New applications of effective field theories

Rothstein

Effective Field Theory:

$\vec{F} = m\vec{a} \Leftarrow S = T - V$ ⇝ ok, but how do I know V for every system?

→ Is there a way to find e.o.m.? ⇒ extremization procedure [action → extrema → classical conf]

↳ CLAIM: the action can be determined by knowing:

1) symm of S
2) symm of ground state (GS)

↳ OK if only interested in gapless dofs (lowest energy)

The mean is GOLDSTONE THEOREM ⇒ Goldstone bosons are those gapless dofs
($\exists$ counter examples)

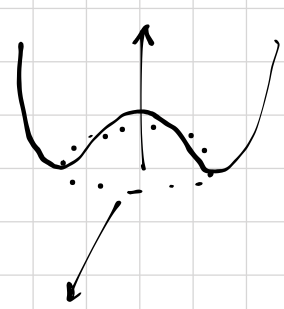
Build the action: ALGORITHM

1) Determine relevant d.o.f.
2) Action consistent with symmetries

⇒ Goldstone Theorem $G \xrightarrow{\text{broken}} H$:

T: unbroken generators

X: broken "



VACUUM MANIFOLD: $T|0\rangle = |0\rangle$

G/H coset space → equivalence class

↳ ex: show that cosets only form a group if H is a normal subgroup ($ghg^{-1} \in H$)

$$(gh)\cdot(g'h') = \underbrace{\underbrace{(ghg^{-1})}_{\in H}(gg')}_{\in G/H} h' = \tilde{g}h' \in G/H$$

$$(gh)^{-1} = h^{-1}g^{-1} = \overbrace{g^{-1}g}^{G} h^{-1}g^{-1} = g^{-1}\overbrace{(ghg^{-1})^{-1}}^{\in H} \in G/H$$

NB $V \to \infty$ limit $\exists$ superselection rule : $\langle \phi_1 | \mathcal{O} | \phi_2 \rangle = 0$ [you cannot pass between two points in the vacuum manifold]

locally is consistent → Goldstone bosons

→ Proof (relativistic): → every broken current couples to a massless particle

$q \to 0 \to e^{iq\cdot x} \to 1$ acting on θ functions

$$\lim_{q\to 0} \int d^dx \; q_\mu \, e^{iq\cdot x} \langle 0 | T[j^\mu(x)\phi(0)] | 0 \rangle = \lim_{q \to 0} \int d^dx\,dt \, e^{iq\cdot x} \, \partial_\mu \langle 0 | T\big(j^\mu(x)\,\phi(0)\big) | 0 \rangle =$$

→ Lorentz invariance

$$= \lim_{q\to 0} q_\mu V^\mu(q^2)$$

$$= \lim_{q\to 0} q_\mu \left(q^\mu F(q^2) \right) = q^2 F(q^2)$$

$$= \lim_{q\to 0} \int d^dx \; e^{iq\cdot x} \, \delta_{\mu 0} \, \langle 0 | [j^0(x), \phi(0)] | 0 \rangle =$$

$$= \delta_{\mu,0} \langle 0 | [Q, \phi] | 0 \rangle \neq 0 \sim \langle 0 | \delta\phi | 0 \rangle$$

then $\lim_{q\to 0} q^2 F(q^2) \neq 0 \Rightarrow F(q^2) \overset{q\to 0}{\sim} \frac{1}{q^2}$ → PARTICLE!

⇒ every broken current produces one Goldstone boson

⇒ NON RELATIVISTIC:

$$\langle 0 | [Q, \phi] | 0 \rangle \neq 0 = \langle 0 | Q\phi(0) - \phi(0) Q | 0 \rangle = \int d^3x \left[\langle 0 | j^0(x,t) \sum_n |n\rangle\langle n| \phi(0)|0\rangle - \dots \right]$$

⇒ Assuming translational invariance:

$$= \int d^3x \left[e^{i\vec{p}_n\cdot\vec{x} - iE_nt} \langle 0 | j^0(0) \sum_n |n\rangle\langle n| \phi(0) | 0 \rangle \dots \right] \neq 0$$

$$= (2\pi)^3 \left[\delta^3(p_n) \, e^{-iE_nt} \langle 0 | j^0(0) \sum_n |n\rangle\langle n| \phi(0) | 0 \rangle - \dots \right] \neq 0$$

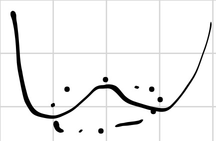
width ≪ energy when $E \to 0$

⇒ ∃ state s.t. $E \to 0$ when $p \to 0$ → can be anything (not necessarily a particle)

↳ e.g.: solid lattice:

$$\left.\begin{array}{l} 3 \text{ boosts} \\ 3 \text{ rotat.} \\ 3 \text{ transl} \end{array}\right\} \text{broken} \Rightarrow \text{only 3 Goldstones (phonons)}$$

⇒ how do we define an action if we don't know how many dofs there are?



broken generators

pions/Goldstone bosons

$$U = e^{i\vec{X}\cdot\vec{\pi}(x)} \in G \quad \text{(local fluctuation on vacuum manifold)}$$

$$\hookrightarrow \; gU(\pi) = U(\pi')\,\underbrace{h(\pi,g)}$$

H leaves everything invariant

$\Rightarrow$ $H \rightarrow \pi$ transform linearly $H: \pi \rightarrow L\pi$

$G \Rightarrow \pi$ " non-lin (eg $\pi \rightarrow \pi + a$)

ex show that this is true: $[T,T] \sim T$

$[T,X] \sim X$

show that this is true first.

$S - TUX$

$[S_i, S_j] = i\,\varepsilon_{ij}{}^{k} S_k$

$[T_a, X_p] - i f_{api} S_i \Rightarrow f_{api} = 0$ if $i = a,b,c.$ because $[T_a, T_b] = i f_{abc} T^c$

(for compact Lie group you can always chose a basis where struct const are totally antisymm.)

$\Rightarrow$ Maurer-Cartan 1-form: $U^{-1}\partial_\mu U = T_a A_{a\mu} + X_a D_\mu \pi^a$

under a G transf.: $*$ $X_a D_{\mu a} \rightarrow h(\pi,g)\, D_{a\mu} X^a\, h^{-1}(\pi,g) \Rightarrow$ transf. as if under H.

ex prove these

$**$: $T A_\mu^a \rightarrow h\, T_a A_{a\mu} h^{-1} + i(\partial_\mu h) h^{-1}$

How do I build an action for π?

$\Rightarrow$ LOCALITY: (ways to define)

* micro-causality: $(x-y)^2 < 0 \quad [O(x), O'(y)] = 0$
* action of finite no. of derivative coupling (∞ no. $\rightarrow$ couples everything $\rightarrow$ against micro-causality)
* GR equivalence: well defined derivative expansion (analytic for $q \rightarrow 0$)

$\hookrightarrow$ NO LOCALITY NO SCIENCE

$$L = (\partial_\mu\phi)^2 + c_1\phi^2 + c_2\phi^4 + \ldots + c_3(\partial_\mu\phi)^n + \ldots \Rightarrow \text{No predictive power } (\infty \text{ no. of terms})$$

$\Rightarrow$ every EFT MUST have some power counting scheme allows to truncate with well defined of "theoretical error"

eg $D=4 \rightarrow [\phi] = M \quad [S] = 0 \quad [O] = M^d \quad \left[\int c_O\, O\, d^4x\right] = 0 \rightarrow c_O = 4 - d$

Then $L = \sum_i \frac{c_i}{(\Lambda)^{d-4}} \mathcal{O}_i \rightarrow \Lambda$: symm. breaking scale

e.g.: $q \ll \Lambda \longrightarrow [\phi(x), \dot{\phi}(x')] \sim \delta^3(x-x') \sim q^3 \Rightarrow [\phi] = M$

① Show that:

- $e^{-A}\frac{d}{dx}e^{A} = \int_0^1 dy\; e^{-yA}\frac{dA}{dx}e^{yA}$
- $e^{A}Be^{-A} = B + [A,B] + \frac{1}{2}[A,[A,B]] + \dots$ s.t. $e^{-A}\frac{d}{dx}e^{A} = A' + \frac{1}{2}[A',A] + \frac{1}{3}[[A',A],A]$

② Construct COSET for $SO(N) \rightarrow SO(N-1)$ up to ∂^4

$\Rightarrow$ from $U^{-1}\partial_\mu U = X^a D_\mu^a(\pi) + T^a A_\mu^a \longrightarrow L = (D_\mu(\pi))^2 + \dots$

$\rightarrow$ suppose $\exists \psi$ transforms linearly in G: $\psi \rightarrow g\psi$

$\psi := U(\pi)\tilde{\psi} \longrightarrow g\,U(\pi)\tilde{\psi} = U(\pi')\,h(g,\pi)\,\tilde{\psi} = U(\pi')\,\tilde{\psi}'$

$\Rightarrow \tilde{\psi} \xrightarrow{G} \underbrace{h}_{H}\tilde{\psi}$

$\Rightarrow D_\mu\tilde{\psi} = (\partial_\mu\tilde{\psi} + A_\mu^a T^a \tilde{\psi}) \longrightarrow$ I can build invariants from this!

$\rightarrow$ SPACETIME SYMM.: non compact group!

$\hookrightarrow$ in this case $[T,T] \sim T$, $[T,X] \sim T + X$

$\Rightarrow U = e^{i\vec{\pi}\cdot\vec{X}}$

$\rightarrow$ Look at non relativistic cases $\longrightarrow$ Galileian symmetries

$\hookrightarrow$ sth in GS $\Rightarrow \exists$ a defect in spacetime:

point particle: $x^i(t) \rightarrow$ breaks boost inv. +3 transl.

string: $y^i(x,t) \longrightarrow$ breaks boost +2 transl + 1 rot...

membrane: $y^i(x^A,t)$

$U \longrightarrow e^{iHt - i\tilde{p}\cdot\vec{X}}\, e^{i\vec{p}\cdot\vec{y}(x,t)}\, e^{i\vec{\eta}\cdot\vec{K}}\, U \Rightarrow$

$\tilde{p}$: unbroken translations

p: broken transl

η: boost Goldstone $\rightarrow$ "framon"

Galileian alg.

$$[K_i, P_j] = i M \delta_{ij}$$
$$[L_i, L_j] = i \varepsilon_{ijk} L_k$$
$$[L_i, K_j] = i \varepsilon_{ijk} K_k$$
$$[H, K_i] = i P_i$$

⇒ COSET for NR p.p. :

broken :

P_i

K_i (boost)

unbroken :

L_i

M

coord / defect coord.

$$\Rightarrow \quad U = e^{iHt}\, e^{-i\vec{p}\cdot\vec{y}(t)}\, e^{i\vec{\eta}(t)\cdot\vec{K}}$$

einbein (NR: 1 ; R: γ)

$$\text{MC 1-form} \Rightarrow U^{-1}\partial_t U = i E\left(P_0 + D\pi^a P^a + D\eta^a K^a + M A_0\right) =$$

$$= e^{-i\vec{\eta}\cdot\vec{K}}\, e^{i\vec{y}(t)\cdot\vec{p}}\, e^{-iHt}\, \partial_t\left[e^{iHt}\, e^{-i\vec{p}\cdot\vec{y}(t)}\, e^{i\vec{\eta}\cdot\vec{K}}\right] =$$

$$= e^{-i\vec{\eta}\cdot\vec{K}}\, e^{i\vec{y}\cdot\vec{p}}\, e^{-iHt}\left((iH)[\dots] + e^{iHt}(-i\dot{\vec{y}}\cdot\vec{p})\dots + \dots(i\dot{\vec{\eta}}\cdot\vec{K})e^{i\vec{\eta}\cdot\vec{K}}\right)$$

$$= \underbrace{e^{-i\vec{\eta}\cdot\vec{K}}(iH)\, e^{i\vec{\eta}\cdot\vec{K}}}_{\text{boost transf for } H} + e^{-i\vec{\eta}\cdot\vec{K}}\left(-i\dot{\vec{y}}(t)\cdot\vec{p}\right)e^{i\vec{\eta}\cdot\vec{K}} + i\dot{\vec{\eta}}\cdot\vec{K} =$$

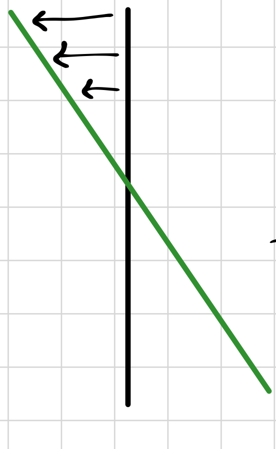
$$= i\left[H - \vec{p}\cdot\vec{\eta} + \frac{1}{2}M\vec{\eta}^2\right] - i\dot{\vec{y}}(t)\cdot\left(\vec{p} - M\vec{\eta}\right) + i\dot{\vec{\eta}}\cdot\vec{K}$$

$E = 1 \Rightarrow$

$$P_0 = H$$
$$A_0 = \frac{1}{2}\vec{\eta}^2 + \dot{\vec{y}}(t)\cdot\vec{\eta}$$
$$D\pi = -\left[\dot{\vec{y}} + \vec{\eta}\right]$$
$$D\eta^a = \dot{\eta}^a$$

⇒ η is a redundant Goldstone boson! ⇒ shouldn't be there!

Suppose a membrane:



→ I can rotate or locally translate!

↳ REVERSE HIGGS MECHANISM

$\Rightarrow [X, P] \sim X'$ (i.e.: [broken gen, unbroken transl] $\sim X'$)

Remove redundant Goldstone $\longrightarrow$ Impose constraint (compatible with symmetries)

$\hookrightarrow$ $D\pi = 0 \Rightarrow \dot{\vec{y}} = -\vec{\eta} \Rightarrow$ the boost is just the velocity of the particle

$\Rightarrow$ ACTION: we have $D\eta^a = -\ddot{y}^a$

then: $S = \int dt\, c_1 (D\eta^a)^2 = \int dt\, c_1 (\ddot{y}^a)^2$ $\Rightarrow$ which doesn't make sense!

NB: locality $\Rightarrow$ differential equation
stability $\Rightarrow$ at most II order derivative } arXiv: 1506.0221

$\Rightarrow (\ddot{y}^a)^2$ is fine as long as it is subleading $\longrightarrow$ OK for lenghts given by the size of the particle

However I'm allowed to consider

$\int c_2 A_0\, dt \longrightarrow$ invariant up to a total derivative $\longleftarrow$ example of Wess-Zumino term.

$$A_0 \propto \vec{\eta}^2 = \dot{\vec{y}}^2$$

$\Rightarrow \int [c_2 \dot{\vec{y}}^2 + c_1 (\ddot{y}^a)^2 + \dots] dt \longrightarrow$ "experimentally": $c_2 = \frac{1}{2} m$

$c_1 = ?$

$$[S] = E \cdot T = L\, M \frac{L}{T^2}\, T = \frac{ML^2}{T}$$

$$[(\ddot{y})^2 dt] = \frac{L^2}{T^4}\, T = \frac{L^2}{T^3}$$

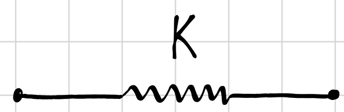
} $[c_1] = T^2 M$

$c_1 \sim \frac{R^2}{V_s^2} M$

R: radius of the particle; M: mass; V_s: speed of sound

$\Rightarrow$ everything breaks down when the perturbation becomes of the same order as the the LO.

2 interacting particles:



$+3$ boosts $\Rightarrow$ inverse Higgs

Goldstone modes: translation of the whole system + rotations $[3 \text{ transl} + 2 \text{ rot}]$ (3D)

1 is unbroken

Gap mode: $\omega = \sqrt{\frac{K}{m}}$ (contractions)

COSET CONSTRUCTION

Broken: $\vec{L}, \vec{T}, \vec{K}$

Unbroken: M, H

$$U = e^{iHt}\, e^{-i\vec{\pi}(t)\cdot\vec{P}}\, e^{i\vec{\eta}(t)\cdot\vec{K}}\, e^{i\vec{L}\cdot\vec{\theta}(t)}$$

$$MC: \quad U^{-1}\partial_t U = \left(H + D\pi^a P^a + D\eta^a K^a + D\theta^i L^i + M A_0\right)$$

$$\Rightarrow \quad D\pi^a = \eta^a + \dot{y}^a$$

$$A_0 = \vec{\eta}\cdot\dot{\vec{X}} + \frac{1}{2}\vec{\eta}^{\,2}$$

$$D\theta^i = -\dot{\theta}^i$$

$$D\eta^i = \dot{\eta}^i + \varepsilon^{ijk}\dot{\eta}^j\theta^k$$

$$D\pi = 0 \Rightarrow \dot{y}^a = -\eta^a$$

a) Using $g\,U(\pi) = U(\pi')\,h(\pi, g)$ determine how $\eta^i, \theta^i, y^i(t)$ transforms under boosts

b) Determine $D\eta^i$ to all orders in θ.

Then:

$$S = \int dt \left[\underbrace{c_1 A_0(t)}_{\text{WZ term}} + I^{ij}\dot{\theta}^i\dot{\theta}^j + B_i\left(D\eta^i = \ddot{x}^i + \ddot{x}^j\theta^k\varepsilon^i_{jk} + \ldots\right)\right]$$

Use e.o.m. to eliminate $\ddot{x}$ "order reduction":

$$S = \int dt\left(-\frac{1}{2}m\dot{x}^i\dot{x}^i + c\,\ddot{x}^i\,\theta^i(x)\right) \qquad x^i \to \frac{c}{m}\theta^i(x) + x^i$$

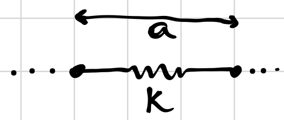
cancels $O(c)$ but generates $O(c^2)$

Write down first non vanishing term beyond canonical kinetic term

Multiple Oscillators

modes: $\omega = 0, \sqrt{\frac{K}{m}}, \sqrt{\frac{3K}{m}} \dots$

What happens if the chain is ∞? "Large N limit"



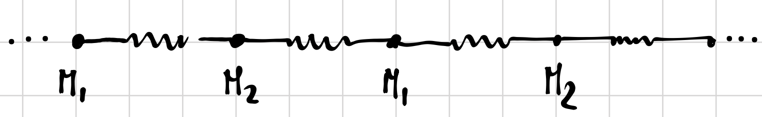
$$W = 2\sqrt{\frac{K}{m}} \sin\left(\frac{qa}{2}\right), \quad q = \frac{2\pi}{L} n \quad L = Na \quad n = 1, \dots, N$$

$\Rightarrow q \to 0 \Rightarrow \omega \sim q \Rightarrow$ waves! New gapless mode!

∞ chain $\Rightarrow$ translational symm.

spontaneously broken! $\to$ PHONONS

Consider a inhomogeneous lattice:



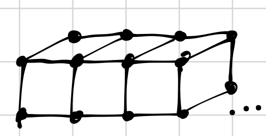
$$\Rightarrow \omega = \left(\frac{2k\mu}{m_1 m_2}\right)^{1/2} a q \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

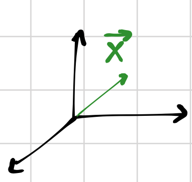
$\hookrightarrow$ I must look at it from larger scales $\Rightarrow$ coarse grain

Build coset for relativ. p.p. ($\varepsilon \neq 1$)

$\Rightarrow$ EFT of solids (1405.7384, 1712.07795) famous

$\rightsquigarrow$ Enhanced Symmetry: rotational and transl. INTERNAL symmetry





Internal Symmetry:

$\vec{R}$: int. rotations

$\vec{Q}$: int. transl.

$\vec{L}$: spacetime rotations

$\vec{P}$: " transl.

$\to \quad \Phi^I(\vec{x}, t)$ s.t. $\langle 0 | \Phi^J(x,t) | 0 \rangle = x^I = \delta^I_a x_a$

$\Rightarrow$ Symm. breaking pattern: $SO(3)_{int} \otimes SO(3)_{ST} \to SO(3)_{int+ST}$

$$T^3_{int} \otimes T^3_{ST} \longrightarrow T^3_{int+ST}$$

Effective Field Theory:

Broken: $Q_i, \vec{P}_i, L_i, R_i, K_i$

Unbroken: $H, M, L_i + R_i, Q_i + P_i$

$\rightarrow \quad U = e^{i\bar{P}\cdot x} e^{i\vec{K}\cdot\vec{\eta}} e^{i\vec{Q}\cdot\vec{\pi}} e^{i\vec{R}\cdot\theta}$

$y_i(t) \xrightarrow{\text{continuous}} \pi(x,t)$

where $\bar{P} = (H, Q^i + P^i)$

$\Rightarrow U^{-1}\partial_\mu U = E^A_{\ \mu}(\nabla_A \pi + \dots)$

$\hookrightarrow$ galileian boosts: $E_0^{\ 0} = 1$, $E_i^{\ j} = \delta_i^{\ j}$, $E_0^{\ i} = \eta^i$, $E_i^{\ 0} = 0$

$\Rightarrow A_0 = \frac{1}{2}\vec{\eta}$ $\qquad \nabla_0 \pi^i = \dot{\pi}^i - \eta^e(\partial^e \pi^i + \theta^j \varepsilon^{ije}) + \eta^i$ $\qquad \nabla_J \eta_I = \partial_J \eta_I$

$A_i = \eta_i$ $\qquad \nabla_j \pi_i = \partial_j \pi_i + \theta_k \varepsilon_{ijk}$ $\qquad \nabla_0 \theta_I = \dot{\theta}_I$

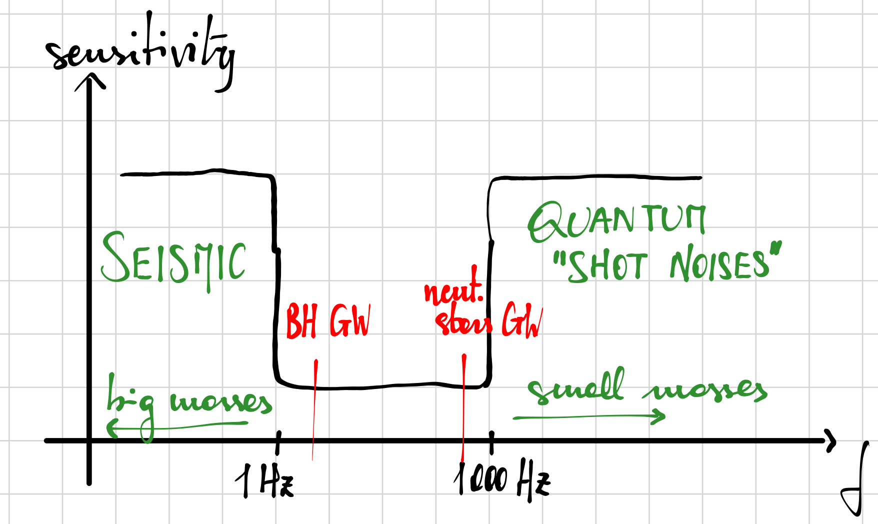
$\nabla_i \theta_j = \partial_i \theta_j$

$\rightarrow$ Inverse Higgs: $\nabla_0 \pi^i = 0 \longrightarrow$ solve for η

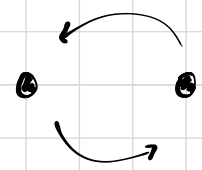
$\varepsilon_{ij} \nabla_j \pi_i = 0 \longrightarrow$ solve for θ

$\rightarrow$ only π remains

GRAVITY



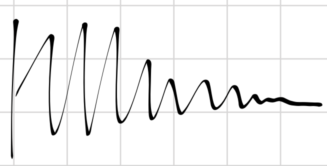
LIGO Bucket



$\Rightarrow$ no radiation: I.S.C.O.: inner stable orbit

radiation: collapse

@ ISCO $\leftarrow$ $f_{\text{plus}} \sim \frac{1}{M}$

Signal:  $\Rightarrow F(M_1, M_2, S_1, S_2, I)$ $\rightarrow$ internal struct.

$\rightarrow$ Use EFT:

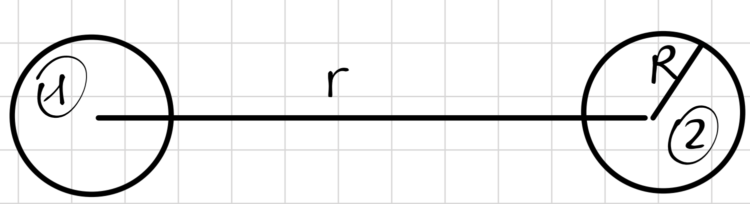
SCALES:

R: radius of compact object

r: radius of orbit $\qquad R \ll r \ll \lambda$

$\lambda \sim \frac{r}{v}$ $\qquad$ as long as $v \ll 1$ we expand in v.

EXPANSION PARAMETERS: $\frac{R}{r}$, $\frac{v}{c}$, $\frac{\hbar}{L}$ $\longrightarrow$ Feynman diag: so small we ignore loops



① $\underline{r \gg R}$: treat system as interacting p.p.

$$S = S_0 + S\left(\frac{R}{r}\right) \rightarrow \text{finite size effects}$$

$(i=1,2)$ $\qquad S_0 = -\sum_i m_i \int d\lambda_i \sqrt{\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\eta_{\mu\nu}} + q_i \int v^\mu A_\mu \, d\lambda_i$ $\qquad x^\mu(\tau)$: world line

<u>FINITE SIZE effects</u> (LO)

Sym: Lorentz, Reparam. inv. (RPI), Gauge inv.

$$v^\mu = \frac{dx^\mu}{d\lambda}$$

$$S_{FS} = \sum_i \int d\lambda_i \; F_{\mu\nu}F^{\mu\nu} C_i^1 \sqrt{v^2} + \sum_i \int d\lambda_i \left(v_i^\mu F_{\mu\nu} F^{\nu\rho} v_{i\rho}\right) \frac{C_i^2}{\sqrt{v_i^2}}$$

NB: $F \rightarrow \bar{F} + \delta F \Rightarrow C_i^1, C_i^2 :=$ EM polarizability

Ex

① EFT

① $S = \int C_1 F^2 \sqrt{V^2} + C_2 (V\cdot F)^2 / \sqrt{V^2}$ (see notes)

$\downarrow$ NR limit

$$\int C_E E^2 + C_B B^2 , \qquad A^\mu = A^\mu_B + \delta A^\mu$$

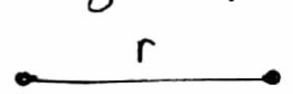
show that $\delta A^\mu(z) = -\frac{C_B}{2\pi}\frac{(\vec z \times \vec B)^i}{|\vec z|^3}\delta_\mu^i - \frac{C_E}{2\pi}\delta_\mu^0 \frac{z^i E_i}{|z|^3}$

$$\vec P = -2C_E \vec E \qquad C_E = -2\pi\alpha_E$$

$$\vec M = -2C_B \vec B \qquad C_B = -2\pi\alpha_B$$

e.g.: perfect conductor: $C_E = \pi R^3$, $C_B = \frac{\pi}{2}R^3$

Consider charged part



1) solve exactly
2) solve through EFT

$$Z[J] = e^{-iW[J]} = \int D\phi\, e^{-i(S+J\phi)} = e^{iE[J]t}$$

$$\mathcal{L} = -\frac{1}{4}F^2 + J\cdot A \qquad J^\mu = \sum_i q_i \int d\lambda\, v_i^\mu \cdot \delta(x^\mu - x_i^\mu(\lambda_i))$$

$\to W[J] = -\frac{i}{2}\int d^4x\, d^4y\, J^\mu(x) G_{\mu\nu}(x-y) J^\nu(y) = -iE[J]t$

$\Rightarrow$ NR limit (Feynman gauge) $\longrightarrow [d^4k] = \frac{d^4k}{(2\pi)^4}$

$$W = -\frac{i}{2}\int \frac{[d^4k]}{k^2+i\epsilon}\int dt\,dt' \Big[e^{ik\cdot(x_1(t)-x_2(t))} \cdot$$

$v^\mu = (1, \vec v)$

$$\cdot \Big\{ q_1 q_2 \big(1 - \vec v_1(t)\cdot \vec v_2(t')\big) + q_1 q_2 \big(1 - \vec v_1(t')\cdot\vec v_2(t)\big) e^{-ik\cdot(x_1(t')-x_2(t))}$$

$$+ \cancel{\text{self energy}} \Big]$$

STATIC LIMIT: $v \to 0$ term:

$$W = -\frac{i}{2}\int \frac{[d^4x]}{k^2+i\epsilon}\, dt\,dt'\, (q_1 q_2)\, e^{ik_0(t-t') - i\vec k\cdot(\vec x_1 - \vec x_2)}$$

indep of t in static limit

$$= -i\int \frac{dk_0}{2\pi}\frac{d^3k}{(2\pi)^3}\, dt\, (q_1 q_2)\, \frac{e^{-i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{k^{0\,2}-\vec{k}^2}\, \cancel{2\pi}\,\delta(-k_0)$$

$$= +it\int \frac{d^3k}{(2\pi)^3}\, q_1 q_2\, \frac{e^{-i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{k}^2} = \frac{i\, q_1 q_2\, t}{4\pi\,|\vec{x}_1-\vec{x}_2|} \Rightarrow$$

$$\Rightarrow \quad E[J] = \frac{q_1 q_2}{4\pi r}$$

NB: $O(v^2)$: $v \longrightarrow k^0 \to 0$

$\longrightarrow \sim v$ corrections

static limit: force is instantaneous!

$\longrightarrow$ moving particle: CORRECTIONS

$\Downarrow$

$$\frac{1}{k^{0\,2}-\vec{k}^2} \sim -\frac{1}{\vec{k}^2} + \frac{k^{0\,2}}{\vec{k}^4} + \dots$$

EX

$$V^{O(v^2)} = -\frac{q_1 q_2}{8\pi r}\left[\vec{V}_1\cdot\vec{V}_2 + \vec{V}_1\cdot\frac{(\vec{x}_1-\vec{x}_2)}{|x_1-x_2|}\; V_2\cdot\frac{(\vec{x}_1-\vec{x}_2)}{|x_1-x_2|}\right]$$

OK only for trivial theory

Now reproduce from EFT:

$E[J] = R + iI \longrightarrow ? \Rightarrow$ DECAY/RADIATION

$$\frac{1}{k^2+i\varepsilon} = P\frac{1}{k^2} + i\pi\,\delta(k^2)$$

$\Rightarrow k_0 = |\vec{k}| \Rightarrow$ ON SHELL $\Rightarrow$ RADIATES!

Consider

$$A_\mu = \bar{A}_\mu + A^p_\mu$$

radiation mode $k^0 \sim |\vec{k}|$

potential $k^0_p \ll |\vec{k}|$

$$\Rightarrow k^\mu_{pot} = \left(k^0 \sim \frac{1}{r} v^{n}, \; \vec{k} \sim \frac{1}{r}\right) \quad n=1$$

$$\Rightarrow k^\mu_{rad} = \left(\frac{v}{r}, \frac{\vec{v}}{r}\right)$$

$$L[A \to \bar{A} + A^p] = -\frac{1}{4}\left[F[\bar{A}+A_p]^2\right] + J\cdot(\bar{A}+A^p)$$

→ <u>Prime EFT Directive</u>: every term in action must scale homogeneously in power counting param.

- How do fields scale?
- How do deriv of fields scale?

$$\Rightarrow \langle A_p A_p \rangle = \int d^4k \, \frac{e^{ik\cdot(x_1-x_2)}}{\cancel{k_0^2} - \vec{k}^2} \Rightarrow A_p \sim v^{1/2}$$

$d^4k \to dk_0 \, d^3k \sim v$

$$\langle \bar{A}(x_2)\bar{A}(x_1)\rangle \sim \int \frac{d^4k}{k^2} e^{ik\cdot(x_1-x_2)} \Rightarrow \bar{A} \sim v$$

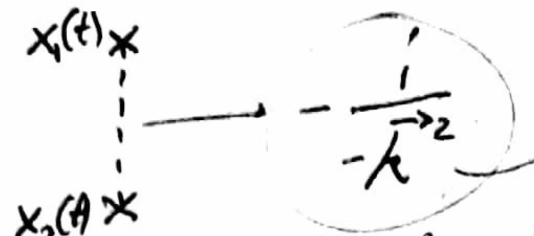
$d^4k \to v^4$, $k^2 \to v^2$

$$\to \overset{LO}{L}[\bar{A}+A^p] = \int d^4x \, \frac{1}{2} \bar{A}_\mu \Box \bar{A}^\mu + \int d^4x \, \frac{1}{2}\left(A^p_\mu (\cancel{\partial_0^2} - \vec{\partial}^2) A^{\mu p}\right) +$$

$\partial_0^2 \to O(v^2)$

$$+ \sum_i q_i \int dt_i \, A^p_0(x_i(t)) \cancel{+ v\bar{A}}$$

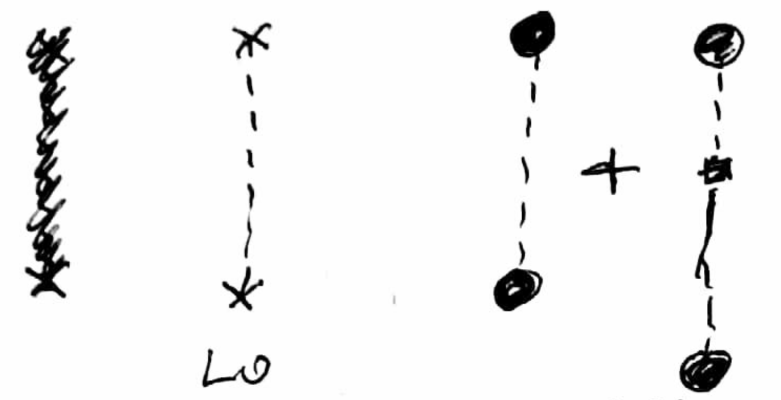
Now compute LO potential:



from $-\vec{\partial}^2$

Coulomb potential!

Now consider $L^{\mathcal{O}(v^2)} = \frac{1}{2}\int d^4x\, A^\mu_p\, \partial_0^2\, A_{\mu p} + \sum_i q_i \int \vec{v}\cdot\vec{A}_p(x_i(t))dt$



$\Rightarrow \int \frac{d^4k}{k^4}(k_0)^2 e^{ik(x_1-x_2)}$

We did everything for A_p, now?

$$\int \mathcal{D}\bar{A}\,\mathcal{D}A_p\, e^{iS[\bar{A},\bar{A}_p] + \int J(A+\bar{A}p)} =$$

$$= \int \mathcal{D}\bar{A}\; e^{iV(x_1-x_2)T + S[\bar{A}] + \int[T,\bar{A}]}$$

From  (Bound state: we eliminated short dist dofs!)

How does radiation couple to the BS:

$$\int J\cdot A = \int dt\left(q_1 \bar{A}(x_1)\cdot v_1 + q_2 \bar{A}(x_2)\cdot v_2\right)$$

→ we removed $x_1 - x_2$ from the syst → we ~~almost have to~~ want the theory of 1 single particle.

→ MULTIPOLE EXPANSION in CM frame

$$\int v^\mu \bar{A}_\mu = \int v^0 \bar{A}_0 - \vec{v}\cdot\bar{A}\, dt \sim \int v^0 \bar{A}^0(0,t) - v^0 \vec{x}\cdot\vec{\partial}\bar{A} - \vec{v}\cdot\bar{A} + \dots$$

NR limit: $v^0 = 1$:

(5) EFT

$$L[\bar{A}] = \sum_i q_i \int dt\, A^0(0,t) - \sum_i q_i \int dt\, \vec{x}_i \cdot \vec{\partial} A^0(0,t) - \sum_i q_i \int dt\, \vec{v}_i \cdot \vec{A}(0,t) + \cdots$$

$\sum q_i$

$$= \underbrace{Q \int A^0(0,t)\, dt}_{\text{MONOPOLE}} - \sum_i q_i \int dt \left[\vec{x}_i \cdot \vec{\partial} A^0(0,t) + \vec{x} \cdot \dot{\vec{A}}(0,t) \right]$$

$$= Q \int A^0\, dt + \underbrace{\int dt \left(q_1 \vec{x}_1 \cdot \vec{E} + q_2 \vec{x}_2 \cdot \vec{E} \right)}_{\int dt\, \vec{p}(t) \cdot \vec{E}} + \cdots$$

DIPOLE !

Power Loss: $\langle 0 | k \rangle_J = \times\!\!\sim\!\sim\!\sim$

$\partial_i A_0 - \partial_0 A_i$

$$\Rightarrow \sum_{pol} |M|^2 = \sum_{pol} |\times\!\!\sim\!\sim\!\sim|^2 =$$

$$= \sum_{pol} \left| \int dt\, \langle 0 | \vec{p}(t) \cdot \vec{E} | k \rangle \right|^2 = \sum_{pol} \int dt\, dt'\, p_i(t) \cdot p_j(t') \langle 0 | E_i(t) | k \rangle \cdot \langle 0 | E_j(t) | k \rangle$$

$\sum \varepsilon_\mu \varepsilon_\nu^* = -g_{\mu\nu}$

$$= \sum_{pol} \int dt\, dt'\, p_i(t) p_j(t')\, e^{ik_0 t} e^{-ik_0 t'} \left(k_0 \varepsilon_i - k_i \varepsilon_0 \right) \left(k_0 \varepsilon_j^* - k_j \varepsilon_0^* \right)$$

$$= -\int dt\, dt'\, p_i(t) p_j(t')\, e^{ik_0(t-t')} \left(-k_0^2 \delta_{ij} + k_i k_j \right)$$

$k_0 = k$

$$\Rightarrow \sum_{pol} \int \frac{d^3k}{(2\pi)^3 2k} |M|^2 = -\int dt\, dt'\, p_i(t) p_j(t') \int \frac{d^3k}{(2\pi)^3 2k} \left(\cancel{\delta_{ij}} k^2 + \frac{1}{3} \delta_{ij} \vec{k}^2 \right)$$

$$= \frac{2}{3} \int dt\, dt'\, p_i(t) p_j(t') \frac{4\pi}{(2\pi)^3} \frac{1}{2} \int_0 dk\, k^3$$

mult by γ energy

$$P = \frac{1}{T} \int_0^\infty dk\, k^4 \frac{1}{3} \frac{1}{2\pi^2}\, dt\, dt'\, e^{-ik(t-t')} \vec{p}(t) \cdot \vec{p}(t')$$

$$= \frac{1}{8}\frac{1}{2\pi^2}\int dt\, dt'\, \ddot{\vec{p}}(t)\cdot\ddot{\vec{p}}(t') \int_{-\infty}^{+\infty} dk\; e^{ik(t-t')}$$ ⑥ EFT

$$= \frac{1}{6\pi}\int dt\, \ddot{\vec{p}}(t)\cdot\ddot{\vec{p}}(t)$$

$$\Rightarrow \quad P_{avg} = \frac{1}{6\pi}\langle \ddot{p}\cdot\ddot{p}\rangle$$

Radiation reaction force

| 1405.5532 YM as coset |

↳ IN-IN formalism $\quad V\cdot A \quad V\cdot A$

→ causal propagator

nothing off shell $\quad$ (at LO : $G(x-y)$ instead of $G^F(x-y)$

$$\longrightarrow \quad iM = \int d\tau\, d\tau'\, (-q^2)\, V_\mu(\tau) V_\nu(\tau') \underbrace{\langle A^\mu(x(\tau))\, A^\nu(x(\tau'))\rangle}_{\frac{1}{2\pi}\delta((x(\tau)-x(\tau'))^2)\cdot\,\theta(x^0(\tau)-x^0(\tau'))}$$

$$\ddot{x}^\mu = \frac{e^2}{4\pi}\left[\frac{2}{3}\dddot{x}^\mu - \dot{x}^\mu(\ddot{x})^2\right] \quad \text{A.D.L.}$$

(as small perturbation))

BH and binary inspiral

- EFT of binary inspiral (NRGR: "non relativ. GR")

1) p.p. approx:

$$S = -m\int d\tau \; + \qquad d\tau = \sqrt{\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}g_{\mu\nu}}\, d\lambda$$

$$+ \int c_1\, R(g)\, d\tau + \int c_2 \frac{V_\mu V_\nu}{\sqrt{V^2}} R^{\mu\nu}(g)\, d\tau$$

everything with symm $\qquad$ they do not contribute: $R^{\mu\nu}=0$ in vacuum

→ LO finite size effect:

$$c_E\int E^2 d\tau + c_B\int B^2 d\tau \qquad C_{\rho\sigma\mu\nu}: \text{Weyl tensor}$$

where $\quad E_{\rho\mu} = C_{\rho\sigma\mu\nu}V^\sigma V^\nu \overset{\text{on-shell}}{\equiv} R_{\rho\sigma\mu\nu}V^\sigma V^\nu$

$$B_{\rho\mu} = \frac{1}{2} * C_{\rho\sigma\mu\nu}V^\sigma V^\nu \equiv \frac{1}{2} * R_{\rho\sigma\mu\nu}V^\sigma V^\nu$$

→ order v^{10} {

NB: $\quad * C_{\rho\sigma\mu\nu} = \varepsilon_{\rho\sigma\alpha\beta} C^{\alpha\beta}{}_{\mu\nu}$

$\Rightarrow$ v expans $\rightarrow$ post Newtonian expansion (PN)

$$v^{2m} \Rightarrow v^{10}: \ 5\text{-PN}$$

Power counting

$$m = M_1 \sim M_2\,, \quad r\,, \quad G_N \sim \frac{1}{M_{pl}^2}$$

$\rightarrow$ Expansion parameters: $\quad v,\ L^{-1}$

$\Rightarrow$ Virial theorem: $\quad \frac{1}{2} m v^2 - G\frac{m^2}{r} \Rightarrow v^2 \sim \frac{m}{r M_{pl}^2}$

$$L \sim mvr \Rightarrow vL \sim \frac{M^2}{M_{pl}^2}$$

$$d\tau \sim \frac{r}{v}$$

$$M d\tau - \frac{Mr}{v} \sim \frac{L}{v^2}$$

Therefore

LO action $\sim L$

$\Rightarrow \quad L[g] = L[\underbrace{\bar{h}}_{\text{radiation}} + \underbrace{H}_{\text{potential}}]$ $\rightarrow$ to be integrated out

$\rightarrow$ work in background field gauge:

$$S_{GF} = M_{pl}^2 \int d^4x \sqrt{\bar{h}}\, T^{\mu} T_{\mu}$$

$$T_{\mu} = D_{\alpha} H^{\alpha}{}_{\mu} - \frac{1}{2}\partial_{\mu} H^{\alpha}{}_{\alpha}$$

$\hookrightarrow \vec{\partial} H \sim \frac{1}{r}$

$\quad \partial_0 H \sim \frac{v}{r}$

$$\Rightarrow \langle H H \rangle \sim \int \frac{d^4k}{\vec{k}^2} \sim v \Rightarrow H \sim \frac{\sqrt{v}}{r}$$

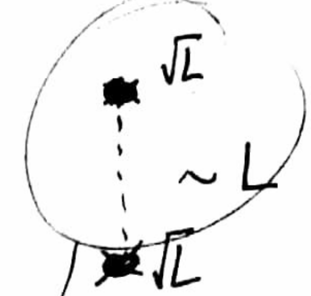
Coupling radiation - worldline: $g_{\mu\nu} = \eta_{\mu\nu} + \frac{H_{\mu\nu}}{M_{pl}}$

$$-M\,d\tau = M\left[1 - \vec{v}^2 + \frac{H_{00}}{M_{pl}} - 2\frac{H_{0i}v^i}{M_{pl}} + \frac{v^i v^j H_{ij}}{M_{pl}}\right]^{1/2}$$

$$\to L \doteq -\frac{M}{2M_{pl}} H_{00} - \frac{M}{M_{pl}} v^i H_{0i} - \frac{M}{4M_{pl}} v^2 H_{00} - \frac{M}{2M_{pl}} v^i v^j H_{ij} - \frac{M}{M_{pl}^2} H_{00}^2 + \dots$$

$$\hookrightarrow L^{\bullet} = -\frac{M}{2M_{pl}} H_{00} \sim \sqrt{vL}\,\frac{\sqrt{v}}{r}$$

$$\Rightarrow S^{\bullet} \sim \int L^{\bullet}\, d\tau \sim \sqrt{L}$$



potential: $iM = \left(-\frac{M_1}{2M_{pl}}\right)\left(-\frac{M_2}{2M_{pl}}\right) \cdot \int \frac{[d^4k]}{(-\vec{k}^2)} (-i)\, P_{00,00}\, e^{ik(x_1-\dots)} dt_1 dt_2$

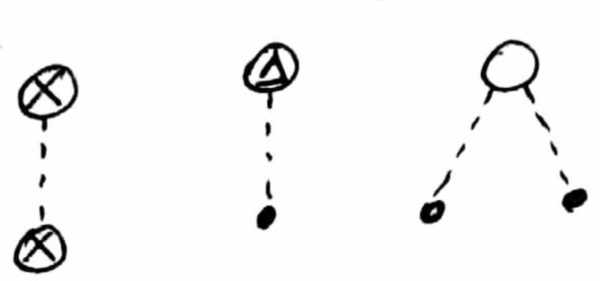
$$P_{\mu\nu;\alpha\beta} = \frac{1}{2}\left\{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \frac{2}{d-2}\eta_{\mu\nu}\eta_{\alpha\beta}\right]$$

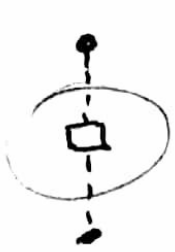
$\Rightarrow$ Newtons Law

$$\hookrightarrow L_1 = -\overbrace{\frac{M}{M_{pl}} v^i H_{0i}}^{\otimes}$$

$$\hookrightarrow L_2 = -\overbrace{\frac{M}{4M_{pl}} v^2 H_{00} - \frac{M}{2M_{pl}} v^i v^j H_{ij}}^{\triangle} \; \overbrace{- \frac{M}{8}\frac{H_{00}^2}{M_{pl}^2}}^{\circ}$$

$\to$ 1PN Potential (E.I.H.)





propagator correct $\frac{1}{k_0^2 - k^2} \to \frac{1}{-\vec{k}^2} + \frac{k_0^2}{\vec{k}^4}$

$$\hookrightarrow S = S_{pp} + S_{BULK} \longrightarrow M_{pl}^2 \int \sqrt{g}\, R\, d^4x$$

$\Rightarrow$ 3-gravitons: $\sim \int d^4x\, H^3 \partial^2 \sim \frac{v^2}{\sqrt{L}} \longrightarrow$ contributes at $\mathcal{O}(v^2)$

→ All in all the potential becomes:

$$V^{(1PN)}_{EIH} = - \frac{G M_1 M_2}{2r}\left[3\left(v_1^2+v_2^2\right) - 7\, v_1\cdot v_2 - \frac{(v_1\cdot r)(v_2\cdot r)}{r^2}\right] + G_N^2 \frac{M_1 M_2 (M_1+M_2)}{2r^2}$$

NB: matching calc lead to $\underline{C_E = C_B = 0}$ → symmetries???

GRAVITATIONAL RADIATION

$$L = V(r_1, v_1, v_2) + \int \sqrt{g}\, R(h) + \int \left[Q_{ab}E^{ab} - \frac{4}{3} J^{ab} B_{ab} + \dots\right]$$

mass quadrupole

current quadrupole

→ power loss formula: $P = \frac{G_N}{5}\langle \dddot{Q}\, \dddot{Q} \rangle$

→ Tree level:

$$-\frac{1}{2M_{pl}}\int d^4x\, T_{\mu\nu}\left[h^{\mu\nu}(0) + x^\rho \partial_\rho h^{\mu\nu}(0) + \frac{1}{2} x^\sigma x^\rho \partial_\sigma\partial_\rho h^{\mu\nu}(0) + \dots\right]$$

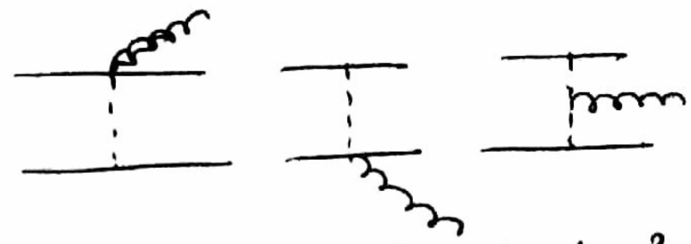
$T_{00} \sim v^0$ LO

$T_{0i} \sim v$

$T_{ij} \sim v^2$

$$\Rightarrow -\frac{1}{2M_{pl}}\int d^4x \left\{\left[\cancel{M h_{00} + 2P^i h_{0i}}\ \overset{M}{T^{00}} h_{00} + 2\overset{\overset{=0 \text{ in rest frame}}{P^i}}{T^{0i}} h_{0i}\right] - \frac{1}{2M_{pl}}\left[\partial_i h_{0k} - \partial_k h_{0i}\right]\underbrace{\left(T^{0i}x^k - T^{0k}x^i\right)}_{L^{ik}} + \dots\right\}$$

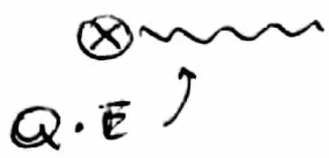
→ ~~They do not radiate~~ higher order terms



(5) EFT

$$M \to (M_1+M_2) + \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2 - G\frac{M_1 M_2}{r}$$

⇒ LO power:



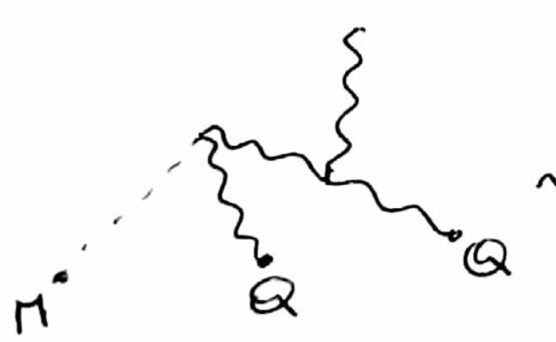
divergences at tree level!

Dimensional regularization: $\omega^2 G_N^2 m^2 \frac{107}{105}\left[\frac{1}{\varepsilon} - \ln\frac{\vec{k}^2}{\mu^2} + \dots\right]$

$$\Rightarrow \mu\frac{d}{d\mu} Q_{ij}(\mu,\omega) = -2(\omega G_N m)^2 \frac{107}{105} Q_{ij}$$

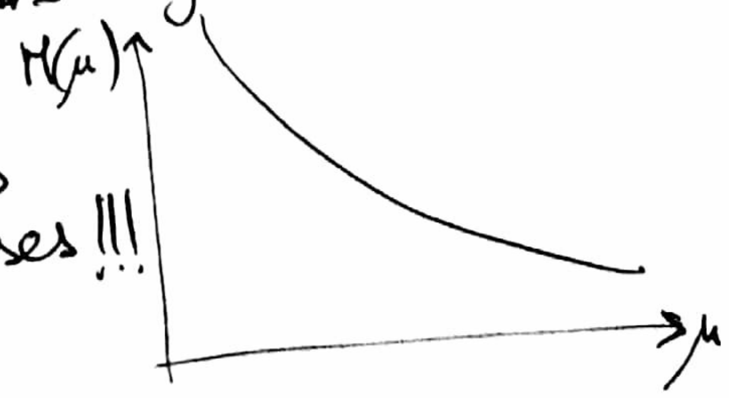
RG-function for quadrupole

$$\left(Q_{ij}(\mu,\omega) = \left(\frac{\mu}{\mu_0}\right)^{-\frac{214}{105}(\omega G_N m)^2} Q_{ij}(\mu_0,\omega) \right)$$



$\sim \frac{1}{UV}$ → MASS counterterms $M(\mu)$

→ it's asymptotically free!!!



⇒ A and B see ≠ masses!!!

(no Landau pole)