IR STRUCT. OF SUSY AND SUGRA

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$$\Rightarrow \quad F^{\mu\nu} = \frac{e}{8\pi} \cdot \frac{\beta^\mu x^\nu - \beta^\nu x^\mu}{(-x^2)} \, \delta(\beta_\mu x^\mu) \qquad \beta_\mu \beta^\mu = 0$$

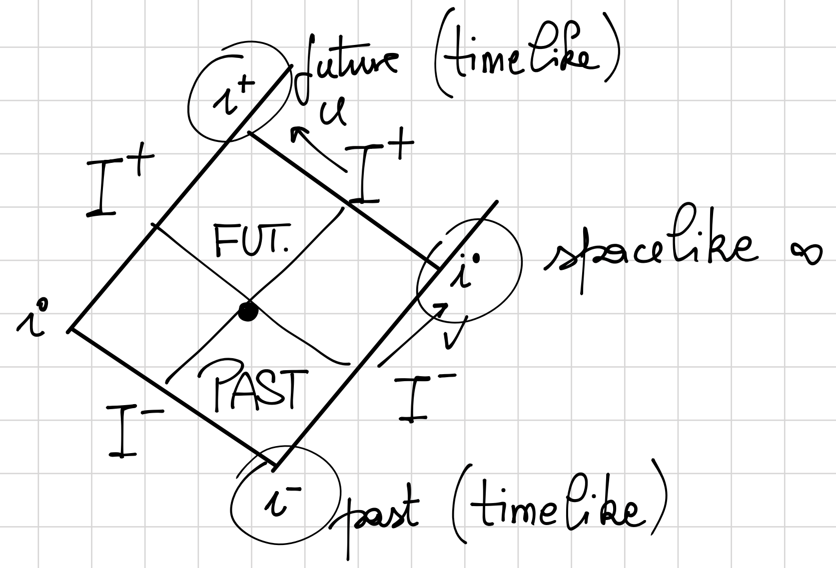
$\hookrightarrow$ behaviour at ∞:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \qquad \eta = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$\downarrow$ conf. transf.

$$\Omega^2(x)\, ds^2$$

$$\left[\Omega \sim \frac{1}{r}\right]$$



$\vec{x} = r\hat{x}$

$u = t - r$

$v = t + r$ $\quad \Rightarrow \quad$ u or v const are massless geodesics

$\Rightarrow$ $M^2 = 0$ start from I^- to I^+

$$\Rightarrow \quad -x^2 = (u+r)^2 - r^2 = u^2 + 2ru$$

Consider

$r \to \infty \quad u = \text{const} \Rightarrow$ on I^+

$$\hookrightarrow \quad \beta_\mu x^\mu = \beta(u+r) - \beta r \hat{\beta}\hat{x}$$

Then:

$$F^{or} = \frac{e^2}{2\pi} \frac{\beta r - \beta\hat{\beta}\cdot\hat{x}(u+r)}{u^2 + 2ru} \delta\left(\beta r(1-\hat{\beta}\hat{x}) - \beta u\right)$$

$$\simeq -\frac{e^2}{2\pi}\beta \frac{(\hat{\beta}\cdot\hat{x}+1)}{2r} \delta\left(\beta r(1-\hat{\beta}\cdot\hat{x}) + \beta u\right) =$$

$$= -\frac{e^2}{2\pi}\frac{1}{r^2}\delta(1-\hat{\beta}\cdot\hat{x}) \quad \text{at } I^+ \quad (r\to+\infty \text{ and } u = \text{const})$$

$$F^{or} = -\frac{e^2}{2\pi}\frac{1}{r^2}\delta(1+\hat{\beta}\hat{x})$$

NB: $I^+_- \Rightarrow I^+$ for $u\to\infty$ ending i^0

$I^-_+ \Rightarrow I^-$ for $v\to\infty$ "

↳ F at I^+_- w/ coord. (θ,φ) = F at I^-_+ w/ coord. $(\pi-\theta, \varphi+\pi)$

(NB) there is a discontinuity at i^0.

NB: $F = \frac{1}{r^2}F^{(2)} + \frac{1}{r^3}F^{(3)} + \dots$

↳ $F_+^{(2)}(\theta,\varphi) = F_-^{(2)}(\pi-\theta, \varphi+\pi)$

Now consider the scattering at future and past:

$$ds^2 = -du^2 - 2\,du\,dr + r^2 d\theta^A d\theta^B \gamma_{AB}$$
$$= -dv^2 + 2r\,dv\,dr + r^2 d\tilde{\theta}^A d\tilde{\theta}^B \gamma_{AB}$$

($\tilde{\theta}$ is antipodal to θ)

Therefore:

$$\int_{\mathcal{I}^+_-} *F\Big|_\theta = Q^+(\theta)$$

$$\int_{\mathcal{I}^-_+} *F\Big|_\theta = Q^-(\tilde{\theta})$$

$$\Rightarrow Q^+(\theta) = Q^-(\tilde{\theta})\Big|_{\theta=\tilde{\theta}}$$

↳ These are <u>changes</u> $\Rightarrow$ Symm.

$$\int_{\mathcal{I}^+_-} *F\, Y_{\ell m} = Q_{\ell m} \quad \infty \text{ changes!}$$

$$\ell = -\infty, \dots, +\infty \qquad m = -\ell, \dots, \ell$$

where

$$\int_{\mathcal{I}^+_-} *F\varepsilon = \lim_{\substack{r\to\infty \\ u\to\infty}} \int d\theta^A d\theta^B \frac{r^2}{2} \epsilon_{ABur} F_{ur}(\theta, r, u)\, \varepsilon(\theta)$$

What are the corresponding Symmetries?

* GAUGE CHOICE $\quad A_r = 0$
* RESIDUAL GAUGE $\quad A_u = 0\Big|_{r=\infty}$
* constraints: $\partial_u F_{ru} + D_A F^A{}_u = j_u$

↳ the only dynam. are A_A. $\to$ 2 polar.

$\Rightarrow\ F_{AB}$ = magn. field on $\mathcal{I}^+ \xrightarrow{u\to\infty}$ magn. field at $\infty = 0$ (no monopoles)

CANONICAL STRUCTURE

Ω sympl. form $\Rightarrow [\omega^i, \omega^j] = i\{\omega^i, \omega^j\}_{PB} = i\Omega^{ij}$

What if ∞ coords? $\Rightarrow$ QFT

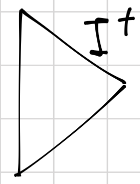
$J^\mu(x) = \delta F^{\mu\nu}\,\delta A_\nu \Longrightarrow$ symplectic current (δ anticomm.)

↳ $\Omega = \int d^3x\, J^0 = \int d^3x\, \underbrace{\delta F^{0i}}_{\delta E^i}\,\delta A_i$

t = const
initial value

What if we take $\mathcal{I}^\pm$ as asymptotic data?

$$\int_{\mathcal{I}^+} *\delta F\wedge\delta A = \int_{\mathcal{I}^+} du\, d^2\Omega\, \sqrt{\gamma}\, \delta F_{uA}\, \delta A_B\, \gamma^{AB}$$



$$\longrightarrow \int_{\mathcal{I}^+_-} *F\varepsilon = -\int_{\mathcal{I}^+} d(*F\varepsilon) + \cancel{\int_{\mathcal{I}^+_+} *F\varepsilon}^{\,0}$$

$$= -\underbrace{\int_{\mathcal{I}^+} \underbrace{d(*F)}_{=*j}\wedge\varepsilon}_{Q_H^+} - \underbrace{\int_{\mathcal{I}^+} *F\wedge d\varepsilon}_{Q_S^+}$$

MAXWELL!

$Q_S^+ \longrightarrow$ "soft" (ε = const $\Rightarrow Q_S^+ \equiv 0$)

Q_H^+ ↳ "hard" (w/ particles)

NB: $A_A = \int \frac{d\omega}{2\pi}\, a_{A,\omega\ell m}\, Y_{\ell m}(\theta)\, e^{-i\omega u} + h.c.$

$$\Rightarrow Q_S^+ = -\int\frac{d\omega}{2\pi}\int du\, \partial_u\left[e^{-i\omega u}\, a^A_{\omega\ell m}\int d\Omega^2\, \partial_A\varepsilon\, Y_{\ell m}\right] =$$

$$= \underbrace{\frac{i}{2}\, a^A_{\omega\ell m}\int d^2\Omega\, \partial_A\varepsilon\, Y_{\ell m}}_{\text{annihil.}} + \underbrace{h.c.}_{\text{creat.}}$$

$\Rightarrow$ Q_S^+ creates/ann. soft (0-freq.) photons !!!

$$[Q^+(\varepsilon), A_A] = i\,\partial_A \varepsilon(\theta)$$

$$[Q^+(\varepsilon), \phi] = q\varepsilon\phi$$

$j^\mu = i\,\phi^* \overleftrightarrow{\partial^\mu} \phi$

$\hookrightarrow$ precisely as a charge!

$$[Q_S(\varepsilon), F_{uA}] = 0$$

$$[Q_S(\varepsilon), \phi] = 0$$

$\hookrightarrow$ c-no. on $\mathcal{H}$?

NB: $\underbrace{Q_H^+ + Q_S^+}_{Q^+} = \underbrace{Q_{H}^- + Q_S^-}_{Q^-}$

However

$$A_A \xrightarrow[u\to\infty]{} -\partial_A C^+(\theta)$$

"boundary photon"

$$[Q_S^\pm(\theta), C^\pm(\theta')] = -i\,\delta^2(\theta-\theta')$$

$\hookrightarrow$ canonically conj.

What happens at the evolution?

$\rightarrow$ $\Theta\, C^+ = C^-$

CPT $\Theta\, C^- = C^+$

$\rightarrow$ equate $C^+ = C^- \Rightarrow Q^+ = Q^-$ ✓

Evolution:

$$U_+ = \exp\Big[-i \sum_{\ell m} C^+_{\ell m} Q^+_{\ell m}\Big]$$

$\hookrightarrow$ $Q^+ = U_+ Q_S^+ U_+^{-1}$

$C^+ = U_+ C^+ U_+^{-1}$

$\Rightarrow$ $U_+ \phi U_+^{-1} = \hat{\phi} = \phi\, e^{iqC(\theta)} \longrightarrow [\hat{\phi}, Q^+] = 0$

$\Rightarrow$ $\hat{A}_+ = \hat{\Omega}^{-1} \hat{A}_- \hat{\Omega}$

Heisenberg evol. oper.

$U_+ = \hat{\Omega}^{-1} U_- \hat{\Omega}$

$\hookrightarrow$ $[\hat{\Omega}, C^+] = [\hat{\Omega}, Q^+] = 0$

$\Rightarrow \frac{\partial}{\partial C_{\ell m}} \hat{\Omega}(C_{\ell m}) = 0 \Longrightarrow \hat{\Omega}$ totally indep. on SOFT dofs

FACTORIZATION

$$(U_- \psi,\ \hat{\phi}_- U_- \Phi) \xrightarrow{S} (U_- \psi,\ \hat{\Omega}^{-1} U_- \phi \underbrace{U_-^{-1} \hat{\Omega} U_-}\, \Phi)$$

free

$$S = S_0^{-1}\, U_-^{-1} \hat{\Omega}\, U_-$$

$$S_0 S_0^{-1}$$

$$= (U_+^{I})^{-1} S_0^{-1} \hat{\Omega}\, U_-^{I}$$

$$S = \underbrace{(U_+^{I})^{-1}}_{\text{soft}}\ \underbrace{\hat{\Omega}}_{\text{no soft}}\ U_-$$