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AdS/CFT correspondence with applications

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Outline:

1) Gauge Field Theory
2) Derivation of AdS/CFT (Maldacena's conjecture)
3) Holographic Wilson loops
4) Holography: dictionary, correlators, ...
5) Applications: probe branes
6) Quantum Hall ferromagnetism

Preamble about gauge fields

Notation: $\eta_{\mu\nu} = \mathrm{diag}(-1,+1,+1,+1)$

$A_\mu(x) \in$ Fundam. rep. of a Lie Algebra ~ gauge groups

slightly more compl. $SU(N)$ or $U(N)$

(in large N limit there aren't many diff.)

Therefore $A_\mu(x)$ is a $N \times N$ Hermitian matrix

↳ connection: ~ $\underbrace{\text{transf. of}}$ $\underbrace{\text{"coordinates"}}$

change of phase of WAVE FUNCTION

$$\phi(x) = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix} \quad \text{s.t.} \quad \phi'(x) = g(x)\,\phi(x)$$

with $g(x) \in U(N)$

Local coord. transf? $\longrightarrow$ Covariant derivative

$$D_\mu \phi(x) = \partial_\mu \phi(x) - i A_\mu(x)\phi(x)$$

then $D_\mu \phi \mapsto g(x) D_\mu \phi \Rightarrow A_\mu(x) \mapsto g(x) A_\mu(x) g^\dagger(x) - i\,\partial_\mu g(x)\, g^\dagger(x)$

Covariant derivative ($N\times N$ Hermitian matrix) $\to$ CURVATURE:

$$F_{\mu\nu}(x) = [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$

$$\mapsto g(x) F_{\mu\nu}(x) g^\dagger(x)$$

* Lagrangians $\Rightarrow$ classical extrema have 0 curv.:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \longrightarrow \text{e.o.m. } F_{\mu\nu} = 0$$

What about coupling const?

$\to$ in front: $\mathcal{L} = -\frac{1}{2g_{ym}^2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ $\to$ important for loop expansion in $O(1/g^{2N})$

$\to$ in the transf: $\partial_\mu - iA_\mu \mapsto \partial_\mu - ig_{ym}A_\mu(x)$

Why $-\frac{1}{2}$ instead of $-\frac{1}{4}$?

* $A_\mu = \sum_{a=1}^{N^2} T^a A_\mu^a \Rightarrow \mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab} \Rightarrow \mathcal{L} = -\frac{1}{4g^2}\sum_a F_{\mu\nu}^a F^{a\mu\nu}$

* otherwise treat A_μ directly as a matrix and write $-\frac{1}{2}$. Then choose the extra $1/2$ according to the rep.

PARALLEL TRANSPORT (comparison between $\phi(x)$ and $\phi(y)$)

$\phi(x) \to \phi(y)$

and $U_C(x,y)\,\phi(y)$

$$\Rightarrow U_C(x,y) = P\, e^{i\int_C d\tau\, \dot{x}^\mu(\tau) A_\mu(x(\tau))}$$

$\hookrightarrow$ path ordered: n-th term in the expansion:

$$U_C(x,y) = \sum_{n=0}^{\infty}\frac{i^n}{n!} P\int d\tau_1\, \dot{x}^\mu A_\mu \dots \int d\tau_n\, \dot{x}^\mu(\tau_n) A_\mu(x(\tau_n))$$

P orders the (classical) matrices (not yet quant. ops.) $\to$ $\dot{x}^\mu(\tau) D_\mu U_C(x,y) = 0$ [covariantly const.]

Gauge transf: $U_c(x,y) \longmapsto g(x)\, U_c(x,y)\, g^\dagger(y)$ (U_c is a $N \times N$ unitary matrix)

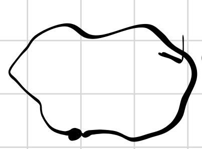
→ now consider a loop. $U_c(x,x) \mapsto g(x)\, U_c(x,x)\, g^\dagger(x)$ → the eigenv. are gauge inv.

↳ average of the eigenvalues: $\frac{1}{N} \operatorname{Tr}\left(U_c(x,x)\right) = W_C$

WILSON LOOP

$$W_C = \operatorname{Tr}\left(e^{i \oint_C d\tau\, \dot{x}^\mu(\tau) A_\mu(x(\tau))} \right) \longleftarrow$$

"diagnostic for the nature of the connection"



C

↳ it does not depend on the base point

QUANTIZATION OF GAUGE FIELDS

$$\langle W_C \rangle = \left\{ \int \mathcal{D}A_\mu\, e^{i \int d^4x \left(-\frac{1}{2g^2_{YM}} F_{\mu\nu}(x) F^{\mu\nu}(x) \right)} W_C \right\} \left\{ \int \mathcal{D}A_\mu\, e^{i \int d^4x \left(-\frac{1}{2g^2_{YM}} F_{\mu\nu} F^{\mu\nu} \right)} \right\}^{-1}$$

(need to add ghosts in perturbation theory)

→ Use a diagnostic tool

$$\langle W_C \rangle = \begin{cases} e^{-\sigma \operatorname{Area}(C)} \longrightarrow \text{CONFINEMENT} \\ e^{-SM \operatorname{Perimeter}(C)} \longrightarrow \text{DECONFINEMENT} \end{cases}$$

⇒ it depends on the scaling law!

↓

before quant. there is no dimensional const ⇒ σ, SM must come from sth:

σ : "string tension"

[if W_C indep. of C ⇒ only topological]

→ in 1 + 1 dim is solvable

in 1+2 it's already unsolvable

Perturbation Theory

→ Taylor expand non quadratic terms (quad ⇒ gauss ⇒ OK)

⇝ everything has to be renormalized:

$$g_{YM}^2\left(\frac{k}{\mu}\right) = \frac{1}{\frac{11}{16}\ln\left(k^2/\mu^2\right)}$$

⇒ ASYMPTOTIC FREEDOM (non pert. at short dist)

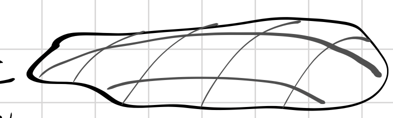
* instantons <u>do not</u> work here...
* CONFINING STRING:

$$\langle W_C \rangle = \sum_{\chi} g^{-\chi} \underbrace{\sum_{\xi \mid \partial\xi = C}}_{\text{disk-like topol.}} \overbrace{\mu(\xi,\chi)}^{\text{disk dep. measure}} e^{-\sigma A(\xi)}$$

⇒ "stat. sum" over surfaces

Euler characteristic ($\chi = 2-2g$)

→ not discrete! (But we don't know how to write it)



⇒ RELATION TO STRING THEORY → doesn't really work...

- Confinement → STRING TENSION
  - area law is OK semi-classical
- Regge trajectories ⇝ We can see them! ⇝ is there a string model underlying strong interac.
- Numerical ⇒ YM can be described by string models
- large N expansion → t'Hooft (1976) $g_s = \frac{1}{N}$

⇝ just evidence! No good models until AdS/CFT!

AdS/CFT

Super YM theory $\Rightarrow$ more fields

$$\langle W_C \rangle = \frac{\int [dA_\mu \ldots]\, e^{i\int d^4x \left(-\frac{1}{g_{YM}^2} F_{\mu\nu}F^{\mu\nu} \ldots\right)} W_C}{\int [dA_\mu \ldots]\, e^{i\int d^4x (\ldots)}}$$

$*$ CONCRETE EXAMPLE

$N=4$ SYM theory

$\rightarrow$ add 6 dim $\rightarrow$ type IIB superstrings on $AdS_5 \times S^5$ (N units of RR flux)

- const. curvature
- $R^4 = g_{YM}^2 N \alpha'$ (radius of S^5 and AdS_5)
  $\hookrightarrow$ (N of $SU(N)$)

$*$ type IIB $\longleftrightarrow$ SYM are perfectly dual! [i.e.: SAME THEORY!]

$\Rightarrow$ R big (small curv) $\rightarrow$ g_{YM} is small

$\rightarrow$ LARGE N PERTURBATION THEORY

$$\underset{(\text{gauge fixed})}{\mathcal{L}} = \mathrm{Tr}\Big\{-\partial_\mu A_\nu \partial^\mu A^\nu + \left(1-\frac{1}{\xi}\right)(\partial_\mu A^\mu)^2 + \partial_\mu \bar{c}\,\partial^\mu c - i g_{YM}\, \partial_\mu \bar{c}\,[A^\mu, c] - i g_{YM}\, \partial_\mu A_\nu [A^\mu, A^\nu] + \frac{1}{2} g_{YM}^2 [A_\mu, A_\nu][A^\mu, A^\nu]\Big\}$$

c are $N\times N$ non Hermitian matrices

$\rightarrow$ keep fields as matrices!

$\rightarrow$ Feynman gauge : $\xi = 1$

we still have global $U(N)$ sym.

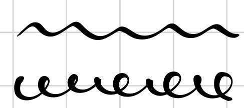
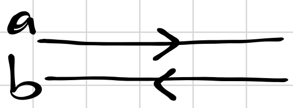
$g_{YM}=0$: "free field theory"

$$* \quad \langle A_\mu^{ab}(x) A_\nu^{cd}(y)\rangle \underset{g_{YM}=0}{=} \delta^{ad}\delta^{bc} \int \frac{d^4k}{(2\pi)^4} \frac{-i\eta_{\mu\nu}}{k^2 - i\epsilon}\, e^{ik_\mu(x-y)^\mu}$$

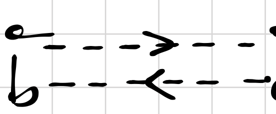
$$* \quad \langle c^{ab}(x)\, \bar{c}^{cd}(y)\rangle \underset{g_{YM}=0}{=} \delta^{ad}\delta^{bc} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - i\epsilon}\, e^{ik_\mu(x-y)^\mu}$$

NB : $U(N) \rightarrow \delta^{ad}\delta^{bc}$
$SU(N) \rightarrow \delta^{ad}\delta^{bc} - \frac{1}{N}\delta^{ab}\delta^{cd}$ $\Rightarrow$ same thing in $N\to\infty$ limit up to NLO

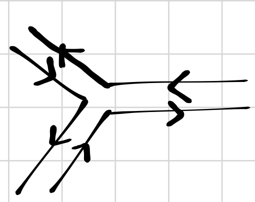
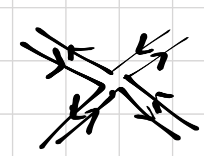
⇒ FAT GRAPH notation

$\langle A_\mu^{ab}(x)\, A_\nu^{cd}(x) \rangle$ (= ) = 

if A_μ are Lie Alg. notations! ⇓ field as matrices!

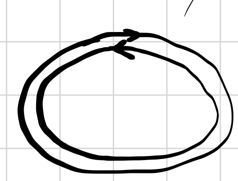
$\langle c^{ab}(x)\, c^{cd}(y) \rangle$ = 

Even vertices can be written this way!

g_{YM}   g_{YM}^2

The usual Feynman diag. is the result of connecting vertices and propagators:

→ everytime we sum color indices then we get a factor N

 (~ )

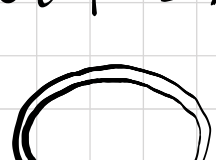
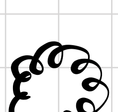
N^2

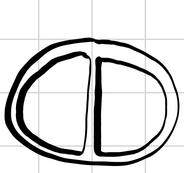
(index loop)

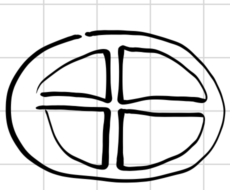
Then define:

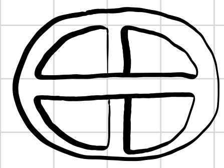
- E = no. of internal lines → "fat" lines
- V_3 = no. of 3 pt vert.
- V_4 = no. of 4 pt vert.
- F = no. of index loops
- $\mathcal{E}$ = no. ext lines

e.g.: VACUUM DIAGRAMS

 $\sim N^2$ ()

 $\sim g_{YM}^2 N^3 = (g_{YM}^2 N) N^2 = \lambda N^2$ ⇒ PLANAR DIAGRAMS

 $\sim g_{YM}^6 N^5 = \lambda^3 N^2$

 $\sim N^0 \Rightarrow$ NON PLANAR DIAGRAM

(cannot draw it on a plane/sphere)

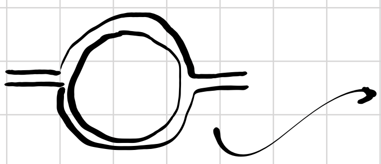
In particular they're behaviour is:

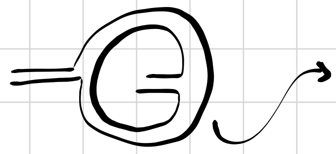
N^{χ} Euler char ($\chi = 2-2g-$holes)

$\Rightarrow$ Planar $\rightarrow$ sphere $\rightarrow$ N^2

Non plan $\rightarrow$ torus $\rightarrow$ N^0

Consider the two point funct.:

$N^1 g_{YM}^2 \sim$  DISK: think of the rupture points as holes in the ext. of the sphere.

$N^0 g_{YM}^2 \sim$  the rupture identify 2 holes as before but one is the interior and on ext.

In general:

$$N^F g_{YM}^{V_3} (g_{YM}^2)^{V_4}$$

$$\chi = F - E + V_3 + V_4$$

$L =$ (no. of momentum $\int$ that remains after we used all δ functions) $= E - V_3 - V_4 + 1$

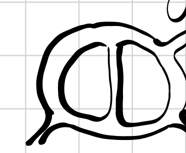
$$\mathcal{E} + 2E = 3V_3 + 4V_4$$

$$\Rightarrow \quad N^F g_{YM}^3 (g_{YM}^2)^4 = \lambda^{L+\mathcal{E}/2-1} N^{\chi - \mathcal{E}/2} \qquad (\lambda = g_{YM}^2 N)$$

LARGE N LIMIT

$\mathcal{E} = 0 \rightsquigarrow$ maximum Euler no. $\Rightarrow$ sphere $\chi = 2$

$\mathcal{E} \neq 0 \rightsquigarrow$ " $\Rightarrow$ sphere + hole $\chi = 1$ → put all ext. lines on the same face



↓

hole at ∞ plane ⟶ Planar limit ('t Hooft 1976)

Therefore in general:

$$\mathcal{A} = \sum_{\chi} N^{\chi - \mathcal{E}/2} \sum_{L} \lambda^{L + \mathcal{E}/2 - 1} \cdot \quad \text{(Feynman diag.)}$$

$\Rightarrow$ SYM ⟷ String Theory

relation with string theory: $N = 1/g_s$

N coincident D3 branes in type IIB superstring theory

NB: odd dim D-branes ⟶ type IIB

even dim D-branes ⟶ type IIA

	0	1	2	3	4	5	6	7	8	9	10
D3	x	x	x	x	o	o	o	o	o	o	o

→ point-like obj. 4-10

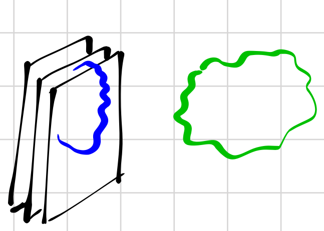
$\underbrace{0\ 1\ 2\ 3}$: Poincaré invar. (enhanced to SO(4,2) thanks to conf. inv.)

$\underbrace{4 \dots 10}$: point-like particles ⇓ SO(6) (spacetime symm.)

NB: brane conf. preserve 16 symm. ⇓ ½ BPS (supercon f. alg. brings the other 16 back)

How to study the configurations?

WEAK COUPLING



⇒ in absence of coupl the closed and open str do not interact ⇒ $\begin{cases} g_s N \ll 1 \to \text{small interactions} \\ \alpha' \text{ s.t. at low energy everything is desc by SYM } \mathcal{N}=4 \end{cases}$

STRONG COUPLING

* black D3 brane : $g_s N \gg 1$
(SUGRA)

* $$S_{SG} = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{g} \left\{ e^{-2\phi} \left(R + 4 (\nabla\phi)^2 \right) - \sum_p \frac{2}{(8-p)!} F_{p+2}^2 \right\}$$

Born-Infeld action at 1st order + floating grav.

$\Rightarrow$ EXACT SOLUTION OF D=10 SUGRA

Metric: $ds^2 = H^{-1/2}(r)\, \eta_{\mu\nu}\, dx^\mu dx^\nu + H^{1/2}(r)\, dx^m dx^m$ [BLACK D3-BRANE SOL.]

$$r = \sum x^m x^m$$

$$e^{2\phi} = g_s^2$$

$$F = (1 + *)\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d(1 - H^{-1}(r))$$

$$H(r) = 1 + \frac{L^4}{r^4} \quad \text{when } L = 4\pi g_s N \alpha'^2$$

The quantum fluctuations lead to $g_s N \gg 1$:

$\Rightarrow$ low energy: $L^4 \gg \alpha'^2 \Rightarrow 4\pi g_s N \gg 1 \Rightarrow g_s N \gg 1$

$\Rightarrow$ weak quantum gravity fluctuations: $L \gg L_{p,10} = \left[\frac{1}{2}(2\pi)^7 g_s^2 \alpha'^4\right]^{1/8}$

("turn off quantum gravity at low energy")

$\hookrightarrow$ $\underline{\underline{N^{\frac{1}{4}} \gg 1}}$

$\downarrow$

we $\underline{\text{need}}$ LARGE N

Now consider: NEAR HORIZON GEOMETRY

$$ds^2 = \frac{r^2}{L^2}\, \eta_{\mu\nu}\, dx^\mu dx^\nu + \frac{L^2}{r^2}\, dx^m dx^m \longrightarrow AdS_5 \times S_5$$

$\Rightarrow$ symm. enhancement due to $\begin{cases} \text{strong coupl.} \Rightarrow \text{near horizon} \\ \text{weak coupl.} \Rightarrow \text{superconf. symm.} \end{cases}$

What are the dofs of the theory?

MALDACENA $\longrightarrow$ dual limits $\begin{cases} \text{strong} \longrightarrow \text{only near horizon gravitons on } AdS_5 \times S_5 \\ \text{weak} \longrightarrow \text{decoupled gravitons from SYM} \end{cases}$

Therefore SYM $\mathcal{N}=4 \longleftrightarrow \underbrace{AdS_5 \times S_5}$

at small radius we can actually recover the entire string spectrum!

$\mathcal{N}=4$ SYM $\longleftrightarrow$ IIB string on $AdS_5 \times S^5$

* $L^2 = \sqrt{\lambda}\,\alpha'$
* N units F_5 flux $=$ $SU(N)$

} conjecture!

EVIDENCE

⇝ both sides have the same symmetries $PSU(2,2|4)$

⇝ multiplets MATCH

⇝ Protected correlation functions ARE IDENTICAL

⇝ Wilson loop, Localization

⇝ deformations from a fully supersymm YM can be found from string theory arguments

⇝ Integrability $\Rightarrow$ anomalous dimensions

To compute corel. func. in $\mathcal{N}=4$ SYM :

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\rangle$$

$$[D, \mathcal{O}_i(x)] = i\left(x^\mu \partial_\mu + \Delta_i\right)\mathcal{O}_i(x) \quad \text{(dilations generator)}$$

* What is the string dual to $\mathcal{O}_i(x)$
* Solve string theory (classical) with the b.c.
* $\varphi_i(x) \longrightarrow \varphi_0(x)$ at boundary of AdS_5 $(r \to \infty)$

$$\Rightarrow Z_{\text{STRING}}[\varphi_i \to \varphi_0] = \langle e^{i\int \varphi_0(x)\mathcal{O}(x)} \rangle$$

$\hookrightarrow$ SOURCE!

$$\parallel$$

i.e.: $\quad e^{i S_{\text{SUGRA}}[\varphi_i \to \varphi_0]}$

Wilson Loop

$$W_C = \langle \mathrm{Tr}\, P e^{i\oint A} \rangle$$

String Theory $\rightarrow$ $W_C = \sum_{\chi} g_s^{-\chi} \sum_{\zeta | \partial\zeta = C} \mu(\zeta, \chi)\, e^{-\sigma A[\zeta]}$

to compute all this beyond classical limit we need to solve string theory

$\mathcal{N}=4$ SYM

Euclidean space

anti-Herm. in Euclidean space

$$S = \int d^4x \; \mathrm{Tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + D_\mu \phi^i D_\mu \phi^i - \frac{1}{2} \sum_{i<j} [\phi^i, \phi^j]^2 + \text{fermions} \right\}$$

$i = 1, \dots, 6 \Rightarrow 6$ scalar fields

ϕ, A_μ $N \times N$ Hermitian matrices

$$\Rightarrow D_\mu \phi^i = \partial_\mu \phi^i - i g_{YM} [A_\mu, \phi^i]$$

$\hookrightarrow$ we're interested in $SU(N)$, but $\sim U(N)$ for $N \to \infty$ (the extra $U(1)$ decouples)

NB: start from the minimum of the potential

$$\sum_{i<j} [\phi^i, \phi^j]^2 \longrightarrow \phi^i = 0 \text{ is a minimum!}$$

$\Downarrow$

DEGENERATE!

$\rightsquigarrow$ choose minimal Higgs mechanism

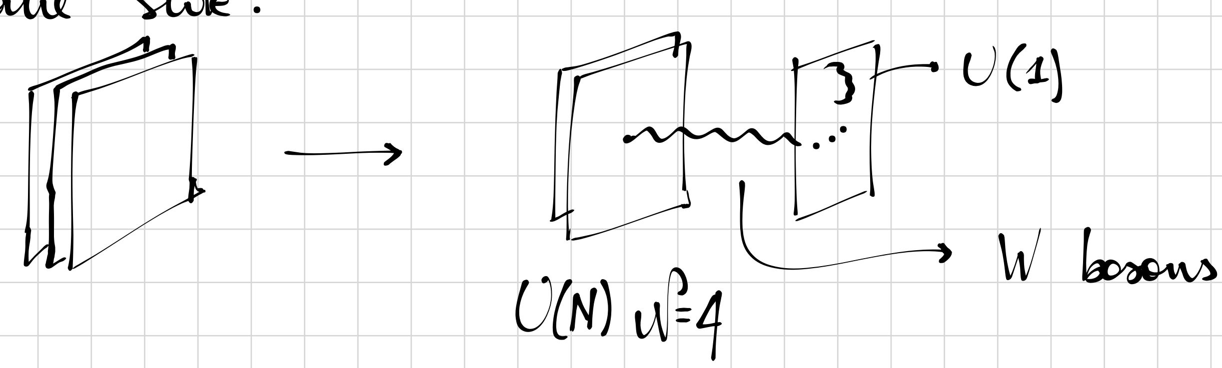
this condensate is still a 0 of the pot! $[\phi^i, \phi^j]_{i<j}$

$N+1$ square matrix : $\phi^6_{N+1,N+1} = \begin{pmatrix} 0 & \\ & \phi^6 \end{pmatrix} \Rightarrow \langle \phi^6 \rangle \neq 0$

$U(N+1)$

$$U(N+1) \longmapsto U(N) \times U(1)$$

On the brane side :



We now have a tunable mass for the W bosons!

$\rightsquigarrow$ brane: no attractive force between stacks $\Rightarrow$ move along flat direction!

NB: W bosons are charged:

$$[A_\mu]_{N+1,a} = [W_\mu]_a$$ $\longrightarrow$ index under the remaining $U(N)$

$\Rightarrow$ fundam rep.

$\rightsquigarrow$ under large N, fundam. rep. fields are "quenched"

$\rightarrow$ they contribute $O(N)$ and NOT $O(N^2)$ $\Rightarrow$ throw away!

Prop / ampl for W-boson at large N:

$$\mathcal{A} = \int [dx^\mu][dT]\, e^{-\int d\tau \left(\frac{\dot{x}^2}{2T} + m^2 T\right) + i\int A}$$

remaining (suppressed) $U(1)$

if interested in W_C: I can forget any W boson lines in $\langle W \rangle$ since they will be suppressed!

W boson strings pull and excite ϕ^6

$$W[X^\mu] = \left\langle \mathrm{Tr}\, \mathcal{P} e^{i\int d\tau\, \dot{X}^\mu(\tau) A_\mu(\tau) + \int d\tau\, |\dot{X}|\, \phi^6(X(\tau))} \right\rangle_{U(N)\ \mathrm{SYM}}$$

$\Rightarrow$ W bosons do not have back reaction!

because of this we can't really say that $\langle W_c \rangle$ is related holonomy

$\rightarrow$ SEMICLASSICAL LIMIT

$$\mathcal{A} = \int [dX^\mu][dT]\, e^{-\int d\tau \left(\frac{\dot{x}^2}{2T} + m^2 T\right)}$$

e.o.m.: $\quad -\dfrac{\ddot{X}}{2T} = 0 \qquad\qquad X^\mu(\tau) = (X_f - X_i)^\mu \dfrac{\tau}{\tau_p} + X_i^\mu$

$$T^2 = \frac{1}{4m^2}\int \dot{x}^2 d\tau \qquad\qquad T = \frac{1}{4m}\frac{(X_f - X_i)^2}{\tau_p}$$

Then $\mathcal{A} \sim e^{-m\sqrt{(x_f - x_i)^2}}$ $\rightsquigarrow$ can we improve it?

$\rightsquigarrow$ Add a term $i\int A$ to the path integral:

$$-\frac{\ddot{x}^\mu}{2T} + \frac{\delta}{\delta x^\mu} \ln W[x^\mu] = 0 \qquad \text{s.t.: } \frac{\delta}{\delta x^\mu} W\Big|_{\substack{\text{straight}\\ \text{line}}} = 0$$

$\Rightarrow$ the straight line is a solution in the semiclass lim:

$$\mathcal{A} \sim e^{-m\sqrt{(x_f - x_i)^2}} \langle W(\text{straight line}) \rangle$$

$\hookrightarrow \infty$ to have gauge invariance

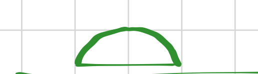
$\rightarrow$ PERTURBATIVE THEORY (small λ): (Feynman gauge $\xi = 1$)

$$W[x^\mu] = N + N\frac{\lambda}{16\pi}\int d\tau_1 d\tau_2 \, \frac{|\dot{x}(\tau_1)||\dot{x}(\tau_2)| - \dot{x}(\tau_1)\cdot\dot{x}(\tau_2)}{(x(\tau_1) - x(\tau_2))^2} + \dots$$

if we plug $x^\mu = (x_f - x_i)^\mu \frac{\tau}{\tau_f} + x_i^\mu$ in : the propagator cancels!!! $\Rightarrow$  $= 0$

NB: $\langle \mathrm{Tr}\, e^{g_{YM}\int(iA+\phi)} \rangle =$

$= \mathrm{Tr}\left\{ \mathbb{I} + g_{YM}^2 \iint \langle (iA+\phi)(iA+\phi)\rangle + \dots \right\}$



fat graph

where $\langle A_\mu^{ab} A_\nu^{cd} \rangle = \frac{1}{4\pi^2}\frac{1}{|x-y|^2}\delta_{\mu\nu}\delta^{ad}\delta^{bc}$

$\downarrow$

SUSY allows the simplification of a lot of Feynman diagrams

$\downarrow$

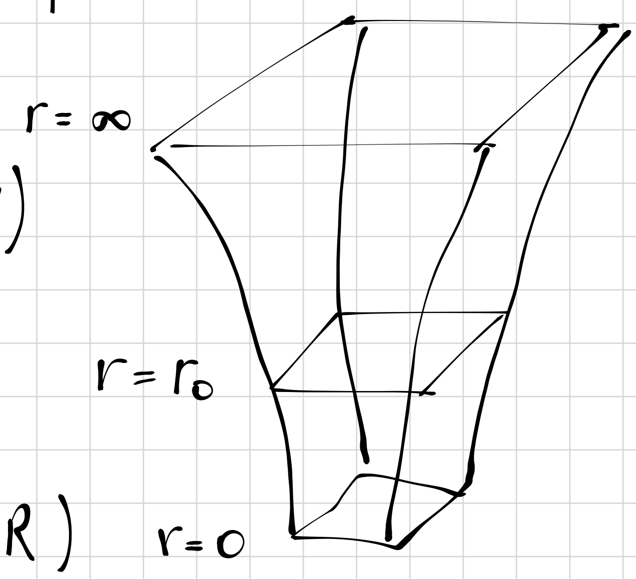
$\mathcal{N}=4$ SYM doesn't backreate on a straight line $\Rightarrow$ cancellation!

Then: $\mathcal{A} \sim N e^{-m\sqrt{(x_f - x_i)^2}}$

[It's a conjecture that this cancellations is OK for all orders!]

What happens on strong coupling side?

→ separate branes in AdS space!



$(r \to r/L)$

$$ds^2 = L^2\left(\frac{dr^2}{r^2} + r^2 dx^{\mu 2} + d\Omega_5^2\right)$$

* branes live (in this case) at $r = 0$

↓

when branes are separated the remaining brane is a PROBE BRANE

large N → switches off dynamics of gravity retaining the gravitational field ⇒ PROBE: "feels" gravity ⟶ place it at $r = r_0$ and it will float there!

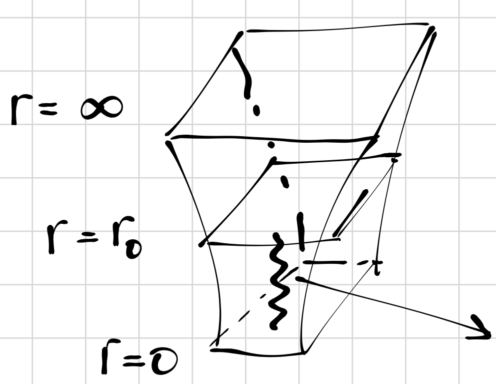
Why? → BORN-INFELD-ACTION

→ tension of D3-brane

it doesn't fall thanks to RR charge

$$S = T_3 \int d^4x \left[-\sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} + \omega_4 \right]$$

↳ analogous to $\langle \phi^6 \rangle \neq 0$ not changing the energy!

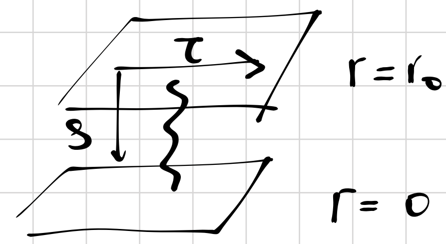


string σ model in AdS background

→ SEMICLASSICAL ⇒ only bosonic (i.e. no K-symm., no RR charges, etc.)

$$\mathcal{A} = e^{-\frac{1}{2\pi\alpha'}\text{Area}} \Rightarrow \text{embedding?}$$

consider W-boson on straight line



$$\Rightarrow X^\mu = (x_f - x_i)^\mu \frac{\tau}{\tau_f} + x_i^\mu$$

$$r = s$$

On the sphere it should look like a point!

$$X^m = r\,\hat{\theta}^m \quad \hookrightarrow \text{unit vector}$$

no $d\Omega_5$!

→ Induced METRIC: $d\sigma^2 = L^2\left(\frac{ds^2}{s^2} + s^2\,\frac{(x_f-x_i)^2}{\tau_p^2}\,d\tau^2\right)$

$$\sqrt{g} = L^2\,\frac{1}{\tau_p}\sqrt{(x_f-x_i)^2}$$

$$L^2 = \sqrt{\lambda}\,\alpha'$$

then $\frac{1}{2\pi\alpha'}\,\text{Area} = \frac{\sqrt{\lambda}\,\alpha'}{2\pi\alpha'}\int_0^{\tau_p} d\tau \int_0^{r_0} ds\,\frac{1}{\tau_p}\sqrt{(x_f-x_i)^2} = \frac{\sqrt{\lambda}}{2\pi}\,r_0\sqrt{(x_f-x_i)^2}$

Then

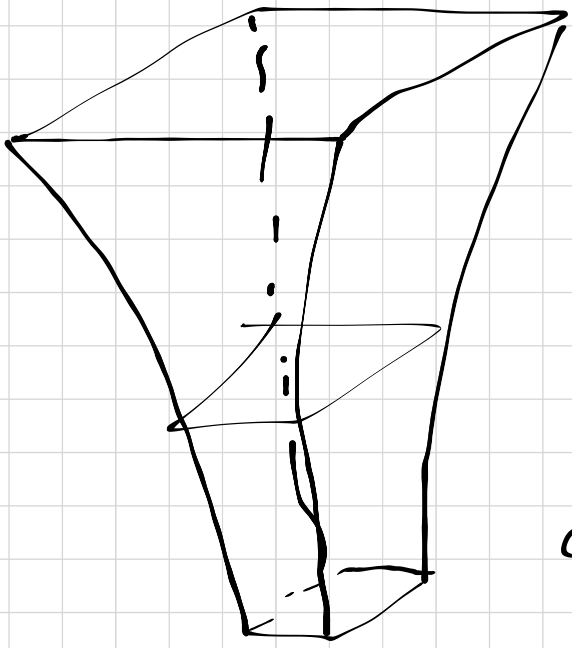
$$A_{\text{string}} = e^{-\frac{\sqrt{\lambda}}{2\pi}\,r_0\sqrt{(x_f-x_i)^2}}$$ (there would be N if $\neq$ disk)

$$A_{\text{field}} = e^{-m\sqrt{(x_f-x_i)^2}}\,N$$

→ they're = if the mass of the W boson:

$$m = \frac{\sqrt{\lambda}}{2\pi}\,r_0$$

The picture is



$$\lambda \ll 1: \quad \mathcal{A} \sim e^{-m\sqrt{(x_f-x_i)^2}}$$

$$\lambda \gg 1: \quad \mathcal{A} \sim e^{-\frac{\sqrt{\lambda}\, r_0}{2\pi}\sqrt{(x_f-x_i)^2}}$$

consistent with: $\quad m = \dfrac{\sqrt{\lambda}\, r_0}{2\pi}, \quad W\left[\begin{smallmatrix}\text{straight}\\ \text{line}\end{smallmatrix}\right] = N$

One W boson is of course charged under the $U(1)$ field on the probe brane.

Landau orbits!

* Field POV:

EUCLIDEAN

$$S = \int d\tau \left[\frac{\dot{x}^2}{4T} + m^2 T + \frac{F_{\mu\nu}}{2} x^\mu \dot{x}^\nu\right] \qquad F_{12} = E$$

$$\rightarrow \begin{cases} \dfrac{\ddot{x}^\mu}{2T} + F^{\mu\nu}\dot{x}^\nu \\ T^2 = \dfrac{1}{4m^2}\displaystyle\int \dot{x}^2 d\tau \end{cases} \Rightarrow \quad x^\mu(\tau) = \frac{m}{E}\left(\cos(2\pi\tau), \sin(2\pi\tau), 0, 0\right)$$

use e.o.m.

$$\Rightarrow S = 2\pi \frac{m^2}{E} - 2\pi E \frac{1}{2}\left(\frac{m}{E}\right) = \pi \frac{m^2}{E}$$

Therefore:

$$\mathcal{A} = \int [dx^\mu][dT]\, e^{-S}\, W[x^\mu]$$

technically it is $-\dfrac{\ddot{x}^\mu}{2T} + F^{\mu\nu}\dot{x}^\nu - \dfrac{\delta \ln W[x]}{\delta x^\mu} = 0$

but $\dfrac{\delta}{\delta x^\mu} W\Big|_{\text{circle}} = 0$

$\longrightarrow$ the circle is a valid semiclass. solution!

Then:

$$\mathcal{A} \overset{\text{semic.}}{=} e^{-\pi\frac{m^2}{E}}\, W[\text{circle}]$$

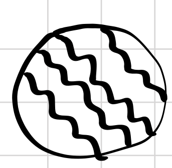
NB: perturbatively can be complicated $\Rightarrow W[x^\mu]$ at 2^{nd} order is not exactly computable.

Now we compute W perturbatively:

$$W = N\left\{ 1 + \frac{\lambda}{16\pi^2}\int d\tau_1 d\tau_2 \frac{|\dot{x}(\tau_1)||\dot{x}(\tau_2)| - \vec{\dot{x}}(\tau_1)\cdot\vec{\dot{x}}(\tau_2)}{\left(x(\tau_1)-x(\tau_2)\right)^2} + \dots \right\}$$

$$= \text{ALONG THE CIRCLE} = N\left\{ 1 + \frac{\lambda}{16\pi^2}\cdot\frac{2\pi^2}{2} + \dots\right\} =$$

$$= N\left\{ 1 + \frac{\lambda}{8} + \dots\right\}$$

e.g.:  $\Rightarrow$ becomes easier! $\longrightarrow$ MATRIX MODELS

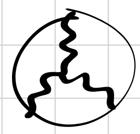
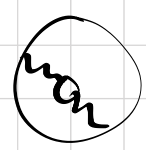
$\rightsquigarrow \langle W(\text{circle})\rangle = \langle \mathrm{Tr}\, e^{M}\rangle_M$ MATRIX

$\hookrightarrow Z = \int d^{N^2}M\, e^{-\frac{8\pi^2}{\lambda}\mathrm{Tr}\, M^2}$

modified Bessel func.

LARGE N: $\langle W\rangle = N\frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \Rightarrow$

With vertex?

 +  $+ \dots = 0$ (assuming symm.)

Now large $N \longrightarrow$ comparison with string order

$$\langle W\rangle \sim e^{\sqrt{\lambda}}\frac{I_N}{\sqrt{\lambda}}\lambda^{-3/4}$$

$\rightsquigarrow$ String Theory Action

$$S = \frac{L^2}{4\pi\alpha'} \int d^2\sigma \left(r\, \partial X \cdot \bar{\partial} X + \frac{\partial r \bar{\partial}}{r^2} \right)$$

$$\Rightarrow r^2 (\partial X^2) + \frac{(\partial r)^2}{r^2}$$

$$r^2 (\bar{\partial} X)^2 + \frac{\bar{\partial} r_0}{r^2} = 0$$

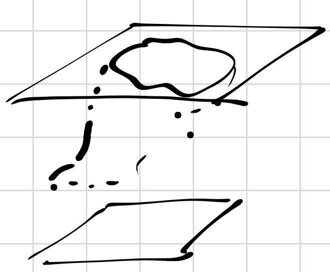
Thus

$$\partial\bar{\partial} r = r^3 \partial X \cdot \bar{\partial} X + \frac{1}{r} \partial r \bar{\partial} r$$

$$\partial (r\, \bar{\partial} X)$$

+ b.c. containing $F_{\mu\nu}$

(AdS space)



$$\rightarrow \vec{X}(\tau, \sigma) = R\left(\cos(2\pi\tau), \sin(2\pi t), 0\right)$$

$$r(\tau, \sigma) = r_0$$

Find solution with symmetries only?

$$\vec{X} = \frac{\cosh(2\pi\sigma_0)}{\cosh(2\pi\sigma)} R\left(\cos(2\pi\tau), \sin(2\pi\tau), 0, 0\right)$$

$$r = r_0 \frac{\tanh 2\pi\sigma}{\tanh 2\pi\sigma_0} \longrightarrow \sinh(2\pi\sigma_0) = \frac{1}{R r_0}$$

Plug everything back into the action:

boundary condition!

$$S = \sqrt{(\underbrace{\sqrt{\lambda}\, r_0 R}_{\text{W boson mass}})^2 + \lambda} - \sqrt{\lambda} - \frac{1}{2} \cdot 2\pi E R^2$$

$$= \frac{\pi m^2}{E} - \sqrt{\lambda} - \cancel{\frac{\lambda E}{4\pi M^2}} \to 0 \text{ in low energy limit}$$

$$\Rightarrow \mathcal{A} = e^{-\frac{\pi M^2}{E} + \sqrt{\lambda}} \longrightarrow \text{Wilson loop contribution}$$

What about the term $-\dfrac{\lambda E}{4\pi M^2}$?

$$\Rightarrow E_c = \frac{2\pi m^2}{\sqrt{\lambda}} \Longrightarrow \text{in Born-Infeld action: } \sqrt{\det(g + 2\pi\alpha' F)}$$

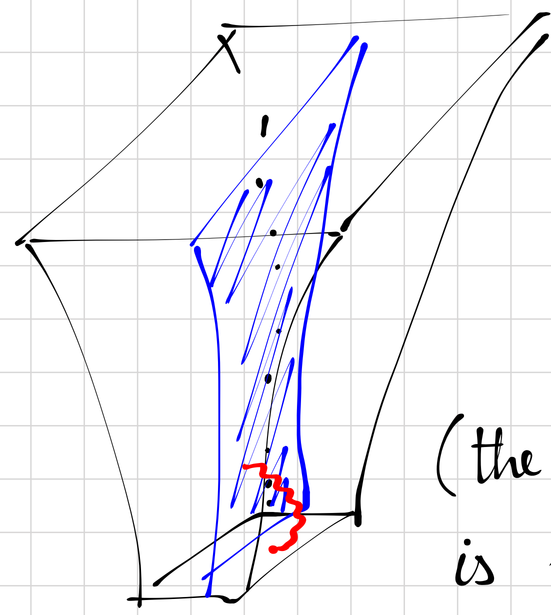
→ E_c is the limit for brane stability: insert induced D3-brane metric and E_c in BI action to see!

PROBE BRANES

* Assume that the no. of probe branes N_F is such that:

$$N_F \ll N$$

(no backreaction)



(the stack of D3s is at the horizon)

- Randall, Karch
- de Wolf, Freedman, Ooguri

Consider D3-branes and D5-branes:

	0	1	2	3	4	5	6	7	8	9
D3	×	×	×	×	o	o	o	o	o	o
D5	×	×	×	o	×	×	×	o	o	o

$\underbrace{0\ 1\ 2}_{2+1\text{-Poinc.}}$, $\underbrace{4\ 5\ 6}_{SO(3)}$, $\underbrace{7\ 8\ 9}_{\widetilde{SO}(3)}$

→ $\frac{1}{2}$ supersymmetric conf.

it's a defect field theory!

↳ enhanced to $Osp(4|4)$

$$AdS_4 \subset AdS_5$$
$$S^2 \subset S^5$$

MAXIMAL

$$\Rightarrow (x^1)^2 + \ldots + (x^6)^2 = 1$$
$$(x^1)^2 + \ldots + (x^3)^2 = 1$$
and $x^4, \ldots, x^6 = 0$

D-brane action (field theory):

$$S_D = \int d^3x \, \{ |D_\mu q|^2 - i\bar{\psi} D\psi \} + \text{interactions}$$

scalar ISO(2,1)
spinor SO(3)

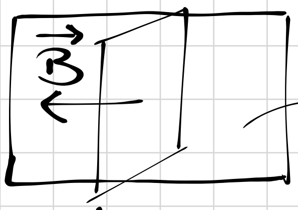
spinor ISO(2,1)
spinor SO(3)

$\Rightarrow$ global $U(1)$ charge corresponding to the inserted D brane

$\Rightarrow$ if $N_F > 1 \Rightarrow$ instead of $U(1)$ we get $U(N_F)$ flavour index

bi-fundamental of $U(N)$ and $U(N_F)$ $\Leftarrow$

see N. Evans et al.
[EM field on branes]



bulk: $\mathcal{N}=4$ SYM

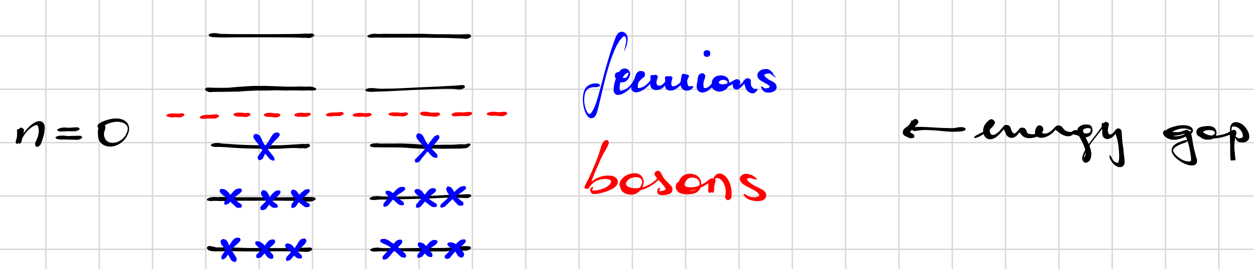
$\hookrightarrow$ defect $\Rightarrow S_D$ field theory lives there!

$\Rightarrow$ the D-brane pinches off and creates a gap with the others $\Rightarrow$ strings have a minimum lenght $\Rightarrow$ MASS GAP! $\longrightarrow$ chiral symmetry breaking

EM field + bos & ferm $\Rightarrow$ Landau level : $\omega_n = \sqrt{B(2n+1)}$ (boson)
$\omega_n = \pm\sqrt{B\,2n}$ (ferm.)

$\longrightarrow$ SUSY is broken!

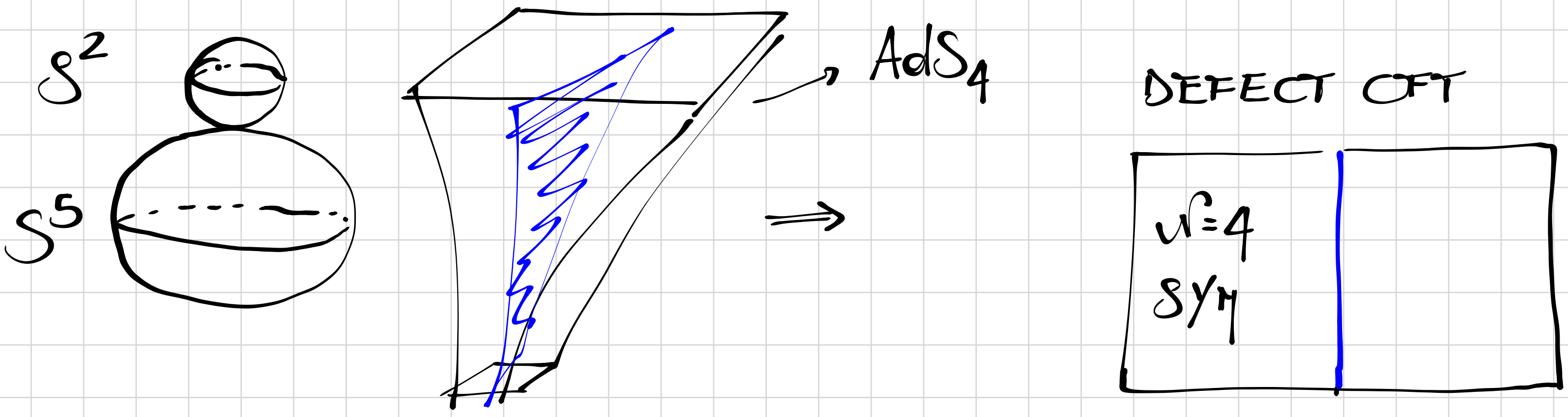
Low energy states :



How to solve degeneracy? $\Rightarrow$ symm. spin wave function (antisymm. in space) $\Rightarrow$ aligned spin $\downarrow$ GRAPHENE

QUANTUM HALL
FERROMAGNETISM

Let's consider again the same setup.



$$S_D = \int d^3x \left\{ |D_\mu q|^2 - i\bar{\psi}\not{D}\psi + \dots \right\}$$

[Finite T approach: AdS BH ⟶ Hawking T]

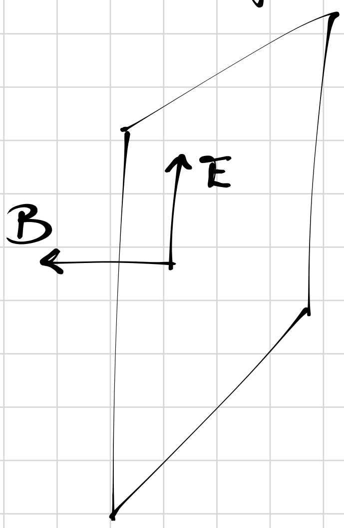
⇒ S^2, S^5 have same radius: $\sum_{i=1}^{6} x_i^2 = 1$, $\sum_{i=1}^{3} x_i^2 = 1$

⇓

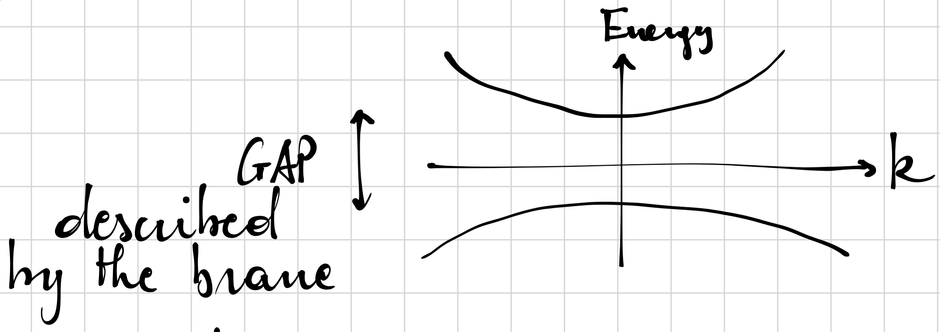
the introduction of the brane breaks the symm. ⟵ SO(3) symmetry ⟶ 4
$\widetilde{SO}(3)$ " ⟶ 9

Now insert a magnetic field B: it "breaks" the defect ⇒ the brane shrinks ⟶ the S^2 cycle shrinks to a point.

Now insert an electric field:



the E field brings the brane towards the brane:



⟶ the Electric field fills the gap

Probe brane $\longrightarrow$ no backreaction on $AdS_5 \times S_5$

* Geometry: $\longrightarrow$ DBI action is the effective action for open strings at disk level:

Dirac - Born - Infeld action:

Chern-Simon

$$S_5 = \frac{T_5}{g_s} \int d^2\sigma \left\{ \sqrt{-\det(g + 2\pi\alpha' F)} + 2\pi\alpha' C^{(4)} \wedge F \right\}$$

from disk ampl.

induced metric from the embedding

$$T_5 = \frac{1}{(2\pi)^5 \alpha'^3} \quad , \quad 4\pi g_s = g_{YM}^2$$

split S^5! S^2 on top!

Now: $$ds^2 = \sqrt{\lambda}\,\alpha' \left[r^2(-dt^2 + dx^2 + dy^2 + dz^2) + \frac{dr^2}{r^2} + d\psi + \sin^2\psi\,(d\theta^2 + \sin^2\theta\, d\phi^2) + \cos^2\psi\,(d\tilde{\theta}^2 + \sin^2\tilde{\theta}\, d\tilde{\phi}^2) \right]$$

then D5-brane: $(t, x, y, r, \theta, \phi)$, $z = 0$

$$\Rightarrow d\sigma^2 = \sqrt{\lambda}\,\alpha' \left[r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}(1 + r^2\psi'(r)^2) + \sin^2\psi(r)(d\theta^2 + \sin^2\theta\, d\phi^2) \right]$$

Gauge fields? find e.o.m. const

$$2\pi\alpha' F = \sqrt{\lambda}\,\alpha' \left[\frac{da(r)}{dr} dr \wedge dt + b\, dx \wedge dy \right]$$

$\Rightarrow$ $\psi(r)$, $a(r)$ will tell us everything we need!

no. of D5-branes

Then:

$$S = -\frac{2\sqrt{\lambda}\, N N_5}{(2\pi)^3} V_{2+1} \int_0^\infty dr\ 2\sin^2\psi(r)\sqrt{b^2 + r^4} \cdot \sqrt{1 + r^2\psi'(r)^2 - a'(r)^2}$$

s.t. other const simplify

$$\Rightarrow a(r) \text{ is cyclic} \Rightarrow \frac{d}{dr}\left(\frac{\delta S}{\delta a'(r)}\right) = 0 \Rightarrow \frac{\delta S}{\delta a'(r)} = q_5\, \zeta$$

Then we plug it back in + Legendre transf:

$$R_5 = S_5 - \int dr\, a'(r)\, \frac{\delta S}{\delta a'(r)}$$

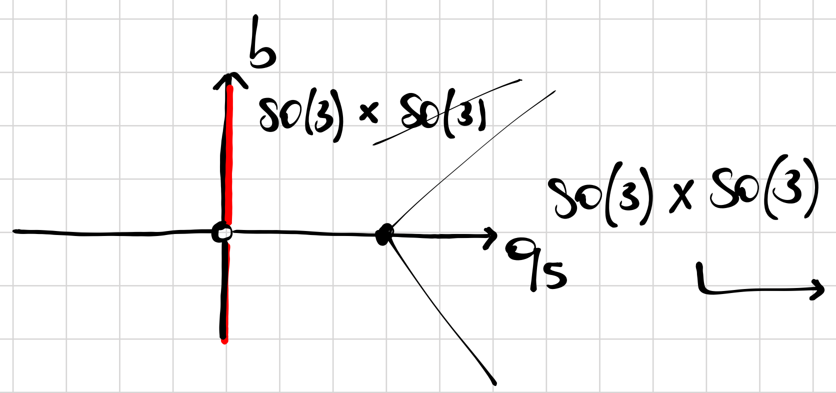
$$\longrightarrow \quad R_5 = -\frac{2\sqrt{\lambda}\, N N_5 V_{2+1}}{(2\pi)^3} \int_0^\infty dr \sqrt{4\sin^2\psi(r)(b+r^4) + q_5^2}\, \sqrt{1+r^2\psi'(r)^2}$$

together with $\psi(r\to\infty) = \frac{\pi}{2}$

NB: $b, q_5 = 0 \implies \psi = \text{const}$ is a sol.

b ON $\implies$ spoils $r\to 0$ behaviour

q_5 ON $\implies$ there's a critical point where $\psi=\text{const}$ works again



$\hookrightarrow$ no charge gap

D7 - brane $\longrightarrow$ wraps 2 S^2 inside S^5 $\to$ at $r=\infty$ one of those pinches off $\to$ D5 b.c.

$$S_7 = \frac{T_7}{g_s}\int d^8\sigma \left[-\sqrt{-\det(g+2\pi\alpha' F)} + \frac{(2\pi\alpha')^2}{2} C^{(4)}\wedge F\wedge F \right]$$

$\longrightarrow \quad AdS_4 \times S^2 \times S^2$

relevant CS term ($C^4 \sim$ axion)

$\rightsquigarrow$ does it compete with the D5 solution?

* Ansatz for metric $\to$ wraps one S^2 more w.r.t. to previous D5
* $C^{(4)} = \lambda\alpha'^2 \left[r^4\, dt\wedge dx\wedge dy\wedge dz + \frac{C(\psi)}{2}\, d\cos\theta\wedge d\phi\wedge d\cos\tilde\theta\wedge d\tilde\phi \right]$

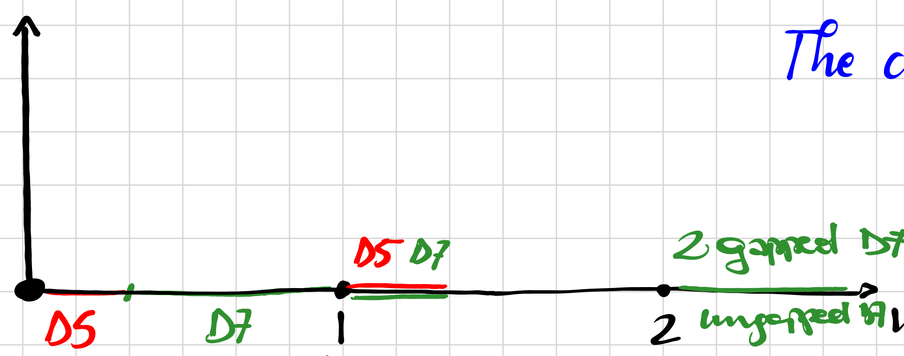
where $C(\psi) = \psi - \frac{1}{4}\sin^4(\psi) - \frac{\pi}{2}$

$$R_7 = -\frac{2\lambda N}{(2\pi)^4} V_{2+1}\int_0^\infty dr \sqrt{4\sin^4\psi\,(f^2+4\cos^4\psi)(b^2+r^2) + (q+2bc)_5^2} \cdot \sqrt{1+r^2\psi'(r)}\, \sqrt{4\sin^4\psi\, f^2(b+r^4)+q^2}$$

D7-brane is lower in energy $\longrightarrow$ preferred state

The chain stops at N_5

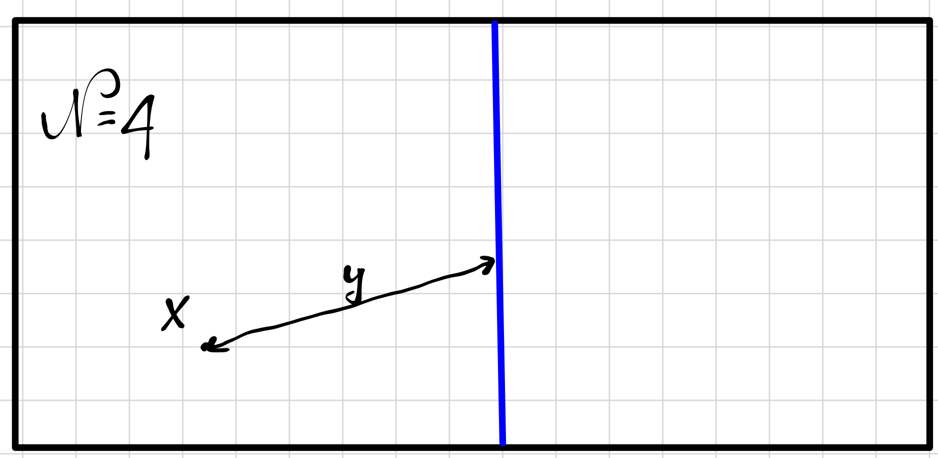
(i.e.: no more than the no. of flavours)



$\nu = \dfrac{q}{b/2\pi}$ (filling fraction)

$\nu = 1$ QUANTUM HALL

DEFECT CFT



$x^\mu = (\vec{x}, y)$

Operators:

$O_I(x)$ in bulk

$\hat{O}_I(x)$ on defect

[$\mathcal{N}=4$ ops.]

$$\Rightarrow \quad [D, O_I(x)] = i(x^\mu \partial_\mu + \Delta_i)\, O_I(x)$$

$$[D, \hat{O}_I(x)] = i(x^\mu \partial_\mu + \hat{\Delta}_I)\, \hat{O}_I(x)$$

$\Rightarrow$ One point function:

$$\langle O_I(x) \rangle = \frac{C_I}{|y|^{\Delta_I}} \longrightarrow \text{distance from defect}$$

R-symm: $SO(6) \longrightarrow SO(3) \times SO(3)$

$\Rightarrow$ CORR FUNC:

$$\langle O_I(x)\, \hat{O}_J(0) \rangle = \frac{d_{IJ}}{|y|^{\Delta_I - \hat{\Delta}_J} (x^\mu x_\mu)^{\hat{\Delta}_J}} \quad \text{(fixed by conf. sym.)}$$

asymptotically this shouldn't depend on the defect

$$\langle O_I(x)\, O_J(x') \rangle = \frac{C_{IJ}\, f(\xi)}{|y|^{\Delta_I}\, |y|^{\Delta_J}} \quad \text{where} \quad \xi = \frac{(x-x')^\mu (x-x')_\mu}{4yy'}$$

$\mathcal{N}=4$ chiral primary operators

$$\mathcal{O}_I(x) = \sum_{i-} C_I^{i_1 \dots i_\Delta} \langle 0 | \mathrm{Tr}\, \phi^{i_1}(x) \phi^{i_2}(x) \dots \phi^{i_\Delta}(x) \, | 0 \rangle \, \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2} \sqrt{\Delta}}$$

irrep of $SO(6)$

't Hooft coupl.

Normalized s.t.: $\sum C_I^{i_1 \dots i_\Delta} C_J^{i_1 \dots i_\Delta} = \delta_{IJ}$

without defect $\langle \mathcal{O}_I(x) \mathcal{O}_J(y) \rangle = \dfrac{\delta_{IJ} \, \delta_{\Delta_I \Delta_J}}{|x-y|^{\Delta_I + \Delta_J}}$

$\Rightarrow$ I has $\dfrac{(3+\Delta)(2+\Delta)^2(1+\Delta)}{12}$ values

$\rightsquigarrow$ superconformal primary

$$[\mathcal{O}(x), S] = 0$$

$\rightsquigarrow$ chiral primary

$$[\mathcal{O}(x), Q^a] = 0 \text{ for some } a$$

(for chiral $\to \frac{1}{2}$ BPS $\Rightarrow$ half of them)

$$\{Q, S\} = \underset{\substack{\downarrow \\ \text{R-symm}}}{T} + \underset{\substack{\downarrow \\ \text{spin}}}{\Sigma} + D$$

$\longrightarrow$ $\{S, [\mathcal{O}, Q^a]\} = 0 \longrightarrow [\mathcal{O}, T + \Sigma + D] = 0$

$\Rightarrow$ some T, Σ cancel the value of D (conf. dim.)

$\Rightarrow$ $C_J^{i_1 \dots i_J}$ form $SO(6)$ sph. harmonics

$\Rightarrow SO(6)$: $Y_I(\hat{x}) = \sum_{i_1 \dots i_\Delta} C_I^{i_1 \dots i_\Delta} \hat{x}^{i_1} \dots \hat{x}^{i_6}$

$(\hat{x} \in S^5)$

$\longrightarrow$ $\nabla^i \hat{x}^j = \dfrac{1}{|x|}\left(\delta_{ij} - \hat{x}^i \hat{x}^j\right) \longrightarrow -\vec{\nabla}^2 Y_I = \dfrac{\Delta_I(\Delta_J + 4)}{x^2} Y_J$

$$-\nabla^2 = -\frac{1}{x^5}\frac{d}{dx}x^5\frac{d}{dx} + \frac{L_{ij}^2}{x^2} \longrightarrow L_{ij} = -i\left(x_i\nabla_j - x_j\nabla_i\right)$$

$$\Rightarrow L_{ij}^2 Y_I = \Delta(\Delta+4)Y_I$$

With S^5 metric: (NB: 2 2-spheres fibered over an interval)

$$\rightarrow \quad ds^2 = d\psi^2 + \cos^2\psi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + \sin^2\psi\left(d\tilde\theta^2 + \sin^2\tilde\theta\, d\tilde\phi^2\right)$$

$$\Rightarrow \left(\frac{1}{\sin^2\psi\cos^2\psi}\frac{d}{d\psi}\sin^2\psi\cos^2\psi\frac{d}{d\psi}\right)Y_I = -\Delta(\Delta+4)Y_I$$

call $z = e^{2i\psi} \Rightarrow \frac{d}{d\psi} = 2iz\frac{d}{dz}$

$$\rightarrow \left[-\left(z\frac{d}{dz}\right)^2 + \left(1+\frac{\Delta}{2}\right)\right]\frac{Y_I(z)}{z-\frac{1}{z}} = 0$$

$$Y_{IA}(z) = \frac{(-1)^{\Delta/2}}{2^{\frac{\Delta-1}{2}}\sqrt{(\Delta+1)(\Delta+2)}}\;\frac{z^{1+\Delta/2} - z^{-1-\Delta/2}}{z - 1/z} \Rightarrow \Delta \text{ even integer}$$

$\Rightarrow$ looks like an $SU(2)$ character: (Taylor exp.)

$$Y_I(z) = \dots\left(z^{\Delta/2} + z^{\Delta/2-1} + \dots + z^{-\Delta/2}\right)$$

$$Y_I(\psi) = \frac{(2+\Delta)!}{2^{\Delta+1/2}}\frac{1}{\sqrt{(\Delta+1)(\Delta+2)}}\sum_{p=0}^{\Delta/2}\frac{(-1)^p\sin^{\Delta-2p}\psi\cos^{2p}\psi}{(2p+1)!\,(1+\Delta-2p)!}$$

$\rightsquigarrow$ there's only one $SO(3)\times SO(3)$ invariant!

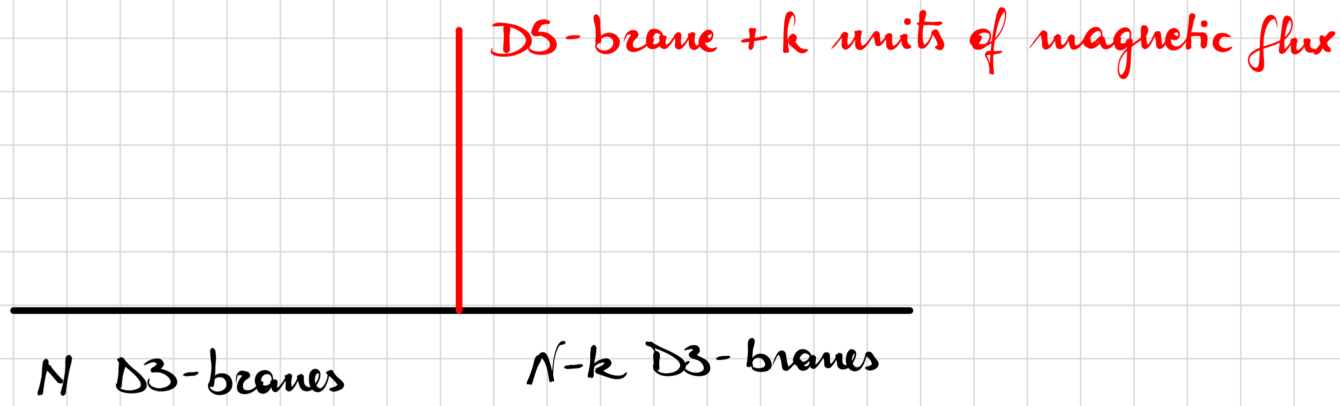
$\rightarrow \quad i_1,\dots,i_\Delta = 1,2,3$ (instead of $1,\dots,6$)

Now go back to D3 - D5 brane system:

Space: $AdS_5 \times S^5$

D5-brane world volume: $AdS_4 \times S^2$

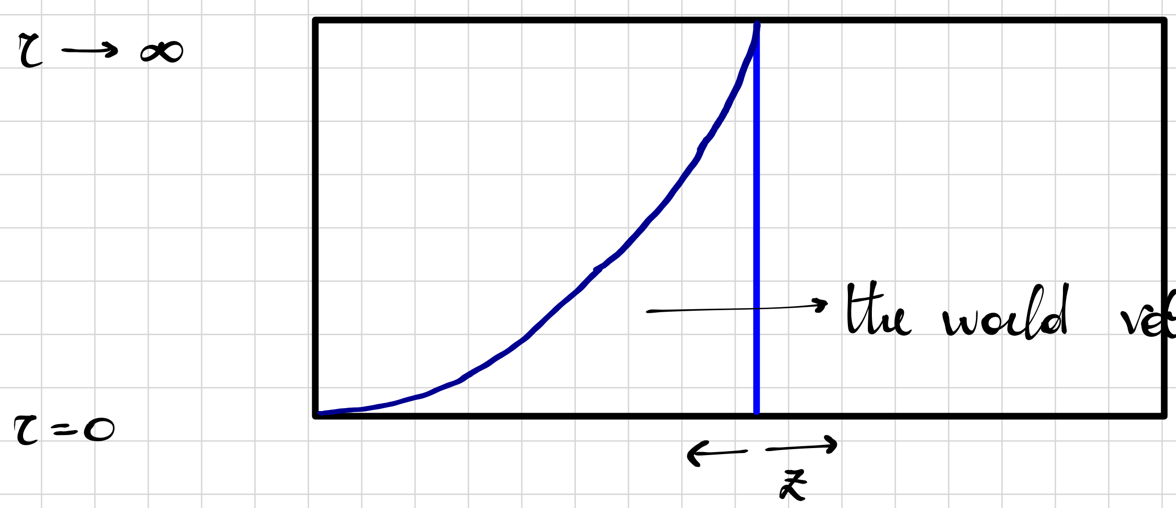
D3-branes can end on D5 branes!



Magnetic fluxes help in the construction:

this time it's relevant!

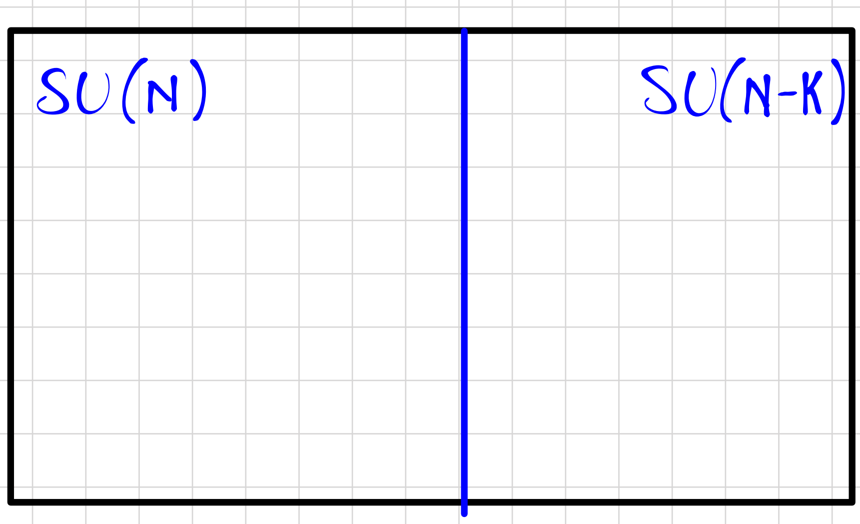
$$S = \frac{T_5}{g_s} \int d^6\sigma \left\{ -\sqrt{-\det(g + 2\pi\alpha' F)} + 2\pi\alpha' F \wedge C^{(4)} \right\}$$



→ the world volume changes as $z = \frac{K}{r}$

but it's still $AdS_4 \times S^2$

Now we have:



→ the definition of defect CFT is still an open problem!

we work in large K approx: $\sqrt{\lambda} \ll K \ll N$ (ok because $\sqrt{\lambda}$ small.

→ string theory is ≠ since $\sqrt{\lambda}$ big)

→ the expansion over $\frac{\lambda}{K^2}$ is meaningful from both sides $\Rightarrow \frac{\lambda}{K^2} \ll 1$

→ how to reproduce 1 pt funct? ⟹ TADPOLE

⇝ condensate of sth → HIGGS [cannot be const however]

$\mathcal{N}=4$ SYM:

$$A_\mu = 0$$
$$\lambda^a_\sigma = 0$$

$$\Rightarrow \nabla^2 \Phi_i - \sum_{j=1}^{6} \left[\Phi_i, \left[\Phi_j, \Phi_i \right] \right] = 0 \qquad (*)$$

dist from def.

$$\Rightarrow SU(N) \text{ charge}: \sum_{i=1}^{6} \left[\Phi_i, \nabla^\mu \Phi_i \right] = 0 \qquad (**)$$

$$\Rightarrow \Phi_i(y) = -\frac{1}{|y|} T_i^{K \times K} \oplus 0^{(N-K)\times(N-K)}$$

SU(2) matr.

$$\Phi_i = -\frac{1}{|y|} \left(\begin{array}{c:c} T_i \; \}K & 0 \\ \hdashline \underbrace{0}_{K} & 0 \end{array} \right) \in \mathrm{Mat}(N \times N) \Rightarrow \text{sol to } (*) \text{ and } (**)$$

the trace of such ops.

$$\rightarrow \langle O_I(x) \rangle = \frac{Y_I(0)\,(2\pi^2)^{\Delta/2}}{\sqrt{\Delta}\,\lambda^{\Delta/2}} (K^2-1)^{\Delta/2}\, K\, \frac{1}{|y|^\Delta} \qquad (\Delta \text{ even integer})$$

STRING theory side:

- dual of the chiral operator?

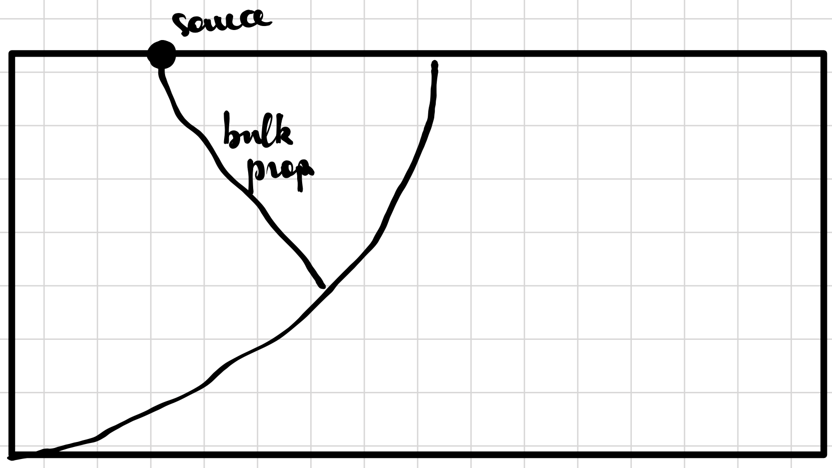
$$h^{AdS}_{\mu\nu} = \frac{2\Delta(\Delta-1)}{\Delta+1} S\, g_{\mu\nu} + \frac{4}{\Delta+1} \nabla_\mu \nabla_\nu S$$

$$h^{S^5}_{\alpha\beta} = 2\Delta S\, g_{\alpha\beta}$$

$$a^{AdS}_{\mu\nu\rho\sigma} = 4\sqrt{g^{AdS}}\, \varepsilon_{\mu\nu\rho\sigma\eta} \nabla^\eta S$$

$\Rightarrow$ find Green funct. for scalar mode:

(arXiv: 1611.04603)



after a lot of struggle this was computed on gauge theory side

string theory naturally gives perturbative corrections

$$\langle O_I(x) \rangle = \frac{Y_I(0)(2\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} \frac{K^{\Delta+1}}{|y|^\Delta} \left(1 + \frac{\lambda}{\pi^2 K^2} \left[\frac{3}{2} + \frac{(\Delta-2)(\Delta-3)}{4(\Delta-1)} \right] + \ldots \right)$$