LACES 2018

SUPERSTRINGS

[R. Russo]

Build a theory s.t.:

i) is consistent with QM $\begin{cases}\text{positive def. } \mathcal{H} \\ \text{special relativity}\end{cases}$

ii) fundam. dof extend in one spacial dim $(p=1)$

Why do we stop at $p=1$?

$\Rightarrow$ diff. to write QM-consistent theory

$\Rightarrow$ ST has also extended obj in NP spectrum.

• Pros:

$\rightarrow$ PERT. SPECTRUM @ $m^2=0$ $\left.\begin{cases}\text{spin } 1 \\ \text{spin } 2\end{cases}\right\}$ + inter. (gauge + Einstein)

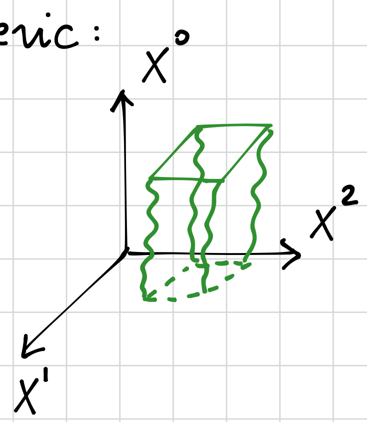
$\rightarrow$ CHIRAL MATTER

$\rightarrow$ SUPERSYMMETRY

$\rightarrow$ Deep relation with math + inspiration for other works $\begin{cases}\text{AdS/CFT} \\ \text{QFT ampl.}\end{cases}$

CLASSICAL THEORY

Take p generic:



$X^M(\xi^a)$ $\quad \xi^a \quad a=0,1,\dots,p;\ M=0,\dots,D-1$

$(p=0 \longrightarrow$ pointlike particles$)$

$\Rightarrow$ choose $SO(1,D-1)$ for SR consistency

$\Rightarrow$ DYNAMICS:

$$p=0: \quad \mathcal{L} \sim \sqrt{\dot{x}^\mu \eta_{\mu\nu} \dot{x}^\nu}$$

$$p \text{ gen.}: \quad S \sim \int d^{p+1}\xi \sqrt{|\det G_{ab}|} \qquad G_{ab} = \partial_a X^M \eta_{MN} \partial_b X^N$$

"induced metric"

Finding e.o.m. gets diff. Then consider:

$$X^M(\xi^a) \qquad h_{ab}(\xi)$$

worldsheet scalars

$$S \sim \int d^{p+1}\xi \left[\partial_a X^M \partial_b X_M h^{ab} - \underbrace{c_p \Lambda_p}_{\sim \text{ cosm. const.}} \right] \sqrt{h} \quad \sim \text{grav. th. on worldsh.}$$

EOM from δh_{ab}:

$$\delta\sqrt{|h|} = -\frac{1}{2}\sqrt{|h|}\,\delta h^{ab} h_{ab} \Rightarrow \sqrt{h}\left[\partial_a X^M \partial_b X_M - \frac{1}{2} h_{ab}\left(\partial_c X^M \partial^c X_M - c_p\Lambda_p\right)\right] = 0$$

$$\Rightarrow \text{take the trace} \quad \partial_a X^M \partial^a X_M - \frac{p+1}{2}\partial_a X^M \partial^a X_M + \frac{p+1}{2} c_p \Lambda_p = 0$$

$$\Leftrightarrow \frac{1-p}{2}\partial_a X^M \partial^a X_M + \frac{p+1}{2} c_p \Lambda_p = 0$$

NB: $p = 1 \Rightarrow \boxed{C_1 = 0} \Rightarrow$ no need for cosm. const.

in general $C_p \propto p-1$

w/o Λ_p there's more symm. to quantize the theory:

$$p=1 \Rightarrow \det\left[\partial_a X^M \partial_b X_M = \frac{1}{2} h_{ab}\,\partial_c X^M \partial^c X_M\right]$$

$$\Leftrightarrow \det G_{ab} = \frac{1}{4} h \left(\partial_c X^M \partial^c X_M\right)^2 \Leftrightarrow \sqrt{\det G_{ab}} = \frac{1}{2}\sqrt{|h|}\left(\partial_c X^M \partial^c X_M\right)$$

(Nambu-Goto $\sim$ Polyakov)

$$\Rightarrow S \sim \int d^2\xi \left(\partial_a X^M \partial_b X_M h^{ab}\right)\sqrt{|h|}$$

List symm:
- GLOBAL : Poincaré $\left(X'^M = \Lambda^M{}_N X^N + a^M\right)$
- LOCAL : diff. on worldvolume $\left(\xi^a \to \xi^a + v^a(\xi)\right)$

$$\delta X^M = -v^a \partial_a X^M$$

$$\delta h_{ab} = -\left(D_a v_b + D_b v_a\right) =$$

$$= -\left(v^c \partial_c h_{ab} + \partial_a v^c h_{cb} + \partial_b v^c h_{ac}\right)$$

$\exists$ also Weyl symmetry: $h_{ab} \to h'_{ab} = e^{2\omega} h_{ab}$

↳ since EOM for h_{ab} is purely algebraic, we can take $\omega = \omega(\xi)$ (LOCAL)

NB we have h_{00}, h_{01}, h_{11} and 3 local gener. of symm. $\Rightarrow$ GAUGE FIXING

$\to$ choose $h_{ab} = \eta_{ab} \Rightarrow S \sim \int d^2\xi \left[-\partial_0 X^\mu \partial_0 X_\mu + \partial_1 X^\mu \partial_1 X^\mu \right]$

$\Rightarrow$ free scalars in 2D.

EOM from δX^μ: $\quad \partial_\tau^2 X^\mu - \partial_\sigma^2 X^\mu = 0$

What about boundary terms:

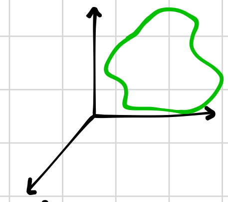
- from $\partial_0 X^\mu \partial_0 X_\mu \longrightarrow$ since $\delta X^\mu = 0$ at τ_{in} and τ_{fin} then we don't need to worry

- from $\partial_1 X^\mu \partial_1 X_\mu \longrightarrow \delta X^\mu \neq 0$ at $\sigma_{in}, \sigma_{fin}$

$\Downarrow$

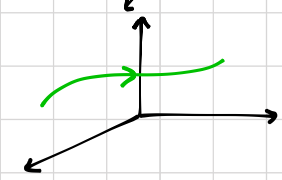
b.c.: $\quad -(\partial_\sigma X^\mu)\, \delta X_\mu \big|_{\sigma_{in}} + (\partial_\sigma X^\mu)\, \delta X_\mu \big|_{\sigma_{fin}} = 0$

i) $X^\mu(\tau, \sigma_{fin}) = X^\mu(\tau, \sigma_{in}) \quad \forall \tau$ (CLOSED STRING)



endpoints travel at speed of light

ii) $\partial_\sigma X^\mu = 0$ at σ_{in} and σ_{fin} (NEUMANN b.c.)



(OPEN STRING)

iii) $X^\mu \big|_{\sigma_{in}} = C^\mu_{in}$; $X^\mu \big|_{\sigma_{fin}} = C^\mu_{fin}$

endpoints are fixed

(DIRICHLET b.c.)

NB: we don't like D b.c. especially on X^0!

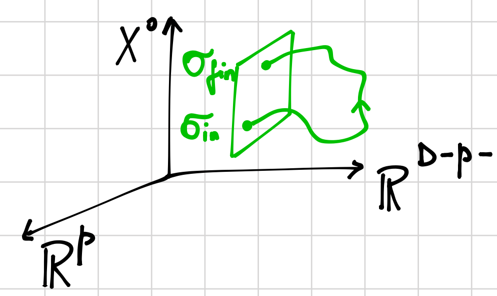
↳ They naturally with Dp-branes → we mix N and D:

$$\partial_\sigma X^\mu \big|_{\sigma_{in},\sigma_{fin}} = 0 \qquad \mu = 0, 1, \dots, p$$

$$X^i \big|_{\sigma_{in},\sigma_{fin}} = c^i \qquad i = p+1, \dots, D-1$$

$$\Downarrow$$

$$\cancel{SO(1, D-1)} \longrightarrow SO(1,p) \times SO(D-(p+1))$$



NB: Open String $\Rightarrow \exists$ D-brane background (D-branes are NOT fundam. obj.)

Therefore:

i) Solve $\partial_\tau^2 X^\eta - \partial_\sigma^2 X^\eta = 0$

$$\partial_a X^\eta \partial_b X_\eta - \frac{1}{2}\eta_{ab}\, \partial_c X^\eta \partial^c X_\eta = 0 \longrightarrow \text{only 2 non-trivial constr.}$$

$$\Rightarrow \quad \partial_\sigma X^\eta \partial_\tau X_\eta = 0$$

$$\partial_\tau X^\eta \partial_\tau X_\eta + \partial_\sigma X^\eta \partial_\sigma X^\eta = 0$$

Choose $\sigma_{in} = 0$, $\sigma_{fin} = 2\pi$ for Closed Strings

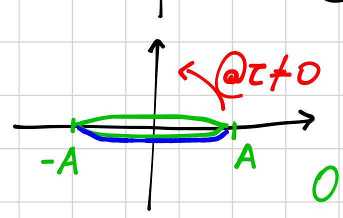
$\sigma_{in} = 0$, $\sigma_{fin} = 2\pi$ for Open Strings

↳ e.g.: $X^0 = A\tau$

$X^1 + iX^2 = A\cos(\sigma)\, e^{i\tau}$

$X^3 = \dots = X^{D-1} = 0$ ⇒ take a snapshot @ $\tau = 0$:

is a sol. of the prev. eqns.



$0 \le \sigma \le 2\pi \rightarrow$ CST

$0 \le \sigma \le \pi \rightarrow$ OST

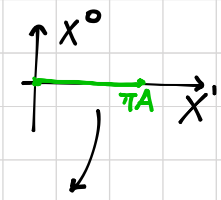
$$\partial_\sigma X^\eta = 0 \Rightarrow (\partial_\tau X^\eta)^2 = 0 = u^2 \rightarrow \text{NULL VEC}$$

$$\Rightarrow \text{SPEED OF LIGHT}$$

What is the coeff in front of the action?

$$S_{GF} = -\frac{T_{F1}}{2}\int d\tau d\sigma \left(\partial_a X^\mu \partial_b X_\mu \eta^{ab}\right) \quad \text{where } T_{F1} \text{ is the STRING TENSION}$$

Use $\underset{(N)}{X^0 = A\tau}$, $\underset{(D)}{X^1 = A\sigma}$, $X^{\mu>1} = 0$ $\rightarrow$



$T_{F1} \sim$ "rest mass energy"

Symm.: $X^0 \longrightarrow X^0 + \text{const.}$

current $\Rightarrow \dfrac{\delta S}{\delta\, \partial X^0} = -T_{F1}\, \partial^a X_0 \rightarrow a=0$ is the charge

$$\frac{\text{Tot. En.}}{\text{Tot. len.}} = \frac{T_{F1}\int_0^\pi d\sigma\, \partial^0 X^0}{A\pi} = T_{F1} \quad \text{(energy density of a static string)}$$

Now take $M^{\mu\nu} = T_{F1}\int d\sigma \underbrace{\left[X^\mu \partial_\tau X^\nu - X^\nu \partial_\tau X^\mu\right]}_{\text{angular mom. dens.}}$

$\Downarrow$

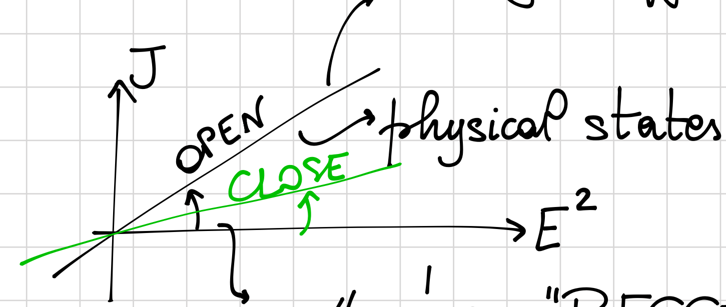
$$M^{12} = J = T_{F1}\frac{\pi}{2}A^2 \quad \text{(open)}$$
$$= T_{F1} A^2 \pi \quad \text{(closed)}$$

$$E = T_{F1}\pi A \quad \text{(open)}$$
$$= T_{F1}\, 2\pi A \quad \text{(closed)}$$

$$\Rightarrow J = E^2 \frac{1}{T_{F1}\, 2\pi} \quad \text{(open)}$$
$$= E^2 \frac{1}{T_{F1}\, 4\pi} \quad \text{(closed)}$$

$\Longrightarrow$

"leading Regge trajectory"



$\alpha' = \dfrac{1}{T_{F1}\, 2\pi}$ "REGGE SLOPE"

$\frac{\alpha'}{2}$ for closed strings

$$\Rightarrow J = \alpha' E^2 \quad \text{(open)}$$
$$= \frac{\alpha'}{2} E^2 \quad \text{(closed)}$$

Introduce $\quad \sigma^{\pm} = \tau \pm \sigma \qquad \partial_{\pm} = \frac{1}{2}\left(\partial_\tau \pm \partial_\sigma\right)$

$\Rightarrow$ e.om. $\quad \partial_+ \partial_- X^\mu = 0 \;\Rightarrow\; X^\mu(\tau,\sigma) = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-)$

$\Rightarrow$ b.c.:

- CLOSED: periodic both in σ^+ and σ^- $\left[\sigma^{\pm} \to \sigma^{\pm} \pm 2\pi\right]$

$$X_L^\mu(\sigma^+) = q_L^\mu - i\sqrt{\frac{\alpha'}{2}}\,\alpha_0^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\alpha_n^\mu}{n} e^{-in\sigma^+}$$

$$X_R^\mu(\sigma^-) = q_R^\mu - i\sqrt{\frac{\alpha'}{2}}\,\tilde{\alpha}_0^\mu \sigma^- + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\tilde{\alpha}_n^\mu}{n} e^{-in\sigma^-}$$

$\downarrow$

b.c. $\Rightarrow \quad \alpha_0 = \tilde{\alpha}_0$

$$X^\mu(\sigma^+,\sigma^-) = q^\mu - i\sqrt{\frac{\alpha'}{2}}\,\alpha_0 \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\left(\alpha_n^\mu e^{-2in\sigma^+} + \tilde{\alpha}_n^\mu e^{-2in\sigma^-}\right)$$

- OPEN: $\quad$ N@$\sigma=0 \;\Rightarrow\; X'^\mu\big|_{\sigma=0} = 0 \;\Rightarrow\; \left[\partial_+ X^\mu = \partial_- X^\mu\right]_{\sigma=0}$

$\Rightarrow \quad \alpha_n = \tilde{\alpha}_n$

D@$\sigma=0 \;\Rightarrow\; X^\mu\big|_{\sigma=0} = C^\mu \;\Rightarrow\; \dot{X}^\mu\big|_{\sigma=0} = 0 \;\Rightarrow\; \partial_+ X = -\partial_- X$

$\Rightarrow \quad \alpha_n = -\tilde{\alpha}_n$

$\longrightarrow$ We can introd. the REFLECTION MATRIX: $\quad \partial_+ X^\mu = R_{(0)}{}^\mu{}_N\, \partial_- X^N$ $\quad$ @$\sigma=0$

$$R_{(0)} = \begin{pmatrix} \overbrace{+ \;\ddots\; +}^{N} & \\ & \underbrace{- \;\ddots\; -}_{D} \end{pmatrix}$$

Then the same arg. must hold at $\sigma = \pi$. For instance:

NN: $\quad \partial_+ X_L(\tau) = \partial_- X_R(\tau)$

$\quad\quad \partial_+ X_L(\tau+\pi) = \partial_- X_R(\tau-\pi) = \partial_+ X_L(\tau-\pi) \;\Rightarrow$ periodicity of 2π

$\left[\partial_+ X_L(\tau+2\pi) = \partial_+ X_L(\tau)\right]$

DD: $\quad \partial_+ X_L(\tau) = -\partial_- X_R(\tau)$

$\quad\quad \partial_+ X_L(\tau+\pi) = -\partial_- X_R(\tau-\pi) = \partial_+ X_L(\tau-\pi) \;\Rightarrow$ Same periodicity

ex: DN and ND

NB: $\alpha_0 \longrightarrow$ linked to momentum: $p^\mu = \int d\sigma P^\mu = T_{F1} \int d\sigma \dot{X}^\mu = \begin{cases} \frac{\alpha_0^\mu}{\sqrt{2\alpha'}} & \text{OPEN} \\ \sqrt{\frac{2}{\alpha'}}\, \alpha_0^\mu & \text{CLOSED} \end{cases}$

OLD COVARIANT QUANTIZATION

$$[X^\mu(\tau,\sigma), P^N(\tau,\sigma')] = i\,\delta(\sigma-\sigma')\,\eta^{\mu N} \Rightarrow \begin{aligned} &[\alpha_m^M, \alpha_n^N] = n\,\delta_{n+m,0}\,\eta^{\mu N} \\ &[\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^N] = n\,\delta_{n+m,0}\,\eta^{\mu N} \\ &[\alpha_m^\mu, \tilde{\alpha}_n^N] = 0 \end{aligned}$$

NB: $[\alpha_m^0, \alpha_n^0] = -n\,\delta_{n,-m} \Rightarrow$ Negative norm Fock space:

$$\alpha_n^\mu |0\rangle = 0 \quad \forall n > 0$$

Then $\alpha_{-n_1}^{N_1} \alpha_{-n_2}^{N_1} \dots |0\rangle$ are states.

$$\alpha_0^\mu |0\rangle \Rightarrow \| \alpha_{-1}^0 |0\rangle \|^2 = \langle 0 | \alpha_1^0 \alpha_{-1}^0 | 0 \rangle = -1 < 0 \quad !!!$$

$$\longrightarrow \mathcal{H}_{FIS} \subset \mathcal{H}_{TOT}$$

$\Rightarrow$ GUPTA-BLEUER QUANT.:

$$\begin{cases} \partial_\tau X^\mu \partial_\sigma X_\mu = 0 \\ \partial_\tau X^\mu \partial_\tau X_\mu + \partial_\sigma X^\mu \partial_\sigma X_\mu = 0 \end{cases} \Leftrightarrow \begin{cases} \partial_+ X^\mu \partial_+ X_\mu = 0 \\ \partial_- X^\mu \partial_- X_\mu = 0 \end{cases}$$

$\hookrightarrow \langle phys | \partial_\pm X^\mu \partial_\pm X_\mu | phys \rangle = 0 \qquad [\langle ph | \partial_\mu A^\mu | ph \rangle = 0]$

"No GHOST THEOREM"

2D CONFORMAL FIELD THEORY

$$S = -\frac{1}{2\pi\alpha'}\int d\sigma^+ d\sigma^- \, \partial_+ X \, \partial_- X$$

Wick rot $\Rightarrow \tau = -i\tau_E \longrightarrow \sigma^{\pm} = \tau \pm \sigma = -i\tau_E \pm \sigma = -i(\tau_E \pm i\sigma) = -i\overset{(-)}{w}$

$$w = \tau_E + i\sigma \qquad \bar{w} = \tau_E - i\sigma = w^*$$

$$\Rightarrow S = \frac{1}{2\pi\alpha'}\int d^2w \, \partial_w X \, \partial_{\bar{w}} X$$

* Symmetries:
- $w \rightarrow w + \text{const}$
- $w \rightarrow e^{i\theta} w$
- $w' = f(w)$ and $\bar{w}' = g(\bar{w})$ $\longrightarrow$ MANY MORE SYMM. w.r.t. POINCARÉ

$\Rightarrow$ NB:

$$\left.\begin{matrix} \eta_{\tau_E\tau_E} = \eta_{\sigma\sigma} = 1 \\ \eta_{\tau_E\sigma} = \eta_{\sigma\tau_E} = 0 \end{matrix}\right\} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \longrightarrow \left.\begin{matrix} \eta_{ww} = \eta_{\bar{w}\bar{w}} = 0 \\ \eta_{w\bar{w}} = \eta_{\bar{w}w} = \frac{1}{2} \end{matrix}\right\} \begin{pmatrix} & 1/2 \\ 1/2 & \end{pmatrix}$$

$$d^2\tau = d^2\tau_E + d^2\sigma = \frac{1}{2} dw d\bar{w} \longrightarrow dw' d\bar{w}' = \underbrace{\partial f \, \bar{\partial} g}_{\Omega(w,\bar{w})} \, dw d\bar{w}$$

$\Omega(w,\bar{w})$: overall factor

$\downarrow$

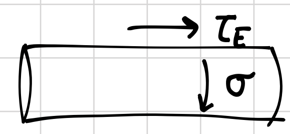
CONFORMAL TRANSF. $\longleftarrow$ lenghts change but angles are the same

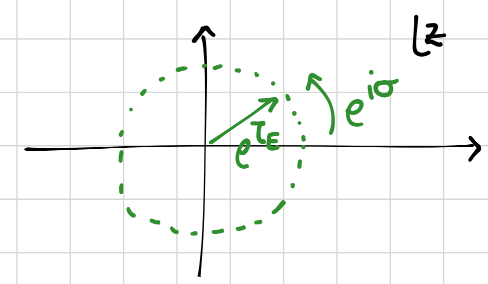
(2D Conf. group is ∞ dimensional)

NB: in string theory the conf. invar. is linked to the local inv. of the ungauged action $\longrightarrow$ we don't want it to break.

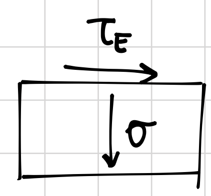
Now consider:

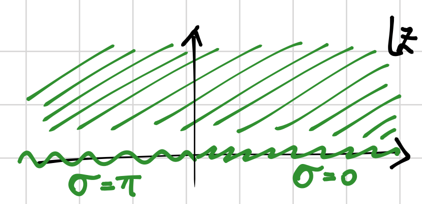
$$z = e^{w}$$
$$\bar{z} = e^{\bar{w}}$$

Worldvolume: CLOSED: $w \rightarrow$ 

$$z = e^{\tau_E + i\sigma} \longrightarrow$$ 

$\Rightarrow$ "RADIAL ORDERING" instead of time ordering

OPEN: $w \longrightarrow$ 

$$z = e^{w} \longrightarrow$$ 

Field transf.:

$$\partial_w X = \frac{\partial z}{\partial w} \partial_z X = z\, \partial_z X \longrightarrow (1, 0)$$

In general:

$$O'(z', \bar{z}') = \left(\frac{\partial z}{\partial z'}\right)^{h} \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} O(z, \bar{z}) \rightarrow \text{"PRIMARY FIELD" of weight } (h, \bar{h})$$

[if only $z' = \dfrac{az + b}{cz + d}$ $(ad - bc = 1)$ then "QUASI-PRIMARY FIELD"]

NB: $\exists$ 1:1 map between local op. and states:

$$\lim_{z \to 0} \partial_z X(z) |0\rangle \quad \text{where} \quad X_L(z) = q_L - i\sqrt{\frac{\alpha'}{2}}\, \alpha_0 \ln z + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{\alpha_n}{n} z^{-n}$$

$$\Rightarrow \lim_{z \to 0} \left(-i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \alpha_n z^{-n-1}\right) |0\rangle = \qquad (\alpha_n |0\rangle \ \forall n > -1)$$

$$= -i\sqrt{\frac{\alpha'}{2}}\, \alpha_{-1} |0\rangle \longrightarrow \underset{\text{OPER.}}{\partial_z X(z)} \longleftrightarrow \underset{\text{STATE}}{\alpha_{-1}|0\rangle}$$

+ the opposite direction.

In CFT: time ordering $\longleftrightarrow$ RADIAL ORDERING

$$[\alpha_n, O] = \alpha_n O - O \alpha_n$$

where $\alpha_n = \oint \frac{dz}{2\pi i} \partial_z X(z)\, z^n \left(i\sqrt{\frac{2}{\alpha'}}\right) \Rightarrow \oint_{|z_1|>|z_2|} \frac{dz_1}{2\pi i} \partial_{z_1} X(z_1)\, z_1^n\, O(z_2, \bar{z}_2) - \oint_{|z_1|<|z_2|} \frac{dz_1}{2\pi i} O(z_2,\bar{z}_2)\, \partial_{z_1} X(z_1)\, z_1^n$

$$= \oint_{z_2} \frac{dz}{2\pi i} \underbrace{\partial_z X(z)\, z^n\, O(z_2, \bar{z}_2)}$$

It entirely depends on the singularities (poles)!

$$\Rightarrow \quad \partial X(z_1)\, \partial X(z_2) \sim \; :\partial X \partial X(z_2): - \frac{\alpha'}{2} \frac{1}{(z_1 - z_2)^2} + \text{finite}$$

"OPE"

Then we have: $T(z) = -\frac{1}{\alpha'} \partial X\, \partial X \qquad \bar{T} = -\frac{1}{\alpha'} \bar{\partial} X\, \bar{\partial} X$

$$\Rightarrow T(z_1)\, \partial X(z_2) = -\frac{1}{\alpha'} \partial X(z_1) \partial X(z_1)\, \partial X(z_2) = -\frac{1}{\alpha'}\, 2 \left(-\frac{\alpha'}{2} \frac{1}{(z_1 - z_2)^2}\right) \partial X(z_1) =$$

$$= \frac{1}{(z_1 - z_2)^2} \partial_{z_1} X(z_1) \sim \frac{\partial_{z_2} X}{(z_1 - z_2)^2} + \frac{1}{z_1 - z_2} \partial(\partial X(z_2)) + \ldots$$

WEIGHT

What about

$$T(z)T(w) = -\frac{1}{(\alpha')^2} \partial X(z) \partial X(z)\, \partial X(w) \partial X(w) = -\frac{2}{(\alpha')^2} \left(-\frac{\alpha'}{2}\right)^2 \frac{1}{(z - w)^4} + \ldots =$$

$$= \frac{1/2}{(z - w)^4} + \ldots = \frac{c}{2} \Rightarrow \text{ANOMALY.}$$

THIS IS NOT A TENSOR!

In general:

$$T(z_1)\,T(z_2) \sim \frac{c/2}{(z_1-z_2)^2} + \frac{2T(z_1)}{(z_1-z_2)^2} + \frac{\partial T(z_1)}{z_1-z_2} \Rightarrow c = 1$$

$$\hookrightarrow T(z) = \sum_n L_n z^{-n-2} \qquad \left[\text{general: } O(z) = \sum_n O_n z^{-n-h}\right]$$

NB: everything holds for the antiholomorphic part: $\overline{T}(\bar{z}) = \sum_n \overline{L}_n \bar{z}^{-n-2}$

CST: $T(z)$ and $\overline{T}(\bar{z})$ are indep

OST: $L_n = \overline{L}_n \Rightarrow T(z) = \overline{T}(\bar{z})$

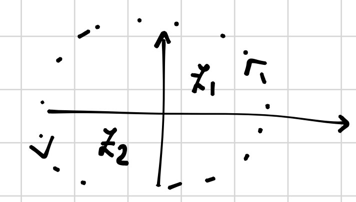
$\Rightarrow$ L_n generates $z \longrightarrow z' = z + \varepsilon z^{n+1}$

$$\Rightarrow O'(z',\bar{z}) = \left(\frac{\partial z}{\partial z'}\right)^h O(z,\bar{z})$$

$$\hookrightarrow [L_n, O] \overset{?}{=} \delta O = O'(z') - O(z)$$

$$\oint_z \frac{dz_1}{2\pi i} \underbrace{T(z_1)\,O(z)}_{OPE} z_1^{n+1} = \delta O(z)$$

Then
$$[L_n, L_m] = \oint_0 \frac{dz_2}{2\pi i} \oint_{z_2} \frac{dz_1}{2\pi i}\, T(z_1)\, z_1^{n+1}\, T(z_2)\, z_2^{m+1} =$$



$$= (n-m)\,L_{n+m} + \frac{n(n^2-1)}{12}\, c\, \delta_{n+m,0}$$

$\Downarrow$

VIRASORO ALGEBRA

$\exists \{L_1, L_0, L_{-1}\} \subset \{L_n\}_n$ s.t. there is NO ANOMALY.

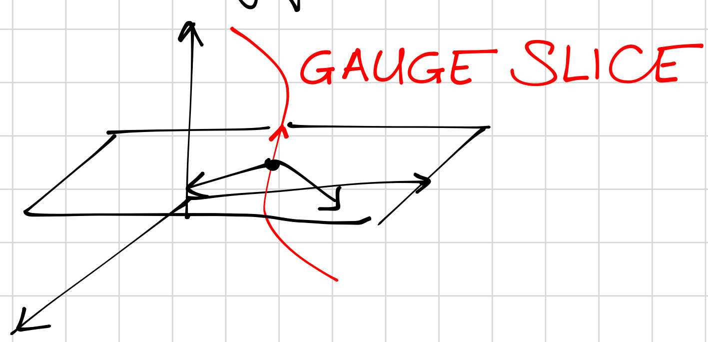
At finite
$$T'(z') = \left(\frac{dz}{dz'}\right)^2 \left[T(z) + \frac{c}{12} S(z,z')\right]$$

$$S(z,z') = -2\left(\frac{dz}{dz'}\right)^{\frac{1}{2}} \frac{d^2}{dz'^2}\left(\frac{dz}{dz'}\right)^{-\frac{1}{2}}$$

COVARIANT QUANTIZATION

$$\int [\mathcal{D}X^\mu][\mathcal{D}h_{ab}]\ e^{-\frac{1}{4\pi\alpha'}\int d^2\sigma\,(\partial_a X^\mu \partial_b X_\mu h^{ab}\sqrt{h})}$$

$\hookrightarrow$ we must find a GAUGE SLICE s.t. all the X and h configurations are inequivalent!



The measure is not invar. under Weyl inv. $\longrightarrow$ can I recover sth. which is?

We have:

$$\delta h_{ab} = D_a \delta v_b + D_b \delta v_a + 2\delta\omega\, h_{ab} =$$
$$= \left(D_a \delta v_b + D_b \delta v_a - h_{ab} D^c \delta v_c\right) + \left(2\delta\omega + D^c \delta v_c\right) h_{ab}$$

We shall insert

$$1 = \int [\mathcal{D}\delta v][\mathcal{D}(2\delta\omega)]\ \delta(h - \hat{h}) \left| \det \frac{\partial(\delta h_{ab})}{\partial(\delta v, \delta\omega)} \right|$$

$$\downarrow$$

$$\int [\mathcal{D}X^\mu][\mathcal{D}h_{ab}][\mathcal{D}\delta v][\mathcal{D}(2\delta\omega)]\ \delta(h - \hat{h}) \left| \det \frac{\partial(\delta h_{ab})}{\partial(\delta v, \delta\omega)} \right| e^{-\frac{1}{4\pi\alpha'}\int d^2\sigma\,(\partial_a X^\mu \partial_b X_\mu h^{ab}\sqrt{h})}$$

Be careful though:

$$(P\delta v)_{ab} = D_a \delta v_b + D_b \delta v_a - h_{ab} D^c \delta v_c$$

$$(2\delta\tilde{\omega})_{\delta v} = 2\delta\omega + D^c \delta\omega_c$$

$\hookrightarrow$ the det becomes:

$$\begin{vmatrix} \frac{\partial \delta P}{\partial \delta v} & \cancel{\frac{\partial \delta P}{\partial \delta \omega}} \to 0 \\ \frac{\partial 2\delta\tilde{\omega}}{\partial \delta v} & \underbrace{\frac{\partial 2\delta\tilde{\omega}}{\partial \delta\omega}}_{=1} \end{vmatrix} = \det\left(\frac{\partial \delta P}{\partial \delta v}\right) = \int [\mathcal{D}b_{ab}][\mathcal{D}c^a]\, e^{\frac{1}{2\pi}\int d^2\sigma\, bPc}$$

Then the exponential is

$$S_{gh} = \frac{1}{2\pi}\int d^2\sigma \sqrt{h}\; h^{ac}\, b_{ab} D_c c^b$$

NB $\frac{\partial \delta P}{\partial \delta v}$ does not look like a Square matrix...

- The object in S_{gh} actually computes

$$\sqrt{\det(P^\dagger P)}$$

- Can $\det(*) = 0$? Yes if P has at least one 0 eigenvalues!

We're actually computing

$$\sqrt{\det'(P^\dagger P)} = \sqrt{\prod_n \lambda_n}$$ → NON ZERO EIGENVAL.

(0 eigenv. must be treated separately)

↳ We end up with

reference metric: $\hat{h} = \eta$

$$\int [\mathcal{D}\delta v\, \mathcal{D}2\delta\omega\, \mathcal{D}b\, \mathcal{D}c\, \mathcal{D}h]\, \delta(h-\eta) \exp\left[-\frac{1}{4\pi\alpha'}\int d^2\sigma\, \partial_a X^\mu \partial_b X_\mu h^{ab}\sqrt{h} + \frac{1}{2\pi}\int d^2\sigma \sqrt{h}\, h^{ac} b_{ab} D_c c^b\right]$$

Now we have to find the e.o.m. for h_{ab} ($c_a = h_{ab} c^b$). We imposed the traceless condition $b_{ab} h^{ab} = 0$ but we can implement it better:

$$S_{gh} = \frac{1}{2\pi}\int d^2\sigma \sqrt{h}\, b_{ab}\left[h^{ac} D_c c^b - \frac{1}{2} h^{ab} D_c c^c\right]$$

↳ equivalent but b_{ab} is arbitrary!

Then $$\frac{\delta S_{gh}}{\delta h^{ab}} = -(\underbrace{2\, b_{zz}\, \partial c^z + \partial b_{zz}\, c^z}_{T_{gh}(z)}) - (\underbrace{2\, b_{\bar z\bar z}\, \bar\partial c^{\bar z} + \bar\partial b_{\bar z\bar z}\, c^{\bar z}}_{\bar T_{gh}(\bar z)})$$

after choosing the ref. metric.

More in general:

$$b_{a_1 a_2 \dots a_\lambda} D_b c^{a_2 \dots a_\lambda} \longrightarrow T(z) = -\left(\lambda b \partial c + (\lambda-1)(\partial b) c\right)$$

Now we can fix the conformal gauge (perform $[\mathcal{D}h]\,\delta(h-\eta)\dots$):

$$\frac{1}{2\pi\alpha'}\underbrace{\int d^2 z}_{(h,\bar{h})=(-1,-1)}\Big[\underbrace{\partial X^\mu}_{h=1}\underbrace{\bar{\partial} X_\mu}_{\bar{h}=1} + \alpha'\big(\underbrace{b}_{h=2}\underbrace{\bar{\partial}}_{\bar{h}=1}\underbrace{c}_{h=-1} + \underbrace{\bar{b}}_{\bar{h}=2}\underbrace{\partial}_{h=1}\underbrace{\bar{c}}_{\bar{h}=-1}\big)\Big]$$

$$\underbrace{b\bar{\partial}c + \bar{b}\partial\bar{c}}_{(h,\bar{h})=(1,1)}$$

OPE:
$$\underbrace{b(z_1)\,c(z_2)}_{h=1} \sim \underbrace{\frac{1}{z_1 - z_2}}_{h=1} \sim \underbrace{c(z_1)\,b(z_2)}_{h=1}$$

$$\Rightarrow T_{gh}(z_1)\,T_{gh}(z_2) \sim \frac{1}{2}\,\frac{\left(1-3(2\lambda-1)^2\right)}{(z_1-z_2)^4} + \dots$$

c_λ [central charge] $\Rightarrow c_2 = -26$

Since $T(z) = T_X(z) + T_{gh}(z)$

$\Downarrow$

d bosons
(d: spacetime dim.)

$$\rightarrow T(z_1)\,T(z_2) \sim \frac{1}{2}\cdot\frac{d-26}{(z_1-z_2)^4} + \dots$$

$\Uparrow$

ANOMALY FREE IF $d = 26$

→ Operator language:

$$b(z) = \sum_n b_n z^{-n-2} \qquad\qquad c(z) = \sum_n c_n z^{-n+1}$$

$$\Rightarrow \{b_n, c_m\} = \delta_{n+m,0} \Rightarrow \text{! VACUUM !}$$

$\hookrightarrow$ $c_1|0\rangle$

$b_1|0\rangle = 0 \Rightarrow \langle 0| b_{-1} = 0$ BUT

$$0 = \langle 0|b_{-1}c_1|0\rangle = \langle 0|\{b_{-1},c_1\}|0\rangle = \langle 0|0\rangle$$

So what is the vacuum?

$$c_n |0\rangle = 0 \quad \forall n \geq \lambda$$

$$b_n |0\rangle = 0 \quad \forall n \geq 1-\lambda$$

$1-\lambda$ λ

STANDARD | PECULIAR | STANDARD

$\lambda = 2 \Rightarrow$ the first object to be NON ZERO is $\langle 0 | \underbrace{c_{-1}\, c_0\, c_1}_{0\text{- modes}} | 0 \rangle = 1$

BRST APPROACH

$$\int [\mathcal{D}\varphi_i][\mathcal{D}b_A \mathcal{D}c^\alpha][\mathcal{D}B] \exp\left[-S_{MAT} - b_A \underbrace{\delta_\alpha G^A}_{} c^\alpha + i B_A G^A\right]$$

index of gauge param

MATTER GHOST

gauge fixing condition

gauge fixing condition

equivalent of $\delta(G^A)$ in FP proc

everything has new GLOBAL SYMMETRY with opposite statistic w.r.t. previous symm.

the gauge param now is the ghost

$$\Rightarrow \quad \delta_B \varphi_i = -\varepsilon c^\alpha \delta_\alpha \varphi_i$$

$$\delta_B b_A = -i\varepsilon B_A$$

$$\delta_B c^\alpha = \frac{1}{2}\varepsilon f^\alpha_{\beta\gamma} c^\beta c^\gamma$$

$$\delta_B B_A = 0$$

$$\hookrightarrow \; -b_A \delta_\alpha G^A c^\alpha + i B_A G^A = -\delta_B (b_A G^A)$$

$\Rightarrow$ combine the NON MATTER FIELD INTO A BRST-EXACT TERM

$\Rightarrow \delta_B^2 = 0$

$\Rightarrow$ PHYSICAL STATES are BRST-CLOSED (not exact) STATES

i.e.: physical states do not depend on the gauge (insertion of BRST-exact terms will not alter the corr. funct.)

In our case:

$$\delta X^\mu = -v^a \partial_a X^\mu \longrightarrow \delta_B X^\mu = -\varepsilon C^a \partial_a X^\mu \longrightarrow -\varepsilon c\, \partial X^\mu - \varepsilon \bar{c}\, \bar{\partial} X^\mu$$

$$\delta v^a = \left[-v^b \partial_b v^a + (\partial_b v^a) v^b \right] \longrightarrow \delta_B c^a = \frac{\varepsilon}{2}\left[-c^b \partial_b c^a + (\partial_b c^a) c^b \right]$$

Global symm. $\Rightarrow \exists\, j_B(z) = c(z) T_X(z) + \frac{1}{2} c(z) T_{gh}(z) =$

$$= c(z) T_X(z) + bc\,\partial c(z)$$

$$\bar{j}_B(\bar{z}) = \bar{c}(\bar{z}) \bar{T}_X(\bar{z}) + \bar{b}\bar{c}\,\bar{\partial}\bar{c}(\bar{z})$$

$\Rightarrow$ BRST CHARGE: $j_B(z) = \sum_n j_n z^{-n-1} \Rightarrow Q_B = \oint_0 \frac{dz}{2\pi i} j_B(z) = j_0$

$\bar{j}_B(\bar{z}) = \sum_n \bar{j}_n \bar{z}^{-n-1} \Rightarrow \bar{Q}_B = \oint_0 \frac{d\bar{z}}{2\pi i} \bar{j}_B(\bar{z}) = \bar{j}_0$

However $T(z_1)\, j_B(z_2) \sim \underbrace{\frac{\#}{(z_1 - z_2)^3}}_{\text{THIS IS NOT THE RIGHT OPE!}} + \dots$

full $T = T_X + T_{gh}$

$\Rightarrow$ we need to improve the result:

$$j_B(z) = c T_X + \frac{1}{2} c T_{gh} + \frac{3}{2} \partial^2 c$$

NB: $\frac{3}{2} \partial^2 c$ does not contrib to Q_B.

CONSERVATION LAW: $\partial_a j_B^a = \bar{\partial} j_B + \partial \bar{j}_B = 0$.

PHYSICAL STATES

Take $j_B(z)$ and write the oper. corresp. to the state:

$$j_B(z_1)\, V^{TOT}(z_2) \sim \frac{O_B}{z_1 - z_2}$$

$$[Q_B, V^{TOT}(z_2)] = \delta_B V^{TOT}(z_2) = \oint_{z_2} \frac{dz_1}{2\pi i}\, j_B(z_1) V^{TOT}(z_2) =$$

$$\sim \oint_{z_2} \frac{dz_1}{2\pi i} \frac{O_B}{z_1 - z_2} = O_B$$

Now suppose to have $V(z)$ s.t. $O_B = 0$, then

$$V \sim V + \delta_B \Omega$$

BOSONIC STRING $\Rightarrow$ always $\exists$ a representative s.t. $\underbrace{c\, V_X(z)}_{\text{only } X}$

$\Rightarrow$ states in BRST cohomology have positive norm.

e.g.: Ground state: $|0; k^\mu\rangle \longrightarrow c_1 |0; k^\mu\rangle \Rightarrow c(z) :e^{ik\cdot X(z)}:$

motion of the center of mass

$\Rightarrow$ left sec. of CST: $j_B(z_1)\left(c e^{ik\cdot X}\right)(z_2) = ?$

- $c\left(-\frac{1}{\alpha'}\partial X^\mu \partial X_\mu\right)(z_1)\; c e^{ik\cdot X}(z_2) \sim$

$c^2 = 0 \Rightarrow$ exp. in z_2

$$\sim \left(-\frac{1}{\alpha'}\right)(z_1 - z_2)(\partial c)\, c \left(e^{ikX}\left(-\frac{\alpha'}{2}\frac{1}{(z_1 - z_2)}\right)^2 (ik_\mu)(ik^\mu)\right)$$

$$= \frac{\alpha'}{4}(\partial c)\, c\, \frac{k^2}{z_1 - z_2}\, e^{ik\cdot X}$$

- $(b c\, \partial c)(z_1)\; c\, e^{ik\cdot X}(z_2) \sim e^{ik\cdot X} \dfrac{c\, \partial c(z_2)}{z_1 - z_2}$

We require $\left[Q_B, c e^{ik\cdot X}(z_2)\right] = 0$. Then we ask the simple pole to vanish:

$$j_B(z_1)\left(c e^{ik\cdot X}\right)(z_2) \sim \frac{1}{z_1 - z_2}\left(\underbrace{\partial c \cdot c \cdot e^{ik\cdot X}\left(\frac{\alpha'}{4}k^2 - 1\right)}_{=0}\right) + \text{regular}$$

$$\Longleftrightarrow$$

$$\boxed{k^2 = \frac{4}{\alpha'}}$$

ON-SHELL COND.

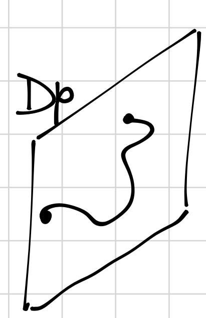
Since we chose $\eta = \begin{pmatrix} - & & & \\ & + & & \\ & & + & \\ & & & \ddots \end{pmatrix}$

$M_{cl}^2 = -k^2 = -\frac{4}{\alpha'}$

$\Downarrow$

TACHYON (closed string) full op: $c\bar{c}\, e^{ik\cdot(X_L + X_R)}$

Now consider OPEN STRING on Dp-brane:



$c e^{ik\cdot X}(z)$ where

$\mu = 0, \dots, p$: Neumann

$i = p+1, \dots, d-1$: Dirichlet

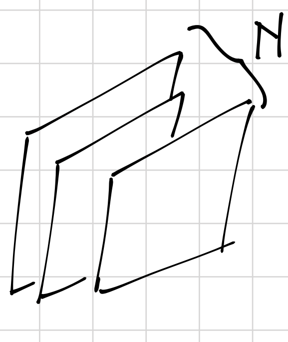
$\Downarrow$

N: $X_L^\mu + X_R^\mu = 2X_L^\mu$

D: $X_L^i + X_R^i = 0$

$$\Rightarrow c(z)\, e^{ik\cdot X}(z) = c(z)\, e^{2ik_\mu X_L^\mu}(z) \Longrightarrow M_{op}^2 = -\frac{1}{\alpha'}$$

$\Rightarrow$ Add indeces to keep track of the branes



N D-branes (label i)

CHAN-PATON factors

$$V^{ij}(z) = T^{ij} c e^{2ik_\mu X_L^\mu}(z)$$

$\Downarrow$

N^2 different states

Now consider

$$c_0 c_1 |0;k\rangle \longrightarrow \partial c\, c\, e^{ik\cdot X}(z)$$

which is very $\simeq$ to the first. However we should also consider

$$b_0 |phys\rangle = 0$$

$$\Rightarrow \quad c_1 |0;k\rangle \checkmark$$
$$c_0 c_1 |0;k\rangle \times$$

Now consider the excited states:

$$\left[\varepsilon_\mu \alpha_{-1}^\mu + \gamma c_{-1} + \beta b_{-1}\right] c_{-1} |0;k\rangle$$

$\Rightarrow$ level 1 states: no. of states $= 26 + 1 + 1 = 28$

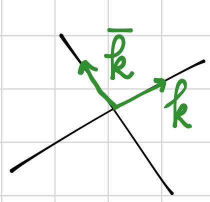
Now impose $[Q_B, V] = 0 \Rightarrow$ 2 unphysical state
2 BRST-exact
24 physical states $\Rightarrow \boxed{k^2 = 0}$

MASSLESS STATES

$$Q_B\left[\varepsilon_\mu \alpha_{-1}^\mu + \gamma c_{-1} + \beta b_{-1}\right] c_1 |0;k\rangle =$$
$$= \left(c_{-1} c_1 \varepsilon \cdot k + \beta c_1 \alpha_{-1} \cdot k\right) |0;k\rangle$$

$k = (E, 0, \dots, E)$
$\bar{k} = (E, 0, \dots, -E)$

NB $\varepsilon_\mu \sim \varepsilon_i,\ k_\mu,\ \bar{k}_\mu$ s.t. $k^2 = \bar{k}^2 = 0,\quad k \cdot \bar{k} = 1$



Then we have to set $\beta = 0 \Rightarrow b_{-1} c_1$ is unphysical

ε_μ prop to $\bar{k}_\mu$ must vanish because unphysical

What are the BRST exact?

I can write $c_{-1} c_1$ as $Q_B(c_1 \bar{k})$ because I have $c_{-1} c_1 \bar{k} \cdot k$

$\Rightarrow$ we are left with $\varepsilon_i \longrightarrow$ 24 STATES!

Q_B comes from worldsheet theory but the same happens in SPACE-TIME where Q_B is given by the gauge transformation!

$$\delta_B^{(\text{spacetime})} A_\mu \sim C^{(\text{spacetime})} k_\mu$$

$$\delta_B^{(\dots)} b^{(\dots)} \sim B \sim k \cdot \alpha$$

$\hookrightarrow$ $\gamma \sim$ spacetime c

$\beta \sim$ " b

$\Rightarrow$ String theory gives rise to GAUGE TH:

N^2 gauge bosons $\Longrightarrow$ $U(N)$ gauge theory

With LEFT and RIGHT sectors: $|0;k\rangle = e^{ik\cdot(X_L+X_R)}|0\rangle$

$\Rightarrow$ L-R must have same mass

require $(L_0-\tilde{L}_0)|\psi\rangle=0$

L_0 is a rot of z (shift of σ)

$$\underbrace{G_{MN}}\; C_1\, \alpha_{-1}^{M}\, \tilde{C}_1\, \tilde{\alpha}_{-1}^{N}\,|0;k\rangle \longrightarrow Q_B[\quad]=0 \Rightarrow \text{MASSLESS}$$

reduce to irreps of LITTLE GROUP $SO(D-2)$ orth. to $k, \bar{k}$

$$G_{MN} \rightarrow \widehat{\square\square} \oplus \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array} \oplus \bullet \longrightarrow G_{(MN)}\; B_{[MN]}\; \phi$$

dilaton

graviton

Kalb-Ramond

$\sim$ open str $\Rightarrow$ gauge th

closed str $\Rightarrow$ gravity

<u>DDF states</u> (Di Vecchia - Del Giudice - Fubini)

* MATTER SECTOR:

* open str: $\alpha_{-1}|0;k\rangle \longrightarrow \varepsilon_M^i\, \partial X^M e^{ik\cdot X}$ $\quad i=1,\dots,D-2$

$D-2$ polar. s.t. they are $\perp$ to $k, \bar{k}$:

$$\varepsilon\cdot k = 0 = \varepsilon\cdot\bar{k}$$

$$\Rightarrow A_n^i = -\oint dz\ \varepsilon^i_\mu\ \partial X^\mu\ e^{-ink\cdot X}$$

$\hookrightarrow [A_n^i, A_m^j] = n\,\delta_{n+m}\,\delta^{ij} \Rightarrow D-2$ harmonic oscill. [w/ only >0 norm]

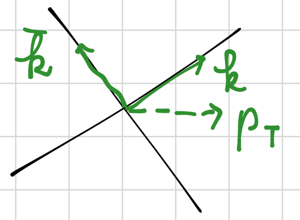
$\Downarrow$

Use these to build states:

$$A_{-n_1}^{i_1} A_{-n_2}^{i_2} \dots |0; p_T\rangle \quad \text{s.t. } p_T^2 = \frac{1}{\alpha'} = 2 \quad (\text{TACHYONIC VACUUM})$$

link between covariant and LC gauge description

$$\Downarrow$$
$$p_T \cdot k = 1$$



How many states do I have?

* Fix the mass $\Rightarrow$ Fix the sum of n_i

$$\text{e.g.: } \sum_i n_i = 2 \Rightarrow A_{-2}^i |0; p_T\rangle,\ A_{-1}^i A_{-1}^j |0; p_T\rangle$$

$\hookrightarrow$ partition funct: $\mathrm{Tr}\left[q^{\sum A_{-n}^i A_n^i}\right] = \mathcal{Z} = f(q) = \sum_p c_p q^p$

$\Rightarrow$ c_p is the degeneration of the level p (no. of states)

$$\hookrightarrow \prod_{n=1}^{\infty} \left(\sum_{k=1}^{\infty} q^{kn}\right)^{D-2} = \prod_{n=1}^{\infty} (1-q^n)^{2-D}$$

$$\Rightarrow C_N = \oint_0 dq\ f(q)\ q^{-N-1} = \oint_0 \exp\left((D-2)\sum_{n=1}^{\infty}\ln(1-q^n)\right)\exp(-(N+1)\ln q)$$

$$C_N \simeq \exp\left[2\sqrt{\frac{\pi^2}{6}(N+1)(D-2)}\right]$$

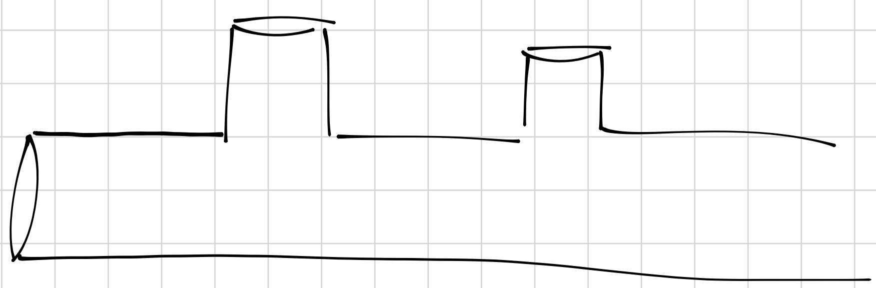
1) The steepest descent is close to $q=1$
2) Expand the $\ln$ and swap the sums
3) Perform $\sum_n$ and $q \longrightarrow 1$
4) $\sum_m \frac{1}{m^2} = \frac{\pi^2}{6}$
5) $\partial_q[\text{exponent}] = 0$

$N \to \infty \Rightarrow C_N \to \infty \Rightarrow \infty$ set of states

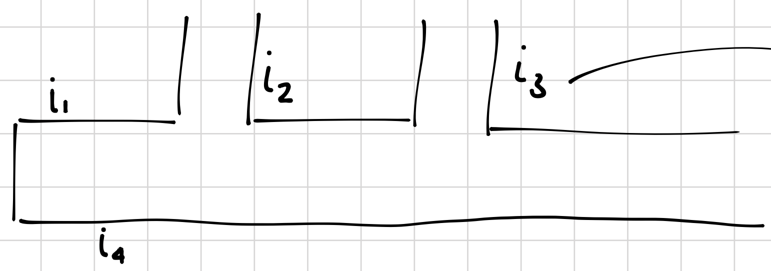
INTERACTING THEORY

Spacetime p.o.v.:

CST:

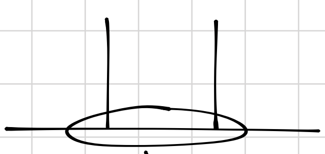


OST:



CHAN-PATON

NB: QFT:



vertices are special

ST: there are no special points

Worldvolume p.o.v.:

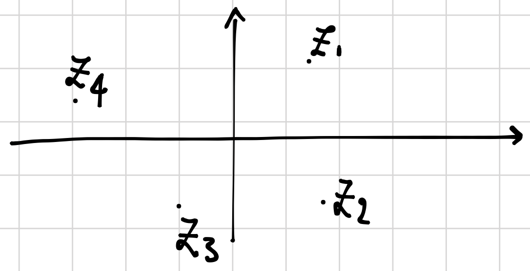
CST: use diff. invariance to choose the position of the states

$$|ph_1\rangle \longrightarrow \lim_{\substack{z\to 0\\ \bar z\to 0}} c(z)\tilde c(\bar z)\, V(z,\bar z)$$

$$|ph_2\rangle \longrightarrow \lim_{\substack{w\to 0\\ \bar w\to 0}} c(w)\tilde c(\bar w)\, V(w,\bar w)$$

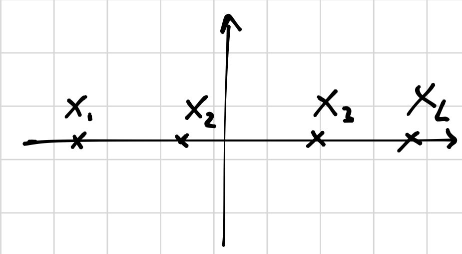
Then

$$\langle\, c(z_1)\tilde c(\bar z_1)V(z_1,\bar z_1)\ c(z_2)\tilde c(\bar z_2)V(z_2,\bar z_2)$$
$$c(z_3)\tilde c(\bar z_3)V(z_3,\bar z_3)\ c(z_4)\tilde c(\bar z_4)V(z_4,\bar z_4)\,\rangle$$

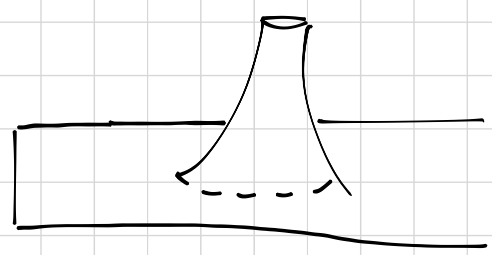


OST $\Rightarrow$ boundary = REAL AXIS:

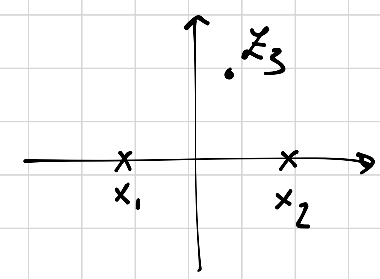
$$\langle c(x_1)V(x_1)\,c(x_2)V(x_2)\ldots\rangle$$



Mixed amplitudes: GAUGE-GRAVITY interactions:



$\longleftrightarrow$



NB: $c V \tilde{c} \tilde{V}$ must be reflected, i.e.: $\tilde{c}\tilde{V} \to c V$ through the reflection matrix.

Let's go back to the 0-mode construction:

$\mathcal{P}$ zero-modes $\longrightarrow$ $\bar{\partial} c = 0 \Rightarrow c = c_1 + c_0 z + c_{-1} z^2 + \ldots$ non sing.

$\hookrightarrow$ the worldvolume of the CLOSED STRING is a SPHERE

$\longrightarrow$ check point at ∞!

2 charts $\longrightarrow$ $z_N = -\frac{1}{z_S} \Rightarrow c(z_S)\frac{\partial}{\partial z_S} = \left(c(z_n)\frac{\partial z_N}{\partial z_S}\right)\frac{\partial}{\partial z_N} = z_N^2\left(c_1 + c_0 z_N^{-1} + c_{-1} z_N^{-2}\right)$

$\Rightarrow$ the only regular part is $\{c_{-1}, c_0, c_1\} \Rightarrow 3$ 0-modes

$\downarrow$

linked to $\{L_{-1}, L_0, L_1\}$

Therefore we should think that the PATH INT $\int [\mathcal{D}c]\, e^{\int b\bar{\partial}c}$ does not depend on these zero-modes.

Therefore

$$\langle cV|_1\, cV|_2\, cV|_3\rangle$$

$[\mathcal{D}c]$ is non zero thanks to them!

[since $\int dc_1 = 0$ and $\int dc_1\, c_1 = 1$, then $[\mathcal{D}c] = \ldots dc_{-2}\, dc_{-1}\, dc_0\, dc_1\, dc_2 \ldots$

$\Rightarrow$ I need to insert $c_{-1} c_0 c_1$ to have $\int dc_1\, dc_0\, dc_{-1}\, c_{-1} c_0 c_1 = 1$

$$\Downarrow$$

$$\langle c_{-1} c_0 c_1 \rangle = 1$$

Take tachyons for instance:

$$\langle c{:}e^{ik\cdot X}\big|_1{:}\; c{:}e^{ik\cdot X}\big|_2{:}\; c{:}e^{ik\cdot X}\big|_3{:} \rangle =$$

$$= \langle c(z_1){:}e^{2ik^1_\mu X^\mu_L(z_1)}{:}\, c(z_2){:}e^{2ik^2_\mu X^\mu_L(z_2)}{:}\, c(z_3){:}e^{2ik^3_\mu X^\mu_L(z_3)}{:} \rangle$$

Now I need

$$e^{2ik_1\cdot X_L(z_1)}\, e^{2ik_2\cdot X_L(z_2)} \sim (z_1 - z_2)^{2\alpha' k_1\cdot k_2}$$

Then

$$\langle c(z_1)\, e^{2ik_1\cdot X_L + 2ik_2\cdot X_L}\, c(z_2) c(z_3)\, e^{2ik_3\cdot X_L} \rangle (z_1 - z_2)^{2\alpha' k_1\cdot k_2} =$$

$$= \langle c(z_1) c(z_2) c(z_3) \rangle \langle {:}e^{2i(k_1\cdot X_L(z_1) + k_2\cdot X_L(z_2) + k_3\cdot X_L(z_3))}{:} \rangle (z_1 - z_2)^{2\alpha' k_1\cdot k_2} (z_1 - z_3)^{2\alpha' k_1\cdot k_3} (z_2 - z_3)^{2\alpha' k_2\cdot k_3}$$

result on 0-modes

$$= \langle \prod_{i=1}^{3} (c_1 + c_0 z_i + c_{-1} z_i^2) \rangle (z_1 - z_2)^{2\alpha' k_1\cdot k_2} (z_1 - z_3)^{2\alpha' k_1\cdot k_3} (z_2 - z_3)^{2\alpha' k_2\cdot k_3}\, \delta^{p+1}(k_1 + k_2 + k_3)$$

$$= (z_1 - z_2)(z_1 - z_3)(z_2 - z_3) \underbrace{\langle c_{-1} c_0 c_1 \rangle}_{1} (z_1 - z_2)^{2\alpha' k_1\cdot k_2} (z_1 - z_3)^{2\alpha' k_1\cdot k_3} (z_2 - z_3)^{2\alpha' k_2\cdot k_3}\, \delta^{p+1}(k_1 + k_2 + k_3)$$

NB $k_i^2 = \frac{1}{\alpha'} \longrightarrow 2\alpha'(k_i\cdot k_j) = \alpha'(k_i + k_j)^2 - \alpha' k_i^2 - \alpha' k_j^2$

GHOST and MATTER compensate!

$$= \delta^{p+1}(k_1 + k_2 + k_3)$$

Now consider

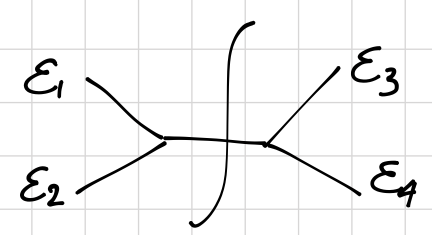
$$k_1^2 = k_3^2 = 0\,,\ k_2 = \frac{1}{\alpha'}$$

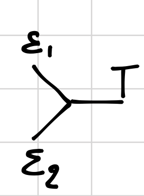
$$\langle c\,\varepsilon_1\cdot\partial X\, e^{ik\cdot X}\big|_1\ c\, e^{ik\cdot X}\big|_2\ c\,\varepsilon_2\cdot\partial X\, e^{ik\cdot X}\big|_3\rangle \sim$$

$$\sim \left[\varepsilon_1\cdot\varepsilon_3 - 2\alpha'\varepsilon_1\cdot k_2\ \varepsilon_3\cdot k_2\right]$$

$\Rightarrow$ if $\varepsilon_1 \sim k_1 \Rightarrow \varepsilon_1\cdot\partial X e^{ik_1\cdot X}$ is a $\underbrace{\text{null state}}_{\text{BRST-exact}}$ and $\langle\ldots\rangle = 0$

The presence of the tachyon makes everything harder $\Rightarrow$ we could truncate/project the spectrum but it is not consistent because it appears as a virtual particle in



$\longrightarrow$ $\Big|$  $\Big|^2 \neq 0 \Rightarrow$ we can have on shell T !

Now consider:

$$\langle \prod_{i=1}^{3}\left(c\,\varepsilon_i\,\partial X\, e^{ik_i\cdot X}\big|_{z_i}\right)\rangle \sim \Big[\underbrace{\varepsilon_1\varepsilon_2\varepsilon_3\cdot k_1 + (\text{cycl.})}_{}\ + 2\alpha'(\varepsilon_1 k_2)(\varepsilon_2 k_3)(\varepsilon_3 k_1)\Big]$$

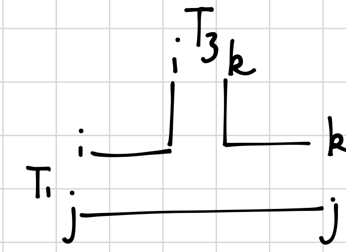
particles 1-2-3 are cyclically symm. but where is the perm 213?

$\Downarrow$

open strings have $PSL(2,\mathbb{R})$ which preserves the ordering

(123) (231) (312) ✓

(213) (132) (321) ✗ $\Rightarrow$ must add them

NB there are CHAN-PATON FACTORS  $\sim \boxed{\mathrm{Tr}(T_1 T_2 T_3)}$

The previous result is therefore 0 unless :

comes from $[213] = -[123]$

$$\mathrm{Tr}(T_1 T_2 T_3)[123] - \mathrm{Tr}(T_2 T_1 T_3)[123] =$$

$$= \mathrm{Tr}\left(\underbrace{[T_1^{a_1}, T_2^{a_2}]}_{\sim f^{a_1 a_2 a_3}} T_3^{a_3}\right)[123]$$

What about the $\propto \alpha'$ contrib ?

$\Rightarrow$ first line $\longrightarrow$ colour-ordered YM

$\Rightarrow$ second line $\Rightarrow$ corrections from UV-complete theory

$$\mathrm{Tr}\,F^2 + \#\,\alpha'\,\mathrm{Tr}\,F^3$$

$$\mathrm{Tr}\left([A,A]\,\partial A\right) \Leftarrow$$ covariant derivative

$\hookrightarrow$ LOCAL SYMMETRY

(we are not imposing it but we are getting it)

4 - point functions:

Naively: $\langle cV|_1 \; cV|_2 \; cV|_3 \; cV|_4 \rangle$

$\hookrightarrow$ are there 0-modes for the b ghost ?

$\bar\partial b = 0 \longrightarrow b = b_{zz}(z) \Rightarrow$ no 0-mode on the sphere

$\Rightarrow \langle \dots \rangle$ is a sphere/disk with 4 punctures : bc has non trivial OPE.

Then choose $b(z) = \prod_{i=1}^{4} \frac{1}{z - z_i}$ (4 poles)

$\downarrow$

regular at $z = \infty$

$\Rightarrow$ we start worrying about b 0-modes only starting from the 4-point

Then change one of the 0-modes for the integration over one of the punctures (to encode the infinitesimal deform. (moduli)):

$\int b_{zz} \mu_{\bar z}^{\;z}$

$\int dz_3 \, \langle cV|_{z_1} \; cV|_{z_2} \; V(z_3) \; cV|_{z_4} \rangle$

VENEZIANO AMPLITUDE

$$V(z) = e^{ik\cdot X(z)} = e^{2ik\cdot X_L(z)}$$

$$\hookrightarrow \int dz \, \langle c e^{2ik_1\cdot X_L}(z_1) \, c e^{2ik_2\cdot X_L}(z_2) \, V(z) \, c e^{2ik_4 \cdot X_L}(z_4) \rangle =$$

$$= \int dz_3 \underbrace{(z_{12} z_{14} z_{24})}_{\text{GHOST}} \underbrace{\frac{(z_{12} z_{34})^{-\alpha' s - 2}}{(z_{13} z_{24})^{-\alpha' s - 1}} \frac{(z_{14} z_{23})^{-\alpha' t - 2}}{(z_{13} z_{24})^{-\alpha' t - 1}}}_{\text{MATTER}}$$

$$\left[\alpha'(k_1+k_2)^2 = -\alpha' s; \quad \alpha'(k_2+k_3)^2 = \alpha' t; \quad \alpha'(k_1+k_4)^2 = \alpha' u \right]$$

$\hookrightarrow$ introduce $z = \dfrac{z_{12} z_{34}}{z_{13} z_{24}} \Rightarrow SL(2,\mathbb{C})$ invariant "cross ratio"

$$= \int \frac{dz_3}{dz} dz \; \ldots = \int_0^1 dz \; z^{-\alpha' s - 2} (1-z)^{-\alpha' t - 2} = \frac{\Gamma(-\alpha' s - 1)\Gamma(-\alpha' t - 1)}{\Gamma(-\alpha'(s+t) - 2)} =$$

$$\hookrightarrow \frac{z_{13}^2 z_{24}}{z_{12} z_{14}}$$

$$\hookrightarrow \underbrace{z_3 \in (z_2, z_4)}_{\text{one colour order}} \Rightarrow z \in (0,1)$$

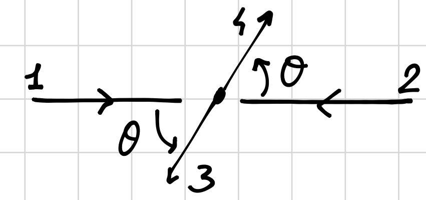
$$= B(-\alpha' s - 1, -\alpha' t - 1)$$

Now we sum over non cyclical perm:

$$A = (2\pi)^{26} \delta^{26}\left(\sum_{i=1}^4 k_i\right) \left\{ B(-\alpha' s - 1, -\alpha' t - 1) + B(-\alpha' s - 1, -\alpha' u - 1) + \right.$$

$$\left. + B(-\alpha' t - 1, -\alpha' u - 1) \right\}$$

Spacetime properties:



Choose $k_1 = \left(\frac{E_{CM}}{2}, \underbrace{\vec{k}_{IN}}_{N}, \underbrace{\vec{0}}_{D}\right)$ $\qquad k_2 = \left(\frac{E_{CM}}{2}, -\vec{k}_{IN}, \vec{0}\right)$

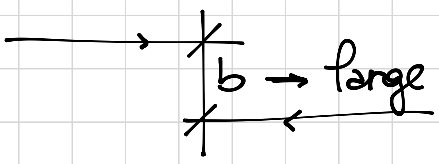
$k_3 = \left(-\frac{E_{CM}}{2}, \vec{k}_{OUT}, \vec{0}\right)$ $\qquad k_4 = \left(-\frac{E_{CM}}{2}, -\vec{k}_{OUT}, \vec{0}\right)$

$$\Rightarrow -\frac{t}{s} = \sin^2\frac{\theta}{2}$$

If energies are $\ll$ than $\alpha' \longrightarrow$ QFT limit

Very large energies $\longrightarrow$ $\alpha' s \gg 1 \Longrightarrow$ STRINGY RESULTS

- REGGE LIMIT: $\alpha' s \gg 1$, $\frac{t}{s} \ll 1$



- FIXED ANGLE: $\alpha' s \gg 1$, $\frac{t}{s}$ fixed
  $|\alpha' t| \gg 1$

We use Stirling approx: $\Gamma(x) \overset{x\to\infty}{\sim} \sqrt{2\pi}\, \exp\Big(\big(x-\tfrac{1}{2}\big)\ln x - (x-1)\Big)$

We can also use

$$\mathcal{A}(s,t,u) = (2\pi)^{26}\,\delta^{26}\Big(\sum_i k_i\Big)\, \overbrace{\Gamma(-\alpha' s-1)\Gamma(-\alpha' t-1)\Gamma(-\alpha' u-1)}^{\sim \exp\left\{-\alpha'\left(s\ln(-\alpha' s) + t\ln(-\alpha' t) + u \ln(-\alpha' u)\right)\right\}}\Big(\sin(\pi\alpha' s) +$$
$$+ \sin(\pi\alpha' t) + \sin(\pi\alpha' u)\Big)$$

Consider the FIXED ANGLE:

$$\mathcal{A}(\dots) \sim e^{\left\{\alpha' E_{CM}^2 \underbrace{\left[\sin^2\frac{\theta}{2} \underbrace{\ln \sin^2\frac{\theta}{2}}_{<0} + \cos^2\frac{\theta}{2}\underbrace{\ln \cos^2\frac{\theta}{2}}_{<0}\right]}_{<0}\right\}}\,[\dots]$$

$\downarrow$

EXPONENTIALLY SUPPRESSED!

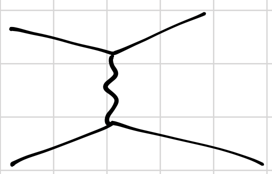
(not point-like object!!! Very $\neq$ from QFT)

Now consider the REGGE LIMIT: $\dfrac{\Gamma(a+b)}{\Gamma(a+c)} \overset{a\to\infty}{\sim} a^{b-c}$

spin of the massless state

$$\Rightarrow B(-\alpha' s-1, -\alpha' t-1) \sim \Gamma(-\alpha' t-1)\,\underbrace{(-\alpha' s)^{1+\alpha' t}}_{\text{REGGE BEHAVIOUR}}$$

$\Rightarrow$ it's a REGGE TRAJECTORY

it's the exchange $\Longrightarrow$ 
of an ∞ series of states!
w/ increasing spin!

SUPERSTRING THEORY

SUSY version of X^μ, h_{ab} in $D=2$ $\longrightarrow$ $\underbrace{X^\mu, \psi^\mu;\ h_{ab}, \chi_a}_{\text{SUGRA in } D=2}$

As before:

$$\begin{array}{ccc} X^\mu,\ h_{ab} & \xrightarrow{G.F.} & X^\mu,\ b,\ c \\ \big\downarrow {\scriptstyle 2D\ SUSY} & & \\ X^\mu, \psi^\mu;\ h_{ab}, \chi_a & \longrightarrow & ? \end{array}$$

Riemann surfaces $\longrightarrow$ Super-Riemann surfaces

z $\qquad\qquad$ $(z, \theta) = Z$

$\Rightarrow$ promote everything to superfield: $X(Z, \bar{Z})$; $B(Z)$; $C(Z)$

We need an extra condition to have SuperRiemannian manifold:

$$D = \frac{\partial}{\partial \zeta} + \zeta \frac{\partial}{\partial z} \quad \text{s.t.} \quad D^2 = \frac{\partial}{\partial z} \neq 0 \text{ everywhere}$$

BRST supercharge:

$$Q_B = \oint dz\, d\zeta\, J_B = \sum_{i=0}^{2} Q_i \longrightarrow \quad Q_0 = \oint dz \quad c\left(T_X + T_\psi + T_{\beta\gamma} + \partial c\, b\right)$$

$$Q_1 = \frac{i}{2} \oint dz \;\; \gamma\left(\psi\, \partial X\right)$$

$$C\left[T_{MAT} + \frac{1}{2} T_{gh}\right]$$

$$Q_2 = -\frac{1}{4} \oint dz \;\; \gamma^2 b$$

FIELDS

$$\psi^\mu = \sum_r \psi_r\, z^{-r-\frac{1}{2}} \qquad \beta = \sum_r \beta_r\, z^{-r-\frac{3}{2}} \qquad \gamma = \sum_r \gamma_r\, z^{-r-\frac{1}{2}}$$

$r \in \mathbb{Z} \Rightarrow$ periodic on cylinder + cuts on plane $\mathbb{C}$

$r \in \mathbb{Z}+\frac{1}{2} \Rightarrow$ antiper " + NO " $\left(\psi^\mu(e^{2\pi i} z) = \psi^\mu(z)\right)$

$\Rightarrow$ NS

$$\partial X \sim \sum_{n\in\mathbb{Z}} \alpha_n\, z^{-n-1}$$

$$\psi \sim \sum_{n\in\mathbb{Z}+\frac{1}{2}} b_n\, z^{-n-\frac{1}{2}}$$

GHOSTS

$$\beta_r\,|0\rangle = 0 \quad \forall r \geq 1-\lambda \quad \longrightarrow \quad 1-\lambda = -\frac{1}{2}$$

$$\gamma_r\,|0\rangle = 0 \quad \forall r \geq \lambda \quad \longrightarrow \quad \lambda = \frac{3}{2}$$

$$\Downarrow$$

$$\langle \tilde{0} | 0 \rangle = ?$$

bc syst $\rightarrow \langle 0 | c_{-1} c_0 c_1 | 0 \rangle = 1 \Rightarrow \langle \tilde{0} | = \langle 0 | c_{-1} c_0 c_1$

think of these insertions as $\delta(c_{-1})\,\delta(c_0)\,\delta(c_1)$ because they are Grassmann variables.

Therefore

$$\beta\gamma \text{ system} \Rightarrow \langle 0| \, \delta(\gamma_{-\frac{1}{2}}) \, \delta(\gamma_{\frac{1}{2}}) |0\rangle$$

NB $\quad \gamma_{\frac{1}{2}} |0\rangle \neq 0$

$$\gamma_{\frac{1}{2}} \, \delta(\gamma_{\frac{1}{2}}) |0\rangle = 0 \quad \text{since } x\,\delta(x) = 0$$

$\hookrightarrow$ $\delta(*)$ changes the def. of creation/annihilation op.

$$\Rightarrow e^{ik\cdot X\sqrt{\frac{\alpha'}{2}}} = e^{ik\cdot X}\left(1 - 3\sqrt{2\alpha'}\, k\psi\right) \Rightarrow \text{is it BRST closed ?}$$

$$\left[Q_1, e^{ik\cdot X}\right] \neq 0 \Rightarrow \underbrace{c\,\delta(\gamma)}_{h=\frac{1}{2}} e^{ik\cdot X} \rightarrow \left[Q_1, c\,\delta(\gamma)\, e^{ik\cdot X}\right] = 0$$

$$\Rightarrow h = \frac{\alpha'}{4}k^2 - 1 + \frac{1}{2} = 0$$

$$\Leftrightarrow k^2 = \frac{2}{\alpha'}$$

TACHYON!

1) How to write $\delta(\gamma)$?
2) Implement cut on plane ?

BOSONIZATION

- $b(z_1)\, c(z_2) \sim \dfrac{1}{z_1 - z_2}$

- Introduce $\quad \chi^1 = \psi^1 + i\psi^2 \qquad \bar{\chi}^1 = \psi^1 - i\psi^2$

$$\tilde{\chi}^1 = \tilde{\psi}^1 + i\tilde{\psi}^2 \qquad \bar{\tilde{\chi}}^1 = \bar{\tilde{\psi}}^1 - i\bar{\tilde{\psi}}^2$$

and same for $\chi^2, \dots, \chi^5$ (split 10 ferm. $\mathbb{R}$ in 5 $\mathbb{C}$ couples)

$$\Rightarrow \bar{\chi}(z_1)\,\chi(z_2) \sim \frac{1}{z_1 - z_2}$$

- consider a boson $S = \frac{1}{2\pi}\int \partial H \,\bar{\partial} H \Rightarrow \partial H(z_1)\, \partial H(z_2) \sim \dfrac{1}{(z_1 - z_2)^2}$

Then $e^{iH(z_1)} e^{-iH(z_2)} \sim \frac{1}{z_1 - z_2}$ $\longrightarrow$ same as $\bar{\chi}(z_1)\chi(z_2) \sim \frac{1}{z_1 - z_2}$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$T_H(z_1) T_H(z_2) \sim \frac{1/2}{(z_1-z_2)^4} + \dots \qquad T_\chi(z_1) T_{\bar\chi}(z_2) \sim \frac{2 \cdot 1/4}{(z_1-z_2)^2} + \dots \rightarrow 2 \text{ ferm } \psi\ \forall \chi$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$C = 1 \quad \xleftrightarrow{\text{DUAL THEORIES}} \quad C = 1$$

$$\Rightarrow \left.\begin{array}{l} \chi \longleftrightarrow e^{iH} \\ \bar\chi \longleftrightarrow e^{-iH} \end{array}\right\} \text{fermions}$$

$$T_\chi \longleftrightarrow T_H$$

NB: $:e^{iH(z_1)}::e^{-iH(z_2)}: \; = \; :e^{i(H(z_1)-H(z_2))}: + \frac{1}{z_1 - z_2}$

$$\sim \frac{1}{z_1 - z_2} + i\,\partial H(z_2) + \dots$$

$$\chi(z_1)\bar\chi(z_2) \sim \frac{1}{z_1-z_2} + :\chi\bar\chi(z_2): + \dots$$

$$\hookrightarrow \underbrace{:\bar\chi\chi(z):}_{\text{two fermions}} \longleftrightarrow i\,\partial H(z)$$

two fermions = 1 boson

$\Downarrow$

CURRENT

We can integrate the current: $i\,\partial H = J$

$$Q = \oint dz\, J = j_0 \Rightarrow \text{it's counting the fermions!}$$

$\hookrightarrow$ the momentum for H is quant.

In general we can do it for $\lambda \neq \frac{1}{2}$ and for β, γ (bosons):

$\lambda \neq \frac{1}{2} \longrightarrow$ choose a boson ϕ (for (b,c)) s.t.

$$S = -\frac{1}{2\pi}\int d^2z \left(\partial\phi\bar\partial\phi + \underbrace{\frac{Q}{4} R(h)\,\phi\sqrt{h}}_{\text{2D gravity + SPACETIME DILATON}}\right) \qquad (Q = 1 - 2\lambda)$$

and in e^{-S} we have $e^{\langle\phi\rangle} e^{\cdots} = g_s e^{\cdots}$

$$\Rightarrow \quad \partial\phi\bar{\partial}\phi + \frac{Q}{4} R\phi\sqrt{h} \Rightarrow \text{"LINEAR DILATON CFT"}$$

(modifies the stress tensor)

$$T = \frac{1}{2}\partial\phi\partial\phi - \frac{Q}{2}\partial^2\phi \quad \Rightarrow C_Q = 1 - 3Q^2$$

$$\lambda = 2 \Rightarrow Q = -3 \Rightarrow C_{-3} = -26$$

What about (β,γ) systems?

$$\beta \longleftrightarrow \partial\xi\, e^{-\varphi}$$

$$\gamma \longleftrightarrow e^{\varphi}\eta$$

free boson $\Rightarrow Q = -2 \quad \left(1 - 2\cdot\frac{3}{2}\right)$

(η,ξ) are fermions $(\sim (b,c))$

$h_\eta = 1 \qquad h_\xi = 0$

$$\Rightarrow \quad \eta(z_1)\xi(z_2) \sim \frac{1}{z_1 - z_2}$$

$$\partial\varphi(z_1)\partial\varphi(z_2) \sim \frac{1}{(z_1-z_2)^2}$$

NB: $h_\beta = \frac{3}{2} \Rightarrow h_{\partial\xi} = 1 \quad h_{e^{-\varphi}} = \frac{1}{2}$

$h_\gamma = -\frac{1}{2} \Rightarrow h_\eta = 1 \quad h_{e^{\varphi}} = -\frac{3}{2}$

$$\Rightarrow \quad h(e^{a\varphi}) = \frac{a(a+Q)}{2}$$

NB: $\xi = \sum_n \xi_n z^{-n} \Longrightarrow \xi_0$ is never present because of $\partial\xi(z)$

(β,γ) generates $\mathcal{H}_S \Rightarrow$ small HS

(φ,η,ξ) " $\mathcal{H}_L \Rightarrow$ large HS

We can now say $\Rightarrow c\,\delta(\gamma)e^{ik\cdot X} \longrightarrow c\,e^{-\varphi}e^{ik\cdot X}$

Let's now write

$$(\psi^\mu, \beta, \gamma) \longrightarrow (\underbrace{H^a, \varphi}_{6\text{ bos.}}, \eta, \xi) \qquad \begin{matrix} \mu = 0, \dots, 9 \\ a = 1, \dots, 5 \end{matrix}$$

$\varphi \downarrow$ $Q = -2$

$e^{i h_a H^a + h_6 \varphi} \Rightarrow$ can be desc. by vectors $(h_1, \dots, h_6)$

e.g.: tachyon $:= (0, \dots, -1)$

e.g.: vector $DX e^{ik\cdot X\sqrt{\frac{\alpha'}{2}}} \longrightarrow c e^{-\varphi} \psi^\mu e^{ik\cdot X}$

we can write it as $e^{i h_a H^a}$
where one of the h_a is $\neq 0$:

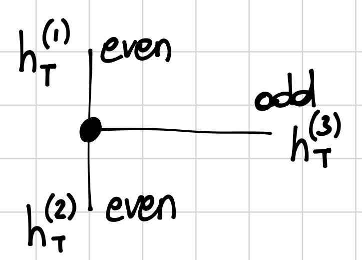
$(\pm 1, 0, 0, 0, 0, -1)$
$(0, \pm 1, 0, 0, 0, -1)$
$\vdots$

points in this 6D lattice are states !!!
↳ each ψ^μ is a lin. comb. of these vectors

GSO PROJECTION

$\Longrightarrow$ keep only $\sum_i h_i = 2n = h_T$

NB do GSO_{even} states generate GSO_{odd}:



$\Rightarrow$ if this ampl. is on shell then we can generate odd states:

$\langle V^{(1)}_{even} \, V^{(2)}_{even} \, V^{(3)}_{odd} \rangle \neq 0$ only if $h_6^{(1)} + h_6^{(2)} + h_6^{(3)} = -2$

and $\sum_{a=1}^{5} h_a^{(i)} = 0 \Rightarrow$ MOMENTUM CONS.

Sum all of them: $h_T^{(1)} + h_T^{(2)} + h_T^{(3)} = -2 \rightarrow$ if $h_T^{(1)}, h_T^{(2)}$ even $\Rightarrow h_T^{(3)}$ even $\Rightarrow \boxed{\langle \dots \rangle \equiv 0}$

What about the second term in

$$e^{ik\cdot \mathcal{X}\sqrt{\frac{\alpha'}{2}}} = e^{ik\cdot X}\left(1 - \zeta\sqrt{2\alpha'}\, k\cdot\psi\right) \ ?$$

We can use it to build different representatives of the same op $\Rightarrow$ PICTURE

$$\Downarrow$$

$$-(\partial\varphi + \eta\zeta)$$

$$\Rightarrow \ -(\partial\varphi + \eta\zeta)(z_1)\ c e^{-\varphi} e^{ik\cdot X}(z_2) \sim \frac{-1}{z_1 - z_2} \Rightarrow \text{picture } -1$$

Now take

$$\zeta c e^{-\varphi} e^{ik\cdot X}(z)$$

$$\Rightarrow \ \{Q_B, \zeta c e^{-\varphi} e^{ik\cdot X}\} := \text{picture changed operator}$$

RAMOND SECTOR

$$\Rightarrow \quad h_a = \pm r \qquad r \in \mathbb{Z} + \tfrac{1}{2}$$

$$e^{\frac{1}{2}(\pm H_1 \pm H_2 \pm H_3 \pm H_4 \pm H_5) - \frac{\varphi}{2}}$$

$$\Downarrow$$

helicity rep. (no 0 entries)

Now take

$$e^{\frac{1}{2}(\pm H_1 \dots) - \frac{\varphi}{2}}(z_1)\ \psi(z) =$$

$$= e^{\frac{1}{2}(\pm H_1 \dots) - \frac{\varphi}{2}}(z_1)\ e^{\pm i H_1}(z) \sim (z - z_1)^{\pm\frac{1}{2}}$$

$$\downarrow$$

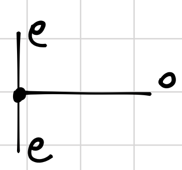
Then $z - z_1 \longrightarrow (z - z_1)e^{2\pi i} \Rightarrow$ multivalued funct.

$$\downarrow$$

RAMOND SECT. $\Big\{$ BOUNDARY CHANGING OPER (SPIN FIELDS)

GSO Proj $\Rightarrow$ only even states!

Then, since we use the same derivation, we know



works as well!

NB: $$V_{-\frac{1}{2}}(z) = c\, e^{\frac{1}{2}(\pm H_1 \dots) - \frac{\varphi}{2}}\, e^{ik\cdot X}(z)$$

$$\hookrightarrow \left[Q_B, V_{-\frac{1}{2}}(z)\right] = 0 \Rightarrow \quad k^2 = 0$$

Choose $h_a = \pm\frac{1}{2} \Rightarrow 2^5 = 32$ possible states

$\to$ 16 GSO_{even} 16 GSO_{odd}

What is the interp?

NS $\;(\pm 1, 0, \dots\;)$
$\quad\;(0, \pm 1, \dots,\;)$
$\quad\;\dots$

$\Big\}\; D-2 = 8$ states $\Rightarrow$ Spacetime MASSLESS VEC.

R $\;(\pm, \pm, \pm, \pm, \pm) \longrightarrow 32$ states $\rightarrow$ MASSLESS SPINOR

$\downarrow$

Dirac spinor $\rightarrow$ 2 Weyl spinor w/ opp. chirality

(GSO $\rightarrow$ chirality projection)

These spinors are Majorana-Weyl!

need level matching

TYPE II THEORIES $\left((NS,\widetilde{NS}), (NS,\widetilde{R}), (R,\widetilde{NS}), (R,\widetilde{R}) \right)$

COMMON

$$(NS,\widetilde{NS}) \to c e^{-\varphi} \psi^M \tilde{c} e^{-\tilde{\varphi}} \tilde{\psi}^N e^{ik\cdot(X_L+X_R)} G_{MN} \quad \text{s.t. } k^2=0 \quad k^M G_{MN} = G_{MN} k^N = 0$$

$$\Rightarrow G_{MN},\ B_{MN},\ \phi$$

$$(NS,\widetilde{R}) \to c e^{-\varphi} \psi^M \tilde{c} \tilde{S}^A_{-\frac{1}{2}} e^{ik\cdot(X_L+X_R)} \Psi^{(1)}_{MA}$$ GRAVITINO (Q_B on this provides eom)

$$e^{\frac{1}{2}(\pm H_1 \pm H_2 \pm H_3 \pm H_4 \pm H_5) - \frac{\varphi}{2}} = S^A_{-\frac{1}{2}} \qquad A = 1,\dots,16 \text{ (Weyl index)}$$

picture

$$(R,\widetilde{NS}) \to \quad IIB \quad c S^A_{-\frac{1}{2}} \tilde{c} e^{-\tilde{\varphi}} \tilde{\psi}^M e^{ik\cdot(X_L+X_R)} \Psi^{(2)}_{MA}$$ 2nd GRAVITINO

↳ IIB: $\mathcal{N} = (2,0)$ → same chirality

IIA: $c S_{A-\frac{1}{2}} \tilde{c} e^{-\tilde{\varphi}} \tilde{\psi}^M e^{ik\cdot(X_L+X_R)} \Psi_M{}^A$ DIFF. CHIRAL!

NS sector ⇒ MUST get rid of TACHYON

↳ IIA: $\mathcal{N} = (1,1)$ → opposite chirality

R sector ⇒ we can change the GSO

→ they have to be in pairs in $\langle \dots \rangle$

$$(R,\widetilde{R}) \to \quad IIB \quad c S^A_{-\frac{1}{2}} \tilde{c} S^B_{-\frac{1}{2}} e^{ik\cdot(X_L+X_R)} M_{AB}$$ → Fierz expand it

$$\Rightarrow M_{AB} = \sum \left(C \Gamma_{M_1 \dots M_N} \right)_{AB} F^{M_1 \dots M_N}$$

→ ODD NO. OF Γ's

$$\Rightarrow \quad F_M \,, \quad F_{M_1M_2M_3} \,, \quad \overline{F}^{\,+}_{M_1\dots M_5} \longrightarrow \text{Self dual}$$

$$\left(\text{NB}: \quad F^M (C\Gamma_M)\,\Gamma_{11} = \Gamma^{11}(C\Gamma_M)\, F^M\right.$$

$$\text{but also: } \Gamma_{11} = i\,\Gamma_0 \dots \Gamma_9 \quad \Rightarrow \quad F_{M_1} \longleftrightarrow F_{M_1\dots M_9}$$

$$\left. F_{M_1\dots M_3} \longleftrightarrow F_{M_1\dots M_7} \right)$$

$$\text{IIA}: \quad c\, S_{A\,-\frac{1}{2}}\, \tilde{c}\, \tilde{S}^{B}_{-\frac{1}{2}}\, e^{ik\cdot(X_L + X_R)}\, M^A{}_B \quad \text{opposite chiral.}$$

$$M^A{}_B = \sum \left(C\Gamma_{M_1\dots M_N}\right)^A{}_B\, F^{M_1\dots M_N} \longrightarrow \underline{\underline{\text{EVEN}}} \text{ NO. OF INDICES}$$

$$\hookrightarrow \quad F_{M_1M_2} \qquad F_{M_1\dots M_4}$$

$\Rightarrow$ bunch of $U(1)$ field strengths.

OPEN STRINGS

$$\partial X_L^M = R^M{}_N\, \bar{\partial} X_R^N \quad \text{where} \quad R = \begin{pmatrix} \mathbb{I}_N & \\ & -\mathbb{I}_D \end{pmatrix}$$

$$\hookrightarrow \quad \alpha^M = R^M{}_N\, \tilde{\alpha}^N$$

NS will work the same way. How is the R sec. behaviour?

$$R^M{}_N \longrightarrow R_{\text{spinor}} \Rightarrow R^M{}_N\,\Gamma^N = R_{sp}^{-1}\,\Gamma^M R_{sp}$$

$$\downarrow$$

$$R_{sp} = (\Gamma_{11}\Gamma_{p+1})(\Gamma_{11}\Gamma_{p+2})\dots(\Gamma_{11}\Gamma_{9-p})$$

$\Rightarrow$ commutes with Γ^M in Neumann direction

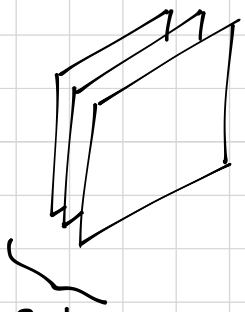
anticomm. " Dirichlet "

Then we use $\tilde{S} = R_{sp}\, S$ for open string:

$$\text{IIA}: \; S_A, \tilde{S}^B \Rightarrow \text{even branes} \quad (D2, D4, D6, D8)$$

$$\text{IIB}: \; S_A, \tilde{S}_B \Rightarrow \text{odd branes} \quad (D1, D3, D5, D7, D9)$$

D3 branes are special



N D3-branes $\longrightarrow$ $SO(1,9) \longrightarrow SO(1,3) \times SO(6)$

$$\text{NS}: \quad \varepsilon_\mu \, c \, \psi^\mu e^{-\varphi} e^{ik\cdot X}, \quad \varepsilon_i \, c \, \psi^i e^{-\varphi} e^{ik\cdot X}$$

$$\text{R}: \quad (\underbrace{h_1, h_2}_{\text{spinor}} \,|\, \underbrace{h_3, h_4, h_5}_{\text{spinor}})$$

$$c\, S^A_{-\frac{1}{2}} e^{ik\cdot X} \Rightarrow$$ 4 gauginos w/ flavour index in spinor rep of $SO(6) \sim SU(4)$

$\Rightarrow$ Dp-branes are minimally coupled to $A_{(R,R)}$ where $F_{(R,R)} = dA_{(R,R)}$

$\downarrow$ charge dens. in extended obj.

$\Downarrow$

POLCHINSKI

tension, charge, ...

solution for classical p-brane: $H(r) = 1 + \dfrac{L^4}{R^4}$

$$\Rightarrow \tau_p = \frac{\sqrt{\pi}}{K_{10}} \left(2\pi\sqrt{\alpha'}\right)^{3-p} \quad \text{(tension)}$$

$$\frac{1}{2K_{10}^2}\int \sqrt{G}\, R$$

$$\mu_p = \sqrt{2\pi}\left(2\pi\sqrt{\alpha'}\right)^{3-p}$$ (charge of the minimal coupling of $A^{(R,R)}_{(p+1)}$ with Dp)

$$\frac{1}{2n!}\int F_n \wedge * F_n$$

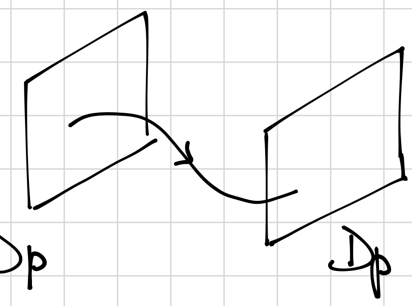
NB: $\boxed{\mu_p \mu_{6-p} = 2\pi}$ (Dirac condition w/ minimal charge)

$\mu_p \to F_{p+2}$, $\mu_{6-p} \to \widetilde{F}_{8-p} = * F_{p+2}$

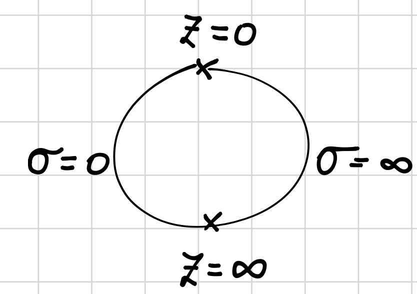
EFFECTIVE Dp-brane ACTION

$$S_{\text{eff}}^{\text{Dp-brane}} = -T_p \int d^{p+1}\xi \, e^{-\Phi} \sqrt{|\det G_{ab}|} + \mu_p \int d^{p+1}\xi \, A_{(p+1)}$$

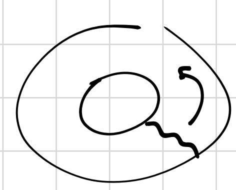
Now consider



$\Rightarrow$



$\hookrightarrow$ vacuum energy $\Rightarrow$ 1L computation
(effective potential)
between the branes



$\tau = \dfrac{\text{length}}{\text{width}}$

from $\dfrac{1}{\sqrt{\det \Pi}}$

propagator

$$\Rightarrow \quad F = -\frac{1}{2} \operatorname{Tr} \left(\ln \Pi \right) = \int_0^\infty d\tau_{op} \, (\ldots)$$

sum over all open string states (incl. ghosts)

do we have UV diverg.?

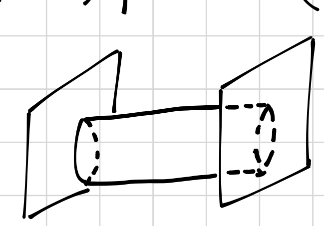
Use $\tau_{op} = \dfrac{1}{\tau_{cl}} = \dfrac{\text{width}}{\text{length}}$

$\Downarrow$

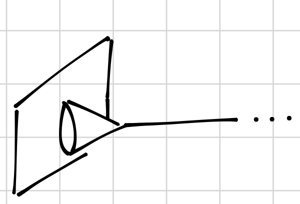
$(\sigma, t)_{op} \longleftrightarrow (t, \sigma)_{cl}$

swapping $0 \leftrightarrow \infty$

$$\int_0^\infty d\tau_{cl} \, (\ldots) \Leftarrow$$



If $\tau_{cl} \to \infty$ we can separate the contrib $\left((NS,\widetilde{NS}), (R,\widetilde{R}), \ldots \right)$



$\Rightarrow$ only $\underbrace{G_{\mu\nu}, \phi}_{T_p}$ and $\underbrace{F_{\mu_1 \ldots \mu_p}}_{\mu_p}$ appear

(no $B_{\mu\nu}$).

$\Rightarrow$ D-branes are sources of grav. fields and RR fields $\Rightarrow$ D-branes are physical states which curve space and couple to RR fields.

Moreover UV div. in op. channel is a IR div. in closed string states $\Rightarrow$ ~ ST is UV safe (the torus ampl. cuts UV div. directly).