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Boundary CFT and D-branes

* OUTLINE
  1. Define BCFT and open strings
  2. Define BS
  3. Define T-duality and branes with EM fields
  4. Define BS for branes with EM fields
  5. Branes bound states and instantons

→ BOUNDARY CFT

* Free boson and T-duality

$$\boxed{S = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left(\partial_\tau X\cdot\partial_\tau X + \partial_\sigma X\cdot\partial_\sigma X\right)}$$

$\Rightarrow$ EOM: $\frac{\delta S}{\delta X} = 0 \longrightarrow \delta S = \frac{-1}{2\pi\alpha'}\int_\Sigma d\tau\, d\sigma\left[-(\partial_\tau^2 + \partial_\sigma^2)X\cdot\delta X + \partial_\sigma\left(\partial_\sigma X\cdot\delta X\right)\right]$

$$\Rightarrow \quad \Box X^\mu(\tau,\sigma) = \partial_w\partial_{\bar w}\left(X_L^\mu(w) + X_R^\mu(\bar w)\right) = 0$$

where $w = \tau - i\sigma$, $\bar w = \tau + i\sigma = w^*$.

$$\Downarrow$$

$$X^\mu(w,\bar w) = x_0^\mu + \frac{a^2}{\pi}p^\mu w + \frac{b^2}{\pi}q^\mu \bar w + \frac{a}{2\pi}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-nw} + \frac{b}{\sqrt{\pi}2}\sum_{n\neq 0}\frac{\tilde\alpha_n^\mu}{n}e^{-n\bar w}$$

$\Rightarrow$ Moreover we have to consider:

$$\Downarrow$$

$$\Sigma = (-\infty,+\infty)\times[0,\pi] \Rightarrow \partial_\sigma X^\mu\big|_{\sigma=0,\pi} = 0.$$

What about $\delta X\big|_{\sigma=0,\pi} = 0$?

→ Suppose to have closed strings s.t.: $X^\mu(w - i\pi, \bar w + i\pi) = X^\mu(w,\bar w)$. Then

$$X^\mu(w,\bar w) = x_0^\mu + \frac{a^2}{\pi}p^\mu(w+\bar w) + \frac{a}{2\pi}\sum_{n\neq 0}\frac{\alpha_n^\mu}{2n}e^{-2nw} + \frac{a}{2\pi}\sum_{n\neq 0}\frac{\tilde\alpha_n^\mu}{2n}e^{-2n\bar w}$$

where $\pi_{CM}^\mu = p^\mu = \frac{1}{\pi}\int_0^\pi d\sigma\,\frac{\delta S}{\delta \dot X_\mu} = \frac{1}{\pi}\cdot\frac{2}{4\pi\alpha'}2a^2p^\mu\cdot\pi = \frac{a^2}{\pi\alpha'}p^\mu \Rightarrow a^2 = \pi\alpha'$

$$\Rightarrow X^\mu(w,\bar w) = x_0^\mu + \alpha' p^\mu(w+\bar w) + \text{osc.}$$

Suppose now to have a compact dimension:

$$X^i(w - i\pi, \bar{w} + i\pi) = X^i(w, \bar{w}) + 2\pi R$$

then we find:

$$p^i + q^i = \frac{2n}{R}\sqrt{\frac{\alpha'}{2}}$$

$$p^i - q^i = \sqrt{\frac{2}{\alpha'}}\, w R$$

$$\Rightarrow M^2 = -(p+q)^\mu (p+q)_\mu = \frac{2}{\alpha'}(p^i)^2 + \frac{4}{\alpha'}(N-1)$$

$$= \frac{2}{\alpha'}(q^i)^2 + \frac{4}{\alpha'}(\tilde{N}-1)$$

where:

$$N = \sum_{k=1}^{\infty} \alpha^\mu_{-k}\alpha_{k,\mu},$$

$$p^i = \left(\frac{n}{R} + \frac{wR}{\alpha'}\right)\sqrt{\frac{\alpha'}{2}},$$

$$q^i = \left(\frac{n}{R} - \frac{wR}{\alpha'}\right)\sqrt{\frac{\alpha'}{2}}.$$

$\Rightarrow$ In the $R \to 0$ limit the compact dimension does not disappear!

Then

$$R \mapsto \frac{\alpha'}{R} \Rightarrow M^2 \text{ unchanged} \to \boxed{\text{"T-DUALITY"}}$$

$\Rightarrow$ What is the action of T-duality on the oscillators?

$$p^i \xmapsto{T} p^i$$

$$q^i \xmapsto{T} -q^i$$

$$\Rightarrow X^i_L(w) + X^i_R(\bar{w}) \xmapsto{T} X^i_L(w) - X^i_R(\bar{w})$$

$\rightarrow$ T-duality: "spacetime parity on right modes".

⇒ What is the action on OPEN STRINGS?

→ Open strings do not wrap around the compact dimension:

↓

The compact dimension disappears

⇒ from a D dimensional theory we get a D-1 theory of open strings. → Its endpoints are constrained on a (D-1)-dimensional plane:

$$\boxed{\partial_\sigma X^i \xmapsto{T} \partial_\tau X^i = 0}$$

⇒ T-duality exchanges the boundary conditions of the endpoints:

$$\begin{cases} \partial_\sigma X^\mu \big|_{\sigma=0,\pi} = 0 & \leftarrow \text{Neumann conditions} \\ \partial_\tau X^\mu \big|_{\sigma=0,\pi} = 0 & \leftarrow \text{Dirichlet conditions} \end{cases}$$

[equivalent to $\delta X^\mu = 0$]

(or in general: $X^\mu \big|_{\sigma=0,\pi} = X_0^\mu$)

If we apply T-duality to p+1 dimensions, we constrain the theory on a (p+1)-dimensional surface:

⇓

Dp-brane := (p+1)-dimensional hypersurface where string endpoints live.

How do we implement the b.c. on the string? $\left[X(\tau,\sigma) = x_0 + \alpha' p\tau + \alpha' q\sigma + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha_n}{n}e^{-n(\tau-i\sigma)} + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n}{n}e^{-n(\tau+i\sigma)}\right]$

1. NEUMANN: $\partial_\sigma X\big|_{\sigma=0} = 0 \Rightarrow \begin{cases} q=0 \\ \alpha_n - \tilde{\alpha}_n = 0 \end{cases}$

2. DIRICHLET: $\partial_\tau X\big|_{\sigma=0} = 0 \Rightarrow \begin{cases} p=0 \longrightarrow \text{CM momentum} \\ \alpha_n + \tilde{\alpha}_n = 0 \end{cases}$

Combining the two endpoints:

1. NN $\Rightarrow \alpha_n - \tilde{\alpha}_n = 0$ and $n \in \mathbb{Z}$,
2. DD $\Rightarrow \alpha_n + \tilde{\alpha}_n = 0$ and $n \in \mathbb{Z} + \frac{1}{2}$,
3. ND $\Rightarrow \alpha_n - \tilde{\alpha}_n = 0$ and $n \in \mathbb{Z} + \frac{1}{2}$,
4. DN $\Rightarrow \alpha_n + \tilde{\alpha}_n = 0$ and $n \in \mathbb{Z}$.

That is:

1. NN $\Rightarrow X(\tau,\sigma) = x_0 + \alpha' p\tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\alpha_n}{n} e^{-n\tau} \cos(n\sigma)$

2. DD $\Rightarrow X(\tau,\sigma) = x_0 + \alpha' q\sigma + \sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\alpha_n}{n} e^{-n\tau} \sin(n\sigma)$

3. ND $\Rightarrow X(\tau,\sigma) = x_0 + i\sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}+\frac{1}{2}} \frac{\alpha_n}{n} e^{-n\tau} \cos(n\sigma)$

4. DN $\Rightarrow X(\tau,\sigma) = x_0 + \sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}+\frac{1}{2}} \frac{\alpha_n}{n} e^{-n\tau} \sin(n\sigma)$

Define:

$$z = e^{w} \quad ; \qquad \bar{z} = e^{\bar{w}}.$$

The canonical commutation relations are:

$$[\alpha_n, \alpha_m] = i n \delta_{n+m,0}$$

which shows that:

$$T(z) = \frac{1}{2} : \partial_z X_L(z) \partial_z X_L(z) : = \frac{1}{2} \sum_{n\in\mathbb{Z}} L_n z^{-n-2},$$

$$\bar{T}(\bar{z}) = \frac{1}{2} : \partial_{\bar{z}} X_R(\bar{z}) \partial_{\bar{z}} X_R(\bar{z}) : = \frac{1}{2} \sum_{n\in\mathbb{Z}} \bar{L}_n \bar{z}^{-n-2}.$$

We need to impose, by both N and D conditions:

$T(z) = \bar{T}(\bar{z}) \Rightarrow L_n = \bar{L}_n \quad [c = \bar{c}] \Rightarrow$ There exists <u>only one</u> Virasoro Algebra

$\Downarrow$

$\mathcal{V}_{T\bar{T}} = V_T \oplus V_{\bar{T}}$

We can then do the same computations for the superstring. Consider:

$$S = -\frac{1}{4\pi\alpha'} \int d\tau\, d\sigma \left(-i\, \bar{\psi}^{\mu} \rho^{\alpha} \partial_{\alpha} \psi_{\mu} \right)$$

where

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \text{ and } \bar{\psi} = \psi^{\dagger}\rho^0, \quad \psi = \begin{pmatrix} \psi \\ \tilde{\psi} \end{pmatrix},$$

then :

$$S = \frac{1}{4\pi\alpha'} \int dw\, d\bar{w} \left(\psi^{\mu} \partial_{w} \psi_{\mu} + \tilde{\psi}^{\mu} \partial_{\bar{w}} \tilde{\psi}_{\mu} \right),$$

s.t.:

CLOSED:

$$\psi^{\mu}(w + 2\pi i) = e^{2\pi i \nu} \psi^{\mu}(w)$$
$$\tilde{\psi}^{\mu}(\bar{w} + 2\pi i) = e^{-2\pi i \tilde{\nu}} \tilde{\psi}^{\mu}(\bar{w})$$

where $\nu, \tilde{\nu} = \begin{cases} 0 \longrightarrow \text{RAMOND} \\ \frac{1}{2} \longrightarrow \text{NEVEU-SCHWARZ} \end{cases}$

$\Downarrow$

4 different L-R sectors!

$$\Rightarrow \psi^{\mu}(z) = \sum_{r \in \mathbb{Z}+\nu} \psi^{\mu}_{r}\, z^{-r-\frac{1}{2}} ;$$

$$\tilde{\psi}^{\mu}(\bar{z}) = \sum_{r \in \mathbb{Z}+\tilde{\nu}} \tilde{\psi}^{\mu}_{r}\, \bar{z}^{-r-\frac{1}{2}}, \quad \text{where } z = e^{-w}, \ \bar{z} = e^{-\bar{w}}.$$

$\Downarrow$

NB: branch cut in R sector!

OPEN :

$$\psi^{\mu}(\tau, 0) = e^{2\pi i \nu} \tilde{\psi}^{\mu}(\tau, 0)$$
$$\psi^{\mu}(\tau, \pi) = \tilde{\psi}^{\mu}(\tau, \pi)$$

$\Rightarrow$ only 2 sectors!

NB.

NS: $\psi^{\mu}_{r} |0\rangle_{NS} = 0 \ \forall r > 0$

R: $\psi^{\mu}_{r} |0\rangle_{R} = 0 \ \forall r > 0 \Rightarrow$ degeneracy for $\{\psi^{\mu}_{0}, \psi^{\nu}_{0}\} = \eta^{\mu\nu} \Rightarrow$ CLIFFORD ALGEBRA

$$|0\rangle_R = |s_1, s_2, s_3, s_4, s_5\rangle_R = |\vec{s}\rangle_R$$

$\Downarrow$

$$Spin(10) \Rightarrow 32 = 16 \oplus 16'$$

NB: we use 5 bosons $H^a(z)$ to express $S_{\alpha} = e^{i s_a H^a(z)}$ which is the SPIN FIELD which creates $|\vec{s}\rangle_R$ from $|0\rangle_{NS}$.

Because of the branch cut, the spacetime field theory is NOT LOCAL. We implement the GSO projection by means of $\Gamma_{11}(-1)^{\sum 4 \cdot m \cdot 4m}$:

$$\boxed{\begin{array}{llll} \text{Type IIA}: & \text{- bosons:} & G_{\mu\nu},\ B_{\mu\nu},\ \phi & (35 \oplus 28 \oplus 1) \\ & \text{- forms:} & C_{\mu},\ C_{\mu\nu\rho} & (8 \oplus 56) \\ & \text{- fermions:} & \psi^{\mu}_{\alpha},\ \tilde{\psi}^{\mu}_{\dot\alpha},\ \lambda_{\alpha},\ \tilde{\lambda}_{\dot\alpha} & (56 \oplus \overline{56} \oplus 8 \oplus \overline{8}) \\ \text{Type IIB}: & \text{- bosons:} & G_{\mu\nu},\ B_{\mu\nu},\ \phi & (35 \oplus 28 \oplus 1) \\ & \text{- forms:} & C,\ C_{\mu\nu},\ C^{+}_{\mu\nu\rho\lambda} & (1 \oplus 28 \oplus 35) \\ & \text{- fermions:} & \psi^{\mu}_{\alpha},\ \chi^{\mu}_{\alpha},\ \lambda_{\alpha},\ \sigma_{\alpha} & (56 \oplus 56 \oplus 8 \oplus 8) \end{array}}$$

We know that from the open strings, the boundary conditions exchange L and R sectors:

SUSY current: $j(z) + \tilde{j}(\bar{z}) \longrightarrow$ only $Q_\alpha + \tilde{Q}_\alpha$ can be conserved

$\Downarrow$

branes are BPS

Under T-duality R modes change the sign:

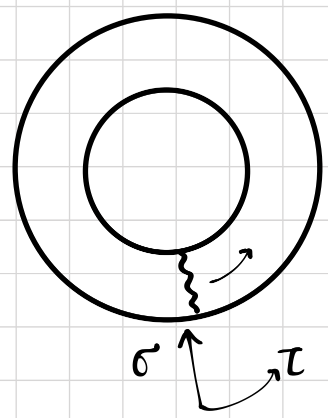
$$Q_\alpha \xmapsto{T} Q_\alpha$$

$$\tilde{Q}_\alpha \xmapsto{T} \beta^{i}_{\alpha\beta} Q_\beta \qquad \text{where } i \text{ is the T-dual coordinate.}$$

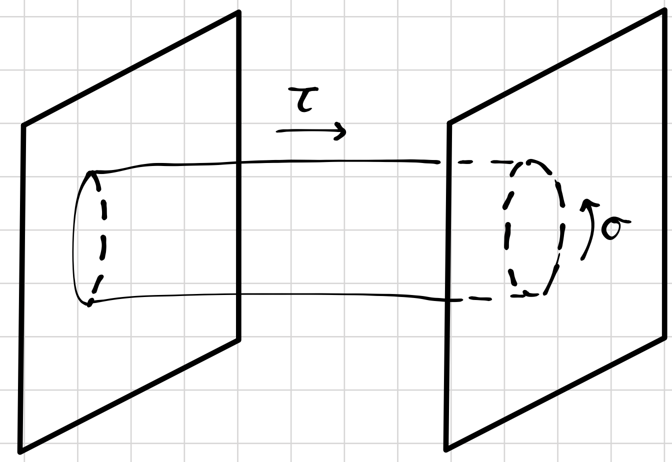
$\Rightarrow$ The unbroken SUSY is $Q_\alpha + \beta^{\perp}\tilde{Q}_\alpha$, where $\beta^{\perp} = \prod_{\{T\text{-dual}\}} \beta^{i}$.

* <u>Boundary states and partition functions</u>

Consider the vacuum 1-loop amplitude for the BOSONIC OPEN STRING:



$\longrightarrow$ under the exchange $(\tau,\sigma) \leftrightarrow (\sigma,\tau)$ we can interpret the amplitude AS A CLOSED STRING being emitted and absorbed by boundaries (D-branes)



$\Downarrow$

OPEN STRING CHANNEL | CLOSED STRING CHANNEL

N: $\partial_\sigma X|_{\sigma=0} = 0 \longrightarrow \partial_\tau X|_{\tau=0} |B_N\rangle = 0 \Rightarrow (\alpha_n + \tilde{\alpha}_{-n})|B_N\rangle = 0$

and $p|B_N\rangle = 0$.

D: $\partial_\tau X|_{\sigma=0} = 0 \longrightarrow \partial_\sigma X|_{\tau=0} |B_D\rangle = 0 \Rightarrow (\alpha_n - \tilde{\alpha}_{-n})|B_D\rangle = 0$.

and s.t.:

$$X|_{\tau=0}|B_D\rangle = x_0 |B_D\rangle$$

Then we have for the bosonic string:

$$|B\rangle = \mathcal{N}\, \delta^{\perp}(y - x_0) \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^{\mu} S_{\mu\nu} \tilde{\alpha}_{-n}^{\nu}\right] |0; k=0, y\rangle$$

where:

$$S_{\mu\nu} = \begin{pmatrix} \eta^{\alpha\beta} & \\ & -\delta_{ij} \end{pmatrix} \longrightarrow \begin{array}{l} \alpha,\beta \in \{\parallel\} \\ i,j \in \{\perp\} \text{ (applied T-duality).} \end{array}$$

How do we fix $\mathcal{N}$?

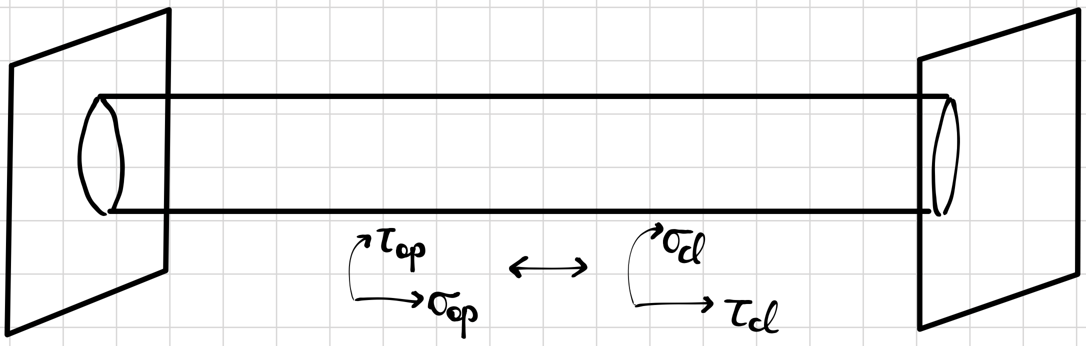
We have to define the D-brane action, which is the Dirac-Born-Infeld action. Let ξ^a be the coordinates on the brane, then:

$$S_{DBI} = -T_p \int d^{p+1}\xi \; e^{-\phi} \left[-\det\left(\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \partial_a X^\mu \partial_b X^\nu B_{\mu\nu} + 2\pi\alpha' F_{ab} \right) \right]^{1/2}.$$

The tension of the Dp-brane is then

$$\tau_p = T_p \, e^{-\langle \phi \rangle}$$

We can then consider the exchange of closed strings between the branes:



Then we find (we assume the superstring framework):

$$\mathcal{A} = \underbrace{(1-1)}_{\text{NSNS - RR} \Rightarrow \text{supersymmetry}} V_{p+1} \frac{1}{2} \int \frac{dt}{t} (2\pi t)^{-(p+1)/2} \left(\frac{t}{2\pi\alpha'} \right)^4 e^{-\frac{ty^2}{8\pi\alpha'^2}}$$

where Y is the separation between the branes. We are now interested in the coupling of massless CLOSED STRINGS $\Rightarrow$ $t \to 0$ in NS-NS sector:

$$\mathcal{A}_{NSNS} = (1-1) V_{p+1} \, 2\pi \left(4\pi^2\alpha' \right)^{3-p} G_{9-p}(Y^2)$$

where

$$G_{9-p}(Y) = \frac{1}{4} \pi^{\frac{p-9}{2}} \Gamma\left(\frac{7-p}{2}\right) Y^{p-7} \qquad \left[\text{NB: } \int dt \, d^p y \, d^{d-p-1} x \; e^{ik_\perp \cdot x} G(x) = \frac{V_{p+1}}{k_\perp^2} \right]$$

is the Green function of the massless string modes. This should then be compared with field theory computation of graviton-dilaton theory in 10 dimensions:

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-\tilde{G}} \left(\tilde{R} - \frac{1}{6} \nabla_\mu \tilde{\phi} \nabla^\mu \tilde{\phi} \right)$$ (then expand to first order $\tilde{G}$ and second order $\tilde{\phi}$)

$$\Rightarrow \mathcal{F}\left[\langle 0 | T\left(\tilde{\phi}(y) \tilde{\phi}(0) \right) | 0 \rangle \right](k) = -\frac{2i\kappa^2}{k^2}$$

$$\mathcal{F}\left[\langle 0 | T\left(h_{\mu\nu}(y) h_{\rho\sigma}(0) \right) | 0 \rangle \right](k) = -\frac{2i\kappa^2}{k^2} \left(\eta_{\mu\sigma}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\sigma} - \frac{1}{4} \eta_{\mu\nu}\eta_{\rho\sigma} \right)$$

Then the D-brane action becomes:

$$S_p = -\tau_p \int d^{p+1}\xi \left(\frac{p-11}{12} \tilde{\phi} - \frac{1}{2} h_{aa} \right)$$

$$\Rightarrow \mathcal{A} = \frac{2i\kappa^2 \tau_p^2}{k_\perp^2} V_{p+1}$$

Therefore:

$$\tau_p^2 = \frac{\pi}{\kappa^2} (4\pi^2 \alpha')^{3-p}.$$

The same can be applied to the R-R contribution (coupling Dp-brane to a (p+1)-form potential):

$$S = -\frac{1}{4\kappa^2} \int d^{10}x \sqrt{-G} \, |F_{p+2}|^2 + \mu_p \int C_{p+1}$$

then the amplitude for the exchange of a (p+1)-form is:

$$\mathcal{A} = -2i\kappa^2 \mu_p^2 \, G_{9-p}(Y)$$

$$\Rightarrow \mu_p^2 = \frac{\pi}{\kappa^2} (4\pi^2 \alpha')^{3-p} = e^{2\phi} \tau_p^2 = T_p^2$$

The same result can be computed from the BS formalism introducing the propagator of the closed string:

$$D = \frac{\alpha'}{4\pi} \int_{|z| \le 1} \frac{d^2 z}{|z|^2} z^{L_0 - 1} \bar{z}^{\bar{L}_0 - 1}$$

and the amplitude

$$\langle B | D | B \rangle = \mathcal{A}$$

Then:

$$|B\rangle = \frac{\sqrt{\pi}}{2} \delta^\perp (y - x_0) (4\pi^2 \alpha')^{3-p/2} \exp\left(-\sum_{k=1}^{\infty} \frac{1}{n} \alpha_{-n}^\mu S_{\mu\nu} \alpha_{-n}^\nu \right) |0; k=0, y\rangle$$