

## Quiz5

Assign

$$P(q_i | q_1 \dots q_{i-1}) = P(q_i | q_{i-1})$$

$$a_{ij} \geq 0; \forall j, i$$

$$\sum_{j=1}^N a_{ij} = 1; \forall i$$

We assume that once a day the weather is observed as being one of the following:

State1: rain or snow

State2: cloudy

State3: sunny.

We postulate that the weather on day  $t$  is characterized by one of the three states above, and the matrix  $A$  of the state-transition probabilities is

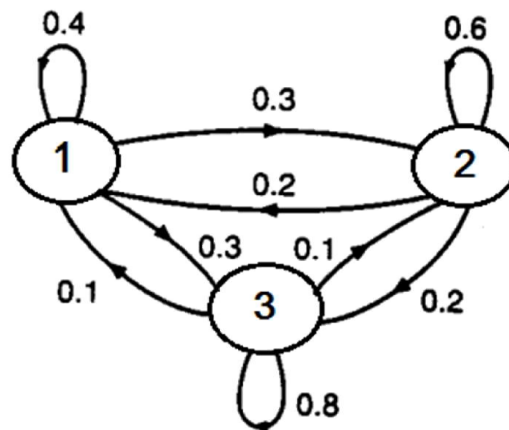
$$A = \{a_{ij}\} = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$$

Problem What is the probability that the weather for 8 consecutive days is “sun-sun-sun-rain-rain-sun-cloudy-sun”?

### Sol<sup>n</sup>

Assign      State1: rain or snow      =      1  
                 State2: cloudy                =      2  
                 State3: sunny                 =      3

According to matrix A, we can draw the transition diagram as show below



Then we set the observations ( $O$ ) as

$O = 3, 3, 3, 1, 1, 3, 2, 3$

Day = 1, 2, 3, 4, 5, 6, 7, 8

So, from  $P(q_i | q_1 \dots q_{i-1}) = P(q_i | q_{i-1})$ , we get

$$\begin{aligned} P[O|\text{Model}] &= P[3, 3, 3, 1, 1, 3, 2, 3 | \text{Model}] \\ &= P[3|0]P[3|3]P[3|3]P[1|3]P[1|1]P[3|1]P[2|3]P[3|2] \\ &= P[3]P[3|3]^2P[1|3]P[1|1]P[3|1]P[2|3]P[3|2] \\ &= \pi_3 \cdot (a_{33})^2(a_{31})(a_{11})(a_{13})(a_{32})(a_{23}) \\ &= (1)(0.8)^2(0.1)(0.4)(0.3)(0.1)(0.2) \end{aligned}$$

where  $\pi_i = P[q_i = i] = 1 \leq i \leq N$

$$= 1.536 \times 10^{-4}$$