Coset constr.

$V_\Phi(g,x) : G \to \mathbb{R}$

$$V_\Phi(g,x) = \phi^k(x)\, g_{kl} \langle \phi^l \rangle$$

NB: $\langle \phi_k \rangle = \underbrace{h_{kl}}_{\hookrightarrow\, h \in H \text{ (unbroken)}} \langle \phi_l \rangle$

if G compact: maxima of V_ϕ define $g(x)$ (degen)

$\to$ COSET $g \cdot h \quad \forall h \in H$

$\phi^k g_{kl} \langle \phi^l \rangle = \phi^k g_{kl} h^l_m \langle \phi^m \rangle$

break degeneracy by choosing a basis

$[T,T] \sim T$

$[T,X] \sim X$

$[X,X] \sim X + T$

$\longrightarrow$ choose $\boxed{U(x) = e^{i\pi^a X^a}}$

These maxima solve $\tilde{\phi}(x) = U(x)\phi(x)$, $\langle \tilde{\phi}(x) \rangle = 0$

$$V_\Phi\left(g(x)\, e^{i\delta^m X^m + i\varepsilon^k T^k}\right) = V_\Phi(g(x)) + O(\delta^2, \varepsilon^2)$$

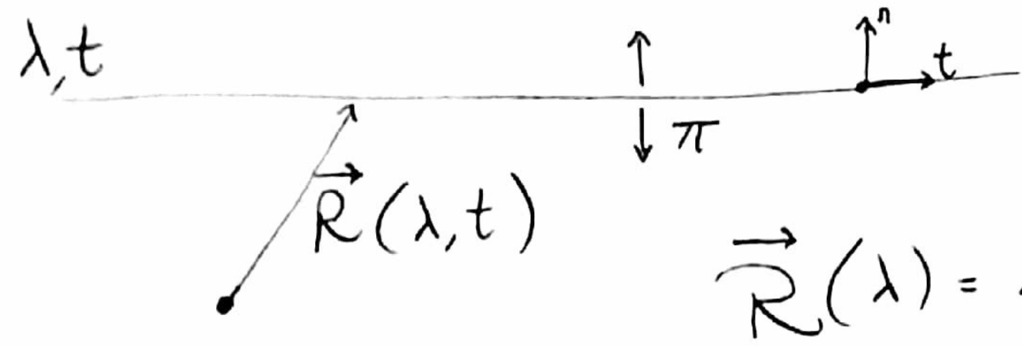
Linear Term: $i\delta^a V_\Phi(g(x)X^a) + i\varepsilon^k V_\Phi(g(x)T^a) = 0$

$\Rightarrow V_\Phi(g(x)X^a) = 0$ (indep

$V_\Phi(g(x)T^a) = 0$

$\Rightarrow \underbrace{\phi^k g_{kl}}_{\tilde{\phi}(x)} X^a_{lm} \langle \phi_m \rangle = 0 \quad \xrightarrow{\simeq} \langle \tilde{\phi} \rangle = 0$

Suppose



$$\vec{R}(\lambda) = \hat{x}\,\lambda$$

$$\Rightarrow \vec{R}(\lambda,t) = \vec{R}(\lambda) + \pi(\lambda,t)\,\overset{\hat{y}}{\hat{n}(\lambda)} =$$

→ do I worry about rotons?

$$\theta(\lambda,t)$$

$$\Rightarrow \vec{R}(\lambda,t) = \vec{R}(\lambda) + \frac{d\hat{R}}{d\lambda} + \underbrace{\theta(\lambda,t)\hat{n}}$$

redef. → NO ROTONS

NB:

$$\left.\begin{array}{l} [J, P_t] = i P_n \rightarrow \text{broken} \\ [J, P_n] = -i P_t \rightarrow \text{unbroken} \end{array}\right\} e^{iHt - iP_t\lambda}\, e^{-iP_n\pi(\lambda,t)}$$

$$=$$

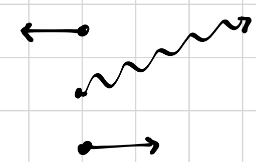
Mode analysis in EFT

$P_n \sim m\,(1, \vec{1})$

SOFT $P_s \sim m\,(v, \vec{v})$ these are QM modes!

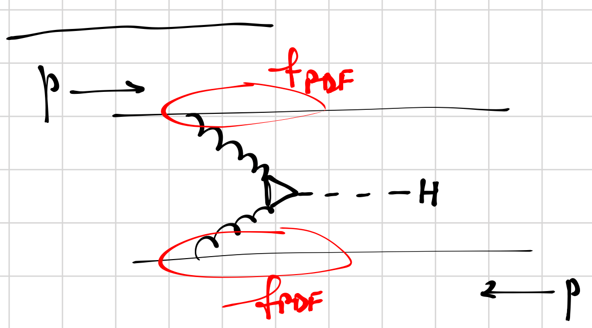
POTENT $P_p \sim m\,(v^2, \vec{v})$

ULTRA SOFT $P_{us} \sim m\,(v^2, \vec{v}^2)$

$\Rightarrow$ 

if P_s it'll break the bound state $\Rightarrow$ order v variation!
↳ cannot observe!

⇝ Potential mode: $\phi^H(t) = e^{imt}\,\psi(t) \rightarrow \dot{\phi}^H(t) = im\,\phi^H + e^{imt}\,\dot{\psi}(t)$ → class. each order scales with mv^2



$p_H \sim m_H\,(1, 1, 1)$

$p_c \sim m_H\left(1, \frac{\Lambda_{QCD}^2}{m_H^2}, \frac{\Lambda}{m_H}\right)$

$p_{\bar{c}} \sim m_H\left(\frac{\Lambda^2}{m_H^2}, 1, \frac{\Lambda}{m_H}\right)$

$p_s \sim m_H\left(\frac{\Lambda}{m_H}, \frac{\Lambda}{m_H}, \frac{\Lambda}{m_H}\right)$

$p_c^2 \sim p^2$

$p_H = p_c + p_{\bar{c}}$

Background response

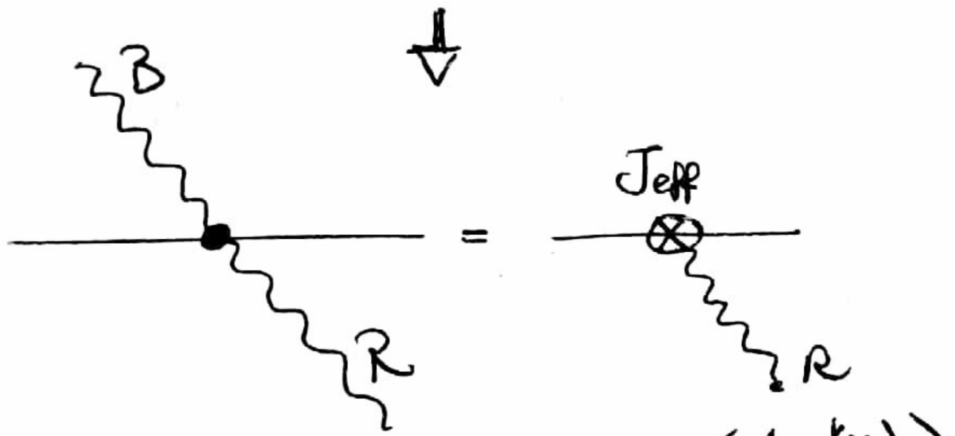
$$S = \int d\lambda \left(c^1 F^2 \sqrt{v^2} + c^2 \frac{(v\cdot F)^2}{\sqrt{v^2}} \right) \qquad v^2 = 1$$

$$= \int dt \left(c^1 \left[\vec{E}^2 - \vec{B}^2 \right] + c^2 \vec{E}^2 \right) =$$

$$= \int dt \left\{ c_E \vec{E}^2 + c_B \vec{B}^2 \right\} = *$$

$$A^\mu = \underbrace{A^\mu_B}_{\text{background}} + \underbrace{A^\mu_R}_{\text{response}} \longrightarrow E^i = -\frac{\partial A^0}{\partial x^i} = E_B^i - \frac{\partial A^0_R}{\partial x_i}$$

$$B^i = \varepsilon^{ijk} \frac{\partial A_k}{\partial x^j} = B^i_B + \varepsilon^{ijk} \frac{\partial A^k_R}{\partial x^j}$$



just R interacting with B

$$* = \int dt \left\{ -2 c_E E_B^i \frac{\partial A^0_R}{\partial x^i} + 2 c_B B_B^i \varepsilon^{ijk} \frac{\partial A^k_R}{\partial x^j} \right\}$$

$$= \int d^4x \left\{ \dots \right\} \delta^3(\vec{x} - \underbrace{\vec{y}(t)}_{\text{worldline}}) = *$$

$\rightarrow$ to be compared with $\int d^4x \, J^\mu(x) A_\mu(x)$

$$\Rightarrow * \overset{\text{IBP}}{=} \int d^4x \underbrace{\left\{ g^{\mu 0} 2 c_E \frac{\partial}{\partial x^i} \left[\delta^3(x - y(t)) E_B^i \right] - \varepsilon^{0\mu j\sigma} 2 c_B \frac{\partial}{\partial x^j} \left[\delta^3(x - y(t)) B_{B\sigma} \right] \right\}}_{J^\mu_{eff}} A_{\mu R}$$

$$\Rightarrow \langle A^\mu_R(z) \rangle_{J_{eff}} = \int d^4x \, G^{R\,\mu\nu}(z - x) \, J^\nu_{eff}(x)$$

(in-in formalism) retarded propagator

$\Rightarrow$ see the response

EFT

$$G_R^{\mu\nu}(z-x) = \frac{g^{\mu\nu}}{4\pi} \cdot \frac{1}{|\vec{z}-\vec{x}|}\,\delta\left(z^0 - x^0 - |\vec{x}-\vec{z}|\right)$$

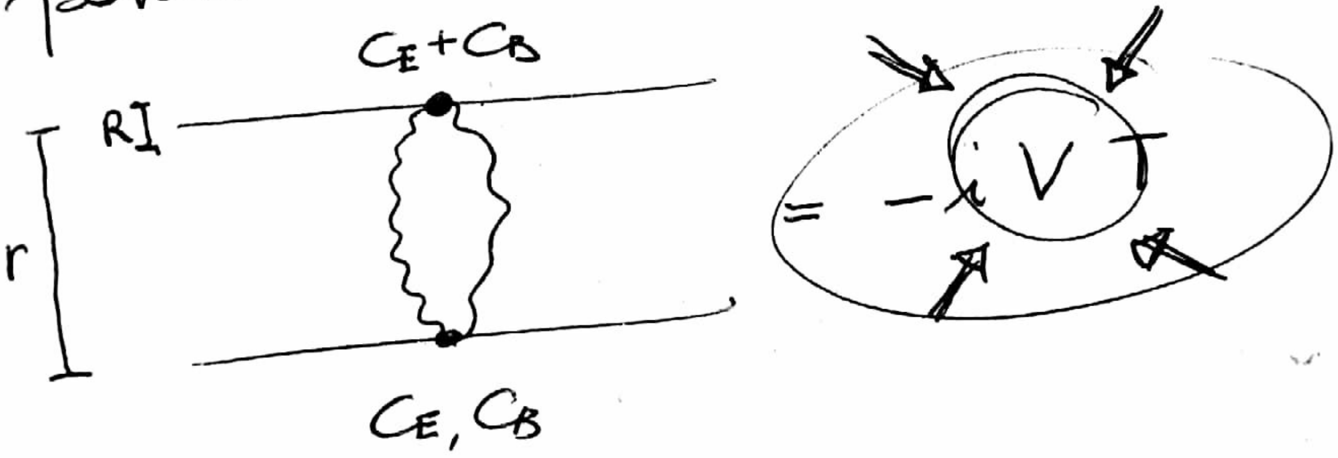
$$\rightarrow \langle A_R^{\mu}(z^0,\vec{z})\rangle = \left[\frac{C_E}{2\pi}\, g^{\mu 0}\, \vec{z}\cdot\vec{E}_B(\vec{0}, z^0 - |\vec{z}|) + \frac{C_B}{2\pi}\left(\vec{z}\times\vec{B}_B(0, z^0-|\vec{z}|)\right)^i \delta_i^{\mu}\right] \cdot \frac{1}{|\vec{z}|^3}$$

$\vec{y}(t)=\vec{0}$

Suppose

$$S^{int} = \sum_{i=1}^{2}\int dt_i \left(C_E \vec{E}^2 + C_B \vec{B}^2\right)^i - \frac{1}{4}\int d^4x\, F_{\mu\nu}F^{\mu\nu}$$

What's the potential between the two particles?



$L^0 \Rightarrow [V]=L^{-1}$

$\Downarrow$

$V \sim \left(\frac{1}{r}\right)^{n+1}$

$[S] = 1\!\!1^0 \Rightarrow [C_E] = L^3 \Rightarrow C_E, C_B \sim R^3$

Then

$$-iT\left(\frac{1}{r}\right)^{n+1} C_{E,B}^2 \Rightarrow \boxed{n=6}$$

coupling constant

$$\Rightarrow V(r) \sim \frac{R^6}{r^7} \quad \text{(Casimir Polder potential Law)}$$

$\left(NB:\ V^{\text{van der Waals}}(r) \sim \frac{R^5}{r^6}\right)$ → must modify the EFT (see Rothstein notes)