Economics Letters 118 (2013) 135–138

Contents lists available at SciVerse ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Measuring the stance of monetary policy in zero lower bound environments

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ARTICLE INFO

Article history:
Received 10 June 2012
Received in revised form
4 October 2012
Accepted 9 October 2012
Available online 16 October 2012

JEL classification:
E43
G12
G13

Keywords:
Zero lower bound
Shadow short rate
Term structure model

ABSTRACT

With interest rates near the zero lower bound, I propose a simple framework to indicate the monetary policy stance as a "shadow short rate". I apply a one-factor model to Japan, provide associated economic intuition, and discuss multiple-factor extensions.

1. Introduction

In this article I propose a simple framework for quantifying the stance of monetary policy in terms of a "shadow short rate" when nominal interest rates within the term structure are near their zero lower bound (ZLB). The framework is relevant to the situation facing many developed economies at present.

The ZLB framework I propose is a tractable and close approximation to the Black (1995) framework for modeling the term structure in ZLB environments. The Black framework obtains non-negative short rates as $\underline{\mathrm{r}}(t) = \max\{\mathrm{r}(t), 0\}$, which represents the "real world" option to hold physical currency if the shadow short rate is negative, and the term structure is then generated from $\underline{\mathrm{r}}(t)$.[1] Unfortunately, practical implementations of the Black framework are relatively complex and/or numerically intensive, particularly as the number of factors increase.

Conversely, my ZLB framework is effectively based on non-negative forward rates obtained using bond options to represent the availability of physical currency. I outline the framework in Section 2. Section 3 demonstrates the framework with a one-factor model applied to Japan. In Section 4, I discuss the important relative advantages for extensions to multiple factors.

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1 A prevalent literature has evolved over several decades using short-rate dynamics designed to avoid negative short rates; e.g. see James and Webber (2000, pp. 226–233). However, such models lack the potential information provided by the shadow short rate in the Black framework and in the present article.

0165-1765/$ – see front matter
doi:10.1016/j.econlet.2012.10.011

2. The non-negative forward rate framework

Consider a finite-step shadow nominal bond with price $\mathrm{P}(t+\tau, \delta)$ at time $t+\tau$ that pays 1 at time $t+\tau+\delta$, where $\tau \geq 0$ is any future horizon from time t and $\delta > 0$ represents the time to maturity. I also assume physical currency is always available at time $t+\tau$ with a price of 1 and will pay 1 at time $t+\tau+\delta$.

To maximize returns, investors will choose the minimum priced investment at time $t+\tau$, i.e. $\min\{1, \mathrm{P}(t+\tau, \delta)\}$. The latter may be expressed as $\mathrm{P}(t+\tau, \delta) - \max\{0, \mathrm{P}(t+\tau, \delta) - 1\}$, which is a terminal boundary condition in two components. Respectively, the boundary condition of $\mathrm{P}(t+\tau, \delta)$ implies a shadow bond price at time t of $\mathrm{P}(t+\tau, \delta)$, and $\max\{0, \mathrm{P}(t+\tau, \delta) - 1\}$ implies a call option price at time t of $\mathrm{C}(t, \tau, \tau+\delta)$, with a strike price of 1 and expiry at time $t+\tau$.

The expression $\mathrm{P}(t, \tau+\delta) - \mathrm{C}(t, \tau, \tau+\delta)$ may be used to obtain forward rates $\underline{\mathrm{f}}(t, \tau)$ that are guaranteed to be non-negative for all maturities. Specifically, the most transparent way to obtain what I will denote currency-adjusted-bond (CAB) forward rates is the following numerical approximation[2]:

$$\underline{\mathrm{f}}(t, \tau) = -\frac{1}{\delta}\left(\log\left[\frac{\mathrm{P}(t, \tau+\delta) - \mathrm{C}(t, \tau, \tau+\delta)}{\mathrm{P}(t, \tau)}\right]\right). \tag{1}$$

2 The expression arises from the standard term structure relationship and intermediate steps as follows: $\underline{\mathrm{f}}(t, \tau) = -\frac{\mathrm{d}}{\mathrm{d}\tau}\log\left[\underline{\mathrm{P}}(t, \tau)\right] \simeq -\frac{1}{\delta}\left(\log\left[\frac{\mathrm{P}(t,\tau+\delta)-\mathrm{C}(t,\tau,\tau+\delta)}{\mathrm{P}(t,\tau)-\mathrm{C}(t,\tau,\tau)}\right]\right)$, and $\mathrm{C}(t, \tau, \tau) = 0$.

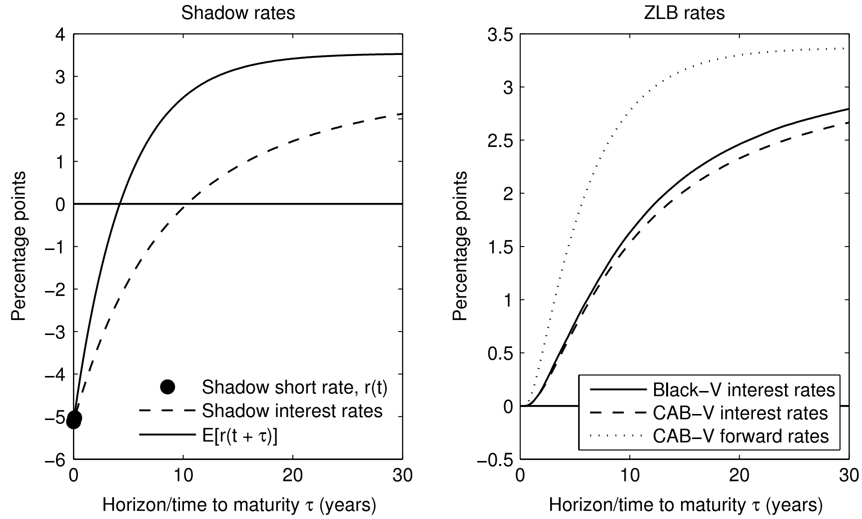

Fig. 1. Panel 1 provides perspectives on the shadow short rate process specified in the text. The shadow short rate $\mathrm{r}(t)$ and its expected path $\mathrm{E}[\mathrm{r}(t+\tau)]$ may be used as indicators of the stance of monetary policy. Panel 2 illustrates that the CAB-Vasicek and Black–Vasicek term structures associated with the given shadow short rate process are very similar.

Corresponding CAB interest rates may be obtained using the standard term structure relationship $\underline{\mathrm{R}}(t,\tau) = \frac{1}{\tau}\int_0^\tau \underline{\mathrm{f}}(t,\upsilon)\,\mathrm{d}\upsilon$ where υ is a dummy integration variable from zero to the time to maturity. Note that $\underline{\mathrm{R}}(t,\tau)$ is conveniently the arithmetic mean of $\underline{\mathrm{f}}(t,\tau)$ when the latter is calculated at uniformly spaced maturities $\Delta\tau$.[3]

3. Application to Japan

I apply my framework to Japan using the Vasicek (1977) model to represent the shadow term structure. What I hereafter call the CAB-Vasicek model provides parsimonious and transparent examples, while the application to Japan allows a ready comparison to the detailed Black–Vasicek examples in Gorovoi and Linetsky (2004) and the empirical application of Ichiue and Ueno (2006).

The diffusion process for the Vasicek (1977) model is $\mathrm{d\,r}(t) = \kappa[\theta - \mathrm{r}(t)]\,\mathrm{d}\,t + \sigma \mathrm{d}W(t)$, where $\mathrm{r}(t)$ is the shadow short rate (the single state variable), κ, θ, and σ are respectively the mean reversion, steady state level, and volatility parameters, $\mathrm{d}W(t)$ are unit normal innovations, and I use a constant market price of γ to allow for risk premiums in my estimation below. The expected path of the shadow short rate conditional on its current value is $\mathrm{E}[\mathrm{r}(t+\tau)] = \theta + \exp(-\kappa\tau)[\mathrm{r}(t) - \theta]$ and the closed-form analytic bond price and bond option price formulas used to obtain CAB-Vasicek rates from Eq. (1) are available from Chaplin (1987) or textbooks (e.g. Hull, 2000, pp. 567–568).

Using the state variable/parameter set from Gorovoi and Linetsky (2004, p. 71) (i.e. $\mathrm{r}(t) = -0.0512/\{\kappa,\theta,\sigma,\gamma\} = \{0.212, 0.0354, 0.0283, 0\}$ estimated for end-February 2002), panel 1 of Fig. 1 illustrates the associated shadow term structure $\mathrm{R}(t,\tau) = -\log[\mathrm{P}(t,\tau)]/\tau$ and the expected path of the shadow short rate $\mathrm{E}[\mathrm{r}(t+\tau)]$. Note that the shadow short rate of −5.12% is the zero maturity interest rate for the shadow term structure, analogous to the policy rate often used as an indicator of the monetary policy stance. When the shadow short rate is negative, its expected path indicates a return to an environment with a policy rate unconstrained by the ZLB. The horizon in this example, as at February 2002, is around four years.

Panel 2 of Fig. 2 plots the CAB-Vasicek forward rate curve, and the CAB-Vasicek and Black–Vasicek term structures associated with the given shadow rate process.[4] The immediate point to note is that, despite using an identical shadow short rate process, CAB-Vasicek interest rates are lower than the Black–Vasicek results.[5] That said, the differences between the two frameworks are very small for this example (a maximum of 14 basis points [bps] at the 30-year maturity) and the subsequent application in Fig. 2. Testing also showed that the differences remain very small for plausible parameter variations.[6] Therefore, the CAB-Vasicek model provides an acceptable approximation to the Black–Vasicek model for monetary policy purposes, while being easier to implement.

In practice, of course, one observes ZLB interest rate data and seeks the underlying shadow short rate and its expected path. To illustrate this process in practice, I apply the CAB-Vasicek model to the month-end Japanese term structure data (zero-coupon government-risk interest rates from Bloomberg) plotted in Fig. 2. The CAB-Vasicek interest rates are obtained using non-linear least squares to jointly estimate the state variables $\mathrm{r}(t)$ for each of the four dates along with the four parameters that apply across all dates (to ensure intertemporal consistency).[7] To check the materiality of differences to the Black–Vasicek model, I obtain the latter results with Monte Carlo simulations using the values of $\mathrm{r}(t)$ and the parameters {0.0704, 0.0561, 0.0179, −0.0168} estimated for the CAB-Vasicek model. The results are indistinguishable from the CAB-Vasicek results shown (i.e. a maximum difference of 4 bps for the 7-year maturity, rising to 32 bps at the 30-year maturity) so

3 That is, $\underline{\mathrm{R}}(t,\tau) = \frac{1}{\tau}\int_0^\tau \underline{\mathrm{f}}(t,\upsilon)\,\mathrm{d}\upsilon \simeq \frac{1}{\tau}\left[\Delta\tau\sum_{i=1}^{I}\underline{\mathrm{f}}(t,i\Delta\tau)\right] = \frac{1}{I}\sum_{i=1}^{I}\underline{\mathrm{f}}(t,i\Delta\tau)$, where $\Delta\tau = \tau/I$.

4 I generate the CAB-Vasicek forward rates using $\{\delta,\Delta\tau\} = \left\{10^{-6}, 10^{-3}\right\}$ here and $\left\{10^{-4}, 10^{-2}\right\}$ for the subsequent estimation, but the results are insensitive to reasonable variations in those finite quantities, using an analytic expression for $\underline{\mathrm{f}}(t,\tau)$ in the limit as $\delta \to 0$ (see Krippner, 2012 for details), and alternative numerical integration techniques for $\underline{\mathrm{R}}(t,\tau)$. I replicate the Gorovoi and Linetsky (2004) results with Monte Carlo simulation.

5 The interest rates are lower because the contingent cashflows $\min\{\mathrm{P}(t+\tau,\delta),1\}$ are discounted with shadow short rates in the CAB framework but effectively with non-negative short rates $\underline{\mathrm{r}}(t)$ in the Black (1995) framework. That difference carries over to higher bond prices and lower forward rates for the CAB framework. Therefore, the CAB-Vasicek model offers minor arbitrage opportunities relative to the Black–Vasicek model, obtainable in principle by selling bonds priced via the CAB-Vasicek framework and investing the proceeds in a rolling investment of $\max\{\mathrm{r}(t), 0\}$.

6 Divergences were always largest for the 30-year maturity, and larger σ values and smaller κ values were the main contributors to wider divergences.

7 This estimation is analogous to yield curve fitting, such as that proposed in Svensson (1995) but with a distinctly different underlying forward rate function.

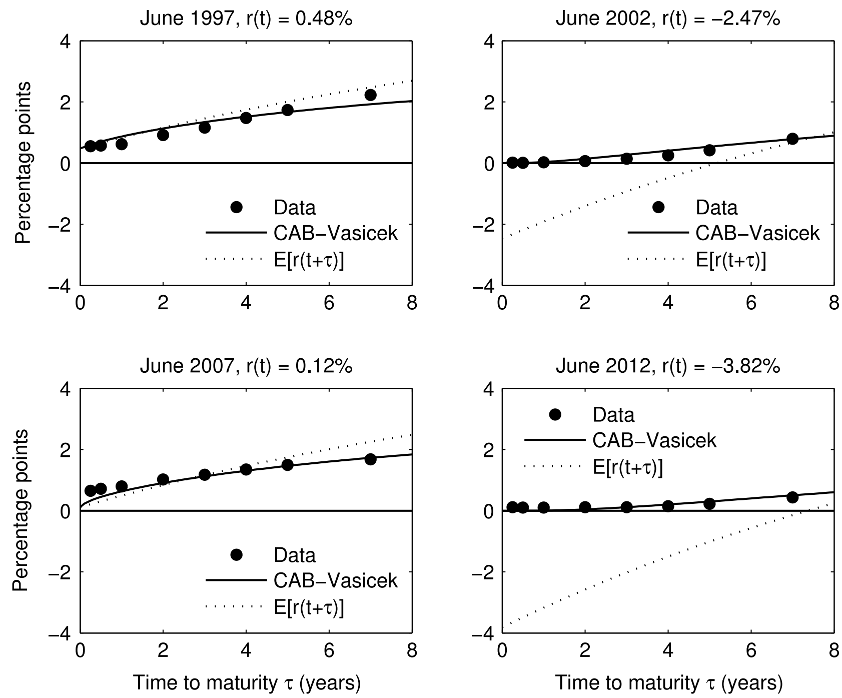

Fig. 2. Month-end Japanese term structure data, estimated CAB-Vasicek term structures, and expected paths of the shadow short rate $E[r(t+\tau)]$. The sub-plot titles provide the estimated CAB-Vasicek shadow short rate $r(t)$ for each date.

I have omitted them for clarity. Also, the magnitude and profile of my CAB-Vasicek shadow short rate estimates up to 2007 are similar to those from Ichiue and Ueno (2006).

From an economic perspective, the shadow short rates $r(t)$ reflect the stances of monetary policy monetary policy prevailing at each date, and changes in $r(t)$ reflect monetary policy changes during the intervening period. Specifically: (1) $r(t)$ is initially positive, and at a level close to the prevailing 0.5% official discount rate; (2) $r(t)$ becomes materially negative following the zero interest rate policy (ZIRP) instigated by the Bank of Japan in February 1999 and quantitative easing policies adopted from December 2001; (3) $r(t)$ becomes mildly positive following the exit from the ZIRP (July 2006)[8]; and (4) $r(t)$ becomes very negative following the re-instigation of the ZIRP (October 2010) and subsequent quantitative easing measures adopted in the wake of the Global Financial Crisis. The negative shadow short rates for June 2002 and June 2012 respectively imply market expectations of a return to normal policy in around five and seven years. In summary, the shadow short rates $r(t)$ indicate the stance of monetary policy in a manner analogous to policy rates, and continue to do so even when the policy rate is constrained by the ZLB.

4. Conclusion and extensions

The results in this article suggest that the CAB-Vasicek framework offers a tractable and close approximation to the Black–Vasicek model for summarizing the stance of monetary policy in a ZLB environment. The estimated shadow short rates from a one-factor CAB-Vasicek model are consistent with the evolution of Japanese monetary policy from the late 1990s.

Two obvious examples of the potential extensions to this article are applying the model to other countries and improving the model estimation. A third and important extension is to multiple factors; first because it is generally accepted that single-factor models are not realistic representations of the term structure; and second because Black–Gaussian models increase substantially in "numerical intensity" (the number of analytic calculations for implementation) as factors are added.[9] Conversely, the numerical intensity of CAB-Gaussian models does not change because closed-form analytic solutions for bond and option prices are available (see Chen, 1995, for example). Finally, if precise Black implementations are required, the CAB framework facilitates more efficient Monte Carlo simulations for one or more factors.

Acknowledgments

For helpful comments associated with this article, I thank Katy Bergstrom, Iris Claus, Toby Daglish, Francis Diebold, Pedro Gomis, Michelle Lewis, Anella Munro, Les Oxley, Peter Phillips, Glenn Rudebusch, Christie Smith, Daniel Thornton, Christopher Waller, participants at the 2011 Reserve Bank of New Zealand Conference, a 2011 Bundesbank seminar, the 2012 New Zealand Econometrics Study Group Meeting, and 2012 seminars at the Federal Reserve Board, the St. Louis and San Francisco Federal Reserves, and the University of Pennsylvania. I also thank anonymous referees for their helpful comments.

References

Black, F., 1995. Interest rates as options. Journal of Finance 50, 1371–1376.
Bomfim, A., 2003. 'Interest rates as options': assessing the markets' view of the liquidity trap. Working Paper. Federal Reserve Board of Governors.

8 However, a time series of $r(t)$ in Krippner (2012) shows mainly mildly negative values around this time. Those results concur with Ichiue and Ueno (2006), and suggest that the term structure around this time was generally shaped as if a mild ZIRP remained in place.

9 Bomfim (2003), Ueno et al. (2006) and Ichiue and Ueno (2007) have respectively used finite-difference grids, Monte Carlo simulations, and interest rate lattices for two-factor Gaussian Black implementations. The numerical intensity of these methods increases to the order of the power of the number of factors. The Gorovoi and Linetsky (2004) approach is semi-analytic, but does not appear to generalize to multiple factors; see Kim and Singleton (2012, pp. 36–37).

Chaplin, G., 1987. A formula for bond option values under an Ornstein–Uhlenbeck model for the spot rate. Working Paper. Department of Statistics and Actuarial Science, University of Waterloo ACTSC 87-15.
Chen, R., 1995. A two-factor, preference-free model for interest rate sensitive claims. Journal of Futures Markets 15 (3), 345–372.
Gorovoi, V., Linetsky, V., 2004. Black's model of interest rates as options, eigenfunction expansions and Japanese interest rates. Mathematical Finance 14 (1), 49–78.
Hull, J., 2000. Options, Futures and Other Derivitives, fourth ed. Prentice Hall.
Ichiue, H., Ueno, Y., 2006. Monetary policy and the yield curve at zero interest: the macro-finance model of interest rates as options. Working Paper. Bank of Japan 06-E-16.
Ichiue, H., Ueno, Y., 2007. Equilibrium interest rates and the yield curve in a low interest rate environment. Working Paper. Bank of Japan 07-E-18.
James, J., Webber, N., 2000. Interest Rate Modelling. Wiley and Sons.
Kim, D., Singleton, K., 2012. Term structure models and the zero bound: an empirical investigation of Japanese yields. Journal of Econometrics 170 (1), 32–49.
Krippner, L., 2012. Measuring the stance of monetary policy in zero lower bound environments. Discussion Paper. Reserve Bank of New Zealand DP2012/04.
Svensson, L., 1995. Estimating forward interest rates with the extended Nelson and Siegel model. Sveriges Riksbank Quarterly Review 3, 13–26.
Ueno, Y., Baba, N., Sakurai, Y., 2006. The use of the Black model of interest rates as options for monitoring the JGB market expectations. Working Paper. Bank of Japan 06-E-15.
Vasicek, O., 1977. An equilibrium characterisation of the term structure. Journal of Financial Economics 5, 177–188.