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Boundary States for GS superstrings in an Hpp wave background. [*]

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ABSTRACT: We construct the boundary states preserving half the global supersymmetries in string theory propagating on a Hpp background.

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1. Introduction and conclusions.

Soon after the conjecture of AdS/CFT correspondence [1], some authors addressed the problem of solving the string theory in $AdS \times S$ backgrounds. The first approach [2] made use of the so-called "killing gauge" for κ-symmetry in order to get the string action in these backgrounds. The actions turned out to be almost intractable, so a renewed attempt was made, using this time light cone gauge [3]; the models still remained not solvable. However, it was recently pointed out [4] that a conformal model describing type IIB superstring propagating in a particular wave metric supported by a *Ramond-Ramond* 5-form background [8]:

$$ds^2 = 2dx^+dx^- \quad -\mathrm{f}^2 x_I^2 dx^+ dx^+ + dx^I dx^I \ , \qquad I = 1, ..., 8 \ , \qquad (1.1)$$

$$F_{+1234} = F_{+5678} = 2\mathrm{f} \qquad\qquad\qquad (1.2)$$

is exactly solvable. This background has several remarkable properties. It preserves the maximal number of 32 supersymmetries [8], and it is related by a Penrose limit to the $AdS_5 \times S^5$ background [8, 9]. It is worth noticing that the metric global symmetry $O(8)$ is broken to $SO(4) \times SO(4)$ by the RR background.

The same properties are true for other backgrounds [5], [6] and we therefore expect that our analysis extends to these cases.

Soon after the discovery of the exact solvability of this string model, an interpretation of its states within the AdS/CFT correspondence [6] as states with large angular momentum $J \sim \sqrt{N}$ and $J - \Delta$ finite in the dual $\mathcal{N} = 4$ SYM. Notice however that the string Hamiltonian in this background $H_{\text{lc string}}$ explains only the leading order anomalous dimension: $\Delta = H_{\text{lc string}} + O\left(1/R\right)$. This interpretation has been extended to other conformal cases in [7].

In this article we derive the boundary states associated with branes which preserve 16 supercharges in this solvable background. Differently from what happens in the flat background, where both the covariant NSR formulation (see [12] for a complete and consistent set of normalizations) and light-cone GS formulation [10] are available, in this case only the latter is at our disposal.

The plan of the paper is the following. In section 2 we give an heuristic derivation of the κ gauge fixed action found by Metsaev and we discuss why it is possible to fix the light-cone gauge. In section 3 after discussing the allowed boundary conditions on the bosonic fields x^I we construct the bosonic boundary states. They turn out to be $O(8)$ invariant, just as the metric is. Finally in section 4 we investigate the conditions to be imposed in the fermionic sector in order to break exactly half the number of global supersymmetric charges. These conditions break explicitly the global symmetry down to $SO(4) \times SO(4)$, as it could be expected due to "fermionic" nature of the RR background. It turns out that $D(-1)$ and $D7$ break more than 16 supersymmetries (actually 24) and that the other branes of type IIB superstring can only have special embeddings and must seat at the origin of the transverse coordinates if they must preserve 16 global supersymmetry charges. They can instead sit anywhere if they are left invariant by 8 charges only.

The branes that can be described on the light cone in this background are always "instantonic" and parallel to the wave. Since the generators J^{-I} do not correspond to isometries, these branes are not in an obvious way related to the ones perpendicular to the wave which are difficult, if not impossible, to describe in the light-cone formalism because even if we could reach the gauge $x^- = p^-\tau$ the resulting σ model would still be interacting. In a similar way it is difficult to perform a double Wick rotation [11] to obtain "physical" branes. It would hence be desirable to solve this theory in other gauges or in a pure spinor formalism [13] since the solution in NSR formalism appears to be difficult. A better understanding of why the D-instanton seems to break more that 16 charges would also be desirable, in particular to clarify whether this happens only due to our ansatz or it because of a more fundamental reason.

Another interesting point would be to compute the $1/R$ corrections to the GS light-cone action in order to find the corrections to the anomalous dimensions in the dual SYM.

2. Green-Schwarz superstrings in an Hpp-wave.

We start considering the lagrangian for a bosonic string propagating on the Hpp wave background (1.1) in the conformal gauge:

$$\mathcal{L} = \frac{1}{2}\left(2\partial_A x^+ \partial^A x^- - f^2 x_I^2 \partial_A x^+ \partial^A x^+ + \partial_A x^I \partial^A x^I \right) \tag{2.1}$$

The variation with respect to x^- of the action eq. (2.1) implies that x^+ obeys the massless free scalar equation; we can therefore impose the light cone gauge (we fix conventionally $2\alpha' = 1$)

$$x^+ = p^+ \tau . \tag{2.2}$$

In this gauge the previous lagrangian becomes, setting $m = p^+ f$,

$$\mathcal{L} = \frac{1}{2}\left(\partial_A x^I \partial^A x^I - m^2 x_I^2\right) , . \tag{2.3}$$

Furthermore, we must take into account the Virasoro conditions, giving in this gauge the following two constraints:

$$\acute{x}^- = \frac{\dot{x}^I \acute{x}^I}{p^+} , \tag{2.4}$$

$$\dot{x}^- = -\frac{\dot{x}^{I2} + \acute{x}^{I2} - m^2 x^{I2}}{2p^+} . \tag{2.5}$$

The gauge fixed lagrangian (2.3) is that of 8 massive two-dimensional free bosons. In view of the fact that the background preserves 32 real supersymmetric charges and that it reduces to flat space in the limit $m \to 0$ it is natural to expect that the fermionic lagrangian (which was first derived in [4] via a super-coset construction) corresponds to the modification of the usual flat-space GS action by mass terms. Since the the RR field in the background eq. (1.2) breaks the $SO(8)$ symmetry to $SO(4) \times SO(4)$, the mass term contains the product $\Pi \equiv \gamma^1 \bar{\gamma}^2 \gamma^3 \bar{\gamma}^4$, which is a symmetric matrix so that the mass term must be "mixed". The fermionic lagrangian reads thus

$$\mathcal{L}_F = \frac{i}{2}\left(S^a \partial_+ S^a + \tilde{S}^a \partial_- \tilde{S}^a\right) - im\, S^a \Pi_{ab} \tilde{S}^b , \tag{2.6}$$

where S^a and $\tilde{S}^a$ [1] $(a = 1, \ldots 8)$ are the canonical GS fields and $\partial_\pm = \partial_0 \pm \partial_1 = \partial_\tau \pm \partial_\sigma$. The coefficient of the mass term was chosen in such a way that x and S have the same plane wave expansion.

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[1] These fields are related to the ones used in [4] by

$$\Gamma^{+-}\theta^1 = \frac{1}{2^{3/4}}S , \quad \Gamma^{+-}\theta^2 = \frac{1}{2^{3/4}}\tilde{S}$$

3. The bosonic sector.

3.1 Boundary conditions and quantization in the bosonic sector.

We start discussing the boundary conditions allowed by the bosonic lagrangian (2.1); in particular the x^I variation implies

$$\delta x^I \, \acute{x}^I|_{\sigma=0}^{\sigma=1} = 0 \qquad (3.1)$$

which allows all usual types of boundary conditions: periodic, Neumann and Dirichlet. The other variations imply

$$\delta x^- \, \acute{x}^+|_0^1 = \delta x^+ \;, \quad \left(\acute{x}^- - \mathrm{f}^2 x_I^2 \acute{x}^+\right)|_0^1 = 0 \qquad (3.2)$$

which are trivially satisfied when (2.2) and (2.4) are used. Moreover the condition (2.4) implies that x^- has always Dirichlet boundary conditions $\acute{x}^-|B\rangle = 0$ at a boundary inserted at fixed τ in the closed channel, both for Neumann and Dirichlet conditions on the x^I, as it is true also for x^+ [10].

The mode expansions for the transverse coordinates x_I corresponding to the different boundary conditions are

$$\begin{aligned}
x_{\mathrm{cl}}^I(\sigma, \tau) = {}& \cos(m)\tau\, x_0^I + m^{-1}\sin(m\tau)\, p_0^I \\
&+ \mathrm{i}\sum_{n\neq 0} \frac{\mathrm{sgn}(n)}{\sqrt{|\omega_n|}} \left(e^{-\mathrm{i}(\omega_n\tau - k_n\sigma)}a_n^I + e^{-\mathrm{i}(\omega_n\tau + k_n\sigma)}\tilde{a}_n^I\right) \;,
\end{aligned} \qquad (3.3)$$

$$\begin{aligned}
x_{NN}^I(\sigma, \tau) = {}& \cos(m\tau)\, x_0^I + m^{-1}\sin(m\tau)\, p_0^I \\
&+ \mathrm{i}\sum_{n\neq 0} \frac{\mathrm{sgn}(n)}{\sqrt{|\omega_{n(o)}|}} e^{-i\omega_{n(o)}\tau}\, \cos\frac{k_n\sigma}{2}a_n^I \;,
\end{aligned} \qquad (3.4)$$

$$\begin{aligned}
x_{DD}^I(\sigma, \tau) = {}& \frac{-q_0^I \sinh(m(\sigma-1)) + q_1^I \sinh(m\sigma)}{\sinh(m)} \\
&+ \mathrm{i}\sum_{n\neq 0} \frac{1}{\sqrt{|\omega_{n(o)}|}} e^{-i\omega_{n(o)}\tau}\, \sin\frac{k_n\sigma}{2}a_n^I \;,
\end{aligned} \qquad (3.5)$$

where we have defined

$$\omega_{\pm|n|} = \pm\sqrt{k_n^2 + m^2}, \;\; \omega_{\pm|n|(o)} = \pm\sqrt{(k_n/2)^2 + m^2}, \;\; k_n \equiv 2\pi n \qquad (3.6)$$

and we have fixed the Dirichlet b.c.'s as $x_{DD}(\sigma=0) = q_0$ and $x_{DD}(\sigma=1) = q_1$.

The previous eq.s are normalized in such a way that the usual commutation relation

$$[x^I(\sigma), \mathcal{P}^J(\sigma')] = i\,\delta(\sigma - \sigma')\,\delta^{IJ} \;, \qquad (3.7)$$

where $\mathcal{P}^I = \dot{x}^I$, imply the following commutators for the modes:

$$[p_0^I, x_0^J] = -i\,\delta^{IJ} \;, \qquad [a_m^I, a_n^J] = [\tilde{a}_m^I, \tilde{a}_n^J] = \mathrm{sgn}(n)\,\delta_{m+n,0}\,\delta^{IJ} \;. \qquad (3.8)$$

The vacuum state of the bosonic part is the direct product of a zero mode vacuum and the Fock vacuum for string oscillation modes, and is altogether defined by

$$a_0^I|0\rangle = 0 , \qquad a_n^I|0\rangle = \tilde{a}_n^I|0\rangle = 0 , \qquad n = 1, 2, ... , \tag{3.9}$$

where we introduced the zero mode creation and annihilation operators

$$a_0^{I\dagger} = \frac{1}{\sqrt{2m}}(p_0^I + \mathrm{i}mx_0^I) , \qquad a_0^I = \frac{1}{\sqrt{2m}}(p_0^I - \mathrm{i}mx_0^I) , \tag{3.10}$$

as usual for an harmonic oscillator.

3.2 Bosonic boundary states.

Neumann conditions. The Neumann boundary condition $\left.\frac{\partial}{\partial\sigma_{open}}x\right|_{\sigma=0} = 0$ for an open string field can be reinterpreted in the closed channel as a condition on the state $|B\rangle$ that represents the insertion of the boundary at a fixed τ:

$$\dot{x}|_{\tau=0}|B\rangle = 0 . \tag{3.11}$$

This equation can be written in terms of the modes:

$$p_0|B\rangle = (a_n + \tilde{a}_{-n})|B\rangle = 0 , \tag{3.12}$$

and can be satisfied by the following boundary state:

$$|B\rangle = e^{-\frac{1}{2}a_0^{2\dagger}} e^{-\sum_{n=1}^{\infty} a_n^\dagger \tilde{a}_n^\dagger}|0\rangle , \tag{3.13}$$

where the zero mode part has become structurally analogous to the non zero mode part because the zero modes have acquired mass.

Dirichlet conditions. The Dirichlet boundary condition in the closed channel

$$(x - q_0)|_{\tau=0}|B\rangle = 0 , \tag{3.14}$$

which also implies that $\acute{x}|_{\tau=0}|B\rangle = 0$, becomes, in terms of the modes,

$$(x_0 - q_0)|B\rangle = (a_n - \tilde{a}_{-n})|B\rangle = 0 . \tag{3.15}$$

The solution to these equations is given by

$$|B\rangle = e^{\frac{1}{2}(a_0^\dagger - i\sqrt{2m}q_0)^2} e^{+\sum_{n=1}^{\infty} a_n^\dagger \tilde{a}_n^\dagger}|0\rangle , \tag{3.16}$$

where again the zero mode part is structurally analogous to the non zero mode part.

General situation. We can summarize the previous discussion by introducing a matrix $M_{IJ} = \text{diag}(\pm 1, \ldots, \pm 1)$, every -1 (+1) entry corresponding to a Neumann (Dirichlet) boundary condition on a transverse field:

$$\left(\partial_+ x^I - M_{IJ} \partial_- x^J \right) |B\rangle = 0 \ . \tag{3.17}$$

This equation can be expressed on the operators as

$$a_n^I - M_{IJ} \tilde{a}_{-n}^J |B\rangle = 0 \tag{3.18}$$

and it is still valid for $n = 0$ with the proviso that in the Dirichlet case we choose $q_0 = 0$. The corresponding boundary state (with $q_0^I = 0$ for simplicity) reads

$$|B\rangle = e^{\frac{1}{2} M_{IJ} a_0^{I\dagger} \tilde{a}_0^{J\dagger}} e^{+\sum_{n=1}^{\infty} M_{IJ} a_n^{I\dagger} \tilde{a}_n^{J\dagger}} |0\rangle \ . \tag{3.19}$$

This description can obviously be generalized to any matrix $M_{(v)} = M^{IJ}$ belonging to $O(8)$.

4. The complete theory.

4.1 Boundary conditions and quantization in the fermionic sector.

The fermionic equations of motion which can be derived from (2.6):

$$\partial_+ S - m\Pi\tilde{S} = 0 \,, \qquad \partial_- \tilde{S} + m\Pi S = 0 \tag{4.1}$$

must be supplemented with boundary conditions satisfying the constraint

$$\left(S^a \, \delta S^a - \tilde{S}^a \, \delta \tilde{S}^a \right)\Big|_{\sigma=0}^{\sigma=1} = 0 \ . \tag{4.2}$$

The allowed boundary conditions are then the usual: periodic (i.e., closed string) or generalized open, i.e. $\tilde{S}^a|_{\sigma=0} = S^a|_{\sigma=0}$ and $\tilde{S}^a|_{\sigma=1} = R_b^a S^b|_{\sigma=1}$, with $RR^T = 1$. The mode expansions with closed string conditions (which are the ones relevant for the boundary state construction) are

$$
\begin{aligned}
S^a(\sigma, \tau) = {} & \cos(m\tau)\, S_0^a + \sin(m\tau)\, (\Pi\tilde{S}_0)^a \\
& + \sum_{n \neq 0} c_n \left(e^{-\mathrm{i}(\omega_n \tau - k_n \sigma)} S_n^a + \mathrm{i}\frac{\omega_n - k_n}{m} e^{-\mathrm{i}(\omega_n \tau + k_n \sigma)} \Pi\tilde{S}_n^a \right) ,
\end{aligned}
\tag{4.3}
$$

$$
\begin{aligned}
\tilde{S}^a(\sigma, \tau) = {} & \cos(m\tau)\, \tilde{S}_0^a - \sin(m\tau)\, (\Pi S_0)^a \\
& + \sum_{n \neq 0} c_n \left(e^{-\mathrm{i}(\omega_n \tau + k_n \sigma)} \tilde{S}_n^a - \mathrm{i}\frac{\omega_n - k_n}{m} e^{-\mathrm{i}(\omega_n \tau - k_n \sigma)} \Pi S_n^a \right) ,
\end{aligned}
\tag{4.4}
$$

where we have defined

$$c_n = \frac{1}{\sqrt{1 + (\frac{\omega_n - k_n}{m})^2}} \ . \tag{4.5}$$

As in the bosonic case, the normalizations are chosen in such a way that the canonical equal-time anti-commutator

$$\{S^a(\sigma),\, S^b(\sigma')\} = \{\tilde{S}^a(\sigma),\, \tilde{S}^b(\sigma')\} = \delta^{ab}\delta_{m+n,0}\,\delta(\sigma - \sigma') \tag{4.6}$$

implies for the modes the relations

$$\{S_n^a, S_m^b\} = \{\tilde{S}_n^a,\ \tilde{S}_m^b\} = \delta^{ab}\delta_{m+n,0}\ . \tag{4.7}$$

The vacuum of the fermionic zero modes sector can be chosen to be the usual set of states $\{|I\rangle,\, |\dot{a}\rangle\}$ which satisfy $S_0^a|I\rangle = \gamma_{a\dot{a}}^I\,|\dot{a}\rangle/\sqrt{2}$ and $S_0^a|\dot{a}\rangle = \gamma_{a\dot{a}}^I\,|I\rangle/\sqrt{2}$ or, as done in [4], to be the state $|0\rangle$ such that

$$(S_0^a + i\,\tilde{S}_0^a)|0\rangle = 0\ , \tag{4.8}$$

which is related to the usual vacuum in such a way that it represents the zero modes part of the flat $D(-1)$ brane. In the non-zero modes sector the prescription is unique:

$$S_n^a|0\rangle = \tilde{S}_n^a|0\rangle\ ,\quad n > 0\ . \tag{4.9}$$

4.2 Constraints on boundary states

As in [10] we look for states which break half of the supercharges and therefore satisfy

$$\left(Q_a + i\eta\, M_{(s)\,ab}\, \tilde{Q}_b\right)|B\rangle = 0\ , \tag{4.10}$$

$$\left(Q_{\dot{a}} + i\eta\, M_{(c)\,\dot{a}b}\, \tilde{Q}_{\dot{b}}\right)|B\rangle = 0\ . \tag{4.11}$$

These are the same conditions as in [10] because they are not affected by the breaking of the $SO(8)$ symmetry, as the reader can convince her/himself by decomposing the charges in $SO(4)$ chiral blocks and then reassembling the expressions written with these blocks. The $SO(8)$ breaking only affects the constraints the matrices M have to satisfy, as we will see.

Differently from what happens in flat space, some of the supersymmetry charges do not commute with the hamiltonian, therefore we must impose explicitely that the constraints (4.10,4.11) be time invariant. From the commutation relations

$$[P^-,\, Q_a] = \Pi_{ab}Q_a\ ,\ \ [P^-,\, \tilde{Q}_a] = \Pi_{ab}\tilde{Q}_a \tag{4.12}$$

we get a further constraint on the matrix $M_{(s)\,ab}$:

$$M_{(s)\,ab} = \left(\Pi M_{(s)}\Pi\right)_{ab}\ . \tag{4.13}$$

The supercharges used in the previous expressions are given by [4]:

$$\frac{1}{2^{3/4}\sqrt{p^+}}Q_a = S_0\ ,\qquad \frac{1}{2^{3/4}\sqrt{p^+}}\tilde{Q}_a = \tilde{S}_0\ , \tag{4.14}$$

$$\frac{\sqrt{p^+}}{2^{1/4}} Q_{\dot a} = p_0^I \bar\gamma^{I \dot a b} S_0^b - m x_0^I \left(\bar\gamma^I \Pi\right)_{\dot a b} \tilde S_0^b$$
$$+ \sum_{n=1}^\infty \left(\sqrt{2\omega_n} c_n \bar\gamma_{\dot a b}^I (a_n^{\dagger I} S_n^b + a_n^I S_n^{b\dagger}) + \frac{im}{\sqrt{2\omega_n} c_n} \left(\bar\gamma^I \Pi\right)_{\dot a b} (\tilde a_n^{\dagger I} \tilde S_n^b - \tilde a_n^I \tilde S_n^{b\dagger}) \right) \; , \tag{4.15}$$

$$\frac{\sqrt{p^+}}{2^{1/4}} \tilde Q_{\dot a} = p_0^I \bar\gamma_{\dot a b}^I \tilde S_0^b + m x_0^I \left(\bar\gamma^I \Pi\right)_{\dot a b} S_0^b$$
$$+ \sum_{n=1}^\infty \left((\sqrt{2\omega_n} c_n \bar\gamma_{\dot a b}^I (\tilde a_n^{\dagger I} \tilde S_n^b + \tilde a_n^I \tilde S_n^{b\dagger}) - \frac{im}{\sqrt{2\omega_n} c_n} \left(\bar\gamma^I \Pi\right)_{\dot a b} (a_n^{\dagger I} S_n^b - a_n^I S_n^{b\dagger}) \right) \; . \tag{4.16}$$

and the light-cone Hamiltonian is given by

$$P^- = -H_{l.c.} = \frac{1}{2p^+} \left(p_0^2 + m^2 \, x_0^2 + 2i \, S_0^a \Pi_{ab} \tilde S_0^b \right)$$
$$+ \frac{1}{p^+} \sum_{n=1}^\infty \omega_n \left(a_n^\dagger a_n + S_n^{a\dagger} S_n^a + \tilde a_n^\dagger \tilde a_n + \tilde S_n^{a\dagger} \tilde S_n^a \right) \; . \tag{4.17}$$

Multiplying (4.10) by $\left(Q^c + i\eta \, M_{(s)\,d}^c \, \tilde Q^d \right)$ and taking the anticommutator, we get as an immediate consequence that

$$\left(M_{(s)} M_{(s)}^T \right)_{ab} = \delta_{ab} \; . \tag{4.18}$$

In a similar way from (4.11), using the anticommutation relations and looking at the terms proportional to P^- we find

$$\left(M_{(c)} M_{(c)}^T \right)_{\dot a \dot b} = \delta_{\dot a \dot b} \; . \tag{4.19}$$

Equation (4.11) can be rewritten as

$$\left(S_0^a + i\eta \, M_{(s)\,b}^a \, \tilde S_0^b \right) |B\rangle = 0 \; , \tag{4.20}$$

which suggests to take the following ansatz:

$$\left(S_n^a + i\eta \, M_{(s)\,b}^a \, \tilde S_{-n}^b \right) |B\rangle = 0 \; , \quad n = 0, \pm 1, \pm 2, \dots \; , \tag{4.21}$$

in order to solve the second defining equation (4.11).

Using the ansatz (4.21) and (3.18) into (4.11), we get three different equations. From the non zero modes structure $a^\dagger S + a S^\dagger$ we find

$$M^{JI} \, (\bar\gamma^J)_{\dot a b} = (M_{(c)} \bar\gamma^I M_{(s)}^T)_{\dot a b} \; . \tag{4.22}$$

This is the same equation arising in flat space [10]. From the non zero modes structure $a^\dagger S - a S^\dagger$, which always enters multiplied by m, we get a new equation which explicitly breaks the $SO(8)$ invariance:

$$M^{IJ} \, (\bar\gamma^J \Pi)_{\dot a b} = -(M_{(c)} \bar\gamma^I \Pi M_{(s)})_{\dot a b} \; . \tag{4.23}$$

Finally, the zero-mode sector yields a further constraint which reads

$$\left[p^I \left(\bar{\gamma}^I M_{(s)} - M_{(c)}\bar{\gamma}^I\right)_{\dot{a}b} + i\eta\, x_0^I \left(-\bar{\gamma}^I \Pi + M_{(c)}\bar{\gamma}^I \Pi M_{(s)}\right)_{\dot{a}b}\right] \tilde{S}_0^b|B\rangle = 0 \ , \qquad (4.24)$$

which can be rewritten in a better way with the help of (4.22,4.23) as

$$\left[\bar{\gamma}^J_{\dot{a}b} \left(\delta^{IJ} - M^{IJ}\right) p^I - i\eta\, x_0^I \left(\delta^{IJ} + M^{IJ}\right) \left(\bar{\gamma}^I \Pi M_{(s)}\right)_{\dot{a}b}\right] \tilde{S}_0^b|B\rangle = 0 \ . \qquad (4.25)$$

4.3 Boundary states.

Since the constraints (4.22,4.23,4.25) are invariant under $SO(4) \times SO(4)$ rotations as Π is, we can look for special solutions from which we can derive the general ones by a rotation. As usual in light-cone formalism branes are "instantonic" since they have Dirichlet boundary condition on $x^+ \propto \tau$. The complete boundary states are given by

$$|B\rangle = \exp\left(\sum_{n=1}^\infty M_{IJ}\, a_n^{I\dagger}\tilde{a}_n^{J\dagger} - i\eta\, M_{(s)\,ab}\, S_n^{a\dagger}\tilde{S}_n^{a\dagger}\right)|B\rangle_0 \ , \qquad (4.26)$$

$$|B\rangle_0 = \left(M_{IJ}|I\rangle\,|\tilde{J}\rangle + i\eta\, M_{(c)\,\dot{a}\dot{b}}|\dot{a}\rangle\,|\tilde{\dot{b}}\rangle\right)\, e^{\frac{1}{2}M_{IJ}\,a_0^{I\dagger}a_0^{J\dagger}}|0\rangle_a \qquad (4.27)$$

The explicit form of the matrices M is given, up to $SO(4) \times SO(4)$ rotations, by the following cases.

D(-1): No solutions preserve 16 supersymmetries. The natural solution $M_{(v)} = M_{(s)} = M_{(c)} = 1_8$ violates (4.23), which in turn means that (4.11) is violated. Eight supercharges are still preserved, thanks to (4.10).

D1: Both spatial Neumann directions must lie in the first four $1, 2, 3, 4$ or second four directions $5, 6, 7, 8$. Moreover the brane must seat in the origin because of the zero modes constraints (4.25). An explicit solution is given by

$$M^a_{(s)\,b} = (\gamma^1\bar{\gamma}^2)_{ab} \ , \quad M^{\dot{a}}_{(c)\,\dot{b}} = (\bar{\gamma}^1\gamma^2)_{\dot{a}\dot{b}} \ , \quad M^{II} = \begin{cases} -1 & I = 1, 2 \\ +1 & I \neq 1, 2 \end{cases} \ , \quad q_0^{3,4,5,6,7,8} = 0 \ . \tag{4.28}$$

Relaxing the condition on the zero-modes q_0, i.e., allowing the brane to sit anywhere, violates (4.25) and thus (4.11). Only 8 charges are then preserved.

D3: No solution preserves 16 supersymmetry charges. The following would-be solution does not satisfy the constraint (4.13) for time-invariance. Three of the four spatial Neumann directions are in either the first four $1, 2, 3, 4$ or second four directions $5, 6, 7, 8$. Again the brane is fixed at the origin of the transverse Dirichlet coordinates: $q_0^{4,6,7,8} = 0$. An explicit solution is

$$M^a_{(s)\,b} = (\gamma^1\bar{\gamma}^2\gamma^3\bar{\gamma}^5)_{ab} \ , \quad M^{\dot{a}}_{(c)\,\dot{b}} = (\bar{\gamma}^1\gamma^2\bar{\gamma}^3\gamma^5)_{\dot{a}\dot{b}} \ , \quad M^{II} = \begin{cases} -1 & I = 1, 2, 3, 5 \\ +1 & I = 4, 6, 7, 8 \end{cases} \ . \tag{4.29}$$

With an even number of directions in the first or second four directions (and arbitrary positions q_0) one obtains a solution preserving the 8 charges corresponding to (4.10).

D5: An even number of directions have to be in each of the two sets of four coordinates. The brane is stuck at the origin of the remaining coordinates: $q_0^{7,8} = 0$ if it must preserve 16 charges (otherwise only 8 are presrved), and

$$M^a_{(s)\,b} = (\gamma^1\bar{\gamma}^2\gamma^3\bar{\gamma}^4\gamma^5\bar{\gamma}^6)_{ab} \; , \quad M^{\dot{a}}_{(c)\,\dot{b}} = (\bar{\gamma}^1\gamma^2\bar{\gamma}^3\gamma^4\bar{\gamma}^5\gamma^6)_{\dot{a}\dot{b}} \; , \quad M^{II} = \begin{cases} -1 \; I \neq 7,8 \\ +1 \; I = 7,8 \end{cases} .$$
(4.30)

D7: no solutions.
The would-be solution $M_{(v)} = -1_8$, $M_{(s)} = \gamma^1\bar{\gamma}^2\gamma^3\bar{\gamma}^4\gamma^5\bar{\gamma}^6\gamma^7$ and $M_{(c)} = \bar{\gamma}^1\gamma^2\bar{\gamma}^3\gamma^4\bar{\gamma}^5\gamma^6\bar{\gamma}^7$ violates (4.23).

As a further check we can notice that applying $P^I = p_0^I$ to (4.11) we get

$$im \left[\left(\bar{\gamma}^I \Pi - M_{(s)}\, \bar{\gamma}^I \Pi\, M_{(c)} \right) \tilde{S}_0 \right]_{\dot{a}} + \left(Q_{\dot{a}} + i\eta\, M_{(c)\,\dot{a}\dot{b}}\, \tilde{Q}_{\dot{b}} \right) p_0^I |B\rangle = 0$$
(4.31)

which is trivially satisfied in Neumann directions.

It is nevertheless worth noticing that the B2B amplitude between two would-be $D(-1)$ boundary states located in arbitrary positions is *zero*, even if they do not preserve 16 supercharges. Indeed, the would-be $D(-1)$ boundary state does belong neither to a short representation nor to a long one: 8 charges are still preserved since (4.10) is still satisfied. Therefore inserting $1 = \frac{1}{2^{5/2}p^+}\{Q_a - i\eta\, M_{(s)\,ac}\, \tilde{Q}_c\, , Q_d + i\eta\, M_{(s)\,bd}\, \tilde{Q}_d\}$ into the boundary-to-boundary amplitude $\langle D(-1), q_0 | e^{-\tau\, L_0^{l.c.}} | D(-1), q_1 \rangle$ we get zero. More generally the same result is valid for the boundary-to-boundary amplitude between two flat space boundary states whose $M_{(s)}$ do satisfy (4.13): in particular it applies to $D3$'s with two directions in each of the two coordinates sets and to the other branes not at the origin of the transverse coordinates.

A. Notation and definitions

We essentially use the Metsaev and Tseylin's conventions [4] which we report here for self consistency. The conventions for the indices are:

$$\mu, \nu, \rho = 0, 1, \ldots, 9 \qquad so(9,1) \text{ vector indices (tangent space indices)}$$
$$I, J, K, L = 1, \ldots, 8 \qquad so(8) \text{ vector indices (tangent space indices)}$$
$$\alpha, \beta, \gamma = 1, \ldots, 16 \qquad so(9,1) \text{ spinor indices in chiral representation}$$
$$a, b, c = 1, \ldots, 8 \qquad so(9,1) \text{ spinor indices surviving the } \kappa \text{ symmetry}$$
$$\text{fixing in chiral representation}$$

$$\dot{a}, \dot{b}, \dot{c} = 1, \ldots, 8 \qquad so(9,1) \text{ spinor indices not surviving the } \kappa \text{ symmetry}$$

$$\text{fixing in chiral representation}$$

$$A, B = 0, 1 \qquad \text{2-d world-sheet coordinate indices}$$

We identify the transverse target indices with tangent space indices, i.e. $x^{\underline{I}} = x^I$, and avoid using the underlined indices in $+$ and $-$ light-cone directions, i.e. adopt simplified notation x^+, x^-. We suppress the flat space metric tensor $\eta_{\mu\nu} = (-, +, \ldots, +)$ in scalar products, i.e. $X^\mu Y^\mu \equiv \eta_{\mu\nu} X^\mu Y^\nu$. We decompose x^μ into the light-cone and transverse coordinates: $x^\mu = (x^+, x^-, x^I)$, $x^I = (x^i, x^{i'})$, where

$$x^{\pm} \equiv \frac{1}{\sqrt{2}}(x^9 \pm x^0). \tag{A.1}$$

The scalar products of tangent space vectors are decomposed as

$$X^\mu Y^\mu = X^+ Y^- + X^- Y^+ + X^I Y^I, \qquad X^I Y^I = X^i Y^i + X^{i'} Y^{i'}. \tag{A.2}$$

The notation $\partial_\pm$, ∂_I is mostly used for target space derivatives[2]

$$\partial_+ \equiv \frac{\partial}{\partial x^+} \qquad \partial_- \equiv \frac{\partial}{\partial x^-}, \qquad \partial_I \equiv \frac{\partial}{\partial x^I}. \tag{A.3}$$

We also use

$$\partial^+ = \partial_-, \qquad \partial^- = \partial_+, \qquad \partial^I = \partial_I. \tag{A.4}$$

The $SO(9,1)$ Levi-Civita tensor is defined by $\epsilon^{01\ldots9} = 1$, so that in the light-cone coordinates $\epsilon^{+-1\ldots8} = 1$. The derivatives with respect to the world-sheet coordinates (τ, σ) are denoted as

$$\dot{x}^I \equiv \partial_\tau x^I, \qquad \acute{x}^I \equiv \partial_\sigma x^I. \tag{A.5}$$

We use the chiral representation for the 32×32 Dirac matrices Γ^μ in terms of the 16×16 matrices γ^μ

$$\Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \bar{\gamma}^\mu & 0 \end{pmatrix}, \tag{A.6}$$

$$\gamma^\mu \bar{\gamma}^\nu + \gamma^\nu \bar{\gamma}^\mu = 2\eta^{\mu\nu}, \qquad \gamma^\mu = (\gamma^\mu)^{\alpha\beta}, \qquad \bar{\gamma}^\mu = \gamma^\mu_{\alpha\beta}, \tag{A.7}$$

$$\gamma^\mu = (1, \gamma^I, \gamma^9), \qquad \bar{\gamma}^\mu = (-1, \gamma^I, \gamma^9), \qquad \alpha, \beta = 1, \ldots 16. \tag{A.8}$$

We adopt the Majorana representation for Γ-matrices, $C = \Gamma^0$, which implies that all γ^μ matrices are real and symmetric, $\gamma^\mu_{\alpha\beta} = \gamma^\mu_{\beta\alpha}$, $(\gamma^\mu_{\alpha\beta})^* = \gamma^\mu_{\alpha\beta}$. As in [4] $\gamma^{\mu_1 \ldots \mu_k}$ are the antisymmetrized products of k gamma matrices, e.g., $(\gamma^{\mu\nu})^\alpha{}_\beta \equiv \frac{1}{2}(\gamma^\mu \bar{\gamma}^\nu)^\alpha{}_\beta - (\mu \leftrightarrow \nu)$, $(\gamma^{\mu\nu\rho})^{\alpha\beta} \equiv \frac{1}{6}(\gamma^\mu \bar{\gamma}^\nu \gamma^\rho)^{\alpha\beta} \pm 5$ terms. Note that $(\gamma^{\mu\nu\rho})^{\alpha\beta}$ are antisymmetric in α, β. We assume moreover the following block decomposition for γ^I:

$$\gamma^I = \begin{pmatrix} 0 & (\gamma^I)^{a\dot{b}} \\ (\gamma^I)^{\dot{a}b} & 0 \end{pmatrix} = \begin{pmatrix} 0 & (\tau^I) \\ (\tau^I)^T & 0 \end{pmatrix} \tag{A.9}$$

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[2]In sections 2, 3.2.3 and 4.1 $\partial_\pm$ indicate world-sheet derivatives.

and the normalization

$$\Gamma_{11} \equiv \Gamma^0 \dots \Gamma^9 = \begin{pmatrix} 1_{16} & 0 \\ 0 & -1_{16} \end{pmatrix} , \quad \gamma^0 \bar{\gamma}^1 \dots \gamma^8 \bar{\gamma}^9 = 1_{16}, \ \gamma^+ = \bar{\gamma}^- = \sqrt{2} \begin{pmatrix} 1_8 & 0 \\ 0 & 0_8 \end{pmatrix}$$
$$(A.10)$$

We use the following definitions

$$\Pi^\alpha{}_\beta \equiv (\gamma^1 \bar{\gamma}^2 \gamma^3 \bar{\gamma}^4)^\alpha{}_\beta , \qquad (\Pi')^\alpha{}_\beta \equiv (\gamma^5 \bar{\gamma}^6 \gamma^7 \bar{\gamma}^8)^\alpha{}_\beta . \tag{A.11}$$

$$\bar{\Pi}_\alpha{}^\beta \equiv (\bar{\gamma}^1 \gamma^2 \bar{\gamma}^3 \gamma^4)_\alpha{}^\beta , \qquad (\bar{\Pi}')_\alpha{}^\beta \equiv (\bar{\gamma}^5 \gamma^6 \bar{\gamma}^7 \gamma^8)_\alpha{}^\beta . \tag{A.12}$$

Note that $\Pi^\alpha{}_\beta = \bar{\Pi}_\beta{}^\alpha$. Because of the relation $\gamma^0 \bar{\gamma}^9 = \gamma^{+-}$ the normalization condition A.10 takes the form $\gamma^{+-} \Pi \Pi' = 1$. Note also the following useful relations (see also [4])

$$(\gamma^{+-})^2 = \Pi^2 = (\Pi')^2 = 1 , \tag{A.13}$$

$$\tau_I \tau_J^T + \tau_J \tau_I^T = 2 \delta_{IJ} 1_8 \tag{A.14}$$

The 32-component positive chirality spinor θ and the negative chirality spinor Q are decomposed in terms of the 16-component spinors as

$$\theta = \begin{pmatrix} \theta^\alpha \\ 0 \end{pmatrix} , \qquad \Gamma^{+-} \theta = \frac{1}{\sqrt{2\sqrt{2}}} \begin{pmatrix} S^a \\ 0_8 \\ 0_{16} \end{pmatrix} , \qquad Q = \begin{pmatrix} 0 \\ Q_\alpha \end{pmatrix} . \tag{A.15}$$

References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1998) 1113] [arXiv:hep-th/9711200].

[2] R. R. Metsaev and A. A. Tseytlin, "Type IIB superstring action in AdS(5) x S(5) background," Nucl. Phys. B **533** (1998) 109 [arXiv:hep-th/9805028].
I. Pesando, "A kappa gauge fixed type IIB superstring action on AdS(5) x S(5)," JHEP **9811** (1998) 002 [arXiv:hep-th/9808020].
R. Kallosh and J. Rahmfeld, "The GS string action on AdS(5) x S(5)," Phys. Lett. B **443** (1998) 143 [arXiv:hep-th/9808038].
I. Pesando, "All roads lead to Rome: Supersolvable and supercosets," Mod. Phys. Lett. A **14** (1999) 343 [arXiv:hep-th/9808146].
I. Pesando, "The GS type IIB superstring action on AdS(3) x S(3) x T**4," JHEP **9902** (1999) 007 [arXiv:hep-th/9809145].
J. Rahmfeld and A. Rajaraman, "The GS string action on AdS(3) x S(3) with Ramond-Ramond charge," Phys. Rev. D **60** (1999) 064014 [arXiv:hep-th/9809164].
J. Park and S. J. Rey, "Green-Schwarz superstring on AdS(3) x S(3)," JHEP **9901** (1999) 001 [arXiv:hep-th/9812062].
M. Kreuzer and J. G. Zhou, "Killing gauge for the 0-brane on AdS(2) x S(2) coset superspace," Phys. Lett. B **472** (2000) 309 [arXiv:hep-th/9910067].

[3] I. Pesando, "On the fixing of the kappa gauge symmetry on AdS and flat background: The lightcone action for the type IIb string on AdS(5) x S(5)," Phys. Lett. B **485** (2000) 246 [arXiv:hep-th/9912284].
R. R. Metsaev and A. A. Tseytlin, "Superstring action in AdS(5) x S(5): kappa-symmetry light cone gauge," Phys. Rev. D **63** (2001) 046002 [arXiv:hep-th/0007036].
R. R. Metsaev, C. B. Thorn and A. A. Tseytlin, "Light-cone superstring in AdS space-time," Nucl. Phys. B **596** (2001) 151 [arXiv:hep-th/0009171].
R. R. Metsaev and A. A. Tseytlin, "Superparticle and superstring in AdS(3) x S(3) Ramond-Ramond background in light-cone gauge," J. Math. Phys. **42** (2001) 2987 [arXiv:hep-th/0011191].

[4] R. R. Metsaev, "Type IIB Green Schwarz superstring in plane wave Ramond Ramond background," hep-th/0112044.
R. R. Metsaev and A. A. Tseytlin, "Exactly solvable model of superstring in plane wave Ramond-Ramond background," arXiv:hep-th/0202109.

[5] A J. G. Russo and A. A. Tseytlin, "On solvable models of type IIB superstring in NS-NS and R-R plane wave backgrounds," arXiv:hep-th/0202179.

[6] D. Berenstein, J. Maldacena and H. Nastase, "Strings in flat space and pp waves from N = 4 super Yang Mills," arXiv:hep-th/0202021.

[7] N. Itzhaki, I. R. Klebanov and S. Mukhi, "PP wave limit and enhanced supersymmetry in gauge theories," arXiv:hep-th/0202153.
J. Gomis and H. Ooguri, "Penrose limit of N = 1 gauge theories," arXiv:hep-th/0202157.

[8] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, "A new maximally supersymmetric background of IIB superstring theory," hep-th/0110242.
M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, "Penrose limits and maximal supersymmetry," hep-th/0201081.
M. Blau, J. Figueroa-O'Farrill and G. Papadopoulos, "Penrose limits, supergravity and brane dynamics," arXiv:hep-th/0202111.

[9] M. Hatsuda, K. Kamimura and M. Sakaguchi, "From Super-$AdS_5 \times S^5$ Algebra to Super-pp-wave Algebra," arXiv:hep-th/0202190.

[10] M. B. Green and M. Gutperle, "Light-cone supersymmetry and D-branes," Nucl. Phys. B **476** (1996) 484 [arXiv:hep-th/9604091].

[11] J. F. Morales, C. Scrucca and M. Serone, "A note on supersymmetric D-brane dynamics," Phys. Lett. B **417** (1998) 233 [arXiv:hep-th/9709063].

[12] M. Billo, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, Nucl. Phys. B **526** (1998) 199 [arXiv:hep-th/9802088].

[13] N. Berkovits and P. Howe, "Ten-dimensional supergravity constraints from the pure spinor formalism for the superstring," arXiv:hep-th/0112160.