An introduction to the SM of EW interactions

Ridolfi

Notice:

* τ^1, τ^2, τ^3 s.t. $\tau^i = \tau^{i\dagger} \Rightarrow e^{i\alpha^i \tau_i} \in SU(2)$ $\left[(\tau^+)^\dagger \neq \tau^+ \right]$
* $\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = R \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$ where $R = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \rightarrow R R^T = \mathbb{I}$ because

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^3 W^{\mu\nu 3} \rightarrow \begin{pmatrix} B_{\mu\nu} \\ W_{\mu\nu}^3 \end{pmatrix} = R \begin{pmatrix} F_{\mu\nu} \\ Z_{\mu\nu} \end{pmatrix} \rightarrow \mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu}$$

thanks to $RR^T = \mathbb{I}$

Hadrons:

β-decay $\longrightarrow$ $N \longrightarrow P + e^- + \bar{\nu}_e$
(udd) (uud)

$\hookrightarrow$ $d \rightarrow u + e^- + \bar{\nu}_e \Rightarrow J_\mu^{(had)} = \bar{u} \gamma_\mu \frac{1}{2} (1 - \gamma_5) d$

$\Rightarrow$ introduce STRANGE HADRONS $(\Lambda, K, \dots)$

$\hookrightarrow$ introduce a s quark $\left(S = -1, Q = -\frac{1}{3}\right)$

Then $J_\mu^{(had)} = \cos\theta_c \, \bar{u} \gamma_\mu \frac{1}{2}(1-\gamma_5) d + \sin\theta_c \, \bar{u} \gamma_\mu \frac{1}{2}(1-\gamma_5) s = \bar{u}_L \gamma_\mu \left(\cos\theta_c \, d_L + \sin\theta_c \, s_L \right) =$

$$= \left(\bar{u}_L \; \bar{d}_L \; \bar{s}_L \right) \gamma_\mu T^+ \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}$$

$$T^+ = \begin{pmatrix} 0 & \cos\theta_c & \sin\theta_c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad T^- = (T^+)^T$$

NB $\quad T^3 = [T^+, T^-] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\cos^2\theta_c & -\sin\theta_c \cos\theta_c \\ 0 & -\sin\theta_c \cos\theta_c & -\sin^2\theta_c \end{pmatrix} \Rightarrow J_\mu^{(had)3} = \bar{u}_L \gamma u_L - \cos^2\theta_c \, \bar{d}_L \gamma_\mu d_L -$

$- \sin^2\theta_c \, \bar{s}_L \gamma_\mu s_L - \sin\theta_c \cos\theta_c \left(\bar{d}_L \gamma_\mu s_L + \bar{s}_L \gamma_\mu d_L \right)$

phenomenologically suppressed $\longleftarrow$ FCNC

$K^+ \longrightarrow \pi^0 + e^+ + \nu_e \quad \Rightarrow \sin\theta_c \; \bar{s} \gamma_\mu \frac{1}{2} (1 - \gamma_5) u$

$K^+ \longrightarrow \pi^+ + e^+ + e^- \quad \Rightarrow \sin\theta_c \cos\theta_c \; \bar{s} \gamma_\mu \frac{1}{2} (1 - \gamma_5) d$

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \dots)}{\Gamma(K^+ \rightarrow \pi^0 \dots)} \approx 0.97 \longrightarrow \text{it's actually } 10^{-5}$$

GIM mechanism $\Rightarrow$ c quark

$$J_\mu^{(had)} = \bar{u}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)d' + \bar{c}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)s'$$

$$\hookrightarrow \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix} = V\begin{pmatrix} d \\ s \end{pmatrix} \qquad V^\dagger V = \mathbb{I}$$

$$\Rightarrow J_\mu^{(had)} = (\bar{u}_L \ \bar{d}_L')\gamma_\mu \tau^+ \begin{pmatrix} u_L \\ d_L' \end{pmatrix} + (\bar{c}_L \ \bar{s}_L')\gamma_\mu \tau^+ \begin{pmatrix} c_L \\ s_L' \end{pmatrix}$$

$$J_\mu^{(had)3} = (\bar{u}_L \ \bar{d}_L')\gamma_\mu \tau_3 \begin{pmatrix} u_L \\ d_L' \end{pmatrix} + (\bar{c}_L \ \bar{s}_L')\gamma_\mu \tau_3 \begin{pmatrix} c_L \\ s_L' \end{pmatrix} =$$

$$= \bar{u}_L\gamma_\mu u_L + \bar{c}_L\gamma_\mu c_L - \bar{d}_L\gamma_\mu d_L - \bar{s}_L\gamma_\mu s_L \quad \text{because } VV^\dagger = \mathbb{I}$$

$\hookrightarrow$ c found in $c\bar{c} \to J/\psi \Rightarrow m \sim 3$ GeV.

($m_c \sim 1.5$ GeV)

NB: R quark are SU(2) singlets $\longrightarrow Y_{u_R} = \frac{4}{3}$ $[Y_{\nu_R} = 0]$

Masses in the SM

- Vector bosons:

$$A^\mu \longrightarrow U A^\mu U^\dagger + \frac{i}{g} U \partial^\mu U^\dagger$$

$\hookrightarrow$ $m^2 A_\mu A^\mu$ not gauge invariant.

- Fermions $\longrightarrow$ chirality!

$$* - m\bar{\psi}\psi = -m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

NB $\begin{cases} \psi_L \to e^{i\alpha}\psi_L \\ \psi_R \to \psi_R \end{cases} \Rightarrow$ NOT gauge invariant!

$\Rightarrow$ Spontaneous breaking of the gauge symmetry [almost: must not break for unitarity]

eg: Scalar QED $\phi(x)$

$$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - m^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu$$

s.t.: $\phi(x) \to e^{ie\Lambda(x)}\phi(x)$ $\qquad D_\mu = \partial_\mu - ieA_\mu$

$\phi^\dagger(x) \to e^{-ie\Lambda}\phi(x)$ $\qquad (\partial^2 + m^2)\phi = J_\phi(\phi, A)$

$A^\mu(x) \to A^\mu(x) + \partial^\mu\Lambda(x)$ $\qquad \partial^2 A^\mu = -J_A^\mu \longrightarrow J_A^\mu = ie\left(\phi^\dagger \partial^\mu\phi - \phi\partial^\mu\phi^\dagger\right)$

$\to$ the symm is realised in Wigner-Weyl: $\phi = 0 \Rightarrow \phi' = 0$

(contrary to Nambu-Goldstone:

$\phi(x) \longrightarrow e^{ie\Lambda(x)}\left[\phi(x) + \frac{v}{\sqrt{2}}\right] - \frac{v}{\sqrt{2}}$ ok only for scalars

$A^\mu \longrightarrow A^\mu + \partial^\mu\Lambda$)

NB: $\left|\phi(x) + \frac{v}{\sqrt{2}}\right|^2$ is invariant under Nambu-Goldstone:

$$V(\phi) = \lambda\left|\phi + \frac{v}{\sqrt{2}}\right|^4 + a\left|\phi + \frac{v}{\sqrt{2}}\right|^2 + b$$

$$V'(0) = 0 \Rightarrow V(\phi) = \lambda\left[\left|\phi + \frac{v}{\sqrt{2}}\right|^2 + \frac{v^2}{2}\right]^2$$

$\Rightarrow$ choose Feynman - t'Hooft gauge fixing: $\mathcal{L} = -\frac{1}{2}\left[\partial_\mu A^\mu - ie\frac{v}{\sqrt{2}}(\phi - \phi^\dagger)\right]^2$

Then

$$\mathcal{L} = \left(D_\mu\left(\phi + \tfrac{v}{\sqrt{2}}\right)\right)^\dagger D^\mu\left(\phi + \tfrac{v}{\sqrt{2}}\right) - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{GF}$$

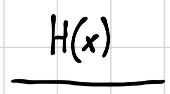
Now:

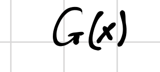
partial integration

$$\mathcal{L}_0 = \partial_\mu\phi^\dagger\partial^\mu\phi + \cancel{ie\tfrac{v}{\sqrt{2}}A_\mu}\partial^\mu(\phi-\phi^\dagger) + \frac{e^2v^2}{\sqrt{2}}A_\mu A^\mu +$$
$$-\frac{1}{2}\partial_\mu A_\nu\partial^\mu A^\nu \underbrace{-\frac{1}{2}\partial_\mu A_\nu\partial^\nu A^\mu - \frac{1}{2}(\partial A)^2}_{\text{TOTAL DERIVATIVE}} +$$
$$+\frac{e^2v^2}{\sqrt{2}}(\phi-\phi^\dagger)^2 + i\cancel{\tfrac{ev}{\sqrt{2}}\partial_\mu A^\mu}(\phi-\phi^\dagger) - \frac{\lambda}{2}v^2(\phi+\phi^\dagger)^2$$

Now call: $\phi(x) = \dfrac{H(x)+iG(x)}{\sqrt{2}}$

$$\Rightarrow \mathcal{L}_0 = \frac{1}{2}\left[\partial_\mu H\partial^\mu H + \partial_\mu G\partial^\mu G - 2\lambda v^2H^2 - e^2v^2G - \partial_\mu A_\nu\partial^\mu A^\nu + e^2v^2A_\mu A^\mu\right]$$

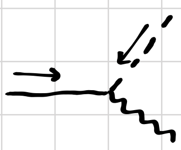
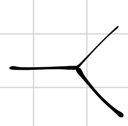
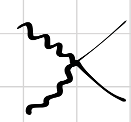
$\Rightarrow$ $H(x)$  $\quad -\dfrac{1}{k^2-2\lambda v^2+i\varepsilon}\quad (m_H^2 = 2\lambda v^2)$

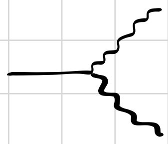
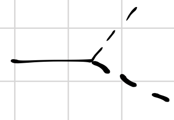
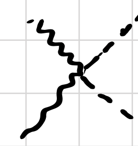
$G(x)$  $\quad -\dfrac{1}{k^2-e^2v^2+i\varepsilon}\quad (m_G^2 = e^2v^2) \longrightarrow$ the Goldstone boson is massive because we broke a local symmetry

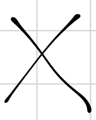
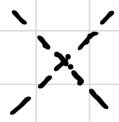
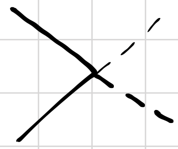
$\quad \dfrac{g^{\mu\nu}}{k^2-e^2v^2+i\varepsilon}\quad (m_A^2 = e^2v^2)$

Now look at the interaction term:

$$\mathcal{L}_I = eA^\mu(H\partial_\mu G - G\partial_\mu H) + e^2v A_\mu A^\mu H(x) - \lambda v H(H^2+G^2) + \frac{e^2}{2}A_\mu A^\mu H^2 + \frac{e^2}{2}A_\mu A^\mu G - \frac{\lambda}{4}(H^4+G^4+2G^2H^2)$$

 $ie(p_G - p_H)^\mu$  $-6\lambda v$  $2e^2g^{\mu\nu}$

 $2e^2v\,g^{\mu\nu}$  $-2\lambda v$  $2e^2g^{\mu\nu}$

 -6λ  -6λ  -2λ

ABELIAN HIGGS MODEL.

$$\Rightarrow \mathcal{L} = -\frac{1}{2}\partial_\mu A_\nu\partial^\nu A^\mu + \frac{1}{2}e^2v^2A_\mu A^\mu + \ldots = -\frac{1}{2}\left[(\partial_\mu A_0)^2 - \frac{1}{2}m_j^2A_0^2\right] + \frac{1}{2}\left[(\partial_\mu A_i)^2 - \frac{1}{2}m_j^2A_i^2\right]$$

How to quantize? $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \longrightarrow [a(\vec{k}), a^\dagger(\vec{q})] = \delta(\vec{k}-\vec{q})$

NB $\mathcal{L}\to-\mathcal{L} \Rightarrow a \leftrightarrow a^\dagger \Rightarrow [a(\vec{k}), a^\dagger(\vec{q})] = -\delta(\vec{k}-\vec{q})$

Then we have: $\left(E_k = \sqrt{m_\gamma^2 + |\vec{k}|^2} \right)$

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2E_k}} \left[a_\mu(\vec{k}) e^{-ik\cdot x} + a_\mu^\dagger(\vec{k}) e^{ik\cdot x} \right] \Rightarrow \left[a_\mu(\vec{k}), a_\nu^\dagger(\vec{q}) \right] = -g_{\mu\nu}\, \delta(\vec{k} - \vec{q})$$

$$\hookrightarrow a_\mu(\vec{k}) = \sum_{i=0}^{3} a(\lambda, \vec{k})\, \varepsilon_\mu(\lambda, \vec{k})$$

$$\varepsilon_\mu(0,\vec{0}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} ; \quad \varepsilon_\mu(1,\vec{0}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} ; \quad \varepsilon_\mu(2,\vec{0}) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} ; \quad \varepsilon_\mu(3,\vec{0}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \varepsilon_\mu(\lambda, \vec{0}) = \delta_\mu^{\ \lambda}$$

$$\hookrightarrow \varepsilon_\mu(\lambda, \vec{k}) = \Lambda_\mu^{\ \nu}(\vec{k})\, \varepsilon_\nu(\lambda, \vec{0}) = \Lambda_\mu^{\ \lambda}(\vec{k})$$

$$\Rightarrow \left[a(\lambda, \vec{k}), a^\dagger(\lambda, \vec{q}) \right] = -g^{\lambda\lambda'}\, \delta(\vec{k} - \vec{q})$$

NB: $\lambda = 0 \Rightarrow \varepsilon_\mu(0, \vec{k}) = \frac{k_\mu}{ev} \longrightarrow$ LONGITUDINAL POLARIZATION

$$\rightarrow A_\mu = \sum_{\lambda=0}^{3} A_\mu(\lambda) \longrightarrow A_\mu(0) = \partial_\mu f(x)$$ SCALAR!

$\Rightarrow \left[a(0, \vec{k}), a^\dagger(0, \vec{q}) \right] = -\delta(\vec{k} - \vec{q}) \Rightarrow$ negative norm states!

$$\hookrightarrow |\vec{k}\rangle = \int d^3p\, f_k(\vec{p})\, a^\dagger(0, \vec{p}) |0\rangle$$

$$\Rightarrow \langle \vec{k} | \vec{k} \rangle = \int d^3p' \int d^3p\, f_k^*(\vec{p}\,')\, f_{\vec{k}}(\vec{p}) \langle 0 | a(0, \vec{p}\,')\, a^\dagger(0, \vec{p}) | 0 \rangle = -\langle 0|0\rangle \int d^3p\, |f_{\vec{k}}(\vec{p})|^2$$

$$\boxed{< 0}$$

$\rightarrow$ must get rid of longitudinal polarization! $\rightarrow$ require $\varepsilon^\mu(\vec{k}) \cdot k_\mu = 0$.