GATE

CRASH COURSE

DS & AI

Algorithms

Greedy Method (Part 02)

(Lecture 7)

By - Aditya sir

Topics to be Covered

1.
2. Intro
3. Applications
4.

Telegram Link for Aditya Jain sir:

https://t.me/AdityaSir_PW

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored working professions in field of Data Science and Analytics
11. Have been mentoring GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

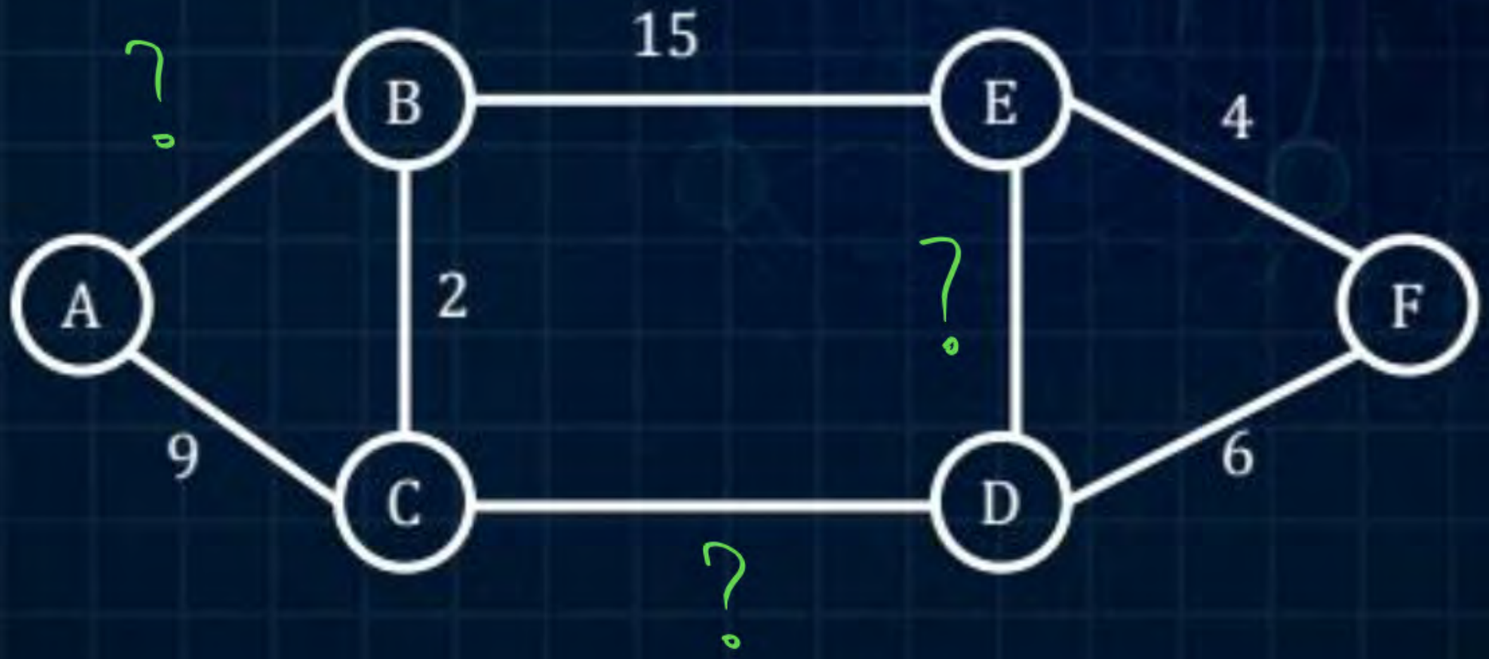
Topic : (Lecture Schedule)

3. Greedy Method

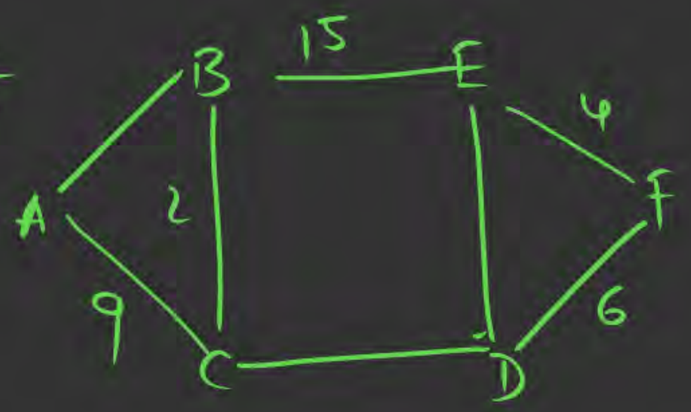
1. Introduction
2. Optimal Merge Patterns
    1. Huffman Coding
3. Minimum Cost Spanning Trees (MCST)
    1. Prims Method
    2. Kruskal's Method
4. Dijkstras Shortest Paths Problem

Question

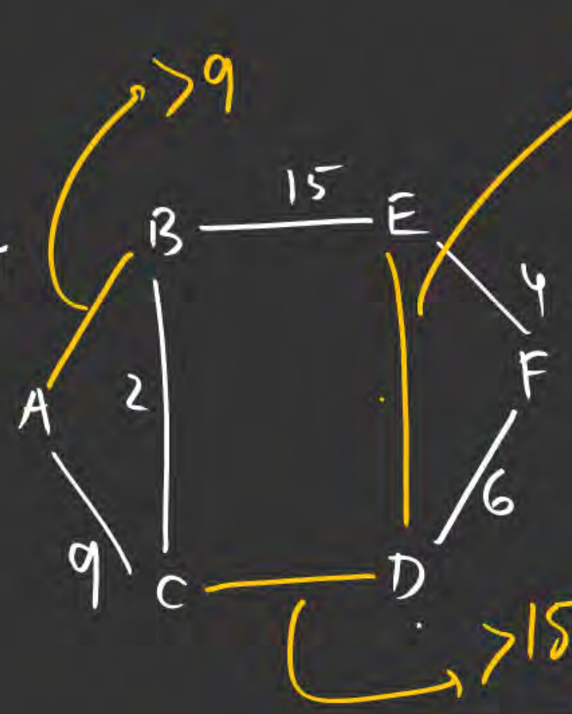
#Q. Consider the following Graph whose minimum cost spanning tree marked with edge has a weight of 36. Minimum possible sum of all edges of the graph G ____.



Soln:-



→ mST



AB → >9 → 10, 11 . . . . → 10

ED → >6 → 7, 8 . . . → 7

CD → >15 → 16, 17 . . . . → 16

Sum of all edges

= 36 + (10 + 7 + 16)

= 36 + 33

= 69 ✓

Question

#Q. Consider the string abbbccddeeee. Each letter in the string must be assigned a binary code satisfying the following properties:

1. For any two letters, the code assigned to one letter must not be a prefix of the code assigned to the other letter.
2. For any two letters of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter.

Among the set of al binary code assignment which satisfy the above two properties, what is the minimum length of the encoded string?

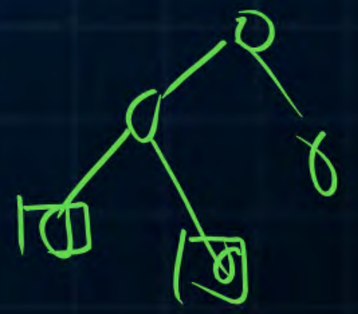
{Huffman Encoding}

A 25

B 23

C 21

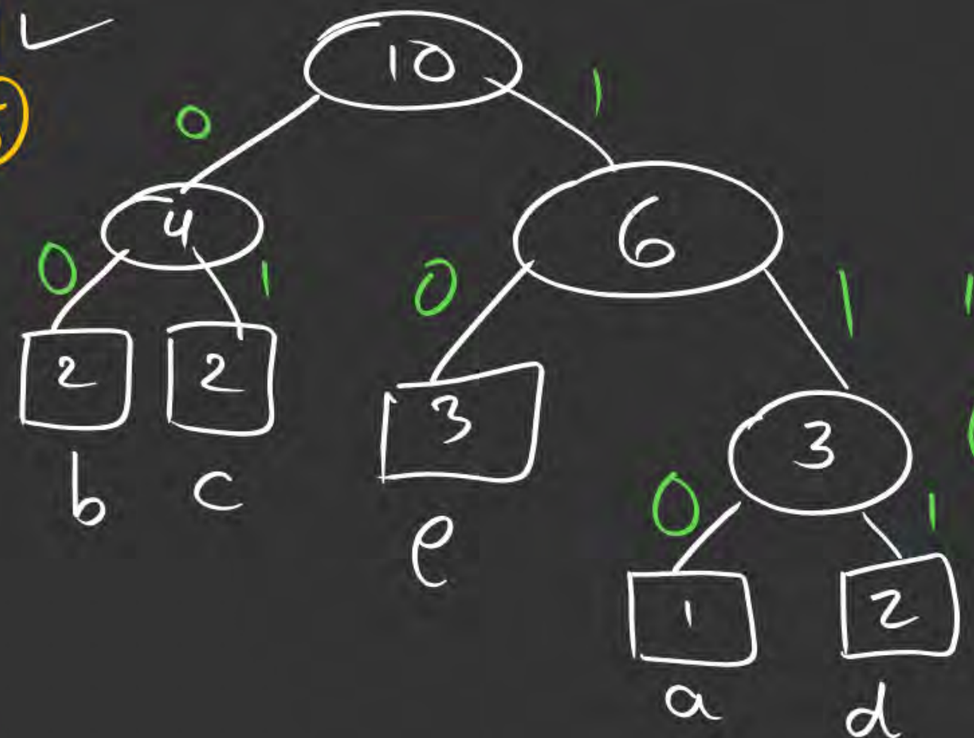
D 30



Soln: text = "abbccddeee"

a = 1 — 1 ✓
b = 2 — 4 ✗
b = 2 — 3 ✗
c = 2
d = 2 — 2 ✓
e = 3 — 5

$len(b) \le len(c) \le len(d)$



a → 110
b → 00
c → 01
d → 111
e → 10

a b b c c d d e e e
110 00 00 01 01 111 111 10 10 10

"11000001011111111010 10"

a → 1
b → 2
c → 2
d → 2 e → 3

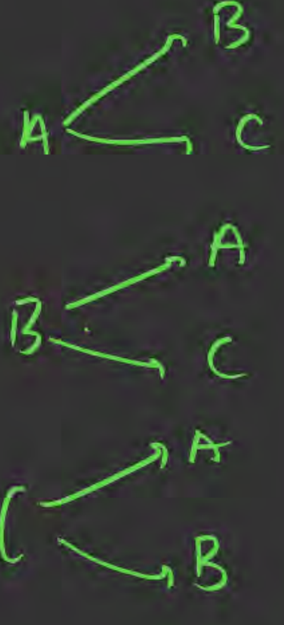
3 + 2x2 + 2x2 + 2x3
+ 3x2
= 3 + 4 + 4 + 6 + 6 = 23

Shortest Path Algos

1) Single Source Shortest Paths (SSSP)
   - a) Dijkstra's SSSP (greedy)
   - b) Bellman Ford (DP)

2) All pairs shortest Paths (APSP)

   - Floyd Warshall (DP)



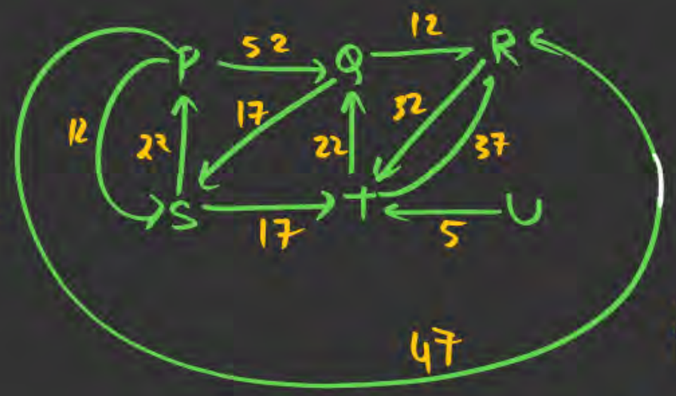
3) Multi-Stage Graph
4) Travelling Salesman Problem

} DP

★ Dijkstra's SSSP

- 1) Shortest Path Cost (Table based)
- 2) Shortest Path (Tree based)

Given $G(V, E)$



destinations

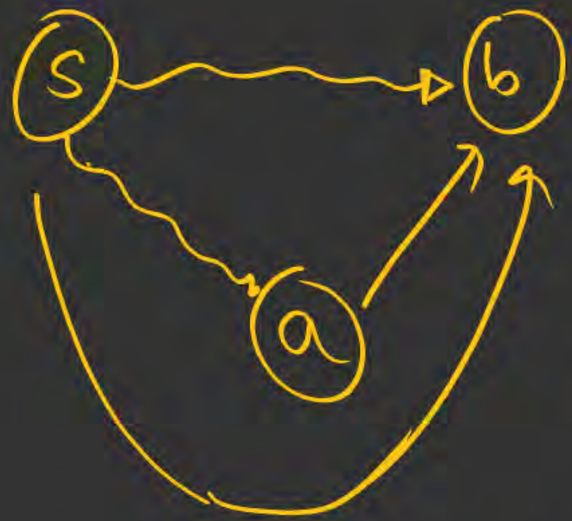
V	P	Q	R	S	T	U
{P}	0	52	47	12	∞	∞
{P, S}	0	52	47	12	29	∞
{P, S, T}	0	51	47	12	29	∞
{P, S, T, R}	0	51	47	12	29	∞
{P, S, T, R, Q}	0	51	47	12	29	∞
{P, S, T, R, Q, U}	0	51	47	12	29	∞

Relaxation

Source: P

$12 \to P = 0$

Relaxation Process



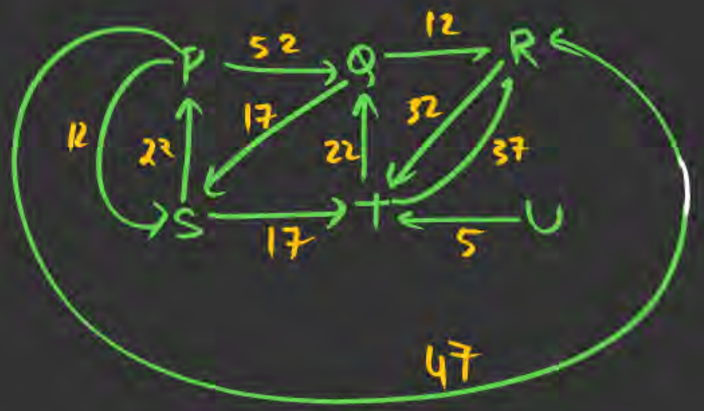
$d[b]$

$d[a]$

$d[b] > d[a] + ab$

$d[b] = d[a] + ab$ Relax

Given $G(V, E)$

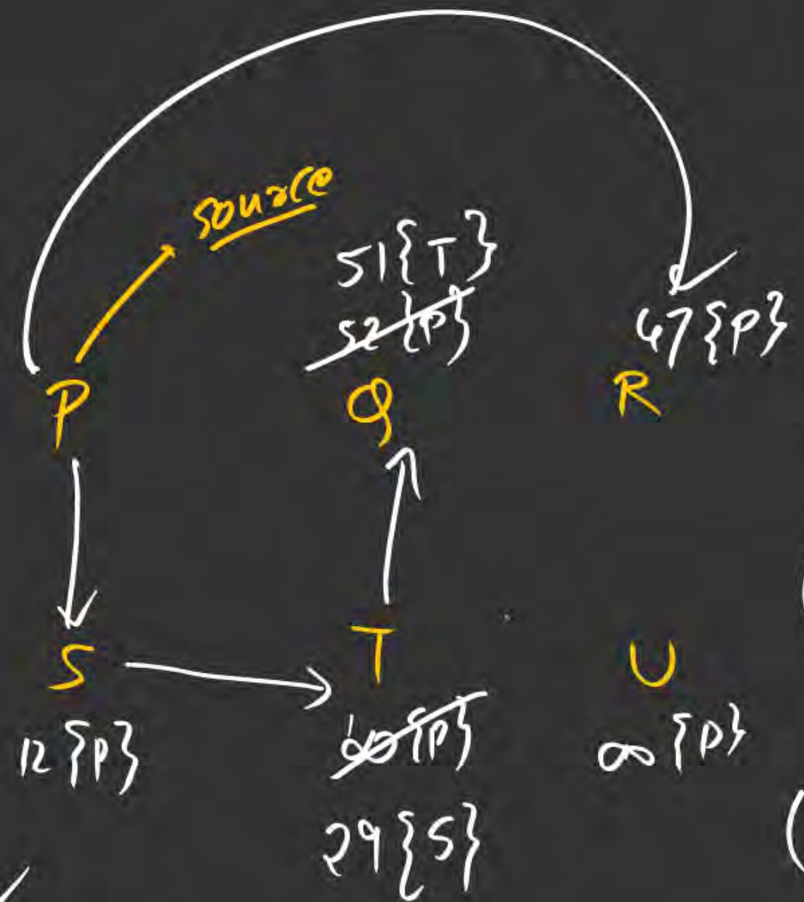


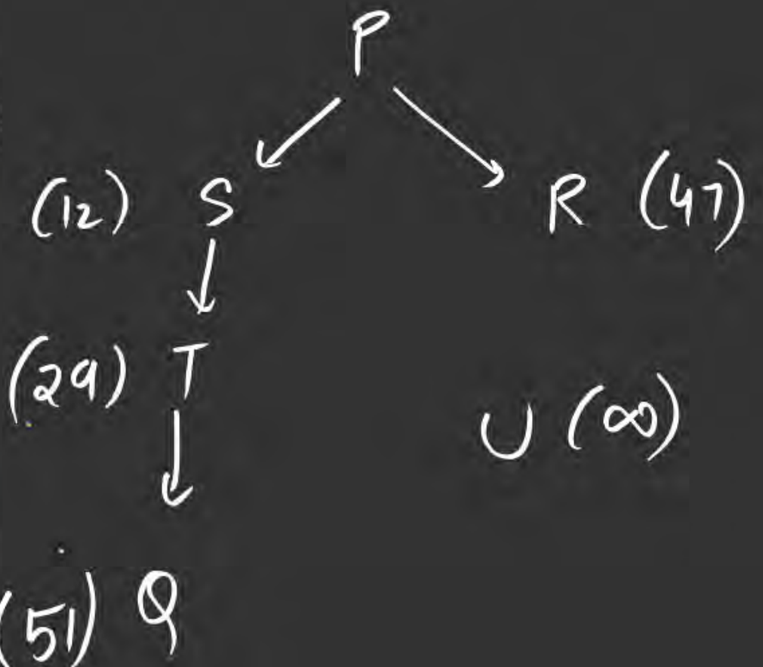
Source: P

$P \rightarrow Q$

$P \rightarrow S \rightarrow T \rightarrow Q$

$12 + 17 + 22 = 51$ ✓





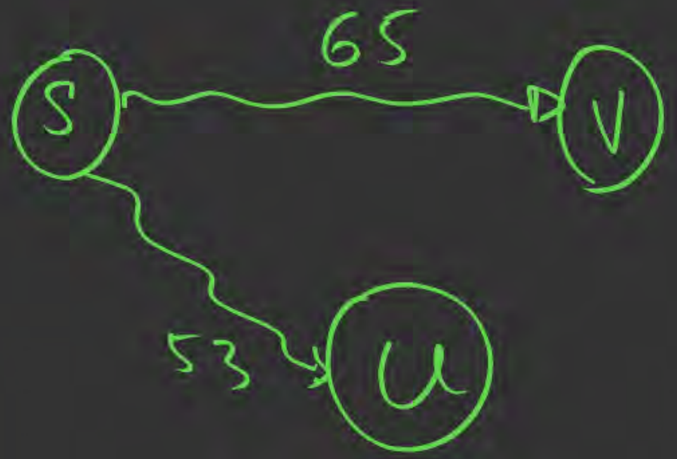
MCQ

*) $uv < 12$

B) $uv > 12$

C) $uv = 12$

D) $uv \geq 12$ ✓



Relax

$d[v] > d[u] + uv$

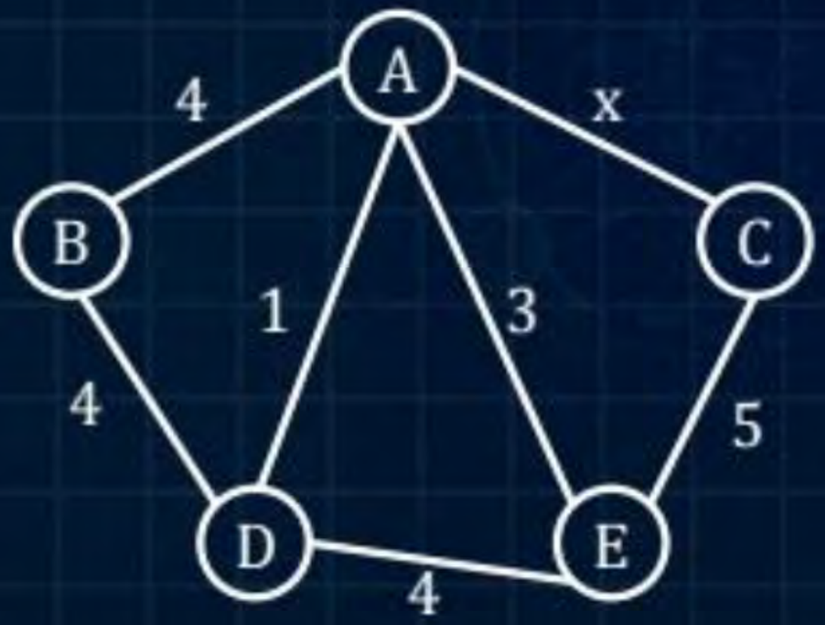
no relax

$d[v] \leq d[u] + uv$

$65 \leq 53 + uv \rightarrow uv \geq 12$

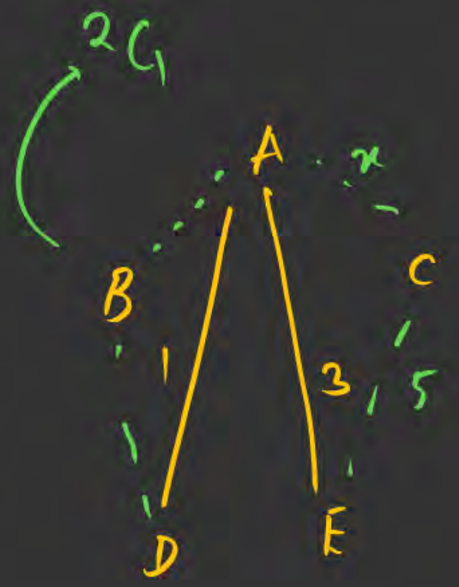
Question

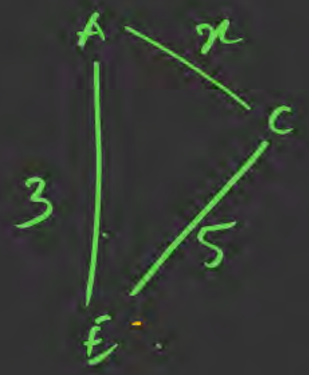
#Q. Consider the following undirected graph G:



Choose a value for x that will maximize the number of minimum weight spanning tree (MWSTs) of G. The number of MWSTs of G this value of x is ____.

Sol^n :-



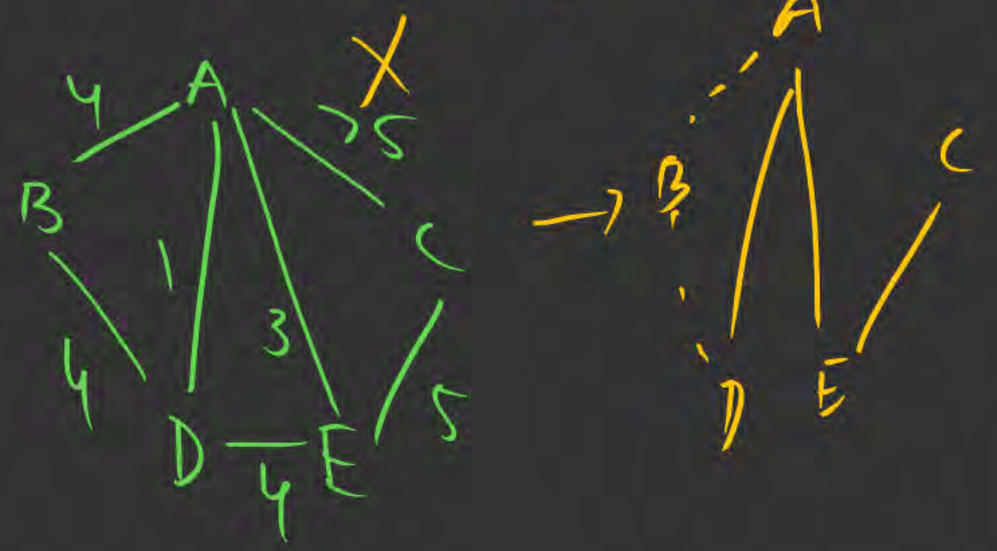


Case 1 : $x > 5$

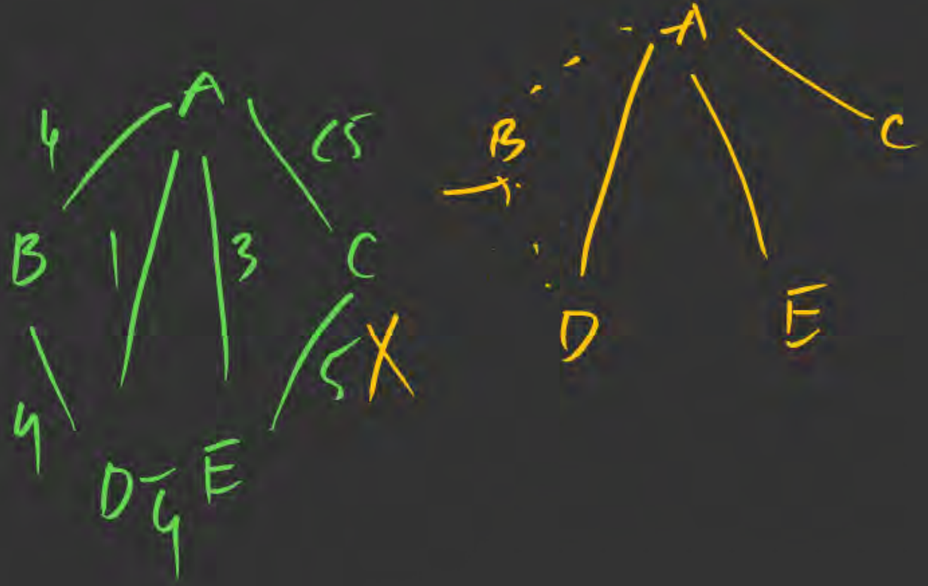
C2 : $x < 5$

C3 : $x = 5$

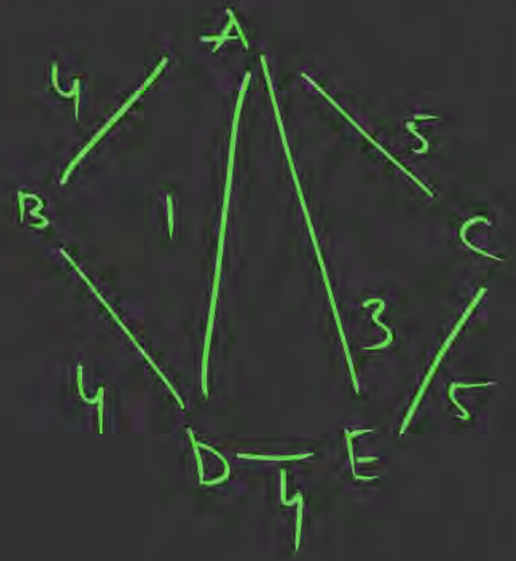
c1: $x > 5$ → (2)

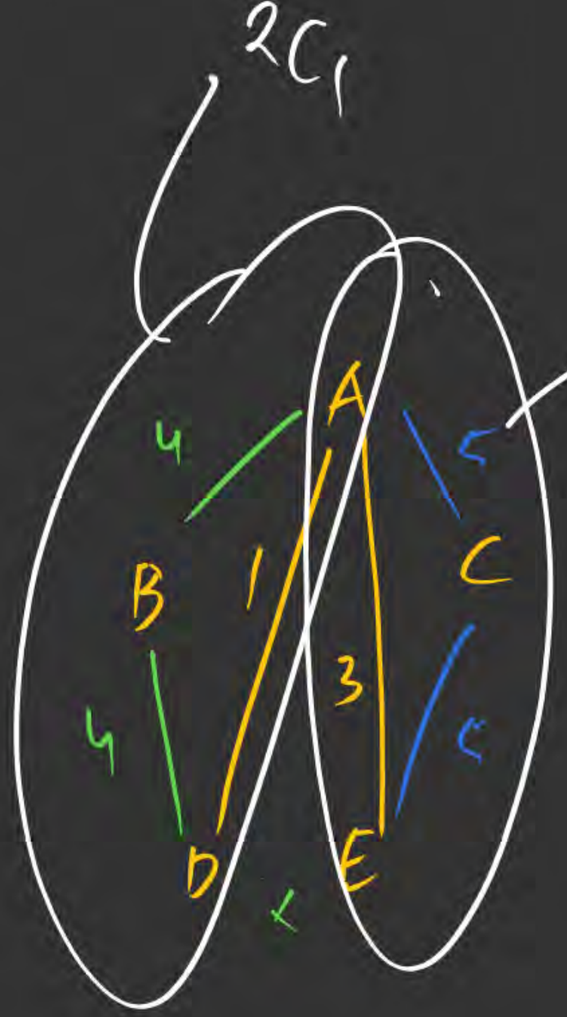


c2: $x < 5$ → (2)



(3) $n = 5$





Total = $^2C_1 = {}^2C_1$

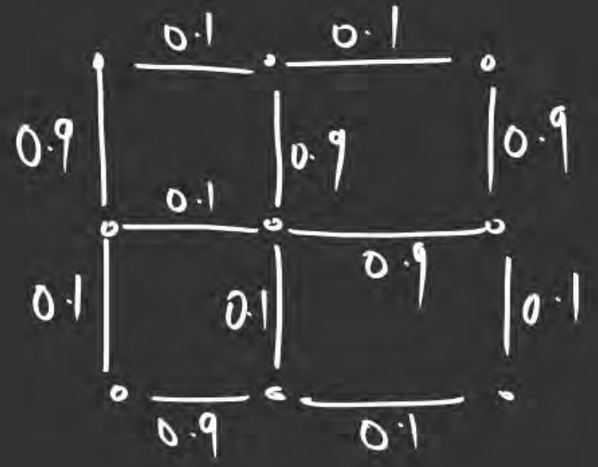
$= 4$

Question

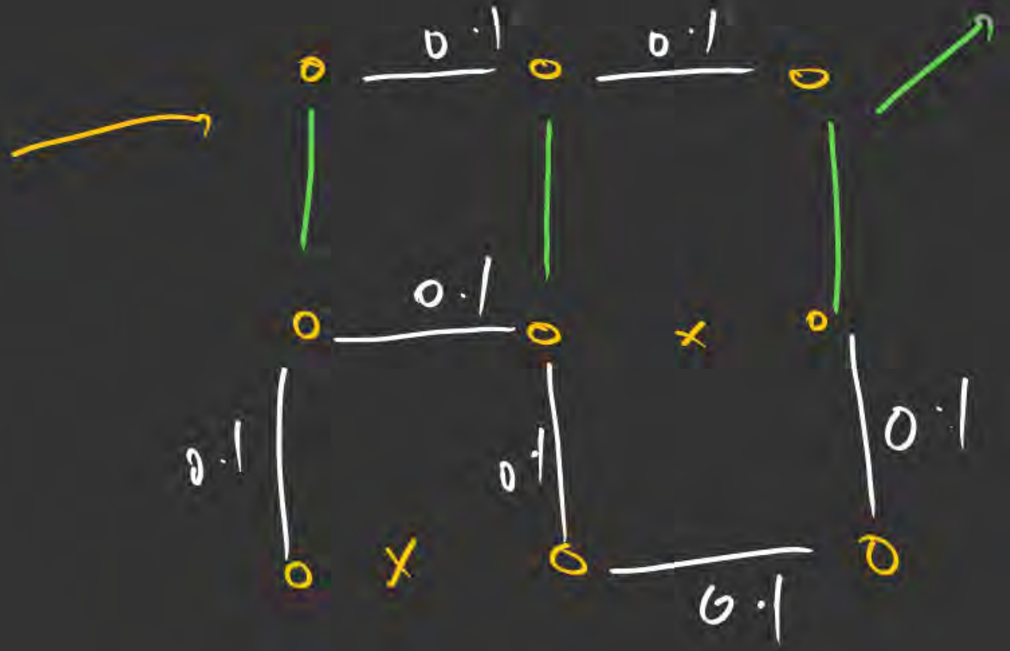
#Q. Consider the following undirected graph with edge weights as shown

The number of minimum-weight spanning trees of the graph is ____.

Soln:-



$n = 9$



$^3C_1 = 3$

Question

#Q. Let W be the minimum weight among all edge weights in an undirected connected graph. Let 'e' be a specific edge of weight 'w'. Which of the following is false?

T i. There is a minimum spanning tree containing 'e' always.

T ii. Every minimum spanning tree has an edge of weight 'w'.

F iii. 'e' is present in every minimum spanning tree.

T iv. If 'e' is not present in a minimum spanning tree named 'T' then there will be a cycle formed by adding 'e' to T.

MSQ

ii


eg:-
A
5
5
B
C
5
W:-5
A
5
B
C
5
A
5
C
B
5

Question

#Q. Let G be a connected undirected weight graph. Consider the following two statements.

S1: There exists a minimum weight edge in G which is present in every minimum spanning tree of G.

S2: If Every edge in G has district weight, then G has a unique minimum spanning tree.

A Both S1 and S are true

B S1 is true and S2 I false

C S1 is false and S2 is true

D Both S1 and S2 are false

Question

#Q. Consider the following directed graph G:

What will be shortest path from B to E by using Dijkstra's algorithms?

(A) B A D C F E → 4+6+12+10+5 = 37

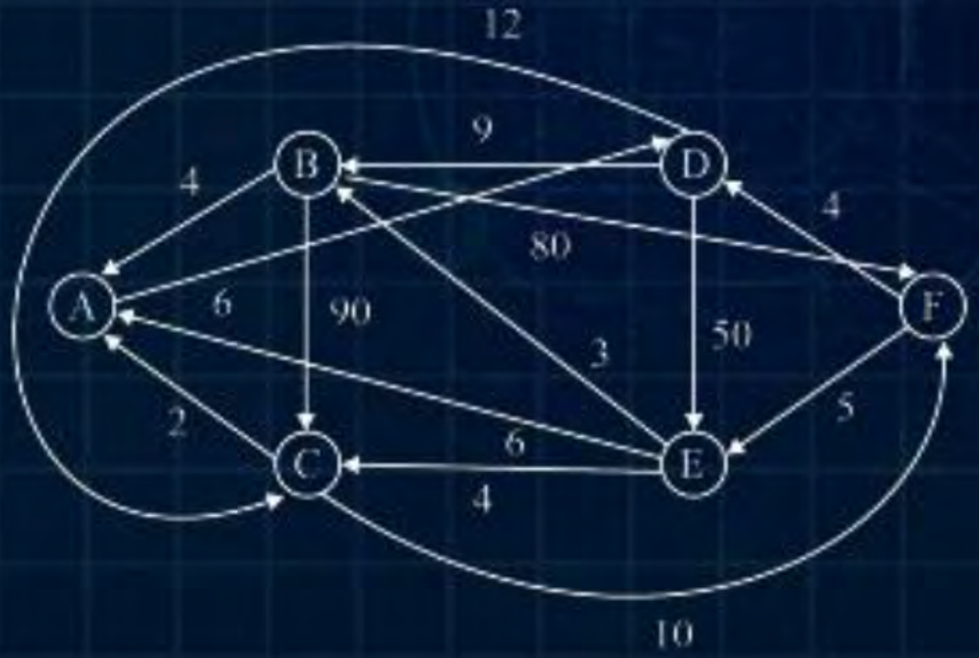
(B) B A D E = 4+6+50 = 60

(C) B A D F E ✗

(D) B D E ✗

Question

#Q. Consider the following directed graph G:
What will be the cost of shortest path from B to E by using Dijkstra's algorithms?

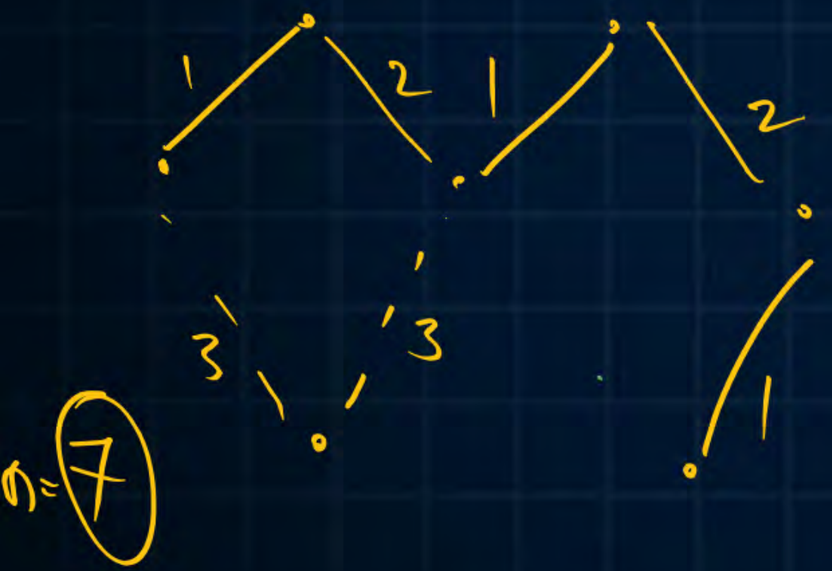


37

HW

Question

#Q. What is the cost the minimum spanning tree for the graph shown below? ~~using prim's algorithm?~~



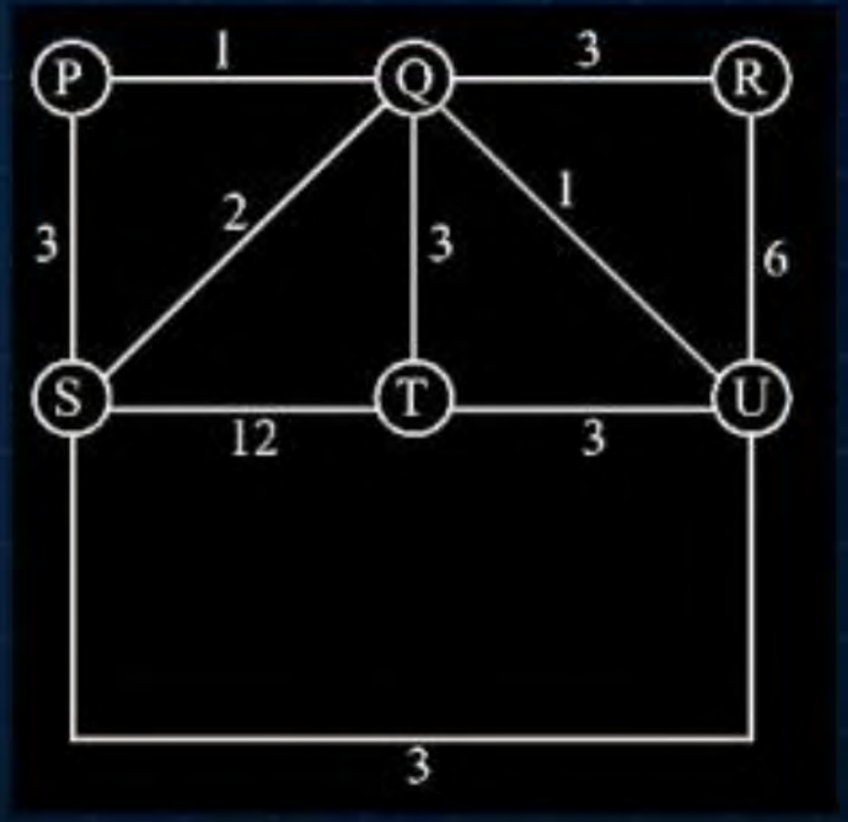
Cost = 1+1+1+2+2+3

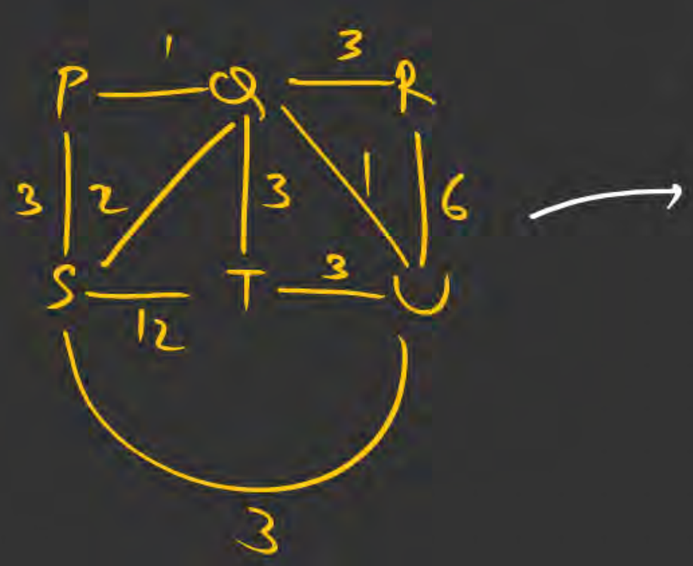
= 10

Question

#Q. Consider the following graph G

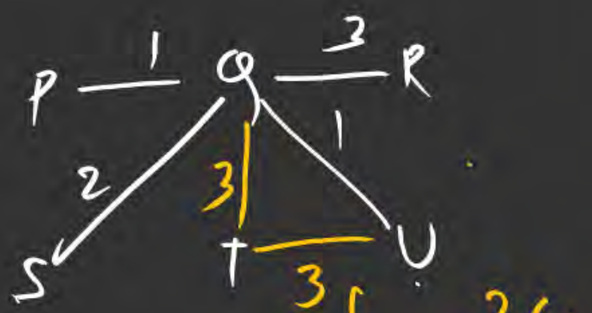
If the cost of MST is P and number of such spanning trees are Y then the value of P+Y is____.





Kruskal

mcst



$^2C_1$

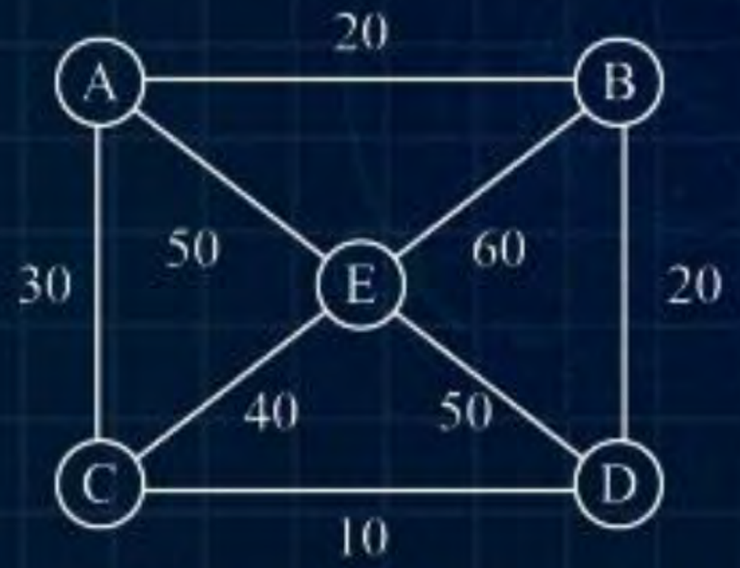
P = cost of mcst = $1+2+1+3+3 = 10$

Y = Count of mcst = $^2C_1 = 2$

$P+Y = 10+2$
$= 12$

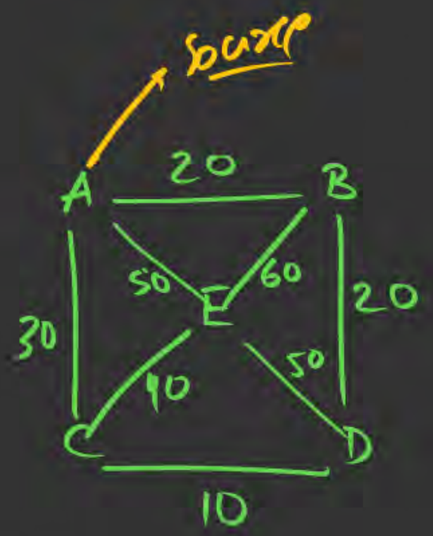
Question

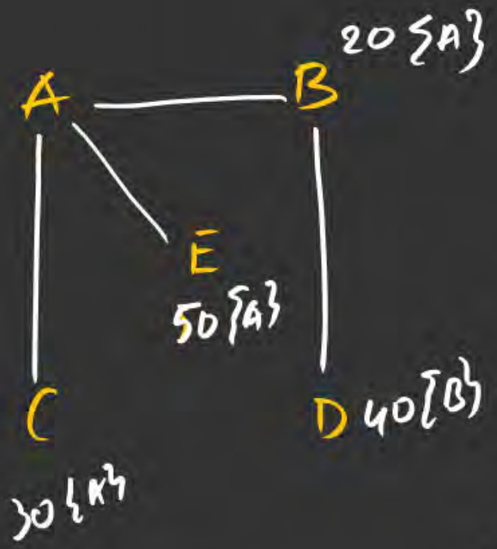
#Q. Assume Dijkstra's algorithm is used to find the shortest path from node 'A' in the following graph G :



The sum of the cost of the edges are not included in any of the shortest path from node A is _____.

Soln:-





A → D

A B D → 40 ✓

✗ A C D → 40

CE — 40

CD — 10

BE — 60

ED — 50

→ 160 ✓

Question

#Q. Let G be the directed , weighted graph shown below:

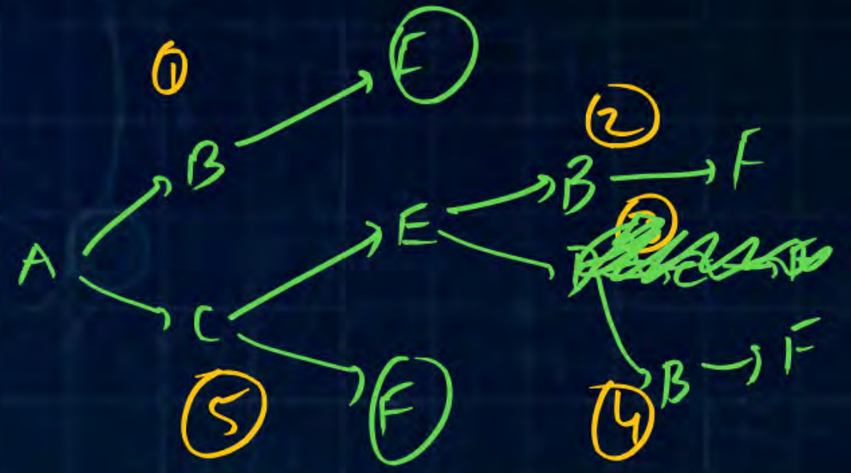
① ABF = 18+3 = 21

② ACEBF
= 6+2+5+3
= 16

If the cost of the shortest path from A to F is X and number of same cost of A to F is Y then the value of X * Y is _____.

HW

③ ACEDBF
= 6+2+4+1+3 = 16



④ ACF = 6+12 = 18

X = 16, Y = 2 → X*Y = 32

Question

#Q. What is the total number of comparisons that will be required in worst case to merge the following sorted files into a single sorted file into a single sorted file by merging together two files at a time

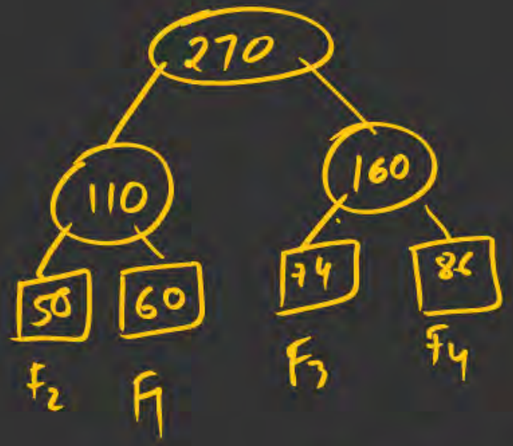
Files	F_1	F_2	F_3	F_4
Number of records	60	50	74	86

2-way merging

Soln :-

$f_1 \rightarrow 60$ — 2
$f_2 \rightarrow 50$ — 1
$f_3 \rightarrow 74$ — 3
$f_4 \rightarrow 86$ — 4

110, 160

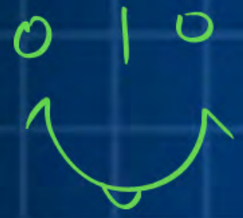


Optimal Soln

1) Total Record Mounts = $(110 + 160 + 270) = 270 + 270 = \underline{540}$

2) Min Comp = $50 + 74 + 110 = 124 + 110 = \underline{234}$

3) max Comp = $(540 - 3) = \boxed{537}$ ✓



Thank

THANK

Keep Hustling!