When we compactify a closed string:

$$X^\mu \longrightarrow (X^i, X^\eta) \qquad \begin{array}{l} i = 0, \dots, d-\ell-1 \Rightarrow d-\ell \text{ coord.} \\ \eta = d-\ell, \dots, d-1 \Rightarrow \ell \text{ coord.} \end{array}$$

then we require, on a torus T^ℓ:

$$X^\eta(\sigma, \tau) = X^\eta(\sigma+\pi, \tau) + 2\pi n R$$

$$\Rightarrow \quad p_L^\eta(m,n) = \frac{m}{R_\eta} + \frac{nR^\eta}{\alpha'}$$

$$p_R^\eta(m,n) = \frac{m}{R_\eta} - \frac{nR^\eta}{\alpha'}$$

Therefore we have:

$$L_0 = \frac{1}{2}\alpha_0^i \alpha_{0i} + \frac{1}{2}\alpha_0^\eta \alpha_{0\eta} + N - 1$$

$$= \frac{\alpha'}{4} p^2 + \frac{\alpha'}{4} p_L^2(m,n) + N - 1$$

$$\bar{L}_0 = \frac{\alpha'}{4} q^2 + \frac{\alpha'}{4} p_R^2(m,n) + \tilde{N} - 1$$

(NB: in this notation $L_0 \equiv :L_0:$)

$$\Rightarrow F = -\frac{1}{2}\operatorname{Tr}\ln(L_0) - \frac{1}{2}\operatorname{Tr}\ln(\bar{L}_0) =$$

$$= -\frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\operatorname{Im}\tau^{1+\frac{D-\ell}{2}}} \operatorname{Tr}\left(q^{L_0}\bar{q}^{\bar{L}_0}\right) =$$

$$= -\frac{1}{2}\int \frac{d^2\tau}{\operatorname{Im}\tau^{1+\frac{D-\ell}{2}}} \operatorname{tr}_L\left(q^{\frac{\alpha'}{4}\hat{p}^2 + \frac{\alpha'}{4}\hat{p}_L^2 + N - 1}\right) \operatorname{tr}_R\left(\bar{q}^{\frac{\alpha'}{4}\hat{q}^2 + \frac{\alpha'}{4}\hat{p}_R^2 + \tilde{N} - 1}\right)$$

$$= -\frac{1}{2}\int \frac{d^2\tau}{\operatorname{Im}\tau^{1+\frac{D-\ell}{2}}} \frac{1}{q\bar{q}} \int \frac{d^{D-\ell}p}{(2\pi)^{D-\ell}} \int \frac{d^{D-\ell}q}{(2\pi)^{D-\ell}}\, q^{\frac{\alpha'}{4}p^2} \bar{q}^{\frac{\alpha'}{4}q^2} \sum_{m,n} q^{\frac{\alpha'}{4}p_L^2} \bar{q}^{\frac{\alpha'}{4}p_R^2} \times$$

$$\times \left(\sum_{k=0}^{\infty} \langle k | q^{\sum_{n=0}^{\infty} n\alpha_n^{\dagger\hat{\mu}}\alpha_n^{\hat{\mu}}} | k \rangle\right)^{D-2} \left(\sum_{k'=0} \langle k' | \bar{q}^{\sum_{n'} n' \alpha_{n'}^{\dagger\hat{\mu}}\alpha_{n'}^{\hat{\mu}}} | k' \rangle\right)^{D-2}$$

$$= -\frac{1}{2}\int \frac{d^2\tau}{\operatorname{Im}\tau^{1+\frac{D-\ell}{2}}} \frac{1}{q\bar{q}} \left|\frac{\pi^2\alpha'}{2}\tau\right|^{-(D-\ell)} \left(\frac{1}{\prod_{n,n'}(1-q^n)(1-\bar{q}^{n'})}\right)^{D-2} \sum_{m,n} q^{\frac{\alpha'}{4}p_L^2} \bar{q}^{\frac{\alpha'}{4}p_R^2}$$

The additional factor

$$\sum_{m,n} q^{\frac{\alpha'}{4} p_L^2(m,n)} \bar{q}^{\frac{\alpha'}{4} p_R^2(m,n)}$$

is the Narain lattice.

However, on an ORBIFOLD such as $T^p/\mathbb{Z}_2$ we can also have:

$$X^\mu(\sigma,\tau) = -X^\mu(\sigma+\pi,\tau) + 2\pi n R$$

as a closed string (half-integer freq.). We are introducing a TWIST OPERATOR s.t.

$$\begin{aligned} g : \mathcal{H} &\longrightarrow \mathcal{H} \\ X &\longmapsto gX = -X \end{aligned}$$

If we want to keep only the g-even states then we include a projector

$$\frac{1+g}{2} \qquad (\text{since } g^2 = \mathrm{id})$$

in the trace:

$$F = -\frac{1}{2} \operatorname{Tr} \ln \left(\frac{1+g}{2} L_0 \bar{L}_0 \right) \qquad [g = g_L \oplus g_R]$$

s.t.

$$\begin{aligned} \operatorname{Tr}\left(g_L q^{L_0}\right) &\sim \sum_{k=0} \langle k | g_L q^{\sum_n n a_n^\dagger a_n} | k \rangle = \\ &= \prod_{n=1}^{\infty} \sum_{k=0}^{\infty} q^{nk} \langle k | g_L | k \rangle = \\ &= \prod_{n=1}^{\infty} \sum_{k=0}^{\infty} (-q^n)^k = \\ &= \prod_{n=1}^{\infty} \frac{1}{1+q^n} . \end{aligned}$$

! the sign !

However

$$F = -\frac{1}{2} \mathrm{Tr} \ln\left(\frac{1+g}{2} L_0 \bar{L}_0\right)$$

is NOT modular inv. w/o including the TWISTED SECTOR! That is we have to consider both:

$$X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$$

$$X^\mu(\sigma,\tau) = -X^\mu(\sigma+\pi,\tau)$$

$$\Downarrow$$

$$F = \underbrace{-\frac{1}{2} \mathrm{Tr}_U \ln\left(\frac{1+g}{2} L_0 \bar{L}_0\right)}_{\text{UNTWISTED SECTOR}} \underbrace{- \frac{1}{2} \mathrm{Tr}_T \ln\left(\frac{1+g}{2} L_0 \bar{L}_0\right)}_{\text{TWISTED SECTOR}}$$

The remaining states are:

$$\alpha^i_{-1} \tilde{\alpha}^j_{-1} |0,\tilde{0}\rangle$$
$$\alpha^\mu_{-1} \tilde{\alpha}^\nu_{-1} |0,\tilde{0}\rangle$$

We <u>projected out</u> $\alpha^i_{-1} \tilde{\alpha}^\mu_{-1} |0,\tilde{0}\rangle$ and $\alpha^\mu_{-1} \tilde{\alpha}^i_{-1} |0,\tilde{0}\rangle$.

Now we consider the compactification:

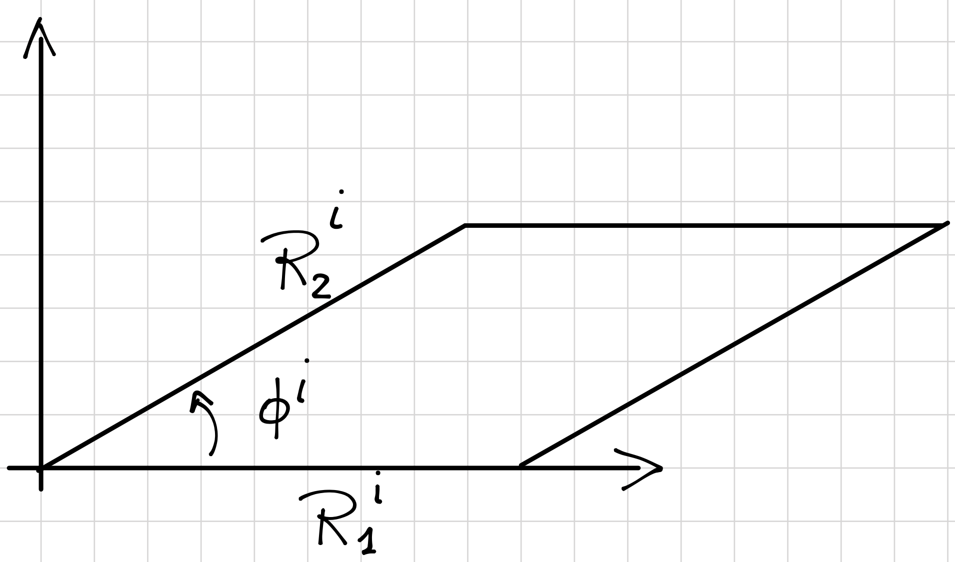
$$\mathcal{M}^{1,9} \longrightarrow \mathcal{M}^{1,3} \times \underbrace{K3}_{\text{CY 2-fold}} \times T^2$$

$$K3 \simeq T^4/\mathbb{Z}_N \quad \text{where } N = 2,3,4,6$$

We must define the action of the twist g.
Consider

$$X \in T^4 \longrightarrow (x_1, x_2) \quad \text{s.t. } x_1 \in T^2_{(1)}, \; x_2 \in T^2_{(2)}$$

and insert a COMPLEX STRUCTURE ON THE TORUS:



$$\Rightarrow \text{def.} \quad U_i = \cos\phi_i + i\,\frac{R_2^i}{R_1^i}$$

Then define the complex coord. ON THE TORUS:

$$\begin{aligned} z_1 &= x_1 + U_1 x_2 \\ z_2 &= x_2 + U_2 x_2 \end{aligned} \quad \longrightarrow \quad Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

s.t.

$$\begin{aligned} g : \; &\mathcal{H} \to \mathcal{H} \\ &Z \to g(Z) = R(v_k)\, Z, \quad k = 0, \dots, N-1 \end{aligned}$$

where $v_k = \left(\frac{k}{N}, -\frac{k}{N}\right)$ and $R(v_k) = \begin{pmatrix} e^{2\pi i \frac{k}{N}} & \\ & e^{-2\pi i \frac{k}{N}} \end{pmatrix}$.

Therefore

$$z_1 \xrightarrow{g} e^{2\pi i \frac{k}{N}} z_1 \,, \qquad k = 0, \dots, N-1$$

$$z_2 \xrightarrow{g} e^{-2\pi i \frac{k}{N}} z_2 \,, \qquad \text{''}$$

e.g.: $T^4/\mathbb{Z}_2$: $\quad z_j \xrightarrow{V_1} e^{(-)^{j+1} i\pi} z_j = - z_j$

Clearly the compactif. of $M^{1,9}$ to $M^{1,3} \times K^3 \times T^2$ leads to a splitting of the little group $SO(8)$ to

$$SO(8) \longrightarrow SO(4) \times \underbrace{SO(4)}_{\text{this is on } K3}$$

Consider Type IIB superstring th.:

$*$ Type IIB $\Rightarrow \mathcal{N} = (2,0) \Rightarrow 2 \times Q_L$

$\rightarrow Q_L$ transform in the 8_s of $SO(8)$

$$Q_L \longrightarrow \exp\left(2\pi i \textstyle\sum_{\mu\nu} \omega^{\mu\nu}\right) Q_L$$

where we define:

8_s : $\quad \left(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right)$ <u>even</u> no. of $+$

8_c : $\quad \left(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right)$ <u>odd</u> no. of $+$

In the splitting $M^{1,9} \longrightarrow M^{1,3} \times T^2 \times T^4/\mathbb{Z}_N$ we assign

$$\left(\zeta_1 ,\ \zeta_2 ,\ \underbrace{\eta_1 ,\ \eta_2}_{T^4} \right)$$

Suppose then to have a spinor λ in the helicity basis (e.g. the SUSY charges Q_L):

$$\lambda \longrightarrow \exp\left(i\pi\, \vec{v}_k \cdot \hat{\eta}\right) \lambda$$

where $\hat{\eta} = (\eta_1, \eta_2)$ is the weight vector on T^4. Since we are considering 8_S:

$$\frac{1}{2}(+,+,+,+) \Rightarrow \lambda \to \exp\left[i\pi\left(\frac{1}{2}\frac{k}{N} - \frac{1}{2}\frac{k}{N}\right)\right]\lambda = \lambda$$

$$\frac{1}{2}(+,+,-,-) \Rightarrow \lambda \to \exp\left[i\pi\left(-\frac{1}{2}\frac{k}{N} + \frac{1}{2}\frac{k}{N}\right)\right]\lambda = \lambda$$

$$\frac{1}{2}(+,-,+,-) \Rightarrow \lambda \to \exp\left[i\pi\left(\frac{1}{2}\frac{k}{N} + \frac{1}{2}\frac{k}{N}\right)\right]\lambda \neq \lambda$$

$$\frac{1}{2}(+,-,-,+) \Rightarrow \lambda \to \exp\left[i\pi\left(-\frac{1}{2}\frac{k}{N} - \frac{1}{2}\frac{k}{N}\right)\right]\lambda \neq \lambda$$

$\vdots$ (exchange all +'s into −'s)

$\Rightarrow$ Only $\frac{1}{2}$ of the supercharges are preserved. Therefore

Type IIB: * in $D=10$: $\mathcal{N}=(2,0)$ $\dim(R(Q))=16$

$\Rightarrow$ 32 SUSY charges

* on K3 ($D=4$):

$\Rightarrow$ 16 SUSY charges

$\boxed{\mathcal{N}=4}$

We now look at the surviving states when compactifying on $K3 \times T^2$:

$$SO(8) \longrightarrow SO(4) \times SO(4)$$

that is:

$$\begin{aligned} 8_v &\longrightarrow (4,1) \oplus (1,4) \\ 8_s &\longrightarrow (2_s, 2_s) \oplus (2_c, 2_c) \end{aligned}$$

From the point of view of the characters this becomes

$$\begin{aligned} V_8 &\longrightarrow V_4 O_4 + O_4 V_4 \\ S_8 &\longrightarrow S_4 S_4 + C_4 C_4 \end{aligned}$$

Since we have

$$\text{Type IIB}: \; F \sim |V_8 - S_8|^2$$

then we find:

"the second char. is $\in T^4$"

$$V_8 - S_8 = V_4 O_4 + O_4 V_4 - S_4 S_4 - C_4 C_4$$

$$\Big\downarrow g$$

$$V_4 O_4 - O_4 V_4 - S_4 S_4 + C_4 C_4$$

Now call

$$\begin{aligned} Q_o &= V_4 O_4 - S_4 S_4 \Rightarrow g(Q_o) = + Q_o \\ Q_v &= O_4 V_4 - C_4 C_4 \Rightarrow g(Q_v) = - Q_v \end{aligned}$$

which are part of the partition function:

$$F \sim \frac{1}{2}\Big\{ |Q_o + Q_v|^2 \sum_{n,m} q^{\frac{\alpha'}{4} p_L^2} \bar{q}^{\frac{\alpha'}{4} p_R^2} + |Q_o - Q_v|^2 \left|\frac{2\eta}{\theta_2}\right|^4 + \underbrace{}_{T^2}$$

to ensure modular inv. $\Longrightarrow$

$$+ |Q_s + Q_c|^2 \left|\frac{2\eta}{\theta_4}\right|^2 + |Q_s - Q_c|^2 \left|\frac{2\eta}{\theta_4}\right|^2 \Big\} \sum_{n,m} q^{\frac{\alpha'}{4} \ell_L^2} \bar{q}^{\frac{\alpha'}{4} \ell_R^2}$$

Suppose now to expand F for $\tau \to \infty$ $\Rightarrow$ take only the massless level. Since

$$\left|\frac{2\eta}{\theta_2}\right|^2 \sim 1 + \ldots$$

then

$$F \sim \frac{1}{2}\left\{ |Q_o + Q_v|^2 + |Q_o - Q_v|^2 \right\} \sum_{m,n} \Lambda_{m,n} =$$

$$= \left\{ |Q_o|^2 + |Q_v|^2 \right\} \sum_{m,n} \Lambda_{mn} .$$

That is

$$Q_o \bar{Q}_o = V_4 O_4 \overline{V_4 O_4} + S_4 S_4 \overline{S_4 S_4} - V_4 O_4 \overline{S_4 S_4} - S_4 S_4 \overline{V_4 O_4}$$

$$Q_v \bar{Q}_v = O_4 V_4 \overline{O_4 V_4} + C_4 C_4 \overline{C_4 C_4} - O_4 V_4 \overline{C_4 C_4} - C_4 C_4 \overline{O_4 V_4}$$

In the first char. (on the left, e.g. $V_4 O_4 \overline{V_4 O_4}$) we read the states we have to consider, while in the 2nd char. we read the multiplicity (NB: $SO(4) \simeq SU(2) \times SU(2)$):

- $V_4 O_4 \bar{V}_4 \bar{O}_4$:

$$V_4 \bar{V}_4 : \quad (2,2) \otimes (2,2) = \underset{\downarrow \; g_{\mu\nu}}{(3,3)} \oplus \underbrace{(3,1) \oplus (1,3)}_{\downarrow \; B_{\mu\nu}} \oplus \underset{\downarrow \; \phi}{(1,1)}$$

$$O_4 \bar{O}_4 : \quad (1,1) \otimes (1,1) = (1,1) \Rightarrow 1 \text{ dof}$$

$$\Rightarrow 1 \times (g_{\mu\nu}, B_{\mu\nu}, \varphi)$$

- $S_4 S_4 \overline{S_4 S_4}$:

$$S_4\overline{S}_4 : \quad (2,1)\otimes(2,1) = \underset{\substack{\downarrow \\ B^+_{MN}}}{(3,1)} \oplus \underset{\substack{\downarrow \\ \varphi}}{(1,1)}$$

$$B^+_{MN} = \frac{1}{2}\,\varepsilon_{MNPQ}\,B_+^{PQ}$$

$$S_4\overline{S}_4 : \quad \frac{4\cdot 3}{2}\cdot\frac{1}{2} + 1 = 4 \text{ dof}$$

$$\Rightarrow \quad 4 \times (B^+_{MN}, \varphi)$$

- $V_4 O_4 \overline{S_4 S_4}$:

$$V_4\overline{S}_4 : \quad (2,2)\otimes(2,1) = \underset{\substack{\downarrow \\ \psi^\mu_L}}{(3,2)} \oplus \underset{\substack{\downarrow \\ \lambda_R}}{(1,2)}$$

$$O_4\overline{S}_4 : \quad (1,1)\otimes(2,1) = (2,1) \Rightarrow 1 \text{ dof}$$

$$\Rightarrow \quad 1 \times (\psi^\mu_L, \lambda_R)$$

- $S_4 S_4 \overline{V_4}\,\overline{O_4}$:

$$S_4\overline{V}_4 : \quad (2,1)\otimes(2,2) = \underset{\substack{\downarrow \\ \psi^\mu_L}}{(3,2)} \oplus \underset{\substack{\downarrow \\ \lambda_R}}{(1,2)}$$

$$S_4\overline{O}_4 : \quad (2,1)\otimes(1,1) = (2,1) \Rightarrow 1 \text{ dof}$$

$$\Rightarrow \quad 1 \times (\psi^\mu_L, \lambda_R)$$

Therefore :

$$Q_o\overline{Q}_o \Rightarrow \quad g_{\mu\nu},\ B_{\mu\nu},\ \varphi,\ 4B^+_{\mu\nu},\ 4\varphi,$$
$$2\psi^\mu_L,\ 2\lambda_R$$

Now we move to $Q_V \overline{Q_V}$:

- $O_4 V_4 \overline{O_4 V_4}$:

$$O_4 \overline{O}_4 : \quad (1,1) \otimes (1,1) = \underset{\varphi}{\underset{\downarrow}{(1,1)}}$$

$$V_4 \overline{V}_4 : \quad (2,2) \otimes (2,2) = (3,3) \oplus (3,1) \oplus (1,3) \oplus (1,1)$$

$$\Rightarrow \underbrace{\left(\frac{5 \cdot 4}{2} - 1\right)}_{\text{symm. traceless}} + \frac{4 \cdot 3}{2} + 1 = 10 + 6 = 16 \text{ dof}$$

$$\Rightarrow \quad 16 \times \varphi$$

- $C_4 C_4 \overline{C_4 C_4}$:

$$C_4 \overline{C}_4 : \quad (1,2) \otimes (1,2) = \underset{B^-_{MN}}{\underset{\downarrow}{(1,3)}} \oplus \underset{\varphi}{\underset{\downarrow}{(1,1)}}$$

$$B^-_{MN} = -\frac{1}{2} \varepsilon_{MNPQ} B_-^{PQ}$$

$$C_4 \overline{C}_4 : \quad \frac{4 \cdot 3}{2} \cdot \frac{1}{2} + 1 = 4 \text{ dof}$$

$$\Rightarrow \quad 4 \times (B^-_{MN}, \varphi)$$

- $O_4 V_4 \overline{C_4 C_4}$:

$$O_4 \overline{C}_4 : \quad (1,1) \otimes (1,2) = \underset{\lambda_R}{\underset{\downarrow}{(1,2)}}$$

$$V_4 \overline{C}_4 : \quad (2,2) \otimes (1,2) = (2,3) \oplus (2,1) \longrightarrow 4 \text{ dof}$$

$$\Rightarrow 4 \times \lambda_R$$

• $C_4 C_4 \overline{O_4 V_4}$:

$$C_4 \overline{O}_4 : \quad (1,2) \otimes (1,1) = \underset{\displaystyle \lambda_R}{\underset{\downarrow}{(1,2)}}$$

$$C_4 \overline{V}_4 \longrightarrow 4 \text{ dof}$$

$$\Rightarrow \quad 4 \times \lambda_R$$

Therefore

$$Q_v \overline{Q}_v : \quad 16\ \varphi\ , \quad 4\, B^-_{MN}\ , \quad 4\ \varphi\ , \quad 8\ \lambda_R$$

Eventually we have that what we found fills a $D=6$ $\mathcal{N}=(2,0)$ SUGRA + tensor mult.:

$$1 \times \text{ SUGRA}: \quad (g_{MN},\ 5\, B^+_{MN},\ 2\, \psi^\mu_L)$$

$$5 \times \text{ TENSOR}: \quad (B^-_{MN},\ 5\, \varphi,\ 2\, \lambda_R)$$

NB: $B_{MN} = B^+_{MN} + B^-_{MN}$.

This is equivalent to Type IIB on $K3 \times T^2 \simeq T^4/\mathbb{Z}_2 \times T^2$ where we keep only the twist-even states. E.g.:

BOSONS:

$$g_{\mu\nu} \longrightarrow g_{MN} \quad \cancel{g_{Mi}} \quad g_{ij}\ {}_{10}$$

$$B_{\mu\nu} \longrightarrow \underset{B^+_{MN} + B^-_{MN}}{B_{MN}} \quad \cancel{B_{Mi}} \quad B_{ij}\ {}_{6} \qquad \Rightarrow 1 \times g_{MN}$$

$$\Rightarrow 5\, B^+_{MN} + 5\, B^-_{MN}$$

$$\phi \longrightarrow \phi\ {}_1 \qquad \Rightarrow 10 + 6 + 1 + 1 + 6 + 1 = 25 \text{ scalars}$$

$$C_0 \longrightarrow C_0\ {}_1 \qquad \text{since } C_{\mu\nu\rho\sigma} = \tfrac{1}{2} \varepsilon_{\mu\nu\rho\sigma} C \rightarrow 1 \text{ scalar}$$

$$C_2 \longrightarrow \overset{B^+_{MN} + B^-_{MN}}{C_{MN}} \quad \cancel{C_{Mi}} \quad C_{ij}\ {}_6$$

$$C^+_4 \longrightarrow C^+_{MNPQ} \quad \cancel{C^+_{MNPi}} \quad C^{+\ +6 \to 3B^+_{MN} + 3B^-_{MN}}_{MNij} \quad \cancel{C^+_{Mijk}} \quad C^+_{ijkl}\ {}^{1}$$

Up to now we considered $K3 \times T^2 = \left(T^4/\mathbb{Z}_2\right) \times T^2$.

Now instead of considering

$$\underbrace{(\rho_1, \phi_1 ; \rho_2, \phi_2 ;}_{T^4} \underbrace{y^i}_{T^2}) \longrightarrow (\rho_1, \phi_1 ; \rho_2, \phi_2 ; y^i + 2\pi R^i)$$
$$\searrow (\rho_1, \phi_1 + 2\pi \tfrac{k}{N} ; \rho_2, \phi_2 - 2\pi \tfrac{k}{N} ; y^i)$$

separately, we want to consider a FREELY ACTING ORBIFOLD (no fixed points):

$$(\rho_1, \phi_1 ; \rho_2, \phi_2 ; y^i) \longrightarrow (\rho_1, \phi_1 + 2\pi \tfrac{k}{N} ; \rho_2, \phi_2 - 2\pi \tfrac{k}{N} ; y^i + 2\pi R^i)$$

[part of Ω - background]

The shift modifies then the structure of the partition function:

$$F \sim \frac{1}{2} \Big\{ |Q_o + Q_v|^2 \, \Lambda^{(4,4)} \sum_{m,n} \Lambda_{mn} +$$
$$+ |Q_o - Q_v|^2 \left|\frac{2\eta}{\theta_2}\right|^4 \sum_{m,n} (-)^m \Lambda_{mn} +$$
$$+ |Q_s + Q_c|^2 \left|\frac{2\eta}{\theta_4}\right|^4 \sum_{m,n} \Lambda_{m,n+\frac{1}{2}} +$$
$$+ |Q_s - Q_c|^2 \left|\frac{2\eta}{\theta_3}\right|^4 \sum_{m,n} (-)^m \Lambda_{m,n+\frac{1}{2}} \Big\}$$

S

T

At the lowest mass levels we can now group

$$\frac{1}{2}\Big\{ |Q_o + Q_v|^2 \sum_{m,n} \Lambda_{m,n} + |Q_o - Q_v|^2 \sum_{m,n} (-)^m \Lambda_{m,n} \Big\} =$$

$$= \frac{1}{2}\Big\{ |Q_o + Q_v|^2 + |Q_o - Q_v|^2 \Big\} \sum_{m,n} \Lambda_{2m,n} + \frac{1}{2}\Big\{ |Q_o + Q_v|^2 - |Q_o - Q_v|^2 \Big\} \sum_{m,n} \Lambda_{2m+1,n}$$

$$= \underbrace{\left(Q_o \bar{Q}_o + Q_v \bar{Q}_v\right)}_{1 \times \text{SUGRA} + 5 \times \text{tensor}} \sum_{m,n} \Lambda_{2m,n} + \underbrace{\left(Q_o \bar{Q}_v + Q_v \bar{Q}_o\right)}_{\boxed{M^2 \sim \frac{1}{R^2}} \ (g_{\mu i}, B_{\mu i}, \dots)} \sum_{m,n} \Lambda_{2m+1,n}$$