An introduction to the SM of EW interactions

Ridolfi

Intro to EW SM

Not EM/str inter: $(Z,A) \to (Z+1,A) + e^- + \bar{\nu}_e$

$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$

$\boxed{\text{L proj.}}$

Parity is not conserved:

$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}}\, \bar{\psi}_p \gamma_\mu (1 - \overset{\sim 1.23}{a}\gamma_5)\psi_n \; \bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_{\nu_e}$$

$$-\frac{G_F}{\sqrt{2}}\, \bar{\psi}_{\nu_\mu} \gamma_\mu (1-\gamma_5)\psi_\mu \; \bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_{\nu_e}$$

mean life:

$\Gamma_n \sim 15$ min

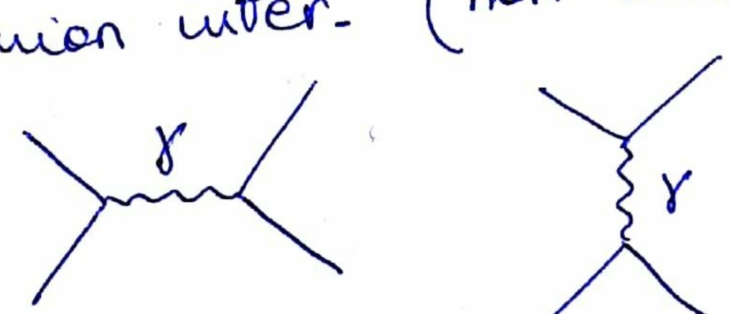
$\Gamma_\mu \sim 10^{-6}$ s

(1) NOT UNITARY
cross sections grow with energy.

(2) NOT RENORMALIZABLE
(G_F has wrong dimensions!)
$\to$ Effective theory

e.g.: QED (unitary, renorm.)

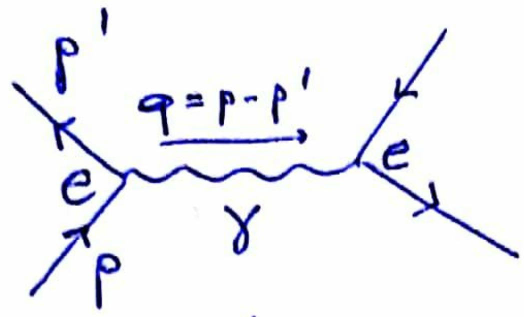
$\to$ 4 fermion inter. (non contact vertex):



$\boxed{m_\gamma = 0}$

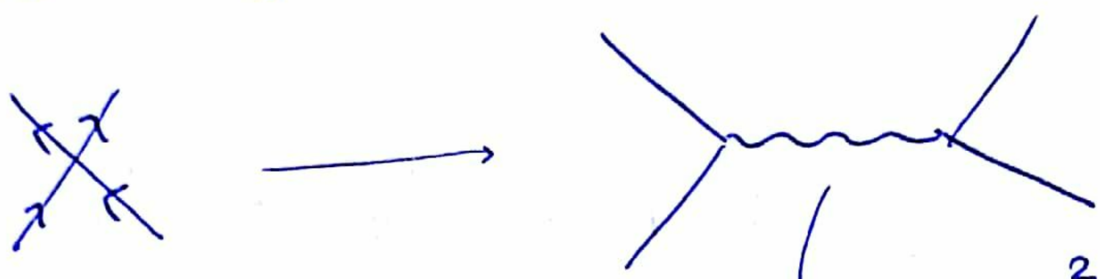
$\Rightarrow$ must be sim. for EW theory.

NB $m_\gamma = 0 \longleftrightarrow \frac{1}{r} \rightarrow$ potential $\longrightarrow$ long range!



$\hookrightarrow \frac{e^2}{q^2}$

EW $\Rightarrow$ <u>short range</u>



$$\frac{g^2}{q^2 - m_W^2}$$

$g \sim 1 \Rightarrow m_W \sim 300\ \text{GeV}$

$g \sim e \Rightarrow m_W \sim 100\ \text{GeV}$

$q^2 \ll m_W^2$: $\sim \boxed{\frac{g^2}{m_W^2} \approx G_F}$

<u>How to write renorm.</u> th.?

YM (1954) $\longrightarrow \boxed{m = 0}$ for gauge invariance

$\downarrow$

it may become a problem!

LEPTONS (Weinberg, 1967)

$\psi_\nu \to \nu$
$\psi_\mu \to \mu$

$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\alpha (1-\gamma_5)\mu \, \bar{e}\gamma^\alpha(1-\gamma_5)\nu_e$$

$$= -\frac{4G_F}{\sqrt{2}} J^{(\mu)}_\alpha J^{\alpha(e)\dagger}$$

PROJECTOR

JUST LEFT FIELDS

$$J_\alpha^{(\mu)} = \bar{\nu}_\mu \gamma_\alpha \frac{1}{2}(1-\gamma_5)\mu$$

$$J_\alpha^{(e)} = \bar{\nu}_e \gamma_\alpha \frac{1}{2}(1-\gamma_5)e$$

$$\left(= \overline{\left[\tfrac{1}{2}(1-\gamma_5)\nu_e\right]}\gamma_\alpha\left[\tfrac{1}{2}(1-\gamma_5)e\right]\right) = \bar{\nu}_{eL}\gamma_\mu e_L$$

$$\left(\partial^\mu J_\mu = 0\right)$$

$\hookrightarrow$ for each generator:

$$J_\mu^A = \bar{\psi}\gamma_\mu T^A \psi \qquad A = 1,\dots,N$$

Noether current

We can therefore consider:

$$l_L^{(e)} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad l_L^{(\mu)} = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$$

$$\Rightarrow \boxed{J_\mu = \bar{l}_L \gamma_\mu \tau^+ l_L} \qquad \tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$J_\mu^\dagger = \bar{l}_L \gamma_\mu \tau^- l_L \qquad \tau^- = (\tau^+)^\dagger = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

then

$$\exists\, J_\mu^3 = \bar{l}_L \gamma_\mu \underbrace{[\tau^+, \tau^-]}_{\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}} l_L$$

$\Rightarrow$ $\boxed{su(2) \text{ ALGEBRA}}$

$$\tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2)$$

we are looking for $SU(2)_L$ invariance

What about R comp?

↳ Trivial rep. of $SU(2)_L$

NB:

$J_\mu^+ , J_\mu^- \rightarrow$ <u>charged current</u> → OK for Fermi Lagr.

$\left(\text{eg: } J_\mu^{+(e)} = \bar{\nu}_L^{(e)} \gamma_\mu e_L \right)$

constr op (for $\bar{\nu}_L^{(e)}$) ; destr op (for e_L)

$$J_\mu^3 = \bar{l}_L \gamma_\mu \tau_3 l_L =$$
$$= \bar{\nu}_L \gamma_\mu \nu_L - \bar{l}_L \gamma_\mu l_L \rightarrow \text{neutral current}$$

?

J_μ^{EM} is a neutral current! Is it the same?
$= \bar{\psi} \gamma_\mu \psi$

$\Rightarrow \boxed{NO}$

↳ J_μ^3 → parity NOT cons
→ neutrino term

⇓

CONTRARY TO J_μ^{EM}

Can we include EM neutral currents?

$$\begin{array}{ccc} SU(2)_L & \otimes & U(1) \\ \downarrow & & \downarrow \\ 3 \text{ gen} & + & 1 \text{ gen} = 4 \end{array}$$

Can we assume $U(1) = U^{EM}(1)$? $\boxed{NO}$

$SU(2)_L \times U(1)^{EM} \Rightarrow$ DO NOT COMMUTE!

Therefore $SU(2)_L \otimes \underbrace{U_Y(1)}_{\text{weak hypercharge}}$

$$\Rightarrow \quad \partial_\mu \psi \longrightarrow D_\mu \psi = \partial_\mu \psi - ig \overset{SU(2)}{\overset{\Downarrow}{T^A W_\mu^A}} \psi - ig' \overset{U(1)}{\overset{\Downarrow}{B_\mu}} \frac{Y}{2} \psi$$

where $\psi \mapsto e^{igT^A \alpha_A(x)} e^{ig' \frac{Y}{2} \beta(x)} \psi(x)$ $\qquad T^A = \frac{\tau^A}{2}$ for doubl.

$T^A = 0$ singl.

Therefore:

FREE $\mathcal{L}_0 = i\bar{l}_L \not{\partial} l_L + i\bar{e}_R \not{\partial} e_R + i\bar{\nu}_R \not{\partial} \nu_R$ (NO MASSES)

INTERACT. $\mathcal{L} = \mathcal{L}_0 + \underset{\substack{\downarrow \\ \text{charged} \\ \text{current}}}{\mathcal{L}_{CC}} + \underset{\substack{\downarrow \\ \text{neutral} \\ \text{current}}}{\mathcal{L}_{NC}}$;

- $\mathcal{L}_{CC} = g \bar{l}_L \gamma_\mu \frac{\tau^1}{2} l_L W_1^\mu + g \bar{l}_L \gamma_\mu \frac{\tau^2}{2} l_L W_2^\mu =$
$= g \bar{l}_L \gamma_\mu \tau^+ l_L \frac{1}{2}(W_1^\mu - iW_2^\mu) + h.c.$
$= \frac{g}{\sqrt{2}} \bar{l}_L \gamma_\mu \tau^+ l_L W^{\mu +} + h.c.$

Now consider:

$$\psi = \begin{pmatrix} \nu_L \\ l_L \\ \nu_R \\ l_R \end{pmatrix} \to \mathcal{L}_{NC} = \bar{\psi} \gamma_\mu \left[g W_\mu^3 T^3 + g' B^\mu \frac{Y}{2} \right] \psi$$

$$T^3 = \begin{pmatrix} \frac{1}{2} & & & \\ & -\frac{1}{2} & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \quad Y = \begin{pmatrix} Y_l & & & \\ & Y_l & & \\ & & Y_{\nu_R} & \\ & & & Y_{l_R} \end{pmatrix}$$

- $\mathcal{L}_{NC} = g \bar{l}_L \gamma_\mu \tau_3 l_L W_3^\mu + \left(g' \bar{l}_L \gamma_\mu \frac{Y_l}{2} l_L + g' \frac{Y_{l_R}}{2} \bar{e}_R \gamma_\mu l_R + g' \frac{Y_{\nu_R}}{2} \bar{\nu}_R \gamma_\mu \nu_R \right) B^\mu$

Now replace

$$\begin{pmatrix} B^\mu \\ W_3^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}$$

Then

$$\mathcal{L}_{NC} = \bar{\psi}\gamma_\mu \left[g\sin\theta_W T_3 + g'\cos\theta_W \frac{Y}{2} \right] \psi A^\mu + \\ + \bar{\psi}\gamma_\mu \left[g\cos\theta_W T_3 - g'\sin\theta_W \frac{Y}{2} \right] \psi Z^\mu$$

Choose A^μ to be the EM sector:

$$J_\mu^{EM} = e\bar{\psi}\gamma_\mu Q \psi \qquad Q = \begin{pmatrix} 0 & & & \\ & -1 & & \\ & & 0 & \\ & & & -1 \end{pmatrix}$$

OK if $eQ = g\sin\theta_W T_3 + g'\cos\theta_W Y/2$

$$\Uparrow$$

$$g\sin\theta_W = g'\cos\theta_W = e$$

$$\Rightarrow \boxed{Q = T_3 + \frac{Y}{2}}$$

$$\rightarrow Y_\ell^{(1)} = 2\left(0 - \tfrac{1}{2}\right) = -1$$

$$Y_\ell^{(2)} = 2\left(-1 - \left(-\tfrac{1}{2}\right)\right) = -1$$

$$Y_{\nu_R} = 2(0-0) = 0$$

$$Y_{\ell_R} = 2(-1-0) = -2$$

it does not interact with anything on any level → at 1st sight it shouldn't exist!

Then $\mathcal{L}_{NC} = A_\mu J^\mu_{EM} + Z_\mu J^\mu_Z$

where
$$J^Z_\mu = e\bar{\psi}\gamma_\mu Q_Z \psi \rightarrow Q_Z = \frac{1}{\cos\theta_W \sin\theta_W}\left[T_3 - Q\sin^2\theta_W\right]$$

Therefore

$\rightarrow \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{NC} + \mathcal{L}_{CC} + \mathcal{L}_{YM}$

$$\mathcal{L}_{YM} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^A_{\mu\nu}W^{\mu\nu}_A$$

$$\begin{pmatrix} B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu + g\,\varepsilon^{ABC} W_{\mu B} W_{\nu C} \end{pmatrix}$$

How to write for A^μ, Z^μ, $W^{\mu\pm}$?

$$\mathcal{L}_{YM} = -\frac{1}{4}\underbrace{F_{\mu\nu}}_{\partial_\mu A_\nu - \partial_\nu A_\mu}F^{\mu\nu} - \frac{1}{4}\underbrace{Z_{\mu\nu}}_{\partial_\mu Z_\nu - \partial_\nu Z_\mu}Z^{\mu\nu} - \frac{1}{2}\underbrace{W^+_{\mu\nu}W^{\mu\nu -}}_{\partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu} +$$

$$+ i\overset{e}{g\sin\theta_W}\left(W^+_{\mu\nu}W^\mu_- A^\nu - W^-_{\mu\nu}W^\mu_+ A^\nu + F_{\mu\nu}W^{\mu}_+ W^{\nu}_-\right)$$

$$+ i g\cos\theta_W\left(\;''\; Z^\nu \;\;''\; Z^\nu + Z_{\mu\nu} \;''\;\right)$$

$$+ \frac{g^2}{2}\left(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right)\Big[\frac{1}{2}W^+_\mu W^+_\nu W^-_\rho W^-_\sigma -$$
$$- W^+_\mu W^-_\nu \left(A_\rho A_\sigma \sin^2\theta_W + \right.$$
$$\left. + Z_\rho Z_\sigma \cos^2\theta_W + 2A_\rho Z_\sigma \sin\theta_W \cdot \cos\theta_W\right)\Big].$$

Where are

- m_W, m_Z?
- m_ψ?
- neutral curr?

. hadrons?

Notice:

* τ^1, τ^2, τ^3 s.t. $\tau^i = \tau^{i\dagger} \Rightarrow e^{i\alpha^i \tau_i} \in SU(2)$ $\left[(\tau^+)^\dagger \neq \tau^+\right]$

* $\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = R \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$ where $R = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \rightarrow R R^T = \mathbb{I}$ because

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^3_{\mu\nu} W^{\mu\nu 3} \longrightarrow \begin{pmatrix} B_{\mu\nu} \\ W^3_{\mu\nu} \end{pmatrix} = R \begin{pmatrix} F_{\mu\nu} \\ Z_{\mu\nu} \end{pmatrix} \rightarrow \mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu}$$

thanks to $RR^T = \mathbb{I}$

Hadrons:

β - decay $\longrightarrow$ $\underset{(udd)}{N} \longrightarrow \underset{(uud)}{P} + e^- + \bar{\nu}_e$

$\hookrightarrow$ $d \longrightarrow u + e^- + \bar{\nu}_e \Rightarrow J_\mu^{(had)} = \bar{u} \gamma_\mu \frac{1}{2}(1-\gamma_5) d$

$\Rightarrow$ introduce STRANGE HADRONS $(\Lambda, K, \dots)$

$\hookrightarrow$ introduce a s quark $(S = -1,\ Q = -\frac{1}{3})$

Then $J_\mu^{(had)} = \cos\theta_c\, \bar{u} \gamma_\mu \frac{1}{2}(1-\gamma_5) d + \sin\theta_c\, \bar{u} \gamma_\mu \frac{1}{2}(1-\gamma_5) s = \bar{u}_L \gamma_\mu \left(\cos\theta_c\, d_L + \sin\theta_c\, s_L\right) =$

$$= \left(\bar{u}_L\ \bar{d}_L\ \bar{s}_L\right) \gamma_\mu T^+ \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}$$

$$T^+ = \begin{pmatrix} 0 & \cos\theta_c & \sin\theta_c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad T^- = (T^+)^T$$

NB $T^3 = \left[T^+, T^-\right] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\cos^2\theta_c & -\sin\theta_c \cos\theta_c \\ 0 & -\sin\theta_c \cos\theta_c & -\sin^2\theta_c \end{pmatrix} \Rightarrow J_\mu^{(had)3} = \bar{u}_L \gamma u_L - \cos^2\theta_c\, \bar{d}_L \gamma_\mu d_L -$

$- \sin^2\theta_c\, \bar{s}_L \gamma_\mu s_L - \sin\theta_c \cos\theta_c \left(\bar{d}_L \gamma_\mu s_L + \bar{s}_L \gamma_\mu d_L\right)$

phenomenologically suppressed $\longleftarrow$ FCNC

$K^+ \longrightarrow \pi^0 + e^+ + \nu_e \quad \Rightarrow \sin\theta_c\ \bar{s} \gamma_\mu \frac{1}{2}(1-\gamma_5) u$

$K^+ \longrightarrow \pi^+ + e^+ + e^- \quad \Rightarrow \sin\theta_c \cos\theta_c\ \bar{s} \gamma_\mu \frac{1}{2}(1-\gamma_5) d$

$$\frac{\Gamma(K^+ \to \pi^+ \dots)}{\Gamma(K^+ \to \pi^0 \dots)} \approx 0.97 \longrightarrow \text{it's actually } 10^{-5}$$

GIM mechanism $\Rightarrow$ c quark

$$J_\mu^{(had)} = \bar{u}\gamma_\mu \frac{1}{2}(1-\gamma_5)d' + \bar{c}\gamma_\mu \frac{1}{2}(1-\gamma_5)s'$$

$$\hookrightarrow \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\vartheta_c & -\sin\vartheta_c \\ \sin\vartheta_c & \cos\vartheta_c \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix} = V\begin{pmatrix} d \\ s \end{pmatrix} \qquad V^\dagger V = \mathbb{I}$$

$$\Rightarrow J_\mu^{(had)} = (\bar{u}_L \ \bar{d}_L')\gamma_\mu \tau^+ \begin{pmatrix} u_L \\ d_L' \end{pmatrix} + (\bar{c}_L \ \bar{s}_L')\gamma_\mu \tau^+ \begin{pmatrix} c_L \\ s_L' \end{pmatrix}$$

$$J_\mu^{(had)3} = (\bar{u}_L \ \bar{d}_L')\gamma_\mu \tau_3 \begin{pmatrix} u_L \\ d_L' \end{pmatrix} + (\bar{c}_L \ \bar{s}_L')\gamma_\mu \tau_3 \begin{pmatrix} c_L \\ s_L' \end{pmatrix} =$$

$$= \bar{u}_L\gamma_\mu u_L + \bar{c}_L\gamma_\mu c_L - \bar{d}_L\gamma_\mu d_L - \bar{s}_L\gamma_\mu s_L \quad \text{because } VV^\dagger = \mathbb{I}$$

$\hookrightarrow$ c found in $c\bar{c} \to J/\psi \Rightarrow m \sim 3$ GeV.

($m_c \sim 1.5$ GeV)

NB: R quark are SU(2) singlets $\longrightarrow$ $Y_{u_R} = \frac{4}{3}$ $[Y_{\nu_R} = 0]$

Masses in the SM

- Vector bosons:

$$A^\mu \longrightarrow U A^\mu U^\dagger + \frac{i}{g} U \partial^\mu U^\dagger$$

$\hookrightarrow$ $m^2 A_\mu A^\mu$ not gauge invariant.

- Fermions $\longrightarrow$ chirality!

$$* - m\bar{\psi}\psi = -m\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$$

NB $\begin{cases} \psi_L \to e^{i\alpha}\psi_L \\ \psi_R \to \psi_R \end{cases} \Rightarrow$ NOT gauge invariant!

$\Rightarrow$ Spontaneous breaking of the gauge symmetry [almost: must not break for unitarity]

eg: Scalar QED $\phi(x)$

$$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - m^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu$$

s.t.: $\phi(x) \to e^{ie\Lambda(x)}\phi(x)$ $\qquad D_\mu = \partial_\mu - ieA_\mu$

$\phi^\dagger(x) \to e^{-ie\Lambda}\phi(x)$ $\qquad (\partial^2 + m^2)\phi = J_\phi(\phi, A)$

$A^\mu(x) \to A^\mu(x) + \partial^\mu\Lambda(x)$ $\qquad \partial^2 A^\mu = -J^\mu_A \longrightarrow J^\mu_A = ie\left(\phi^\dagger \partial^\mu\phi - \phi\partial^\mu\phi^\dagger\right)$

$\rightarrow$ the symm is realised in Wigner-Weyl: $\phi = 0 \Rightarrow \phi' = 0$

(contrary to Nambu-Goldstone:

$$\phi(x) \longrightarrow e^{ie\Lambda(x)}\left[\phi(x) + \frac{v}{\sqrt{2}}\right] - \frac{v}{\sqrt{2}}$$

ok only for scalars

$$A^\mu \longrightarrow A^\mu + \partial^\mu\Lambda$$

)

NB: $\left|\phi(x) + \frac{v}{\sqrt{2}}\right|^2$ is invariant under Nambu-Goldstone:

$$V(\phi) = \lambda\left|\phi + \frac{v}{\sqrt{2}}\right|^4 + a\left|\phi + \frac{v}{\sqrt{2}}\right|^2 + b$$

$$V'(0) = 0 \rightarrow V(\phi) = \lambda\left[\left|\phi + \frac{v}{\sqrt{2}}\right|^2 + \frac{v^2}{2}\right]^2$$

$\Rightarrow$ choose Feynman-t'Hooft gauge fixing: $\mathcal{L} = -\frac{1}{2}\left[\partial_\mu A^\mu - ie\frac{v}{\sqrt{2}}(\phi - \phi^\dagger)\right]^2$

Then

$$\mathcal{L} = \left(D_\mu\left(\phi+\frac{v}{\sqrt{2}}\right)\right)^\dagger D^\mu\left(\phi+\frac{v}{\sqrt{2}}\right) - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{GF}$$

Now:

partial integration

$$\mathcal{L}_0 = \partial_\mu\phi^\dagger\partial^\mu\phi + \cancel{ie\frac{v}{\sqrt{2}}A_\mu\partial^\mu(\phi-\phi^\dagger)} + \frac{e^2v^2}{\sqrt{2}}A_\mu A^\mu +$$
$$-\frac{1}{2}\partial_\mu A_\nu\partial^\mu A^\nu \underbrace{-\frac{1}{2}\partial_\mu A_\nu\partial^\nu A^\mu - \frac{1}{2}(\partial A)^2}_{\text{TOTAL DERIVATIVE}} +$$
$$+\frac{e^2v^2}{\sqrt{2}}(\phi-\phi^\dagger)^2 + \cancel{i\frac{ev}{\sqrt{2}}\partial_\mu A^\mu(\phi-\phi^\dagger)} - \frac{\lambda}{2}v^2(\phi+\phi^\dagger)^2$$

Now call: $\phi(x) = \dfrac{H(x)+iG(x)}{\sqrt{2}}$

$$\Rightarrow \mathcal{L}_0 = \frac{1}{2}\left[\partial_\mu H\partial^\mu H + \partial_\mu G\partial^\mu G - 2\lambda v^2H^2 - e^2v^2G - \partial_\mu A_\nu\partial^\mu A^\nu + e^2v^2A_\mu A^\mu\right]$$

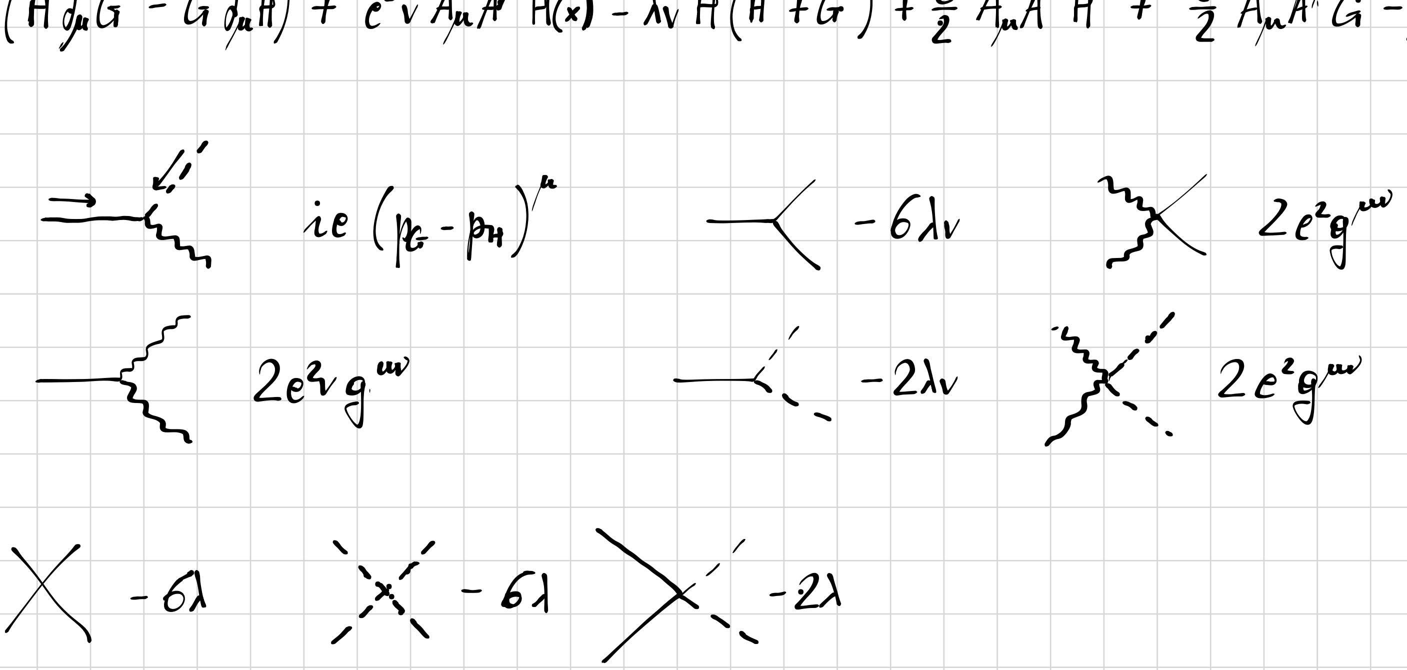
$\Rightarrow$ $H(x)$: $\quad -\dfrac{1}{k^2-2\lambda v^2+i\varepsilon} \quad (m_H^2 = 2\lambda v^2)$

$G(x)$: $\quad -\dfrac{1}{k^2-e^2v^2+i\varepsilon} \quad (m_G^2 = e^2v^2) \longrightarrow$ the Goldstone boson is massive because we broke a local symmetry

$\mu \; A \; \nu$: $\quad \dfrac{g^{\mu\nu}}{k^2-e^2v^2+i\varepsilon} \quad (m_A^2 = e^2v^2)$

Now look at the interaction term:

$$\mathcal{L}_I = eA^\mu(H\partial_\mu G - G\partial_\mu H) + e^2v\,A_\mu A^\mu H(x) - \lambda v\,H(H^2+G^2) + \frac{e^2}{2}A_\mu A^\mu H^2 + \frac{e^2}{2}A_\mu A^\mu G - \frac{\lambda}{4}(H^4+G^4+2G^2H^2)$$



ABELIAN HIGGS MODEL.

$$\Rightarrow \mathcal{L} = -\frac{1}{2}\partial_\mu A_\nu\partial^\nu A^\mu + \frac{1}{2}e^2v^2A_\mu A^\mu + \dots = -\frac{1}{2}\left[(\partial_\mu A_0)^2 - \frac{1}{2}m_\gamma^2A_0^2\right] + \frac{1}{2}\left[(\partial_\mu A_i)^2 - \frac{1}{2}m_\gamma^2A_i^2\right]$$

How to quantize? $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \longrightarrow [a(\vec{k}), a^\dagger(\vec{q})] = \delta(\vec{k}-\vec{q})$

NB $\mathcal{L}\to-\mathcal{L} \Rightarrow a\leftrightarrow a^\dagger \Rightarrow [a(\vec{k}), a^\dagger(\vec{q})] = -\delta(\vec{k}-\vec{q})$

Then we have: $\left(E_k = \sqrt{m_\gamma^2 + |\vec{k}|^2} \right)$

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2E_k}} \left[a_\mu(\vec{k}) e^{-ik\cdot x} + a_\mu^\dagger(\vec{k}) e^{ik\cdot x} \right] \Rightarrow \left[a_\mu(\vec{k}), a_\nu^\dagger(\vec{q}) \right] = -g_{\mu\nu}\, \delta(\vec{k}-\vec{q})$$

↳ $a_\mu(\vec{k}) = \sum_{i=0}^{3} a(\lambda,\vec{k})\, \varepsilon_\mu(\lambda,\vec{k})$

$$\varepsilon_\mu(0,\vec{0}) = \begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad \varepsilon_\mu(1,\vec{0}) = \begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad \varepsilon_\mu(2,\vec{0}) = \begin{pmatrix}0\\0\\1\\0\end{pmatrix};\quad \varepsilon_\mu(3,\vec{0}) = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \rightarrow \varepsilon_\mu(\lambda,\vec{0}) = \delta_\mu^{\ \lambda}$$

↳ $\varepsilon_\mu(\lambda,\vec{k}) = \Lambda_\mu^{\ \nu}(\vec{k})\, \varepsilon_\nu(\lambda,\vec{0}) = \Lambda_\mu^{\ \lambda}(\vec{k})$

$$\Rightarrow \left[a(\lambda,\vec{k}), a^\dagger(\lambda',\vec{q}) \right] = -g^{\lambda\lambda'}\, \delta(\vec{k}-\vec{q})$$

NB: $\lambda = 0 \Rightarrow \varepsilon_\mu(0,\vec{k}) = \frac{k_\mu}{m_\gamma}$ ⟶ LONGITUDINAL POLARIZATION

→ $A_\mu = \sum_{\lambda=0}^{3} A_\mu(\lambda) \longrightarrow A_\mu(0) = \partial_\mu f(x)$ SCALAR!

⇒ $\left[a(0,\vec{k}), a^\dagger(0,\vec{q}) \right] = -\delta(\vec{k}-\vec{q})$ ⇒ negative norm states!

↳ $|\vec{k}\rangle = \int d^3p\, f_k(\vec{p})\, a^\dagger(0,\vec{p})|0\rangle$

$$\Rightarrow \langle \vec{k}|\vec{k}\rangle = \int d^3p' \int d^3p\, f_k^*(\vec{p}')\, f_{\vec{k}}(\vec{p}) \langle 0| a(0,\vec{p}')\, a^\dagger(0,\vec{p})|0\rangle = -\langle 0|0\rangle \int d^3p\, |f_{\vec{k}}(\vec{p})|^2$$

$$\boxed{< 0}$$

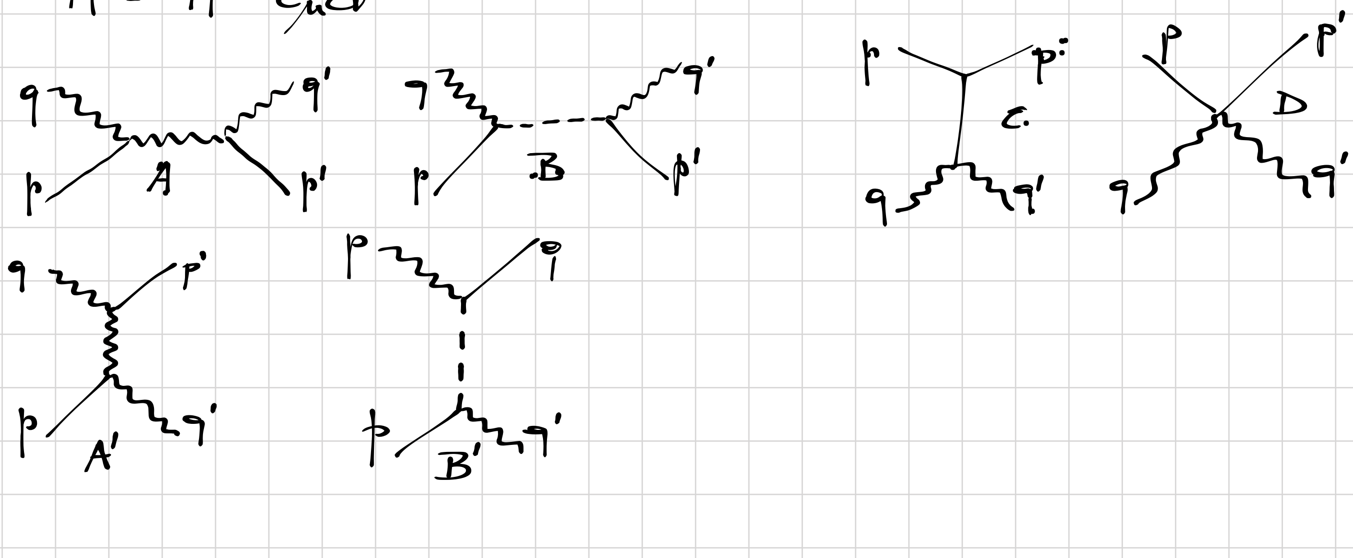
→ must get rid of longitudinal polarization! → require $\varepsilon_\mu(\lambda,\vec{k})\, k^\mu = 0$

We know $\varepsilon_\mu \varepsilon^{\mu *} = -1$ and

$$\sum_{\lambda=1}^{3} \varepsilon_\mu(\lambda,\vec{k})\, \varepsilon_\nu^*(\lambda,\vec{k}) = A g_{\mu\nu} + B k_\mu k_\nu = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_\gamma^2}$$

e.g.: $H(p) + \gamma(q,\varepsilon) \longrightarrow H(p') + \gamma(q',\varepsilon')$

$$\mathcal{M} = \mathcal{M}^{\mu\nu} \varepsilon_\mu \varepsilon_\nu^*$$



$$M_A^{\mu\nu} = (2e^2v)^2 g^{\nu\rho} \frac{g^{\rho\sigma}}{(p+q)^2 - e^2v^2} g^{\sigma\mu} = 4e^4v^2 \frac{g^{\mu\nu}}{(p+q)^2 - e^2v^2}$$

$$M_B^{\mu\nu} = ie\,(p+q+p')^\nu\, ie\,(-p-q-p)^\mu \left(-\frac{1}{(p+q)^2 - e^2v^2}\right) = -e^2 \frac{(2p+q)^\mu (p+q+p')^\nu}{(p+q)^2 - e^2v^2}$$

$$\Rightarrow M_{A+B}^{\mu\nu} = \frac{1}{(p+q)^2 - e^2v^2}\, 4e^4v^2 \left\{ g^{\mu\nu} - \frac{(\overbrace{p+q}^{=p'+q'}+p')^\nu (2p+q)^\mu}{4e^2v^2} \right\}$$

NB: $(p+q+p')^\nu \varepsilon_\nu^* = (2p'+q')^\nu \varepsilon_\nu^* = 2(p'+q')^\nu \varepsilon_\nu^*$ (because $q'^\nu \varepsilon_\nu^* = 0$)

$(2p+q)^\mu \varepsilon_\mu = 2(p+q)^\mu \varepsilon_\mu$

$$M_{A+B}^{\mu\nu} = 4e^4v^2 \frac{g^{\mu\nu} - \frac{(p+q)^\nu (p+q)^\mu}{e^2v^2}}{(p+q)^2 - e^2v^2}$$

→ this tensor is the sum over PHYSICAL polarization

⇒ including the Goldstone projects out unphysical polarization

NB Goldstone bosons must NOT appear in asymptotic states

We chose $\phi(x) = \frac{H(x) + iG(x)}{\sqrt{2}}$, now consider:

$$\phi(x) + \frac{v}{\sqrt{2}} = e^{ie\theta(x)} \frac{H(x)+v}{\sqrt{2}} \longrightarrow e^{-ie\theta(x)} e^{ie\theta(x)} \frac{H(x)+v}{\sqrt{2}} = \frac{H(x)+v}{\sqrt{2}}$$

$$A_\mu \longrightarrow A'_\mu = A_\mu - \partial_\mu \theta(x)$$

⇒ UNITARY GAUGE: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu H \partial^\mu H + \frac{e^2}{2}(A_\mu A^\mu)(H+v)^2 - \frac{\lambda}{4}\left[(H+v)^2 - v^2\right]^2$

→ no unphys dofs but renormalizability is not manifest.

$$\Delta^{\mu\nu}(k) = \frac{1}{k^2 - e^2v^2 + i\varepsilon}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{e^2v^2}\right)$$ → not the correct power counting

NB we could use:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial_\mu A^\mu \ldots)^2 \longrightarrow \Delta^{\mu\nu}(k) = \frac{1}{k^2 - m^2 + i\varepsilon}\left(g^{\mu\nu} - \frac{(1-\xi)k^\mu k^\nu}{k^2 - \xi m^2}\right)$$

$\xi = 1$: Feynman

$\xi = 0$: Landau

$\xi \to \infty$: Unitary

SM $\rightarrow \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \longrightarrow V(\phi) = \lambda \left[\left|\phi + \frac{v}{\sqrt{2}}\right|^2 - \frac{v^2}{2} \right]^2$

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$\longrightarrow$ No mass for γ:

$$\phi + \frac{v}{\sqrt{2}} \rightarrow e^{ig\alpha^i \frac{\tau^i}{2}} e^{ig'\beta(x)\frac{Y_\phi}{2}} \left(\phi + \frac{v}{\sqrt{2}}\right) \rightarrow \text{no EM breaking } \phi \rightarrow e^{ieQ\alpha}\phi$$

$$Q = T_3 + \frac{Y_\phi}{2} \longrightarrow Q\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow 1)\ v_1 = 0 \Rightarrow Y_\phi = 1$$

$$2)\ v_2 = 0 \Rightarrow Y_\phi = -1$$

$$V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \phi + \frac{V}{\sqrt{2}} = \frac{1}{\sqrt{2}} e^{i\frac{\tau^i}{2}\vartheta^i(x)} \begin{pmatrix} 0 \\ H(x)+v \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H(x)+v \end{pmatrix}$$

$$D_\mu\left(\phi + \frac{V}{\sqrt{2}}\right) = \partial_\mu \phi - ig\frac{\tau^i}{2} W_\mu^i \left(\phi + \frac{V}{\sqrt{2}}\right) - ig' B_\mu \frac{1}{2}\left(\phi + \frac{V}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial_\mu H \end{pmatrix} - \frac{ig}{2} \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ H+v \end{pmatrix} - \frac{ig'}{2} B_\mu \begin{pmatrix} 0 \\ H+v \end{pmatrix} \right]$$

$$\cancel{\Rightarrow \left| D_\mu\left(\phi + \frac{V}{\sqrt{2}}\right)\right|^2}$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial_\mu H \end{pmatrix} - \frac{i}{2}(H+v) \begin{pmatrix} g(W_\mu^1 - iW_\mu^2) \\ g' B_\mu - g W_\mu^3 \end{pmatrix} \right]$$

$$\Rightarrow \left| D_\mu\left(\phi + \frac{V}{\sqrt{2}}\right)\right|^2 \Bigg|_{H=0} = \text{MASS TERM} =$$

$$= \frac{1}{2}\frac{v^2}{4}\left\{ g^2 (W_\mu^1 + iW_\mu^2)(W^{\mu 1} - iW^{\mu 2}) + (g' B_\mu - g W_\mu^3)^2 \right\}$$

$$= \frac{v^2}{4}\left[g^2 W_\mu^+ W^{-\mu} + \frac{1}{2}(g^2 + g'^2) Z_\mu Z^\mu \right]$$

$$\Rightarrow \begin{cases} m_W^2 = \dfrac{v^2 g^2}{4} \\ m_Z^2 = \dfrac{g^2 + g'^2}{4} v^2 \end{cases}$$

β-decay $\Rightarrow$

$$\left.\begin{aligned} M &= \left(\frac{g}{2\sqrt{2}} \bar{u}_L \gamma_\mu d_L\right) \frac{g^{\mu\nu}}{q^2 - m_W^2} \left(\frac{g}{2\sqrt{2}} \bar{e}_L \gamma_\nu \nu_L\right) \\ M_F &= -\frac{G_F}{\sqrt{2}} \bar{u}\gamma_\mu (1-\gamma_5) d \, \bar{e}\gamma^\mu (1-\gamma_5)\nu \end{aligned}\right\} \quad q^2 \to 0 \to \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}}$$

$$\Rightarrow \boxed{v \simeq 247\ \text{GeV}}$$

FERMION MASSES

$$\mathcal{L}_{\text{Fermion}} = \sum_{k=1}^{5} \bar{\psi}_k\, i \not{D}\, \psi_k$$

1 generations

repr.

$k=1$: $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $Y_1 = \frac{1}{3}$

$k=2$: u_R $Y_2 = \frac{4}{3}$

$k=3$: d_R $Y_3 = -\frac{2}{3}$

$k=4$ $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ $Y_4 = -1$

$k=5$ e_R $Y_5 = -2$

$$D_\mu = \partial_\mu - ig T^i W_\mu^i - ig' \frac{Y}{2} B_\mu$$

ACCIDENTAL SYMM:

$$\psi_k \longrightarrow e^{i\alpha_k} \psi_k \qquad \underline{[U(1)]^5 \text{ symmetry}}$$

constant

5 cons. curr:

$J_\mu^1 = \bar{u}_L \gamma_\mu u_L + \bar{d}_L \gamma_\mu d_L$

$J_\mu^2 = \bar{u}_R \gamma_\mu u_R$

$J_\mu^3 = \bar{d}_R \gamma_\mu d_R$

$J_\mu^4 = \bar{\nu}_L \gamma_\mu \nu_L + \bar{e}_L \gamma_\mu e_L$

$J_\mu^5 = \bar{e}_R \gamma_\mu e_R$

$\partial_\mu J^{\mu k} = 0 \quad \forall k = 1, \dots, 5$

Linear comb:

$$J_\mu^Y = \sum Y_k J_\mu^k$$ hyperch. ✓

$$J_\mu^B = \frac{1}{3}(J_\mu^1 + J_\mu^2 + J_\mu^3)$$

$$= \frac{1}{3}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)$$ baryon no. ✓

$$J_\mu^L = J_\mu^4 + J_\mu^5 =$$

$$= \bar{\nu}_L \gamma_\mu \nu_L + \bar{e}\gamma_\mu e$$ lepton no. ✓

$$J_\mu^{B5} = J_\mu^1 - J_\mu^2 - J_\mu^3 =$$

$$= -(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)$$ axial curr. ×

ANOMALOUS

$$J_\mu^{L5} = J_\mu^4 - J_\mu^5$$ ×

If more than one gener.:

$$\mathcal{L}_{Fermion} = \sum_{f=1}^{n} \sum_{k=1}^{5} \bar{\psi}_k^{(f)} \, i\not{D} \, \psi_k^{(f)}$$

we can mix families with unitary matrices:

$$\psi_k^{(f)} \longrightarrow U^{fg} \psi_k^{(g)}$$

→ NB must break some of the symm.

EXPLICIT BREAKING of ACCIDENTAL SYMMETRIES

$$q' = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} \quad u'_R \quad d'_R \quad l' = \begin{pmatrix} \nu'_L \\ e'_L \end{pmatrix} \quad l'_R$$

$\forall$ family of fermions!

$$\mathcal{L}_{YUKAWA} = -\bar{q}' h'_D d'_R \left(\phi + \frac{v}{\sqrt{2}}\right) - \bar{q}' h'_u u'_R \left(\phi + \frac{v}{\sqrt{2}}\right)^c - \bar{l}' h'_L e_R \left(\phi + \frac{v}{\sqrt{2}}\right) + h.c.$$

$n \times n$ matrix in generation space

- $SU(2)$ invariant
- $U(1)_Y$ "

NB: $\phi^c = \varepsilon \phi^*$

$\hookrightarrow \varepsilon = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$\to$ Yukawa inter. break most of the accidental symm.!

$$\mathcal{L}_{Yuk} = -\frac{v}{\sqrt{2}} \Big[\bar{d}_L' h_D' d_R' + \bar{u}_L' h_U' u_R' + \\ + \bar{e}_L' h_L' e_R' + h.c. \Big] + \underline{\text{interac with Higgs}}$$

<u>Singular value decomp</u>:

$$h' = U^\dagger h V \qquad \text{s.t.} \qquad UU^\dagger = \mathbb{I} \quad VV^\dagger = \mathbb{I}$$

where h is diag with positive entries

$\Rightarrow$ use to diag $h'_{D,U,L}$:

$$\mathcal{L}_Y = -\frac{v}{\sqrt{2}} \Big[\bar{d}_L' U_D^\dagger h_D V_D d_R' + \bar{u}_L' U_U^\dagger h_U V_U u_R' + \\ + \bar{e}_L' U_L^\dagger h_L V_L e_R + h.c. \Big] + \dots$$

Now we define:

$$(\bullet) \qquad d_R = V_D d_R' \qquad u_R = V_U u_R' \qquad l_R = V_L l_R'$$
$$d_L = U_D d_L' \qquad u_L = U_U u_L' \qquad l_L = U_L l_L'$$

$\rightarrow$ diagonal mass terms:

$$\mathcal{L}_Y = -\frac{v}{\sqrt{2}} \Big\{ \underline{\bar{d}_L h_D d_R} + \bar{u}_L h_U u_R + \bar{l}_L h_D l_R + h.c. \Big\} + \dots$$

$$-\frac{vh}{\sqrt{2}} \bar{\psi} \psi$$

What happens to the rest of $\mathcal{L}$ under $(\bullet)$?

$$\mathcal{L}_{Ferm} = \mathcal{L}_0 + \mathcal{L}_{nc} + \mathcal{L}_{cc}$$

- $\bar{u}_L' i \partial\!\!\!/ u_L' = \bar{u}_L i \partial\!\!\!/ u_L$ because $U_L^\dagger U_L = \mathbb{I}$
- same for $\mathcal{L}_{nc}$
- charged interact.: $\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_L' \gamma^\mu d_L' + h.c. =$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_L \gamma^\mu \underbrace{U_U U_D^+}_{\text{unitary matrix}} d_L + h.c.$$

$U_U U_D^+ = V$ CABIBBO - KOBAYASHI - MASKAWA matrix

While

$$\mathcal{L}_{cc}^{(lep)} = \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L' \gamma_\mu l_L'$$

they do not rotate since $\nexists \nu_R$

$\Rightarrow$ I assume $\nu_L' = U_L \nu_L' \Rightarrow \mathcal{L}_{co}^{(lep)} = \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma_\mu l_L$

no generation mixing

Then we have

param : $g \quad g' \quad \lambda \quad v$

masses : $m_u^f \quad m_d^f \quad m_e^f$

CKM : how many param? $VV^+ = \mathbb{I}$ $n \times n$ matrix

$$V = e^{iH} \longrightarrow \text{Herm.}: \; n + 2\frac{n(n-1)}{2} = n^2 \text{ in principle}$$

where

$$n^2 = N_{angles} + N_{phases} = \frac{1}{2}n(n-1) + \underbrace{\frac{1}{2}n(n+1)}_{= N_{phases}}$$

$$N_{angles} = \binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{1}{2}n(n-1)$$

Not all N_{phases} are measurable:

$$\begin{aligned} u_L^f &\longrightarrow e^{i\alpha_f} u_L^f \\ d_L^f &\longrightarrow e^{i\beta_f} d_L^f \end{aligned} \Rightarrow V_{fg} \longrightarrow V_{fg}\, e^{i(\beta_g - \alpha_f)}$$

redef. for $2n-1$ phases

$$\rightarrow N_{phases}^{true} = \frac{1}{2}n(n+1) - (2n-1) =$$

$$= \frac{n^2+n-4n+2}{2} = \frac{(n-1)(n-2)}{2}$$

$\longrightarrow n = 1,2 \Rightarrow N_{phases}^{true} = 0$

$n = 3 \Rightarrow N_{phase}^{true} = 1 \longrightarrow$ if $V \neq V^*$ (complex) then $\boxed{\text{CP violated}}$ (1964)

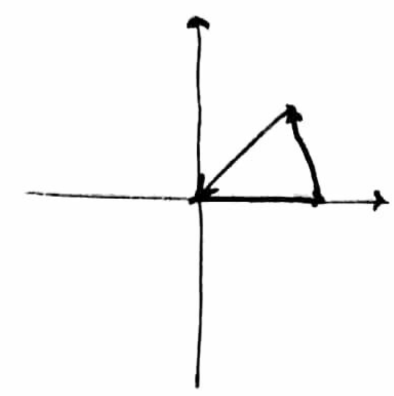
Then $n = 3 \Rightarrow$ 3 angles 1 phase:

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^2 A(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^2(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

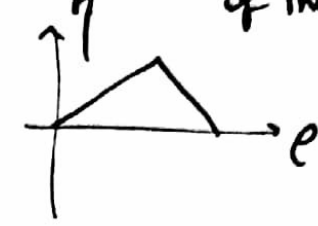
Unitarity:

$$\xi_i^{\alpha\beta} = V_{i\alpha} V_{i\beta}^*$$

$$\sum_{i=1}^{n} \xi_i^{\alpha\beta} = \delta^{\alpha\beta} \rightarrow \alpha \neq \beta \Rightarrow$$

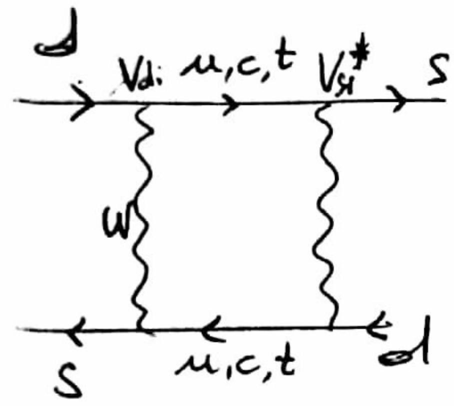


apart from one of them



=

Mixing $K_0 - \bar{K}_0$:
$(s\bar{d})$ $(\bar{s}d)$



$$\mathcal{L}_{eff}^{\Delta S=2} = \frac{G_F^2 m_W^2}{64\pi^2} \bar{s}\gamma^\mu(1-\gamma_5)d\,\bar{s}\gamma_\mu(1-\gamma_5)d \cdot \sum_{i=u,c,t} \sum_{j=u,c,t} \xi_i \xi_j^* F(x_i, x_j)$$

$$(V_{di} V_{si}^* = \xi_i)$$

$$x_i = \frac{m_i^2}{m_W^2}$$

$$\Delta M = 2|M_{12}|, \quad M_{12} = -\frac{1}{2m_K} \langle K^0 | \mathcal{L}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$$

$$\Rightarrow \Delta M = \frac{G_F^2 m_W^2}{64\pi^2 m_K} \left| \sum_{i,j} \xi_i \xi_j F(x_i, x_j) \right| \langle K^0 | (\bar{s}\gamma_\mu(1-\gamma_5)d)^2 | \bar{K}^0 \rangle$$

$$x_i, x_j \ll 1 \quad (m_u \sim 0, \; m_t \ll m_W)$$

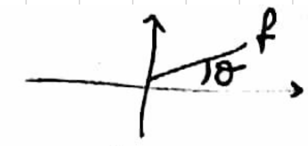
$$F(x_i, x_j) = \frac{4 x_i x_j}{x_i - x_j} \log \frac{x_i}{x_j} \qquad F(x_i, x_j) \simeq 4$$

$$\rightarrow \Delta M = \frac{G_F^2 \xi_c^2 m_c^2}{16\pi^2 m_K} \underbrace{\langle K^0 | (\bar{s}\gamma_\mu(1-\gamma_5)d)^2 | \bar{K}^0 \rangle}_{\frac{8}{3} f_K^2 m_K B_K} \qquad B_K \sim 1$$

unknown

decay

e.g. $e^+e^- \longrightarrow f\bar{f}$



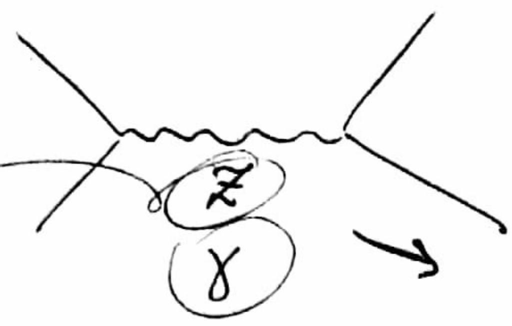
F : $\cos\theta > 0$
B : $\cos\theta < 0$

④ EW

Fwd - bwd asymm :

$$A_{FB}(f) = \frac{\sigma(F) - \sigma(B)}{\sigma(F) + \sigma(B)} = f(\theta_W)$$

→ at least NLO

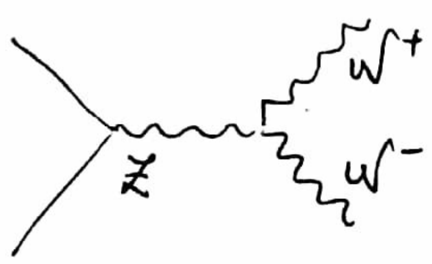


10^6

$\rightarrow \frac{1}{s - m_Z^2}$ or $\frac{1}{s}$

→ LEP1 was close to $m_Z^2 \Rightarrow Z$ domin.

LEP2 @ 215 GeV → $e^+e^- \longrightarrow W^+W^-$



LOOP EFFECTS

- SM param:
  - gauge-Higgs sector: $g \quad g' \quad \lambda \quad v$

$$\left[\text{related to } G_F \quad \alpha_{EM} \quad m_Z \quad m_H\right]$$

G_F → μ decay (EM rad. corr.)

m_H → 125.9 GeV, $m_H^2 = 2\lambda v^2$

m_Z → 91.1876 GeV (e^+e^- collisions)

⇓

$m_W = 80.385$ GeV
↳ $m_W^2 = \frac{1}{4} g^2 v^2 \rightarrow$
⇒ $v = 247$ GeV

→ strong evidence of $100\,\text{GeV} < m_H < 200\,\text{GeV}$ was there ~~a lot~~ before the discovery

- $n_q = n_\ell$ (no. of families)

↳ <u>ANOMALIES</u> : symm. anomalous at radiative corr

$$\underbrace{\partial_\mu J^\mu = 0}_{\text{tree}} \rightarrow \underbrace{\partial_\mu J^\mu \neq 0}_{\text{radiative}}$$

<u>Axial current</u>:

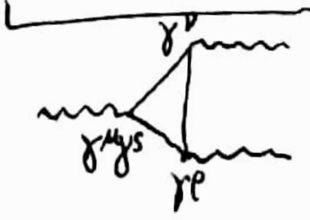
QED:

$$J^\mu = \bar\psi \gamma^\mu \psi \Rightarrow \partial_\mu J^\mu = 0 \qquad (\psi \rightarrow e^{i\alpha}\psi)$$

$$J^{\mu 5} = \bar\psi \gamma^\mu \gamma^5 \psi \Rightarrow \partial_\mu J^{\mu 5} \neq 0 \qquad (\psi \rightarrow e^{i\alpha\gamma_5}\psi)$$

$$\partial_\mu J^{\mu 5} = 2im\bar\psi\gamma_5\psi \quad \text{(partially conserved)}$$

if we add radiative corrections ⇒ $\partial_\mu J^{\mu 5} = 2im\bar\psi\gamma_5\psi + \underbrace{\frac{1}{(4\pi)^2}\varepsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}}$



<u>Non abelian</u>: $\partial_\mu J^{\mu 5} = \# \, F\tilde F + \ldots$ (any field strenght)

↳ $\# \propto \mathrm{Tr}\left(\{T^a, T^b\} T^c\right)$

⇒ SM: $\frac{\tau^a}{2}$, Y →
- 3 SU(2) gener ⇒ $\mathrm{Tr}(\ldots) = 0$
- $\mathrm{Tr}(\{\tau^a,\tau^b\}Y) \propto \mathrm{Tr}(Y) \sim \mathrm{Tr}\,Q \Rightarrow \boxed{n_q - n_\ell = 0}$
- $\mathrm{Tr}(Y^3) = 0$

<u>Therefore</u>:

- masses:

$$\left.\begin{array}{lll} 3 & u & \text{quarks} \\ 3 & d & '' \end{array}\right\} \begin{array}{l} \text{light}: u, d \\ \text{heavy}: c, b, t \end{array} \text{(s between)}$$

$\Lambda_{QCD} \sim 200$ MeV

EW

$$\alpha_{QCD} = \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda_{QCD}}}$$

3 ~~charged~~ charged leptons

- CKM : 4 angles +1 phase

NB: i) <u>neutrinos</u> :

$m_\nu \ll m_e$ → out of SM

ν_R ?

$\boxed{\text{Majorana mass term}}$ → see-saw mechanism

$M \bar{\nu}_R^c \nu_R + h.c.$

→ NB very large !

ii) Hierarchy → naturaleness

↳ why are there diff scales (EW, GUT, ...)?

↳ mass corrections:

$$m_H^2 \sim \Lambda^2 f(\lambda, g, g', \dots) \rightarrow \text{fine tuning}$$

$$m_f \sim m_f \ln \frac{\Lambda}{m_f}$$