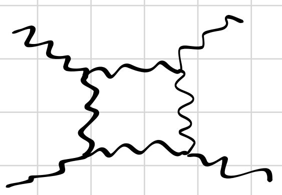
Amplitudeology - ex.

→ Build an accelerator to detect W, Z bosons:

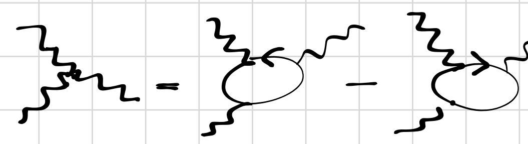
* hard process centre of mass energy: $s x_1 x_2$
* $M_Z \sim 100$ GeV ; $x_1 = x_2 = 0.1$ (maxima in the valence pdf)

↳ $s x_1 x_2 = M_Z^2 \to s = 1$ TeV

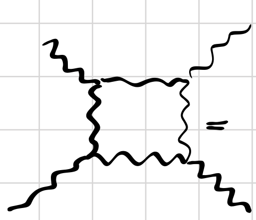
- Trace decomposition



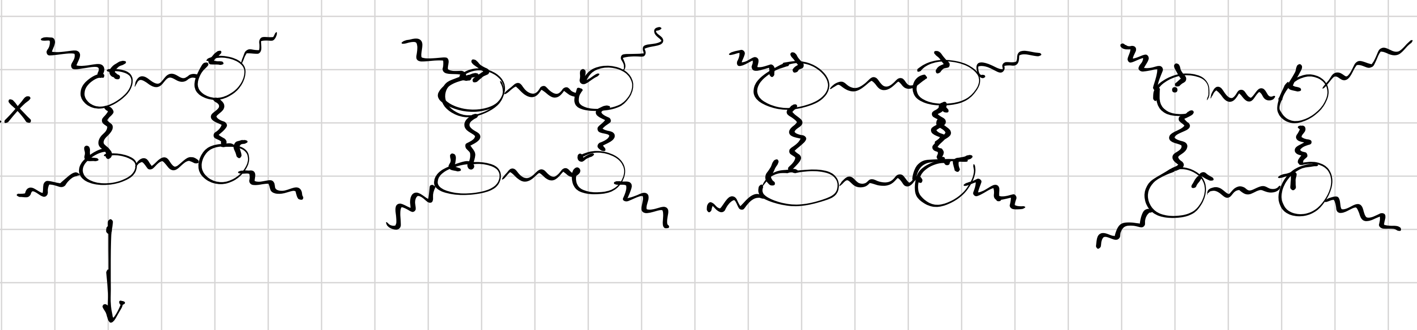
→ use

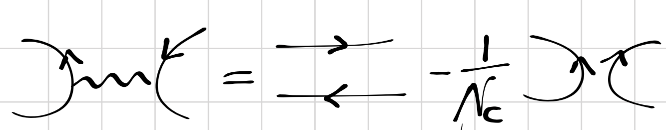


→ technically $2^4 = 16$ comp. but we can reduce them:

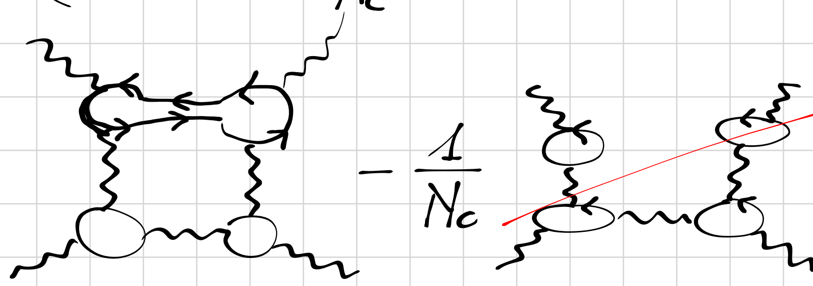


$= 2\times$

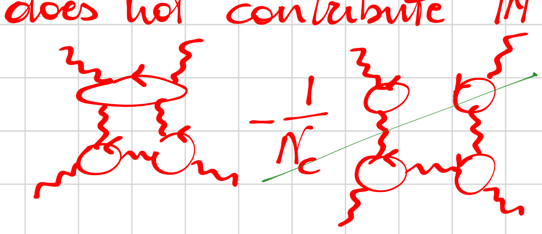




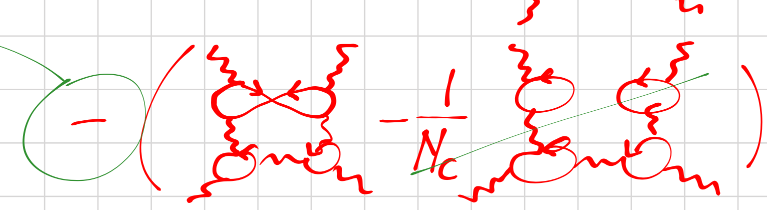
$\Rightarrow$



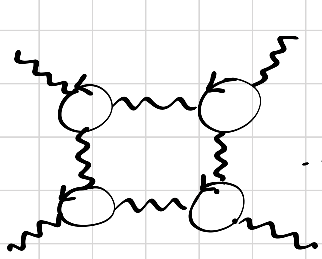
= does not contribute IN THE END



$f_{abc} = \mathrm{Tr}(T_a T_b T_c) - \mathrm{Tr}(T_c T_b T_a)$

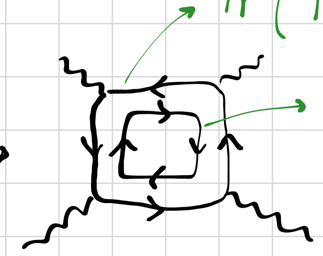


Then:

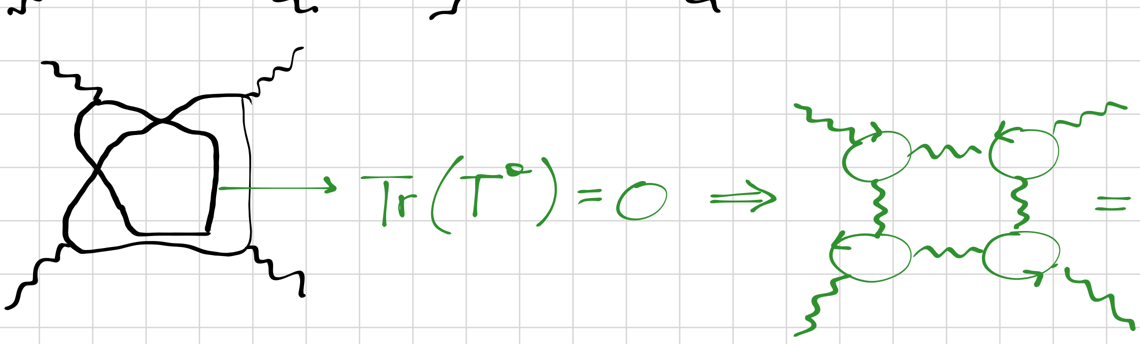


$\mathrm{Tr}(T^a T^b T^c T^d)$

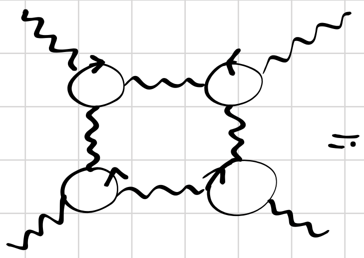
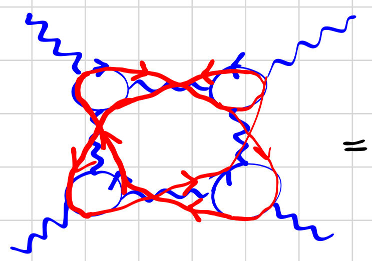
$\mathrm{Tr}(\mathbb{I}) = N_c$

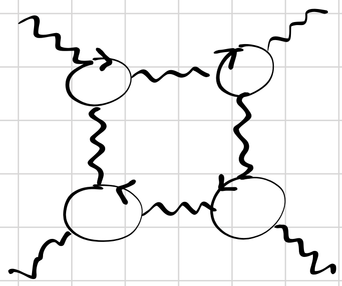
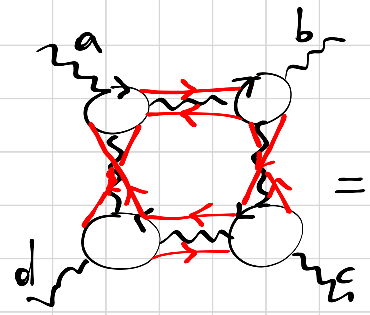


$\mathrm{Tr}(T^a) = 0 \Rightarrow$

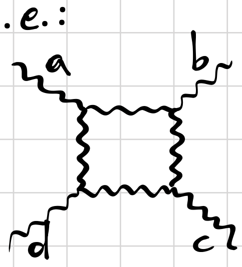


$= 0$

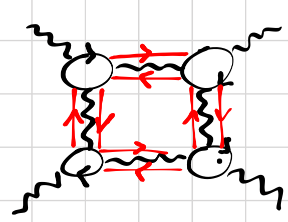
 =  $= \mathrm{Tr}[T^a T^c]\,\mathrm{Tr}[T^b T^d]$

 = 

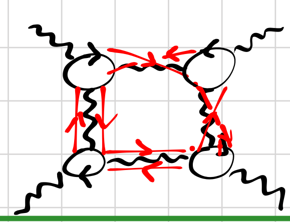
$= \mathrm{Tr}[T^a T^b]\,\mathrm{Tr}[T^c T^d]$

i.e.: 

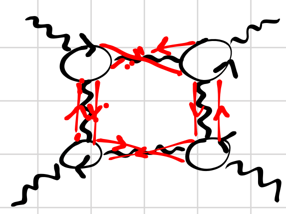
$\longrightarrow$



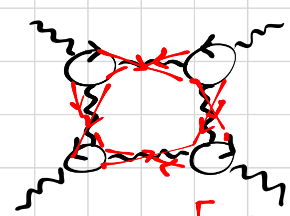
$\mathrm{Tr}(T^a T^b T^c T^d)$



$\boxed{\begin{gathered}\mathrm{Tr}[T^a T^c T^d]\,\mathrm{Tr}[T^b] \\ = 0\end{gathered}}$



$\mathrm{Tr}[T^a T^d]\,\mathrm{Tr}[T^b T^c]$



$\mathrm{Tr}[T^a T^c]\,\mathrm{Tr}[T^b T^d]$

- Polarization vectors:

$$\varepsilon^i_{\alpha\dot\alpha}(q) = \frac{q_\alpha \tilde\lambda^i_{\dot\alpha}}{\varepsilon^{\beta\gamma} q_\gamma \lambda^i_\beta} \longrightarrow \varepsilon_i \cdot k_i = 0 \qquad (k^i_{\alpha\dot\alpha} = \lambda_\alpha \lambda_{\dot\alpha})$$

$$\mathrm{Tr}(\varepsilon_{\alpha\dot\alpha} k^{\alpha\dot\alpha}) = 0$$

$$\hookrightarrow \varepsilon_{\alpha\dot\alpha} k_{\beta\dot\beta} \varepsilon^{\alpha\beta} \varepsilon^{\dot\alpha\dot\beta} = \frac{q_\alpha \tilde\lambda^i_{\dot\alpha}}{\varepsilon^{\beta\gamma} q_\beta \lambda^i_\gamma} \lambda^i_\beta \tilde\lambda^i_{\dot\beta} \varepsilon^{\alpha\beta} \varepsilon^{\dot\alpha\dot\beta}$$

$$\Rightarrow \varepsilon^{\dot\alpha\dot\beta} \tilde\lambda^i_{\dot\alpha} \tilde\lambda^i_{\dot\beta} = 0$$

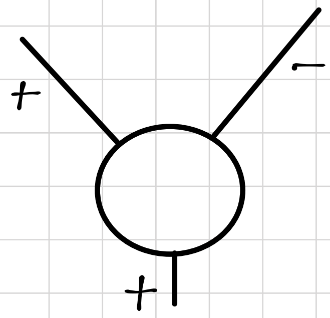
- Shouten identities

$$\lambda^\alpha_k = c_1 \lambda^\alpha_i + c_2 \lambda^\alpha_j$$

$$\hookrightarrow \langle ik \rangle = \cancelto{0}{c_1 \langle ii \rangle} + c_2 \langle ij \rangle \longrightarrow c_2 = \frac{\langle ik \rangle}{\langle ij \rangle}$$

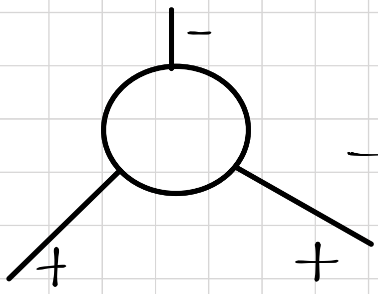
$$\langle jk \rangle = c_1 \langle ji \rangle + \cancelto{0}{c_2 \langle jj \rangle} \longrightarrow c_1 = \frac{\langle jk \rangle}{\langle ji \rangle}$$

$$\hookrightarrow |k\rangle = \frac{\langle jk \rangle}{\langle ji \rangle} |i\rangle + \frac{\langle ik \rangle}{\langle ij \rangle} |j\rangle$$



$\longrightarrow$ it's made of $[\;\;]$ $\longrightarrow$ $[n,1\rangle$ sets all $[\;\;]=0$

$\rightsquigarrow$ VANISHES



$\longrightarrow$ it's made of $[\;\;]$

$\hookrightarrow$ $[n,1\rangle$ chooses all $\langle\;\;\rangle=0$

$\Rightarrow$ this is OK

$*$ $$(+-+-+-) = c_1(+++---) + c_2(++-+--)$$

$$(+\{-+\}-\{+-\}) = 2(+-+-+-) + 4(+--++-) =$$
$$= 2(+-+-+-) + 4(++-+--)$$

$$\hookrightarrow (+-+-+-) = -4(++-+--)$$

$$c_1 = 0$$
$$c_2 = -4$$

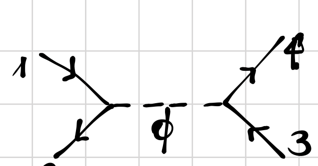
\- $3 \parallel 4$ limit of NMHV of $A_6(+++---)$

$$A_6(\overset{1\,2\,3\,4\,5\,6}{+++---}) = i\left\{\frac{\langle 6|(1+2)|3]^3}{\langle 61\rangle\langle 12\rangle[34][45]\,s_{612}\,\langle 2|(6+1)|5]} + \dots\right\}$$

$$\longrightarrow \frac{\langle 6|(1+2)|p]^3 z^{3/2}}{\langle 61\rangle\langle 12\rangle\sqrt{z(1-z)}\,[p\hat{p}]\dots}$$

$$k_p = k_3 + k_4$$

$$\lambda_3 \sim \sqrt{z}\,\lambda_p \qquad \lambda_4 \sim \sqrt{1-z}\,\lambda_p$$
$$\tilde{\lambda}_3 \sim \sqrt{z}\,\tilde{\lambda}_p \qquad \tilde{\lambda}_4 \sim \sqrt{1-z}\,\tilde{\lambda}_p$$

$*$ 

$$\longrightarrow (ig\,\bar{u}_4 v_3)\frac{-i}{(p_1+p_2)^2}(ig\,\bar{u}_2 v_1)$$

$$ig^2[43]\frac{1}{\langle 12\rangle[21]}\langle 21\rangle = ig^2\frac{[43]}{[12]}$$

with spinors $|\;\rangle$ and $|\;]$ are just the spinors