Artificial Intelligence

Propositional logic & Reasoning under uncertainty
1500+ series

Lecture - 06

By - Siddharth Sabharwal Sir

P(C)

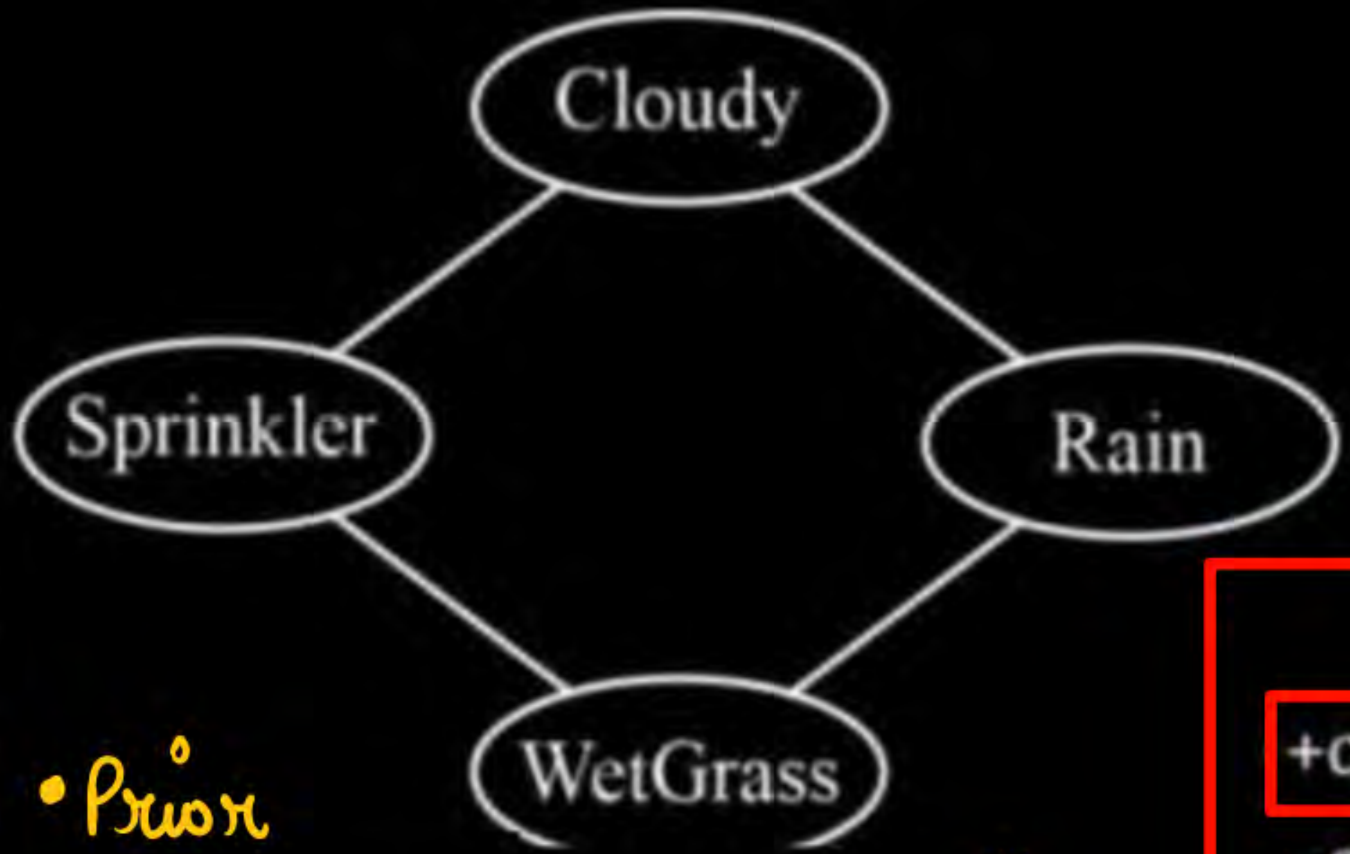
+c	0.5
−c	0.5

P(S|C)

+c	+s	0.1
	−s	0.9
−c	+s	0.5
	−s	0.5

P(R|C)

+c	+r	0.8
	−r	0.2
−c	+r	0.2
	−r	0.8



P(W|S, R)

+s	+r	+w	0.99
		+w	0.01
	−r	+w	0.90
		+w	0.10
+s	+r	+w	0.90
		+w	0.10
	−r	+w	0.01
		+c	0.99

• Prior

estimate $P[C/rw] \Rightarrow \frac{2}{3}$ ✓

Samples:

+c,	−s,	+r,	+w
−c,	+s,	+r,	+w
−c,	+s,	+r,	−w
+c,	−s,	+r,	+w
−c,	−s,	−r,	+w

P(C)

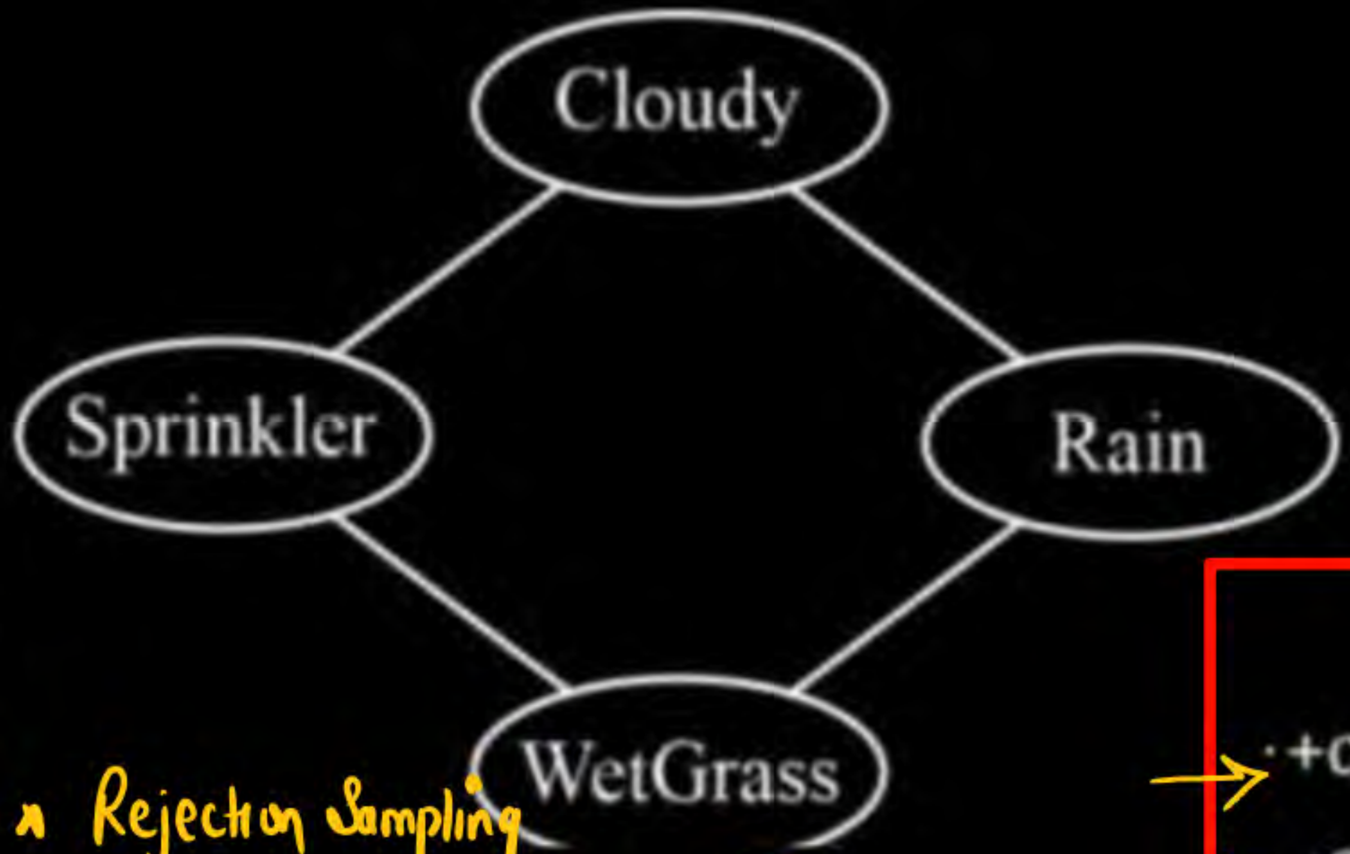
+c	0.5
−c	0.5

P(S|C)

+c	+s	0.1
	−s	0.9
−c	+s	0.5
	−s	0.5

P(R|C)

+c	+r	0.8
	−r	0.2
−c	+r	0.2
	−r	0.8



P(W|S, R)

+s	+r	+w	0.99
		+w	0.01
	−r	+w	0.90
		+w	0.10
+s	+r	+w	0.90
		+w	0.10
	−r	+w	0.01
		+c	0.99

* Rejection Sampling

estimate P[C/rw] ⇒ 2/3

Samples:

+c,	−s,	+r,	+w
−c,	+s,	+r,	+w
−c,	+s,	+r,	−w
+c,	−s,	+r,	+w
−c,	−s,	−r,	+w

- weight of any sample $\Rightarrow$ Product of P (evidence/Parent)

$$P = \frac{.72 + .72}{.72 + .72 + .198}$$

$\boxed{.879}$ ✓

P(C)

+c	0.5
−c	0.5

P(S|C)

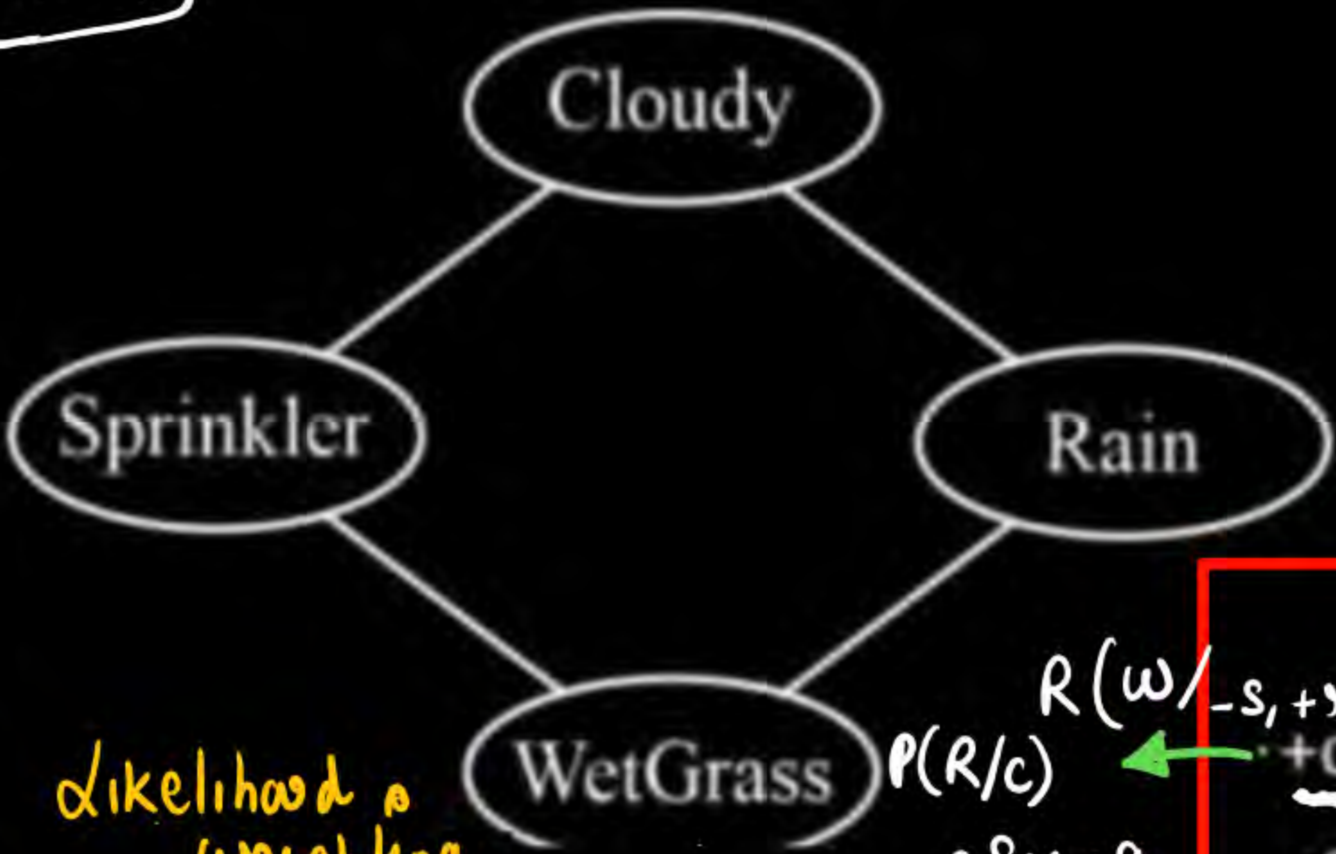
+c	+s	0.1
	−s	0.9
−c	+s	0.5
	−s	0.5

P(R|C)

+c	+r	0.8
	−r	0.2
−c	+r	0.2
	−r	0.8

P(W|S, R)

+s	+r	+w	0.99
		−w	0.01
	−r	+w	0.90
		−w	0.10
−s	+r	+w	0.90
		−w	0.10
	−r	+w	0.01
		−w	0.99



Likelihood weighting

estimate P[C/rw] ⇒

we have 2 evidence r, w

$P(R/-c) \times P(W/SR)$

$.2 \times .99$ $\quad .198$

$R(W/-s,+r)$ $\quad .72$

$P(R/c)$

$.8 \times .9$

Samples:

+c,	−s,	+r,	+w	
−c,	+s,	+r,	+w	
−c,	+s,	+r,	−w	×
+c,	−s,	+r,	+w	
−c,	−s,	−r,	+w	×

$P(W/R-s)\,P(R/c)$

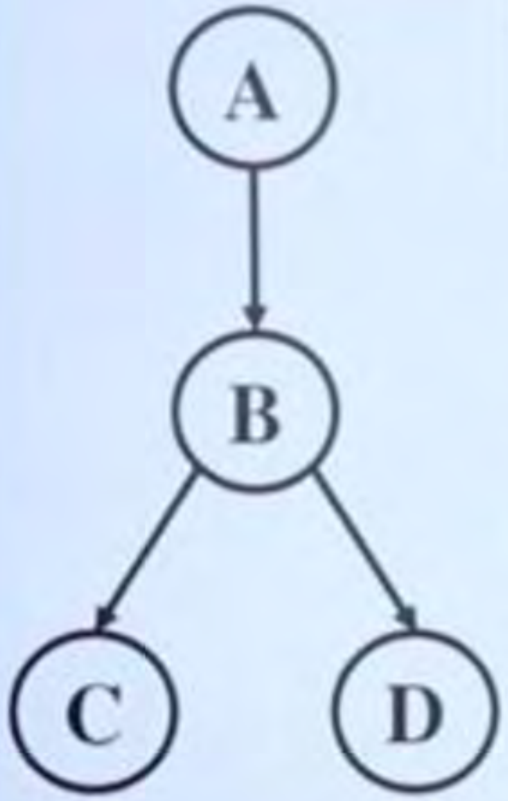
$.9 \times .8 \quad .72$

A	P(A)
a	1/5
¬a	4/5

A	B	P(B|A)
a	b	1/5
¬a	b	1/2
a	¬b	4/5
¬a	¬b	1/2

B	C	P(C|B)
b	c	1/4
¬b	c	3/4
b	¬c	2/5
¬b	¬c	3/5

B	D	P(D|B)
b	d	1/2
¬b	d	4/5
b	¬d	1/2
¬b	¬d	1/5



s_1: $(\neg a, \neg b, c, d)$;

s_2: $(a, \neg b, \neg c, d)$;

s_3: $(\neg a, \neg b, c, \neg d)$;

s_4: $(\neg a, \neg b, \neg c, \neg d)$

s_5: $(\neg a, b, c, d)$;

s_6: $(a, \neg b, \neg c, \neg d)$;

s_7: $(\neg a, b, \neg c, d)$;

s_8: $(a, b, \neg c, \neg d)$

→ only 3 Sample Pass

Cross out the samples that would be rejected by rejection sampling to estimate $P(a \mid \neg c, \neg d)$

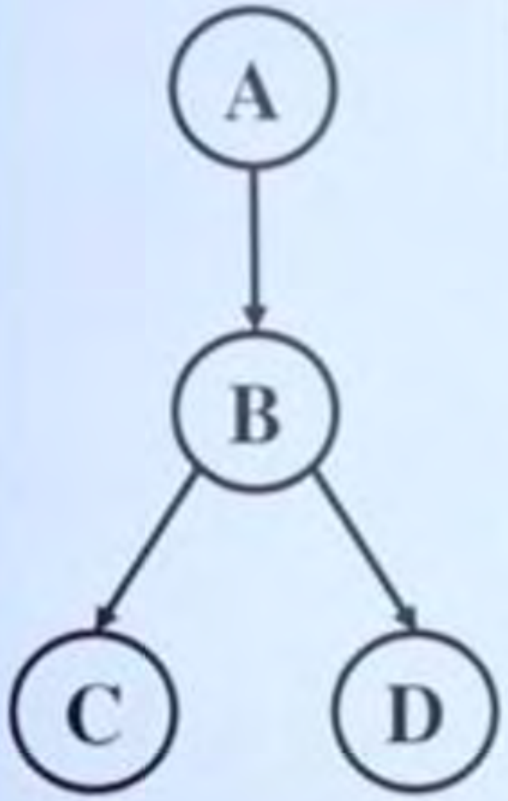
→ • $(P(a/\neg c, \neg d) = 2/3)$

A	P(A)
a	1/5
¬a	4/5

A	B	P(B|A)
a	b	1/5
¬a	b	1/2
a	¬b	4/5
¬a	¬b	1/2

B	C	P(C|B)
b	c	1/4
¬b	c	3/4
b	¬c	2/5
¬b	¬c	3/5

B	D	P(D|B)
b	d	1/2
¬b	d	4/5
b	¬d	1/2
¬b	¬d	1/5



likelihood weighting

s_1: $(\neg a, \neg b, c, d)$; ×

s_2: $(a, \neg b, \neg c, d)$; ×

s_3: $(\neg a, \neg b, c, \neg d)$; ×

s_4: $(\neg a, \neg b, \neg c, \neg d)$ ✓

↳ $P[\neg c/\neg b]\, P[\neg d/\neg b] = 3/5 \times 1/5$

s_5: $(\neg a, b, c, d)$; × ✓

• s_6: $(a, \neg b, \neg c, \neg d)$; → $3/5 \times 1/5$

s_7: $(\neg a, b, \neg c, d)$; ×

• s_8: $(a, b, \neg c, \neg d)$ ✓ → $2/5 \times 1/2$

Cross out the samples that would be rejected by rejection sampling to estimate $P(a \mid \neg c, \neg d)$

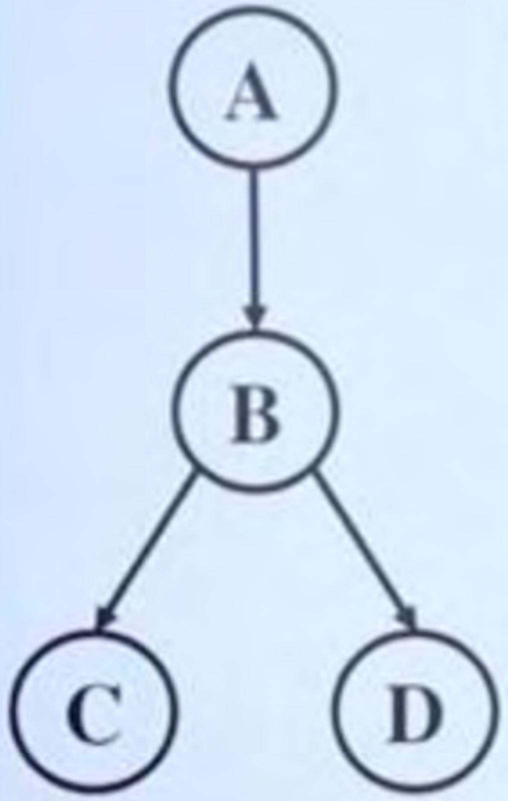
$$\frac{3/25 + 1/5}{3/25 + 3/25 \times \frac{1}{5}} \Rightarrow \frac{8}{11}$$

A	P(A)
a	1/5
¬a	4/5

A	B	P(B|A)
a	b	1/5
¬a	b	1/2
a	¬b	4/5
¬a	¬b	1/2

B	C	P(C|B)
b	c	1/4
¬b	c	3/4
b	¬c	2/5
¬b	¬c	3/5

B	D	P(D|B)
b	d	1/2
¬b	d	4/5
b	¬d	1/2
¬b	¬d	1/5



s_1: $(\neg a, \neg b, c, d)$;
s_2: $(a, \neg b, \neg c, d)$;
s_3: $(\neg a, \neg b, c, \neg d)$;
s_4: $(\neg a, \neg b, \neg c, \neg d)$
s_5: $(\neg a, b, c, d)$;
s_6: $(a, \neg b, \neg c, \neg d)$;
s_7: $(\neg a, b, \neg c, d)$;
s_8: $(a, b, \neg c, \neg d)$

Cross out the samples that would be rejected by rejection sampling to estimate $P(a \mid \neg c, \neg d)$

Samples:

s_1: $(\neg a, \neg b, c, d)$;

s_2: $(a, \neg b, \neg c, d)$;

s_3: $(\neg a, \neg b, c, \neg d)$;

s_4: $(\neg a, \neg b, \neg c, \neg d)$;

s_5: $(\neg a, b, c, d)$;

s_6: $(a, \neg b, \neg c, \neg d)$;

s_7: $(\neg a, \boxed{b, \neg c}, d)$;

s_8: $(a, b, \neg c, \neg d)$;

s_9: $(\neg a, \neg b, \neg c, d)$;

s_{10}: $(a, \neg b, c, \neg d)$;

Queries:

$P(\neg c) \rightarrow \frac{6}{10}$

$P(\neg a, b) \rightarrow 2/10$

$P(\neg b \mid \neg d) \rightarrow \frac{4}{5}$

$P(a \mid b, \neg c) \rightarrow 1/2$

Likehihood weigthing

Samples:

s_1: $(\neg a, \neg b, c, d)$; ✗

s_2: $(a, \neg b, \neg c, d)$; ✗

.2 $P(\neg d/\neg b)$ ← s_3: $(\neg a, \neg b, c, \neg d)$; ✓

.2 $P(\neg d/\neg b)$ ← s_4: $(\neg a, \neg b, \neg c, \neg d)$; ✓

s_5: $(\neg a, b, c, d)$; ✗

.2 $P(\neg d/\neg b)$ ← s_6: $(a, \neg b, \neg c, \neg d)$; ✓

s_7: $(\neg a, b, \neg c, d)$; ✗

.5 $P(\neg d/b)$ ← s_8: $(a, b, \neg c, \neg d)$; NOT

s_9: $(\neg a, \neg b, \neg c, d)$; ✗

.2 $P(\neg d/\neg b)$ ← s_{10}: $(a, \neg b, c, \neg d)$; ✓

Queries:

$P(\neg c)$ → 6/10

$P(\neg a, b)$ → 2/10

No evidence thus all Samples of equal weight

$P(\neg b \mid \neg d)$ → $\frac{.8}{.8+.5}$

weight $P[\neg d/\text{Parent}]$

$P(a \mid b, \neg c)$

Likehilood weigthing

Samples:

s_1: $(\neg a, \neg b, c, d)$;

s_2: $(a, \neg b, \neg c, d)$;

s_3: $(\neg a, \neg b, c, \neg d)$;

s_4: $(\neg a, \neg b, \neg c, \neg d)$;

s_5: $(\neg a, b, c, d)$;

s_6: $(a, \neg b, \neg c, \neg d)$;

s_7: $(\neg a, b, \neg c, d)$; ✓

s_8: $(a, b, \neg c, \neg d)$; ✓

s_9: $(\neg a, \neg b, \neg c, d)$;

s_{10}: $(a, \neg b, c, \neg d)$;

Queries:

$P(\neg c)$ → 6/10

$P(\neg a, b)$ → 2/10

No evidence thus all Samples of equal weight

$P(\neg b \mid \neg d)$ weight $P[\neg d / \text{Parent}]$

$P(a \mid b, \neg c)$ → $P(b/A)\ P(\neg C/B)$

$P(b/\neg A)\ P(\neg C/B) \rightarrow \frac{1}{2} \times 2/5 = 1/5$

$P(b/a)\ P(\neg C/B) = 1/5 \times 2/5 = 2/25$

$$P = \frac{2/25}{2/25 + \frac{1}{5}} = .285$$

P(C)

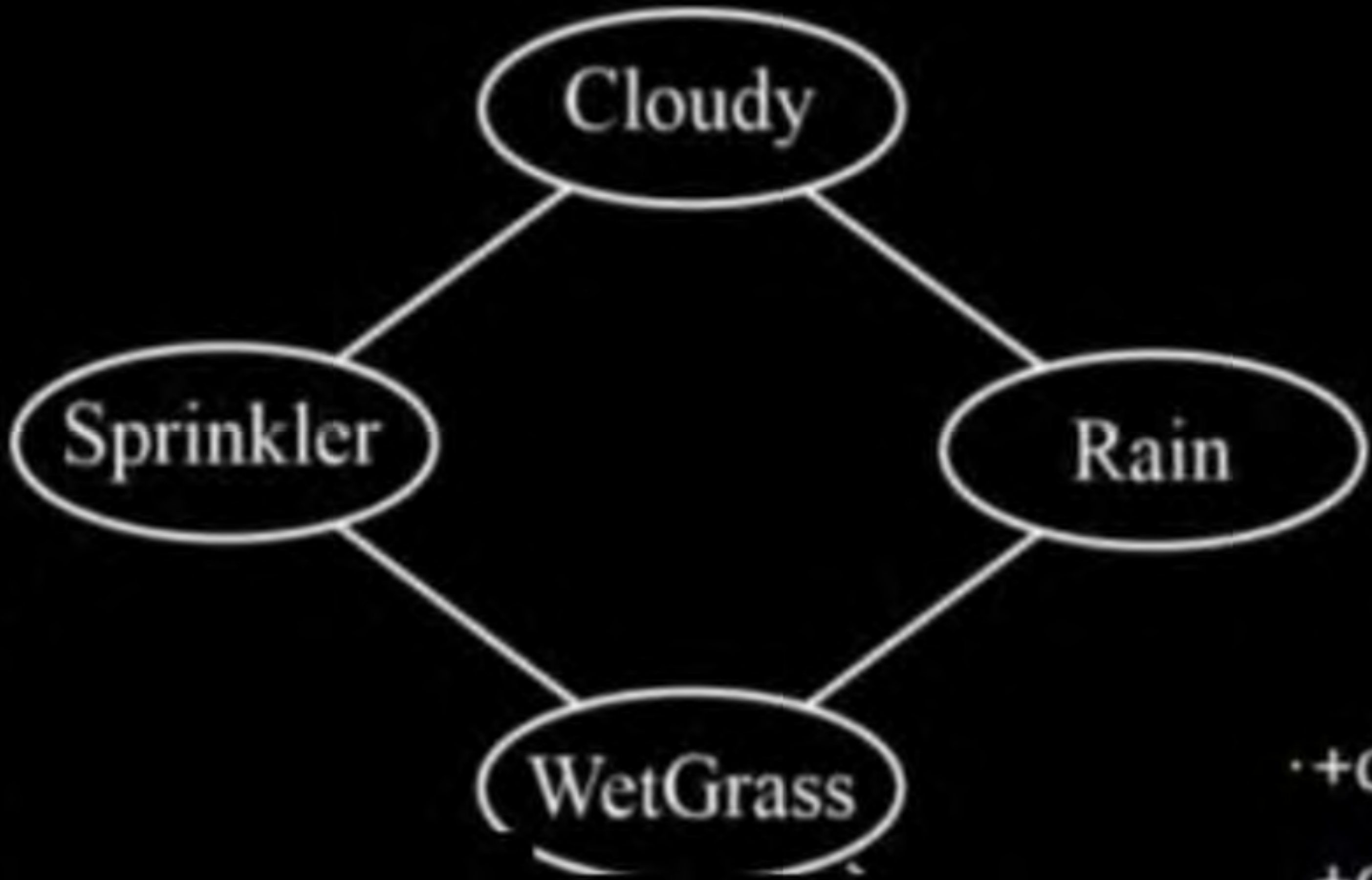
+c	0.5
−c	0.5

P(S|C)

+c	+s	0.1
	−s	0.9
−c	+s	0.5
	−s	0.5

P(R|C)

+c	+r	0.8
	−r	0.2
−c	+r	0.2
	−r	0.8



P(W|S, R)

+s	+r	+w	0.99
		+w	0.01
	−r	+w	0.90
		+w	0.10
+s	+r	+w	0.90
		+w	0.10
	−r	+w	0.01
		+c	0.99

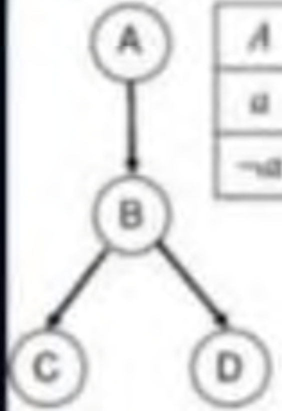
Samples:

+c, −s, +r, +w
+c, +s, +r, +w
−c, +s, +r, −w
+c, −s, +r, +w
−c, −s, −r, +w

Consider the Bayes Net containing four Boolean random variables (A, B, C, D), with the following convention:

$A = True \Rightarrow A = a, and\ A = False \Rightarrow A = \neg a$

and similarly for the other variables. The conditional probability tables for the nodes in the network are also indicated in the figure. The following samples were generated through likelihood weighting:



A	$P(A)$
a	1/5
$\neg a$	4/5

A	B	$P(B\|A)$
a	b	1/5
$\neg a$	b	1/2
a	$\neg b$	4/5
$\neg a$	$\neg b$	1/2

B	C	$P(C\|B)$
b	c	1/4
$\neg b$	c	3/4
b	$\neg c$	2/5
$\neg b$	$\neg c$	3/5

B	D	$P(D\|B)$
b	d	1/2
$\neg b$	d	4/5
b	$\neg d$	1/2
$\neg b$	$\neg d$	1/5

$s_1: (\neg a, \neg b, \neg c, \neg d); s_2: (\neg a, b, \neg c, \neg d);\ s_3: (\neg a, \neg b, \neg c, d); s_4: (\neg a, b, \neg c, d)$

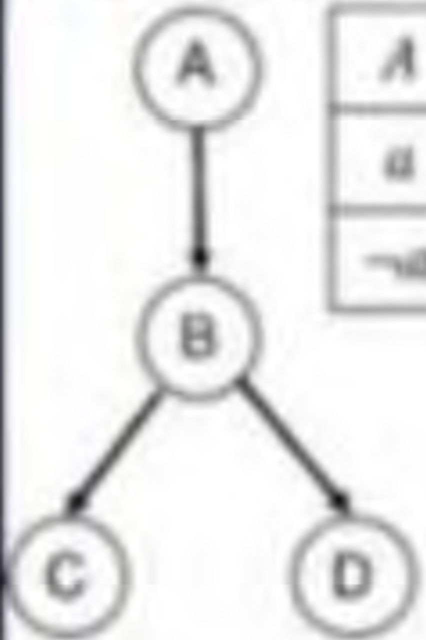
Estimate the likelihood weight of each sample and thereby estimate $P(b|\neg a, \neg c)$

(A)	$s_1: 0.48, s_2: 0.32, s_3: 0.48, s_4: 0.32, P(b\|\neg a, \neg c) = 0.4$
(B)	$s_1: 0.48, s_2: 0.32, s_3: 0.48, s_4: 0.32, P(b\|\neg a, \neg c) = 0.64$
(C)	$s_1: 0.32, s_2: 0.48, s_3: 0.48, s_4: 0.32, P(b\|\neg a, \neg c) = 0.64$
(D)	$s_1: 0.48, s_2: 0.32, s_3: 0.32, s_4: 0.32, P(b\|\neg a, \neg c) = 0.4$

Consider the Bayes Net containing four Boolean random variables (A, B, C, D), with the following convention:

$A = True \Rightarrow A = a, and\ A = False \Rightarrow A = \neg a$

and similarly for the other variables. The conditional probability tables for the nodes in the network are also indicated in the figure. The following samples were generated through likelihood weighting:



A	P(A)
a	1/5
$\neg a$	4/5

A	B	P(B|A)
a	b	1/5
$\neg a$	b	1/2
a	$\neg b$	4/5
$\neg a$	$\neg b$	1/2

B	C	P(C|B)
b	c	1/4
$\neg b$	c	3/4
b	$\neg c$	2/5
$\neg b$	$\neg c$	3/5

B	D	P(D|B)
b	d	1/2
$\neg b$	d	4/5
b	$\neg d$	1/2
$\neg b$	$\neg d$	1/5

$s_1: (\neg a, \neg b, \neg c, \neg d); s_2: (\neg a, b, \neg c, \neg d); s_3: (\neg a, \neg b, \neg c, d); s_4: (\neg a, b, \neg c, d)$

Estimate the likelihood weight of each sample and thereby estimate

$P(b|\neg a, \neg c)$

$S_1 \Rightarrow P[\neg a]\, P[\neg c / \neg b]$

$= \frac{4}{5} \times 3/5 = 12/25 \quad .48$

$S_2 = P[\neg a]\, P[\neg c / b] =$

$= 4/5 \times 2/5 \quad .32$

$S_3 = P[\neg a]\, P[\neg c / \neg b]$

$= 4/5 \times 3/5 \quad .48$

$S_4 = P[\neg a]\, P[\neg c / b]$

$= 4/5 \times 2/5 \quad .32$

$.32 + .32 / (.48 + .32)\, 2 \rightarrow .4$

Topic : Proposition and predicate & Bayesian network

#Q. Assume the following Bayes* net, and the corresponding distributions over the variables in the Bayes' net:

$$\left\{ \frac{S_2+S_6}{S_2+S_3+S_4+S_6+S_8} \right. \Leftarrow P[+A/+c,-d]$$

$$A \longrightarrow B \longrightarrow C \longrightarrow D$$

5/8

$S_2 \to P(c/-b)\,P(-d/c) = \frac{5}{6} \times \frac{3}{4}$

$S_3 \to P(c/b)\,P(-d/c) = \frac{1}{2} \times \frac{5}{6}$

5/12

P(4)	
−a	3/4
+a	1/4

P(B|A)		
−a	−b	2/3
−a	+b	1/3
+a	−b	4/5
+a	+b	1/5

P(C|B)		
−b	−c	1/4
−b	+c	3/4
+b	−c	1/2
+b	+c	1/2

P(D|C)		
−c	−d	1/8
−c	+d	7/8
+c	−d	5/6
+c	+d	1/6

+a	+b	−c	−d ✓
+a	−b	+c	−d ✓
−a	+b	+c	−d ✓
−a	−b	+c	−d ✓

+a	−b	−c	+d ✗
+a	+b	+c	−d ✓
−a	+b	−c	+d ✗
−a	−b	+c	−d ✓

$S_6 = S_3$ 5/12

$S_8 = S_2$ 5/8

$S_4 = P(c/-b)\,P(-d/c) = \frac{5}{6} \times \frac{3}{4} = \frac{5}{8}$

(a) You are given the following samples:

+a	+b	−c	−d	+a	−b	−c	+d
+a	−b	+c	−d	+a	+b	+c	−d
−a	+b	+c	−d	−a	+b	−c	+d
−a	−b	+c	−d	−a	−b	+c	−d

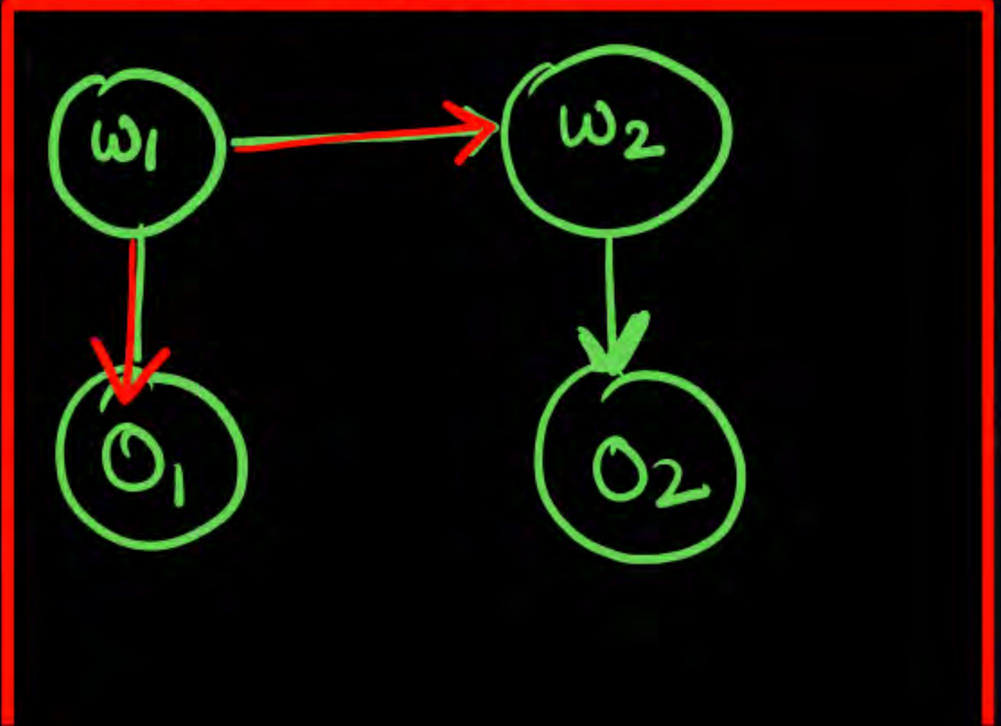
(i) Assume that these samples came from performing Prior Sampling, and calculate the sample estimate of P(+c).

5/8

(ii) Now we will estimate P(+c | +a, −d). Above, clearly cross out the samples that would not be used when doing Rejection Sampling for this task, and write down the sample estimate of P(+c | +a, −d) below.

2/3

#Q. Consider the following Hidden Markov Model. O_1 and O_2 are supposed to be shaded.



initial ✓

W1	P(W1)
0	0.3
1	0.7

Wt	Wt+1	P(Wt+1|Wt)
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t\|W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

$P(\omega_2 / O_1=a, O_2=b)$

$$\frac{P(\omega_2, O_1=a, O_2=b)}{P(O_1=a, O_2=b)} \Rightarrow P(O_2=b/\omega_2, O_1=a)\, P(O_1=a, \omega_2) = P(O_2=b/\omega_2)$$

$$\frac{P(\omega_2, O_1=a, O_2=b)}{P(O_1=a, O_2=b)}$$

$$P(O_1=a, O_2=b, \omega_1 \omega_2) + P(O_1=a, O_2=b, \omega_1, \tilde{\omega}_2) + P(O_1=a, O_2=b, \tilde{\omega}_1, \omega_2) + P(O_1=a, O_2=b, \tilde{\omega}_1, \tilde{\omega}_2)]$$

$$P(O_1=a/\omega_1)\,P(O_2=b/\omega_2)\,P(\omega_2/\omega_1)\ P(\omega_1)$$

$$.5 \times .5 \times \ .2 \times .7$$

$$.5 \times .1 \times .8 \times .7$$

$$.9 \times .5 \times .6 \times .3$$

$$.9 \times .1 \times .4 \times .3$$

$$.1548$$

$$\text{Ans} = \frac{.116}{.1548}$$

$$\frac{P(\omega_2, O_1=a, O_2=b)}{P(O_1=a, O_2=b)}$$

$$\Rightarrow P(O_2=b/\omega_2, O_1=a)\, P(O_1=a, \omega_2)$$

$$P(O_2=b/\omega_2)\left[P(O_1=a, \omega_1, \omega_2) + P(O_1=a, \tilde{\omega}_1, \omega_2)\right]$$

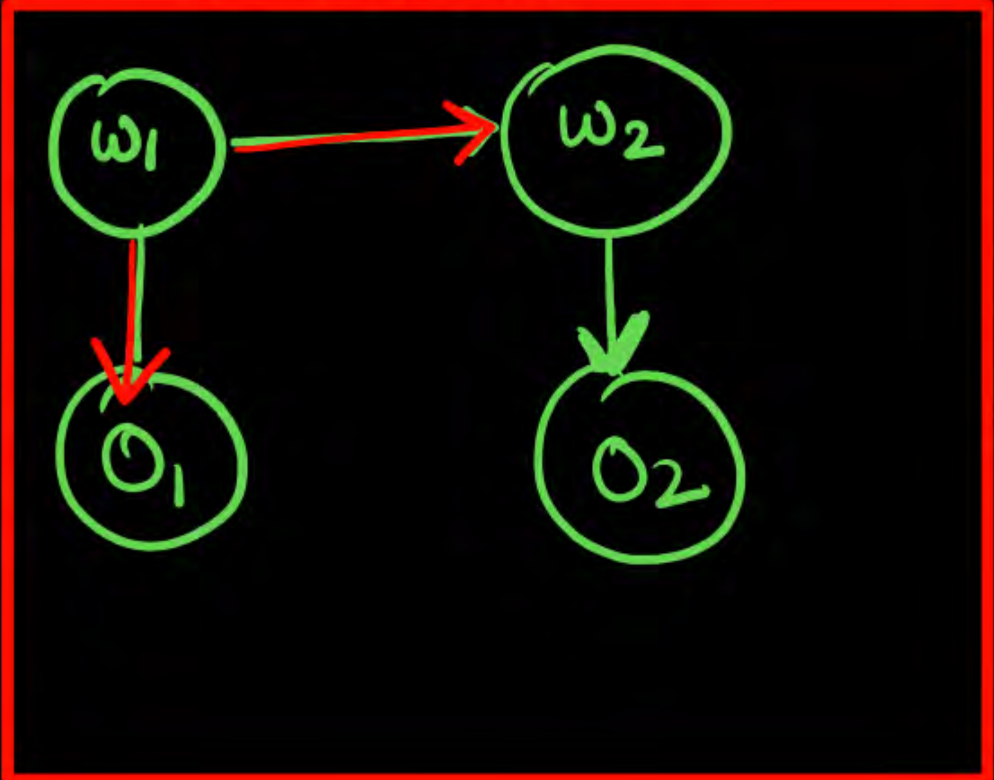
$$P[O_2=b/\omega_2]\left[P(O_1=a/\omega_1)\, P(\omega_2/\omega_1)\, P(\omega_1) + P(O_1=a/\tilde{\omega}_1)\, P(\omega_2/\tilde{\omega}_1)\, P(\tilde{\omega}_1)\right]$$

$$= .5\left[.5 \times .2 \times .7 + .9 \times .6 \times .3\right]$$

$$= .116$$

$$P(O_1=a, O_2=b, \omega_1 \omega_2) + P(O_1=a, O_2=b, \omega_1, \tilde{\omega}_2) + P(O_1=a, O_2=b, \tilde{\omega}_1, \omega_2) + P(O_1=a, O_2=b, \tilde{\omega}_1, \tilde{\omega}_2)]$$

#Q. Consider the following Hidden Markov Model. O_1 and O_2 are supposed to be shaded.



initial ✓

W1	P(W1)
0	0.3
1	0.7

✓

Wt	Wt+1	P(Wt+1|Wt)
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t\|W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

$P(\omega_2 | O_1 = a, O_2 = b)$

Suppose that we observe $\boxed{O_1 = a \text{ and } O_2 = b.}$ ✓

Using She forward algorithm, compute the probability distribution $\boxed{P(W_2|O_1 = a, O_2 = b)}$ one step at a time.

Topic : Proposition and predicate & Bayesian network

#Q. Consider the following Hidden Markov Model. O_1 and O_2 are supposed to be shaded.

[redacted]

W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1} \mid W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t \mid W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

[redacted]

Topic : Proposition and predicate & Bayesian network

#Q. Consider a Bayesian network with three binary variables A B and C. The join probability distribution P(A, B, C) is given in the table below:

A	B	c	P(A, B, C)
0	0	0	0.3
0	0	1	0.7
0	1	0	0.3
0	1	1	0.3
1	0	0	0.3
1	0	1	0.7
1	1	0	0.3
1	1	1	0.3

Using variable elimination, calculate the conditional probability $P(A = 1 | B = 1)$.

Topic : Proposition and predicate & Bayesian network

#Q. Consider a Bayesian network representing climates patterns, there are two variables: W(weather) and R(rain). The conditional probability tables are follows:

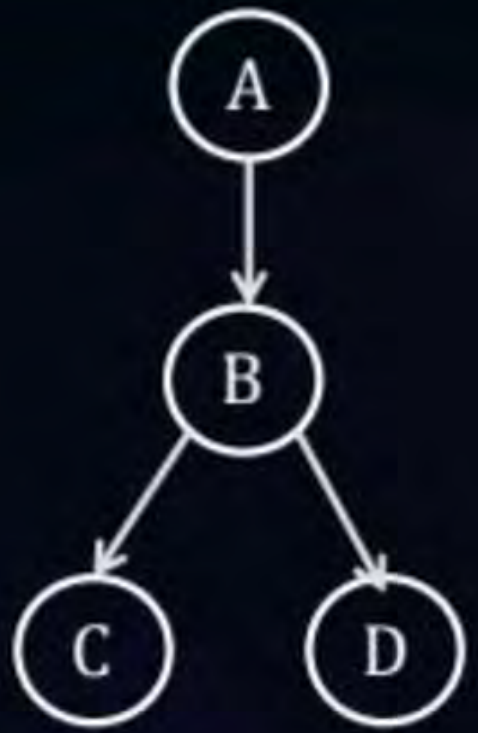
W(weather)	Probability P(W)
Sunny	0.6
Cloudy	0.4

Rain(R)	W(weather)	Probability (R =1|W)
1	Sunny	0.3
1	Cloudy	0.7

Compute the marginal probability P(R= 1)__________. (Upto to 2 decimal)

Topic : Proposition and predicate & Bayesian network

#Q. Given the following Bayesian Network consisting of four Bernoulli random variables and the associated conditional probability tables:



	P(·)
A = 0	0.6
A =1	0.4

	P (B = 0|.)	P (B = 1|.)
A = 0	0.6	0.6
A =1	0.4	0.4

	P (C = 0|.)	P (C = 1|.)
B = 0	1	0
B =1	0	1

	P (D = 0|.)	P (B = 1|.)
B = 0	0.6	0.6
B =1	1	1

The value of P(A=1, B=0, C=0, D = 1) ____(Rounded off to three decimal places)

Topic : Proposition and predicate & Bayesian network

#Q. Consider the process of likelihood weighting in approximate Bayesian inference.
Which of the above statements is/are correct?

A. Likelihood weighting samples from the prior distribution of variables.

B. Likelihood weighting assigns weights to samples based on how well they match the observed evidence.

C. Likelihood weighting always guarantees exact posterior probabilities.

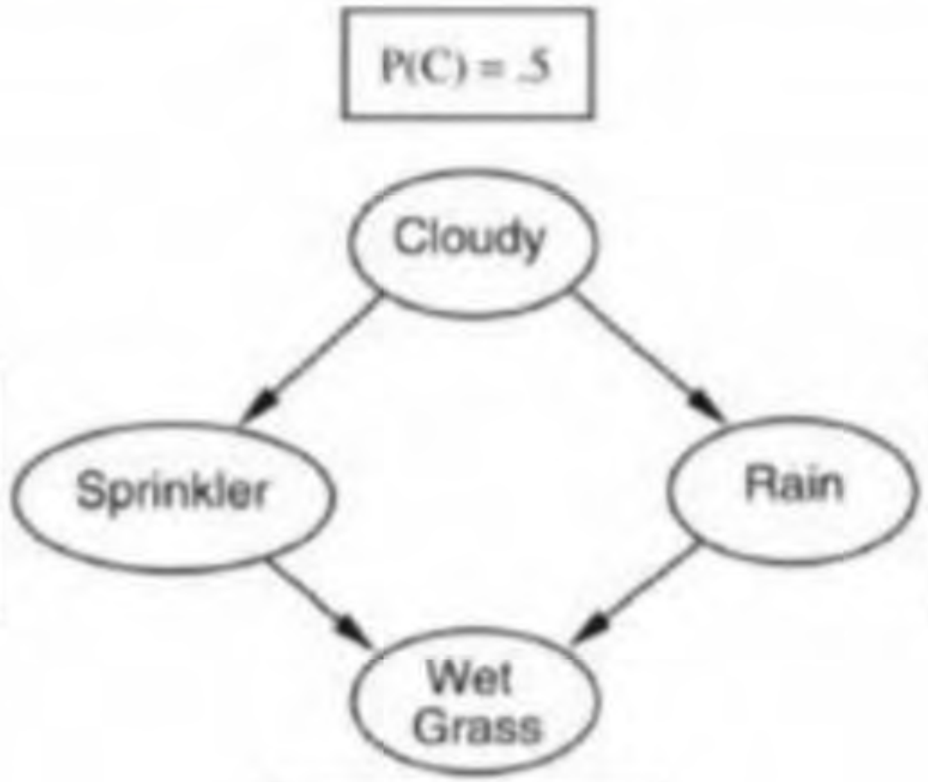
D. Likelihood weighting is computationally efficient for large Bayesian networks.

#Q. Consider a Bayesian network with two variables: A and B. The joint probability distribution is given as follows:

Using variable elimination, calculate the marginal probability P(B = 1).

A	B	P(A,B)
0	0	0.2
0	1	0.3
1	0	0.1
1	1	0.4

3) Consider the following Bayesian Network. Suppose you are doing likelihood sampling to determine P(S| ¬ C,W).



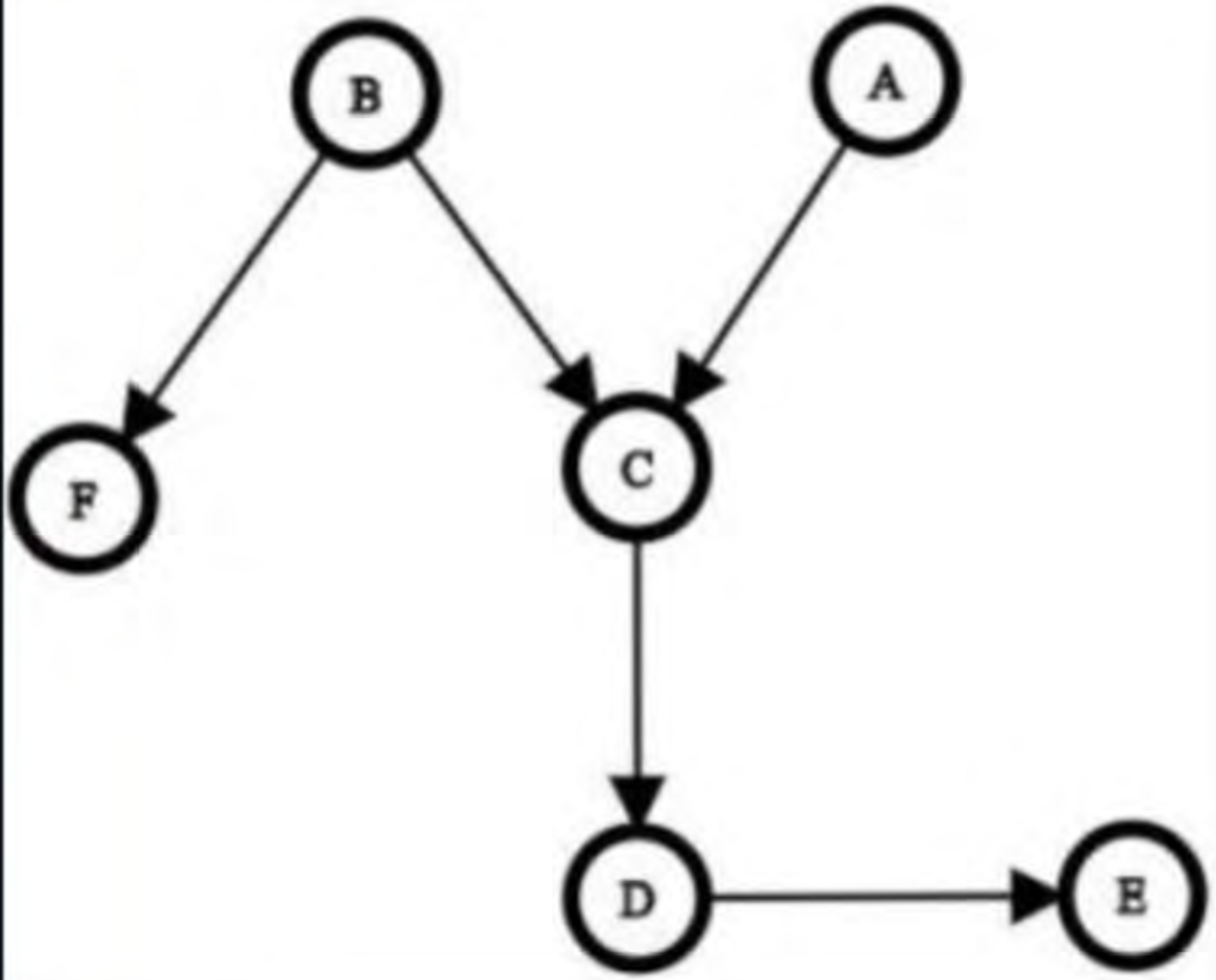
C	P(S)
T	.10
F	.50

C	P(R)
T	.80
F	.20

S	R	P(W)
T	T	.99
T	F	.90
F	T	.90
F	F	.00

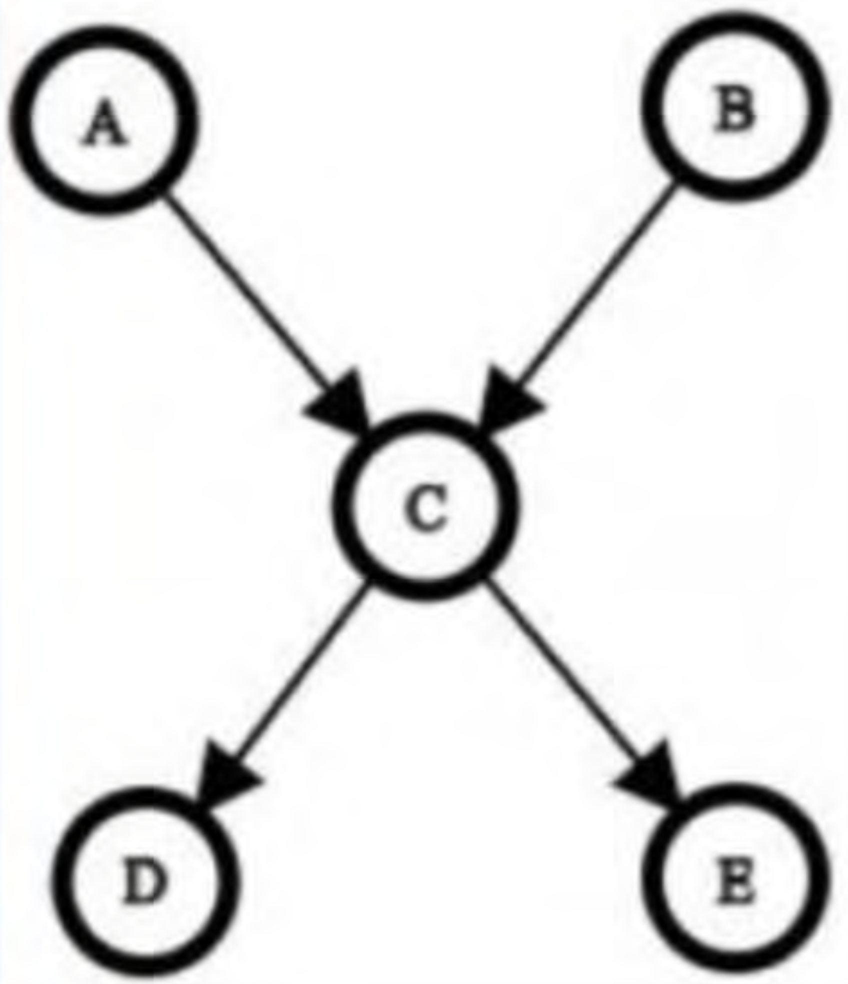
Let the weight for the sample (¬ C,S, ¬R, W) be w. What is 100w? (Round off your answer to the closest integer)

2) Consider the following Bayesian Network. Given evidence about C which of the following pair of variables are conditionally independent.



- [ ] A and B
- [ ] A and D
- [ ] D and E
- [ ] A and F

9) Consider the following Bayesian Network with the given information. What is the value of $P(\neg a \mid d, e)$ up to 3 decimal places?



$P(a) = 0.001$, $P(b) = 0.002$

$P(c \mid a, b) = 0.95$, $P(c \mid a, \neg b) = 0.94$, $P(c \mid \neg a, b) = 0.29$, $P(c \mid \neg a, \neg b) = 0.001$

$P(d \mid c) = 0.9$, $P(d \mid \neg c) = 0.05$

$P(e \mid c) = 0.7$, $P(e \mid \neg c) = 0.01$

Recall that during Gibbs Sampling, samples are generated through an iterative process.

Assume that the only evidence that is available is A = +a. Clearly fill in the circle(s) of the sequence(s) below that could have been generated by Gibbs Sampling.

Sequence 1

1 :	+a	−b	−c	+d
2 :	+a	−b	−c	+d
3 :	+a	−b	+c	+d

Sequence 2

1 :	+a	−b	−c	+d
2 :	+a	−b	−c	−d
3 :	−a	−b	−c	+d

Sequence 3

1 :	+a	−b	−c	+d
2 :	+a	−b	−c	−d
3 :	+a	+b	−c	−d

Sequence 4

1 :	+a	−b	−c	+d
2 :	+a	−b	−c	−d
3 :	+a	+b	−c	+d

THANK - YOU