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Anomalies

Anomalies are related to the notion of symm. breaking. There are several mechanisms to break the symmetry:

* SOFT BREAKING: addition of a "small" term into $\mathcal{L}$ s.t. it breaks the existent symmetry
* SPONTANEOUS SYMMETRY BREKING ['39 Heisenberg in cond. mat. '60s Goldstone in qft]
* ANOMALOUS BREAKING: disappearance of class. symm. at quantum level.

$$\text{CLASSICAL} \xrightarrow[\;\not\;]{\text{SYMMETRY}} \text{QUANTUM}$$

needs "regularisation" to be promoted! (otherwise it cannot be realised)

there can be both local and global anomalous symm. with $\neq$ outcomes (they must not lead to inconsistent theories!) $\Leftarrow$

e.g.: suppose S invariant $\Rightarrow$ current

$$\partial_\mu J^\mu_\alpha = 0 \text{ (classically)}$$

$\rightarrow$ it might be that:

$$\partial_\mu \langle J^\mu_\alpha(x) \rangle = a_\alpha(x)$$

at quantum level.

e.g.: suppose $D = 2$ and interacting theory s.t.:

$$\begin{cases} \gamma^0 = \sigma^1 \\ \gamma^1 = \sigma^2 \\ \gamma^3 = -i\sigma^3 \end{cases} \Longrightarrow \begin{aligned} J^\mu &= \bar{\psi}\gamma^\mu\psi \\ J^\mu_5 &= i\bar{\psi}\gamma^\mu\gamma^5\psi \end{aligned} \quad \text{s.t.:} \quad \partial_\mu J^\mu = \partial_\mu J^\mu_5 = 0. \quad [\text{NB: } J^\mu_5 = \underset{\text{dual!}}{\varepsilon^{\mu\nu}} J_\nu]$$

Then at quant. level $\Rightarrow$

$$\langle J^\mu(x) J^\nu(0) \rangle = g^{\mu\nu}\Pi_1(x) - \square^{-1}\left(\partial^\mu\partial^\nu\Pi_2(x) + \partial^{(\mu}\varepsilon^{\nu)\rho}\partial_\rho\Pi_3(x)\right)$$

$$\langle J^\mu_5(x) J^\nu(0) \rangle = \varepsilon^{\mu\nu}\Pi_1(x) - \square^{-1}\left(\varepsilon^{\mu\rho}\partial_\rho\partial^\nu\Pi_2(x) + g^{\mu\nu}\square\Pi_3(x) - 2\partial^\mu\partial^\nu\Pi_3(x)\right)$$

$\rightarrow$ imposing conservation, we get:

$$\begin{cases} \Pi_1 = \Pi_2, \quad \Pi_3 = 0 \\ \Pi_1 = 0 \end{cases} \Rightarrow \Pi_1 = \Pi_2 = \Pi_3 \Rightarrow \text{either the THEORY IS TRIVIAL or it is ANOMALOUS!}$$

Consider for example:

* YM theory in $D=4$ spacetime dimension:

   - G: gauge group
   - ψ^R, A_μ^R: fields in rep. R of G
   - $A_\mu^R = A_\mu^a T_a^R$ where T_a^R are generators of G in rep. R
   - $\delta A_\mu^R = \partial_\mu \varepsilon^R - i\left[A_\mu^R, \varepsilon^R\right] \qquad (\varepsilon^R = \varepsilon^a T_a^R)$

     $\delta \psi^R = i\varepsilon^R \psi^R$

     e.g.: $\delta\phi^a = i\varepsilon^b (T_b^{adj})^a{}_c \phi^c = -\varepsilon^b f_{bc}{}^a \phi^c \Rightarrow \delta\phi^R = -\varepsilon^b f_{bc}^a T_a^R \phi^c =$

     $= i\varepsilon^b [T_b^R, T_c^R]\phi^c = i[\varepsilon^R, \phi^R]$

⇒ Gauge transf.:

$$\delta\psi(x) = i\,\varepsilon^a(x)\,(T_a^R)\,\psi(x)$$

$$\delta A_\mu^a(x) = \partial_\mu \varepsilon^a(x) + f_{bc}^a A_\mu^b(x)\varepsilon^c(x)$$

$$\Downarrow$$

$$(D_\mu)^{ab} = \delta^{ab}\partial_\mu - f^{ab}{}_c A_\mu^c$$

Moreover:

add $\mathcal{L}_{Matter}[\psi, \bar\psi]$ to YM action:

$$\frac{\delta \mathcal{L}_{matt}}{\delta A_\mu^a} = (J^\mu_{matt})_a \longrightarrow (D_\mu F^{\mu\nu})_a = -(J^\nu_{matt.})_a \quad \text{s.t.} \quad \underline{\underline{D_\mu (J^\mu_{matt}) = 0}}$$

ABELIAN ANOMALY

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \underbrace{-i \bar{\psi} \not{D} \psi}_{= \mathcal{L}_{matt}} \quad \text{where} \quad \not{D} = \not{\partial} - i \not{A}^R$$

$\Rightarrow$ How to compute correlation functions?

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \; e^{i \int d^4x \, \mathcal{L}[\psi, \bar{\psi}, A_\mu^R]} \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n)$$

Consider $\mathcal{O}_i(x_i) \sim (J^{\mu_i}(x_i))_{a_i}$:

$\Rightarrow$ it's enough to study

$$e^{iW[A_\mu]} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \; e^{i \int d^4x \, \mathcal{L}_{matt}[\psi, \bar{\psi}, A_\mu]}$$

and take funct. deriv. w.r.t. $A_{\mu_i}^{a_i}(x_i)$ since $(J^\mu_{matt}(x))_a = \frac{\delta \mathcal{L}_{matt}}{\delta A_\mu^a}$.

[arbitrary S-matrix elements are made of current correlators and gauge field propagators $\Rightarrow$ considering the above is not a loss in generality]

Now consider:
$$\psi(x) \mapsto U(x) \psi(x)$$
$$\bar{\psi}(x) \mapsto \bar{\psi}(x) \bar{U}(x)$$
$\Rightarrow$ the action is NOT invariant unless U is const. or $U(x)$ is a gauge transf. $[\bar{U} = (i\gamma^0) U^\dagger (i\gamma^0)]$

- $U(x) = e^{i\varepsilon^a(x) T_a} \Rightarrow \bar{U}(x) = U^{-1}(x)$:

$$\mathcal{D}\psi' \mathcal{D}\bar{\psi}' = \mathcal{D}\psi \mathcal{D}\bar{\psi} \, (\det \mathcal{U})^{-1} (\det \bar{\mathcal{U}})^{-1}$$

where $\langle x | \overset{(-)}{\mathcal{U}} | y \rangle = \overset{(-)}{U}(x) \, \delta^4(x-y)$.

$$\Rightarrow \mathcal{D}\psi' \mathcal{D}\bar{\psi}' = \mathcal{D}\psi \mathcal{D}\bar{\psi} \Rightarrow \text{the measure is invariant!}$$

$\downarrow$

no anomalies for <u>non chiral</u> coupling of gauge fields to matter

- $U(x) = e^{i\varepsilon^a(x) T_a \gamma_5} \rightarrow \bar{U}(x) = U(x)$:

$$\Rightarrow \mathcal{D}\psi' \mathcal{D}\bar{\psi}' = \mathcal{D}\psi \mathcal{D}\bar{\psi} \, (\det \mathcal{U}(x))^{-2} \longrightarrow \text{NON TRIVIAL JACOBIAN}$$

where:

$$(\det \mathcal{U})^{-2} = e^{\mathrm{Tr}\,\ln \mathcal{U}^{-2}} = e^{-2\mathrm{Tr}\ln \mathcal{U}} = \exp\left(-2\int d^4x \, \langle x|\mathrm{tr}\ln U(x)\, \delta^4(0)|x\rangle\right) =$$

$$= \exp\left(-2\int d^4x \; i\varepsilon^a(x)\, \mathrm{tr}(T_a\gamma_5)\, \delta^4(0)\right) = \exp\left(-2\delta^4(0)\int d^4x \; \varepsilon^a(x)\, \mathrm{Tr}(T_a\gamma_5)\right)$$

In general:

$$\mathcal{D}\psi'\mathcal{D}\bar{\psi}' = \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{i\int d^4x\, \varepsilon^a(x)\, a_a(x)}$$

$\longrightarrow$ in this case we first need to regularize the $\delta^4(0)$ factor [UV divergence! Short distance behaviour]

$$\Rightarrow (\det \mathcal{U})^{-2} = \exp\left(-2\lim_{\Lambda\to\infty}\int d^4x\, \mathrm{Tr}\left[\langle x|\varepsilon^a(x) T_a\gamma_5\, f\left(-\tfrac{\not{D}^2}{\Lambda^2}\right)|x\rangle\right]\right) =$$

$f(0) = 1, \; f(\infty) = 0$

$s f'(s) = 0$ at $s = 0, \infty$ $[f(s) = e^{-s}]$

$\langle \chi|\not{D}|x\rangle = \not{D}\langle \chi|x\rangle$

$$= \exp\left(-2\lim_{\Lambda\to\infty}\int d^4x \frac{d^4q}{(2\pi)^4}\, \mathrm{Tr}\left[\langle x|q\rangle\langle q|\ldots|x\rangle\right]\right) =$$

$$= \exp\left(-2\lim_{\Lambda\to\infty}\int d^4x\; \varepsilon^a(x)\int\frac{d^4p}{(2\pi)^4}\, \langle x|p\rangle\, \mathrm{tr}\left(T_a\gamma_5\, \langle p|f\left(-\tfrac{\not{D}^2}{\Lambda^2}\right)|x\rangle\right)\right) =$$

$$= \exp\left(-2\lim_{\Lambda\to\infty}\int d^4x\, \varepsilon^a(x)\int\frac{d^4p}{(2\pi)^4}\, e^{ip\cdot x}\, \mathrm{tr}\left(T_a\gamma_5\, f\left(-\left(-i\underset{\substack{\downarrow \\ \text{expand around } q^2}}{q}+\tfrac{\not{D}}{\Lambda}\right)^2\right)\right)\Lambda^4\right) \sim$$

$$\sim \exp\left(-2\int d^4x\, \varepsilon^a(x)\underbrace{\int\frac{d^4q}{(2\pi)^4}\cdot\frac{1}{2}f''(q^2)}_{\frac{i}{32\pi^2}}\, \mathrm{Tr}\left(T_a\gamma_5\, (-\not{D}^2)^2\right)\right) =$$

$$= \exp\left(-\frac{1}{16\pi^2}\int d^4x\; \varepsilon^a(x)\; \varepsilon^{\mu\nu\rho\sigma}\, \mathrm{Tr}\left(T_a F_{\mu\nu}F_{\rho\sigma}\right)\right) =$$

Then $a_a = -\frac{1}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(T_a F_{\mu\nu}F_{\rho\sigma}\right)$. Consider $G = U(1) \ni T$

* CLASSICAL THEORY:

$\mathcal{L}_{matt} = -\bar{\psi}\not{D}\psi$ under $\psi'(x) = e^{i\varepsilon^a T\gamma_5}\psi(x)$ with $\varepsilon^a = $ const is a symm!

$$\Rightarrow J_5^\mu = i\bar{\psi}\gamma^\mu\gamma_5\psi \rightarrow \partial_\mu J_5^\mu(x) = 0.$$

* QUANTUM THEORY:

$$Z = \int \mathcal{D}\psi'\mathcal{D}\bar{\psi}'\, e^{\frac{i}{\hbar}\int d^4x\, \mathcal{L}_{matt}[\psi',\bar{\psi}',A_\mu]} = \int\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{i\int d^4x\,\varepsilon^a(x)a_a}\; e^{\frac{i}{\hbar}\int d^4x\{\mathcal{L}_{matt}[\psi,\bar{\psi},A_\mu]-\varepsilon^a\partial_\mu J_5^\mu\}}$$

$$\sim\int\mathcal{D}\psi\mathcal{D}\bar{\psi}\left[1+i\int d^4x\,\varepsilon^a(x)a_a+O(\varepsilon^2)\right]e^{\frac{i}{\hbar}\int d^4x\{\mathcal{L}_{matt}[\psi,\bar{\psi},A_\mu]-\varepsilon^a(x)\,\partial_\mu J_5^\mu(x)\}}$$

$\Rightarrow$ therefore AT QUANTUM LEVEL:

$$[T, T_a] = 0 \Rightarrow \text{anomaly is gauge inv!}$$

from gauge group

$$\underbrace{\partial_\mu \langle J_5^\mu(x) \rangle_A}_{\text{fixed } A_\mu \text{ background}} = \frac{\hbar}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr}\left(T \, F_{\mu\nu} F_{\rho\sigma}\right) \boxed{\neq 0}$$

since $F_{\mu\nu}$ contains A_μ $\langle \ldots \rangle_A$ is superflous!

Summary:

- classical symmetry (global)
  - functional measure not invariant
    - quantum anomaly!

e.g.: "abelian symmetry" (chiral $U(1)$ transf)

$\Rightarrow$ RELATION TO INSTANTONS

The abelian anomaly is the ANOMALOUS VARIATION OF AN EFFECTIVE ACTION:

$W[A_\mu]$ depends only on A_μ $\longrightarrow$ define sth with fermions! (We are looking at anom. on matter fields (not gauge))

$\Downarrow$

Introduce $J[\chi, \bar\chi] = \int d^4x \left(\bar\chi \psi + \chi \bar\psi\right)$ as source for fermions:

$$\psi(x) = \frac{\delta J}{\delta \bar\chi} \quad ; \quad \bar\psi(x) = \frac{\delta J}{\delta \chi}$$

$\Rightarrow$ def $\tilde W[A_\mu, \chi, \bar\chi]$ after functional integral on $\psi, \bar\psi$ (fixed A_μ).

$\rightarrow$ EFFECTIVE ACTION: $\tilde\Gamma[A_\mu, \psi, \bar\psi] = \tilde W[A_\mu, \chi, \bar\chi] - \int d^4x \, \bar\chi \frac{\delta J}{\delta \bar\chi} - \int d^4x \, \chi \frac{\delta J}{\delta \chi}$ (Legendre transf)

$\Downarrow$

obeys Slavnov-Taylor:

with anomaly: $\delta_\varepsilon \tilde\Gamma = \varepsilon \int d^4x \, a(x)$

In abelian anomaly:

$$\int d^4x\, a(x) \sim \int d^4x\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = \frac{64\pi^2}{g^2}\nu$$

$$\hookrightarrow \delta_\varepsilon \tilde{\Gamma} = -4 C_R \nu \varepsilon$$

Instanton no.

where $\mathrm{Tr}(T_a^R T_b^R) = C_R \delta_{ab}$

RELATION WITH INDEX OF DIRAC OPERATOR:

Grassmann var.

→ consider Euclidean space: $\quad i \not{D}_E \varphi_k = \lambda_k \varphi_k \quad$ where $\psi = \sum_k c_k \varphi_k\,,\ \bar{\psi} = \sum_k \bar{c}_k \varphi_k^\dagger$

→ since $[T, \not{D}_E] = 0 \Rightarrow T\varphi_k = t_k \varphi_k$

Therefore

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{-\int d^4x\, \bar{\psi} \not{D}_E \psi} = \det \not{D}_E = \prod_k \lambda_k$$

and

$$\gamma_5 \not{D}_E = -\not{D}_E \gamma_5\,, \quad [T, \gamma_5] = 0 \quad \Rightarrow \quad \begin{aligned} i\not{D}_E(\gamma_5 \varphi_k) &= -\lambda_k(\gamma_5 \varphi_k) \\ T(\gamma_5 \varphi_k) &= \lambda_k (\gamma_5 \varphi_k) \end{aligned}$$

Now define

$$\varphi_{k,\pm} = \frac{\mathbb{I} \pm \gamma_5}{2} \varphi_k \longrightarrow \not{D}_E^2 \varphi_{k,\pm} = \pm \lambda_k^2 \varphi_{k,\pm} \quad \text{for } \lambda_k \neq 0$$

while if $\lambda_k = 0 \Rightarrow \not{D}_E \varphi_0 = 0$ and $\not{D}_E^2 \varphi_{0,\pm} = 0$

$\Rightarrow \lambda_0 = 0$ eigenspace:

$$\begin{cases} n_+ \text{ times } \varphi_{0,+} \\ n_- \text{ times } \varphi_{0,-} \end{cases}$$

Therefore with the regularization:

$$\lim_{\Lambda\to\infty} \mathrm{Tr}\left(\gamma_5 T f\left(-\frac{\not{D}_E^2}{\Lambda^2}\right)\right) = \lim_{\Lambda\to\infty} \sum_k \langle \varphi_k | \gamma_5 T f\left(-\frac{\not{D}_E^2}{\Lambda^2}\right) | \varphi_k \rangle =$$

$$= \lim_{\Lambda\to\infty} \sum_k f\left(\frac{\lambda_k^2}{\Lambda^2}\right) \langle \varphi_k | \gamma_5 T | \varphi_k \rangle =$$

$$= \sum_{u=1}^{n_+} t_u - \sum_{v=1}^{n_-} t_v$$

Consider $T = \mathbb{I}$:

$$\lim_{\Lambda\to\infty} \mathrm{Tr}\left(\gamma_5 f\left(-\frac{\not{D}_E^2}{\Lambda^2}\right)\right) = n_+ - n_- = \mathrm{index}(i\not{D}_E)$$

Since

$$\int d^4x\, a(x) = -2\, \mathrm{index}(i\not{D}_E) \longrightarrow \mathrm{index}(i\not{D}_E) = \frac{1}{32\pi^2}\int d^4x\, \varepsilon^{\mu\nu\rho\sigma}\, \mathrm{Tr}_R\left(F_{\mu\nu}F_{\rho\sigma}\right).$$

In general, for $d = 2n$:

$$\mathrm{index}(i\not{D}_E) = \frac{(-1)^n}{n!}\frac{1}{(4\pi)^n}\int d^{2n}x\, \varepsilon^{\mu_1\dots\mu_{2n}}\, \mathrm{Tr}\left[F_{\mu_1\mu_2}\cdots F_{\mu_{2n-1}\mu_{2n}}\right].$$

ANOMALOUS GAUGE FIELDS

Consider a local symmetry:

$$S[\phi^r + \delta\phi^r] = S[\phi^r] \quad \text{under} \quad \delta A_\mu^a = D_\mu \varepsilon^a \text{ in adj rep.}$$

such that:

$$\prod_r D(\phi^r + \delta\phi^r) = \prod_r D\phi^r\, e^{i\varepsilon\int d^4x\, \mathcal{A}(x)}$$

Therefore:

$$\delta_\varepsilon W[\phi^r, A_\mu] = \varepsilon^a \int d^4x\, \mathcal{A}_a(x)$$

$$= \int d^4x\, \delta A_\mu^a \frac{\delta W}{\delta A_\mu^a} = \int d^4x\, D_\mu\varepsilon^a \frac{\delta W}{\delta A_\mu^a} = -\int d^4x\, \varepsilon^a(x)\, D_\mu J_a^\mu$$

$$\Rightarrow \left(D_\mu \langle J^\mu\rangle\right)^a = -\mathcal{A}^a(x) \implies \text{anomalous current !}$$

From the diagrammatic POV:

coupling $\dots A_\mu J^\mu \dots$

$$\frac{\delta}{\delta A_\mu^a}\cdots W[A]\Big|_{A_\mu=0} = \langle J_a^\mu \dots\rangle$$

$\hookrightarrow \Gamma^{\mu_1,\dots,\mu_n}_{a_1,\dots,a_n}(x_1\dots x_n)$ (only connected)

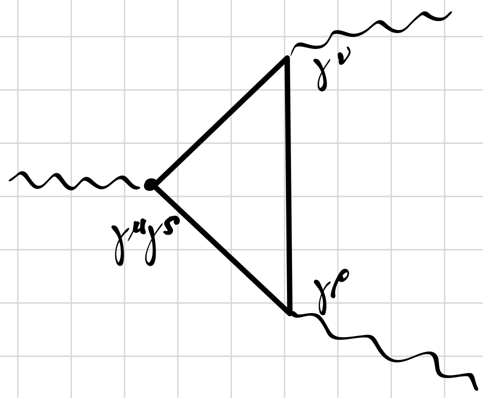
Consider the abelian anomaly:

$$\partial_\mu\langle J_5^\mu\rangle = -a(x) \Rightarrow \partial_\mu\left[\delta_{A_\mu}\delta_{A_\nu}\langle J_5^\mu\rangle\right] = -\partial_\mu T_5^{\mu\nu\rho}(x,y,z) = -\delta_{A_\mu}\delta_{A_\nu}\, a(x)$$

Then the anomalous

$$-i\,(p+q)_\mu \Gamma_5^{\mu\nu\rho}(-p-q,p,q) = -\frac{1}{2\pi^2}\mathrm{Tr}\left(T\, T_a T_b\right)\varepsilon^{\nu\rho\lambda\sigma} p_\lambda p_\sigma$$

comes from:



⇒ No other anomalies since $U(1) \Rightarrow$ bilinear in $A_\mu \to$ at most $2\ \delta A_\mu$!

If non abelian symmetry ⟶ more legs (i.e.: more functional derivatives)!

$$D_\mu \langle J^\mu \rangle_a = -\mathcal{A}_a \to \text{Define:} \quad \Gamma^{\mu\nu\rho}_{abc} = -\langle J^\mu_a J^\nu_b J^\rho_c \rangle$$

$$\Pi^{\rho\nu}_{ab} = i \langle J^\rho_a J^\nu_b \rangle$$

$$A^{\nu\rho}_{abc} = \delta_{A^b_\nu} \delta_{A^c_\rho} \mathcal{A}_a$$

Therefore:

$$\partial_\mu \Gamma^{\mu\nu\rho}_{abc}(x,y,z) + f_{abc}\left[\delta^4(x-y)\,\Pi^{\nu\rho}_{dd}(y,z) - \delta^4(x-z)\,\Pi^{\nu\rho}_{ff}(y,z)\right] = -\mathcal{A}^{\nu\rho}_{abc}(x,y,z)$$

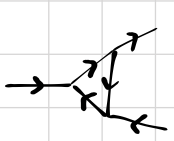
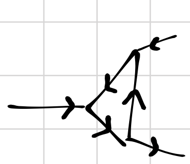
Since

$$\mathcal{A}_a = 4c\,\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[T_a T_b T_c\right]\partial_\mu A^b_\nu \partial_\rho A^c_\sigma + O(A^3)$$

$$\Rightarrow -i\,(p+q)_\mu\, \Gamma^{\mu\nu\rho}_{abc}(-p-q,p,q)\Big|_{\text{anomaly}} = 8c\,\varepsilon^{\nu\rho\sigma\lambda} p_\sigma p_\lambda D^{R}_{abc}$$

AXIAL ANOMALY

$$\left.\begin{aligned} J^\mu_{5a} &= i\bar\psi\gamma^\mu\gamma_5 T_a \psi \\ J^\mu_a &= i\bar\psi\gamma^\mu T_a\psi \end{aligned}\right\} \longrightarrow \text{compute } \langle J^\mu_{5a} J^\nu_b J^\rho_c \rangle$$

 + 

Then:

Pauli-Villars regularization ⇒ $-i\,(p+q)_\mu \Gamma^{\mu\nu\rho}_{5abc}(-p-q,p,q) = -\frac{1}{2\pi^2}\varepsilon^{\nu\rho\lambda\sigma} p_\lambda p_\sigma \mathrm{Tr}\left(T_a \{T_b T_c\}\right)$

→ if anomalous GLOBAL chiral symmetry ⇒ NO inconsistency

→ NB: vector currents:

$$p_\nu \Gamma^{\mu\nu\rho}_{abc}(-p-q,p,q) = q_\rho \Gamma^{\mu\nu\rho}_{abc}(-p-q,p,q) = 0 \Rightarrow \text{gauge fields NOT anomalous!}$$

What if chiral matter? $\Rightarrow \psi_L, \psi_R$:

$$-i(p+q)_\mu T^{\mu\nu\rho}_{abc}(-p-q,p,q) = -\frac{1}{12\pi^2}\,\mathrm{Tr}\left(T_{(a}T_bT_{c)}\right)\varepsilon^{\nu\rho\lambda\sigma}p_\lambda p_\sigma \text{ " }+\mathcal{O}(A^3)\text{"}$$

because

$\bar\psi(\not\partial - i\not A P_L)\psi$ does not transf. covariantly!

$\hookrightarrow$ the measure of $\mathcal{D}\psi\mathcal{D}\bar\psi$ does not transf. nicely $\Rightarrow$ anomaly!

Since anomalies are always FINITE (not div) and LOCAL:

full anomaly: $$\mathcal{A}^L_a(x) = -\frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[T_a\,\partial_\mu\left(A_\nu\partial_\rho A_\sigma - \frac{i}{4}A_\nu[A_\rho,A_\sigma]\right)\right]$$

$\Rightarrow$ Does it vanish?

$$\mathcal{A}_a(x) = \sum_i \mathcal{A}^L_{a,i} + \sum_j \mathcal{A}^R_{a,j}$$

$\hookrightarrow$ cancels for pseudoreal rep.'s:

- $SO(2n+1)$, $SO(4n)$
- $USp(2n)$
- G_2, F_4, E_7, E_8
- $SU(2)$ (real)
- $SO(4n+2)|_{n>1}$, E_6 (luck...)

Final remark:

$$-\left(D_\mu\frac{\delta}{\delta A_\mu}\right)^a W[A] = \mathcal{A}^a \rightarrow \text{Define: } G_a = -\left(D_\mu\,\delta_{A_\mu}\right)^a \Rightarrow [G_a,G_b] = f_{ab}{}^c\,G_c(x)\,\delta^4(x-y)$$

$\rightarrow$ WZ consistency relation: $$G_a(x)\mathcal{A}_b(y) - G_b(y)\mathcal{A}_a(x) = f_{ab}{}^c\,\delta^4(x-y)\,\mathcal{A}_c(x).$$