Data Science & Artificial Intelligence

Algorithms

Test Series 1500+

Lecture - 05 06

By- Aditya sir

Recap of Previous Lecture

Topic

Topic

Sorting

Heaps

Misc

} Questions

Topics to be Covered

- Topic
- Topic

Misc

[MCQ]

#Q.2 The minimum number of interchanges needed to convert the array into a max-heap is.

110,40, 61,38,33,31, 23, 26,28,32,27, 30,91

A 0

B 2

C 1

D 3

Ans: B

Heap Creation

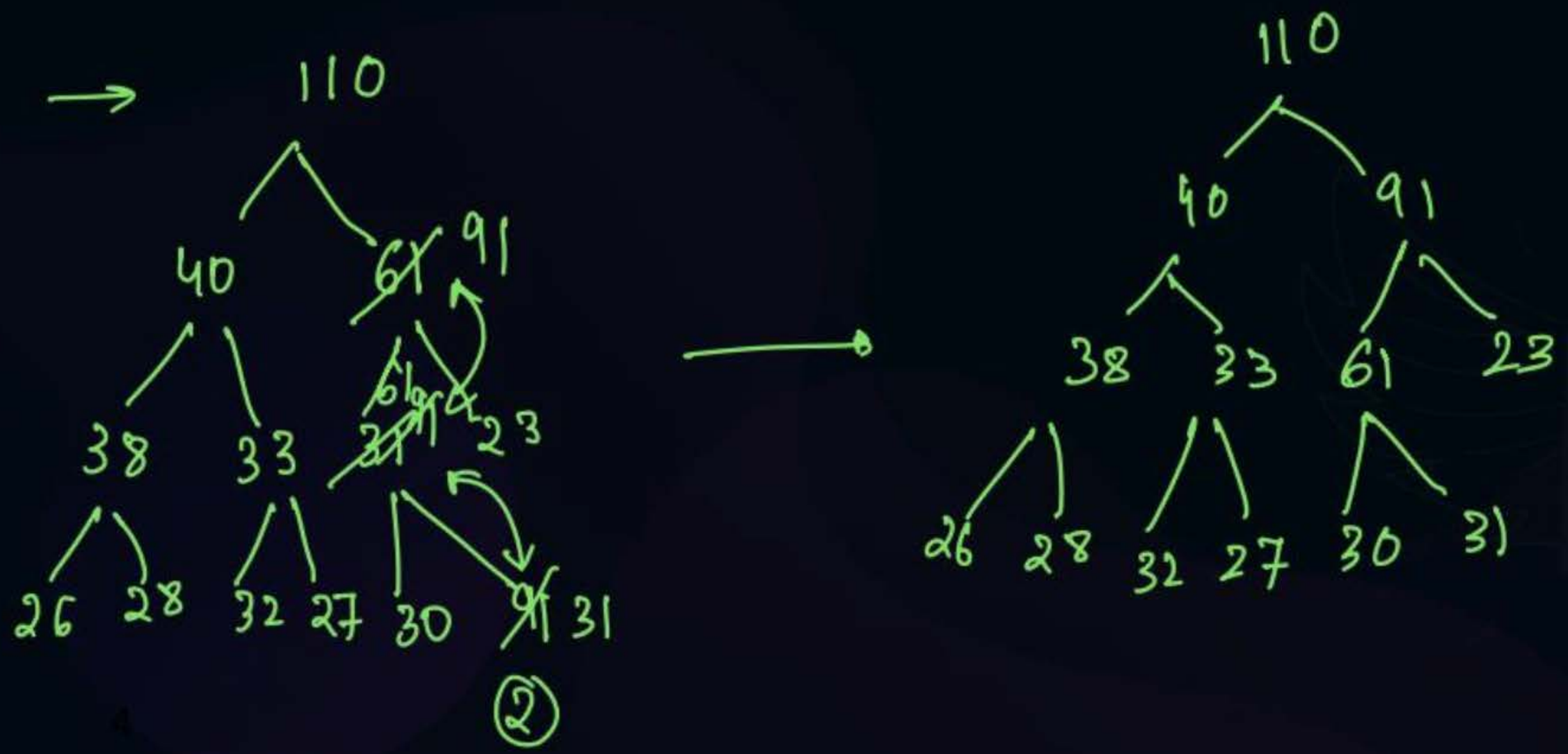
→1> Insertion mtd.

→2> Heapify

Build Heap.

Sol:- 1) Appr 1: Insertion method (Max-Heap)

A: 110 40 61 38 33 31 23 26 28 32 27 30 91

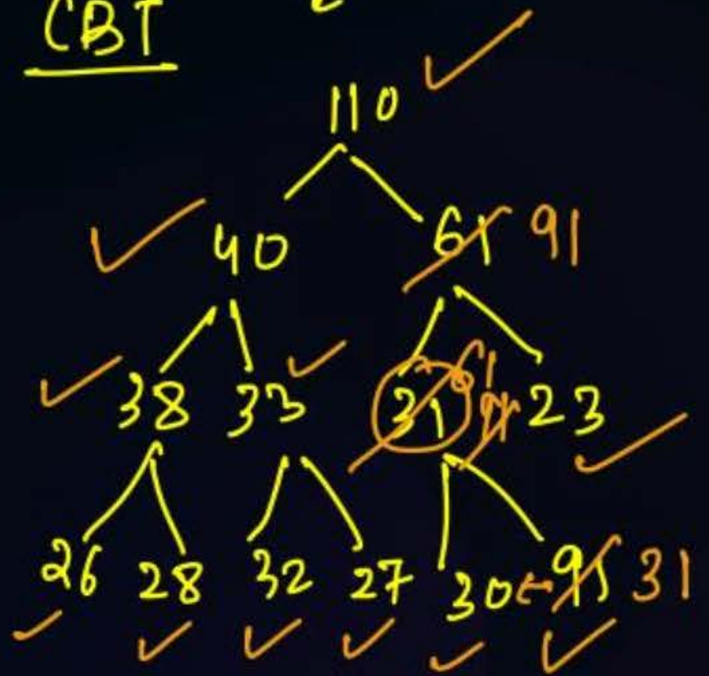


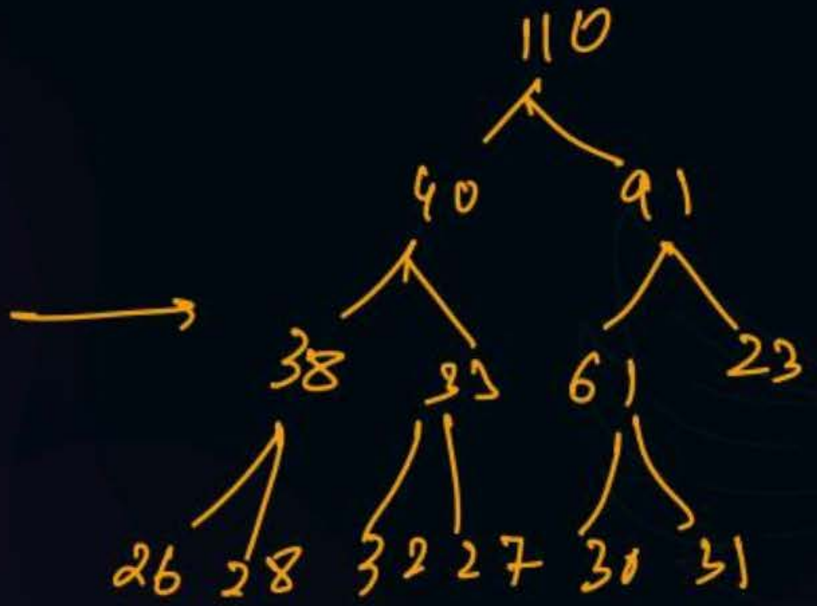
2 interchanges

(2) Build / Heapify:

A: 110 40 61 38 33 31 23 26 28 32 27 30 91

CBT





Max-Heap

2 interchanges

[NAT]

#Q.5 The number of possible min -heap containing each value from {34,25,9,12,50,5,18} exactly once.

0%

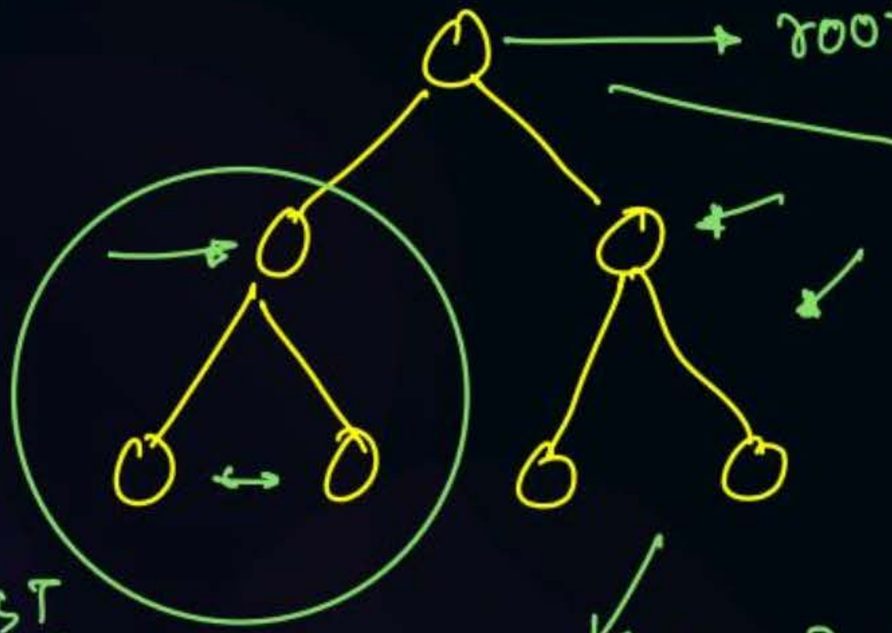
Ans: 80

Soln: {34, 25, 9, 12, 50, 8, 18}

n = 7

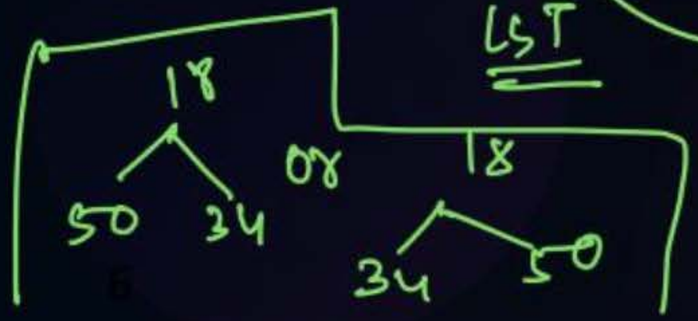
Min-Heap / Max-Heap

Appr1:



6 $*2!*2!$ (3, 0, 0)

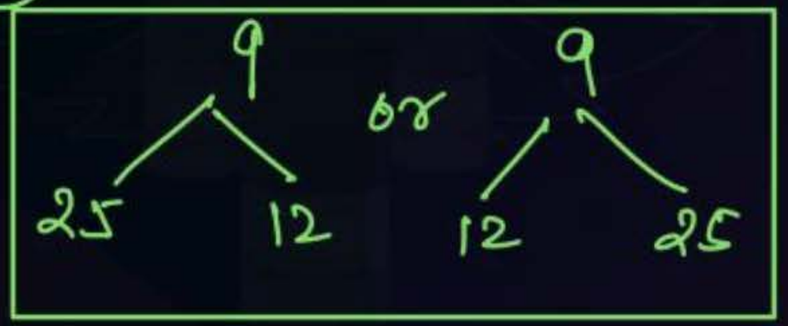
LST



remaining 3 elem

34, 50, 18

9 25 12



Total min-Heaps : ${}^{6}C_3 * 2! * 2!$

$$= \frac{6!}{3! * 3!} \times 2 \times 2$$

$$= \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 6} \times 2 \times 2$$

$$= 20 \times 4$$

$$= \boxed{80}$$

Appr 2 : Formula / Recurrence based :

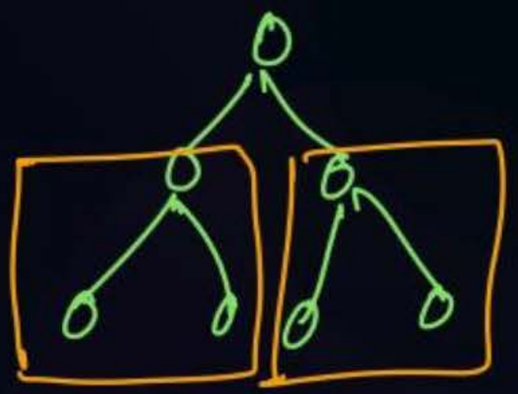
$$T(n) = {}^{n-1}C_k * T(k) * T(n-1-k)$$

where n = total no. of elems

k = no. of elems in left subtree.

eg:- $n = 7$

$k = 3$



$$\Rightarrow {}^{7-1}C_3 * T(3) * T(7-1-3)$$

$$= {}^{6}C_3 * T(3) * T(3)$$

$T(3)$: $n = 3$, $k = 1$ $\Rightarrow$ ${}^{2}C_1 * T(1) * T(1)$

Base case : $T(1) = 1$

$T(3) = {}^{2}C_1 \times 1 \times 1$

$= 2$

$T(7) = {}^{6}C_3 \times T(3) \times T(3)$

$= {}^{6}C_3 * 2 \times 2 = 80$

(Q.2) For prev Question, if $n = 15$ diff elems are given

How many distinct Max-Heaps ?

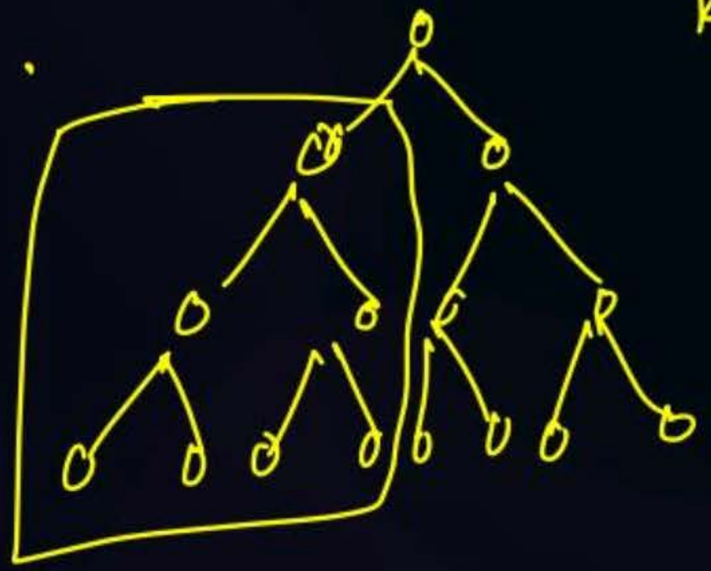
$n = 15$

$k = 7$

$T(15) = {}^{14}C_7 \times T(7) \times T(7)$

$= {}^{14}C_7 \times 80 \times 80$

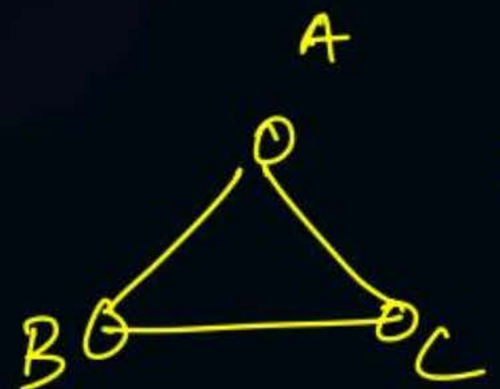
⇒ Appr2.



[NAT] Imp

#Q.8 Suppose, G is a undirected connected complete graph with 5 vertices. How many BFS traversals are possible for Graph G?___

Soln: $n = 3$



#

$\Rightarrow$

BFS traversals:

- A B C
- A C B
- B A C
- B C A
- C A B
- C B A

$3! = 6$

Complete

$n = 5$

$5 * 4 * 3 * 2 * 1$

$= 5!$

$= 120$

[MSQ]

#Q10. Bubble sort is

A. In place sorting technique ✓

B. Outplace sorting technique ✗

C. Unstable sorting technique ✗

D. Stable sorting technique ✓

Bubble Sort

→ 1) Stable

2) Inplace → O(1)

Ans: A, D

[MCQ]

#Q.7 Let P be a binary min-heap consisting of n elements implemented as an array. What is the worst case time complexity of an optimal algorithm to find the maximum element in H?

A $\Theta(\log n)$

B $\Theta(n \log n)$ ✗

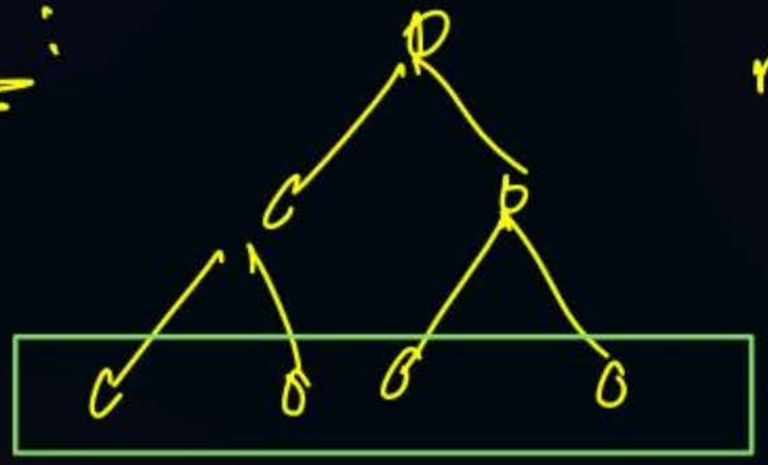
C $\Theta(n)$ ✓ → Best algo for WC

D $\Theta(1)$

Soln:- 1) Appr 1 :

min-Heap

n elems



n elem Heap $\approx (n/2)$ elems are leaf

→ max elem (always a leaf node)

↳ Linear search $(n/2)$ elems to find max

$\Rightarrow O(n/2) = O(n)$

② Min-Heap (array) → Heap Sort → incr order

TC: $O(n \log_2 n)$

last elem
a[n] → max

3rd Appr: Heapify:

Min-Heap (CBT) → Heapify → Max-Heap

Heapify: $O(n)$

Max-Heap → Root: $O(n) + O(1) = O(n)$

$O(1)$ (max)

[NAT]

#Q15. Consider the following array A with 8 elements:

What is the index value of elements 19 after second pass of selection sort?

Ans: 1

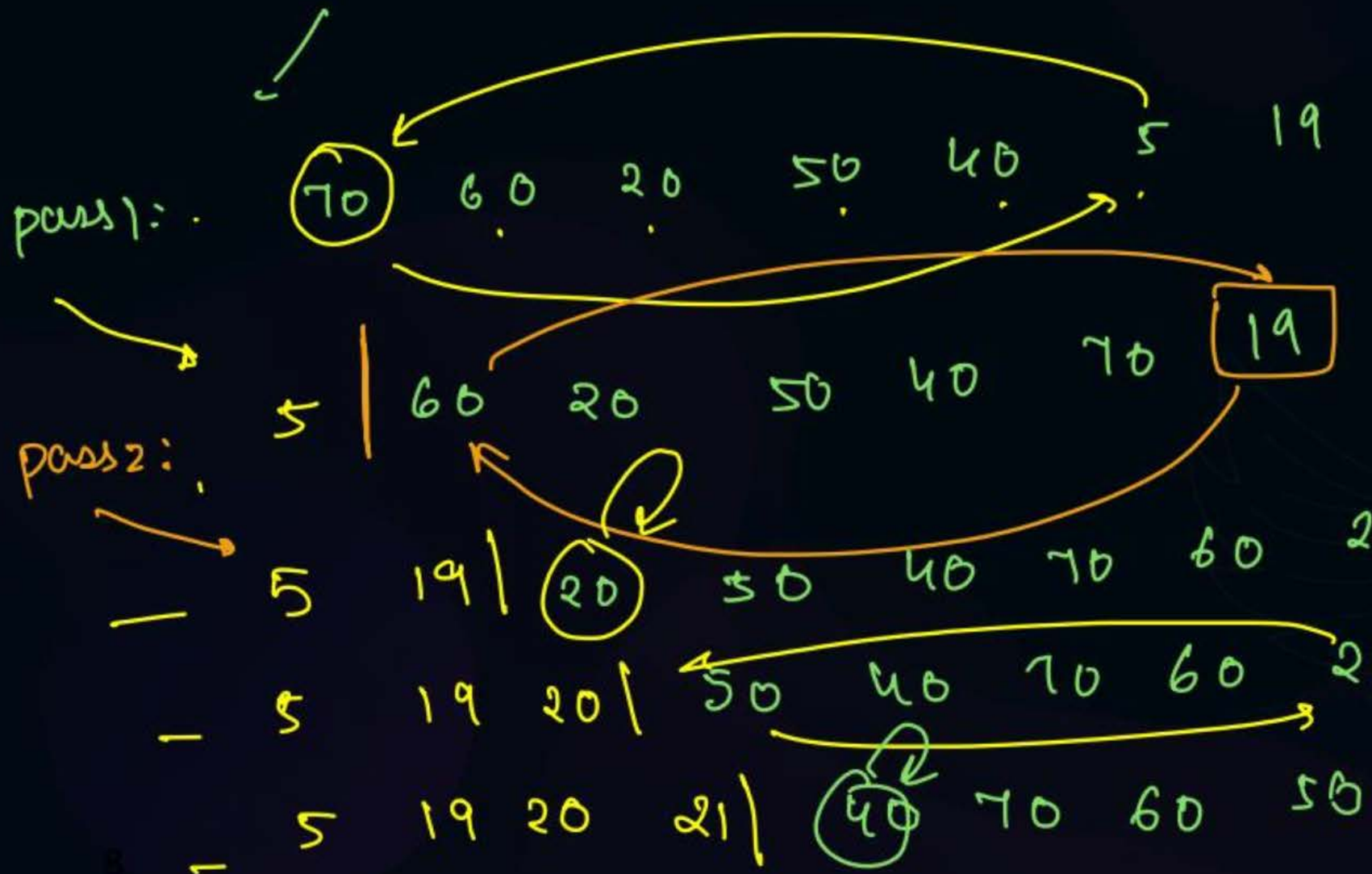
1st pass → 1st min

2nd pass → 2nd min

Soln:-

i/p: 70 60 20 50 40 5 19 21

selection sort



→ pass 2 O/p.

— 5 19 20 21 40 | 70 60 50

— 5 19 20 21 40 50 | 60 70

push 7 —

→ sorted o/p :

(n-1) passes.

Topic : Analysis of algorithm

#Q. Suppose, $f(n) = \sum_{i=1}^{n} O(n^2), g(n) = \sum_{i=1}^{n^2} O(n)$

Which of the following is/are corrected?

$f \geq f$ ✓

$f > f$ ✗

Ans: AB C

$f =_A g$

$F = O(g)$

$f = O(g)$

$F = \Omega(g)$

A $f(n) = \theta(g(n))$ ✓

B $f(n) = O(g(n))$ ✓

C $f(n) = \Omega(f(n))$ ✓

D $f(n) = \omega(f(n))$ ✗

$f(n) = \Omega(g(n)) \rightarrow T$

$f(n) = \omega(g) \rightarrow F$

Soln :-

$$f(n) = \sum_{i=1}^{n} O(n^2) \Rightarrow O(n^3)$$

```
for( i: 1 ⟶ n)
{
    O(n²)  ⇒
}
```

$$g(n) = \sum_{i=1}^{n^2} O(n) \Rightarrow O(n^3)$$

```
For( i: 1 ⟶ n²)      → n² times
{
    O(n)
}
```

$f(n) = \Theta(n^3)$

$g(n) = \Theta(n^3)$

Approx:- $\sum_{i=1}^{n} O(n^2) = \underbrace{O(n^2) + O(n^2) - - - O(n^2)}_{n \text{ times}}$

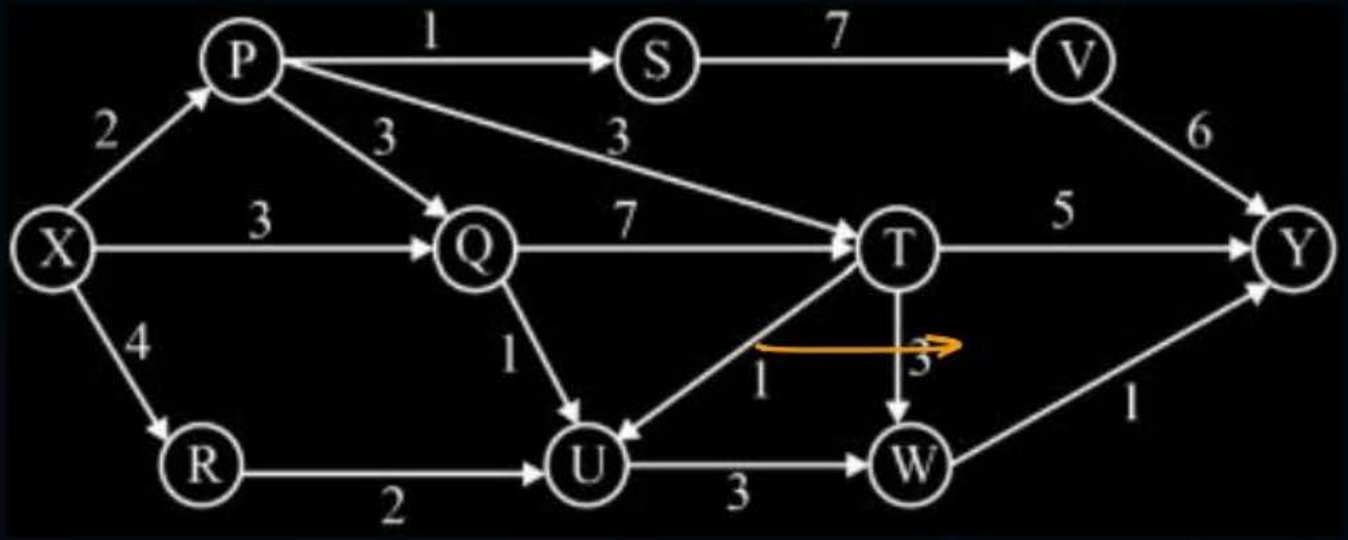
$= O(n^3)$

$\sum_{i=1}^{n^2} O(n) = \underbrace{O(n) + O(n) - - - O(n)}_{n^2 \text{ times}}$

$= \underline{\underline{O(n^3)}}$

[NAT]

#Q18. Consider the following graph G



A) The minimum distance from X to Y is ____ (where X is source and Y is destination) reported by Dijkstra.

B) What is the path reported by Dijkstra for X → Y ? / Spanning Tree appr)

Soln :- Matrix based appr Dijkstra → SSSP

Source = X

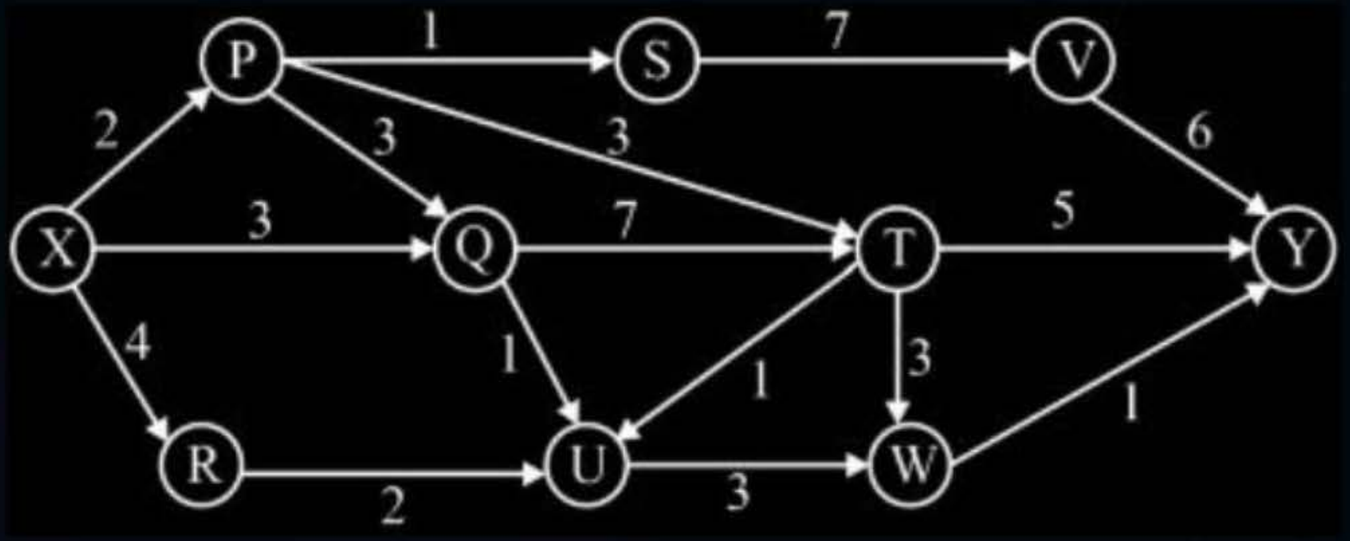
visited set	P	Q	R	S	T	U	V	W	X	Y
{X}	(2)	3	4	(∞)	(∞)	∞	∞	∞	0	∞
{X, P}	2	(3)	4	3	5	∞	∞	∞	0	∞
{X, P, Q}	2	3	4	(3)	5	4	∞	∞	0	∞
{X, P, Q, S}	2	3	(4)	3	5	4	10	∞	0	∞
{X, P, Q, S, R}	2	3	4	3	5	(4)	10	∞	0	∞
{X, P, Q, S, R, U}	2	3	4	3	(5)	4	10	7	0	(∞)
{X, P, Q, S, R, U, T}	2	3	4	3	5	4	10	(7)	0	10

{x, P, Q, S, R, U, T, W}	2	3	4	3	5	4	10	7	0	8
{x P, Q, S, R, U, T, W, Y}	2	3	4	3	5	4	10	7	0	8
{x, P, Q, S, R, U, T, W, Y, V}	2	3	4	3	5	4	10	7	0	8

$X \rightarrow Y \longrightarrow \boxed{8}$

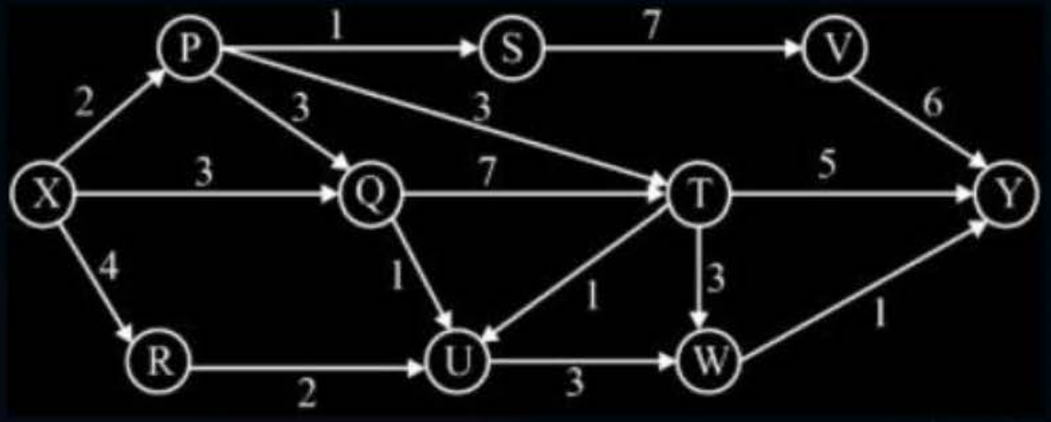
[NAT]

#Q18. Consider the following graph G

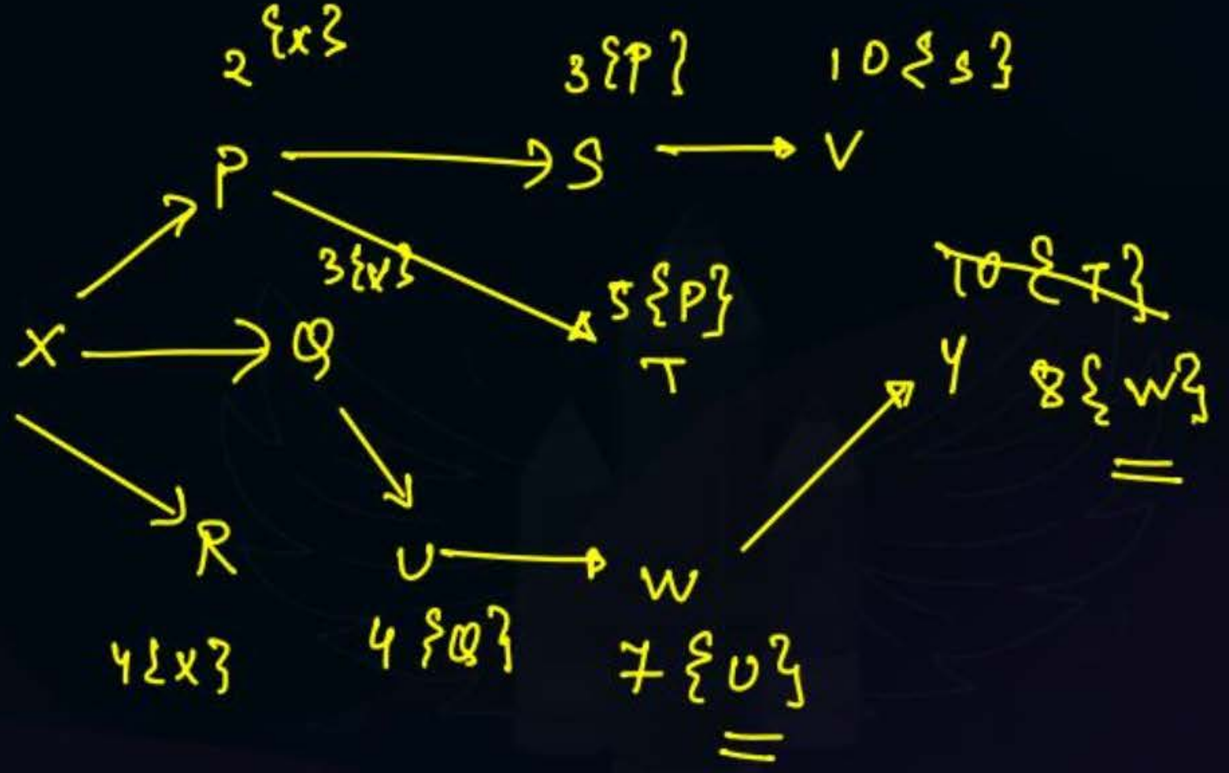


The minimum distance from X to Y is ____ (where X is source and Y is destination)

B) Spanning Tree approach :



Spanning Tree by Dj SSSP

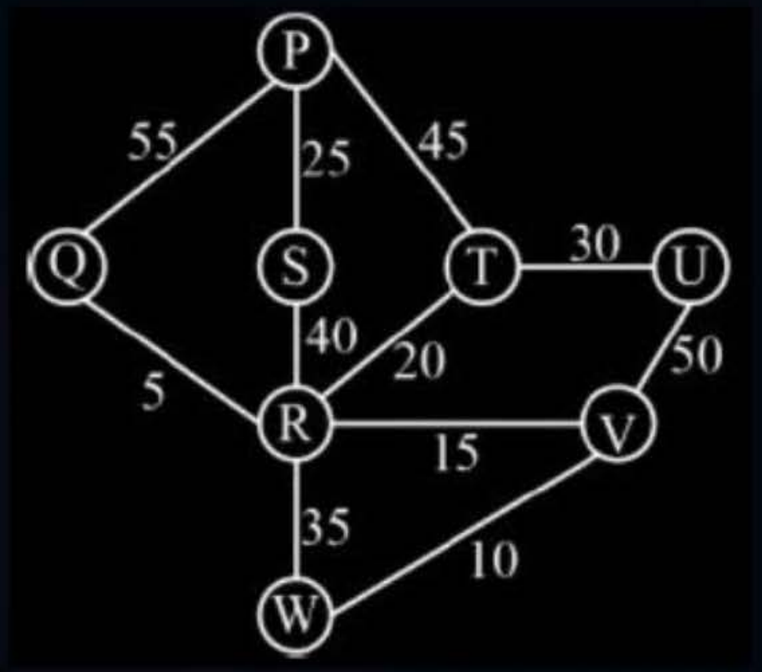


X → Y: Cost : 8

X → Q → U → W → Y

[NAT]

#Q19. Consider the following graph G (starting from P)



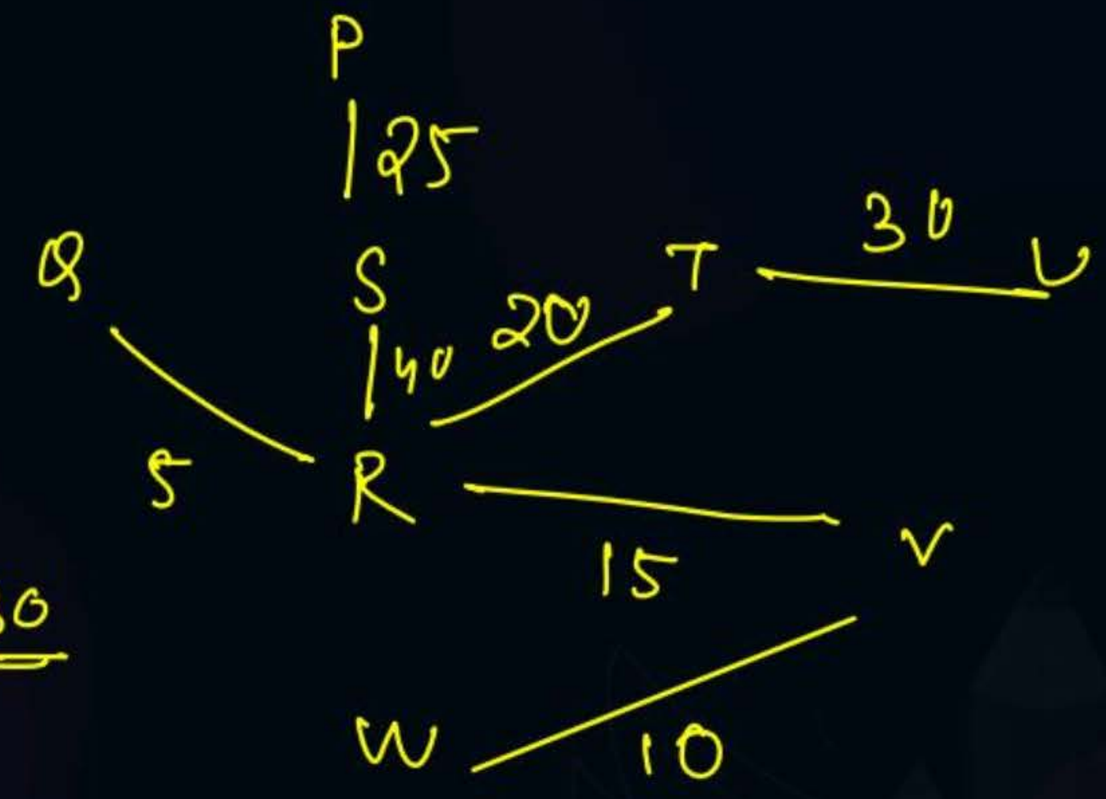
→ every edge : distinct cost

↳ MCST is also unique.

The cost of minimum cost spanning tree is _____.

Soln :- 1) Prims

MCST ⟹



Cost = 5 + 10 + 15 + 20 + 30

+ 40 + 25

= 145

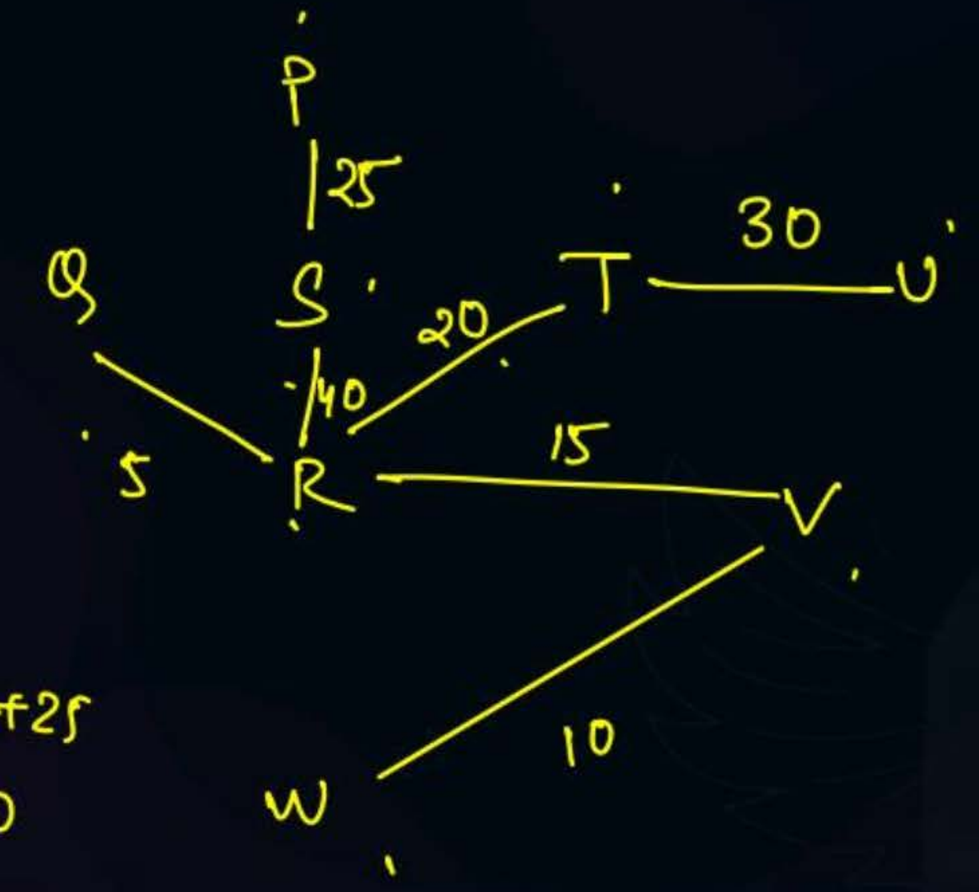
* 27 Kruskal

$n = 8$

edges in MCST

$= 8 - 1$

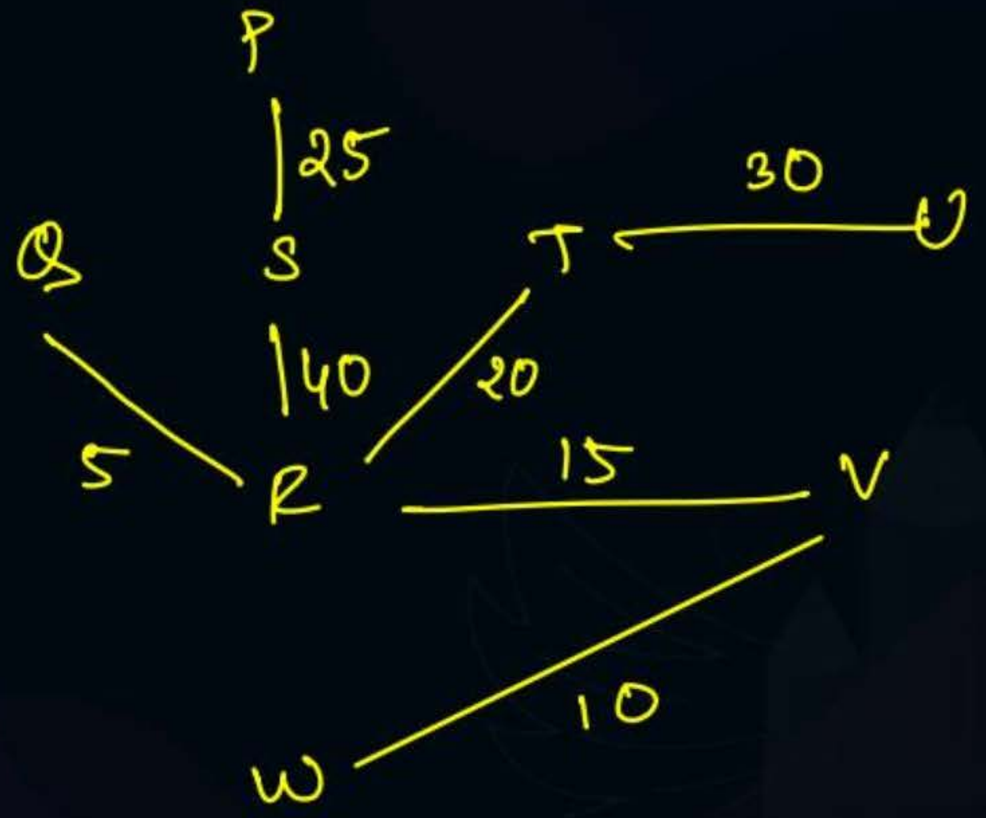
$= 7$



Cost = $5 + 10 + 15 + 20 + 30 + 25 + 40$

$= \boxed{145}$

3) Dijkstra MCST



MCST

Cost = 145

2 mins Summary

Topic

Topic

Topic

Topic

Heaps

Graphs

Sorting

Shortest Path

Misc

} Questions

THANK - YOU

Telegram Link for Aditya Jain sir:
https://t.me/AdityaSir_PW