GATE

CRASH COURSE

ALL BRANCHES

Engineering Mathematics

Differential Equation (Part 02)
(Lec 11)

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Topics *to be covered*

DIFFERENTIAL EQUATIONS (Part 2)

1. Exact D Eq.
2. CF & PI Methods
3. Cauchy L.D.E Solving Method.

D.Eqn :-
- → O.D.Eq → Consist only O-Derivatives
- → P.D.Eq → " " Partial "

order of D Eqn :- The highest order Derivative occurring in a DEq is called it's order

Degree of D Eqn :- Degree is the exponent of highest order Derivative when D.Eq is represented in the polynomial form in terms of Derivatives.

eg

$$\frac{d^2y}{dx^2} = \sqrt{\left(\frac{dy}{dx}\right)^3 + y} \Rightarrow \boxed{\left(\frac{d^2y}{dx^2}\right)^{2} = \left(\frac{dy}{dx}\right)^{3} + y}$$

order = 2, Degree = 2, NL

Conclusions, ① No. of A Const in G. Sol = order of DEq.

② To form a DEq we can diff it → Easy task

③ There should not be any A Const in D.Eq. (T)

④ There should not be any Derivative in the Sol of DEq. (T)

⑤ Solution of DEqⁿ:- The Relationship b/w y & x satisfying the given DEq is called Sol. Types
- G. Sol. → Consist A. Const
- P. Sol. → it has No A. Const.

⑥ To find the Sol of DEq we should integrate it → Tough task

LINEAR D.Eq. of 1^{st} order

Type I: it's G. Form is $\boxed{\dfrac{dy}{dx} + Py = Q}$ — (1) where P & Q are func^n of $\underline{x}$ alone

& it's I.F $= e^{\int P\,dx}$ & it's solution is

$$\boxed{y(\text{IF}) = \int Q(\text{IF})\,dx + C}$$

it is LDE in $\underline{y}$ & $\underline{x}$

Type II: it is given as $\dfrac{dx}{dy} + Px = Q$ where P & Q are func^n of $\underline{y}$ alone

& $\text{IF} = e^{\int P\,dy}$ & Sol^n is $\boxed{x(\text{IF}) = \int Q(\text{IF})\,dy + C}$

it is LDE in $\underline{x}$ & $\underline{y}$

Bernoullie DEqu^n^ → it can be converted into Linear form. [Let the soln is $y = f(x)$]

Non Linear D Eq
① If Degree is more than 1
② If Exponent of y is other than one
③ If Exponent of any Derivative is other than one
④ If Product of y & it's derivative exist.

Linear DEq → when out DEq is free from all the above 4 Properties then it is called L D Eq.

$$\left(\frac{d^3x}{dt^3}\right)^{I} + 2x^{5\ II} + x\frac{d^2x}{dt^2}^{IV} + \left(\frac{dx}{dt}\right)^{2\ III} = 4t^3$$

$x = f(t)$

$O = 3$, $Dg = 1$, NL

The solution of $\frac{dy}{dx} = (4x + y + 1)^2$ is

(a) $\frac{1}{2}\tan^{-1}\left(\frac{4x + y + 1}{2}\right) = c$

(b) $\frac{1}{2}\tan^{-1}\left(\frac{4x + y + 1}{2}\right) = x + c$

(c) $\frac{1}{2}\tan^{-1}(4x + 4y + 1) = x + c$

(d) $\frac{1}{2}\tan^{-1}(4x + y + 1) = c$

Put $4x+y+1 = t$

$$4 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 4 = t^2 \Rightarrow \int \frac{dt}{4+t^2} = \int dx + c$$

$$\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right) = x + c$$

$$\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + c$$

HOMOGENEOUS D.Eq

it is of the type $\boxed{\dfrac{dy}{dx} = \dfrac{f(x,y)}{g(x,y)}}$ — (1) where $f(x,y)$ & $g(x,y)$ are Homog func^n of same degree n

i.e $f(\lambda x, \lambda y) = \lambda^n \cdot f(x,y)$ & $g(\lambda x, \lambda y) = \lambda^n \cdot g(x,y)$

To solve (1) Put $y = vx \Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}(vx) = v + x\dfrac{dv}{dx}$

after that you can use Variable separable Method

The solution of differential equation $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$, is

(a) $\frac{x}{y}-2\log x+3\log y=c$

(b) $\frac{y}{x}-2\log y+3\log x=c$

(c) $\frac{x}{y}+2\log x-3\log y=c$ ✓

(d) $\frac{y}{x}+2\log y-3\log x=c$

$(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$ — (1)

$\frac{dy}{dx}=\frac{x^2y-2xy^2}{x^3-3x^2y}$ — (1)

Put $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx}=\frac{v-2v^2}{1-3v}$

$x\frac{dv}{dx}=\frac{v-2v^2}{1-3v}-v=\frac{v^2}{1-3v}$

$\left(\frac{1-3v}{v^2}\right)dv=\left(\frac{1}{x}\right)dx \Rightarrow$

$\frac{\partial M}{\partial y}=x^3-4xy$ i.e. (1) is Not Exact

$\frac{\partial N}{\partial x}=-3x^2+6xy$

$$\int\left(\frac{1}{v^2}-\frac{3}{v}\right)dv=\int\left(\frac{1}{x}\right)dx+c_1$$

$$-\frac{1}{v}-3\ln v=\ln x+\ln c$$

$$-\frac{x}{y}-3\ln\left(\frac{y}{x}\right)=\ln x+\ln c$$

$$-\frac{x}{y}-3(\ln y-\ln x)-\ln x=\ln c$$

$$\frac{x}{y}=2\ln x-3\ln y$$

EXACT D.Eq

Consider a 1^{st} DEq of the type $\boxed{Mdx + Ndy = 0}$ — ①

this DEq is called Exact (solvable) if $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ N. Cond^n

& it's solution is given as

$$\boxed{\int_{y=\text{const.}} (Mdx) + \int (\text{those terms of } N \text{ which are free from } x)\, dy = C}$$

The differential equation

$$(27x^2 + ky\cos x)dx + (2\sin x - 27y^3)dy = 0$$

is exact for $k =$ ____.

$M = 27x^2 + ky\cos x$, $N = 2\sin x - 27y^3$

$\frac{\partial M}{\partial y} = k\cos x$, $\frac{\partial N}{\partial x} = 2\cos x$

So for Exact $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow k = 2$

(ii) Also find the Sol of this D Eq.

$(27x^2 + 2y\cos x)dx + (2\sin x - 27y^3)dy = 0$ —— (1)

$O = 1$

NL

Sol of (1) is $\int_{y=\text{const.}} (27x^2 + 2y\cos x)dx + \int (-27y^3)dy = 0$

$\Rightarrow 9x^3 + 2y\sin x - \frac{27}{4}y^4 = C$

The differential equation

$$(\alpha xy^3 + y\cos x)dx + (x^2y^2 + \beta\sin x)dy = 0$$

is exact for

(a) $\alpha = \frac{3}{2}, \beta = 1$ (b) $\alpha = 1, \beta = \frac{3}{2}$

(c) $\alpha = \frac{2}{3}, \beta = 1$ (d) $\alpha = 1, \beta = \frac{2}{3}$

$M = \alpha x y^3 + y\cos x$, $\frac{\partial M}{\partial y} = 3\alpha x y^2 + \cos x$

$N = x^2y^2 + \beta \sin x$, $\frac{\partial N}{\partial x} = 2xy^2 + \beta\cos x$

For Exact: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$3\alpha x y^2 + \cos x = 2xy^2 + \beta \cos x$

$\Rightarrow 3\alpha = 2$, $\beta = 1$

$\alpha = \frac{2}{3}$

i.e. (c)

Q. Find the soln of $\left(\frac{3}{2}xy^3 + y\cos x\right)dx + \left(x^2y^2 + \sin x\right)dy = 0$ O = 1

NL

Degree = 1

$\because$ it is Exact so it's soln is

$$\int_{y=\text{const}} \left(\frac{3}{2}xy^3 + y\cos x\right)dx + \int (\quad 0 \quad)dy = C$$

$$\frac{3}{2}\frac{x^2}{2}\cdot y^3 + y\sin x = C \quad \underline{\text{Ans}}$$

Q. Find the soln of $\boxed{y'' = x} \Rightarrow \int y''dx = \int x\,dx + C_1 \Rightarrow \int y' = \int \frac{x^2}{2} + \int C_1 \Rightarrow y = \frac{x^3}{6} + C_1x + C_2$ //

Q. " " " $y'' = y \Rightarrow \int y''dx = \int y\,dx$

or $y' = ??$ Not possible

L.D.E of HIGHER ORDER with Constant Coefficient

[CF + PI]

Imp

Q: Solve $\boxed{\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = 0}$ —(1)

it is Linear, 2^{nd} order, const Coeff.

By hit & trial: let us take soln as,

$y = e^x$ ✓, $y = e^{-x}$ ✗, $y = e^{2x}$ ✗, $y = e^{-2x}$ ✗

$y = xe^x$ ✓, & so on .....

Q: Solve: $\boxed{\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^{3x}}$

it's soln are

$y = e^x$ ✗, $y = xe^x$ ✗, $y = \dfrac{e^{3x}}{4}$ ✓

Soln of LHS only are e^x & $xe^x \simeq CF$

∴ RHS only is $e^{3x}/4 \simeq PI$

So Complete Soln is $y = CF + PI$

① if DE is of the type $\boxed{f(D)\cdot y = 0}$ then G. Sol is $y = CF$

② if DE is of the type $\boxed{f(D)\,y = Q}$ then G. Sol is $y = CF + PI$

where Q is a func^n of $\underline{\quad x \quad}$ alone

③ CF = Sol of the LHS only

PI = " " RHS only

④ To find Auxillary equ^n of given D Eq. we should Replace $D \to m$ & $y \to 1$ in LHS

AEq of ① is: $m^2 - 2m + 1 = 0$

⑤ In this chapter, we have,

$\frac{d}{dx} = D, \quad \frac{d^2}{dx^2} = D^2, \quad \frac{d^3}{dx^3} = D^3$ - - - -

eg $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$\Rightarrow D^2y - 2Dy + y = 0$

or $(D^2 - 2D + 1)y = 0$ — ①

Methods of Calculating CF → (Complementary function) →

Case I: if Roots of A Eq are Real & distinct i.e $m = m_1, m_2, m_3, m_4 \ldots$

$$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \ldots$$

Case II → if Roots of AEq are Real & Repeated i.e $m = a, a, b, b, b, c, d \ldots$

$$CF = (c_1 + c_2 x) e^{ax} + (c_3 + c_4 x + c_5 x^2) e^{bx} + \underbrace{c_6 e^{cx} + c_7 e^{dx} + \ldots}_{\text{Case I}}$$

Case III → if Roots of AEq are Complex i.e $m_1 = \alpha + i\beta$, $m_2 = \alpha - i\beta$.

then $CF = c_1 e^{(\alpha + i\beta)x} + c_2 e^{(\alpha - i\beta)x} \implies \boxed{e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)}$

Q: Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

Sol: $D^2y - 2Dy + y = 0$

or $(D^2 - 2D + 1)y = 0$ —— (1)

AEq is $m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0$

or $m = 1, 1$

So By Case II: $CF = (c_1 + c_2 x)e^{1x}$

$PI = 0$

So Complete Sol $y = CF = (c_1 + c_2 x)e^{x}$

i.e. $y = c_1(e^{x}) + c_2(xe^{x})$

Note: (1) i.e. $y = c_1 y_1 + c_2 y_2$

where $y_1 = e^{x}$ & $y_2 = xe^{x}$

are also the sol of (1)

(2) Here y_1 & y_2 are (L.I) Sol^n of given D Eq

$\because \exists$ No linear Relationship b/w the Solutions.

For the differential equation $\frac{d^2x}{dt^2}+6\frac{dx}{dt}+8x = 0$ with initial conditions $x(0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0$, the solution is

(a) $x(t) = 2e^{-6t} - e^{-2t}$ (b) $x(t) = 2e^{-2t} - e^{-4t}$

(c) $x(t) = -e^{-6t} + 2e^{-4t}$ (d) $x(t) = 1e^{-2t} + 2e^{-4t}$

$\Rightarrow x = f(t)$ is it L.D.E of 2nd order with Const Coeff in $\underline{x}$ & $\underline{t}$

Let $\frac{d}{dt} = D$, $\frac{d^2}{dt^2} = D^2$ - - - - -

S. form is $D^2x + 6Dx + 8x = 0$

$(D^2+6D+8)x = 0$ —— (1)

A Eqn $m^2+6m+8 = 0 \Rightarrow m = -2 \;\&\; -4$

$CF = C_1e^{(-2)t} + C_2e^{(-4)t}$ & $PI = 0$

Soln (1) is $x = CF + PI = C_1e^{-2t} + C_2e^{-4t}$

ie $x(t) = C_1e^{-2t} + C_2e^{-4t} \Rightarrow 1 = C_1 + C_2$

$\frac{dx}{dt} = -2C_1e^{-2t} - 4C_2e^{-4t} \Rightarrow 0 = -2C_1 - 4C_2$

The solutions of the differential equations

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$$ are

(e) None

(a) $e^{-(1+i)x}, e^{-(1-i)x}$

(b) $e^{(1+i)x}, e^{(1-i)x}$

(c) $e^{-(1+i)x}, e^{(1-i)x}$

(d) $e^{(1+i)x}, e^{-(1-i)x}$

L. Relationship :-

$c_1y_1+c_2y_2+c_3y_3+\dots+c_ny_n=0$

S. form is $(D^2+2D+2)y$ — (1)

AEq is $m^2+2m+2=0$

$$m=\frac{-2\pm\sqrt{4-8}}{2}$$

$$m=-1\pm 1i \quad (\alpha\pm i\beta) \quad \alpha=-1,\ \beta=1$$

is $m_1=-1+i$, $m_2=-1-i$

So $CF=c_1e^{(-1+i)x}+c_2e^{(-1-i)x}$

$=c_1e^{-(1-i)x}+c_2e^{-(1+i)x}$

2nd possible Ans will be ?

$CF=e^{(-1)x}[c_1\cos 1.x+c_2\sin 1.x]$

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$$\lambda^3(\lambda+4)^2(\lambda^2+2\lambda+5)^2=0$$

(a) $y(x)=c_1+c_2x+c_3x^2+c_4e^{-4x}+c_5xe^{4x}+e^{x}\{c_6\cos 2x+c_7\sin 2x+c_8x\cos 2x+c_9x\sin 2x\}$

(b) $y(x)=c_1+c_2x+c_3x^2+c_4e^{-4x}+c_5xe^{-4x}+e^{-x}\{c_6\cos 2x+c_7\sin 2x\}+e^{x}\{c_8x\cos 2x+c_9x\sin 2x\}$

(c) $y(x)=c_1+c_2x+c_3x^2+c_4e^{-4x}+c_5xe^{4x}+e^{x}\{c_6\cos 2x+c_7\sin 2x\}+e^{-x}\{c_8x\cos 2x+c_9x\sin 2x\}$

(d) $y(x)=c_1+c_2x+c_3x^2+c_4e^{-4x}+c_5xe^{-4x}+e^{-x}\{c_6\cos 2x+c_7\sin 2x+c_8x\cos 2x+c_9x\sin 2x\}$

AE is $\lambda^3(\lambda+4)^2(\lambda^2+2\lambda+5)^2=0$

$\lambda=0,0,0,\ -4,-4,\ -1\pm 2i$ (Repeat)

$\alpha=-1,\ \beta=2$

$CF=(c_1+c_2x+c_3x^2)e^{0x}$

$+(c_4+c_5x)e^{-4x}$

$+e^{-x}\left[(c_6+c_7x)\cos 2x+(c_8+c_9x)\sin 2x\right]$

PI = (soln of RHS)

Consider the DEq $\boxed{f(D)y = Q}$ — (1) then $PI = \frac{1}{f(D)} Q$

where $\frac{1}{f(D)}$ is an operator (and generally it represents integration)

Case I: if $Q = e^{ax}$ then $\frac{1}{f(D)}(e^{ax}) = \frac{1}{f(a)} e^{ax}$

if $f(a) = 0$ then $PI = \frac{1}{f(D)}(e^{ax}) = \frac{x}{f'(D)}(e^{ax}) = \frac{x}{f'(a)}(e^{ax})$

if $f''(a) \neq 0$ then $PI = \frac{1}{f(D)}(e^{ax}) = \cdots\cdots = \frac{x^2}{f''(a)} e^{ax}$ & so on....

Case II: if $Q = \sin ax$ or $\cos ax$ then replace $D^2 \to -a^2$ in D^r only

ie $\frac{1}{f(D^2)}(\sin ax) = \frac{1}{f(-a^2)} \sin ax$

$= \frac{x}{f'(-a^2)}(\sin ax)$ ......

Case III: if $Q = x^m$, $m \in I^+$ then use Binomial Th of following type

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}(x^3) + \ldots\ldots$

where $n = -$ve int or fraction.

eg $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots$

What is the initial value if $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x}$,

with $y(0) = 2, \left(\frac{dy}{dx}\right)_{x=0} = 1$?

(a) $y(x) = \left(\frac{13}{4} + \frac{1}{2}x\right)e^{-x} - \frac{5}{4}e^{-3x}$ (As per students)

(b) $y(x) = \left(\frac{13}{4} + \frac{1}{2}x\right)e^{-3x} - \frac{5}{4}e^{-x}$

(c) $y(x) = \left(\frac{13}{4} + \frac{1}{2}x\right)e^{-x} + \frac{5}{4}e^{-3x}$

(d) $y(x) = \left(\frac{13}{4} - \frac{1}{2}x\right)e^{-x} - \frac{5}{4}e^{-3x}$

S. form: $(D^2+4D+3)y = e^{-x}$

$PI = \frac{1}{f(D)}(x) \Rightarrow \frac{1}{D^2+4D+3}(e^{-1x})$

$= \frac{x}{2D+4}(e^{-1x}) = \frac{x}{2(-1)+4}(e^{-x})$

$= \frac{x}{2}e^{-x}$

AE is $m^2+4m+3=0$

$m^2+3m+m+3=0 \Rightarrow m=-1,-3$

$CF = c_1e^{-x}+c_2e^{-3x}$ & G.Soln is $y = c_1e^{-x}+c_2e^{-3x}+\frac{x}{2}e^{-x}$

How use Boundary Condn answer in → Do yourself

The particular integral of

$(D^2 - 9)y = e^{3x} + \sin 2x$ is

(a) $\frac{e^{3x}}{6} + \frac{\sin 2x}{13}$ (b) $\frac{e^{3x}}{6} - \frac{1}{32}\sin 2x$

(c) $\frac{e^{3x}}{6} - \frac{1}{13}\sin 2x$ (d) $\frac{xe^{3x}}{6} - \frac{1}{13}\sin 2x$ ✓

$$PI = \frac{1}{D^2-9}(e^{3x}) = \frac{x}{2D}(e^{3x}) = \frac{x}{2(3)}e^{3x}$$

The particular integral of the differential equation $(D^2-4D+3)y=\cos x$ is

(a) $\frac{\cos x-2\sin x}{10}$ (b) $\frac{\cos x+2\sin x}{10}$

(c) $\frac{2\cos x-4\sin x}{5}$ (d) $\frac{2\cos x+4\sin x}{5}$

$$PI=\frac{1}{f(D)}\,Q=\frac{1}{D^2-4D+3}(\cos 1x)$$

$$=\frac{1}{(-1^2)-4D+3}(\cos x)$$

$$=\frac{1}{-4D+2}(\cos x)$$

$$=\frac{1}{2}\left[\frac{1}{1-2D}\cos x\right]$$

$$=\frac{1}{2}\left[\frac{1+2D}{1-4D^2}\cos x\right]$$

$$=\frac{1}{2}\left[\frac{(1+2D)}{1-4(-1^2)}\right]\cos x$$

$$=\frac{1}{10}\left[\cos x+2(-\sin x)\right]$$

The respective expressions for complimentary function and particular integral part of the solution of the differential equation

$$\frac{d^4y}{dx^4} + 3\frac{d^2y}{dx^2} = 108x^2 \text{ are}$$

(a) $\left[c_1 + c_2x + c_3 \sin\sqrt{3x} + c_4 \cos\sqrt{3x}\right]$ and $\left[3x^4 - 12x^2 + c\right]$

(b) $\left[c_2 + c_3 \sin\sqrt{3x} + c_4 \cos\sqrt{3x}\right]$ and $\left[5x^4 - 12x^2 + c\right]$

(c) $\left[c_1 + c_3 \sin\sqrt{3x} + c_4 \cos\sqrt{3x}\right]$ and $\left[3x^4 - 12x^2 + c\right]$

(d) $\left[c_1 + c_2x + c_3 \sin\sqrt{3x} + c_4 \cos\sqrt{3x}\right]$ and $\left[5x^4 - 12x^2 + c\right]$

S. Form is $(D^4+3D^2)y = 108x^2$

$$PI = \frac{1}{f(D)}Q = \frac{1}{D^4+3D^2}(108x^2) = \frac{1}{3D^2\left(1+\frac{D^2}{3}\right)}(108x^2)$$

$$= \frac{108}{3}\left[\frac{1}{D^2}\left(1+\frac{D^2}{3}\right)^{-1}(x^2)\right]$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$$

$$= 36\left(\frac{1}{D^2}\left\{1-\left(\frac{D^2}{3}\right)+\left(\frac{D^2}{3}\right)^2-\left(\frac{D^2}{3}\right)^3+\cdots\right\}\right](x^2)$$

$$= 36\left(\frac{1}{D^2}\left\{x^2-\frac{2}{3}+0-0+0\cdots\right\}\right]$$

$$PI = 36\left[\frac{1}{D}\left(\frac{x^3}{3}-\frac{2}{3}x\right)\right] = 36\left[\frac{x^4}{12}-\frac{x^2}{3}\right]$$

$$= 3x^4 - 12x^2$$

Cauchy's L.D.E with variable coeff

consider the D.Eq. $\left[a_0 x^2 D^2 + a_1 x D + a_2\right] y = Q$ —— (1)

where $D = \frac{d}{dx}$. it is LDE with variable coeff in y & x

Now use, $xD = D_1$, $x^2 D^2 = D_1(D_1 - 1)$, $x^3 D^3 = D_1(D_1-1)(D_1-2)$ ----

DEq (1) will convert into LDE with Constant coeff in y & z

where $D = \frac{d}{dx}$, $D_1 = \frac{d}{dz}$ & the transformation is $x = e^z$ or $z = \log x$

$x = e^z \Rightarrow z = \ln x$

(A)

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{dz}\left(\frac{1}{x}\right)$$

$$x\frac{dy}{dx} = \frac{dy}{dz}$$

$$x\frac{d}{dx} = \frac{d}{dz}$$

$$\boxed{xD = D_1}$$

(A) $\boxed{x^2D^2 = D_1(D_1 - 1)}$

Proof: No Need

General solution of the Cauchy-Euler equation :

$$x^2 \frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 16y = 0 \text{ is}$$

it is not with Const Coeff so CF+PI ×

(a) $y = C_1x^2 + C_2x^4$

(b) $y = C_1x^2 + C_2x^{-4}$

(c) $y = (C_1 + C_2 \ln x)x^4$

(d) $y = C_1x^4 + C_2x^{-4} \ln x$

Given D Eq is $\left[x^2D^2 - 7xD + 16\right]y = 0$ → (1)

Using Cauchy's Transformations,

$\left[D_1(D_1-1) - 7D_1 + 16\right]y = 0$

$\boxed{(D_1^2 - 8D_1 + 16)y = 0}$ → (2)

Now it is LDE in y & z with Const Coeff

AEq of (2), $m^2 - 8m + 16 = 0 \Rightarrow m = 4, 4$

$CF = (C_1 + C_2 z)e^{4z}$

$PI = 0$

G. Soln of (1) is $y = CF + PI$

$y = (C_1 + C_2 z)e^{4z}$

$y = (C_1 + C_2 \log x)(x)^4$

2019 : 2

The solution of the differential equation

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x \text{ is}$$

(a) $y = (C_1 + C_2x)\log x + 2\log x + 3$

(b) $y = (C_1 + C_2x^2)\log x + \log x + 2$

(c) $y = (C_1 + C_2x)\log x + \log x + 2$

(d) $y = (C_1 + C_2 \log x)x + \frac{x}{2}(\log x)^2$

$(x^2D^2 - xD + 1)y = \log x$ — (1)

$(D_1(D_1-1) - D_1 + 1)y = e^z$

$(D_1^2 - 2D_1 + 1)y = e^z$ — (2)

AE is $m^2 - 2m + 1 = 0 \Rightarrow m = 1, 1$

$CF = (C_1 + C_2 z)e^z = (C_1 + C_2\log x).x$

$PI = \frac{1}{f(D_1)}(e^z) = \frac{1}{(D_1-1)^2}(e^z)$

$= \frac{z^2}{2}e^z = \frac{1}{2}x(\log x)^2$

i.e. sol is $y = (C_1 + C_2\log x)x + \frac{x}{2}(\log x)^2$

Thank

THANK

Keep Hustling!