# ECB Project - Coding Appendix

## Contents

# Preliminaries

## Setup

```r
# --------- 1. Clear memory
rm(list=ls())

# --------- 2. Load here package to enable finding script loading other packages
require(here)

# --------- 3. Set directory
getwd()
setwd("...")

# --------- 4. Load packages & helper functions for plots and tables
source(here("scripts/packages.R"))
source(here("helpers/raw_plotter.R"))
source(here("helpers/stationarity_tables.R"))
source(here("helpers/taylor_tables.R"))
source(here("helpers/struct_breaks_tables.R"))
source(here("helpers/roll_TR_plotter.R"))
source(here("helpers/pseudo_outofsample_tables.R"))
source(here("helpers/pseudo_outofsample_plots.R"))
source(here("helpers/actual_forecast_displays.R"))

# Api key for data
fredr_set_key("e0169694a62c1337f1969e3872605eca")

# --------- 5. Dates (to automatically get the latest data from API calls
start_date <- "1999-01-01"
end_date <- Sys.Date()

# For replication
set.seed(2025)
```

## Interactive Option Selection

- Use Hamilton Filter:
  - TRUE: Selects Hamilton method for output gap estimation
  - FALSE: Selects Hodrick-Prescott method for output gap estimation
- Use Inflation Expectations:
  - TRUE: The models used for forecasting will use 12-month ahead inflation expectations from the ECB survey of professional forecasts (average).
  - FALSE: The models used for forecasting will use realised inflation
- Use Formula
  - Formula 1: Actual interest rate regressed on inflation and output gaps
  - Formula 2: Shadow interest rate regressed on inflation and output gaps

- Formula 3: Actual interest rate regressed on the one-quarter lag of the interest rate and on inflation and output gaps
- Formula 4: Shadow interest rate regressed on the one-quarter lag of the shadow interest rate and on inflation and output gaps

- Format:
  - html: For outputting in console or knitting to html
  - latex: For knitting to pdf

```r
source(here("scripts/options_config.R"))

#temp remove soon
format <- "html"
format <- "latex"
```

```r
source(here("scripts/taylor_rule_formulas.R"))
```

# Data

## Sources & Explanations

- FRED Data Source: European Central Bank, ECB Deposit Facility Rate for Euro Area [ECBDFR], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/ECBDFR. The data is the Deposit Facility and is directly reprinted from the ECB. It's in Percent and not seasonally adjusted. The ECB Monetary Policy is steered through this rate
- Shadow Interest Rate: The Shadow rate was developed by **wuTimeVaryingLowerBound2017** and does quantify a hypothetical removal of the ZLB. The rate does not track 1:1 on the deposit facility in the data before the ZLB, instead being closer to the refinancing rate. (maybe need to adjust, oui je crois faux, surtout la partie sur refinancing rate)
- GDP: The GDP Data is quarterly real GDP in 2010 Euros in million retrieved from FRED via Eurostat. The data is seasonally adjusted.
- Potential GDP: Either estimated with the Hodrick-Prescott filter or the Hamilton filter, based on the formulas described in equation (1):

$$\begin{aligned} \text{HP Filter:} \quad & \min_{\tau} \left( \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left[ (\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}) \right]^2 \right) \\ \text{Hamilton:} \quad & y_t = \beta_0 + \sum_{j=1}^{p} \beta_j y_{t-h-j+1} + v_t \quad \text{(where } v_t \text{ is the cycle)} \end{aligned} \tag{1}$$

- Inflation:
  - Realised: The Inflation data is from Eurostat and measures HICP monthly data (annual rate of change). It's the index that the ECB uses for Inflation and is not seasonally adjusted, but the fact that it represents the year-on-year change in prices implies there is no seasonality.
  - Expectations: Expected Inflation is the ECB's survey of professional forecasters. It tracks professional forecasts of HICP inflation 12 months in advance.

## Loading & Preparation Data

To get this data, we run the following code using API calls to FRED, EuroStat, and DBnomics. We convert each variable to the quarterly average and then store them all to one dataframe named "data". It includes a date column, the deposit rate, the shadow rate, inflation, inflation gap (which is just inflation minus the ECB target, which is set at 2%), expected inflation, expected inflation gap, real GDP, GDP output gap computed with the HP filter and GDP output gap computed with the Hamilton filter.

```r
#copypaste data script and explain
source(here("scripts/data.R"))
```

## Options Configuration

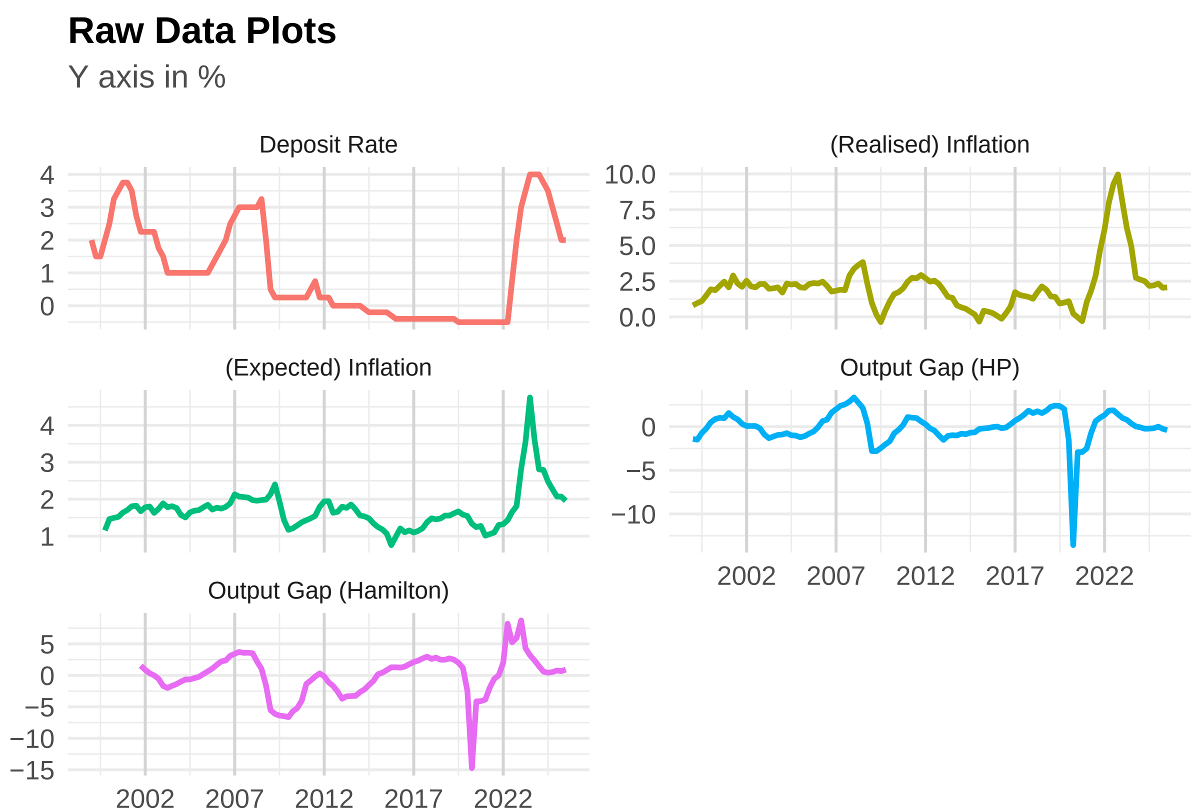
We then implement the chosen options on using inflation expectations or not, and on using either the HP filter or the Hamilton filter for output gap.

```r
source(here("scripts/options_implement.R"))
```

- CONFIGURATION: Using Hamilton Filter for output gap estimation.
- CONFIGURATION: Using realised inflation in Taylor Rule forecasting.

### Raw Data Plots

```r
# Who cares about the method here, only result matters
raw_data_plotter(data = data)
```




### Data Properties

In this section, we run various tests to understand the properties of our main variables, interest rate, inflation and output gap. These include both ADF and KPSS stationarity tests (using the tseries package) and a cointegration test (using the aTSA package) to see if the interest rate is cointegrated with inflation. We wrap these in custom functions that automatically report the results and intepret them based on the relevant p-value.

```r
# copy paste both stat and coint funcs and explain
source(here("helpers/stationarity_tests.R"))
```

Table 1: Summary of Stationarity Tests (ADF & KPSS)

| Variable | ADF p-value | KPSS p-value | Result |
|---|---|---|---|
| Interest Rate | 0.4350 | 0.0359 | Non-Stationary I(1): ADF confirms unit root, KPSS rejects stationarity. |
| Inflation | 0.2257 | 0.1000 | Conflicting Results: ADF confirms unit root, while KPSS suggests stationarity. |
| Output Gap | 0.0131 | 0.1000 | Stationary I(0): ADF rejects unit root, KPSS confirms stationarity. |
| Interest Rate (1st Diff) | 0.0100 | 0.1000 | Stationary I(0): ADF rejects unit root, KPSS confirms stationarity. |
| Inflation (1st Diff) | 0.0100 | 0.1000 | Stationary I(0): ADF rejects unit root, KPSS confirms stationarity. |

Table 2: Cointegration Test Results

| Variables | p-value | Result |
|---|---|---|
| Interest Rate & Inflation | 0.0585 | Not Cointegrated |

```r
# -----  Run Tests -----

# Use helper to run stationarity checks for our main variables
test_rate <- check_stationarity(data$rate, "Interest Rate")
test_inflation <- check_stationarity(data$inflation, "Inflation")
test_output_gap <- check_stationarity(na.omit(data$output_gap), "Output Gap")

# Given results for rate and inflation, run tests on 1st diffs
test_rate_diff <- check_stationarity(diff(data$rate), "Interest Rate (1st Diff)")
test_inflation_diff <- check_stationarity(diff(data$inflation), "Inflation (1st Diff)")

# Since rate and inflation are I(1), run a cointegration test
coint_test = check_coint(data$rate, data$inflation,
            var_name1 = "Interest Rate", var_name2 = "Inflation")


# ----- Print Results -----

# Combine stationarity results
all_stationarity_results <- rbind(test_rate,
                                  test_inflation,
                                  test_output_gap,
                                  test_rate_diff,
                                  test_inflation_diff)

# Tables
generate_stat_tests_table(stat_tests = all_stationarity_results)
```

```r
generate_coint_tests_table(coint_test = coint_test)
```

```r
# Cleanup
rm(all_stationarity_results, coint_test, test_inflation,
   test_inflation_diff, test_output_gap, test_rate, test_rate_diff)
```

## Taylor Rule Estimation

We now run basic Taylor Rule estimations using OLS on the complete dataset in order to get some information on the historical Taylor coefficients.

### Without Lags

The "No Lag" Taylor Rule specifications follow the regression expressed in equation (2). We run 6 variations total of this, starting with the base case using the actual deposit rate, realised inflation (gap) and the output gap. We then mix cases where we use the shadow rate, expected inflation (gap) and finally including both expected and realised inflation (gap).

$$i_t = \pi^* + \beta(\pi_t - \pi^*) + \gamma(y_t - \bar{y_t}) \tag{2}$$

```r
TR <- lm(rate ~ realised_inflation_gap + output_gap, data = data)
TRsr <- lm(shadowrate ~ realised_inflation_gap + output_gap, data = data)

models <- list(TR, TRsr)
taylor_regression_to_table(models, caption = "No Lag, No Expectations")
```

Table 3: No Lag, No Expectations

| | TR | TR w/ SR |
|---|---|---|
| (Intercept) | 0.8460 | -0.8296 |
| | (0.8943) | (2.6530) |
| realised_inflation_gap | 0.1946 | 0.6812 |
| | (0.4318) | (0.6058) |
| output_gap | 0.0839 | -0.0528 |
| | (0.1632) | (0.2763) |
| N | 96 | 96 |
| R2 | 0.1722 | 0.1146 |

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

```r
TR_e <- lm(rate ~ exp_inflation_gap + output_gap, data = data)
TRsr_e <- lm(shadowrate ~ exp_inflation_gap + output_gap, data = data)
TR_ie <- lm(rate ~ realised_inflation_gap + exp_inflation_gap +
                   output_gap, data = data)
TRsr_ie <- lm(shadowrate ~ realised_inflation_gap + exp_inflation_gap +
                           output_gap, data = data)

models <- list(TR_e, TRsr_e, TR_ie, TRsr_ie)
taylor_regression_to_table(models, caption = "No Lag, with
                                              Inflation Expectations")
```

Table 4: No Lag, with Inflation Expectations

| | TR | TR w/ SR | TR | TR w/ SR |
|---|---|---|---|---|
| (Intercept) | 1.3522 *** | 0.3098 | 1.3470 *** | 0.1984 |
| | (0.3169) | (1.5706) | (0.3707) | (1.6114) |
| exp_inflation_gap | 1.7282 *** | 3.7705 ** | 1.7165 *** | 3.5220 ** |
| | (0.3556) | (1.3951) | (0.3483) | (1.1795) |
| output_gap | 0.0711 | -0.0015 | 0.0671 | -0.0872 |
| | (0.0469) | (0.1872) | (0.0811) | (0.2131) |
| realised_inflation_gap | | | 0.0146 | 0.3120 |
| | | | (0.1278) | (0.3423) |
| N | 96 | 96 | 96 | 96 |
| R2 | 0.6180 | 0.3914 | 0.6182 | 0.4089 |

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

## Lagged Models

The lagged Taylor Rule specifications follow the regression expressed in equation (3) which is the same as (2) but also including a one-quarter lag for the interest rate. We then run the same 6 variations as for the models without the lag.

$$i_t = \pi^* + \phi i_{t-1} + \beta(\pi_t - \pi^*) + \gamma(y_t - \bar{y_t}) \tag{3}$$

```r
lTR <- lm(rate ~ rate_lag + realised_inflation_gap + output_gap, data = data)
lTRsr <- lm(shadowrate ~ shadowrate_lag + realised_inflation_gap +
                         output_gap, data = data)

models <- list(lTR, lTRsr)
taylor_regression_to_table(models, caption = "Interest Rate Lag,
                                              No Expectations")
```

```r
lTR_e <- lm(rate ~ rate_lag + exp_inflation_gap + output_gap, data = data)
lTRsr_e <- lm(shadowrate ~ shadowrate_lag + exp_inflation_gap +
                           output_gap, data = data)
lTR_ie <- lm(rate ~ rate_lag + realised_inflation_gap + exp_inflation_gap +
                    output_gap, data = data)
lTRsr_ie <- lm(shadowrate ~ shadowrate_lag + realised_inflation_gap +
                            exp_inflation_gap + output_gap, data = data)

models <- list(lTR_e, lTRsr_e, lTR_ie, lTRsr_ie)
taylor_regression_to_table(models, caption = "Interest Rate Lag, with
                                              Inflation Expectations")
```

Table 5: Interest Rate Lag, No Expectations

| | TR | TR w/ SR |
|---|---|---|
| (Intercept) | 0.0512 | -0.0756 |
| | (0.0406) | (0.0592) |
| rate_lag | 0.9195 *** | |
| | (0.0381) | |
| realised_inflation_gap | 0.0877 * | 0.2493 *** |
| | (0.0348) | (0.0343) |
| output_gap | 0.0219 | -0.0113 |
| | (0.0163) | (0.0184) |
| shadowrate_lag | | 0.9535 *** |
| | | (0.0173) |
| N | 96 | 96 |
| R2 | 0.9597 | 0.9828 |

*** p < 0.001; ** p < 0.01; * p < 0.05.

### Checking for structural breaks

The previous Taylor Rule estimation are most likely flawed if there have been structural breaks in the economic system that would influence the way the ECB reacts to the economy. Given this, we run both a Chow test and a Bai-Perron test in order to see if there are any using the "strucchange" package. The Chow test is run on two dates where we suspect therehave been breaks which are in the third quarter of 2012, representing the start of the Zero Lower Bound, and in the first quarter of 2020, representing the start of the COVID-19 pandemic. The Bai-Perron test, however, works by finding any structural breaks present in the overall sample which minimise the Aikake Information Criterion.

```r
#only paste chow one
source(here("helpers/struct_breaks_tests.R"))

# 1st Suspected break: Start of ZLB in 2012 Q3
#  -> R = 55 in evaluation chunk

# Suspected break: Covid in 2020 Q1
#  -> R = 85 in evaluation chunk

# Run test using helper function
chow_tests(data, break1 = 55, break2 = 85,
           events_name <- c("ZLB Start", "COVID-19 Start"))
```

Table 6: Interest Rate Lag, with Inflation Expectations

| | TR | TR w/ SR | TR | TR w/ SR |
|---|---|---|---|---|
| (Intercept) | 0.1805 * | 0.0012 | 0.1343 *** | -0.0876 |
| | (0.0791) | (0.0852) | (0.0376) | (0.0594) |
| rate_lag | 0.8612 *** | | 0.8750 *** | |
| | (0.0436) | | (0.0381) | |
| exp_inflation_gap | 0.2381 ** | 0.1295 | 0.1530 *** | -0.0548 |
| | (0.0734) | (0.1038) | (0.0434) | (0.0618) |
| output_gap | 0.0449 * | 0.0592 | 0.0234 | -0.0106 |
| | (0.0218) | (0.0474) | (0.0165) | (0.0181) |
| shadowrate_lag | | 0.9631 *** | | 0.9586 *** |
| | | (0.0287) | | (0.0187) |
| realised_inflation_gap | | | 0.0768 * | 0.2528 *** |
| | | | (0.0378) | (0.0346) |
| N | 96 | 96 | 96 | 96 |
| R2 | 0.9547 | 0.9714 | 0.9614 | 0.9829 |

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

```r
#only paste bp one
source(here("helpers/struct_breaks_tests.R"))

# Run test using helper function
bp_tests(data)
```

## Rolling Estimation

Given that there have been structural breaks (according to our tests), we decide to run a rolling estimation scheme for the Taylor Rule formula selected in the setup options configuration. To do this, we use a custom function which runs the chosen model firstly on the first data "window" (which can be selected to be however many quarters desired) and then moves this window forward in time by one quarter at a time, running the

Table 7: Chow tests for suspected structural breaks

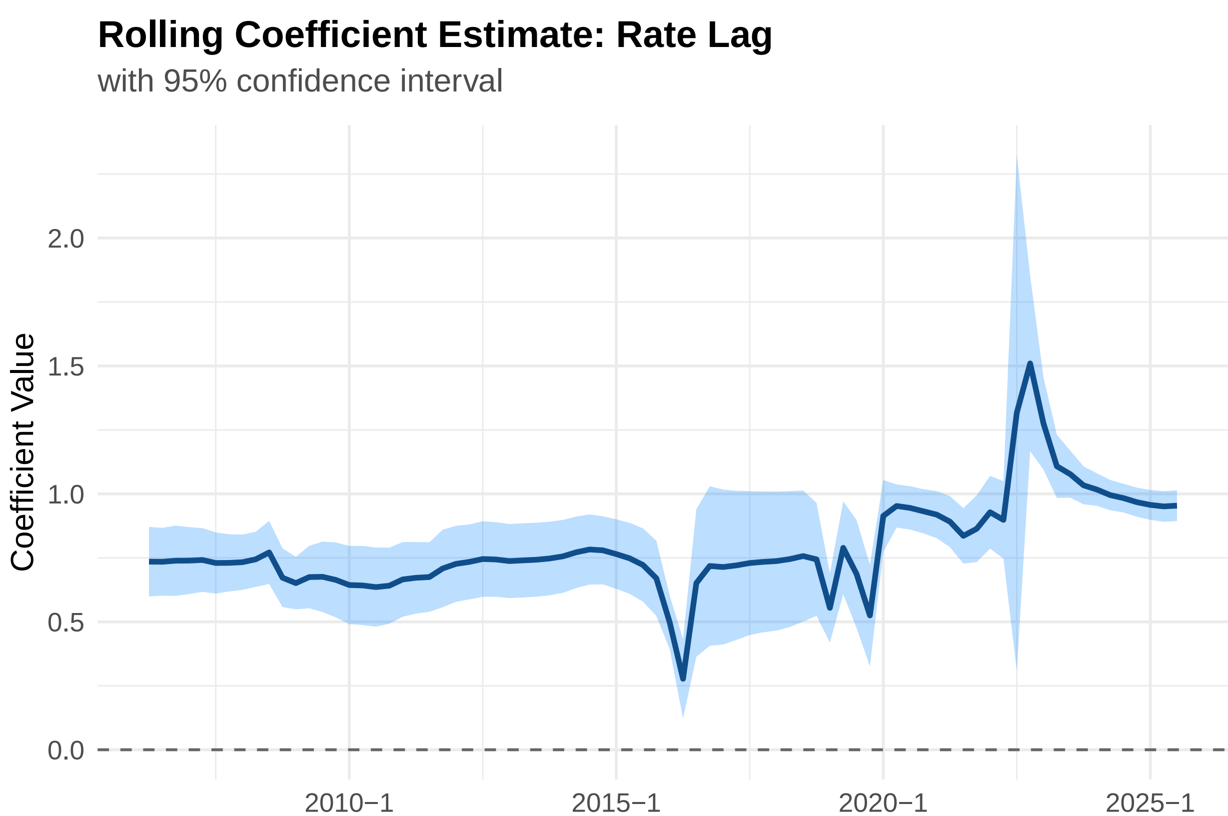
| Event | Date | p-value |
|---|---|---|
| ZLB Start | 2012 Q3 | 0.0018 |
| COVID-19 Start | 2020 Q1 | 0.1230 |

Table 8: Bai-Perron test for multiple breaks

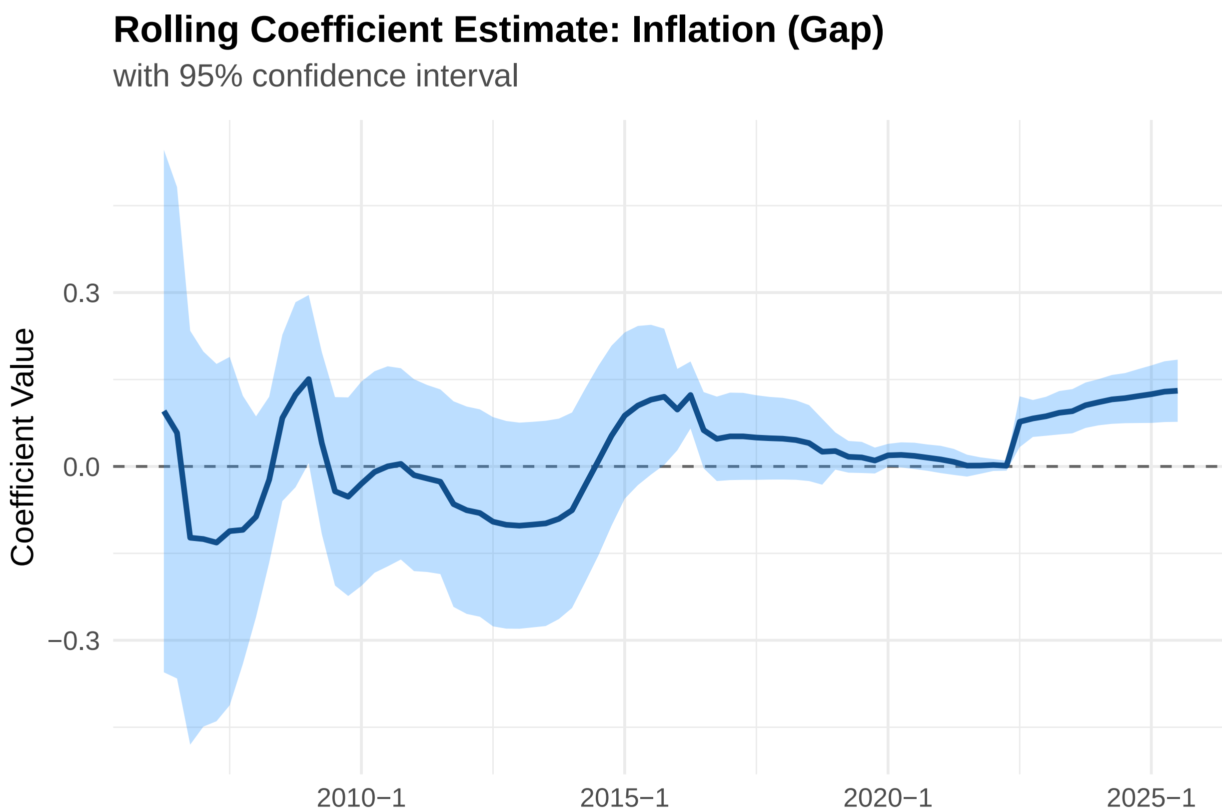
| Detected Breaks |
|---|
| 2006 Q2 |
| 2019 Q2 |

estimation for each loop. We then use another custom function to plot the resulting coefficients through time, including confidence intervals.

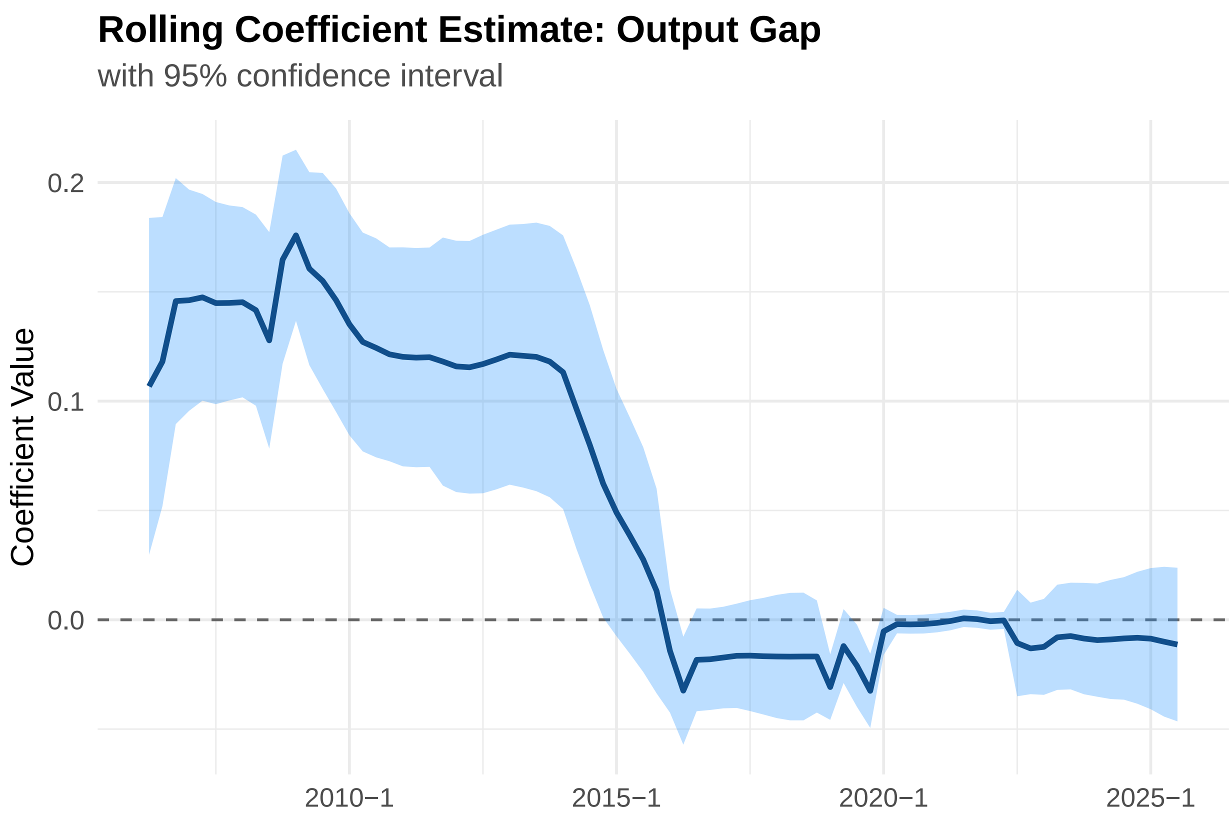
```
source(here("helpers/roll_TR_estimator.R"))

# Estimate a rolling-window (W in quarters) Taylor Rule specification
TR_roll <- estimate_rolling_TR(data, model_formula, W = 30)

# Plotting (note: this part is not modular, obviously)
plot_rolling_coefs(TR_roll, "rate_lag", var_name_title="Rate Lag")
```




```
plot_rolling_coefs(TR_roll, "inflation_gap", var_name_title="Inflation (Gap)")
```




```r
plot_rolling_coefs(TR_roll, "output_gap", var_name_title="Output Gap")
```


Rolling Coefficient Estimate: Output Gap
with 95% confidence interval
Coefficient Value
0.2
0.1
0.0
2010−1
2015−1
2020−1
2025−1

# Forecasting Model Evaluation

## Methodology

Our method of forecasting consists of two steps. First, we run our custom auto ARIMA function on the inputs of the Taylor Rule (inflation and output gap). This function replicates the one in the "forecast" package, though does not loop over the differentiation parameter as we want to set that value manually based on the stationarity tests run above. We additionally run this function on the interest rate in order to serve as our benchmark model for relative forecasting performance evaluation. In order to compute the forecasts based on the fitted ARIMA models, we use a custom function that replicates the same use-case in the "forecast" package. At the same time as the ARIMA fits, we use OLS to compute the Taylor Rule coefficients. The second step is then to do the actual forecasting, where we predict future values of the interest rate based on the forecasted values of the inputs using the fitted ARIMA models, weighted by the coefficients computed with OLS.

```
source(here("helpers/auto_ARIMA_replic.R"))
```

## Pseudo Out of Sample Estimation

In order to asses the performance of this methodology, we use a pseudo out-of-sample rolling estimation scheme of direct forecasts for all Taylor Rule formulas and for the benchmark ARIMA. We forecast up to a horizon of 10 quarters ahead. To do this, we loop over quarters starting just before the evaluation sample, where we then implement our methodology to forecast up to the farthest horizon. We then store these values and the loop starts again, adding one more quarter from the evaluation sample, and dropping the very first quarter in the overall sample. For the lagged models, we additionally need to include an iterative forecasting step, where we loop over each horizon in order to use the previous point forecast to compute the next one. In order to speed up the process, we include a simple parallelization procedure which runs these loops in parallel on a fourth the the users' CPU cores.

```
#add explain parameters
R = 85 # Chow: Structural breaks at R=55 and R=85
cat("Evaluation sample starts after ",as.character(data$quarter[R]),".",sep="")
```

```
## Evaluation sample starts after 2020 Q1.
```

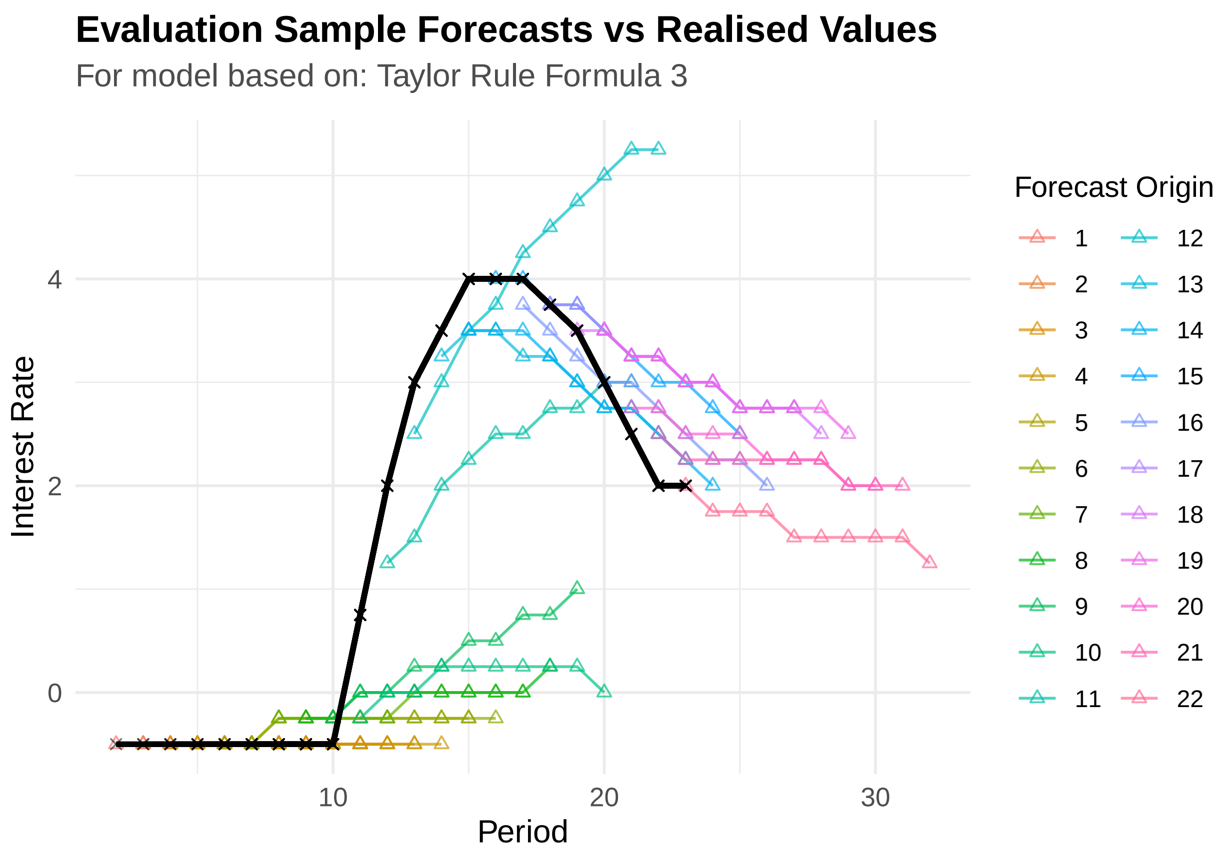
```
P = nrow(data) - R #but will effectively be: P = T-h-R
H = 10 # Number of different horizons (takes 10 to go until 2027 Q4)
```

```
source(here("scripts/pseudo_out_of_sample_estimation.R"))
```

## Spaghetti Plots

In order to visualize the pseudo out-of-sample evaluation exercise, we use spaghetti plots of the point forecasts which allow to compare how often the model "misses" the realised values of the interest rate between different horizons and dates. We do the plot only on the on the Taylor Rule formula selected in the options configuration in order to avoid clutter.

```
spaghetti_plotter(evals = eval_all_models,
                  model = model,
                  model_name = model_name)
```

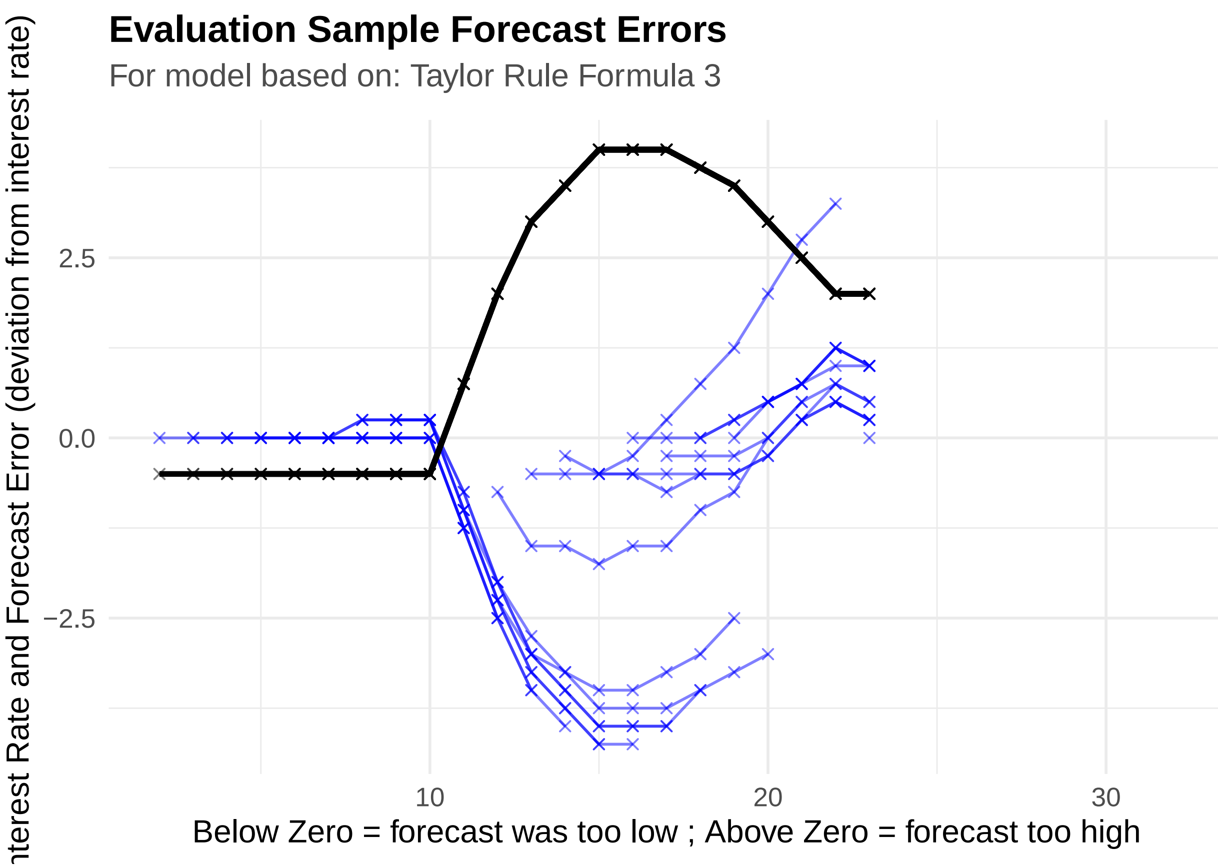



## Forecast Errors

### Plots of FE

Similar to the spaghetti plot of the point forecasts, we essentially use the same plot but computing the forecast errors as the difference between our point forecast and the realised value.

```
FE_spaghetti_plotter(evals = eval_all_models)
```

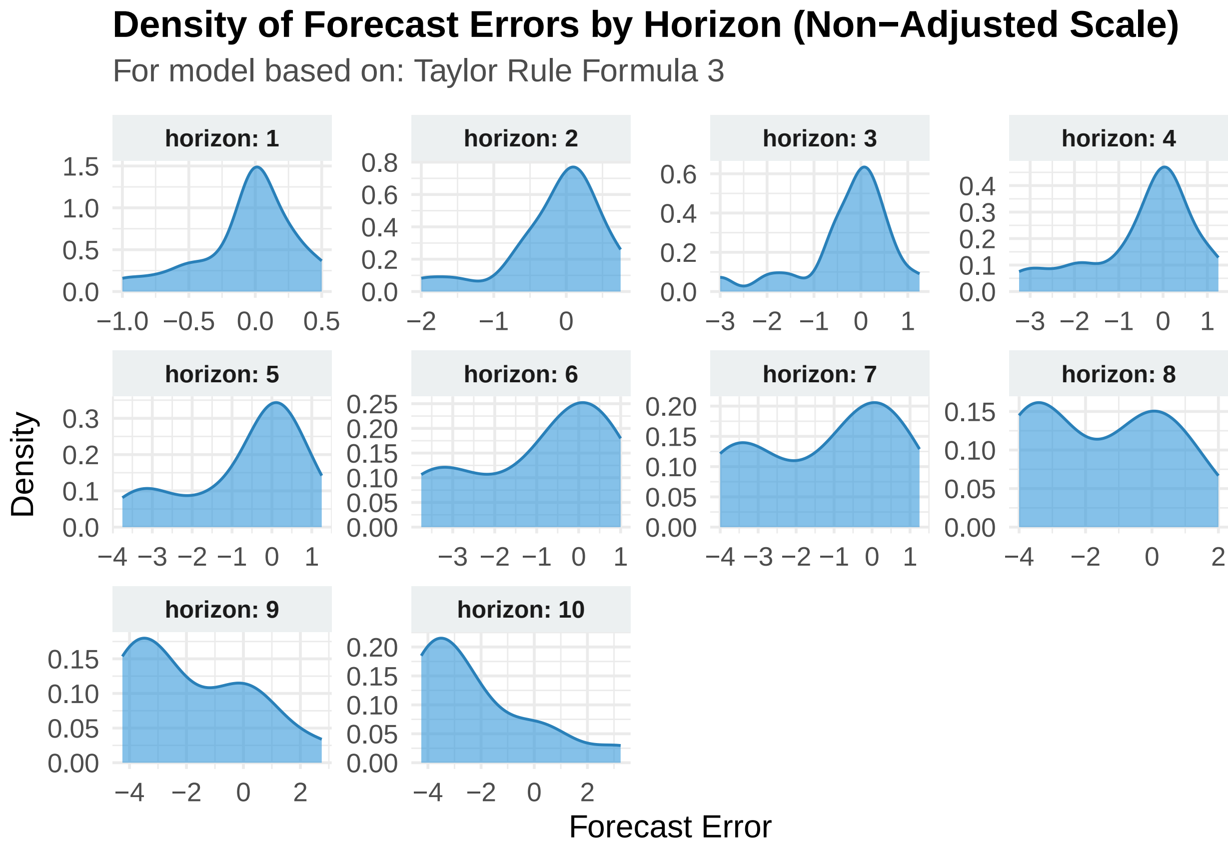



### Density of FE

We can then plot the distribution of these forecast errors using a density plot. We include 3 versions: one that is unscaled in order to compare the absolute values, one that is scaled in order to compare relative differences, and one that plots density in a way to compare both.

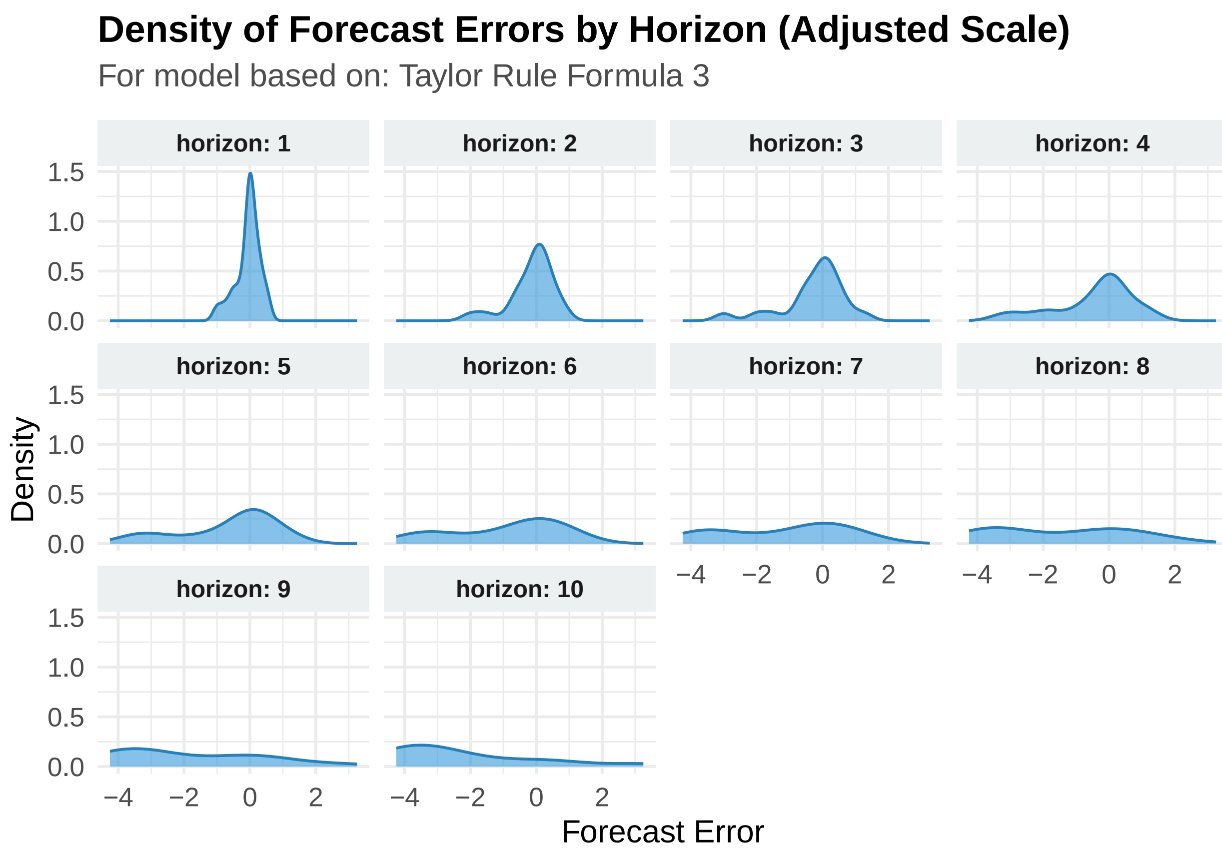
```
FE_density_plotter_unscaled(evals = eval_all_models)
```

```
## Warning: Removed 45 rows containing non-finite outside the scale range
## (`stat_density()`).
```




```r
FE_density_plotter_scaled(evals = eval_all_models)
```

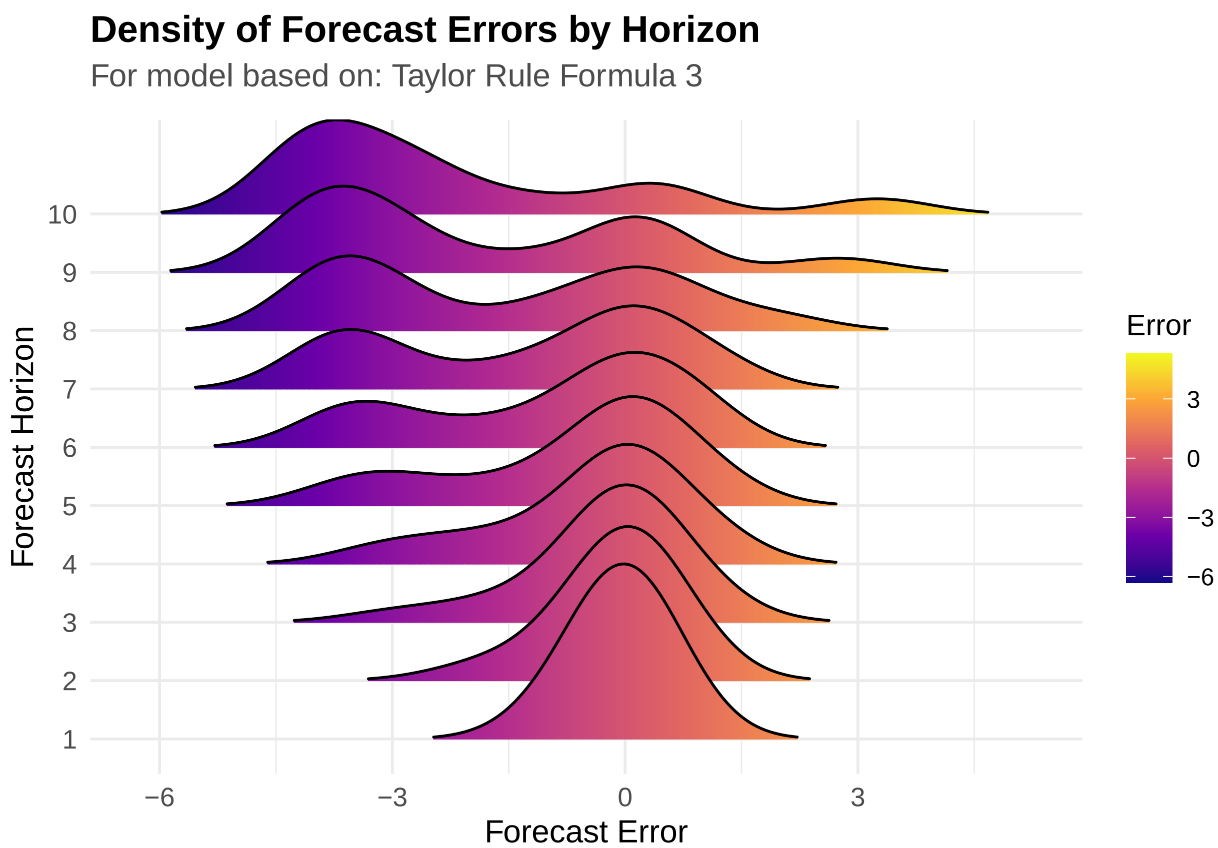
```
## Warning: Removed 45 rows containing non-finite outside the scale range
## (`stat_density()`).
```




```r
FE_density_plotter_ridges(evals = eval_all_models)
```

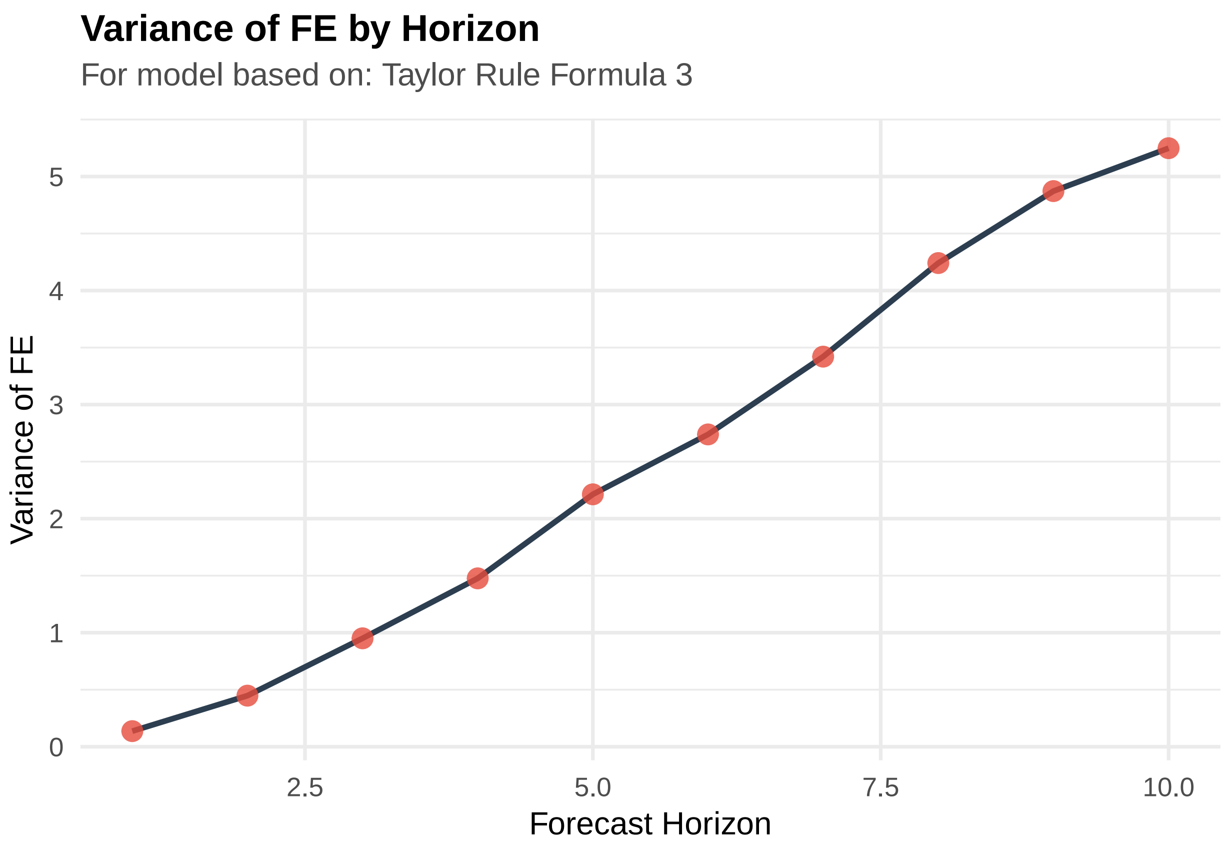
```
## Warning: `stat(x)` was deprecated in ggplot2 3.4.0.
## i Please use `after_stat(x)` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```

```
## Picking joint bandwidth of 0.688
```




**Variance of FE**

```
FE_variance_plotter(var_by_horizon)
```




### Horizon 1 Autocorrelation of FE

Model 1, 2, 3 are autocorrelated at h=1. Include results in paper, not here in the appendix. Well maybe not idk.

```
#Durbin Watson tests for first autocorrelation at h=1

# Select Forecast Columns
formula_cols <- grep("^F_TR_FORMULA_", names(eval_all_models), value = TRUE)

# Automatically set max horizon for loop
max_h <- max(eval_all_models$horizon)

# Loop for each model
for (e_model_name in formula_cols) {

  # Extract and regress errors
  all_forecast_errors <- eval_all_models[["actuals"]] - eval_all_models[[e_model_name]]
  h1_errors_vector <- all_forecast_errors[eval_all_models$horizon == 1]
  temp_model <- lm(h1_errors_vector ~ 1)

  # Durbin-Watson Test
  dw_test_result <- durbinWatsonTest(temp_model)

  # Print output
```

```
  cat(sprintf("\n--- Durbin-Watson Test for Model: %s (h=1) ---\n", e_model_name))
  print(dw_test_result)
  cat(sprintf("DW Statistic: %.4f\n", dw_test_result$statistic))}
```

— Durbin-Watson Test for Model: F_TR_FORMULA_1 (h=1) — lag Autocorrelation D-W Statistic p-value 1 0.9175895 0.1455016 0 Alternative hypothesis: rho != 0

— Durbin-Watson Test for Model: F_TR_FORMULA_2 (h=1) — lag Autocorrelation D-W Statistic p-value 1 0.8986753 0.1798937 0 Alternative hypothesis: rho != 0

— Durbin-Watson Test for Model: F_TR_FORMULA_3 (h=1) — lag Autocorrelation D-W Statistic p-value 1 0.4973872 1.002973 0.022 Alternative hypothesis: rho != 0

— Durbin-Watson Test for Model: F_TR_FORMULA_4 (h=1) — lag Autocorrelation D-W Statistic p-value 1 -0.07327455 2.115385 0.804 Alternative hypothesis: rho != 0

#### Overall Autocorrelation of FE

We don't reject H0 -> h+1 are uncorrelated 1,2, are autocorrelated, 3 and 4 are not.

```
#Ljung Box Test, with Columns & max lags set in Durbin Watson code chunk

# Loop through each TR
for (e_model_name in formula_cols) {

    cat(sprintf("--- Checking Errors for Model: %s ---\n", e_model_name))

    # Calculate the errors of the model
    all_errors <- eval_all_models[["actuals"]] - eval_all_models[[e_model_name]]

    # Loop over all forecast horizons to test for autocorrelation
    for (h in 1:max_h){

        # select errors for each horizon
        h_errors <- all_errors[eval_all_models$horizon == h]

        # max lag according to slides around sqrt T
        T_errors <- length(h_errors)
        max_lag <- 4 #round(sqrt(T_errors)) # use 4 or 5 chose 4 because 1 year has 4 quarters

        # Ljung-Box Test
        lb_test_result <- Box.test(h_errors,
                                   lag = max_lag,
                                   type = "Ljung-Box")

        #Print Results
        cat(sprintf("Horizon h=%d (N=%d, Lags=%d): Q=%.2f, p-value=%.3f\n",
                    h, T_errors, max_lag, lb_test_result$statistic, lb_test_result$p.value))}
        cat("\n")}
```

— Checking Errors for Model: F_TR_FORMULA_1 — Horizon h=1 (N=22, Lags=4): Q=47.79, p-value=0.000 Horizon h=2 (N=22, Lags=4): Q=45.55, p-value=0.000 Horizon h=3 (N=22, Lags=4): Q=44.69, p-value=0.000 Horizon h=4 (N=22, Lags=4): Q=40.67, p-value=0.000 Horizon h=5 (N=22, Lags=4): Q=27.72, p-value=0.000 Horizon h=6 (N=22, Lags=4): Q=16.42, p-value=0.003 Horizon h=7

(N=22, Lags=4): Q=10.25, p-value=0.036 Horizon h=8 (N=22, Lags=4): Q=9.62, p-value=0.047 Horizon h=9 (N=22, Lags=4): Q=7.04, p-value=0.134 Horizon h=10 (N=22, Lags=4): Q=8.80, p-value=0.066

— Checking Errors for Model: F_TR_FORMULA_2 — Horizon h=1 (N=22, Lags=4): Q=43.55, p-value=0.000 Horizon h=2 (N=22, Lags=4): Q=31.45, p-value=0.000 Horizon h=3 (N=22, Lags=4): Q=18.25, p-value=0.001 Horizon h=4 (N=22, Lags=4): Q=5.79, p-value=0.215 Horizon h=5 (N=22, Lags=4): Q=2.22, p-value=0.695 Horizon h=6 (N=22, Lags=4): Q=1.52, p-value=0.824 Horizon h=7 (N=22, Lags=4): Q=1.55, p-value=0.819 Horizon h=8 (N=22, Lags=4): Q=1.09, p-value=0.896 Horizon h=9 (N=22, Lags=4): Q=0.95, p-value=0.918 Horizon h=10 (N=22, Lags=4): Q=0.74, p-value=0.947

— Checking Errors for Model: F_TR_FORMULA_3 — Horizon h=1 (N=22, Lags=4): Q=8.50, p-value=0.075 Horizon h=2 (N=22, Lags=4): Q=15.23, p-value=0.004 Horizon h=3 (N=22, Lags=4): Q=15.87, p-value=0.003 Horizon h=4 (N=22, Lags=4): Q=18.15, p-value=0.001 Horizon h=5 (N=22, Lags=4): Q=17.36, p-value=0.002 Horizon h=6 (N=22, Lags=4): Q=16.86, p-value=0.002 Horizon h=7 (N=22, Lags=4): Q=16.02, p-value=0.003 Horizon h=8 (N=22, Lags=4): Q=14.72, p-value=0.005 Horizon h=9 (N=22, Lags=4): Q=12.30, p-value=0.015 Horizon h=10 (N=22, Lags=4): Q=8.83, p-value=0.065

— Checking Errors for Model: F_TR_FORMULA_4 — Horizon h=1 (N=22, Lags=4): Q=1.82, p-value=0.770 Horizon h=2 (N=22, Lags=4): Q=9.02, p-value=0.061 Horizon h=3 (N=22, Lags=4): Q=11.89, p-value=0.018 Horizon h=4 (N=22, Lags=4): Q=12.75, p-value=0.013 Horizon h=5 (N=22, Lags=4): Q=11.97, p-value=0.018 Horizon h=6 (N=22, Lags=4): Q=11.28, p-value=0.024 Horizon h=7 (N=22, Lags=4): Q=11.98, p-value=0.017 Horizon h=8 (N=22, Lags=4): Q=12.30, p-value=0.015 Horizon h=9 (N=22, Lags=4): Q=12.41, p-value=0.015 Horizon h=10 (N=22, Lags=4): Q=11.81, p-value=0.019

#### Errors Normally Distributed

Run the Jarque-Bera test using a custom function that explain loop and package where the actual test function is made

```r
# in final markdown, input just the relevant code, i.e. the tests made
# who cares about code to create tables, that stays in helpers
source(here("helpers/pseudo_outofsample_tests.R"))
```

```r
# Run with helper functions
# missing benchmark
generate_all_jb_reports(
  formula_cols =  formula_cols,
  eval_all_models = eval_all_models,
  max_h = max_h)
```

Table 9: Jarque–Bera Test Results

| horizon | F_TR_FORMULA_1 | F_TR_FORMULA_2 | F_TR_FORMULA_3 | F_TR_FORMULA_4 |
|---|---|---|---|---|
| 1 | 1.3147 (0.518 ) | 1.4536 (0.483 ) | 3.0503 (0.218 ) | 17.8432 (0.000 ***) |
| 2 | 1.5416 (0.463 ) | 1.3227 (0.516 ) | 8 (0.018 **) | 2.9683 (0.227 ) |
| 3 | 1.7299 (0.421 ) | 1.6514 (0.438 ) | 7.5554 (0.023 **) | 2.5318 (0.282 ) |
| 4 | 1.4723 (0.479 ) | 30.1444 (0.000 ***) | 2.75 (0.253 ) | 2.2488 (0.325 ) |
| 5 | 1.4886 (0.475 ) | 58.129 (0.000 ***) | 2.3399 (0.310 ) | 2.194 (0.334 ) |
| 6 | 1.5736 (0.455 ) | 67.3941 (0.000 ***) | 1.8789 (0.391 ) | 2.4866 (0.288 ) |
| 7 | 0.9188 (0.632 ) | 64.089 (0.000 ***) | 1.6653 (0.435 ) | 3.159 (0.206 ) |
| 8 | 0.3858 (0.825 ) | 56.1628 (0.000 ***) | 1.3626 (0.506 ) | 2.9565 (0.228 ) |
| 9 | 0.3242 (0.850 ) | 42.1115 (0.000 ***) | 1.3005 (0.522 ) | 3.2125 (0.201 ) |

| | | | | |
|---|---|---|---|---|
| 10 | 1.5089 (0.470 ) | 30.9638 (0.000 ***) | 3.1801 (0.204 ) | 3.0005 (0.223 ) |

### Absolute Performance: Efficiency & Bias

In order to evaluate the absolute performance of our model with the results from the pseudo out-of-sample estimation, we use Mincer-Zarnowitz regressions to check for both efficiency and bias. The method is to regress the actual realised values of the interest rate on the forecasted ones, as seen in equation (4). We then run a joint test, on the null hypothesis that $\alpha = 0$ (model is unbiased) and $\beta = 1$ (model is efficient). We run this with a custom function that loops over each horizon, in order to assess the performance of our models in the shorter vs longer term, and do so on all Taylor Rule formulas as well as on the benchmark

$$i_t^{actual} = \alpha + \beta i_t^{forecast} + \epsilon_t \tag{4}$$

```r
# in final markdown, input just the relevant code, i.e. the tests made
# who cares about code to create tables, that stays in helpers
source(here("helpers/pseudo_outofsample_tests.R"))
```

```r
# Call MZ-test helper function 4 times.
#  Note: These reports is wrapped in trycatch as it sometimes fails
#         If it does fail, simply decrease R in order to have more
#          observations, removing potential multicolinearity.

# MZ Report 1: Actual Rate, No Lag
mz_report_1 <- tryCatch({
  generate_absolute_performance_report(
    F_model = F_TR_1,
    Actual_values = Actuals,
    H = H,
    model_caption = "Mincer-Zarnowitz Test: Actual Rate, No Lag",
    format = format)}, error = function(e) {
  message("Error generating MZ Report (Actual Rate, No Lag): ", e$message)
  message("Skipping this report and continuing...")
  return(NULL)})

# MZ Report 2: Shadow Rate, No Lag (
mz_report_2 <- tryCatch({
  generate_absolute_performance_report(
    F_model = F_TR_2,
    Actual_values = Actuals,
    H = H,
    model_caption = "Mincer-Zarnowitz Test: Shadow Rate, No Lag",
    format = format)}, error = function(e) {
  message("Error generating MZ Report (Shadow Rate, No Lag): ", e$message)
  message("Skipping this report and continuing...")
  return(NULL)})

# MZ Report 3: Actual Rate, with Lag
mz_report_3 <- tryCatch({
  generate_absolute_performance_report(
    F_model = F_TR_3,
    Actual_values = Actuals,
    H = H,
```

```r
    model_caption = "Mincer-Zarnowitz Test: Actual Rate, with Lag",
    format = format)}, error = function(e) {
  message("Error generating MZ Report (Actual Rate, with Lag): ", e$message)
  message("Skipping this report and continuing...")
  return(NULL)})

# MZ Report 4: Shadow Rate, with Lag
mz_report_4 <- tryCatch({
  generate_absolute_performance_report(
    F_model = F_TR_4,
    Actual_values = Actuals,
    H = H,
    model_caption = "Mincer-Zarnowitz Test: Shadow Rate, with Lag",
    format = format)}, error = function(e) {
  message("Error generating MZ Report (Shadow Rate, with Lag): ", e$message)
  message("Skipping this report and continuing...")
  return(NULL)})

# MZ Report 5: Benchmark
mz_report_BM <- tryCatch({
  generate_absolute_performance_report(
    F_model = F_BM,
    Actual_values = Actuals,
    H = H,
    model_caption = "Mincer-Zarnowitz Test: Benchmark ARIMA",
    format = format)}, error = function(e) {
  message("Error generating MZ Report (Benchmark ARIMA): ", e$message)
  message("Skipping this report and continuing...")
  return(NULL)})

list(mz_report_1, mz_report_2, mz_report_3, mz_report_4, mz_report_BM)
```

[[1]]

[[2]]

[[3]]

[[4]]

[[5]]

## Relative Performance (against benchmark)

In order to evaluate the relative performance of our model against the benchmark ARIMA, we first compute the Mean Squared Forecast Errors (MSFE) for each horizon. This allows us to see the average performance of each estimation. For a deeper comparison, we implement Diebold-Mariano tests on each horizon. This test compares the forecast errors of both models by computing a loss differential, which is the difference of the squared forecast errors between the main model and the benchmark. The test statistic is as reported in equation (5) and assumes that $\widehat{d_{h,t}}$ is covariance stationary. We include three versions of the test, one where the alternative hypothesis is that the two models bring about different losses, one where the alternative hypothesis is that the tested model is greater than the benchmark, and one where the alternative hypothesis is that the tested model is worse than the benchmark (greater loss). The null hypothesis is the same for all three, being that the two models bring about the same loss.

Table 10: Mincer-Zarnowitz Test: Actual Rate, No Lag

| h | Alpha | Beta | pv(Joint) |
|---|---|---|---|
| **1** | 1.2076 | 0.3116 | 0.184 |
| **2** | 1.2531 | 0.3795 | 0.756 |
| **3** | 0.9920 | 0.8033 | 0.857 |
| **4** | 0.8109 | 1.2440 | 0.218 |
| **5** | 0.8868 | 1.4210 | 0.007 *** |
| **6** | 1.0360 | 1.4710 | 0.000 *** |
| **7** | 1.2366 | 1.3372 | 0.000 *** |
| **8** | 1.5904 | 1.0953 | 0.000 *** |
| **9** | 2.1531 | 0.6397 | 0.000 *** |
| **10** | 2.8582 | 0.0750 | 0.004 *** |

*Note:* pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast. * Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

Table 11: Mincer-Zarnowitz Test: Shadow Rate, No Lag

| h | Alpha | Beta | pv(Joint) |
|---|---|---|---|
| **1** | 1.8200 | -0.3633 | 0.000 *** |
| **2** | 1.8598 | -0.2299 | 0.000 *** |
| **3** | 1.7793 | -0.0457 | 0.000 *** |
| **4** | 1.8299 | 0.0095 | 0.000 *** |
| **5** | 1.9916 | -0.0130 | 0.000 *** |
| **6** | 2.1898 | -0.0409 | 0.000 *** |
| **7** | 2.4185 | -0.0665 | 0.000 *** |
| **8** | 2.6798 | -0.0894 | 0.000 *** |
| **9** | 2.8483 | -0.0617 | 0.000 *** |
| **10** | 3.0489 | -0.0389 | 0.000 *** |

*Note:*
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

Table 12: Mincer-Zarnowitz Test: Actual Rate, with Lag

| h | Alpha | Beta | pv(Joint) |
|---|---|---|---|
| **1** | 0.0547 | 1.0015 | 0.793 |
| **2** | 0.2077 | 0.9484 | 0.793 |
| **3** | 0.4454 | 0.8750 | 0.731 |
| **4** | 0.7365 | 0.7927 | 0.655 |
| **5** | 1.1190 | 0.6535 | 0.492 |
| **6** | 1.4765 | 0.5449 | 0.372 |
| **7** | 1.8810 | 0.3823 | 0.108 |
| **8** | 2.2834 | 0.1895 | 0.023 ** |
| **9** | 2.6784 | 0.0002 | 0.001 *** |
| **10** | 3.0516 | -0.1807 | 0.000 *** |

*Note:*
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

Table 13: Mincer-Zarnowitz Test: Shadow Rate, with Lag

| h | Alpha | Beta | pv(Joint) |
|---|---|---|---|
| **1** | 0.0681 | 0.9143 | 0.087 * |
| **2** | 0.1930 | 0.7986 | 0.186 |
| **3** | 0.4392 | 0.6389 | 0.007 *** |
| **4** | 0.7626 | 0.4854 | 0.000 *** |
| **5** | 1.1300 | 0.3526 | 0.000 *** |
| **6** | 1.4534 | 0.2523 | 0.000 *** |
| **7** | 1.8018 | 0.1659 | 0.000 *** |
| **8** | 2.1810 | 0.0875 | 0.000 *** |
| **9** | 2.6095 | 0.0196 | 0.000 *** |
| **10** | 3.0870 | -0.0420 | 0.000 *** |

*Note:*
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

Table 14: Mincer-Zarnowitz Test: Benchmark ARIMA

| h | Alpha | Beta | pv(Joint) |
|---|---|---|---|
| **1** | 0.1487 | 0.9447 | 0.382 |
| **2** | 0.3943 | 0.8503 | 0.193 |
| **3** | 0.7096 | 0.7125 | 0.197 |
| **4** | 1.0656 | 0.5675 | 0.290 |
| **5** | 1.4463 | 0.4072 | 0.146 |
| **6** | 1.8131 | 0.2622 | 0.144 |
| **7** | 2.1639 | 0.1192 | 0.006 *** |
| **8** | 2.4781 | -0.0143 | 0.001 *** |
| **9** | 2.7540 | -0.1508 | 0.000 *** |
| **10** | 2.9915 | -0.2735 | 0.000 *** |

*Note:*
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

$$
\begin{aligned}
DM &= \frac{\bar{\hat{d}}_h}{\sigma^{\hat{2}}_{\bar{\hat{d}}_h}} \\
&= \frac{\frac{1}{P}\sum_{t=R}^{T-h} \hat{d}_{h,t}}{\sigma^{\hat{2}}_{\bar{\hat{d}}_h}} \\
\text{where } \widehat{d_{h,t}} &= \widehat{e^2_{1,t+h|t}} - \widehat{e^2_{2,t+h|t}}
\end{aligned}
\tag{5}
$$

```r
# in final markdown, input just the relevant code, i.e. the tests made
# who cares about code to create tables, that stays in helpers
source(here("helpers/pseudo_outofsample_tests.R"))
```

```r
# Call DM-test helper function 4 times.

# Report 1: Actual Rate, No Lag
report_1 <- generate_relative_performance_report(
  FE_TR_model = FE_TR_1,
  FE_BM_model = FE_BM,
  H = H,
  model_caption = "MSFE Comparison, Trained on Actual Rate, No Lag",
  format = format)

# Report 2: Shadow Rate, No Lag
report_2 <- generate_relative_performance_report(
  FE_TR_model = FE_TR_2,
  FE_BM_model = FE_BM,
  H = H,
  model_caption = "MSFE Comparison, Trained on Shadow Rate, No Lag",
```

Table 15: MSFE Comparison, Trained on Actual Rate, No Lag

| h | MSFE TR | MSFE BM | Ratio | DM Two-Sided | DM Greater | DM Lesser |
|---|---|---|---|---|---|---|
| **1** | 3.9517 | 0.1591 | 24.8393 | 0.000 *** | 1.000 | 0.000 *** |
| **2** | 3.9732 | 0.6458 | 6.1521 | 0.032 ** | 0.984 | 0.016 ** |
| **3** | 3.4469 | 1.6062 | 2.1459 | 0.386 | 0.807 | 0.193 |
| **4** | 3.1086 | 2.8224 | 1.1014 | 0.915 | 0.542 | 0.458 |
| **5** | 3.0938 | 4.3368 | 0.7134 | 0.603 | 0.302 | 0.698 |
| **6** | 3.2794 | 5.8787 | 0.5578 | 0.124 | 0.062 * | 0.938 |
| **7** | 3.5234 | 7.4570 | 0.4725 | 0.000 *** | 0.000 *** | 1.000 |
| **8** | 4.0667 | 9.0333 | 0.4502 | 0.001 *** | 0.000 *** | 1.000 |
| **9** | 4.8839 | 10.1786 | 0.4798 | 0.000 *** | 0.000 *** | 1.000 |
| **10** | 5.8798 | 11.5721 | 0.5081 | 0.000 *** | 0.000 *** | 1.000 |

*Note:* TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark of the interest rate using an ARIMA model. Ratio < 1 indicates that the TR model has lower MSFE.

*DM Test Alternative Hypotheses (H_A):* * 'DM Greater' tests if the TR model is significantly more accurate than the BM model. † 'DM Lesser' tests if the TR model is significantly less accurate than the BM model.

```r
  format = format)

# Report 3: Actual Rate, with Lag
report_3 <- generate_relative_performance_report(
  FE_TR_model = FE_TR_3,
  FE_BM_model = FE_BM,
  H = H,
  model_caption = "MSFE Comparison, Trained on Actual Rate, with Lag",
  format = format)

# Report 4: Shadow Rate, with Lag
report_4 <- generate_relative_performance_report(
  FE_TR_model = FE_TR_4,
  FE_BM_model = FE_BM,
  H = H,
  model_caption = "MSFE Comparison, Trained on Shadow Rate, with Lag",
  format = format)

list(report_1, report_2, report_3, report_4)
```

[[1]]

[[2]]

[[3]]

[[4]]

Table 16: MSFE Comparison, Trained on Shadow Rate, No Lag

| h | MSFE TR | MSFE BM | Ratio | DM Two-Sided | DM Greater | DM Lesser |
|---|---|---|---|---|---|---|
| **1** | 8.7557 | 0.1591 | 55.0357 | 0.000 *** | 1.000 | 0.000 *** |
| **2** | 9.4762 | 0.6458 | 14.6728 | 0.002 *** | 0.999 | 0.001 *** |
| **3** | 9.4969 | 1.6062 | 5.9125 | 0.030 ** | 0.985 | 0.015 ** |
| **4** | 12.0493 | 2.8224 | 4.2692 | 0.209 | 0.895 | 0.105 |
| **5** | 16.4410 | 4.3368 | 3.7910 | 0.313 | 0.844 | 0.156 |
| **6** | 22.9412 | 5.8787 | 3.9024 | 0.363 | 0.819 | 0.182 |
| **7** | 30.3125 | 7.4570 | 4.0650 | 0.384 | 0.808 | 0.192 |
| **8** | 40.6792 | 9.0333 | 4.5032 | 0.336 | 0.832 | 0.168 |
| **9** | 48.6339 | 10.1786 | 4.7781 | 0.256 | 0.872 | 0.128 |
| **10** | 59.1154 | 11.5721 | 5.1084 | 0.144 | 0.928 | 0.072 * |

*Note:* TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark of the interest rate using an ARIMA model. Ratio < 1 indicates that the TR model has lower MSFE.
*DM Test Alternative Hypotheses (H_A):* [*] 'DM Greater' tests if the TR model is significantly more accurate than the BM model. [†] 'DM Lesser' tests if the TR model is significantly less accurate than the BM model.

Table 17: MSFE Comparison, Trained on Actual Rate, with Lag

| h | MSFE TR | MSFE BM | Ratio | DM Two-Sided | DM Greater | DM Lesser |
|---|---|---|---|---|---|---|
| **1** | 0.1335 | 0.1591 | 0.8393 | 0.491 | 0.246 | 0.754 |
| **2** | 0.4435 | 0.6458 | 0.6866 | 0.196 | 0.098 * | 0.902 |
| **3** | 0.9719 | 1.6062 | 0.6051 | 0.136 | 0.068 * | 0.932 |
| **4** | 1.5987 | 2.8224 | 0.5664 | 0.104 | 0.052 * | 0.948 |
| **5** | 2.5347 | 4.3368 | 0.5845 | 0.037 ** | 0.018 ** | 0.982 |
| **6** | 3.4632 | 5.8787 | 0.5891 | 0.008 *** | 0.004 *** | 0.996 |
| **7** | 4.7305 | 7.4570 | 0.6344 | 0.009 *** | 0.004 *** | 0.996 |
| **8** | 6.2083 | 9.0333 | 0.6873 | 0.002 *** | 0.001 *** | 0.999 |
| **9** | 7.9732 | 10.1786 | 0.7833 | 0.013 ** | 0.006 *** | 0.994 |
| **10** | 9.7356 | 11.5721 | 0.8413 | 0.080 * | 0.040 ** | 0.960 |

*Note:* TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark of the interest rate using an ARIMA model. Ratio < 1 indicates that the TR model has lower MSFE.
*DM Test Alternative Hypotheses (H_A):* [*] 'DM Greater' tests if the TR model is significantly more accurate than the BM model.
[†] 'DM Lesser' tests if the TR model is significantly less accurate than the BM model.

Table 18: MSFE Comparison, Trained on Shadow Rate, with Lag

| h | MSFE TR | MSFE BM | Ratio | DM Two-Sided | DM Greater | DM Lesser |
|---|---|---|---|---|---|---|
| **1** | 0.1591 | 0.1591 | 1.0000 | 1.000 | 0.500 | 0.500 |
| **2** | 0.5536 | 0.6458 | 0.8571 | 0.715 | 0.357 | 0.643 |
| **3** | 1.5969 | 1.6062 | 0.9942 | 0.946 | 0.473 | 0.527 |
| **4** | 3.5822 | 2.8224 | 1.2692 | 0.371 | 0.815 | 0.185 |
| **5** | 6.7917 | 4.3368 | 1.5661 | 0.230 | 0.885 | 0.115 |
| **6** | 11.0846 | 5.8787 | 1.8856 | 0.076 * | 0.962 | 0.038 ** |
| **7** | 17.4023 | 7.4570 | 2.3337 | 0.079 * | 0.960 | 0.040 ** |
| **8** | 26.3667 | 9.0333 | 2.9188 | 0.100 | 0.950 | 0.050 * |
| **9** | 37.3973 | 10.1786 | 3.6741 | 0.096 * | 0.952 | 0.048 ** |
| **10** | 51.5048 | 11.5721 | 4.4508 | 0.056 * | 0.972 | 0.028 ** |

*Note:* TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark of the interest rate using an ARIMA model. Ratio $< 1$ indicates that the TR model has lower MSFE.

*DM Test Alternative Hypotheses (H_A):* [*] 'DM Greater' tests if the TR model is significantly more accurate than the BM model.

[†] 'DM Lesser' tests if the TR model is significantly less accurate than the BM model.

Table 19: For model based on: Taylor Rule Formula 3

| Horizon: Quarter | Taylor Rule Forecast | Benchmark Forecast | Inflation Forecast | Output Gap Forecast |
|---|---|---|---|---|
| **1: 2025 Q4** | 2.00 | 2.00 | 2.17 | 1.24 |
| **2: 2026 Q1** | 1.75 | 2.25 | 2.03 | 1.38 |
| **3: 2026 Q2** | 1.75 | 2.25 | 2.10 | 1.33 |
| **4: 2026 Q3** | 1.75 | 2.25 | 2.09 | 1.11 |
| **5: 2026 Q4** | 1.75 | 2.25 | 2.09 | 0.79 |
| **6: 2027 Q1** | 1.50 | 2.25 | 2.08 | 0.42 |
| **7: 2027 Q2** | 1.50 | 2.25 | 2.08 | 0.07 |
| **8: 2027 Q3** | 1.50 | 2.25 | 2.07 | -0.22 |
| **9: 2027 Q4** | 1.50 | 2.25 | 2.07 | -0.42 |
| **10: 2028 Q1** | 1.25 | 2.25 | 2.07 | -0.52 |

## Actual Forecast Model

As described earlier, our methodology consists of two steps where we first fit ARIMA models to the Taylor Rule inputs as well as estimate the coefficients, and then use the forecasted inputs and the coefficients to compute our point forecast for the interest rate. In this section however, we run this method on the entire sample (Q1 1999 to Q3 2025) making forecasts up to 10 quarters ahead (Q1 2028).

### Forecasting

In order to do this, we use a custom function that first fits the ARIMA models for the inputs, uses these to make forecasts of said inputs, estimates the Taylor Rule coefficients and then computes the point forecast for the interest rate. Note that for the lagged models, we then use these point forecasts to compute the next forecast iteratively. It also fits an ARIMA model for the interest rate, so that we have a simple benchmark model to see if there are large differences in estimates.

```r
source(here("helpers/actual_forecast_estimator.R"))

final_forecasts <- our_predict(data = data, formula = model_formula, H = H)
```

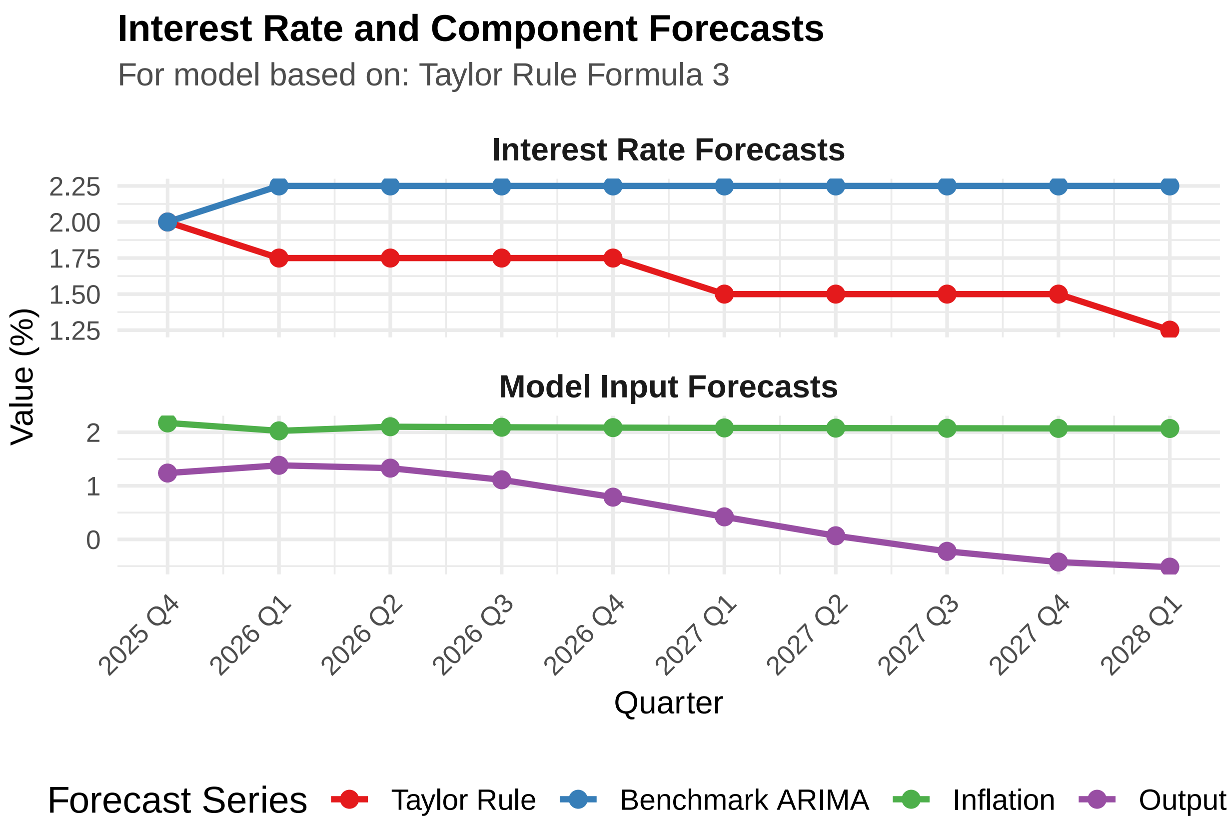
The ARIMA model fitted to the 'Inflation Gap' variable is specified as ARIMA(1, 1, 4).

The ARIMA model fitted to the 'Output Gap' variable is specified as ARIMA(2, 0, 3).

The ARIMA model fitted to the 'Interest Rate' variable is specified as ARIMA(2, 1, 0).

```r
display_forecasts(final_forecasts,
                  caption = paste("For model based on:", model_name),
                  format = format)
```

```r
plot_forecasts(final_forecasts)
```

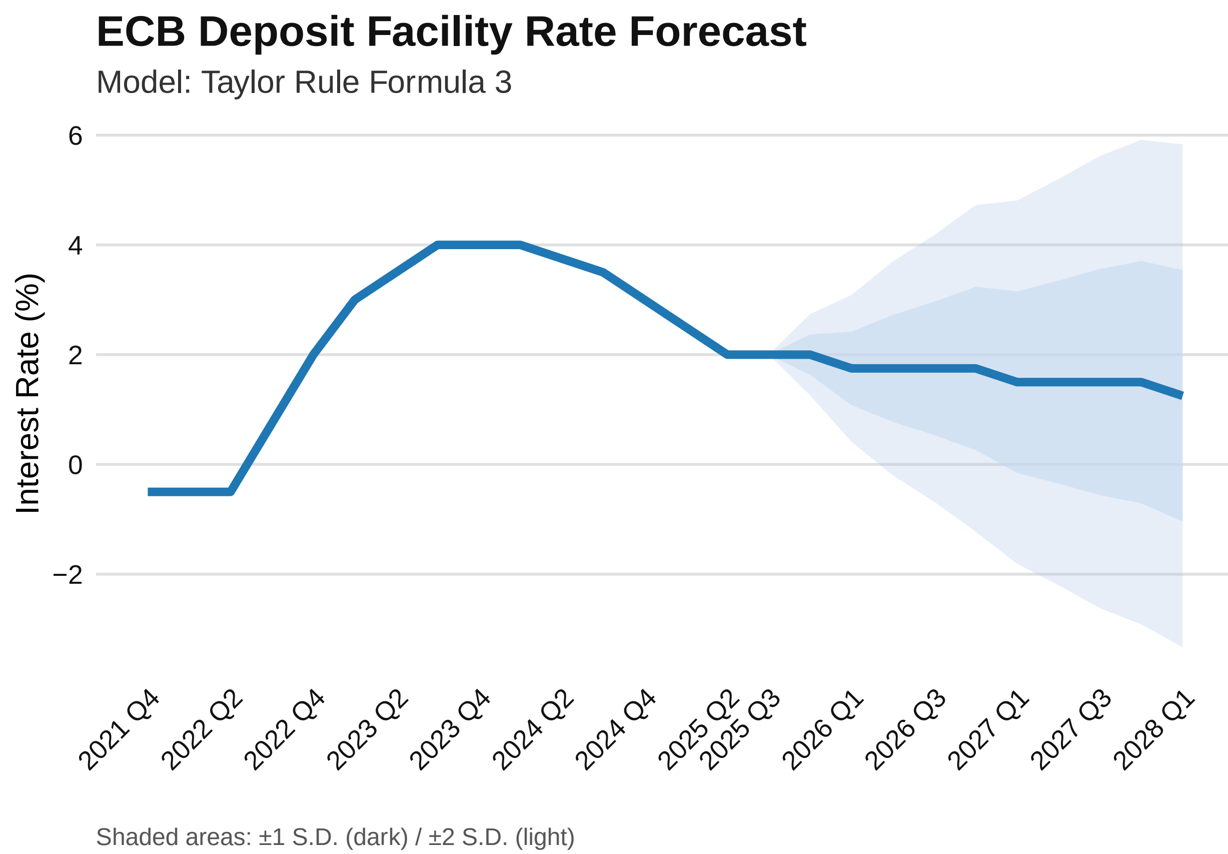



### Prediction Intervals

To compute the prediction intervals for our point forecasts, we use the estimated variance of errors computed in the pseudo out-of-sample evaluation exercise assuming that the distribution of errors will be the same for our actual future forecast. We include two intervals, one being one standard deviation away from the point forecasts, and the other being two standard deviations away. This is computed as in equation (6).

$$\begin{aligned} PI_1^{upper} &= i_t^{forecast} + \sigma_e \\ PI_1^{lower} &= i_t^{forecast} - \sigma_e \\ PI_2^{upper} &= i_t^{forecast} + 2\sigma_e \\ PI_2^{lower} &= i_t^{forecast} - 2\sigma_e \end{aligned} \tag{6}$$

```r
source(here("helpers/actual_forecast_estimator.R"))

# Computes prediction intervals
final_interval <- our_predict_intervals(estimated_variance = var_by_horizon,
                                        forecast = final_forecasts)

# Plots prediction intervals
plot_forecasts_pred_int(data, intervals = final_interval)
```


ECB Deposit Facility Rate Forecast
Model: Taylor Rule Formula 3
Interest Rate (%)
6
4
2
0
−2
2021 Q4
2022 Q2
2022 Q4
2023 Q2
2023 Q4
2024 Q2
2024 Q4
2025 Q2
2025 Q3
2026 Q1
2026 Q3
2027 Q1
2027 Q3
2028 Q1
Shaded areas: ±1 S.D. (dark) / ±2 S.D. (light)