satisfy

$$X^i|_{\sigma=0} = y^i \qquad i = p+1, ..., d-1 \tag{3.58}$$

Translating these conditions in the closed channel we get the constraints that the boundary state must satisfy to describe a Dp-brane with an external field

$$(\partial_\tau X_\alpha + \hat{F}_{\beta\alpha}(X)\partial_\sigma X^\beta)|_{\tau=0}|B_X\rangle = 0 \tag{3.59}$$

$$(X^i|_{\tau=0} - y^i)|B_X\rangle = 0 \qquad i = p+1, ..., D-1 \tag{3.60}$$

Substituting in the previous equation the mode expansion we get the overlap conditions in terms of the oscillators. Since they can be easily solved only in the case of a constant $F_{\mu\nu}$ in the following we limit ourselves to a constant field [6]. In this case eqs.(3.59) and (3.60) become:

$$\begin{cases} \hat{p}^\beta|B_X\rangle = 0 \quad ; \quad \hat{q}^i|B_X\rangle = y^i|B_X\rangle \\ \left\{ (\,1\!\!1 + \hat{F})^\alpha{}_\beta \alpha_n^\beta + (\,1\!\!1 - \hat{F})^\alpha{}_\beta \tilde{\alpha}_{-n}^\beta \right\} |B_X\rangle = 0 \\ (\alpha_n^i - \tilde{\alpha}_{-n}^i)|B_X\rangle = 0 \end{cases} \tag{3.61}$$

with $n \neq 0$. They are satisfied by the following boundary state

$$|B_X\rangle = N_p(F)\delta^{(d_\perp)}(\hat{q} - y)\exp\left[-\sum_{n=1}^\infty \frac{1}{n}\alpha_{-n}\cdot M \cdot \tilde{\alpha}_{-n}\right]|0\rangle_\alpha|0\rangle_{\tilde{\alpha}}|p=0\rangle \;, \tag{3.62}$$

where
$$M^\mu{}_\nu \equiv ([(\,1\!\!1 - \hat{F})(\,1\!\!1 + \hat{F})^{-1}]^\alpha{}_\beta; -\delta^i{}_j) \tag{3.63}$$

and $N_p(F)$ is a normalization factor depending on the external field that we will determine later. Proceeding in an analogous way we can get the overlap conditions for the fermionic oscillators

$$\left\{ (\,1\!\!1 + \hat{F})^\alpha{}_\beta \psi_t^\beta - i\eta(\,1\!\!1 - \hat{F})^\alpha{}_\beta \tilde{\psi}_{-t}^\beta \right\} |B_\psi, \eta\rangle = 0 \tag{3.64}$$

for any half-integer [integer] value of t for the NS [R] sector. The boundary state satisfying the previous conditions is equal to:

$$|B_\psi, \eta\rangle_{\mathrm{NS}} = (-i)\exp\left[i\eta \sum_{t=1/2}^\infty \psi_{-t} \cdot M \cdot \tilde{\psi}_{-t}\right]|0\rangle \;, \tag{3.65}$$

---

[6]A boundary state with non constant gauge field has been recently considered [15].

for the NS sector and

$$|B_\psi, \eta\rangle_{\mathrm{R}} = -\exp\left[\mathrm{i}\eta \sum_{t=1}^{\infty} \psi_{-t} \cdot M \cdot \widetilde{\psi}_{-t}\right] |B_\psi, \eta\rangle_{\mathrm{R}}^{(0)} \quad, \tag{3.66}$$

for the R sector, where the superscript $^{(0)}$ denotes the zero-mode contribution that is equal to:

$$|B_\psi, \eta\rangle_{\mathrm{R}}^{(0)} = k(F)\mathcal{M}_{AB}^{(\eta)} |A\rangle|\widetilde{B}\rangle \quad, \tag{3.67}$$

$k(F)$ is a normalization constant to be determined and

$$\mathcal{M}^{(\eta)} = C\Gamma^0\Gamma^1 \dots \Gamma^p \left(\frac{1 + \mathrm{i}\eta\Gamma_{11}}{1 + \mathrm{i}\eta}\right) U \quad, \tag{3.68}$$

C is the charge conjugation matrix and U is equal to

$$U =; \mathrm{e}^{-1/2\hat{F}^{\alpha\beta}\Gamma_\alpha\Gamma_\beta}; \tag{3.69}$$

where the symbol ; ; means that we have to expand the exponential and then antisymmetrize the indices of the Γ-matrices. The boundary state for the matter fields is then obtained by inserting in the first equation in (1.2) the bosonic boundary state in eq.(3.62) and the fermionic one given in eq.(3.65) for the NS sector and in eq.(3.66) for the R sector. The complete boundary state is finally obtained by performing the appropriate GSO-projections [7] on the state given in eq.(1.1) where the ghost contribution is unchanged with respect to the case with $F = 0$.

Analogously we get also the conjugate boundary state

$$\langle B_X| = \langle p = 0|\langle 0|_\alpha\langle 0|_{\widetilde{\alpha}} \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n} \alpha_n \cdot M \cdot \widetilde{\alpha}_n\right] \quad, \tag{3.70}$$

for the bosonic coordinate,

$$\langle B_\psi, \eta|_{\mathrm{NS}} = \mathrm{i}\,\langle 0|\, \exp\left[\mathrm{i}\,\eta \sum_{t=1/2}^{\infty} \psi_t \cdot M \cdot \widetilde{\psi}_t\right] \tag{3.71}$$

for the NS-NS sector, and

$$\langle B_\psi, \eta|_{\mathrm{R}} = -\langle B, \eta|_{\mathrm{R}}^{(0)} \exp\left[\mathrm{i}\,\eta \sum_{t=1}^{\infty} \psi_t \cdot M \cdot \widetilde{\psi}_t\right] \tag{3.72}$$

for the R-R sector, where

$$\langle B, \eta|_{\mathrm{R}}^{(0)} = (-1)^p k(\hat{F})\langle A| \langle\widetilde{B}| \left(C\Gamma^0\Gamma^1 \dots \Gamma^p \frac{1 + \mathrm{i}\eta\Gamma_{11}}{1 - \mathrm{i}\eta} U\right)_{AB} \tag{3.73}$$

---

[7]See eqs.(1.9) and Ref. [4] for more details.

with U given in eq.(3.69).

The coupling of a Dp-brane with the massless fields of the closed superstring can be obtained by saturating the boundary state with the states corresponding to those fields. In the following we will show that the structure of those couplings is the same as that obtained from the DBI action and actually the comparison with what comes from the DBI action allows us to fix the normalization constants $N_p(F)$ and $k(F)$ appearing in the boundary state. We want to stress, however, that the normalization constants $N_p(F)$ and $k(F)$ could also be independently determined by requiring that the interaction between two branes be the same if we compute it in the open or in the closed string channel.

The coupling of a Dp-brane with a specific massless field Ψ can be computed by saturating the boundary state with the corresponding field $\langle\Psi|$ ($\langle\Psi|$ can be $\langle\Psi_h|, \langle\Psi_B|, \langle\Psi_\phi|$ corresponding respectively to the graviton, antisymmetric tensor and dilaton or $\langle C_{(n)}|$ corresponding to a R-R state). By proceeding in this way we get the following couplings:

$$
\begin{aligned}
J_\phi &\equiv \frac{1}{2\sqrt{2}} J^{\mu\nu} \left(\eta_{\mu\nu} - k_\mu \ell_\nu - k_\nu \ell_\mu\right) \phi \\
&= \frac{N_p(F)}{2\sqrt{2}} V_{p+1} T_p \left[\frac{d - 2p - 4}{2} + \mathrm{Tr}\left(\hat{F}(\mathbb{1} + \hat{F})^{-1}\right)\right] \phi \ ;
\end{aligned}
\tag{3.74}
$$

for the dilaton,

$$
J_h \equiv J^{\mu\nu} h_{\mu\nu} = -N_p(F) V_{p+1} T_p \left[(\eta + \hat{F})^{-1}\right]^{\alpha\beta} h_{\beta\alpha}
\tag{3.75}
$$

for the graviton,

$$
\begin{aligned}
J_B &\equiv \frac{1}{\sqrt{2}} J^{\mu\nu} B_{\mu\nu} = -\frac{N_p(F)}{2\sqrt{2}} V_{p+1} T_p \left[(\eta - \hat{F})(\eta + \hat{F})^{-1}\right]^{\alpha\beta} B_{\beta\alpha} \\
&= -\sqrt{2} N_p(F) V_{p+1} \frac{T_p}{2} \left[(\eta + \hat{F})^{-1}\right]^{\alpha\beta} B_{\beta\alpha}
\end{aligned}
\tag{3.76}
$$

for the NS-NS 2-form potential and

$$
J_{C_n} \equiv \langle C_{(n)}|B\rangle_R = -\frac{C_{\mu_1\ldots\mu_n}}{16\sqrt{2}(n)!} V_{p+1} N_p(F) k(F) \frac{T_p}{2} (1 - (-1)^{p+n})
$$

$$
Tr\left(\Gamma^{\mu_n\cdots\mu_1}\Gamma^0 \ldots \Gamma^p; \mathrm{e}^{-1/2\hat{F}_{\alpha\beta}\Gamma^\alpha\Gamma^\beta};\right)
\tag{3.77}
$$

for the R-R states. The details of these calculations are given in Ref. [16]. The trace in eq.(3.77) can be easily computed by expanding the exponential term. The first term of the expansion of the exponential gives the coupling of the boundary state with a $(p+1)$-form potential of the R-R sector and is given by

$$
J_{C_{(p+1)}} = \frac{\sqrt{2} T_p N_p(F) k(F)}{(p+1)!} V_{p+1} C_{\alpha_0\ldots\alpha_p} \varepsilon^{\alpha_0\ldots\alpha_p}
\tag{3.78}
$$

12

where we have used that for $d = 10$ the Γ matrices are 32×32 dimensional matrices, and thus $Tr(\mathbb{1}) = 32$ and that only the term with $n = p + 1$ gives a non-vanishing contribution. Here $\varepsilon^{\alpha_0 \dots \alpha_p}$ indicates the completely antisymmetric tensor on the D brane world-volume [8]. From Eq. (3.78) we can immediately deduce that the charge of a Dp-brane with respect to the R-R potential $C_{(p+1)}$ is

$$\sqrt{2} T_p \, N_p(F) k(F) \tag{3.79}$$

The next term in the expansion of the exponential of Eq. (3.77) yields the coupling of the Dp brane with a $(p-1)$-form potential which is given by

$$J_{C_{(p-1)}} = \frac{\sqrt{2} N_p(F) k(F)}{(p-1)!} V_{p+1} \frac{T_p}{2} C_{\alpha_0 \dots \alpha_{p-2}} \hat{F}_{\alpha_{p-1} \alpha_p} \varepsilon^{\alpha_0 \dots \alpha_p} \quad . \tag{3.80}$$

where we have explicitly used the fact that only the term with $n = p - 1$ gives a non-vanishing contribution. By proceeding in the same way, one can easily evaluate also the higher order terms generated by the exponential which describe the interactions of the D-brane with potential forms of lower degree. All these couplings can be encoded in the following term

$$\sum_{n=0}^{\ell_{\max}} \langle C_{(n)} | B \rangle_R = \sqrt{2} T_p \, N_p(F) k(F) \int_{V_{p+1}} \left[\sum_{\ell=0}^{\ell_{\max}} C_{(p+1-2\ell)} \wedge e^{\hat{F}} \right]_{p+1} \tag{3.81}$$

where $\ell_{\max}$ is $p/2$ for the type IIA string and $\frac{p+1}{2}$ for the type IIB string. We have defined $\hat{F} = \frac{1}{2} \hat{F}_{\alpha\beta} \, d\xi^\alpha \wedge d\xi^\beta$, while

$$C_{(n)} = \frac{1}{n!} C_{\alpha_1 \dots \alpha_n} \, d\xi^{\alpha_1} \wedge \dots \wedge d\xi^{\alpha_n} \tag{3.82}$$

$C_{\alpha_1 \dots \alpha_n}$ is the pullback of the n-form potential on the D-brane world-volume. The square bracket in Eq. (3.81) means that in expanding the exponential form one has to pick up only the terms of total degree $(p+1)$, which are then integrated over the $(p+1)$-dimensional world-volume.

The couplings of a Dp-brane with the massless states of closed superstring that have been extracted from the boundary state can be compared with those computed from its low energy effective action. The low energy effective action describing a Dp-brane consists of a sum of two terms. The first one is the so called Dirac-Born-Infeld action, which in the string frame is given by

$$S_{DBI} = -\frac{T_p}{\kappa} \int_{V_{p+1}} d^{p+1}\xi \, e^{-\varphi} \sqrt{-\det\left[G_{\alpha\beta} + \mathcal{B}_{\alpha\beta} + \hat{F}_{\alpha\beta} \right]} \quad . \tag{3.83}$$

where we are considering only its bosonic part, $2\kappa^2 = (2\pi)^7 (\alpha')^4 g_s^2$, ($g_s$ being the string coupling) and $G_{\alpha\beta}$ and $\mathcal{B}_{\alpha\beta}$ are respectively the pullbacks of the space-time metric and of the NS-NS antisymmetric tensor on the D-brane world volume:

$$G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X) \qquad\qquad \mathcal{B}_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \mathcal{B}_{\mu\nu}(X) \tag{3.84}$$

---

[8]Our convention is that $\varepsilon^{0 \dots p} = -\varepsilon_{0 \dots p} = 1$.

Notice that the action in eq.(3.83) is written using the string metric $G_{\mu\nu}$, and should be considered together with the gravitational bulk action in the string frame

$$S_{bulk} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\varphi} \left[R(G) + 4(\nabla\varphi)^2 - \frac{1}{12}(H_3)^2 \right] - \right.$$

$$\left. - \sum_n \frac{1}{2 \cdot (n+1)!} (F_{(n+1)})^2 \right\} \quad . \tag{3.85}$$

where H_3 $[F_{n+1}]$ is the field strenght corresponding to the NS-NS [R-R] 2-form [n-form] potential. In order to compare the couplings described by this action with the ones obtained from the boundary state, it is first necessary to rewrite S_{DBI} in the Einstein frame. In fact, like any string amplitude computed with the operator formalism, also the couplings J_h, J_ϕ and J_B are written in the Einstein frame. Furthermore, it is also convenient to introduce canonically normalized fields

$$G_{\mu\nu} = e^{\varphi/2} g_{\mu\nu} \quad , \quad \varphi = \sqrt{2}\kappa\,\phi \quad , \quad \mathcal{B}_{\alpha\beta} = \sqrt{2}\kappa B_{\alpha\beta}e^{\varphi/2} \quad . \tag{3.86}$$

Inserting these fields in Eq. (3.83) we get

$$S_{DBI} = -\frac{T_p}{\kappa} \int d^{p+1}\xi\, e^{-\kappa\phi(3-p)/(2\sqrt{2})} \sqrt{-\det\left[g_{\alpha\beta} + \sqrt{2}\kappa B_{\alpha\beta} + \hat{F}_{\alpha\beta}e^{-\kappa\phi/\sqrt{2}} \right]} \quad . \tag{3.87}$$

By expanding the metric around the flat background as

$$g_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \tag{3.88}$$

and keeping only the terms which are linear in the fields h, ϕ and B one gets from eq.(3.87) the following expression:

$$S_{DBI} \simeq - T_p \int_{V_{p+1}} d^{p+1}\xi \sqrt{-\det\left[\eta + \hat{F} \right]} \left\{ \left[(\eta + \hat{F})^{-1} \right]^{\alpha\beta} h_{\beta\alpha} \right. \tag{3.89}$$

$$\left. - \frac{1}{2\sqrt{2}} \left[3 - p + \text{Tr}\left(\hat{F}(\eta + \hat{F})^{-1} \right) \right] \phi + \frac{1}{\sqrt{2}} \left[(\eta + \hat{F})^{-1} \right]^{\alpha\beta} B_{\beta\alpha} \right\} \quad .$$

Comparing these couplings with those obtained in eqs.(3.74), (3.75) and (3.76) we can fix the normalization constant $N_p(F)$ to be

$$N_p(F) = \sqrt{-\det(\eta + \hat{F})} \tag{3.90}$$

Therefore the normalization factor of the boundary state turns out to be proportional to the Dirac-Born-Infeld Lagrangian.

The second term is the Wess-Zumino action which is given by

$$S_{WZ} = \mu_p \int_{V_{p+1}} \sum_{n=0}^{\ell_{max}} C_{(p+1-2n)} \wedge e^{\hat{F}} \tag{3.91}$$

14

This has exactly the same structure as in eq.(3.81) obtained by saturating the boundary state with the R-R state.

Comparing eq.(3.91) with eq.(3.81) we can fix the other normalization constant to be

$$k(F) = \frac{1}{\sqrt{\det(\eta + \hat{F})}} \tag{3.92}$$

and we can see that the coupling of a Dp-brane with the R-R (p+1)-form potential is given by

$$\sqrt{2}T_p = \mu_p \tag{3.93}$$

This is exactly the R-R charge carried by the p-brane classical solution of the low-energy string effective action in ten dimensions. In conclusion we have explicitly shown that by projecting the boundary state $|B\rangle$ with an external field onto the massless states of the closed string spectrum, one can reconstruct the linear part of the low-energy effective action of a Dp brane. This is the sum of the Dirac-Born-Infeld part (3.89) and the Wess-Zumino term (3.81) which are produced respectively by the NS-NS and the R-R components of the boundary state.

For the sake of completeness we conclude this section by giving the asymptotic behaviour of the fields generated by a Dp-brane with an arbitrary external field on it, determined by computing the quantity

$$\langle P_x|D|B\rangle \tag{3.94}$$

where D is the closed superstring propagator and P_x are the projectors of the closed superstring massless sector that can be found in Ref. [16]. For the massless NS-NS fields we get

$$\delta\phi = \frac{T_p V_{p+1}}{2\sqrt{2}k_\perp^2}\sqrt{-\det(\eta + \hat{F})}\left(\frac{(d-2p-4)}{2} + Tr\hat{F}(\mathbb{1} + \hat{F})^{-1}\right), \tag{3.95}$$

for the dilaton,

$$\delta h^{\mu\nu} = -\frac{T_p V_{p+1}}{4k_\perp^2}\sqrt{-\det(\eta + \hat{F})}\left\{2\left(\eta + \hat{F}\right)^{-1\ \alpha\beta} + 2\left(\eta + \hat{F}\right)^{-1\ \beta\alpha} + \right.$$

$$\left. +\frac{\eta^{\alpha\beta}}{2}\left(-(1+p) + Tr\hat{F}(\mathbb{1} + \hat{F})^{-1}\right), \frac{\delta^{ij}}{2}\left(-(1+p) + Tr\hat{F}(\mathbb{1} + \hat{F})^{-1}\right)\right\} \tag{3.96}$$

for the graviton, and

$$\delta B^{\alpha\beta} = -\frac{T_p V_{p+1}}{\sqrt{2}k_\perp^2}\sqrt{-\det(\eta + \hat{F})}\left[\left(\eta + \hat{F}\right)^{-1\ \beta\alpha} - \left(\eta + \hat{F}\right)^{-1\ \alpha\beta}\right] \tag{3.97}$$

for the antisymmetric tensor. Here, as usual $\alpha, \beta \in (0, ..., p)$, $i, j \in (p+1, ..., d-1)$ and $k_\perp$ is the transverse momentum emitted from the brane. For the massless R-R fields instead the long range fluctuaction around the background values differs from

the expressions of the couplings contained in eq.(3.81) simply for the inclusion of the effect of the propagator, which generate a factor $1/k_\perp^2$.

From the previous expressions we can reconstruct the long distance behaviour of the various fields in configuration space using the same notation as in Refs. [4]. For the metric tensor which is connected to the graviton field by the relation in eq.(3.88) we get

$$\delta \tilde{h}^{\mu\nu}(r) = -\frac{Q_p}{4r^{7-p}} \sqrt{-\det(\eta + \hat{F})} \left\{ 2 \left(\eta + \hat{F} \right)^{-1 \ \alpha\beta} + \left(\eta + \hat{F} \right)^{-1 \ \beta\alpha} + \right.$$

$$\left. +\frac{\eta^{\alpha\beta}}{2} \left(-(1+p) + Tr\hat{F}(\eta + \hat{F})^{-1} \right), \frac{\delta^{ij}}{2} \left(-(1+p) + Tr\hat{F}(\eta + \hat{F})^{-1} \right) \right\} \quad (3.98)$$

where r is the radial coordinate in the transverse space. For the dilaton field φ which is connected to the canonically normalized field by the second eq. in (3.86) we get

$$\delta\varphi(r) = \frac{Q_p}{4r^{7-p}} \sqrt{-\det(\eta + \hat{F})} \left(3 - p + Tr\hat{F}(\eta + \hat{F})^{-1} \right), \quad (3.99)$$

and for the antisymmetric tensor we get

$$\delta\mathcal{B}^{\alpha\beta}(r) = -\frac{Q_p}{2r^{7-p}} \sqrt{-\det(\eta + \hat{F})} \left[\left(\eta + \hat{F} \right)^{-1 \ \beta\alpha} - \left(\eta + \hat{F} \right)^{-1 \ \alpha\beta} \right] \quad (3.100)$$

The field $\mathcal{B}$ is related to B in eq.(3.97) by eq.(3.86).

4   T-duality and transverse excitations

It is by now well known[9] that, compactifying $d - p - 1$ directions and performing on them a T-duality transformation, the abelian gauge potential A_μ that lives on a D9-brane and that describes the massless excitations of an open string with NN boundary conditions in all directions is transformed as follows: its longitudinal components (those along the directions in which no T-duality transformation is performed) give rise to a gauge field A_α living on the world volume of a Dp-brane, while its transverse components (those along the directions transformed by T-duality) become the coordinates of the Dp-brane. As the original gauge field, that is related to the massless excitations of an open string, corresponds to the collective coordinates of the D9-brane, so after the T-duality transformation both its longitudinal components and its transverse ones, that become the coordinates of the Dp-brane, correspond to the collective coordinates of the Dp-brane.

In the following we want to see how this conclusion can be reached from the point of view of the boundary state. To this purpose let us compare the overlap conditions for a boundary state describing a Dp-brane with an external electric field,

---

[9]See for instance sect. 4 of Ref. [4]

that for simplicity we take with only one non vanishing component F^{0k} (here k is one of the longitudinal directions), with those for the boundary state describing a $D(p-1)$-brane boosted in the transverse direction k discussed in the section 2. In other words the direction k is one of the longitudinal directions of the Dp-brane, while is one of the transverse directions of the boosted $D(p-1)$-brane. Remember that a T-duality transformation in a longitudinal direction transforms a Dp-brane into a $D(p-1)$-brane. Therefore if we perform it on the Dp-brane that is described by the boundary state defined through the overlap conditions given in the eq.(3.59) by remembering that, as discussed in Ref. [4], T-duality acts on the coordinate X^k by changing its τ derivative (Dirichlet-like boundary conditions in the open channel) with minus its σ derivative (Neumann-like boundary conditions), we obtain that eq.(3.59) becomes:

$$(\partial_\tau X^0 - \hat{F}_k{}^0 \partial_\tau X^k)|_{\tau=0}|B\rangle = 0 \tag{4.101}$$

$$(\partial_\sigma X^k - \hat{F}(X)_0{}^k \partial_\sigma X^0)|_{t=0}|B\rangle = 0 \tag{4.102}$$

By making the identification

$$\hat{F}_0{}^k = \hat{F}_k{}^0 = -v \tag{4.103}$$

the previous equations exactly reproduce eqs.(2.15) and (2.18). We can therefore conclude that the boundary state for a Dp-brane with an external electric field having only a non zero component in the direction k is equivalent, through a T-duality transformation along the direction X^k, to the boundary state of a $D(p-1)$-brane boosted and delocalized along the transverse direction k.

Analogously it can be shown that the boundary state for a Dp-brane with an external magnetic field having $F_{\alpha k}$ as the only non vanishing component ($\alpha \neq 0$ and $k \neq 0$ are both longitudinal components) is transformed by a T-duality transformation on the coordinate X^k to that of a $D(p-1)$-brane rotated in the (α, k) plane and delocalized in the direction X^k. In this case in fact from eq.(3.59) we get

$$(\partial_\tau X^\alpha - \hat{F}_k{}^\alpha \partial_\tau X^k)|_{\tau=0}|B\rangle = 0 \tag{4.104}$$

$$(\partial_\sigma X^k - \hat{F}_\alpha{}^k \partial_\sigma X^\alpha)|_{t=0}|B\rangle = 0 \tag{4.105}$$

Then by making the identification

$$\hat{F}_\alpha{}^k = -\hat{F}_k{}^\alpha = \tan\omega \tag{4.106}$$

the previous equations reproduce the overlap conditions for a $D(p-1)$-brane which is rotated in the (α, k) plane and delocalized in the direction X^k, that are given in eq.(2.36) and in the equation obtained by taking the σ derivative of eq.(2.37).

This result shows that after a T-duality transformation the T-dualized components of the gauge field $U(1)$ are correctly reinterpreted as the transverse coordinates of a $D(p-1)$-brane. In fact if we consider a boost in the direction k and start with $X^k = 0$ we get

$$\begin{cases} X'^0 = \frac{X^0}{\sqrt{1-v^2}} \\ X'^k = \frac{vX^0}{\sqrt{1-v^2}} \end{cases} \Rightarrow \partial_{0'} X'^k = v \tag{4.107}$$

Comparing eqs.(4.107) with eq.(4.103) we get

$$\hat{F}_0{}^k = -\partial_{0'} X'{}^k \Rightarrow \hat{A}^k \sim -(X')^k \qquad (4.108)$$

Analogously considering a rotation in the (α, k) plane and starting with $X^k = 0$ we get

$$\begin{cases} X'{}^\alpha = X^\alpha \cos\omega \\ X'{}^k = -X^\alpha \sin\omega \end{cases} \Rightarrow \partial_{\alpha'} X'{}^k = -\tan\omega \qquad (4.109)$$

that compared with eq. (4.106) gives

$$F_\alpha{}^k = -\partial_{\alpha'} X'{}^k \Rightarrow \hat{A}^k \sim -X^k \qquad (4.110)$$

showing that the transverse components of the gauge fields become the transverse collective coordinates of the Dp-brane. Notice that in deriving both eqs.(4.108) and (4.110) we have assumed that $\partial_k A^0$ and $\partial_k A^\alpha$ are equal zero. This is a consequence of the fact that the D$(p-1)$-brane is delocalized in the k direction.

Since in the static gauge the $p+1$ coordinates of the world volume of a Dp-brane can be fixed to be $X^\alpha = \xi^\alpha$, a Dp-brane is described by the transverse coordinate X^i and by the gauge field A_α corresponding to a total of 10 degrees of freedom of which only 8 are physical. Turning on a longitudinal gauge field in the boundary state as we have done in eqs.(3.62), (3.65) and (3.66) we have included in the description of the Dp-brane only some of its excitations. Following the previous discussion we can also include a dependence on its transverse coordinates and therefore have a boundary state depending not only on a longitudinal gauge field, but also on the transverse coordinates of the Dp-brane. From the previous discussion it is then immediately clear how this can be done: we must start from the boundary state of a D9-brane in which $9-p$ directions are compactified and then perform a T-duality transformation along all of them. This can be easily done and we arrive at the following bosonic boundary state:

$$|B_X\rangle = \frac{T_9}{2}\sqrt{-\det(\eta_{\alpha\beta} + \widetilde{F}_{\alpha\beta})}\, e^{-\sum_{n=1} \frac{1}{n}\alpha_{-n}\cdot M(\hat{F}, X^\perp)\cdot\widetilde{\alpha}_{-n}} |0\rangle_\alpha |0\rangle_{\widetilde{\alpha}} |k=0\rangle \qquad (4.111)$$

where $M(\hat{F}, X^\perp)$ is the following 10×10 matrix

$$M(\hat{F}, X^\perp)_{\mu\nu} = \begin{pmatrix} [(1-\widetilde{F})(1+\widetilde{F})^{-1}]^{\alpha\beta} & 2\partial_\beta X^i(1+\widetilde{F})^{-1\,\alpha\beta} \\ 2\partial_\beta X^i(1+\widetilde{F})^{-1\,\beta\alpha} & -I + \partial_\alpha X^i \partial_\beta X^j(1+\widetilde{F})^{-1\,\alpha\beta} \end{pmatrix} \qquad (4.112)$$

and where we have defined

$$\widetilde{F}^{\alpha\beta} = \hat{F}^{\alpha\beta} + \partial^\alpha X^i \partial^\beta X_i \qquad (4.113)$$

The expression of the matrix $M(\hat{F}, X^\perp)$ has been determined by observing that under a T-duality transformation over $(9-p)$ coordinates the exponential factor appearing in the boundary state changes as follows

$$\alpha^\mu_{-n} M_{\mu\nu} \widetilde{\alpha}^\nu_{-n} \rightarrow \alpha^\alpha_{-n} M_{\alpha\beta} \widetilde{\alpha}^\beta_{-n} - \alpha^\alpha_{-n} M_{\alpha j} \widetilde{\alpha}^j_{-n} + \alpha^j_{-n} M_{j\alpha} \widetilde{\alpha}^\alpha_{-n} - \alpha^i_{-n} M_{ij} \widetilde{\alpha}^j_{-n} \qquad (4.114)$$

18

where $\alpha, \beta \in (0, ...p)$ and $i, j \in (p + 1, ...9 - p)$. Then in terms of the matrix appearing in the exponential factor of the boundary state a T-duality transformation simply acts by changing the sign of all the columns corresponding to the directions affected by the T-duality transformation.

Saturating the previous boundary state with the various massless fields of the bosonic sector of the closed superstring we reproduce the couplings that one can read from the DBI action, for the case $\partial^\alpha X^j \neq 0 \; \forall \alpha, j$ which are in fact given by the following expression

$$-\frac{T_p}{2} \int d^{p+1}\xi \sqrt{-\det(\eta_{\alpha\beta} + \widetilde{F}_{\alpha\beta})} \left\{ \frac{\chi}{\sqrt{2}} \left[(p - 3) - (1 + \widetilde{F})^{-1 \; \beta\delta} \hat{F}_{\delta\beta} \right] + \right.$$

$$2(1 + \widetilde{F})^{-1 \; \beta\delta} \left(h_{\delta\beta} + h_{ij}\partial_\delta X^i \partial_\beta X^j + h_{\delta i}\partial_\beta X^i + h_{\beta i}\partial_\delta X^i \right)$$

$$\left. +\sqrt{2}(1 + \widetilde{F})^{-1 \; \beta\delta} \left(B_{\delta\beta} + B_{ij}\partial_\delta X^i \partial_\beta X^j + B_{\delta i}\partial_\beta X^i + B_{\beta i}\partial_\delta X^i \right) \right\} \tag{4.115}$$

5 (F,Dp) bound states from the boundary state

In this section we are going to show that the boundary state with an external *electric* field on it can be used to obtain the long distance behaviour of the various fields describing the bound state (F, Dp) formed by a fundamental string and a Dp-brane [16]. This type of bound state is a generalization of the dyonic string solution of Schwarz [17] and has been recently discussed from the supergravity point of view [8, 9, 10]. The (F, Dp) bound state can be obtained from a Dp-brane by turning on an *electric* field $\hat{F}$ on its world volume. With no loss in generality we can choose $\hat{F}$ to have non vanishing components only in the directions X^0 and X^1 so that it can be represented by the following $(p + 1) \times (p + 1)$ matrix

$$\hat{F}_{\alpha\beta} = \begin{pmatrix} 0 & -f & & & \\ f & 0 & & & \\ & & 0 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}. \tag{5.1}$$

Using this expression in Eq. (3.63) one can easily see that the longitudinal part of the matrix M appearing in the boundary state is given by

$$M_{\alpha\beta} = \begin{pmatrix} -\frac{1+f^2}{1-f^2} & \frac{2f}{1-f^2} & & & \\ -\frac{2f}{1-f^2} & \frac{1+f^2}{1-f^2} & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix} \tag{5.2}$$

19

while the transverse part of M is simply minus the identity in the remaining $(9 - p)$ directions. Furthermore, using eq.(5.1) one finds

$$- \det \left(\eta + \hat{F} \right) = 1 - f^2 \quad . \tag{5.3}$$

Since we want to describe configurations of branes with arbitrary R-R charge, we multiply the entire boundary state by an overall factor of x. Later we will see that the consistency of the entire construction will require that x be an integer, and also that the electric field strength f cannot be arbitrary.

Let us now begin our analysis by studying the projection in eq.(3.94) in the NS-NS sector. Using eqs. (3.95)-(3.97), we find the long-distance behavior of the NS-NS massless fields generated by the (F,Dp) bound state. For the dilaton we get

$$\delta \phi = \mu_p \frac{V_{p+1}}{k_\perp^2} x \frac{f^2(p - 5) + (3 - p)}{4 \sqrt{1 - f^2}} \tag{5.4}$$

For the antisymmetric Kalb-Ramond field we find that, since the matrix $M_{\mu\nu}$ is symmetric except in the block corresponding to the 0 and 1 directions (see Eq. (5.2)), its only non-vanishing component is B_{01} whose long-distance behavior is given by

$$\delta B_{01} = \mu_p \frac{V_{p+1}}{k_\perp^2} \frac{x \, f}{\sqrt{1 - f^2}} \quad . \tag{5.5}$$

Finally, the components of the metric tensor are

$$\begin{aligned}
\delta h_{00} &= -\delta h_{11} = \mu_p \frac{V_{p+1}}{k_\perp^2} x \frac{f^2(p - 1) + (7 - p)}{8\sqrt{2}\sqrt{1 - f^2}} \quad , \\
\delta h_{22} &= \ldots = \delta h_{pp} = \mu_p \frac{V_{p+1}}{k_\perp^2} x \frac{f^2(9 - p) + (p - 7)}{8\sqrt{2}\sqrt{1 - f^2}} \quad , \\
\delta h_{p+1,p+1} &= \ldots = \delta h_{99} = \mu_p \frac{V_{p+1}}{k_\perp^2} x \frac{f^2(1 - p) + (p + 1)}{8\sqrt{2}\sqrt{1 - f^2}} \quad .
\end{aligned} \tag{5.6}$$

Let us now turn to the R-R sector. In this case, after the insertion of the closed string propagator, we have to saturate the R-R boundary state with the projectors on the various R-R massless fields that can be found in Ref. [16]. Due to the structure of the R-R component of the boundary state describing the bound state (F,Dp), it is not difficult to realize that the only projectors that can give a non vanishing result are those corresponding to a $(p + 1)$-form and to a $(p - 1)$-form with all indices along the world-volume directions. In particular, we find that the long distance behavior of the $(p + 1)$-form is given by

$$\delta C_{01 \cdots p} \equiv \langle P^{(C)}_{01 \cdots p}| \, D \, |B\rangle_{\rm R} = -\mu_p \frac{V_{p+1}}{k_\perp^2} x \quad . \tag{5.7}$$

Similarly, given our choice of the external field, we find that the only non vanishing component of the $(p - 1)$-form emitted by the boundary state is $C_{23 \cdots p}$ whose long-distance behavior turns out to be

$$\delta C_{23 \cdots p} \equiv \langle P^{(C)}_{23 \cdots p}| \, D \, |B\rangle_{\rm R} = -\mu_p \frac{V_{p+1}}{k_\perp^2} x \, f \quad . \tag{5.8}$$

20

Notice that if $p = 1$ this expression has to be interpreted as the long-distance behavior of the R-R scalar which is usually denoted by χ.

In all our previous analysis, the two parameters x and f that appear in the boundary state seem to be arbitrary. However, this is not so at a closer inspection. In fact, they are strictly related to the electric charges of the (F,Dp) configuration under the Kalb-Ramond field and the R-R $(p+1)$-form potential. It is well-known that these charges must obey the Dirac quantization condition, *i.e.* they must be integer multiples of the fundamental unit of (electric) charge of a p-dimensional extended object μ_p. In our notations this quantization condition amounts to impose that the coefficients of $-\mu_p \frac{V_{p+1}}{k_\perp^2}$ in Eqs. (5.5) and (5.7) be integer numbers. This implies that

$$x = n \quad \text{and} \quad -\frac{xf}{\sqrt{1-f^2}} = m \tag{5.9}$$

with n and m two integers. While the restriction on x had to be expected from the very beginning because x simply represents the number of Dp branes (and hence of boundary states) that form the bound state, the restriction on the external field f is less trivial. In fact, from Eq. (5.9) we see that f must be of the following form

$$f = -\frac{m}{\sqrt{n^2 + m^2}} \quad . \tag{5.10}$$

This is precisely the same expression that appears in the analysis of Ref. [18] on the dyonic string configurations, and is also consistent with the results of Ref. [8, 9, 10].

Using Eq. (5.9), we can now rewrite the long distance behavior of the massless fields produced by a (F,Dp) bound state in a more suggestive way. In doing so, we also perform the Fourier transformation for $d = 10$ to work in configuration space. For later convenience, we also introduce the following notations

$$\Delta_{m,n} = m^2 + n^2 \tag{5.11}$$

Then, using Eq. (5.4) and assuming for the time being that the dilaton has vanishing vacuum expectation value, after some elementary steps, we obtain that the long-distance behavior of the dilaton is

$$\varphi = \sqrt{2}\,\kappa\,\phi \simeq -\frac{n^2\,(p-3) + 2\,m^2}{4\,\Delta_{m,n}}\,\frac{Q_p}{r^{7-p}} \quad , \tag{5.12}$$

where Q_p can be found in Ref. [16].

Since we are going to compare our results with the standard supergravity description of D-branes, we have reintroduced the field φ which differs from the canonically normalized dilaton ϕ by a factor of $\sqrt{2}\,\kappa$ (see eq.(3.86)). Similarly, recalling that $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa\,h_{\mu\nu}$, from Eq. (5.6) we find

$$g_{00} = -g_{11} \simeq -1 + \frac{6m^2 - n^2\,(p-7)}{8\,\Delta_{m,n}}\,\frac{Q_p}{r^{7-p}} \quad ,$$

$$g_{22} = \ldots = g_{pp} \simeq 1 + \frac{2m^2 - n^2\,(7-p)}{8\,\Delta_{m,n}}\,\frac{Q_p}{r^{7-p}} \quad, \tag{5.13}$$

$$g_{p+1,p+1} = \ldots = g_{99} \simeq 1 + \frac{2m^2 + n^2\,(p+1)}{8\,\Delta_{m,n}}\,\frac{Q_p}{r^{7-p}} \quad. $$

Rescaling the Kalb-Ramond field by a factor of $\sqrt{2}\,\kappa$ to obtain the standard supergravity normalization and using eq.(5.5), we easily get

$$\mathcal{B} = \sqrt{2}\kappa\,B \simeq -\frac{m}{\Delta_{m,n}^{1/2}}\,\frac{Q_p}{r^{7-p}}\,dx^0 \wedge dx^1 \quad. \tag{5.14}$$

Finally, repeating the same steps for the R-R potentials (5.7) and (5.8) we find

$$\mathcal{C}_{(p+1)} = \sqrt{2}\kappa\,C_{(p+1)} \simeq -\frac{n}{\Delta_{m,n}^{1/2}}\,\frac{Q_p}{r^{7-p}}\,dx^0 \wedge \ldots \wedge dx^p \quad, \tag{5.15}$$

and

$$\mathcal{C}_{(p-1)} = \sqrt{2}\kappa\,C_{(p-1)} \simeq \frac{m\,n}{\Delta_{m,n}}\,\frac{Q_p}{r^{7-p}}\,dx^2 \wedge \ldots \wedge dx^p \quad. \tag{5.16}$$

Eqs. (5.12)-(5.16) represent the leading long-distance behavior of the massless fields emitted by the (F,Dp) bound state. Proceeding as in Ref. [16] where we have assumed that the exact solution can be written in terms of powers of the usual harmonic function

$$H(r) = 1 + \frac{Q_p}{r^{7-p}} \tag{5.17}$$

and also of the new harmonic function

$$H'(r) = 1 + \frac{n^2}{\Delta_{m,n}}\,\frac{Q_p}{r^{7-p}} \tag{5.18}$$

introduced in Ref. [9], from eqs.(5.12)-(5.16) we can infer that in the exact brane-solution corresponding to the (F, Dp) bound state the dilaton is

$$\mathrm{e}^{\varphi} = H^{-1/2}\,H'^{(5-p)/4} \quad, \tag{5.19}$$

the metric is

$$\begin{aligned}
ds^2 = \ & H^{-3/4}\,H'^{(p-1)/8}\left[-\left(dx^0\right)^2 + \left(dx^1\right)^2\right] \\
& + \ H^{1/4}\,H'^{(p-9)/8}\left[\left(dx^2\right)^2 + \cdots + (dx^p)^2\right] \\
& + \ H^{1/4}\,H'^{(p-1)/8}\left[\left(dx^{p+1}\right)^2 + \cdots + \left(dx^9\right)^2\right] \quad,
\end{aligned} \tag{5.20}$$

the Kalb-Ramond 2-form is

$$\mathcal{B} = \frac{m}{\Delta_{m,n}^{1/2}}\,\left(H^{-1} - 1\right)\,dx^0 \wedge dx^1 \quad, \tag{5.21}$$

and finally the R-R potentials are

$$\mathcal{C}_{(p+1)} = \frac{n}{\Delta_{m,n}^{1/2}} \left(H^{-1} - 1 \right) dx^0 \wedge \cdots \wedge dx^p \quad, \tag{5.22}$$

and

$$\mathcal{C}_{(p-1)} = -\frac{m}{n} \left(H'^{-1} - 1 \right) dx^2 \wedge \cdots \wedge dx^p \quad. \tag{5.23}$$

In the case $p = 1$, the last equation has to be replaced by

$$\chi = -\frac{m}{n} \left(H'^{-1} - 1 \right) \tag{5.24}$$

where χ is the R-R scalar field also called axion.

In writing this solution we have assumed that all fields except the metric have vanishing asymptotic values. This explains why we have subtracted the 1 in the last four equations. Our solution exactly agrees with the one recently derived in Ref. [9] from the supergravity point of view [10]. Moreover, eq.(5.24) can be shown to exactly agree with the axion field of the dyonic string solution of Schwarz [17] in the case of vanishing asymptotic background values for the scalars ($\varphi_0 = \chi_0 = 0$). One can compute the vacuum amplitude between two boundary states at a distance r from each other. This calculation has been performed in Ref. [16] where it has been found that the two branes do not interact.

We can therefore conclude that the boundary state with an external electric field (eq.(5.1)), really provides the complete conformal description of the BPS bound states formed by fundamental strings and Dp-branes.

6 Compactified boundary state

In this section we construct the boundary state describing a Dp-brane which has all directions compactified on circles. From it, by decompactifying some directions, we can obtain the one in which only certain directions are compactified. For the sake of simplicity we take radii all equal to R, but from the formulas that we will get, it will be trivial to extend our results to arbitrary radii. Before we write the boundary state we first want to introduce a convenient notation. In the compactified case it is convenient to introduce position and momentum operators separately for momentum and winding degrees of freedom requiring for them the following commutation relations:

$$[q_w^\mu, p_w^\nu] = i\eta^{\mu\nu} \qquad ; \qquad [q_n^\mu, p_n^\nu] = i\eta^{\mu\nu} \tag{6.25}$$

[10] Actually, in comparing our results with those of Ref. [9], we find total agreement except for the overall sign in the Kalb-Ramond 2-form. Our sign however agrees with the dyonic string solution of Schwarz [17] when we put $p = 1$.

where the subscripts n and w correpond respectively to the momentum and to the winding degrees of freedom and the other commutators are all vanishing. By denoting with $|n^\mu, w^\nu\rangle$ an eigenstate of the two "momentum" operators

$$p_n^\rho |n, w\rangle = \frac{n^\rho}{R} |n, w\rangle \quad ; \quad p_w^\rho |n, w\rangle = \frac{w^\rho R}{\alpha'} |n, w\rangle \tag{6.26}$$

it is easy to convince oneself that the previous state can also be written as follows

$$|n, w\rangle = e^{iq_n \cdot n/R} e^{iq_w \cdot w R/\alpha'} |0, 0\rangle \quad ; \tag{6.27}$$

where $|0, 0\rangle$ is the state with zero momentum and winding number. The state in eq.(6.27) is normalized as:

$$\langle n, w | n', w' \rangle = \Phi \, \delta_{nn'} \delta_{ww'} \tag{6.28}$$

where Φ is the "self-dual" volume that has the following properties:

$$\Phi = 2\pi R \quad if \quad R \to \infty \quad ; \quad \Phi = \frac{2\pi\alpha'}{R} \quad if \quad R \to 0 \tag{6.29}$$

Let us use this formalism to write the boundary state for the compactified case. In this case the part corresponding to the non-zero modes is unchanged, while the one corresponding to the zero modes of the bosonic coordinate becomes [19]:

$$|\Omega\rangle = \mathcal{N}_p \prod_{\alpha=0}^{p} \left[\sum_{w^\alpha} e^{i(q_w^\alpha - y^\alpha)w_\alpha R/\alpha'} \right] \prod_{i=p+1}^{d-1} \left[\sum_{n^i} e^{i(q_n^i - y^i)n_i/R} \right] |n = 0, w = 0\rangle \tag{6.30}$$

where the parameters y^α and y^i correspond respectively to Wilson lines turned on along the world volume of the brane and to the position of the brane in the transverse directions.

The previous boundary state satisfies the overlap conditions:

$$\left(e^{i(R/\alpha')q_w^\alpha} - e^{i(R/\alpha')y^\alpha} \right) |\Omega\rangle = p_n^\alpha |\Omega\rangle = 0 \quad , \quad \alpha = 0 \ldots p \tag{6.31}$$

and

$$\left(e^{iq_n^i/R} - e^{iy^i/R} \right) |\Omega\rangle = p_w^i |\Omega\rangle = 0 \quad , \quad i = p + 1 \ldots 9 - p \ . \tag{6.32}$$

The overall normalization can be determined by comparing the calculation of the brane interaction done in the closed and open string channels. In this way one gets the following relation [19]:

$$\mathcal{N}_p^2 \frac{\alpha'}{4} \Phi^d = \frac{V C_1}{2\pi} \tag{6.33}$$

where

$$V = (2\pi R)^{p+1} \left(\frac{2\pi\alpha'}{R} \right)^{d-p-1} \quad ; \quad C_1 = (2\pi)^{-d}(2\alpha')^{-d/2} \tag{6.34}$$

24

From eq.(6.33) we get

$$\mathcal{N}_p = \sqrt{\frac{2VC_1}{\pi\alpha'(2\pi R)^d}} (2\pi R)^{d-p-1} \left[\left(\frac{2\pi R}{\Phi}\right)^{d/2} (2\pi R)^{p+1-d} \right] \qquad (6.35)$$

After some calculation it is easy to see that the part of $\mathcal{N}_p$ that is not contained in the square bracket just reproduces the normalization of the boundary state in the uncompactified case and therefore we get:

$$\mathcal{N}_p = \frac{T_p}{2} \left[\left(\frac{2\pi R}{\Phi}\right)^{d/2} (2\pi R)^{p+1-d} \right] \qquad (6.36)$$

where T_p is defined in eq.(1.1). Let us show now that the previous normalization factor $\mathcal{N}_p$ reduces to $T_p/2$ in the decompactified limit. In the decompactification limit ($R \to \infty$) one can easily check the following relations:

$$\sum_{w^\alpha} e^{i(q_w^\alpha - y^\alpha)w_\alpha R/\alpha'} |0,0\rangle \to |0,0\rangle \qquad (6.37)$$

and

$$\sum_{n^i} e^{i(q_n^i - y^i)n_i/R/} |0,0\rangle \to R \int dk e^{i(q_n - y)k} |0,0\rangle =$$

$$(2\pi R) \int \frac{dk}{2\pi} e^{i(q_n - y)k} |0,0\rangle = 2\pi R \delta(q-y) |0,0\rangle \qquad (6.38)$$

where in the first relation we have taken into account that in the limit $R \to \infty$ only the term with $w = 0$ survives and in the second relation we have substituted in the decompactification limit a sum with an integral by introducing $k = n/R$. Using the two previous relations and the first equation in (6.29) it is easy to see that the normalization factor reduces to $T_p/2$ that is the correct one in the decompactified limit. If we decompactify only the time and the transverse directions the zero mode contribution in eq.(6.30) becomes

$$\frac{T_p}{2} \left(\frac{2\pi R}{\Phi}\right)^{p/2} \prod_{\alpha=1}^{p} \left[\sum_{w^\alpha} e^{i\theta^\alpha w^\alpha} |n^\alpha = 0, w^\alpha\rangle \right] |k^0 = 0\rangle \prod_{i=p+1}^{d-1} \left[\delta(q^i - y^i) |k^i = 0\rangle \right] \quad (6.39)$$

where $\theta = -yR/\alpha'$.

7 Stable non BPS states in type I theory

In this section, following the analysis of Sen [20, 21, 22, 23, 24] and the approach of Frau et al. [25], we construct the boundary state corresponding to the stable non-BPS particle of type I theory that is related through the heterotic-type I duality to the state that belongs to the first excited level of the $SO(32)$ heterotic theory and that transforms according to the spinor representation of $SO(32)$. Let us start by

reminding some properties of the $SO(32)$ heterotic string and of its duality with type I theory. The heterotic string is a theory of closed oriented strings with a local gauge invariance in ten dimensions. It can be considered as a combination of the bosonic string and the superstring. In fact it has a left sector, that is the same as the left sector of a bosonic string in which 16 of the 26 coordinates are compactified and that is described by a lefthanded coordinate $X^\rho(\tau - \sigma)$ where $\rho = 0 \ldots 25$ and a righthanded one, that is the same as the one of the superstring and that is described by the righthanded coordinates $\widetilde{X}^\mu(\tau + \sigma)$ and $\widetilde{\psi}^\mu(\tau + \sigma)$ with $\mu = 0 \ldots 9$. We can combine $\widetilde{X}^\mu(\tau + \sigma)$ from the righthanded sector with the first ten coordinates of the lefthanded sector $X^\mu(\tau - \sigma)$ to obtain the usual closed string coordinate:

$$X^\mu(\tau, \sigma) = \frac{1}{2}\left(\widetilde{X}^\mu(\tau + \sigma) + X^\mu(\tau - \sigma)\right) \quad \mu = 0 \ldots 9 \ . \tag{7.40}$$

In addition we are left with 16 compact righthanded coordinates $X^A(\tau - \sigma)$ with $A = 1 \ldots 16$ and of course with the lefthanded fermionic coordinate $\widetilde{\psi}^\mu(\tau + \sigma)$ with $\mu = 0 \ldots 9$.

The mass spectrum of the heterotic string in the NS sector is determined by the mass-shell conditions

$$\left(L_0 - 1\right)|\Psi\rangle = \left(\widetilde{L}_0 - \frac{1}{2}\right)|\Psi\rangle = 0 \ , \tag{7.41}$$

where we have used the values of the intercept of the bosonic theory ($a = -1$) in the left sector and of the NS superstring ($a = -1/2$) in the right sector. By expanding $\widetilde{L}_0$ in modes, one easily finds from the second equality in Eq. (7.41) that the mass M of a state is given by

$$M^2 = \frac{4}{\alpha'}\left(\widetilde{N} - \frac{1}{2}\right) \tag{7.42}$$

where $\widetilde{N}$ is the total number of right moving oscillators. From the first equality of Eq. (7.41) one can derive a generalized level matching condition which relates $\widetilde{N}$ to the total number of left moving oscillators N that are present in a given state. This condition reads as follows

$$\widetilde{N} + \frac{1}{2} = N + \frac{1}{2}\sum_A (p^A)^2 \ , \tag{7.43}$$

where we have conveniently measured the internal momenta p^A in units of $\sqrt{2\alpha'}$. Additional restrictions come from the GSO projection that one has to perform in the right sector of the theory in order to have a consistent model. The GSO projection on NS states selects only half-integer occupation numbers $\widetilde{N}$. Since the left occupation number N is always integer as in the bosonic theory, in order to be able to satisfy Eq. (7.43), the internal momenta p^A have to be quantized. In particular, the quantity $\sum_A (p^A)^2$ must be an even number. This condition implies that the internal coordinates X^A must be compactified on an *even* 16-dimensional

lattice. The modular invariance of the one-loop partition function requires that this lattice be also *self-dual*. It can be shown that there exist only two 16-dimensional lattices satisfying both these properties: the root lattice of $E_8 \times E_8$, and a Z_2 sublattice of the weight lattice of $SO(32)$. Since we are interested in the heterotic theory with gauge group $SO(32)$ here we will focus only on the second lattice which is denoted by Γ_{16} and is defined by

$$(n_1, \ldots, n_{16}) \in \Gamma_{16} \quad \text{and} \quad \left(n_1 + \frac{1}{2}, \ldots, n_{16} + \frac{1}{2}\right) \in \Gamma_{16} \iff \sum_i n_i \in 2Z \quad (7.44)$$

The lowest states of the NS sector are massless and, because of eq.(7.42), all such states must have $\widetilde{N} = 1/2$, so that their right-moving part is simply $\widetilde{\psi}^i_{-1/2}|\widetilde{k}\rangle$ where $i = 2, \ldots, 9$ labels the directions transverse to the light-cone. On the other hand, the level matching condition in eq.(7.43) requires that

$$N + \frac{1}{2} \sum_A \left(p^A\right)^2 = 1 \quad (7.45)$$

This condition can be satisfied either by taking $N = 1$ and $p^A = 0$, or by taking $N = 0$ and the momenta p^A to be of the form $P = (\pm 1, \pm 1, 0, \ldots, 0)$ (or any permutation thereof with only both plus or both minus signs). The first choice gives 16 states $\alpha^A_{-1}|k; 0\rangle$, while the second one contributes with $16 \times 15 \times 2 = 480$ states $|k; P\rangle$. Altogether we have 496 massless states that carry a spacetime vector index from the right-moving part and span the adjoint representation **496** of $SO(32)$. Those states correspond to the gauge fields of $SO(32)$. At the massless level we have 64 more bosonic states which correspond to $\widetilde{N} = 1/2$, $N = 1$ and $p^A = 0$, and are given by

$$\alpha^i_{-1}|k; 0\rangle \otimes \widetilde{\psi}^j_{-1/2}|\widetilde{k}\rangle \quad (7.46)$$

where the indices i, j run along the transverse directions $2, \cdots, 9$. These states are singlets with respect to the gauge group but are space-time tensors. Decomposing them into irreducible components, we get a graviton, a dilaton and an antisymmetric two-index tensor. In conclusion, the bosonic massless states of the heterotic theory are in the following representations

$$(\mathbf{1}; \mathbf{1}) \oplus (\mathbf{35}; \mathbf{1}) \oplus (\mathbf{28}; \mathbf{1}) \oplus (\mathbf{8}; \mathbf{496}) \quad (7.47)$$

where in each term the two labels refer to the Lorentz and gauge group respectively. By analyzing the R sector, one finds an equal number of fermionic massless states that complete the $N = 1$ supersymmetric multiplets.

Let us now consider the first excited level of the NS sector that consists of states with $\widetilde{N} = 3/2$ and mass squared $M^2 = 4/\alpha'$. The states satisfying the condition $\widetilde{N} = 3/2$ are the following

$$\widetilde{\psi}^i_{-3/2}|\widetilde{k}\rangle \quad \to \quad 8 \text{ states} \quad (7.48)$$

$$\widetilde{\alpha}^i_{-1}\widetilde{\psi}^j_{-1/2}|\widetilde{k}\rangle \quad \to \quad 64 \text{ states} \quad (7.49)$$

$$\widetilde{\psi}^i_{-1/2}\widetilde{\psi}^j_{-1/2}\widetilde{\psi}^\ell_{-1/2}|\widetilde{k}\rangle \quad \to \quad 56 \text{ states} \quad (7.50)$$

27

By putting together the antisymmetric part of eq.(7.49) together with the states in eq.(7.50) we obtain a massive three-form transforming according to the representation **84** of the Lorentz group, while the remaining states transform together as a symmetric two-index tensor in the representation **44**. The level matching condition (7.43) imposes the constraint

$$N + \frac{1}{2} \sum_A \left(p^A \right)^2 = 2 \qquad (7.51)$$

There are 73,764 ways to satisfy this requirement! The complete list of the corresponding states can be found for example on pag. 342 of Ref. [26] where it is shown that they transform as scalars, spinors, second-rank antisymmetric tensors, fourth-rank antisymmetric tensors and second-rank symmetric traceless tensors of $SO(32)$. Here we focus on the 2^{15} states that are obtained by taking in eq.(7.51) $N = 0$ and momenta p^A of the form $(\pm\frac{1}{2}, \pm\frac{1}{2}, \ldots, \pm\frac{1}{2})$ with an even number of plus signs. Notice that these momenta define a point in the lattice Γ_{16}, since they satisfy the second condition of eq.(7.44), and correspond to the *spinor* representation of $SO(32)$. By combining these left modes with the right-moving ones of eqs. (7.48) - (7.50), we then obtain bosonic states transforming as

$$(\mathbf{44}; \mathbf{2^{15}}) \oplus (\mathbf{84}; \mathbf{2^{15}}) \quad . \qquad (7.52)$$

Analyzing the first excited level of the R sector, one can find 128 massive fermionic states which transform in the spinor representation of $SO(32)$ and complete the $N = 1$ supersymmetry multiplets. Thus, altogether the spinors of $SO(32)$ appear with 256 different polarizations, 128 bosonic and 128 fermionic, corresponding to a *long* multiplet of the $N = 1$ supersymmetry algebra in ten dimensions. These states are not BPS, but, nevertheless, stable. In fact, since there are no spinors of $SO(32)$ at the massless level, they are the lighest states with these quantum numbers and therefore cannot decay.

There is by now a strong evidence that the $SO(32)$ heterotic string is dual to the type I theory [27] [11]. They have the same spectrum of massless states, their low-energy effective actions can be transformed into each other through the following transformations on the metric and the dilaton:

$$G_{\mu\nu}^H = e^{\phi_I} G_{\mu\nu}^I \quad , \quad \phi_H = -\phi_I \quad . \qquad (7.53)$$

Since the second equation in (7.53) implies that the strong coupling limit of one theory is related to the weak coupling limit of the other theory, the fact that the perturbative massive spectra of the two theories are totally different from each other is not in contradiction with the fact that the two theories are non-perturbatively equivalent. For instance in the type I theory all states transform according to the adjoint representation of the gauge group $SO(32)$, while in the heterotic string the

---

[11]See also the second paper in Ref. [11].

perturbative states transform according to all the representations of $SO(32)$. But, if we have in one of the two theory some state that cannot decay, then we expect it to be present in both theories at all values of the coupling constant. Identifying such states in both theories is a check of duality. For instance an heterotic string wrapped around a compact dimension breaks $1/2$ of supersymmetry and is charged under the antisymmetric 2-form of the gravitational multiplet, the charge being simply its winding number. This is a BPS configuration and should also appear in type I theory. A natural candidate is the D string of type I theory that is charged with respect to the R-R 2-form of type I that corresponds under the strong/weak duality to the antisymmetric 2-form potential of the gravitational multiplet of the heterotic string. But, since an heterotic string is a BPS configuration with tension equal to

$$\tau_H = \frac{1}{2\pi\alpha'} \tag{7.54}$$

it should match the tension of the D string of type I theory given by

$$\tau_{D1} = \frac{1}{2\pi\alpha' g_I} \tag{7.55}$$

In fact, if we use the metric relation in eq.(7.53), we see that the two tensions in eqs.(7.54) and (7.55) transform into each other. The identification between D string of type I and the fundamental heterotic $SO(32)$ string can be tested at a deeper level by showing that the world sheet structure of a (wrapped) heterotic string is exactly reproduced by the world-sheet dynamics on a (wrapped) D string [27]. Moreover, if the heterotic $SO(32)$ string theory is dual to the type I theory, then we must be able to find in the latter one a description of the massive stable state transforming according to the 2^{15} spinor representation of the heterotic $SO(32)$ theory. In heterotic string units its mass is given by:

$$M_H = \frac{2}{\sqrt{\alpha'}} f(g_H) \quad , \quad f(0) = 1 \tag{7.56}$$

By going to type I units we expect its mass to be given by:

$$M_I = \frac{2}{\sqrt{\alpha'}} \tilde{f}(g_I) \quad , \quad \tilde{f}(g) = f(1/g) \tag{7.57}$$

that in the weak coupling regime of type I theory becomes:

$$M_I = \frac{2}{\sqrt{\alpha'}} \tilde{f}(g_I \to 0) \tag{7.58}$$

where $\tilde{f}(0)$ cannot be determined in type I perturbation theory.

From the heterotic $SO(32)$ point of view the stable state is just an excitation of the fundamental string. Therefore in the type I theory we expect that a D string should be involved in the description of this state. But a D string alone is not be

sufficient because, on the one hand, it is a BPS configuration and on the other hand it is dual to the fundamental heterotic string with winding number equal to 1, while the stable state is not charged under the gravitational 2-form of the heterotic string and therefore should be dual to a configuration of type I theory that is neutral under the 2-form RR potential of type I theory. Because of this the next simple possibility is that the stable state is a combination of a D string and an anti D string:

$$|A> = |D1> + |\overline{D1}> \quad . \tag{7.59}$$

But such a system is unstable because has tachyonic open string excitations. This can be easily seen by noticing that a change in sign in front of the R-R spin structure, necessary in order to describe an anti D string in the closed string channel, corresponds to a sign change of the $NS(-1)^F$ spin structure in the open string channel. As a consequence the NS sector of the open strings of the systems $1 - \bar{1}$ and $\bar{1} - 1$ contains only states that are odd under $(-1)^F$ and their lowest states are tachyons described in the -1 picture by the states:

$$|k>_{-1} \lambda_{1\bar{1}} \quad , \qquad |k>_{-1} \lambda_{\bar{1}1} \quad , \tag{7.60}$$

where we have introduced the following notation for the open string connecting D strings and anti D strings:

$$\text{for a 1-1 string} \quad \rightarrow \quad \lambda_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{7.61}$$

$$\text{for a } \bar{1}\text{-}\bar{1} \text{ string} \quad \rightarrow \quad \lambda_{\bar{1}\bar{1}} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{7.62}$$

$$\text{for a 1-}\bar{1} \text{ string} \quad \rightarrow \quad \lambda_{1\bar{1}} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{7.63}$$

$$\text{for a } \bar{1}\text{-1 string} \quad \rightarrow \quad \lambda_{\bar{1}1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{7.64}$$

The linear combination of the two tachyons in eq.(7.60) that is even under the Ω projection corresponding to the sum of the two states in eq.(7.60) will survive in the type I theory. In conclusion in type I theory the state in eq.(7.59) is unstable. But, even if we would find a way of eliminating the tachyon state, we will be left with the problem that the state in eq.(7.59) cannot represent the stable state of the heterotic string because both the D string and the anti D string carry the quantum number of the spinor representation of $SO(32)$ and therefore their bound state cannot transform itself under the spinor representation. By introducing, however, a Wilson line along the compactified direction around which the anti D string is wrapped we can make it to transform as a scalar of $SO(32)$. Therefore, if instead of the one in eq.(7.59) we introduce the state

$$|A> = |D1> + |\overline{D1}'> \quad , \tag{7.65}$$

30

where the prime denotes the Wilson line, it will have the correct quantum numbers for representing in type I theory the stable state of the heterotic $SO(32)$. This is the state proposed by Sen [22] to represent in type I theory the stable non BPS perturbative massive state in eq.(7.52) in the $SO(32)$ heterotic string.

Up to now we have not really used the fact that one direction has been compactified. In the following instead we want to consider the $D1/\overline{D1}$ system at the particular radius :

$$R = \sqrt{\frac{\alpha'}{2}} \quad . \tag{7.66}$$

This value is special for two reasons. From the mass-shell condition $L_0 - 1/2 = 0$ of the NS sector of the open type I theory we get the following mass spectrum:

$$\alpha' M^2 = N - 1/2 + \frac{(n+1/2)^2}{R^2} \quad , \tag{7.67}$$

where N is the oscillator number, n is a integer and the quantity $(n+1/2)/R$ is the Kaluza-Klein momentum obtained from the general expression $p = n/R + \theta/(2\pi R)$ for $\theta = \pi$ corresponding to Z_2 Wilson lines. For the critical radius in eq.(7.66) the lighest string excitations, corresponding to the values $n = 0, -1$, are massless and not tachyonic. But since in the limit of infinite radius they give rise to tachyons, we will call them "tachyons" also in the case we are considering.

The second reason is that the conformal field theory generated by X^1 and ψ^1, where the direction 1 corresponds to the compactified one, admits several representations. One of them is the one obtained by just using X^1 and ψ^1. The other two can be obtained by fermionizing X^1 in terms of two additional fermions ξ and η:

$$\mathrm{e}^{\pm iX^1/\sqrt{2\alpha'}} \simeq \frac{1}{\sqrt{2}} (\xi \pm i\eta) \quad , \qquad \eta\xi \simeq i\partial X^1/(2\alpha') \quad . \tag{7.68}$$

The three fermions ψ^1, η and ξ are completely on equal footing and can be regrouped in three different ways. The first is the one in which we combine ξ and η as in eq.(7.68) and we represent them in terms of the scalar field X^1 together with the fermionic field ψ^1. The other two correspond in combining either ξ and ψ^1 or η and ψ^1 in terms of respectively the scalar fields ϕ and ϕ' as follows

$$\mathrm{e}^{\pm i\phi^1/\sqrt{2\alpha'}} \simeq \frac{1}{\sqrt{2}} \left(\xi \pm i\psi^1\right) \quad , \qquad \psi^1\xi \simeq i\partial\phi/(2\alpha') \tag{7.69}$$

and

$$\mathrm{e}^{\pm i\phi'/\sqrt{2\alpha'}} \simeq \frac{1}{\sqrt{2}} \left(\eta \pm i\psi^1\right) \quad , \qquad \psi^1\eta \simeq i\partial\phi'/(2\alpha') \quad , \tag{7.70}$$

ϕ and ϕ' are bosonic fields compactified on a circle with radius given in eq.(7.66).

The advantage of using ϕ and η instead of X^1 and ψ^1 is that in this case it is possible to explicitly encode the tachyonic background in the conformal field theory

and to move the system from the unstable to a stable situation. In order to show this let us start from the "tachyonic" states in the -1 picture:

$$|T_\pm\rangle = \mathrm{e}^{\pm\frac{\mathrm{i}}{\sqrt{2\alpha'}}X^1}|0\rangle_{-1}\otimes\sigma_1 \qquad (7.71)$$

and look at the explicit form of their vertex operators $\mathcal{V}_{T_\pm}$ in the various representations. Using the bosonization formulas (7.68), (7.69) and (7.70) it is easy to see that

$$\mathcal{V}_{T_\pm}^{(-1)} = \mathrm{e}^{\pm\frac{\mathrm{i}}{\sqrt{2\alpha'}}X^1} \simeq \frac{1}{\sqrt{2}}(\xi\pm\mathrm{i}\eta) \simeq \left[\pm\frac{\mathrm{i}}{\sqrt{2}}\eta + \frac{1}{2}\left(\mathrm{e}^{\frac{\mathrm{i}}{\sqrt{2\alpha'}}\phi} + \mathrm{e}^{-\frac{\mathrm{i}}{\sqrt{2\alpha'}}\phi}\right)\right] \ , \quad (7.72)$$

where for simplicity we have understood the superghost part and the Chan-Paton factor given by the Pauli matrix σ_1. From these relations we immediately realize that the states $|T_\pm\rangle$ in eq.(7.71) can also be written either as

$$|T_\pm\rangle = \frac{1}{\sqrt{2}}(\xi_{-\frac{1}{2}}\pm\mathrm{i}\,\eta_{-\frac{1}{2}})|0\rangle_{-1}\otimes\sigma_1 \ , \qquad (7.73)$$

or as

$$|T_\pm\rangle = \left[\pm\frac{\mathrm{i}}{\sqrt{2}}\,\eta_{-\frac{1}{2}}|0\rangle_\phi + \frac{1}{2}\left(|+\tfrac{1}{2}\rangle_\phi + |-\tfrac{1}{2}\rangle_\phi\right)\right]|0\rangle_{-1}\otimes\sigma_1 \ , \qquad (7.74)$$

where we have denoted by $|\ell\rangle_\phi$ the vacuum of ϕ with momentum ℓ. In particular, in the latter representation the combination

$$|\mathcal{T}\rangle \equiv \frac{1}{\sqrt{2}\,\mathrm{i}}\left(|T_+\rangle - |T_-\rangle\right) = \eta_{-\frac{1}{2}}|0\rangle_\phi|0\rangle_{-1}\otimes\sigma_1 \qquad (7.75)$$

is formally identical to a massless vector state at zero momentum in the -1 picture with ψ replaced by η. This implies that the deformation induced by $|\mathcal{T}\rangle$ corresponding to change the vacuum expectation value of the "tachyonic state" can be described by the introduction of Wilson lines, that, however, should not be confused with the Z_2 ones introduced above. In the 0 picture $\mathcal{V}_{T_\pm}$ becomes

$$\mathcal{V}_{T_\pm}^{(-1)} = \mathrm{e}^{\pm\frac{\mathrm{i}}{\sqrt{2\alpha'}}X^1} \to \mathcal{V}_{T_\pm}^{(0)} = \pm\mathrm{i}\,\psi^1\mathrm{e}^{\pm\frac{\mathrm{i}}{\sqrt{2\alpha'}}X^1} \ ; \qquad (7.76)$$

and then, by using the bosonization formulas in eqs.(7.68), (7.69), and (7.70) we can easily obtain the (ϕ, η) description of $\mathcal{V}_{T_\pm}^{(0)}$:

$$\mathcal{V}_{T_\pm}^{(0)} = \pm\mathrm{i}\psi^1(\xi\pm\mathrm{i}\eta)\sqrt{\alpha'} = \mp\left(\partial\phi\pm\mathrm{i}\partial\phi'\right) \qquad (7.77)$$

that implies

$$\mathcal{V}_{\mathcal{T}}^{(0)} = \frac{\mathrm{i}}{\sqrt{2\alpha'}}\,\partial\phi\otimes\sigma^1 \ . \qquad (7.78)$$

This is identical to the vertex of the usual gauge boson at zero momentum, where ϕ plays the role of the coordinate X. From this equation, it is clear that $\mathcal{V}_{\mathcal{T}}^{(0)}$ represents

a marginal operator which can be used to modify the theory. In particular we can modify the theory by introducing Wilson lines along ϕ parametrized by

$$W(\theta) = \frac{1}{2} Tr \left(e^{i \frac{\theta}{2\sqrt{2\alpha'}} \oint d\sigma \, \partial_\sigma \phi \otimes \sigma^1} \right) \tag{7.79}$$

where in a qualitative sense, the constant θ is equivalent to the tachyon vacuum expectation value since it multiplies the "tachyon" vertex operator $\mathcal{V}_T^{(0)}$. By using the expansion:

$$\partial \phi = -2w \sqrt{\frac{\alpha'}{2}} + \dots \quad , \tag{7.80}$$

where ... correspond to terms that do not give any contribution when we perform the integral in eq.(7.79), we get

$$W(\theta) = \cos\left(\theta \pi w / 2\right) \tag{7.81}$$

where w is the total winding number of the closed string state as seen by the operator $W(\theta)$.

Let us now construct the boundary state of type IIB theory corresponding to the stable state following very closely the procedure described in Ref. [25]. In the language of the boundary state the proposal of Ref. [22] corresponding to eq.(7.65), is given by the superposition of the boundary states describing respectively the D string and the anti D string. To describe this system we introduce the following boundary states

$$|B, +\rangle_{\text{NS}} \equiv |D1, +\rangle_{\text{NS}} + |D1', +\rangle_{\text{NS}} \tag{7.82}$$

$$|B, +\rangle_{\text{R}} \equiv |D1, +\rangle_{\text{R}} - |D1', +\rangle_{\text{R}} \tag{7.83}$$

where the $'$ indicates the presence of the $\mathbf{Z}_2$ Wilson line. Note that the minus sign in Eq. (7.83) accounts for the fact that one of the two members of the pair is an anti D-string. Using the explicit expressions for the boundary state, we have [25]

$$|B, +\rangle_{\text{NS}} = \frac{T_1}{2} \sqrt{\frac{2\pi R}{\Phi}} \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \cdot \hat{S}^{(1)} \cdot \tilde{\alpha}_{-n}\right] \exp\left[+i \sum_{r=1/2}^{\infty} \psi_{-r} \cdot \hat{S}^{(1)} \cdot \tilde{\psi}_{-r}\right]$$

$$\exp\left[-\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n}\right] \exp\left[+i \sum_{r=1/2}^{\infty} \psi_{-r} \tilde{\psi}_{-r}\right] |\Omega\rangle_{\text{NS}} \tag{7.84}$$

where we have denoted by $\hat{S}^{(1)}$ the D-string S-matrix for all non-compact directions and have separately indicated in the second line the contribution of the bosonic and fermionic non-zero modes of the compact direction (*i.e.* the modes of X, ψ and $\tilde{\psi}$). Due to the presence of the $\mathbf{Z}_2$ Wilson line, the vacuum $|\Omega\rangle_{\text{NS}}$ is given by

$$|\Omega\rangle_{\text{NS}} = \delta^{(8)}(q^i)|k^0 = 0\rangle \left(\sum_w |0, w\rangle + \sum_w (-1)^w |0, w\rangle \right) \prod_{i=2}^{9} |k^i = 0\rangle$$

$$= 2\, \delta^{(8)}(q^i)|k^0 = 0\rangle \sum_w |0, 2w\rangle \prod_{i=2}^{9} |k^i = 0\rangle \tag{7.85}$$

where for simplicity we have set to zero the coordinates y^i of the D-strings. Analogously, in the R-R sector we have

$$|B, +\rangle_R \;=\; \frac{T_1}{2}\sqrt{\frac{2\pi R}{\Phi}}\,\exp\!\Big[-\sum_{n=1}^{\infty}\frac{1}{n}\,\alpha_{-n}\cdot\hat{S}^{(1)}\cdot\tilde{\alpha}_{-n}\Big]\exp\!\Big[+\mathrm{i}\sum_{n=1}^{\infty}\psi_{-n}\cdot\hat{S}^{(1)}\cdot\tilde{\psi}_{-n}\Big]$$
$$\exp\!\Big[-\sum_{n=1}^{\infty}\frac{1}{n}\,\alpha_{-n}\tilde{\alpha}_{-n}\Big]\exp\!\Big[+\mathrm{i}\sum_{n=1}^{\infty}\psi_{-n}\tilde{\psi}_{-n}\Big]|D1, +\rangle_R^{(0)}\,|\Omega\rangle_R \qquad (7.86)$$

where

$$|D1, +\rangle^{(0)R} = \Big(C\Gamma^0\Gamma^1\frac{1+\mathrm{i}\Gamma_{11}}{1+\mathrm{i}}\Big)_{AB}|A>|\tilde{B}> \qquad (7.87)$$

and

$$|\Omega\rangle_R \;=\; \delta^{(8)}(q^i)|k^0=0\rangle\left(\sum_w|0,w\rangle-\sum_w(-1)^w|0,w\rangle\right)\prod_{i=2}^{9}|k^i=0\rangle$$
$$=\; 2\,\delta^{(8)}(q^i)|k^0=0\rangle\sum_w|0,2w+1\rangle\prod_{i=2}^{9}|k^i=0\rangle \quad . \qquad (7.88)$$

Let us now suppose that the radius R is equal to the critical radius given in eq.(7.66) and rewrite the boundary state using a parametrization along the compact direction in terms of the coordinate ϕ instead of X^1 as we have done in eqs.(7.84) and (7.86). This will also allow the introduction of $U(1)$ Wilson lines corresponding to a non vanishing vacuum expectation value for the tachyon. We are now in the position of writing the boundary state which describes the D-string – anti D-string pair in the presence of a non vanishing tachyon v.e.v. This is given by Eqs. (7.84) and (7.86) with the oscillators α_n, $\tilde{\alpha}_n$, ψ_r, $\tilde{\psi}_r$ of the compact direction replaced by ϕ_n, $\tilde{\phi}_n$, η_r and $\tilde{\eta}_r$, and with a vacuum that carries an explicit dependence on the parameter θ according to eq.(7.81). In particular, in the NS-NS sector we have

$$|B(\theta), +\rangle_{NS} \;=\; \frac{T_1}{2}\sqrt{\frac{2\pi R_c}{\Phi}}\,\exp\!\Big[-\sum_{n=1}^{\infty}\frac{1}{n}\,\alpha_{-n}\cdot\hat{S}^{(1)}\cdot\tilde{\alpha}_{-n}\Big]\exp\!\Big[+\mathrm{i}\sum_{r=1/2}^{\infty}\psi_{-r}\cdot\hat{S}^{(1)}\cdot\tilde{\psi}_{-r}\Big]$$
$$\exp\!\Big[-\sum_{n=1}^{\infty}\frac{1}{n}\,\phi_{-n}\tilde{\phi}_{-n}\Big]\exp\!\Big[+\mathrm{i}\sum_{r=1/2}^{\infty}\eta_{-r}\tilde{\eta}_{-r}\Big]|\Omega(\theta)\rangle_{NS} \qquad (7.89)$$

where

$$|\Omega(\theta)\rangle_{NS} = 2\,\delta^{(8)}(q^i)|k^0=0\rangle\sum_{w_\phi}\cos(\pi\theta w_\phi)\,|0,2w_\phi\rangle\prod_{i=2}^{9}|k^i=0\rangle \quad . \qquad (7.90)$$

Analogously, in the R-R sector we have

$$|B(\theta), +\rangle_R \;=\; \frac{T_1}{2}\sqrt{\frac{2\pi R_c}{\Phi}}\,\exp\!\Big[-\sum_{n=1}^{\infty}\frac{1}{n}\,\alpha_{-n}\cdot\hat{S}^{(1)}\cdot\tilde{\alpha}_{-n}\Big]\exp\!\Big[+\mathrm{i}\sum_{n=1}^{\infty}\psi_{-n}\cdot\hat{S}^{(1)}\cdot\tilde{\psi}_{-n}\Big]$$
$$\exp\!\Big[-\sum_{n=1}^{\infty}\frac{1}{n}\,\phi_{-n}\tilde{\phi}_{-n}\Big]\exp\!\Big[+\mathrm{i}\sum_{n=1}^{\infty}\eta_{-n}\tilde{\eta}_{-n}\Big]|D1, +\rangle_R^{(0)}\,|\Omega(\theta)\rangle_R \qquad (7.91)$$

34

where

$$|\Omega(\theta)\rangle_{\mathrm{R}} = 2\,\delta^{(8)}(q^i)|k^0 = 0\rangle \sum_{w_\phi} \cos\left(\pi\theta(w_\phi + \frac{1}{2})\right) |0, 2w_\phi + 1\rangle \prod_{i=2}^{9} |k^i = 0\rangle \quad . \quad (7.92)$$

Notice that at $\theta = 0$ the boundary states (7.89) and (7.91) reduce to the original ones written in eqs.(7.84) and (7.86). However the idea is that, for arbitrary values of θ, only the boundary states (7.89) and (7.91) must be used.

In the computation of the interaction between two boundary states of the type as in eqs.(7.89) and (7.91) we must perform the GSO projection that acts on the boundary state in terms of the variables ϕ and η differently than on the original variables X^1 and ψ^1. From eq.(7.69) one can immediately read the action of $(-1)^{\tilde{F}}$ getting in the NS-NS sector

$$(-1)^{\tilde{F}} \quad : \quad \tilde{\alpha}_n^\mu \to \tilde{\alpha}_n^\mu \ , \quad \tilde{\psi}_r^\mu \to -\tilde{\psi}_r^\mu \ , \quad \tilde{\phi}_n \to -\tilde{\phi}_n \ , \quad \tilde{\eta}_r \to \tilde{\eta}_r \qquad (7.93)$$

and similarly for $(-1)^F$ on the left moving oscillators. Notice that the action of $(-1)^{\tilde{F}}$ on ϕ looks very much like T-duality because it amounts to change the relative sign between its left and right moving oscillators. It acts also on the zero modes as follows:

$$\sqrt{\frac{\alpha'}{2}}\left(\frac{n_\phi}{R} - \frac{w_\phi R}{\alpha'}\right) = n_\phi - \frac{w_\phi}{2} \longrightarrow -\left(n_\phi - \frac{w_\phi}{2}\right) \qquad (7.94)$$

implying that

$$n_\phi \to \frac{w_\phi}{2} \quad , \qquad w_\phi \to 2n_\phi \qquad (7.95)$$

This implies

$$(-1)^{\tilde{F}} \quad : \quad |n_\phi, w_\phi\rangle \ \to \ |\frac{w_\phi}{2}, 2n_\phi\rangle \quad . \qquad (7.96)$$

Of course, similar considerations apply also for the boundary states in the R-R sector. Using the previous rules rules, one can easily see, for example, that

$$\begin{aligned}
(-1)^{\tilde{F}}|B(\theta), +\rangle_{\mathrm{NS}} \ = \ & -T_1 \sqrt{\frac{2\pi R_c}{\Phi}} \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n}\, \alpha_{-n} \cdot \hat{S}^{(1)} \cdot \tilde{\alpha}_{-n}\right] \\
& \exp\left[-\mathrm{i} \sum_{r=1/2}^{\infty} \psi_{-r} \cdot \hat{S}^{(1)} \cdot \tilde{\psi}_{-r}\right] \\
& \exp\left[+\sum_{n=1}^{\infty} \frac{1}{n} \phi_{-n}\tilde{\phi}_{-n}\right] \exp\left[+\mathrm{i} \sum_{r=1/2}^{\infty} \eta_{-r}\tilde{\eta}_{-r}\right] \\
& \delta^{(8)}(q^i)|k^0 = 0\rangle \sum_{w_\phi} \cos(\pi\theta w_\phi) |w_\phi, 0\rangle \prod_{i=2}^{9} |k^i = 0\rangle \quad .
\end{aligned} \qquad (7.97)$$

Let us now compute the vacuum amplitude of the theory defined on the world-volume of our D-string – anti D-string pair. In the boundary state formalism this amplitude is simply given by

$$\mathcal{A}(\theta) = \langle B(\theta), +| \, P_{\mathrm{GSO}} \, D \, |B(\theta), +\rangle \qquad (7.98)$$

where the GSO projection operator is given in eqs.(1.9) and D is the closed string propagator

$$D = \frac{\alpha'}{4\pi} \int \frac{d^2 z}{|z|^2} z^{L_0 - a} \bar{z}^{\tilde{L}_0 - a} \tag{7.99}$$

with intercept $a_{\text{NS}} = 1/2$ in the NS-NS sector, and $a_{\text{R}} = 0$ in the R-R sector. Using the explicit expressions of the boundary states written above, and performing standard manipulations, one finds [25]

$$\mathcal{A}_{\text{NS-NS}}(\theta) = \frac{V R_c}{2\pi\alpha'} \int_0^\infty \frac{dt}{t^4} \left[\left(\sum_{w_\phi} \cos^2(\pi\theta w_\phi) \, q^{w_\phi^2} \right) \frac{f_3^8(q)}{f_1^8(q)} - \sqrt{2} \, \frac{f_4^7(q) \, f_3(q)}{f_1^7(q) \, f_2(q)} \right] \tag{7.100}$$

and

$$\mathcal{A}_{\text{R-R}}(\theta) = -\frac{V R_c}{2\pi\alpha'} \int_0^\infty \frac{dt}{t^4} \left[\sum_{w_\phi} \cos^2\left(\pi\theta(w_\phi + \frac{1}{2}) \right) \, q^{\left(w_\phi + \frac{1}{2}\right)^2} \right] \frac{f_2^8(q)}{f_1^8(q)} \tag{7.101}$$

where V is the (infinite) length of the time direction

It is interesting to observe that the contribution of the NS-NS$(-1)^F$ spin structure (*i.e.* the second term in Eq. (7.100)) does not depend on the tachyon v.e.v. θ. This is a direct consequence of the fact that this spin structure arises from the overlap between $|B(\theta), +\rangle_{\text{NS}}$, whose vacuum contains states with only winding numbers, and $(-1)^{\tilde{F}}|B(\theta), +\rangle_{\text{NS}}$, whose vacuum instead contains states with only Kaluza-Klein numbers (see eqs.(7.90) and (7.97)). Therefore, in the NS-NS$(-1)^F$ spin structure there is no contribution from the bosonic zero modes of the compact direction ϕ, and hence no dependence on the tachyon v.e.v. θ.

If one performs the modular transformation $t \to 1/t$, the entire amplitude $\mathcal{A}(\theta)$ can be interpreted as the one-loop vacuum energy of the open strings living in the world-volume of the D-string – anti D-string pair.

In particular, one sees that the θ-independent NS-NS$(-1)^F$ spin structure of the closed string channel goes into the R spin structure of the open string channel, that indeed has been shown in Ref. [22] to be independent of the tachyon v.e.v.

At $\theta = 1$ a remarkable simplification occurs: the R-R contribution to $\mathcal{A}$ vanishes and the entire vacuum amplitude becomes

$$\begin{aligned} \mathcal{A}(\theta = 1) &= \frac{V R_c}{2\pi\alpha'} \int_0^\infty \frac{dt}{t^4} \left[\left(\sum_{w_\phi} q^{w_\phi^2} \right) \frac{f_3^8(q)}{f_1^8(q)} - \sqrt{2} \, \frac{f_4^7(q) \, f_3(q)}{f_1^7(q) \, f_2(q)} \right] \\ &= \frac{V}{4\pi R_c} \int_0^\infty \frac{dt}{t^4} \left(\sum_{w_\phi} q^{w_\phi^2} \right) \left[\frac{f_3^8(q)}{f_1^8(q)} - \frac{f_4^8(q)}{f_1^8(q)} \right] \end{aligned} \tag{7.102}$$

that is obtained using the following identities

$$f_2(q) \, f_3(q) \, f_4(q) = \sqrt{2} \quad , \quad f_1(q) \, f_3^2(q) = \sum_{n=-\infty}^{+\infty} q^{n^2} \tag{7.103}$$

36

Notice that with these manipulations we have managed to reconstruct the typical combination of f-functions that is produced by the *usual* GSO projection of the NS-NS sector. Thus, one is lead to think that a simpler underlying structure may actually exist at $\theta = 1$. In fact the amplitude $\mathcal{A}(\theta = 1)$ can be factorized in terms of a new boundary state according to [25]

$$\mathcal{A}(\theta = 1) = \langle \widetilde{B}, +| \, P_{GSO} \, D \, |\widetilde{B}, +\rangle \tag{7.104}$$

where

$$
\begin{aligned}
|\widetilde{B}, \pm\rangle \;\; = \;\; & \frac{T_1}{2} \sqrt{\frac{\pi \alpha'}{R_c \Phi}} \, \exp\Big[-\sum_{n=1}^{\infty} \frac{1}{n} \, \alpha_{-n} \cdot \hat{S}^{(1)} \cdot \tilde{\alpha}_{-n}\Big] \exp\Big[\pm i \sum_{r=1/2}^{\infty} \psi_{-r} \cdot \hat{S}^{(1)} \cdot \tilde{\psi}_{-r}\Big] \\
& \exp\Big[+\sum_{n=1}^{\infty} \frac{1}{n} \, \alpha_{-n}\tilde{\alpha}_{-n}\Big] \exp\Big[\mp i \sum_{r=1/2}^{\infty} \psi_{-r}\tilde{\psi}_{-r}\Big]|\widetilde{\Omega}\rangle
\end{aligned}
\tag{7.105}
$$

with

$$|\widetilde{\Omega}\rangle = 2\,\delta^{(8)}(q^i)|k^0 = 0\rangle \sum_w |w, 0\rangle \prod_{i=2}^{9} |k^i = 0\rangle \quad . \tag{7.106}$$

Of course, the simple factorization of a vacuum amplitude does not allow to conclude that the two boundary states that have been used are completely equivalent. However, a detailed analysis of correlation functions shows that the new boundary state $|\widetilde{B}, +\rangle$, which is written in terms of the original degrees of freedom for the compact direction (*i.e.* X and ψ), is equivalent to the boundary state of Eq. (7.89) for $\theta = 1$. For details see Ref. [25].

Based on these results, we can conclude that in order to describe the D-string – anti D-string pair at $R = R_c$ in terms of X and ψ, we have to use the original boundary states of eqs.(7.84) and (7.86) if $\theta = 0$, whereas we have to use the new boundary state of Eq. (7.105) if $\theta = 1$. Of course, at this particular value of θ there is no R-R sector, as we have explicitly shown. It is interesting to observe that $|B, +\rangle_{\mathrm{NS}}$ and $|\widetilde{B}, +\rangle$ can be related to each other by means of a "generalized T-duality". Indeed, as is clear from eqs.(7.84) and (7.105), we may go from $|B, +\rangle_{\mathrm{NS}}$ to $|\widetilde{B}, +\rangle$ by changing the sign to the right moving oscillators of the compact direction, and consequently by changing the vacuum from $|0, 2w\rangle$ to $|w, 0\rangle$ since the radius of X satisfies eq.(7.66) (see also eq.(7.96)). Like the true T-duality, also this "generalized T-duality" transforms a longitudinal direction into a transverse one, so that the new boundary state $|\widetilde{B}, +\rangle$ describes a D0-brane with a compact transverse direction. However, unlike the true T-duality, the "generalized T-duality" switches off the R-R sector. This fact suggests that, more than a symmetry of the theory, this "generalized T-duality" has to be regarded simply as an effective way of implementing the change of the tachyon v.e.v. from $\theta = 0$ to $\theta = 1$ on the original boundary states, which can be justified by introducing the new fields ϕ and η through the bosonization procedure as it has been done above [25].

Up to now we have worked at the critical radius given in eq.(7.66). As pointed out in Ref. [25] the decompactification limit is ill defined on the original boundary

states ($|B, +\rangle_{NS}$ and $|B, +\rangle_R$) because their vacuum contains only odd or even winding numbers, but it is perfectly well defined on the new one ($|\tilde{B}, +\rangle_{NS}$) at $\theta = 1$. In fact when $R \to \infty$ we get

$$
\begin{aligned}
|\tilde{\Omega}\rangle &= 2\,\delta^{(8)}(q^i)\,|k^0 = 0\rangle\,2\pi R \int \frac{dk^1}{2\pi} |k^1\rangle \prod_{i=2}^{9} |k^i = 0\rangle \\
&= 4\pi R\,\delta^{(9)}(q^i) \prod_{\mu=0}^{9} |k^\mu = 0\rangle
\end{aligned}
\tag{7.107}
$$

which corresponds to the vacuum structure of a 0 brane. Furthermore, combining the factor of $4\pi R$ from Eq. (7.107) with the prefactors of $|\tilde{B}, +\rangle$ in eq.(7.105), we see that the complete normalization of the boundary state becomes

$$
\frac{T_1}{2} \sqrt{\frac{\pi\alpha'}{R\Phi}}\,4\pi R \quad \longrightarrow \quad \frac{\sqrt{2}\,T_1\,(2\pi\sqrt{\alpha'})}{2} = \frac{\sqrt{2}\,T_0}{2}
\tag{7.108}
$$

where we have used the asymptotic behavior of Φ for $R \to \infty$ (see eq.(6.29) and the explicit expression of the tensions T_p (see Eq. (1.1)). Thus, in the decompactification limit our system is described by

$$
\begin{aligned}
|\tilde{B}, +\rangle &= \frac{\sqrt{2}\,T_0}{2} \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n}\,\alpha_{-n} \cdot S^{(0)} \cdot \tilde{\alpha}_{-n}\right] \exp\left[+\mathrm{i}\sum_{r=1/2}^{\infty} \psi_{-r} \cdot S^{(0)} \cdot \tilde{\psi}_{-r}\right] \\
&\quad \delta^{(9)}(q^i) \prod_{\mu=0}^{9} |k^\mu = 0\rangle \ .
\end{aligned}
\tag{7.109}
$$

By performing the usual GSO projection, we then obtain the complete boundary state

$$
|\tilde{B}\rangle \equiv P_{\mathrm{GSO}}\,|\tilde{B}, +\rangle = \frac{1}{2}\Big[|\tilde{B}, +\rangle - |\tilde{B}, -\rangle\Big]
\tag{7.110}
$$

which describes a D0-brane in the Type IIB theory. Since there is no R-R sector, this D-particle is non-supersymmetric and non-BPS. Moreover, from eq.(7.109) we explicitly see that its tension (or mass) is a factor of $\sqrt{2}$ bigger than the tension of the ordinary supersymmetric D-particle of the Type IIA theory.

It turns out that the non-supersymmetric D particle is still an unstable configuration of type IIB theory. This is due to the fact that the absence of a R-R sector implies the absence of GSO projection in the open string channel. Therefore one gets an open string tachyon that denotes instability. However, this tachyon is eliminated by the Ω projection by going from type IIB to type I theory. In conclusion the 0 brane that we have constructed above is a stable configuration of type I theory.

Acknowledgements

Appendix

Let γ^i be the eight 16×16 γ-matrices of $SO(8)$. Starting from these matrices we can construct a chiral representation for the 32×32 Γ-matrices of $SO(1,9)$, *i.e.*

$$
\begin{aligned}
\Gamma^i &= \begin{pmatrix} 0 & \gamma^i \\ \gamma^i & 0 \end{pmatrix} = \sigma^1 \otimes \gamma^i \ , \\
\Gamma^9 &= \begin{pmatrix} 0 & \gamma^1 \cdots \gamma^8 \\ \gamma^1 \cdots \gamma^8 & 0 \end{pmatrix} = \sigma^1 \otimes \left(\gamma^1 \cdots \gamma^8 \right) \ , \\
\Gamma^0 &= \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} = i\sigma^2 \otimes \mathbb{1} \ ,
\end{aligned}
\tag{A.1}
$$

where σ^a's are the standard Pauli matrices. One can easily verify that these matrices satisfy $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$. Other useful matrices are

$$
\begin{aligned}
\Gamma_{11} &= \Gamma^0 \ldots \Gamma^9 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} = \sigma^3 \otimes \mathbb{1} \ , \\
C &= \begin{pmatrix} 0 & -i\,\mathbb{1} \\ i\,\mathbb{1} & 0 \end{pmatrix} = \sigma^2 \otimes \mathbb{1} \ ,
\end{aligned}
\tag{A.2}
$$

where C is the charge conjugation matrix such that

$$
(\Gamma^\mu)^T = -C\,\Gamma^\mu\,C^{-1} \ . \tag{A.3}
$$

Let $A, B, \ldots$ be 32-dimensional indices for spinors in ten dimensions, and $|A\rangle|\widetilde{B}\rangle$ denote the vacuum of the Ramond fields $\psi^\mu(z)$ and $\widetilde{\psi}^\mu(\bar{z})$ with spinor indices A and B in the left and right sectors respectively, that is

$$
|A\rangle|\widetilde{B}\rangle = \lim_{z,\bar{z}\to 0} S^A(z)\,\widetilde{S}^B(\bar{z})|0\rangle \tag{A.4}
$$

where S^A ($\widetilde{S}^B$) are the left (right) spin fields [28], and $|0\rangle$ the Fock vacuum of the Ramond fields. The action of the Ramond oscillators ψ_n^μ and $\widetilde{\psi}_n^\mu$ on the state $|A\rangle|\widetilde{B}\rangle$ is given by

$$
\psi_n^\mu |A\rangle|\widetilde{B}\rangle = \widetilde{\psi}_n^\mu |A\rangle|\widetilde{B}\rangle = 0 \tag{A.5}
$$

where n is a positive integer and

$$
\begin{aligned}
\psi_0^\mu |A\rangle|\widetilde{B}\rangle &= \frac{1}{\sqrt{2}} (\Gamma^\mu)^A_{\ C} \, (\mathbb{1})^B_{\ D} \, |C\rangle|\widetilde{D}\rangle \\
\widetilde{\psi}_0^\mu |A\rangle|\widetilde{B}\rangle &= \frac{1}{\sqrt{2}} (\Gamma_{11})^A_{\ C} \, (\Gamma^\mu)^B_{\ D} \, |C\rangle|\widetilde{D}\rangle \ .
\end{aligned}
\tag{A.6}
$$

It is easy to check that this action correctly reproduces the anticommutation properties of the fermionic oscillators, and in particular that $\{\psi_0^\mu, \psi_0^\nu\} = \{\widetilde{\psi}_0^\mu, \widetilde{\psi}_0^\nu\} = \eta^{\mu\nu}$, and $\{\psi^\mu, \widetilde{\psi}^\nu\} = 0$. On the conjugated state $\langle A|\langle\widetilde{B}|$ we have analogously

$$\langle A|\langle\widetilde{B}|\,\psi_n^\mu \;=\; \langle A|\langle\widetilde{B}|\widetilde{\psi}_n^\mu = 0 \tag{A.7}$$

if $n < 0$, and

$$\langle A|\langle\widetilde{B}|\,\psi_0^\mu \;=\; -\frac{1}{\sqrt{2}}\langle C|\langle\widetilde{D}|\psi_0^\mu\,(\Gamma^\mu)^A{}_C\,(\mathbb{1})^B{}_D$$

$$\langle A|\langle\widetilde{B}|\,\psi_0^\mu\,\widetilde{\psi}_0^\mu \;=\; \frac{1}{\sqrt{2}}\langle C|\langle\widetilde{D}|\,(\Gamma_{11})^A{}_C\,(\Gamma^\mu)^B{}_D \quad. \tag{A.8}$$

We now use these definitions to derive the fermionic structure of the boundary state $|B,\eta\rangle_R^{(0)}$ in eq.(1.7), which has to satisfy the following overlap equation (see eq.(7.230) of Ref. [4])

$$\left(\psi_0^\mu - i\eta\,S^\mu{}_\nu\widetilde{\psi}_0^\nu\right)|B,\eta\rangle_R^{(0)} = 0 \tag{A.9}$$

where $S^\mu{}_\nu$ is the matrix defined in Eq. (1.4). If we write

$$|B_\psi,\eta\rangle^{(0)} = \mathcal{M}_{AB}\,|A\rangle|\widetilde{B}\rangle \tag{A.10}$$

then, Eq. (A.9) for $n = 0$ implies that the 32×32 matrix $\mathcal{M}$ has to satisfy the following equation

$$(\Gamma^\mu)^T\,\mathcal{M} - i\,\eta S^\mu{}_\nu\Gamma_{11}\,\mathcal{M}\,\Gamma^\nu = 0 \quad. \tag{A.11}$$

Using our previous definitions, one finds that a solution is

$$\mathcal{M} = C\,\Gamma^0\cdots\Gamma^p\,\frac{1 + i\eta\,\Gamma_{11}}{1 + i\eta} \quad. \tag{A.12}$$

In the same way we can determine the conjugated state $\langle B_\psi,\eta|_R^{(0)}$, which satisfies the conjugated equation

$$\langle B_\psi,\eta|_R^{(0)}\left(\psi_0^\mu + i\eta S^\mu{}_\nu\widetilde{\psi}_0^\nu\right) = 0 \quad. \tag{A.13}$$

Writing

$$\langle B_\psi,\eta|_R^{(0)} = \langle A|\langle\widetilde{B}|\,\mathcal{N}_{AB} \tag{A.14}$$

and using eqs.(A.8), we can rewrite eq.(A.13) as

$$-\,(\Gamma^\mu)^T\,\mathcal{N} + i\,\eta S^\mu{}_\nu\left(\Gamma^{11}\right)^T\mathcal{N}\Gamma^\nu = 0 \tag{A.15}$$

which is satisfied by

$$\mathcal{N}_{AB} = (-1)^p\left(C\Gamma^0...\Gamma^p\frac{1 + i\eta\Gamma^{11}}{1 - i\eta}\right)_{AB} \tag{A.16}$$

References

[1] M.J. Duff, R.R. Khuri and J.X. Lu, "String solitons," *Phys. Rep.* **259** (1995) 213; K.S. Stelle, "BPS Branes in Supergravity," hep-th/9803116; R. Argurio, "Brane Physics in M-Theory," hep-th/9807171.

[2] J. Polchinski, S. Chaudhuri and C.V. Johnson, "Notes on D-branes," hep-th/9602052; J. Polchinski, "TASI lectures on D-branes," hep-th/9611050.

[3] J. Polchinski, "Dirichlet-branes and Ramond-Ramond Charges," *Phys. Rev. Lett.* **75** (1995) 4724, hep-th/9510017.

[4] P. Di Vecchia and A. Liccardo, "D branes in string theories, I," Lectures given at the NATO Advanced Study Institute, Akureyri, Iceland, August 1999, hep-th/9912161.

[5] M. Billó, D. Cangemi and P. Di Vecchia, "Boundary state for moving D-branes," *Phys. Lett.* **B400** (1997) 63, hep-th/9701190.

[6] C.G. Callan and I.R. Klebanov, "D-brane boundary dynamics ," *Nucl. Phys.* **B465** (1996) 473, hep-th/9511173.

[7] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, "String loops corrections to beta functions," *Nucl. Phys.* **B288** (1987) 525; "Adding holes and cross-caps to the superstring," *Nucl. Phys.* **B293** (1987) 83; "Loop corrections to superstring equations of motion ," *Nucl. Phys.* **B308** (1988) 221.

[8] J.X. Lu and S. Roy, "(m, n) String-Like Dp-Brane Bound States," hep-th/9904112.

[9] J.X. Lu and S. Roy, "Non-Threshold (F, Dp) Bound States," hep-th/9904129.

[10] J.X. Lu and S. Roy, " $((F, D1), D3)$ Bound State and its T Dual Daughters," hep-th/9905014; "$(F, D5)$ Bound State, $SL(2, Z)$ Invariance and the Descendent States in Type IIB/A String Theory, " hep-th/9905056.

[11] A. Sen, "Non-BPS states and branes in string theory," hep-th/9904207; A. Lerda and R. Russo, "Stable non-BPS states in string theory: a pedagogical review," hep-th/9905006; J.H. Schwarz, "TASI lectures on non-BPS D-brane systems," hep-th/9908144.

[12] C. Bachas, " D-brane dynamics," *Phys.Lett.* **B374** (1996) 37, hep-th/9511043.

[13] M.B. Green and Gutperle, "Light-cone supersymmetry and D-branes," *Nucl.Phys.* **B476** (1996) 484, hep-th/9604091.

[14] M.R. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, "D-branes and short distances in string theory," *Nucl. Phys.* **B485** (1997) 85, hep-th/9608024.

[15] K. Hashimoto,"Corrections to D-brane Action and Generalized Boundary," hep-th/9909027; "Generalized Supersymmetric Boundary State," hep-th/9909095.

[16] P. Di Vecchia, M. Frau, A. Lerda and A. Liccardo, "(F, Dp) bound states from the boundary state," hep-th/9906214. To be published in Nuclear Physics.

[17] J.H. Schwarz, "An $SL(2, Z)$ multiplet of type IIB superstrings," *Phys. Lett* **B360** (1995) 13, Erratum *Phys. Lett.* **B364** (1995) 252, hep-th/9508143.

[18] C. Schmidhuber, "D-brane actions," *Nucl. Phys.* **B467** (1996) 146, hep-th/9601003.

[19] M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo," Scattering of closed strings from many D-branes," *Phys. Lett.* **B400** (1997) 52, hep-th/9702037.

[20] A. Sen, "Stable non-BPS states in string theory," *JHEP* **9806** (1998) 007, hep-th/9803194; "Tachyon condensation on the brane antibrane system," *JHEP* **9808** (1998) 012, hep-th/9805170.

[21] A. Sen, "Stable non-BPS bound states of BPS D-branes," *JHEP* **9808** (1998) 010, hep-th/9805019.

[22] A. Sen,"$SO(32)$ spinors of type I and other solitons on brane-antibrane pair," *JHEP* **9809** (1998) 023, hep-th/9808141.

[23] A. Sen, "Type I D-particle and its interactions," *JHEP* **9810** (1998) 021, hep-th/9809111.

[24] A. Sen,"BPS D-branes on non-supersymmetric cycles," *JHEP* **9812** (1998) 021, hep-th/9812031.

[25] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, "Stable non-BPS D-branes in type I string theory," hep-th/9903123.

[26] M.B. Green, J.H. Schwarz and E. Witten, "Superstring Theory," Cambridge University Press, 1987.

[27] J. Polchinski and E. Witten,"Evidence for heterotic-Type I string duality," *Nucl. Phys.* **B460** (1996) 525, hep-th/9510169.

[28] D. Friedan, E. Martinec and S. Shenker, "Conformal invariance, supersymmetry and string theory," *Nucl. Phys.* **B271** (1986) 93.