ECB Tests 6

Contents

Data

The Data from Fred come from here: European Central Bank, ECB Deposit Facility Rate for Euro Area [ECBDFR], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/ECBDFR. The data is the Deposit Facility and is directly reprinted from the ECB. It's in Percent and not seasonally adjusted. The ECB Monetary Policy is steered through this rate

The Shadow rate was developed by **wuTimeVaryingLowerBound2017** and does quantify a hypothetical removal of the ZLB. The rate does not track 1:1 on the deposit facility in the data before the ZLB, instead being closer to the refinancing rate. (maybe need to adjust)

The GDP Data is real GDP in 2010 Euros in million retrieved from FRED via Eurostat. The data is seasonally adjusted.

The Inflation data is from Eurostat and called HICP. It's the index the ECB uses for Inflation and isn't seasonally adjusted.

Expected Inflation is the ECB's survey of professional forecasters. It forecasts the HICP 12 months in advance.

Main

```r
# --- 1. ECB Deposit Facility Rate & Shadow Rate ---
ecb_rate_daily <- fredr(series_id = "ECBDFR", observation_start = as.Date(start_date))
ecb_rate_q <- ecb_rate_daily %>%
  mutate(quarter = as.yearqtr(date)) %>%
  group_by(quarter) %>%
  summarise(rate = last(value)) %>%
  mutate(date = as.Date(quarter))
# Wu-Xia Shadow Rate
shadow_rate_daily = as.data.frame(readMat("data/shadowrate_ECB.mat"))
colnames(shadow_rate_daily) <- c("DATE", "shadowrate")
shadow_rate_daily$DATE <- as.Date(paste0(shadow_rate_daily$DATE, "01"), format="%Y%m%d")
shadow_rate_daily$quarter <- as.yearqtr(as.Date(shadow_rate_daily$DATE))
shadow_rate_daily$month <- as.yearmon(as.Date(shadow_rate_daily$DATE))
quarterly_shadow = aggregate(shadowrate ~ quarter, data=shadow_rate_daily, FUN=mean, na.rm=T)
monthly_shadow = aggregate(shadowrate ~ month, data=shadow_rate_daily, FUN=mean, na.rm=T)

# --- 2. HICP Inflation (Euro Area) ---
inflation_data <- get_eurostat("prc_hicp_manr", filters = list(geo = "EA", coicop = "CP00"), type = "la
inflation_q <- inflation_data %>%
  filter(time >= start_date) %>%
  dplyr::select(date = time, inflation = values) %>%
  mutate(quarter = as.yearqtr(date)) %>%
  group_by(quarter) %>%
  summarise(inflation = mean(inflation, na.rm = TRUE)) %>%
  mutate(date = as.Date(quarter))

#inflation expectations
inflation_exp <- rdb(ids = "ECB/SPF/M.U2.HICP.POINT.P12M.Q.AVG")
#inflation_exp <- rdb(ids = "ECB/SPF/M.U2.HICP.POINT.P24M.Q.AVG")
inflation_exp_q <- inflation_exp %>%
  mutate(quarter = as.yearqtr(period)) %>%
  group_by(quarter) %>%
```

```r
  summarise(exp_inflation = last(original_value)) %>%
  mutate(date = as.Date(quarter))

#P12M : 12-month ahead forecasts
inflation_q$exp_inflation = c(rep(NA,3),as.numeric(inflation_exp_q$exp_inflation),NA)
#P24M : 24-month ahead forecasts
#inflation_q$exp_inflation = c(rep(NA,7),as.numeric(inflation_exp_q$exp_inflation[1:101]))

# --- 3. Real GDP and Estimated Output Gap ---
# a) Real GDP for the Euro Area. The series ID is CLVMNACSCAB1GQE_A.
gdp_q <- fredr(
  series_id = "CLVMEURSCAB1GQEA19",
  observation_start = as.Date(start_date)) %>%
  mutate(quarter = as.yearqtr(date)) %>%
  dplyr::select(quarter, real_gdp = value) %>%
  mutate(log_real_gdp = log(real_gdp))

# b) Estimate Potential GDP (the trend) using the HP Filter on the log of real GDP.
# The lambda value of 1600 is standard for quarterly data.
hp_gdp <- hpfilter(gdp_q$log_real_gdp, freq = 1600)
gdp_q$potential_gdp_log <- as.numeric(hp_gdp$trend)
ham_gdp_cycle <- filter_hamilton(gdp_q$log_real_gdp, p = 4, horizon = 8)
gdp_q$potential_gdp_log_ham <- gdp_q$log_real_gdp - ham_gdp_cycle

# Combine all data into a single data frame
data <- ecb_rate_q %>%
  dplyr::select(quarter, rate) %>%
  left_join(inflation_q, by = "quarter") %>%
  left_join(gdp_q, by = "quarter") %>%
  left_join(quarterly_shadow, by = "quarter")

# Create model variables
data <- data %>%
  mutate(
    realised_inflation_gap = inflation - 2.0,
    exp_inflation_gap = exp_inflation -2.0,
    output_gap_hp = 100 * (log_real_gdp - potential_gdp_log),
    output_gap_ham = 100 * (log_real_gdp - potential_gdp_log_ham),
    rate_lag = lag(rate, 1),
    shadowrate = case_when(
      quarter < "2012 Q3" | quarter >= "2022 Q3" ~ rate,
      TRUE ~ shadowrate),
    shadowrate_lag = lag(shadowrate, 1))

# Remove last row since no output data
data = subset(data, quarter < "2025 Q3")

# Clean environment
rm(gdp_q, hp_gdp, ecb_rate_daily, ecb_rate_q, inflation_data, inflation_q,
   inflation_exp, inflation_exp_q, monthly_shadow, quarterly_shadow, shadow_rate_daily)
```

Data Configuration

```r
# Choices in setup chunk

# --------- 1. Filter selection for output gap estimation ----------

# TRUE  = Use Hamilton Filter (newer, arguably more robust)
# FALSE = Use HP Filter (classic approach)

# Applying selection
if (USE_HAMILTON_FILTER) {
    data$output_gap <- data$output_gap_ham
  message(">> CONFIGURATION: Using Hamilton Filter for output gap estimation.")
} else {
  data$output_gap <- data$output_gap_hp
  message(">> CONFIGURATION: Using HP Filter for output gap estimation.") }
```

```
## >> CONFIGURATION: Using Hamilton Filter for output gap estimation.
```

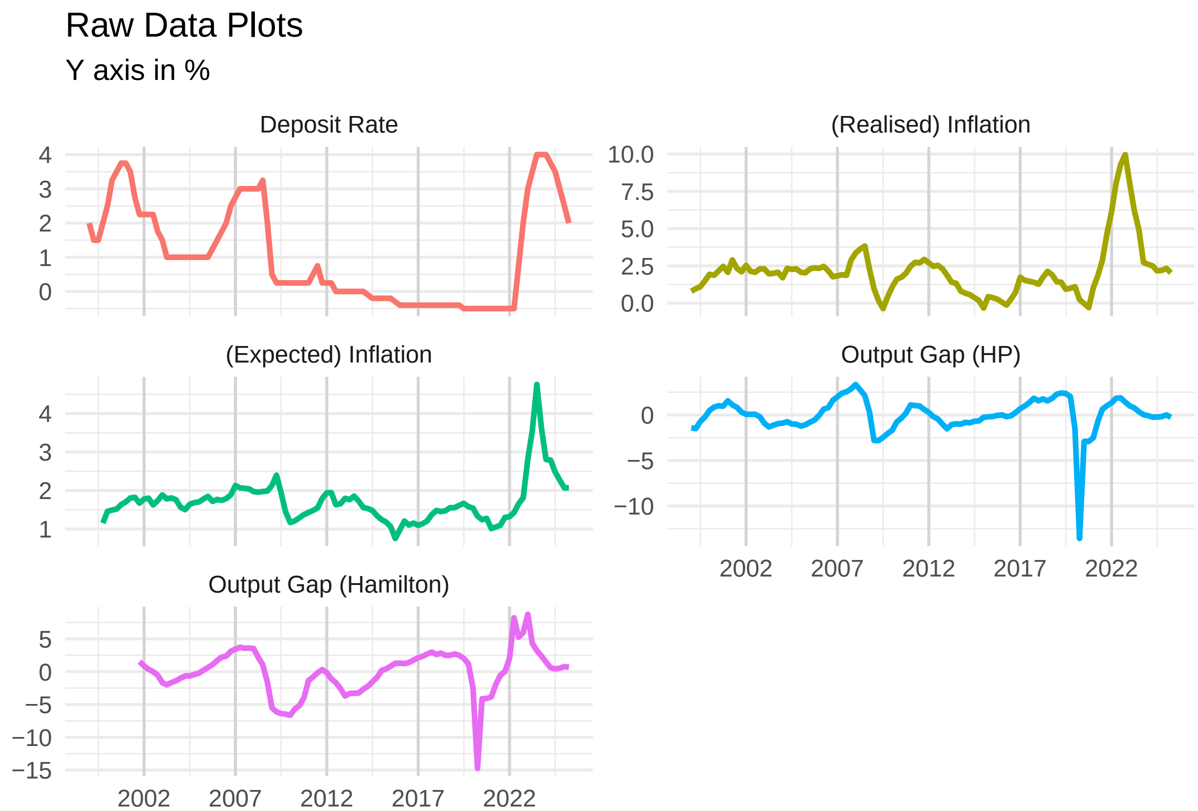
```r
# --------- 2. Inflation expectations choice ----------

# TRUE  = Use inflation expectations from ECB survey of professional forecasts
# FALSE = Use realised inflation

# Applying selection
if (USE_INFLATION_EXPECTATIONS) {
    data$inflation_gap <- data$exp_inflation_gap
  message(">> CONFIGURATION: Using inflation expectations in Taylor Rule forecasting.")
} else {
  data$inflation_gap <- data$realised_inflation_gap
  message(">> CONFIGURATION: Using realised inflation in Taylor Rule forecasting.") }
```

```
## >> CONFIGURATION: Using realised inflation in Taylor Rule forecasting.
```

Raw Data Plots

```r
# Data must be in long format for a faceted plot
plot_data <- data %>%
  pivot_longer(cols = c(rate, inflation, exp_inflation, output_gap_hp, output_gap_ham),
               names_to = "series",
               values_to = "value") %>%
  mutate(series = factor(series,
    levels = c("rate", "inflation", "exp_inflation", "output_gap_hp", "output_gap_ham"),
    labels = c("Deposit Rate", "(Realised) Inflation", "(Expected) Inflation", "Output Gap (HP)", "Outpu

ggplot(plot_data, aes(x = date, y = value)) +
  geom_line(aes(color = series), linewidth = 1) +
  facet_wrap(~ series, scales = "free_y", ncol = 2) +
  scale_x_date(date_breaks = "5 years", date_labels = "%Y") +
  labs(title = "Raw Data Plots", subtitle = "Y axis in %", x = "", y = "") +
```

```r
  theme_minimal() + theme(
    legend.position = "none",
    panel.grid.major.x = element_line(linewidth = 0.525, color = "grey83"))
```




```r
rm(plot_data)
```

Data Properties

```r
# Helper function to run ADF and KPSS tests and return dataframe fit for table

check_stationarity <- function(y, var_name) {

  # Running tests and getting pvalues
  adf_result <- tseries::adf.test(y, alternative = "stationary")
  kpss_result <- tseries::kpss.test(y, null = "Level")
  adf_p <- adf_result$p.value
  kpss_p <- kpss_result$p.value

  # Automatically assign interpretation (yes, modular!)
  interpretation <- ""
  if (adf_p < 0.05 && kpss_p > 0.05) {
    interpretation <- "Stationary I(0): ADF rejects unit root, KPSS confirms stationarity." }
  else if (adf_p > 0.05 && kpss_p < 0.05) {
```

```r
    interpretation <- "Non-Stationary I(1): ADF confirms unit root, KPSS rejects stationarity." }
  else if (adf_p > 0.05 && kpss_p > 0.05) {
    interpretation <- "Conflicting Results: ADF confirms unit root, while KPSS suggests stationarity." }
  else { # adf_p < 0.05 && kpss_p < 0.05
    interpretation <- "Conflicting Results: ADF rejects unit root, while KPSS rejects stationarity." }

  # Export results
  data.frame(Variable = var_name,
            `ADF p-value` = adf_p,
            `KPSS p-value` = kpss_p,
             Result = interpretation, check.names = FALSE) }


# Similar helper function, but for cointegration tests
check_coint <- function(y1,y2, var_name1, var_name2){

  # Run test and get pvalue
  coint_test <- aTSA::coint.test(y1, y2, output = FALSE)
  p_value <- coint_test[1, 3] #type 1

  # Again, modular(!) interpretation
  interpretation <- ""
  if (p_value < 0.05) {
    interpretation <- "Cointegrated"}
  else {
    interpretation <- "Not Cointegrated" }

  # Export results
  data.frame(Variables = paste(var_name1, "&", var_name2),
             `p-value` = p_value,
             Result = interpretation, check.names = FALSE) }
```

```r
# -----  Run Tests -----

# Use helper to run stationarity checks for our main variables
test_rate <- check_stationarity(data$rate, "Interest Rate")
test_inflation <- check_stationarity(data$inflation, "Inflation")
test_output_gap <- check_stationarity(na.omit(data$output_gap), "Output Gap")

# Given results for rate and inflation, run tests on 1st diffs
test_rate_diff <- check_stationarity(diff(data$rate), "Interest Rate (1st Diff)")
test_inflation_diff <- check_stationarity(diff(data$inflation), "Inflation (1st Diff)")

# Since rate and inflation are I(1), run a cointegration test
coint_test = check_coint(data$rate, data$inflation,
            var_name1 = "Interest Rate", var_name2 = "Inflation")


# ----- Print Results -----

# Combine stationarity results
all_stationarity_results <- rbind(test_rate,
                                  test_inflation,
```

Table 1: Summary of Stationarity Tests (ADF & KPSS)

Variable	ADF p-value	KPSS p-value	Result
Interest Rate	0.2900	0.0325	Non-Stationary I(1): ADF confirms unit root, KPSS rejects stationarity.
Inflation	0.2319	0.1000	Conflicting Results: ADF confirms unit root, while KPSS suggests stationarity.
Output Gap	0.0144	0.1000	Stationary I(0): ADF rejects unit root, KPSS confirms stationarity.
Interest Rate (1st Diff)	0.0100	0.1000	Stationary I(0): ADF rejects unit root, KPSS confirms stationarity.
Inflation (1st Diff)	0.0100	0.1000	Stationary I(0): ADF rejects unit root, KPSS confirms stationarity.

Table 2: Cointegration Test Results

Variables	p-value	Result
Interest Rate & Inflation	0.0433	Cointegrated

```
                                 test_output_gap,
                                 test_rate_diff,
                                 test_inflation_diff)

# Tables
kable(all_stationarity_results, digits = 4, format = format, booktabs = TRUE,
      caption = "Summary of Stationarity Tests (ADF \\& KPSS)") %>%
      kable_styling(latex_options = "scale_down",
      position = "center") %>%
        column_spec(1, border_right = TRUE) %>%
        column_spec(4, width = "6cm")
```

```
kable(coint_test, digits = 4, format = format, booktabs = TRUE,
      caption = "Cointegration Test Results") %>%
      kable_styling(latex_options = "scale_down",
      position = "center") %>%
        column_spec(1, border_right = TRUE)
```

Taylor Rule Estimation

Without Lags

$$i_t = \pi^* + \beta(\pi_t - \pi^*) + \gamma(y_t - \bar{y_t})$$

```
TR <- lm(rate ~ realised_inflation_gap + output_gap, data = data)
TRsr <- lm(shadowrate ~ realised_inflation_gap + output_gap, data = data)

table1 <- export_summs(TR, TRsr, vcov = sandwich::NeweyWest,
          model.names = c("TR", "TR w/ SR"), digits = 4)
huxtable::caption(table1) <- "No Lag, No Expectations"
table1
```

Table 3: No Lag, No Expectations

	TR	TR w/ SR
(Intercept)	0.8345	-0.8601
	(0.9055)	(2.6782)
realised_inflation_gap	0.1963	0.6857
	(0.4289)	(0.6062)
output_gap	0.0824	-0.0567
	(0.1628)	(0.2758)
N	95	95
R2	0.1726	0.1159

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

```
TR_e <- lm(rate ~ exp_inflation_gap + output_gap, data = data)
TRsr_e <- lm(shadowrate ~ exp_inflation_gap + output_gap, data = data)
TR_ie <- lm(rate ~ realised_inflation_gap + exp_inflation_gap + output_gap, data = data)
TRsr_ie <- lm(shadowrate ~ realised_inflation_gap + exp_inflation_gap + output_gap, data = data)

table2 <- export_summs(TR_e, TRsr_e, TR_ie, TRsr_ie, vcov = sandwich::NeweyWest,
          model.names = c("TR", "TR w/ SR", "TR", "TR w/ SR"), digits = 4)
huxtable::caption(table2) <- "No Lag, with Inflation Expectations"
table2
```

Lagged Models

$$i_t = \pi^* + \phi i_{t-1} + \beta(\pi_t - \pi^*) + \gamma(y_t - \bar{y_t})$$

Table 4: No Lag, with Inflation Expectations

	TR	TR w/ SR	TR	TR w/ SR
(Intercept)	1.3440 ***	0.2867	1.3381 ***	0.1715
	(0.3179)	(1.5603)	(0.3714)	(1.5918)
exp_inflation_gap	1.7238 ***	3.7581 **	1.7107 ***	3.5044 **
	(0.3523)	(1.3756)	(0.3466)	(1.1674)
output_gap	0.0707	-0.0026	0.0662	-0.0898
	(0.0468)	(0.1871)	(0.0810)	(0.2133)
realised_inflation_gap			0.0163	0.3171
			(0.1277)	(0.3433)
N	95	95	95	95
R2	0.6177	0.3904	0.6180	0.4086

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

```
lTR <- lm(rate ~ rate_lag + realised_inflation_gap + output_gap, data = data)
lTRsr <- lm(shadowrate ~ shadowrate_lag + realised_inflation_gap + output_gap, data = data)

table3 <- export_summs(lTR, lTRsr, vcov = sandwich::NeweyWest,
          model.names = c("TR", "TR w/ SR"), digits = 4)
huxtable::caption(table3) <- "Interest Rate Lag, No Expectations"
table3
```

```
lTR_e <- lm(rate ~ rate_lag + exp_inflation_gap + output_gap, data = data)
lTRsr_e <- lm(shadowrate ~ shadowrate_lag + exp_inflation_gap + output_gap, data = data)
lTR_ie <- lm(rate ~ rate_lag + realised_inflation_gap + exp_inflation_gap + output_gap, data = data)
lTRsr_ie <- lm(shadowrate ~ shadowrate_lag + realised_inflation_gap + exp_inflation_gap + output_gap, da

table4 <- export_summs(lTR_e, lTRsr_e, lTR_ie,lTRsr_ie, vcov = sandwich::NeweyWest,
          model.names = c("TR", "TR w/ SR", "TR", "TR w/ SR"), digits = 4)
huxtable::caption(table4) <- "Interest Rate Lag, with Inflation Expectations"
table4
```

Checking for structural breaks

```
# Formula to test for breaks
break_formula = rate ~ rate_lag + inflation_gap + output_gap

# Suspected break: Start of ZLB in 2012 Q3
breakpoint1_obs = 55 #R = 55 in evaluation chunk
```

Table 5: Interest Rate Lag, No Expectations

	TR	TR w/ SR
(Intercept)	0.0507	-0.0777
	(0.0410)	(0.0618)
rate_lag	0.9189 ***	
	(0.0390)	
realised_inflation_gap	0.0879 *	0.2498 ***
	(0.0350)	(0.0344)
output_gap	0.0218	-0.0116
	(0.0163)	(0.0185)
shadowrate_lag		0.9530 ***
		(0.0176)
N	95	95
R2	0.9594	0.9828

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

```r
breakpoint1_date <- data$quarter[breakpoint1_obs]

# Suspected break: Covid
breakpoint2_obs = 85 #R = 85 in evaluation chunk
breakpoint2_date <- data$quarter[breakpoint2_obs]

# Chow test (rejecting the null means there are structural breaks)
chow_test1 <- sctest(break_formula, type = "Chow", point = breakpoint1_obs, data = data)
chow_test2 <- sctest(break_formula, type = "Chow", point = breakpoint2_obs, data = data)


# Table with Chow results (for suspected breaks)
chow_df <- data.frame(
  Event = c("ZLB Start", "COVID-19 Start"),
  Date = c(as.character(breakpoint1_date), as.character(breakpoint2_date)),
  `p-value` = c(chow_test1$p.value, chow_test2$p.value),check.names = FALSE)

kable(chow_df, digits = 4, format = format, booktabs = TRUE,
      caption = "Chow tests for suspected structural breaks") %>%
      kable_styling(latex_options = "scale_down",
      position = "center") %>%
        column_spec(1, border_right = TRUE)
```

Table 6: Interest Rate Lag, with Inflation Expectations

	TR	TR w/ SR	TR	TR w/ SR
(Intercept)	0.1804 *	0.0009	0.1341 ***	-0.0896
	(0.0796)	(0.0869)	(0.0378)	(0.0622)
rate_lag	0.8606 ***		0.8742 ***	
	(0.0438)		(0.0392)	
exp_inflation_gap	0.2387 **	0.1296	0.1536 ***	-0.0546
	(0.0733)	(0.1039)	(0.0435)	(0.0622)
output_gap	0.0449 *	0.0592	0.0233	-0.0108
	(0.0218)	(0.0474)	(0.0165)	(0.0182)
shadowrate_lag		0.9630 ***		0.9581 ***
		(0.0287)		(0.0190)
realised_inflation_gap			0.0770 *	0.2532 ***
			(0.0378)	(0.0347)
N	95	95	95	95
R2	0.9544	0.9712	0.9611	0.9828

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

```r
# Estimate Bai-Perron test & output results
BP_test = breakpoints(break_formula, data = data)
BP_test_res = summary(BP_test)

# Optimal values (modular!)
bic_values <- BP_test_res$RSS[2, ]
optimal_m <- as.numeric(names(bic_values)[which.min(bic_values)])

# Make a table out of results (also modular!)
if (optimal_m == 0) {
  bp_df <- data.frame(
    `Detected Break Dates` = "No structural breaks detected",
    check.names = FALSE)
  } else {
```

Table 7: Chow tests for suspected structural breaks

Event	Date	p-value
ZLB Start	2012 Q3	0.0020
COVID-19 Start	2020 Q1	0.0365

Table 8: Bai-Perron test for multiple breaks

Detected Break Dates
2006 Q2
2019 Q1

```r
  break_obs <- na.omit(BP_test_res$breakpoints[optimal_m, ])
  detected_dates <- data$quarter[break_obs]
  bp_df <- data.frame(
    `Detected Break Dates` = as.character(detected_dates),
    check.names = FALSE) }

kable(bp_df, format = format, booktabs=TRUE,
  caption = "Bai-Perron test for multiple breaks") %>%
  kable_styling(latex_options = "scale_down",
      position = "center")
```

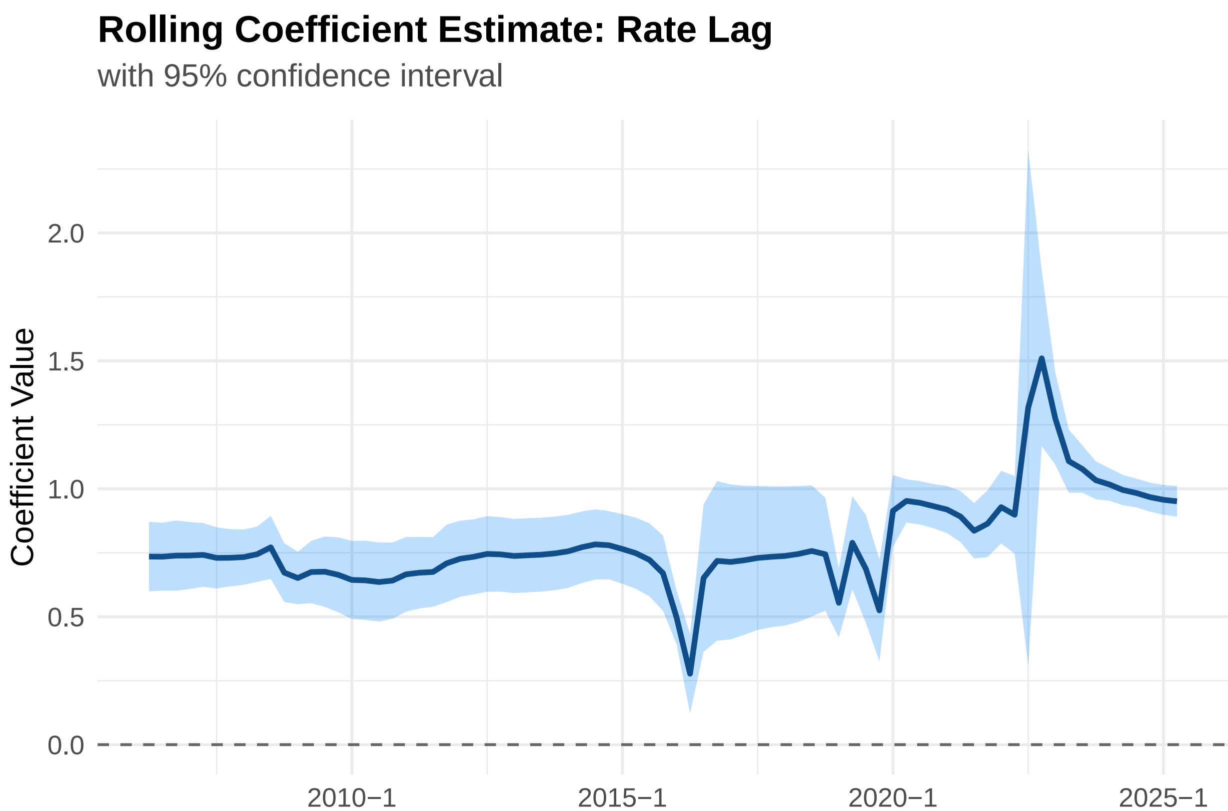
```r
# Note: breaks_obs shows in which row the BP breaks are
```

Rolling Estimation (for structural breaks)

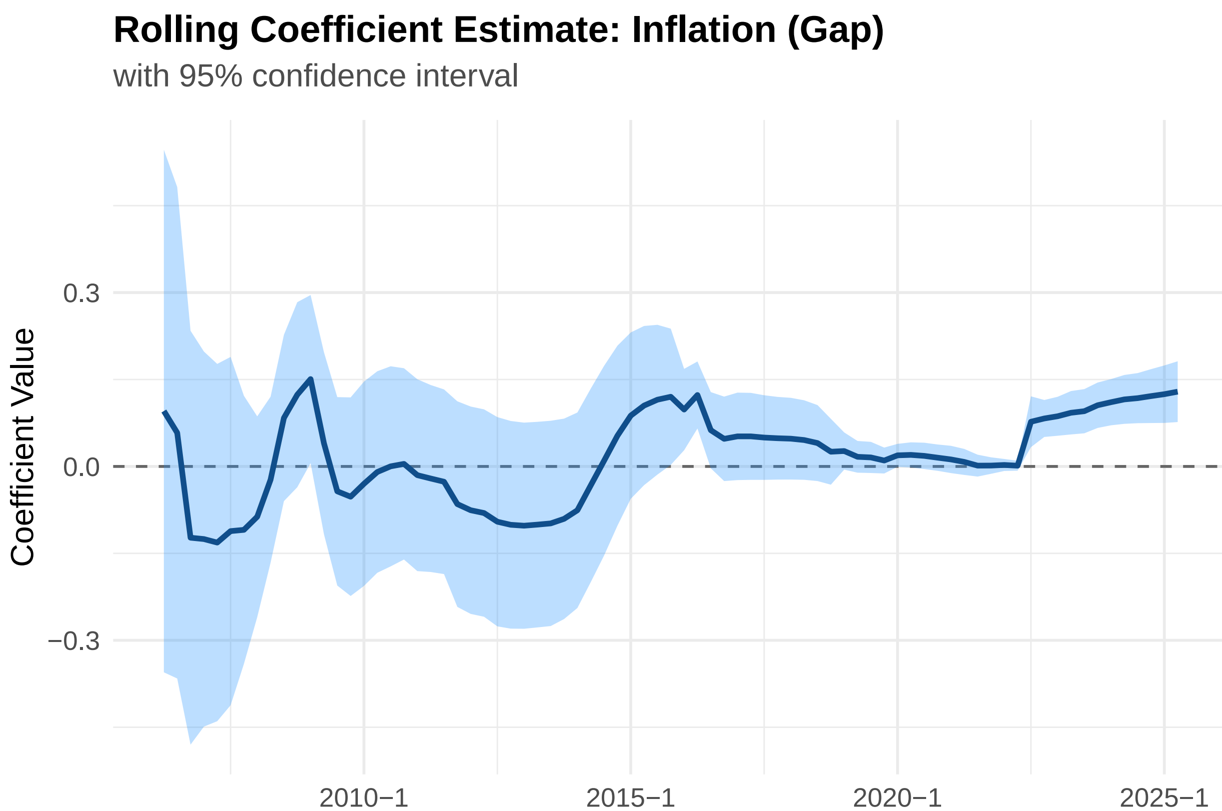
```r
# This function automatically plots the output from the rolling estimation loop
plot_rolling_coefs <- function(data, var_name, var_name_title=var_name) {

  # 1. Dynamically create the column names for CIs
  lower_col <- paste0(var_name, "_lower")
  upper_col <- paste0(var_name, "_upper")

  # 2. Create the plot, using the .data[[]] pronoun to find the
  # columns based on the variable names (modularity)

  plot <- ggplot(data, aes(x = date)) +

    # Add a dashed line at y=0 for reference
    geom_hline(yintercept = 0, linetype = "dashed", color = "grey40", linewidth = 0.5) +

    # Add the 95% confidence interval ribbon
    geom_ribbon(aes(ymin = .data[[lower_col]], ymax = .data[[upper_col]]),
                fill = "dodgerblue", alpha = 0.3) +

    # Add the coefficient estimate line
    geom_line(aes(y = .data[[var_name]]),
              color = "dodgerblue4", linewidth = 1) +

    # Aesthetic
    labs(title = paste("Rolling Coefficient Estimate:", var_name_title),
        subtitle = "with 95% confidence interval",
      x = "",
      y = "Coefficient Value") +
```

```
    theme_minimal() +
    theme(plot.title = element_text(face = "bold", size = 14, margin = margin(b=5)),
          plot.subtitle = element_text(size = 12, color = "grey30", margin = margin(b=10)),
          axis.title = element_text(size = 12),
          axis.text = element_text(size = 10))
      return(plot) }
```

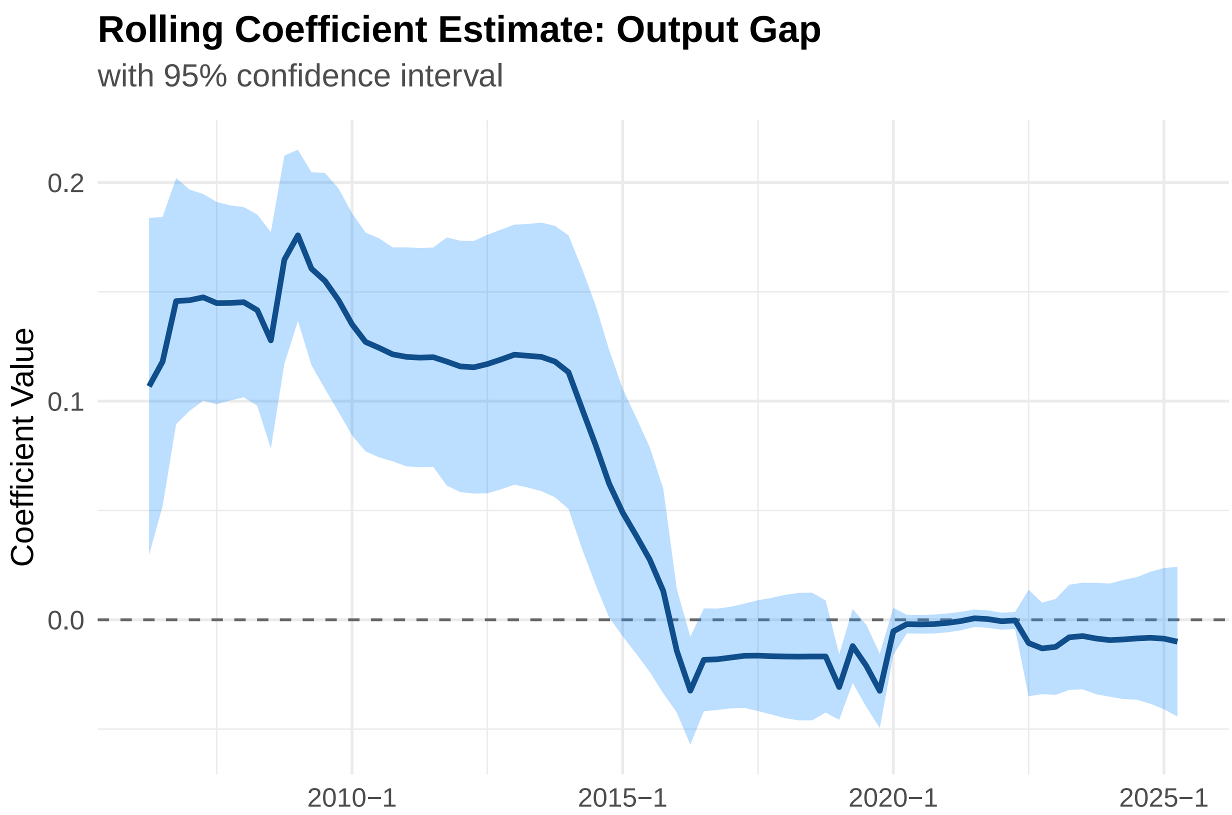
```
# Set rolling window (in quarters) & Looping Parameter
W = 30
L = nrow(data) - W + 1
formula = rate ~ rate_lag + inflation_gap + output_gap

# Preparation of result data, dates, var names, and confidence intervals
var_names <- attr(terms(formula), "term.labels")
window_end_dates <- data$quarter[W:nrow(data)] # First window [1:W] ends at data$date[W]
TR_roll <- data.frame(date = window_end_dates)
TR_roll[var_names] <- NA
lower_col_names <- paste0(var_names, "_lower")
upper_col_names <- paste0(var_names, "_upper")

# Looped estimation of TR, outputs coefficients and CIs
for (l in 1:L) {
  # 1. Define splits (with rolling scheme)
  rolled_data <- data[l:(W + l - 1), ]

  # 2. Estimate TR on split data, using whatever formula is desired
  TR_estimate <- lm(formula, data = rolled_data)

  # 3. Pull out coefficients & compute confidence intervals
  all_coefs <- coef(TR_estimate)
  all_cis <- confint(TR_estimate)

  TR_roll[l, var_names] <- all_coefs[var_names]
  TR_roll[l, lower_col_names] <- all_cis[var_names, 1]
  TR_roll[l, upper_col_names] <- all_cis[var_names, 2]    }

# Plotting (note: this part is not modular, obviously)
plot_rolling_coefs(TR_roll, "rate_lag", var_name_title="Rate Lag")
```




```r
plot_rolling_coefs(TR_roll, "inflation_gap", var_name_title="Inflation (Gap)")
```




```
plot_rolling_coefs(TR_roll, "output_gap", var_name_title="Output Gap")
```


Rolling Coefficient Estimate: Output Gap
with 95% confidence interval
Coefficient Value
0.2
0.1
0.0
2010−1
2015−1
2020−1
2025−1

Forecasting Model Evaluation

Helpers

```r
#-------------------------------------------------------------------
# Helper function for adding p-value significance stars
#-------------------------------------------------------------------

  format_p_values_with_stars <- function(p) {
    stars <- case_when(
      p < 0.01 ~ "***",
      p < 0.05 ~ "**",
      p < 0.10 ~ "*",
      TRUE     ~ "")
    paste0(format(round(p, 4), nsmall = 3), " ", stars)}

#-------------------------------------------------------------------
# HELPER FUNCTION FOR MINCER-ZARNOWITZ REPORTING
#-------------------------------------------------------------------
# This function runs the Mincer-Zarnowitz regression (Actuals ~ Forecasts)
# for each horizon h and tests the joint null hypothesis H0: (alpha, beta) = (0, 1).
#-------------------------------------------------------------------

generate_mincer_zarnowitz_report <- function(F_model,
                                             Actual_values,
                                             H,
                                             model_caption,
                                             format = "html") {
  # Pre-allocate storage for results
  mz_results <- data.frame(
    Horizon = 1:H,
    Alpha = numeric(H),
    Beta = numeric(H),
    P_Value_Joint_Test = numeric(H))

  for (h in 1:H) {
    # 1. Create a clean data frame for this horizon
    #    This pairs the forecasts and actual values and removes any NAs,
    #    ensuring they remain perfectly aligned.
    df_h <- data.frame(
      actuals = Actual_values[[h]],
      forecasts = F_model[[h]] ) %>%
      na.omit()

    # Check if we have enough data to run the regression (at least 2 obs)
    if (nrow(df_h) > 2) {
      # 2. Run MZ regression
      mz_reg <- lm(actuals ~ forecasts, data = df_h)

      # 3. Get coefficients
      coeffs <- summary(mz_reg)$coefficients
      mz_results$Alpha[h] <- coeffs[1, 1]
      mz_results$Beta[h]  <- coeffs[2, 1]
```

```
      # Using NW errors as seen in class, with lag selection h-1
      v_matrix <-
        if (h == 1) {
          # h=1: No autocorrelation, use standard "White" (HC) errors
          sandwich::vcovHC(mz_reg, type = "HC3")
        } else {
          # h>1: Use Newey-West, manually setting lag = h-1
          sandwich::NeweyWest(mz_reg, lag = h - 1)}

      # 4. Test Joint Hypothesis H0: Alpha = 0 AND Beta = 1 and store pvalues
      test_joint <- linearHypothesis(mz_reg,
                         c("(Intercept) = 0", "forecasts = 1"), vcov. = v_matrix)
      mz_results$P_Value_Joint_Test[h] <- test_joint$"Pr(>F)"[2]

    } else {
      # Not enough data to run regression for this horizon
      mz_results$Alpha[h] <- NA_real_
      mz_results$Beta[h]  <- NA_real_
      mz_results$P_Value_Joint_Test[h] <- NA_real_ } }

  # Format the results for the table
  mz_results <- mz_results %>%
    mutate(Alpha = round(Alpha, 4),
           Beta = round(Beta, 4),
           P_Value_Joint_Test = format_p_values_with_stars(P_Value_Joint_Test))

  # Create the table
  table_output <- kable(
    mz_results,
    format = format,
    booktabs = TRUE,
    caption = model_caption,
    digits = 4,
    col.names = c("h", "Alpha", "Beta", "pv(Joint)"),
    escape = FALSE ) %>%
    kable_styling(
      latex_options = c("striped", "scale_down"),
      position = "center") %>%
    column_spec(1, bold = TRUE, border_right = TRUE) %>%
    column_spec(4, monospace = TRUE) %>%
    footnote(
      general = "pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p
      symbol = c(
        "Signif. codes:  '***' 0.01,  '**' 0.05,  '*' 0.1"),
      general_title = "Note:",
      symbol_title = "",
      footnote_as_chunk = TRUE,
      threeparttable = TRUE)
  return(table_output) }



#-------------------------------------------------------------------
```

```r
# HELPER FUNCTION FOR REPORTING DM tests
#----------------------------------------------------------------------

# This function creates the DM tests and kable output
generate_report_table <- function(FE_TR_model, FE_BM_model, H, model_caption, format = "html") { MSFE_TR
  MSFE_BM = numeric(H)

  # Calculate MSFEs
  for (h in 1:H) {
    # Ensure errors are cleaned of NAs
    fe1 <- na.omit(FE_TR_model[[h]])
    fe2 <- na.omit(FE_BM_model[[h]])

    MSFE_TR[h] = mean((fe1)^2)
    MSFE_BM[h] = mean((fe2)^2)}

  # Run DM Tests
  DMpvalues = matrix(, nrow = H, ncol = 3)
  colnames(DMpvalues) <- c("DM_Two_Sided", "DM_Greater", "DM_Lesser")
  for (h in 1:H){
    # Note: dm.test needs the *full* (un-omitted) error vectors
    # to align them properly, hence using the original list inputs
    x1 = dm.test(e1 = FE_BM_model[[h]], e2 = FE_TR_model[[h]], h = h)
    x2 = dm.test(e1 = FE_BM_model[[h]], e2 = FE_TR_model[[h]], h = h, alternative = "greater")
    x3 = dm.test(e1 = FE_BM_model[[h]], e2 = FE_TR_model[[h]], h = h, alternative = "less")
    DMpvalues[h, 1] = round(x1$p.value, digits = 4)
    DMpvalues[h, 2] = round(x2$p.value, digits = 4)
    DMpvalues[h, 3] = round(x3$p.value, digits = 4)}

  # Create final table data
  forecast_comparison <- data.frame(
    Horizon = 1:H,
    MSFE_TR = MSFE_TR,
    MSFE_BM = MSFE_BM) %>%
    mutate(Ratio_TR_vs_BM = MSFE_TR / MSFE_BM)

  forecast_comparison <- bind_cols(forecast_comparison, as.data.frame(DMpvalues))

  final_data_formatted <- forecast_comparison %>%
    mutate(across(starts_with("DM_"), format_p_values_with_stars))

  # Create the kable table
  table_output <- kable(
    final_data_formatted,
    format = format,
    booktabs = TRUE,
    caption = model_caption,
    digits = 4,
    col.names = c("h", "MSFE TR", "MSFE BM", "Ratio", "DM Two-Sided", "DM Greater", "DM Lesser"),
    escape = FALSE) %>%
    kable_styling(
      latex_options = c("striped", "scale_down"),
      position = "center") %>%
```

```r
    column_spec(1, bold = TRUE, border_right = TRUE) %>%
    column_spec(5:7, monospace = TRUE) %>%
    footnote(
      general = "TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark
      symbol = c(
        "'DM Greater' tests if the TR model is significantly more accurate than the BM model.",
        "'DM Lesser' tests if the TR model is significantly less accurate than the BM model."),
      general_title = "Note:",
      symbol_title = "DM Test Alternative Hypotheses (H_A):",
      footnote_as_chunk = TRUE,
      threeparttable = TRUE)

  return(table_output)}
```

Estimation

```r
#parameters
R = 85 # Chow: Structural breaks at R=55 and R=85
cat("Evaluation sample starts after", as.character(data$quarter[R]))
```

Evaluation sample starts after 2020 Q1

```r
P = nrow(data) - R #but will effectively be: P = T-h-R
H = 10 #number of different horizons (takes 10 to go until 2027 Q4)

#note: we are doing a recursive estimation scheme for out-of-sample tests
#note: we are doing direct forecasts

#-------------------------------------------------------------------
# 1. DEFINE THE TAYLOR RULE (TR) MODEL FORMULAS
#-------------------------------------------------------------------

# TR specifications (using either current inflation or inflation expectations
#                     according to configuration, same with HP vs Hamilton)
formula_1 <- rate ~ inflation_gap + output_gap
formula_2 <- shadowrate ~ inflation_gap + output_gap
formula_3 <- rate ~ rate_lag + inflation_gap + output_gap
formula_4 <- shadowrate ~ shadowrate_lag + inflation_gap + output_gap

#-------------------------------------------------------------------
# 2. PRE-ALLOCATE STORAGE FOR ALL RESULTS
#-------------------------------------------------------------------

# We need 4 lists for the TR models, 1 list for the shared benchmark
init_storage_list <- function(H, P) {
  storage <- vector("list", length = H)
  for (h in 1:H) {
    storage[[h]] <- rep(NA_real_, P)}
  return(storage)}

# Storage for realised values
```

```r
Actuals <- init_storage_list(H, P)

# Storage for Forecasts
F_TR_1 <- init_storage_list(H, P) # Model 1: shadowrate, no lag
F_TR_2 <- init_storage_list(H, P) # Model 2: rate, no lag
F_TR_3 <- init_storage_list(H, P) # Model 3: shadowrate, with lag
F_TR_4 <- init_storage_list(H, P) # Model 4: rate, with lag
F_BM   <- init_storage_list(H, P) # Benchmark: ARIMA

# Storage for Forecast Errors
FE_TR_1 <- init_storage_list(H, P) # Model 1: shadowrate, no lag
FE_TR_2 <- init_storage_list(H, P) # Model 2: rate, no lag
FE_TR_3 <- init_storage_list(H, P) # Model 3: shadowrate, with lag
FE_TR_4 <- init_storage_list(H, P) # Model 4: rate, with lag
FE_BM   <- init_storage_list(H, P) # Benchmark: ARIMA


#-----------------------------------------------------------------
# 3. SETUP & RUN THE PARALLEL BACKTESTING LOOP
#-----------------------------------------------------------------
num_cores <- detectCores() - 1
cl <- makeCluster(num_cores)
registerDoParallel(cl)

# .export sends read-only objects to each core
# .packages loads libraries on each core
worker_results <- foreach(
  p = P:1,
  .packages = c("forecast", "stats", "dplyr"),
  .export = c("data", "H", "formula_1", "formula_2", "formula_3", "formula_4")
) %dopar% {

  # 1. Define splits (with rolling scheme)
  training <- data[(1 + nrow(data) - R - p):(nrow(data) - p), ]
  testing <- data[(nrow(data) - (p - 1)):nrow(data), ]

  # --- 2. Fit common models only once ---
  # note: d=1 for interest and inflation as non-stationary
  inflation_arma <- my.auto.arima(training$inflation_gap, max.p=4, max.q=4, d=1)
  outputgap_arma <- my.auto.arima(training$output_gap, max.p=4, max.q=4, d=0)
  interest_arma <- my.auto.arima(training$rate, max.p=4, max.q=4, d=1) # Benchmark

  # --- 3. Get common forecasts only once (all H horizons) ---
  inflation_forecasts <- my.forecast(inflation_arma, h = H)
  outputgap_forecasts <- my.forecast(outputgap_arma, h = H)
  BMpredicted_rates <- my.forecast(interest_arma, h = H)

  # --- 4. Fit the 4 TR models ---
  TR_model_1 <- lm(formula_1, data = training)
  TR_model_2 <- lm(formula_2, data = training)
  TR_model_3 <- lm(formula_3, data = training)
  TR_model_4 <- lm(formula_4, data = training)
```

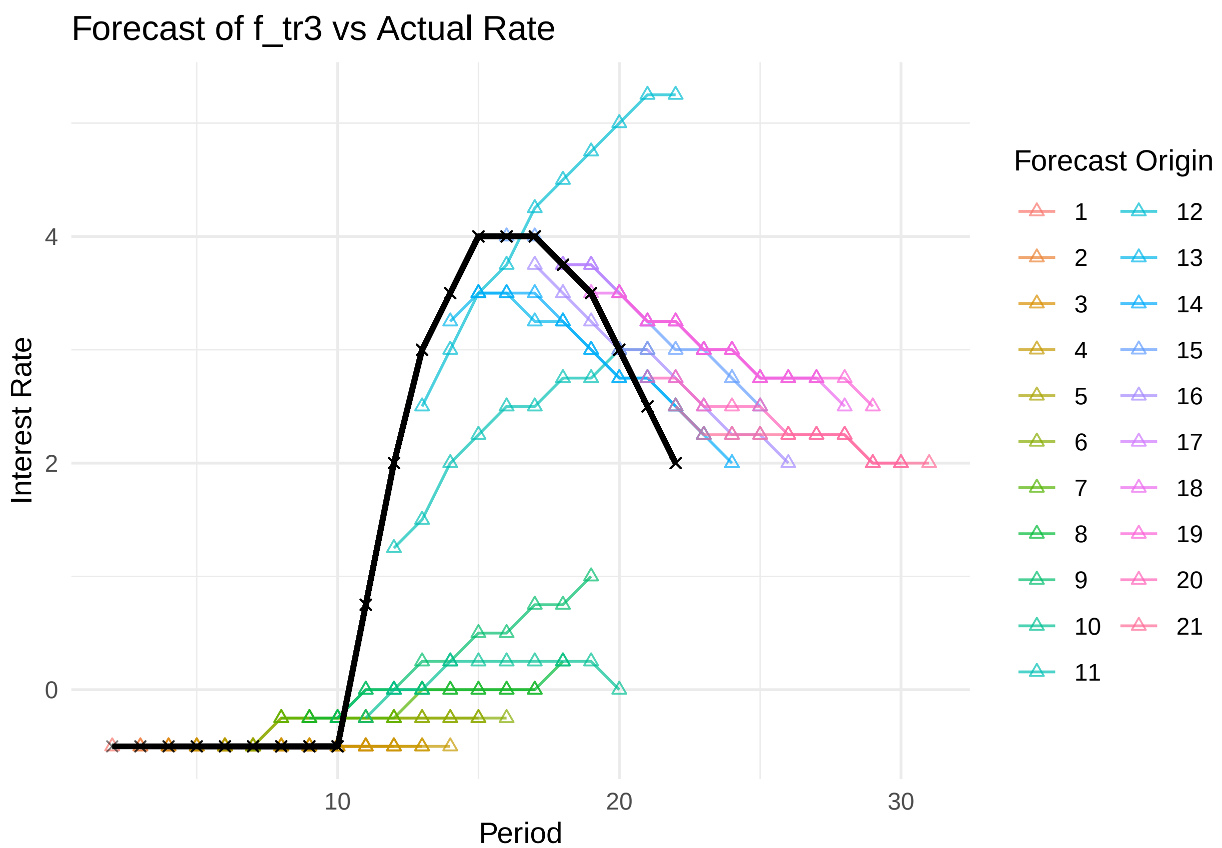
```r
# --- 5. Build forecast input data & get forecasts for non-lagged models ---
# These are direct forecasts
new_data_base <- data.frame(
  inflation_gap = inflation_forecasts,
  output_gap = outputgap_forecasts)

TR_preds_1 <- round(pmax(predict(TR_model_1, new_data_base), min(data$rate)) / 0.25) * 0.25
TR_preds_2 <- round(pmax(predict(TR_model_2, new_data_base), min(data$rate)) / 0.25) * 0.25
BM_preds <- round(pmax(BMpredicted_rates, min(data$rate)) / 0.25) * 0.25

# --- 6. Get forecasts for lagged models via iteration ---
# We must loop 1 step at a time, feeding forecasts back in.

# a) Pre-allocate storage for H forecasts
TR_preds_3 <- numeric(H)
TR_preds_4 <- numeric(H)

# b) Get the last known lag from the training set (lag for h=1 forecast)
current_rate_lag  <- last(training$rate)
current_shadowrate_lag <- last(training$shadowrate)

# Loop for iterative forecasting
for (h in 1:H) {
  # --- Prepare dataset for predictions ---
  new_data_3_h <- data.frame(
    inflation_gap = inflation_forecasts[h],
    output_gap = outputgap_forecasts[h],
    rate_lag = current_rate_lag)
  new_data_4_h <- data.frame(
    inflation_gap = inflation_forecasts[h],
    output_gap = outputgap_forecasts[h],
    shadowrate_lag = current_shadowrate_lag )

  # Get the forecast values (keep for lag, and then round for actual prediction)
  pred_3_h <- predict(TR_model_3, new_data_3_h)
  TR_preds_3[h] <- round(pmax(pred_3_h, min(data$rate)) / 0.25) * 0.25
  pred_4_h <- predict(TR_model_4, new_data_4_h)
  TR_preds_4[h] <- round(pmax(pred_4_h, min(data$rate)) / 0.25) * 0.25

  # Update lag for h+1
  current_shadowrate_lag <- pred_4_h
  current_rate_lag <- pred_3_h }

# --- 7. Get actual values in evaluation sample  ---
actual_rates <- testing$rate[1:H]

# --- 8. Return all FORECASTS and ACTUALS from the worker ---
list(f_tr1 = TR_preds_1,
     f_tr2 = TR_preds_2,
     f_tr3 = TR_preds_3,
     f_tr4 = TR_preds_4,
     f_bm  = BM_preds,
     actuals = actual_rates) }
```

```r
# --- Stop the Cluster ---
stopCluster(cl)
rm(cl)

#-----------------------------------------------------------------
# 4. UNPACK PARALLEL RESULTS INTO STORAGE LISTS
#-----------------------------------------------------------------

# 'worker_results' is a list of P lists. We need to re-organize it.
for (i in 1:P) {
  # i=1 corresponds to p=P, i=2 to p=P-1, ... i=P to p=1
  # This 'storage_index' matches the loop order
  storage_index <- i
  p_results <- worker_results[[i]]

  for (h in 1:H) {
    # Get the raw values for this h
    actual_val <- p_results$actuals[h]
    f_tr1_val  <- p_results$f_tr1[h]
    f_tr2_val  <- p_results$f_tr2[h]
    f_tr3_val  <- p_results$f_tr3[h]
    f_tr4_val  <- p_results$f_tr4[h]
    f_bm_val   <- p_results$f_bm[h]

    # Store Actuals (for MZ)
    Actuals[[h]][storage_index] <- actual_val

    # Store Forecasts (for MZ)
    F_TR_1[[h]][storage_index] <- f_tr1_val
    F_TR_2[[h]][storage_index] <- f_tr2_val
    F_TR_3[[h]][storage_index] <- f_tr3_val
    F_TR_4[[h]][storage_index] <- f_tr4_val
    F_BM[[h]][storage_index]   <- f_bm_val

    # Calculate and Store Errors (for MSFE/DM)
    FE_TR_1[[h]][storage_index] <- f_tr1_val - actual_val
    FE_TR_2[[h]][storage_index] <- f_tr2_val - actual_val
    FE_TR_3[[h]][storage_index] <- f_tr3_val - actual_val
    FE_TR_4[[h]][storage_index] <- f_tr4_val - actual_val
    FE_BM[[h]][storage_index]   <- f_bm_val  - actual_val } }

#-----------------------------------------------------------------
# 5. RENDER RESULTS MORE INTUITIVE FOR FURTHER ANALYSIS
#-----------------------------------------------------------------

# Convert the forecast lists (F_TR_x) into single dataframes
forecast_to_df <- function(forecast_list, period) {
  # Convert each element to numeric (benchmark is ts object, which is bad)
  numeric_list <- lapply(forecast_list, function(x) as.numeric(x)) #just make each list inside numeric
  df <- as.data.frame(numeric_list)
  # Add period and horizon
  df$period <- period #first list is all horizon 1 forecasts, gives this to all observations
  df$horizon <- 1:nrow(df) #counts rows and gives each the horizon corresponding to it
```

```
  df}

# Apply to all forecasting models
df_all <- do.call(rbind, lapply(seq_along(worker_results), function(i) {
  forecast_to_df(worker_results[[i]], period = i) }))
df_all[] <- lapply(df_all, function(x) as.numeric(x))
```

Spaghetti Plots

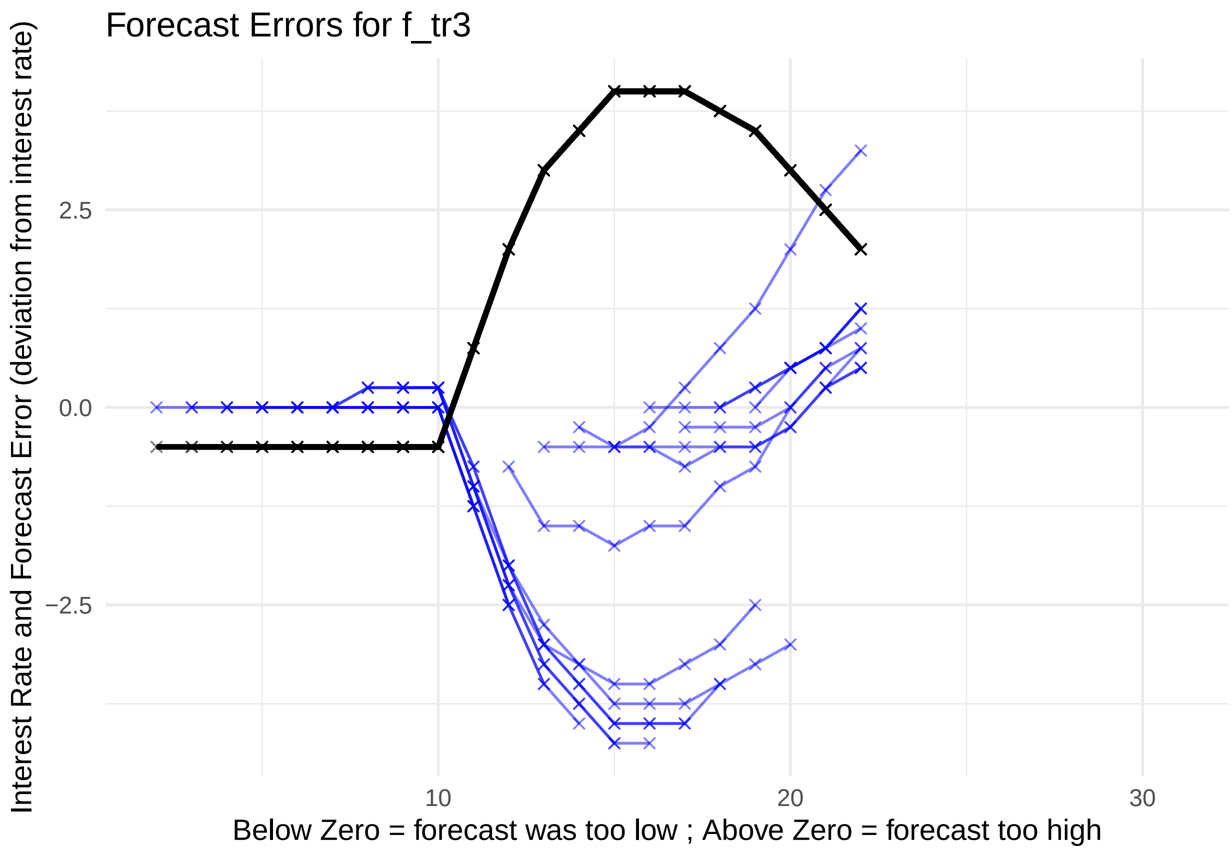
```
# Select the model to plot
model <- "f_tr3"

# period = date the forecast was made
# date_of_forecast = the future date we are predicting
df_all$date_of_forecast <- df_all$period + df_all$horizon

# Spaghetti plot with color per period
ggplot(df_all, aes(x = date_of_forecast, y = .data[[model]], group = period, color = factor(period))) +
  geom_line(alpha = 0.7) +    # forecast lines
  geom_point(shape = 2, alpha = 0.7) +

  # Actuals as black baseline
  geom_line(aes(y = actuals), color = "black", size = 1) +
  geom_point(aes(y = actuals), color = "black", shape = 4, alpha = 0.5) +

  labs(title = paste("Forecast of", model, "vs Actual Rate"),
       x = "Period",
       y = "Interest Rate",
       color = "Forecast Origin") +
  theme_minimal()
```



Plots of FE

```
df_all_3 <- df_all

# Compute forecast error
df_all_3$forecast_error <- df_all_3[[model]] - df_all_3$actuals

#compute date_of_forecast for x-axis
df_all_3$date_of_forecast <- df_all_3$period + df_all_3$horizon

ggplot(df_all_3, aes(x = date_of_forecast, group = period)) +
  # Forecast error lines
  geom_line(aes(y = forecast_error), color = "blue", alpha = 0.5) +
  geom_point(aes(y = forecast_error), color = "blue", alpha = 0.5, shape = 4) +

  # Actuals line
  geom_line(aes(y = actuals), color = "black", size = 1) +
  geom_point(aes(y = actuals), color = "black", shape = 4, alpha = 0.5) +

  labs(
    title = paste("Forecast Errors for", model),
    x = "Below Zero = forecast was too low ; Above Zero = forecast too high",
    y = "Interest Rate and Forecast Error (deviation from interest rate)"
```

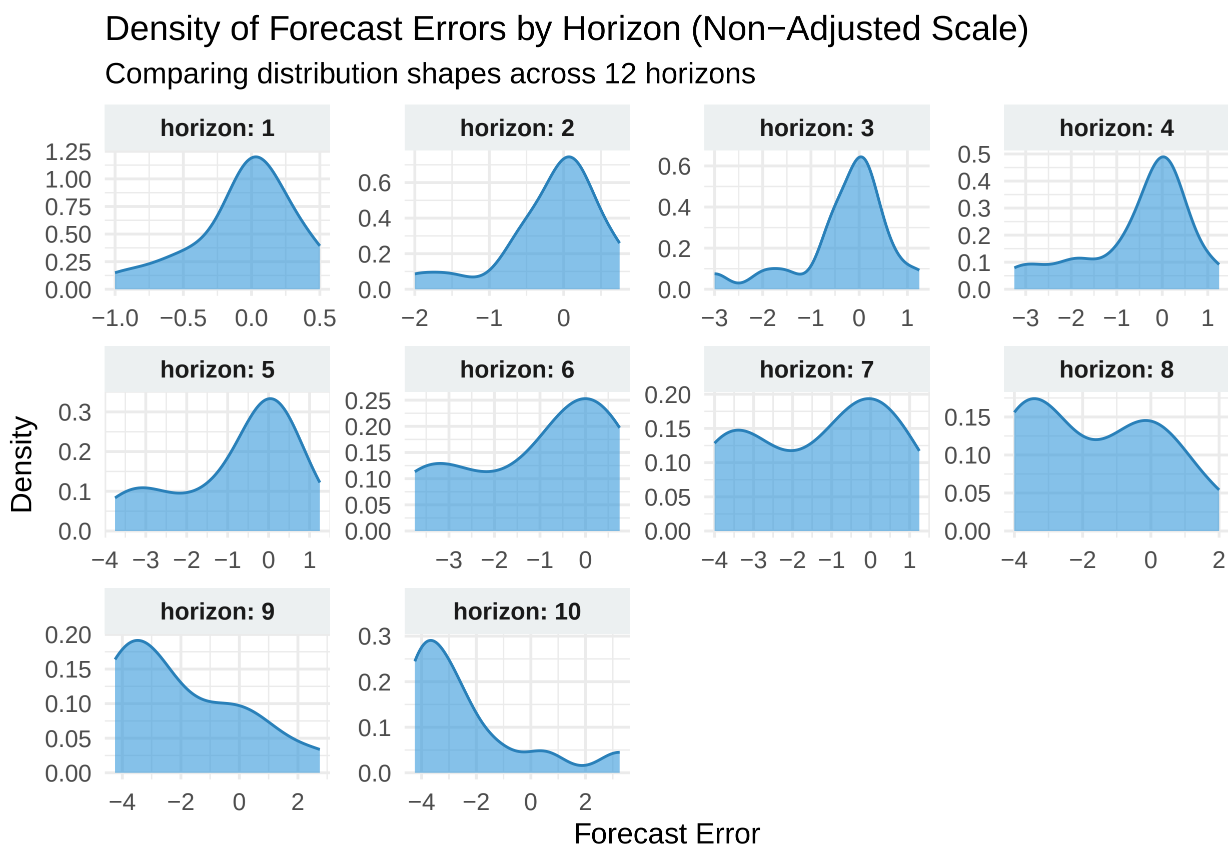
```r
  ) +
  theme_minimal()
```



Density of FE

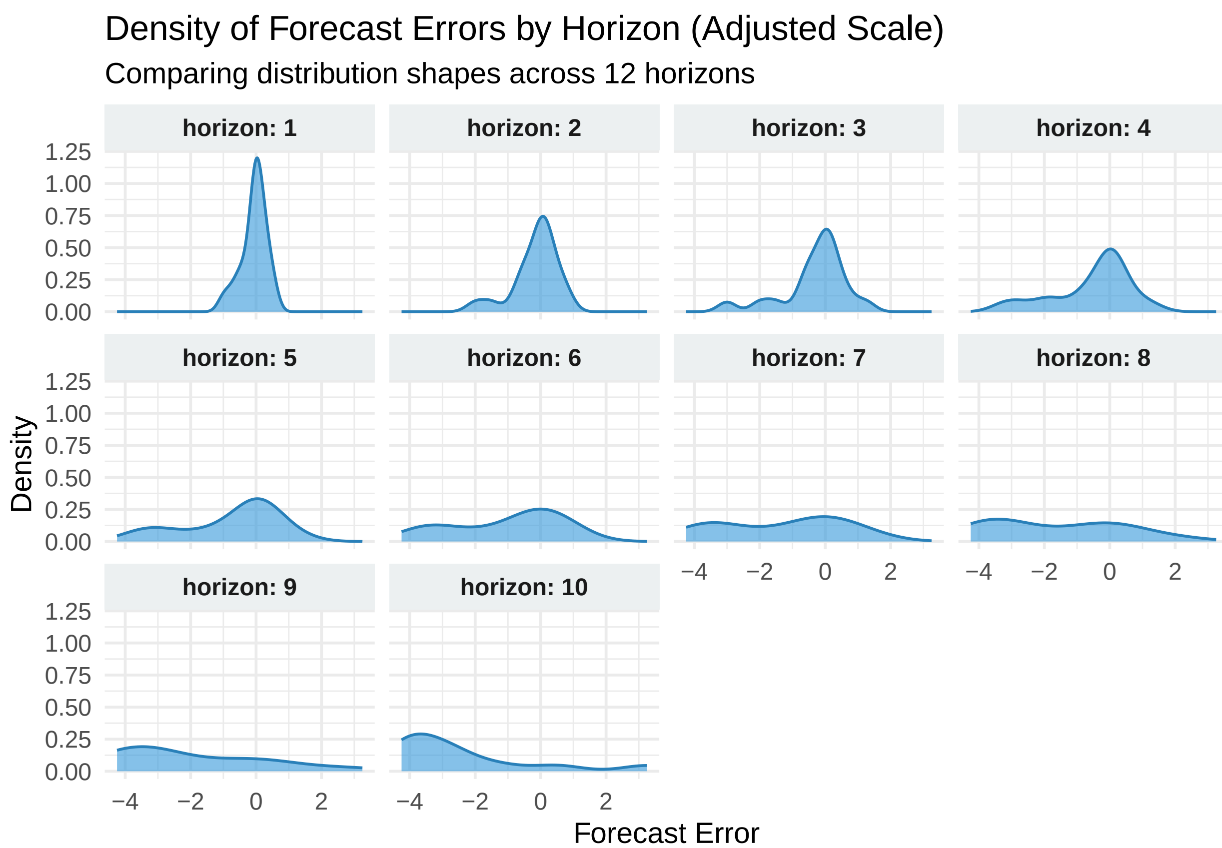
```r
# Version 1: Non-Adjusted Scales
plot_facet <- ggplot(df_all_3 %>% filter(horizon <= H), aes(x = forecast_error)) +
  geom_density(fill = "#3498db", color = "#2980b9", alpha = 0.6) +
  facet_wrap(~horizon, ncol = 4, labeller = label_both, scales = "free") +
  labs(
    title = "Density of Forecast Errors by Horizon (Non-Adjusted Scale)",
    subtitle = "Comparing distribution shapes across 12 horizons",
    x = "Forecast Error",
    y = "Density") +
  theme_minimal() +
  theme(
    strip.background = element_rect(fill = "#ecf0f1", color = NA), # Nice gray boxes for labels
    strip.text = element_text(face = "bold"))
print(plot_facet)
```

```
## Warning: Removed 45 rows containing non-finite outside the scale range
## (`stat_density()`).
```




```r
# Version 2: Adjusted scales
plot_facet <- ggplot(df_all_3 %>% filter(horizon <= H), aes(x = forecast_error)) +
  geom_density(fill = "#3498db", color = "#2980b9", alpha = 0.6) +
  facet_wrap(~horizon, ncol = 4, labeller = label_both) +
  labs(title = "Density of Forecast Errors by Horizon (Adjusted Scale)",
    subtitle = "Comparing distribution shapes across 12 horizons",
    x = "Forecast Error",
    y = "Density") +
  theme_minimal() +
  theme(strip.background = element_rect(fill = "#ecf0f1", color = NA),
    strip.text = element_text(face = "bold"))
print(plot_facet)
```

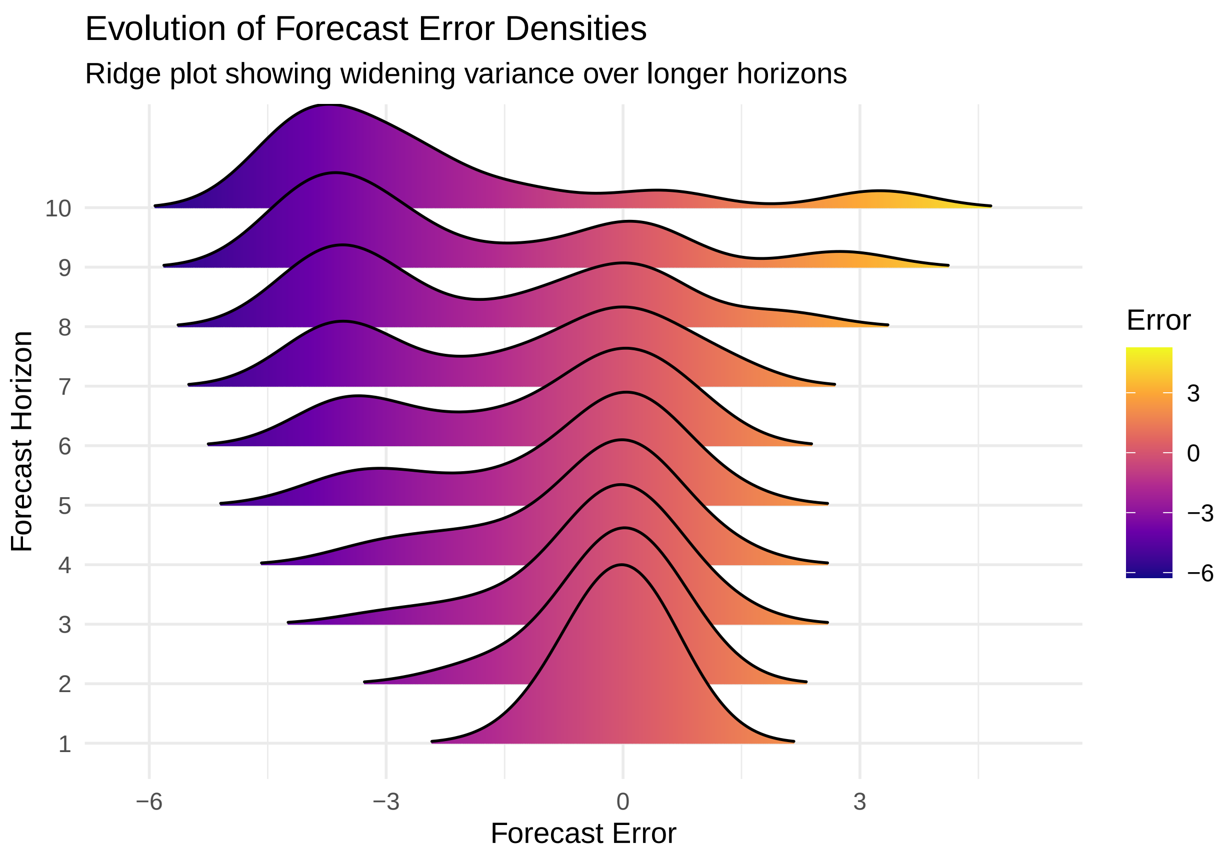
```
## Warning: Removed 45 rows containing non-finite outside the scale range
## (`stat_density()`).
```




```r
# Version 3: "Ridges"
plot_ridge <- ggplot(df_all_3 %>% filter(horizon <= H),
                       aes(x = forecast_error, y = as.factor(horizon), fill = stat(x))) +
    geom_density_ridges_gradient(scale = 3, rel_min_height = 0.01) +
    scale_fill_viridis_c(name = "Error", option = "C") +
    labs(title = "Evolution of Forecast Error Densities",
      subtitle = "Ridge plot showing widening variance over longer horizons",
      x = "Forecast Error",
      y = "Forecast Horizon") +
    theme_minimal()

print(plot_ridge)
```

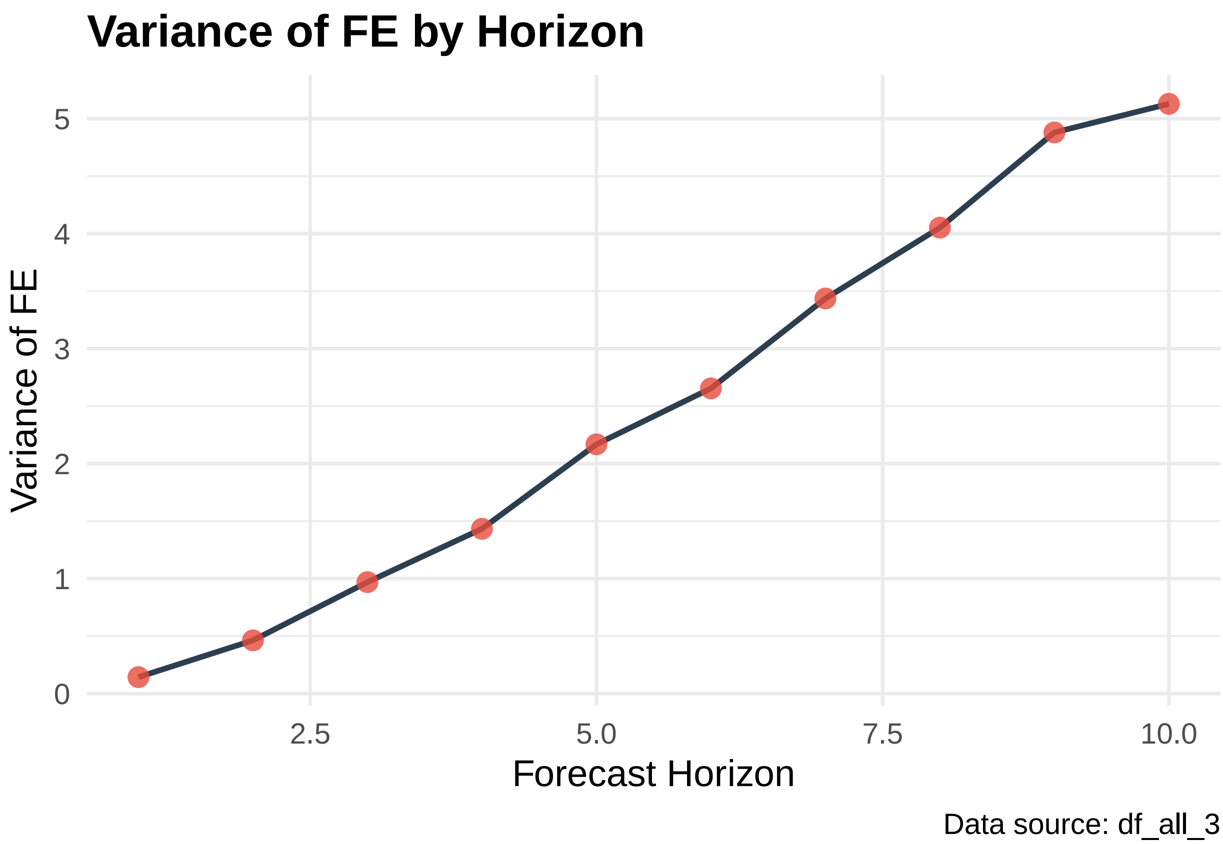
```
## Warning: `stat(x)` was deprecated in ggplot2 3.4.0.
## i Please use `after_stat(x)` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```

```
## Picking joint bandwidth of 0.664
```



Variance of FE

```
# This gives us the mean forecast error for the h step ahead forecast
var_by_horizon <- df_all_3 %>%
  group_by(horizon) %>%
  summarize(
    mean_fe = mean(forecast_error, na.rm=T),
    var_fe = sd(forecast_error, na.rm=T)^2, n = n() )

ggplot(var_by_horizon, aes(x = horizon, y = var_fe)) +
  geom_line(color = "#2c3e50", size = 1) +
  geom_point(color = "#e74c3c", size = 3, alpha = 0.8) +
  theme_minimal(base_size = 14) +
  scale_y_continuous(labels = scales::comma) +
  labs(title = "Variance of FE by Horizon",
    x = "Forecast Horizon",
    y = "Variance of FE",
    caption = "Data source: df_all_3") +
  theme(plot.title = element_text(face = "bold"),
        plot.subtitle = element_text(color = "gray50"),
        panel.grid.minor.x = element_blank() )
```



Absolute Performance: Efficiency & Bias

```
# Call MZ-test helper function 4 times.

# MZ Report 1: Actual Rate, No Lag
mz_report_1 <- generate_mincer_zarnowitz_report(
  F_model = F_TR_1,
  Actual_values = Actuals,
  H = H,
  model_caption = "Mincer-Zarnowitz Test: Actual Rate, No Lag",
  format = format)

# MZ Report 2: Shadow Rate, No Lag (
#  Note: This report is wrapped in trycatch as it sometimes fails
#         If it does fail, simply decrease R in order to have more
#          observations, thus no problem in multicolinearity.
mz_report_2 <- tryCatch({
  generate_mincer_zarnowitz_report(
    F_model = F_TR_2,
    Actual_values = Actuals,
    H = H,
    model_caption = "Mincer-Zarnowitz Test: Shadow Rate, No Lag",
    format = format)}, error = function(e) {
  message("  Error generating MZ Report (Shadow Rate, No Lag): ", e$message)
```

```r
  message(">> Skipping this report and continuing...")
  return(NULL)})

# MZ Report 3: Actual Rate, with Lag
mz_report_3 <- generate_mincer_zarnowitz_report(
  F_model = F_TR_3,
  Actual_values = Actuals,
  H = H,
  model_caption = "Mincer-Zarnowitz Test: Actual Rate, with Lag",
  format = format)

# MZ Report 4: Shadow Rate, with Lag
mz_report_4 <- generate_mincer_zarnowitz_report(
  F_model = F_TR_4,
  Actual_values = Actuals,
  H = H,
  model_caption = "Mincer-Zarnowitz Test: Shadow Rate, with Lag",
  format = format)

# MZ Report 5: Benchmark
mz_report_BM <- generate_mincer_zarnowitz_report(
  F_model = F_BM,
  Actual_values = Actuals,
  H = H,
  model_caption = "Mincer-Zarnowitz Test: Benchmark ARIMA",
  format = format)

list(
  mz_report_1,
  mz_report_2,
  mz_report_3,
  mz_report_4,
  mz_report_BM)
```

[[1]]

[[2]]

[[3]]

[[4]]

[[5]]

Relative Performance (against benchmark)

```r
# Call DM-test helper function 4 times.

# Report 1: Actual Rate, No Lag
report_1 <- generate_report_table(
  FE_TR_model = FE_TR_1,
  FE_BM_model = FE_BM,
  H = H,
```

Table 9: Mincer-Zarnowitz Test: Actual Rate, No Lag

h	Alpha	Beta	pv(Joint)
1	1.1720	0.3204	0.2184
2	1.2241	0.3856	0.7727
3	0.9667	0.8076	0.8783
4	0.7954	1.2455	0.2386
5	0.8566	1.4292	0.0103 **
6	1.0184	1.4755	0.0000 ***
7	1.2284	1.3395	0.0000 ***
8	1.6036	1.0913	0.0000 ***
9	2.2021	0.6248	0.0001 ***
10	2.9629	0.0414	0.0030 ***

Note:
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

Table 10: Mincer-Zarnowitz Test: Shadow Rate, No Lag

h	Alpha	Beta	pv(Joint)
1	1.8201	-0.3634	0.000 ***
2	1.8580	-0.2293	0.000 ***
3	1.7647	-0.0424	0.000 ***
4	1.8174	0.0116	0.000 ***
5	1.9914	-0.0130	0.000 ***
6	2.2061	-0.0425	0.000 ***
7	2.4557	-0.0697	0.000 ***
8	2.7436	-0.0941	0.000 ***
9	2.9320	-0.0671	0.000 ***
10	3.1602	-0.0452	0.000 ***

Note:
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

Table 11: Mincer-Zarnowitz Test: Actual Rate, with Lag

h	Alpha	Beta	pv(Joint)
1	0.0568	1.0019	0.7940
2	0.2189	0.9525	0.7692
3	0.4630	0.8861	0.7036
4	0.7537	0.8270	0.6691
5	1.1378	0.6906	0.5505
6	1.4947	0.5933	0.4541
7	1.9075	0.4131	0.1574
8	2.3049	0.2375	0.0148 **
9	2.7110	0.0277	0.0010 ***
10	3.0873	-0.1496	0.0003 ***

Note:
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

Table 12: Mincer-Zarnowitz Test: Shadow Rate, with Lag

h	Alpha	Beta	pv(Joint)
1	0.0533	0.9137	0.0771 *
2	0.1926	0.7985	0.1967
3	0.4409	0.6390	0.0084 ***
4	0.7725	0.4865	0.0000 ***
5	1.1435	0.3537	0.0000 ***
6	1.4715	0.2535	0.0000 ***
7	1.8170	0.1656	0.0000 ***
8	2.2158	0.0884	0.0000 ***
9	2.6636	0.0189	0.0000 ***
10	3.1629	-0.0418	0.0000 ***

Note:
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

Table 13: Mincer-Zarnowitz Test: Benchmark ARIMA

h	Alpha	Beta	pv(Joint)
1	0.1403	0.9439	0.4025
2	0.3980	0.8509	0.1991
3	0.7262	0.7193	0.2696
4	1.0868	0.5906	0.3892
5	1.4665	0.4339	0.2697
6	1.8291	0.3006	0.1350
7	2.1804	0.1495	0.3461
8	2.4846	0.0308	0.0078 ***
9	2.7628	-0.1282	0.0000 ***
10	2.9597	-0.3873	0.0000 ***

Note:
pv(Joint) is the p-value for the joint hypothesis H_0: (Alpha, Beta) = (0, 1). A high p-value means we fail to reject the null hypothesis of an unbiased, efficient forecast.
* Signif. codes: '***' 0.01, '**' 0.05, '*' 0.1

```
  model_caption = "MSFE Comparison, Trained on Actual Rate, No Lag",
  format = format)

# Report 2: Shadow Rate, No Lag
report_2 <- generate_report_table(
  FE_TR_model = FE_TR_2,
  FE_BM_model = FE_BM,
  H = H,
  model_caption = "MSFE Comparison, Trained on Shadow Rate, No Lag",
  format = format)

# Report 3: Actual Rate, with Lag
report_3 <- generate_report_table(
  FE_TR_model = FE_TR_3,
  FE_BM_model = FE_BM,
  H = H,
  model_caption = "MSFE Comparison, Trained on Actual Rate, with Lag",
  format = format)

# Report 4: Shadow Rate, with Lag
report_4 <- generate_report_table(
  FE_TR_model = FE_TR_4,
  FE_BM_model = FE_BM,
  H = H,
  model_caption = "MSFE Comparison, Trained on Shadow Rate, with Lag",
  format = format)

list(report_1, report_2, report_3, report_4)
```

[[1]]

Table 14: MSFE Comparison, Trained on Actual Rate, No Lag

h	MSFE TR	MSFE BM	Ratio	DM Two-Sided	DM Greater	DM Lesser
1	4.0655	0.1637	24.8364	0.0001 ***	0.9999	0.0001 ***
2	4.0938	0.6781	6.0369	0.0355 **	0.9823	0.0177 **
3	3.5461	1.6776	2.1137	0.4047	0.7977	0.2023
4	3.1944	2.8924	1.1044	0.9159	0.5421	0.4579
5	3.1434	4.4596	0.7049	0.5990	0.2995	0.7005
6	3.3438	5.9961	0.5577	0.1229	0.0614 *	0.9386
7	3.6083	7.7500	0.4656	0.0120 **	0.0060 ***	0.9940
8	4.1964	9.0357	0.4644	0.0020 ***	0.0010 ***	0.9990
9	5.0865	10.7260	0.4742	0.0002 ***	0.0001 ***	0.9999
10	6.1823	12.0156	0.5145	0.0000 ***	0.0000 ***	1.0000

Note: TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark of the interest rate using an ARIMA model. Ratio < 1 indicates that the TR model has lower MSFE. *DM Test Alternative Hypotheses (H_A):*
[*] 'DM Greater' tests if the TR model is significantly more accurate than the BM model.
[†] 'DM Lesser' tests if the TR model is significantly less accurate than the BM model.

[[2]]

[[3]]

[[4]]

Table 15: MSFE Comparison, Trained on Shadow Rate, No Lag

h	MSFE TR	MSFE BM	Ratio	DM Two-Sided	DM Greater	DM Lesser
1	8.8750	0.1637	54.2182	0.0001 ***	1.0000	0.0000 ***
2	9.6375	0.6781	14.2120	0.0024 ***	0.9988	0.0012 ***
3	9.6678	1.6776	5.7627	0.0371 **	0.9815	0.0185 **
4	12.3715	2.8924	4.2773	0.2245	0.8878	0.1122
5	17.0404	4.4596	3.8211	0.3261	0.8369	0.1631
6	23.9844	5.9961	4.0000	0.3694	0.8153	0.1847
7	31.9167	7.7500	4.1183	0.3853	0.8073	0.1927
8	43.1384	9.0357	4.7742	0.3166	0.8417	0.1583
9	51.8942	10.7260	4.8382	0.2251	0.8875	0.1125
10	63.5208	12.0156	5.2865	0.0521 *	0.9739	0.0261 **

Note: TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark of the interest rate using an ARIMA model. Ratio < 1 indicates that the TR model has lower MSFE. *DM Test Alternative Hypotheses (H_A):*
* 'DM Greater' tests if the TR model is significantly more accurate than the BM model.
† 'DM Lesser' tests if the TR model is significantly less accurate than the BM model.

Table 16: MSFE Comparison, Trained on Actual Rate, with Lag

h	MSFE TR	MSFE BM	Ratio	DM Two-Sided	DM Greater	DM Lesser
1	0.1399	0.1637	0.8545	0.5402	0.2701	0.7299
2	0.4625	0.6781	0.6820	0.1832	0.0916 *	0.9084
3	1.0099	1.6776	0.6020	0.1328	0.0664 *	0.9336
4	1.6319	2.8924	0.5642	0.1117	0.0559 *	0.9441
5	2.6250	4.4596	0.5886	0.0449 **	0.0224 **	0.9776
6	3.6172	5.9961	0.6033	0.0143 **	0.0072 ***	0.9928
7	5.0292	7.7500	0.6489	0.0143 **	0.0072 ***	0.9928
8	6.5804	9.0357	0.7283	0.0000 ***	0.0000 ***	1.0000
9	8.5817	10.7260	0.8001	0.0220 **	0.0110 **	0.9890
10	10.5417	12.0156	0.8773	0.1656	0.0828 *	0.9172

Note: TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark of the interest rate using an ARIMA model. Ratio < 1 indicates that the TR model has lower MSFE.
DM Test Alternative Hypotheses (H_A): * 'DM Greater' tests if the TR model is significantly more accurate than the BM model.
† 'DM Lesser' tests if the TR model is significantly less accurate than the BM model.

Table 17: MSFE Comparison, Trained on Shadow Rate, with Lag

h	MSFE TR	MSFE BM	Ratio	DM Two-Sided	DM Greater	DM Lesser
1	0.1637	0.1637	1.0000	1.0000	0.5000	0.5000
2	0.5781	0.6781	0.8525	0.7065	0.3533	0.6467
3	1.6678	1.6776	0.9941	0.9464	0.4732	0.5268
4	3.7257	2.8924	1.2881	0.3371	0.8315	0.1685
5	7.0993	4.4596	1.5919	0.1935	0.9033	0.0967 *
6	11.6367	5.9961	1.9407	0.0913 *	0.9544	0.0456 **
7	18.5458	7.7500	2.3930	0.1470	0.9265	0.0735 *
8	27.9643	9.0357	3.0949	0.1615	0.9193	0.0807 *
9	40.1971	10.7260	3.7476	0.1525	0.9238	0.0762 *
10	55.4635	12.0156	4.6160	0.1044	0.9478	0.0522 *

Note: TR refers to the forecast made with an estimated Taylor Rule. BM refers to a benchmark of the interest rate using an ARIMA model. Ratio <1 indicates that the TR model has lower MSFE.

DM Test Alternative Hypotheses (H_A): * 'DM Greater' tests if the TR model is significantly more accurate than the BM model.

† 'DM Lesser' tests if the TR model is significantly less accurate than the BM model.

Actual Forecast Model

Helpers

```r
#------------------------------------------------------------------
# Helper function for displaying our final forecast results (table)
#------------------------------------------------------------------

display_forecasts <- function(forecast_list,
                              caption = "Interest Rate Forecasts",
                              format = "html") {

  # Determine the number of horizons and corresponding quarters
  H <- length(forecast_list$TR_Forecast)

  forecast_quarters <- seq(from = last(data$quarter) + 0.25,
                           by = 0.25,
                           length.out = H)

  horizon_quarter_label <- paste0(1:H, ": ", as.character(forecast_quarters))

  # Create a data frame for display
  forecast_df <- data.frame(
    Horizon_Quarter = horizon_quarter_label,
    Taylor_Rule_Forecast = round(forecast_list$TR_Forecast,2),
    Benchmark_ARIMA_Forecast = round(forecast_list$BM_Forecast,2),
    Inflation_Gap_Forecast = round(forecast_list$Inflation_Forecast,2),
    Output_Gap_Forecast = round(forecast_list$OutputGap_Forecast,2))

  # Create the table
  table_output <- kable(
    forecast_df,
    format = format,
    digits = 4,
    col.names = c("Horizon: Quarter", "Taylor Rule Forecast", "Benchmark Forecast",
                  "Inflation Forecast", "Output Gap Forecast"),
    caption = caption,
    booktabs = TRUE) %>%
  kable_styling(
    latex_options = "striped",
    position = "center") %>%
  column_spec(1, bold = TRUE, border_right = TRUE)
    return(table_output) }



#------------------------------------------------------------------
# Helper function for plotting our final forecast results
#------------------------------------------------------------------

plot_forecasts <- function(forecast_list,
                           title = "Interest Rate and Component Forecasts") {
```

```r
# Create the data frame for plotting w/ numerical quarters
H <- length(forecast_list$TR_Forecast)
forecast_quarters_yearqtr <- seq(from = last(data$quarter) + 0.25,
                                 by = 0.25,
                                 length.out = H)

# b) numeric version (for plotting)
forecast_quarters_numeric <- as.numeric(forecast_quarters_yearqtr)

# c) character version (for labels)
forecast_quarters_labels <- as.character(forecast_quarters_yearqtr)

forecast_df <- data.frame(
  Quarter = forecast_quarters_numeric,
  "Taylor Rule" = forecast_list$TR_Forecast,
  "Benchmark ARIMA" = forecast_list$BM_Forecast,
  "Inflation" = forecast_list$Inflation_Forecast,
  "Output Gap" = forecast_list$OutputGap_Forecast,
  check.names = FALSE )

# Long format for ggplot
forecast_long <- forecast_df %>%
  pivot_longer(cols = -Quarter,
    names_to = "Forecast_Type",
    values_to = "Value") %>%
  mutate(Plot_Group = case_when(
         Forecast_Type %in% c("Taylor Rule", "Benchmark ARIMA") ~ "Interest Rate Forecasts",
         Forecast_Type %in% c("Inflation", "Output Gap") ~ "Model Input Forecasts"),
    Plot_Group = factor(Plot_Group, levels = c("Interest Rate Forecasts", "Model Input Forecasts")),
    Forecast_Type = factor(Forecast_Type, levels = c("Taylor Rule", "Benchmark ARIMA", "Inflation", "(

# Actual plot
plot <- ggplot(forecast_long, aes(x = Quarter, y = Value, color = Forecast_Type)) +
  geom_line(linewidth = 1.1) +
  geom_point(size = 2.5) +
  facet_wrap(~ Plot_Group, ncol = 1, scales = "free_y") +
  # Prettyness
  labs(title = title,
       x = "Quarter",
       y = "Value (%)",
       color = "Forecast Series") +
  scale_x_continuous(breaks = forecast_quarters_numeric,
                     labels = forecast_quarters_labels) +
  theme_minimal(base_size = 14) +
  theme(legend.position = "bottom",
        plot.title = element_text(face = "bold", size = 14),
        plot.subtitle = element_text(size = 12),
        strip.text = element_text(face = "bold", size = 12),
        axis.text.x = element_text(angle = 45, hjust = 1)  ) +
        scale_color_brewer(palette = "Set1")
return(plot) }
```

Forecasting

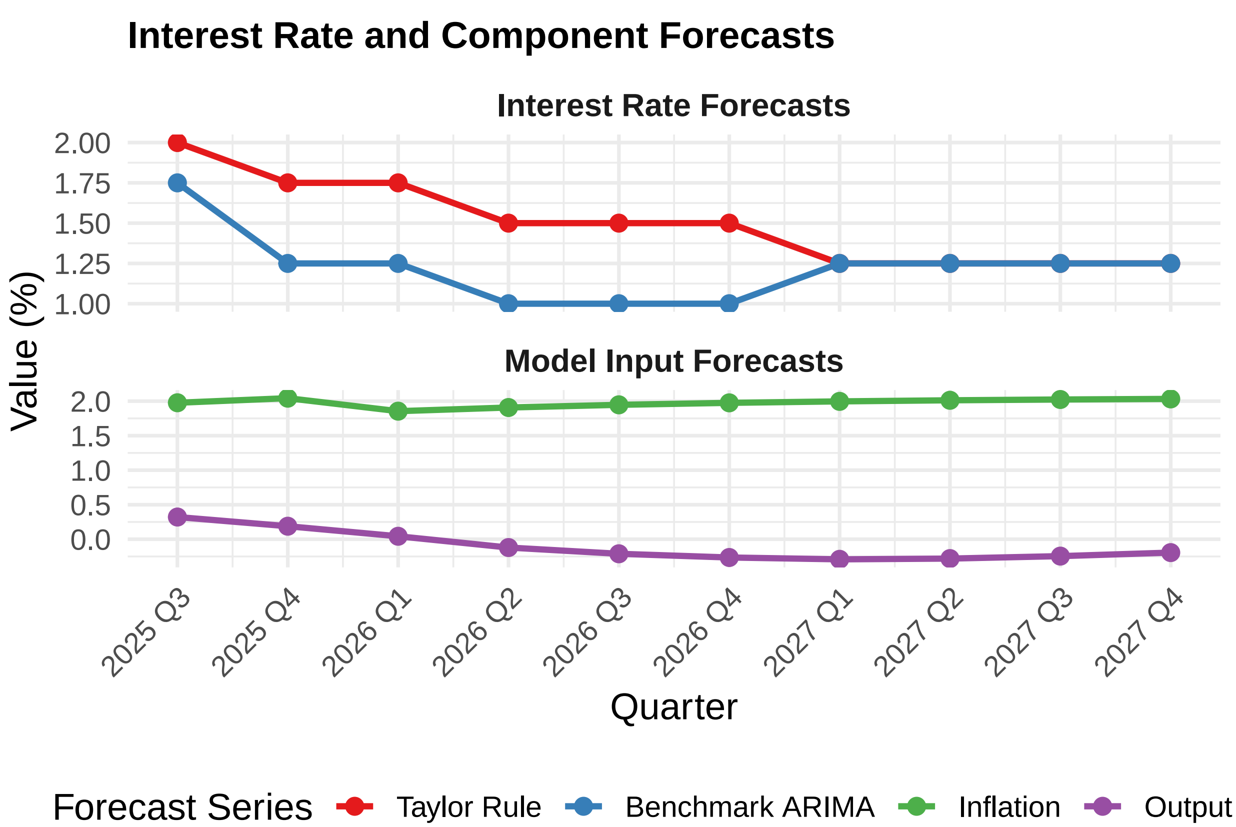
```r
# Formula 4 seems to work best
our_predict <- function(data,formula,H){

  # --- 1. Fit inputs and benchmark models  ---
   inflation_arma <- my.auto.arima(data$inflation_gap, max.p=4, max.q=4, d=1)
   outputgap_arma <- my.auto.arima(data$output_gap, max.p=4, max.q=4, d=0)
   interest_arma <- my.auto.arima(data$rate, max.p=4, max.q=4, d=1) # Benchmark

  # --- 2. Get forecasts of inputs (all H horizons) ---
   inflation_forecasts <- my.forecast(inflation_arma, h = H)
   outputgap_forecasts <- my.forecast(outputgap_arma, h = H)
   BMpredicted_rates <- my.forecast(interest_arma, h = H)

  # --- 3. Fit TR model
   TR_model <- lm(formula, data = data)

  # --- 4. Build forecast input data frame (iteratively for lags) ---

   # Allocate storage for full horizon
   TR_preds <- numeric(H)

   # Get last known lags (starting point for lagged models)
   current_shadowrate_lag <- last(data$shadowrate)
   current_rate_lag <- last(data$rate)

  for (h in 1:H) {
     new_data_h <- data.frame(
        inflation_gap = inflation_forecasts[h],
        #exp_inflation_gap = exp_inflation_forecasts[h],
        output_gap = outputgap_forecasts[h],
        shadowrate_lag = current_shadowrate_lag,
        rate_lag = current_rate_lag)

     # Get forecasted values
     pred_h <- predict(TR_model, new_data_h)
     TR_preds[h] <- round(pmax(pred_h, min(data$rate)) / 0.25) * 0.25

     # Update the lag for h+1
     current_rate_lag <- pred_h }

   # --- 5. Compute forecast for BM ---
    BM_preds <- round(pmax(BMpredicted_rates, min(data$rate)) / 0.25) * 0.25

  return(list(TR_Forecast = TR_preds,
              BM_Forecast = BM_preds,
              Inflation_Forecast = inflation_forecasts + 2, #to add back target
              OutputGap_Forecast = outputgap_forecasts  ))}


final_forecasts <- our_predict(data = data, formula = formula_3, H = H)
```

Table 18

Horizon: Quarter	Taylor Rule Forecast	Benchmark Forecast	Inflation Forecast	Output Gap Forecast
1: 2025 Q3	2.00	1.75	1.98	0.32
2: 2025 Q4	1.75	1.25	2.04	0.19
3: 2026 Q1	1.75	1.25	1.85	0.04
4: 2026 Q2	1.50	1.00	1.91	-0.12
5: 2026 Q3	1.50	1.00	1.95	-0.21
6: 2026 Q4	1.50	1.00	1.98	-0.26
7: 2027 Q1	1.25	1.25	2.00	-0.29
8: 2027 Q2	1.25	1.25	2.01	-0.28
9: 2027 Q3	1.25	1.25	2.02	-0.24
10: 2027 Q4	1.25	1.25	2.03	-0.19

```r
display_forecasts(final_forecasts,
                  caption = "",
                  format = format)
```

```r
plot_forecasts(final_forecasts)
```



Prediction Intervals

```
prediction <- var_by_horizon
final_interval <- final_forecasts
prediction$sd_fe <- sqrt(prediction$var_fe)

final_interval$sd <- prediction$sd_fe
final_interval$upper_1_sd <- final_interval$sd + final_interval$TR_Forecast
final_interval$lower_1_sd <- final_interval$sd*(-1) + final_interval$TR_Forecast

final_interval$upper_2_sd <- final_interval$sd*2 + final_interval$TR_Forecast
final_interval$lower_2_sd <- final_interval$sd*2*(-1) + final_interval$TR_Forecast

final_interval <- as.data.frame(final_interval)
```

```
ggplot(final_interval, aes(x = seq_len(nrow(final_interval)), y = TR_Forecast, group = 1)) +
  geom_ribbon(aes(ymin = lower_1_sd, ymax = upper_1_sd), fill = "lightgrey", alpha = 0.6) + # intervall
  geom_ribbon(aes(ymin = lower_2_sd, ymax = upper_2_sd), fill = "lightgrey", alpha = 0.2) + # intervall
  geom_line(color = "#1f78b4", size = 1) +  # line
  geom_point(color = "#e31a1c", size = 3) + # points
  labs(title = "Prelinimary Forecast plot", x = "h step ahead") +
  theme_minimal() +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))
```