GATE

DATA SCIENCE + CS & IT

Engineering Mathematics

SUPER 1500

Lec : 01 Linear - 1

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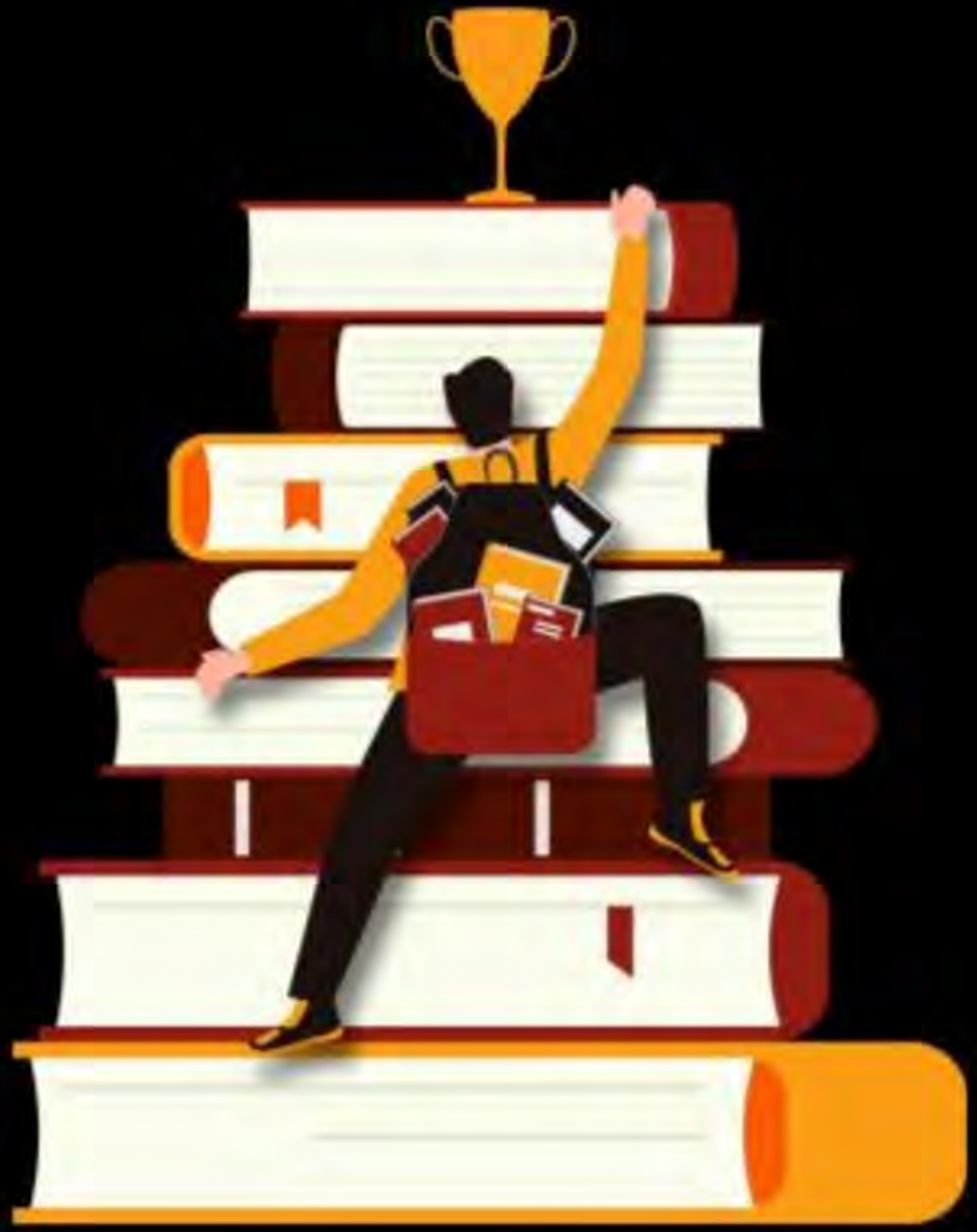
Topics to be covered

LINEAR ALGEBRA

1. Determinants
2. General Properties of Matrix



Let A be a 10 × 10 matrix in which each row has eaxctly one entry to 1 the remaining nine entries of the row being 0 which of the following is not possible value for the determinant of the matrix.

(a) 0
(b) –1
(c) 10
(d) 1

let $A_{2\times 2}$ then various possibilities for A ;

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Downarrow$ $|A| = 0$

$\Downarrow$ $|A| = 1$

$\Downarrow$ $|A| = -1$

If $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ then which of the following is a factor of Δ.

(a) $a + b$

(b) $a - b$

(c) abc

(d) $a + b + c$

$R_2 - R_1$ & $R_3 - R_1$

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$$

$R_3 - R_2$

$$\Delta = (b-a)(c-a) \begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 0 & c-b \end{vmatrix}$$

$$= (b-a)(c-a)\left[c-b\right]$$

$$\Delta = (a-b)(b-c)(c-a)$$

Note: $\Delta = \frac{1}{abc} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix} = \frac{abc}{abc} \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$

$$= - \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Consider the matrix

$$J_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

which is obtained by reversing the order of the columns of the identity matrix I_6.

Let $\boxed{P = I_6 + \alpha J_6}$, where α is a non-negative real number. The value of α for which $\det(P) = 0$ is _____

(a) 0 (b) -1 (c) ✓ 1 (d) 3

Let $P_{2\times 2} = I_2 + \alpha J_2$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \alpha \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \alpha \\ \alpha & 0 \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix}$

so $\boxed{|P| = (1-\alpha^2)^1}$

Let $P_{4\times 4} = I_4 + \alpha J_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \alpha \\ 0 & 0 & \alpha & 0 \\ 0 & \alpha & 0 & 0 \\ \alpha & 0 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & \alpha & 0 \\ 0 & \alpha & 1 & 0 \\ \alpha & 0 & 0 & 1 \end{bmatrix} \Rightarrow \boxed{|P| = (1-\alpha^2)^2}$

Similarly $P_{6\times 6} = I_6 + \alpha J_6 \Rightarrow \boxed{|P| = (1-\alpha^2)^3}$ & so on

So $|P| = 0 \Rightarrow \alpha = \pm 1$ Hence Ans $\alpha = 1$

(M-II) using observation: if we take $\alpha = 1$ then $|P| = 0$ Hence Ans $\alpha = 1$

Which one of the following does NOT equal

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} ? = \boxed{(x-y)(y-z)(z-x)}$$

(a) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

(c) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

(d) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

Let us take (a) :-

$$\begin{vmatrix} 1 & x^2+x & x \\ 1 & y^2+y & y \\ 1 & z^2+z & z \end{vmatrix} + \begin{vmatrix} 1 & x^2+x & 1 \\ 1 & y^2+y & 1 \\ 1 & z^2+z & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} + \begin{vmatrix} 1 & x & x \\ 1 & y & y \\ 1 & z & z \end{vmatrix} + 0$$

$$= -\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + 0 \neq \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

If A is a square matrix or order n then |Adj (Adj A| = ______.

(a) $|A|$

(b) $|A|^n$

(c) $|A|^{n-1}$

(d) $|A|^{(n-1)^2}$

$|\text{adj adj} A| = ?$

if $A_{n\times n}$ then $\boxed{|\underbrace{\text{adj adj adj} \ldots \text{adj}}_{r \text{ times}} A| = |A|^{(n-1)^r}}$

$\Rightarrow |\text{adj adj} A| = (|A|)^{(n-1)^2}$

eg $|\text{adj} A| = |A|^{(n-1)}$

The value of the determinant $\begin{vmatrix} {}^5C_1 & {}^5C_2 & {}^5C_3 \\ {}^7C_1 & {}^7C_2 & {}^7C_3 \\ {}^9C_1 & {}^9C_2 & {}^9C_3 \end{vmatrix}$ is

(a) 420 (b) 240 (c) 120 (d) None

HINT: ${}^nC_1 = n$, ${}^nC_2 = \frac{n(n-1)}{2}$, ${}^nC_3 = \frac{n(n-1)(n-2)}{3\times2\times1}$

$\boxed{{}^nC_r = {}^nC_{n-r}} \Rightarrow {}^5C_3 = {}^5C_2 = 10$

${}^7C_3 = \frac{7\times6\times5}{3\times2} = 35$, ${}^9C_3 = 84$

$$|A| = \begin{vmatrix} 5 & 10 & 10 \\ 7 & 21 & 35 \\ 9 & 36 & 84 \end{vmatrix} = 5\times7\times9 \begin{vmatrix} 1 & 2 & 2 \\ 1 & 3 & 5 \\ 1 & 4 & 84/9 \end{vmatrix}$$

$$|A| = 315 \begin{vmatrix} 1 & 2 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & 22/3 \end{vmatrix}$$

$$= 315\left(\frac{22}{3} - 6\right) = 315 \times \frac{4}{3}$$

$$= 105 \times 4 = 420$$

If A and B are matrices of determinant 1 then

(a) Determinant of A + B is 2

(b) Determinant of A + B is 1

(c) Determinant of A + B is 0

(d) Nothing can be concluded about the determinant of A + B

$|A| = 1$, $|B| = 1$ then $|A+B| = ? \neq |A| + |B|$

ie No formula exist for $|A+B|$

Let a 3 × 3 matrix A have determinant value 5. If $B = 4A^2$ then the determinant valueo f B is equal to ________.

$|B| = ?$

(a) 20
(b) 100
(c) 320
(d) 1600

Ans, $|A|_{3\times3} = 5$ & $B = 4A^2$

So B is also a Mat of 3×3

Hence, $|B| = |4A^2| = 4^3|A^2|$

$= 64\,|A|^2 = 64 \times 25$

The value of the determinant $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ will be ______.

(a) $2a^2b^2c^2$ (b) $-2a^2b^2c^2$

(c) $4a^2b^2c^2$ (d) $-4a^2b^2c^2$

$$= abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

$$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix}$$

$$= a^2b^2c^2 \left[-1 \{0-4\} \right] = 4a^2b^2c^2$$

The determinant of a matrix has 720 terms (in the unsimplified form). The order of the matrix is ___.

(a) 5 (b) 6 (c) 7 (d) 8

We know that, Max No. of terms in the General Expansion of $|A|_{n\times n} = n!$

So ATQ, $n! = 720$

$n = 6$

M is a squaqre matrix or order n and its determinant value is 5. If all the elements of M are multiplied by 2, its determinant value becomes 40. The value of n is

(a) 2 (b) 3

(c) 4 (d) 5

$\underset{n\times n}{|M|} = 5$ & let $A = 2M$

$|A| = |2M| = 2^n |M|$

$40 = 2^n \times 5$

$2^n = 8 \Rightarrow n = 3$

If the sum of the diagonal elements of a 2 × 2 matrix is –6, then the maximum possible value of determinant of the matrix is ____9____.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow a+d=-6$

$\& \ |A| = ad - bc$

$\& \ |A|_{max} = ?$

Th: "if sum of two numbers is constant then their product will be max if they are equal"

Let $A = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$

$a+d=-6 \Rightarrow a=d=-3$

$|A| = ad \Rightarrow |A|_{max} (\text{at } a=d) = a^2 = (-3)^2 = 9$

Qs (2014) if Trace of 2×2 Symm Mat is 14 then find the Max Value of it's Det ?

$$A = \begin{bmatrix} x & z \\ z & y \end{bmatrix}$$

$x+y=14$ (Const) $\Rightarrow x=y=7$

$$\boxed{|A| = xy - z^2}$$

for Max $|A|$ we should take $z=0$

so $|A|_{max} = xy - 0$

$= (7)(7)$

$= 49$

$$\boxed{x+y=14}$$

Let $x=9, y=5 \Rightarrow xy=45$

Let $x=3, y=11 \Rightarrow xy=33$

Let $x=8, y=6 \Rightarrow xy=48$

But at $x=7, y=7 \Rightarrow xy=49$ Ans

If A and B are two matrices of order 3 × 5 and 5 × 3 respectively then determinant of the matrix 4BA equals

(a) $4|B|/A|$ (b) $4^3|B||A|$

(c) $4^5|A||B|$ (d) None

$|4BA| = ?$

$\because$ A & B are Rectangular Matrices

& $|A| = DNE$, $|B| = DNE$

So (d) is the Ans.

(WRONG APP)

$B_{5\times3} A_{3\times5} = (BA)_{5\times5}$

$|4BA| = 4^5 \cdot |BA|$

$\neq 4^5 |B| \cdot |A|$

X and Y are non-zero square matrices of size n × n. If XY = $O_{m \times n}$

(a) $|X| = 0$ and $|Y| \neq 0$ (b) $|X| \neq 0$ and $|Y| = 0$

(c) $|X| = 0$ and $|Y| = 0$ (d) $|X| \neq 0$ and $|Y| \neq 0$

ATQ, $X \neq O, Y \neq O$, X & Y are sq Mat

And, $\boxed{XY = O}$ — (1)

Let $|Y| \neq 0 \Rightarrow Y^{-1}$ exist

So $(XY)Y^{-1} = OY^{-1} \Rightarrow X = O$??

ie (a) is WRONG

Let $|X| \neq 0 \Rightarrow X^{-1}$ exist

So By (1) $X^{-1}(XY) = X^{-1}O \Rightarrow Y = O$

again it is WRONG

∴ Correct choice is (c)

If A and B are non-zero square matrices, then AB = 0 implies

(a) A and B are orthogonal

(b) A and B are singular

(c) B is singular

(d) A is singular

$A \neq 0,\ B \neq 0$

$A \& B = $ Sq. Mat

ATQ $AB = 0 \Rightarrow |A| = 0 \ \& \ |B| = 0$

$\Rightarrow$ Both are singular

The number of different matrices that can be formed with elements 0, 1, 2 and 3; each matrix having 4 elements is

(a) 2×4^4 (b) 3×4^4

(c) 4×4^4 (d) 3×2^4

Various types of matrices;

$\begin{bmatrix} - & - \\ - & - \end{bmatrix}_{2\times 2} \longrightarrow 4^4$ Matrices

or

$\begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix}_{4\times 1} \longrightarrow 4^4$ "

or

$\begin{bmatrix} - & - & - & - \end{bmatrix}_{1\times 4} \longrightarrow 4^4$ "

So Req Matrices $= 4^4 + 4^4 + 4^4 = 3 \times 4^4$

In order to make a Matrix with 4 elements we have to fill 4 places using 0, 1, 2, 3

So No. of ways to fill 4 places $= \underset{P_1}{\underline{4}} \times \underset{P_2}{\underline{4}} \times \underset{P_3}{\underline{4}} \times \underset{P_4}{\underline{4}} = 4^4$

RA

The number of singular matrices of order 2, where each element is either 0 or 1 is ____. 10

Total possible Matrices of 2×2 are $= \underset{P_1}{\underline{2\text{ ways}}} \times \underset{P_2}{\underline{2\text{ ways}}} \times \underset{P_3}{\underline{2\text{ ways}}} \times \underset{P_4}{\underline{2\text{ ways}}} = 16$ Matrices

for eg $\underset{1}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}, \underset{4}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}, \underset{6}{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}}, \underset{4}{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}, \underset{1}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}$

Non Singular Matrices are ; $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

ie Singular Matrices $= 16 - 6 = 10$

If X and Y are two singular matrices such that XY = Y and YX = X then $X^2 + Y^2$ equals

(a) X + Y ✓
(b) XY
(c) YX
(d) 2(X + Y)
(e) 2I

ATQ, $XY = Y$ & $YX = X$

where X & Y are singular

then $X^2 + Y^2 = ?$

(WRONG APP) →

~~$XY = Y \Rightarrow X = I$ ∵ $|X| = 0$~~

~~$YX = X \Rightarrow Y = I$ ∵ $|Y| = 0$~~

~~So $X^2 + Y^2 = I^2 + I^2 = I + I = 2I$~~

$XY = Y$ — ①, $YX = X$ — ②

$$X^2 + Y^2 = XX + YY$$
$$= X(YX) + Y(XY)$$
$$= (XY)X + (YX)Y$$
$$= YX + XY$$
$$= X + Y \text{ i.e. (a) } ✓$$

The matrices $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

commute under multiplication.

(a) If $a = b$ or $\theta = n\pi$, n is an integer

(b) always

(c) never

(d) If $a\cos\theta \neq b\sin\theta$

$AB = BA$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} a\cos\theta & -b\sin\theta \\ a\sin\theta & b\cos\theta \end{bmatrix} = \begin{bmatrix} a\cos\theta & -a\sin\theta \\ b\sin\theta & b\cos\theta \end{bmatrix}$$

$-b\sin\theta = -a\sin\theta$

$(a-b)\sin\theta = 0 \rightarrow a = b$

or

$\sin\theta = 0 \Rightarrow \theta = n\pi$

A sequence x[n] is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ for } n \geq 2.$$

The initial conditions are $x[0] = 1$, $x[1] = 1$ and $x[n] = 0$ for $n < 0$. The value of $x[12]$ is ....

$x(n) = 0,\ n<0,\ x_0 = x_1 = 1$

for $n=2$, $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

so $\begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\Rightarrow x_2 = 2$

for $n=3$,

$\begin{bmatrix} x_3 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^3 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

$\begin{bmatrix} x_3 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\Rightarrow x_3 = 3$ & $x_2 = 2$

i.e. $x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \ldots\ldots x_{11}, \; x_{12}$

$= 1 \quad 1 \quad 2 \quad 3 \quad 5 \quad 8 \ldots\ldots, \; = 233$

i.e. FIBONACCI Series

If A & B are two matrices of same order then which of the following is true.

(a) $(A + B)^2 = A^2 + 2AB + B^2$

(b) $(A - B)^2 = A^2 - 2AB + B^2$

(c) $(A + B)^2 + (A - B)^2 = 2A^2 + 2B^2$

(d) $(A + B)(A - B) = A^2 - B^2$

Let us take (a):

$$(A+B)^2 = (A+B)(A+B)$$
$$= A^2 + \underbrace{AB + BA} + B^2$$

Option (d):

$$(A+B)(A-B) = A^2 \underbrace{- AB + BA} - B^2$$

W.K. that Matrix Multi is not Commutative in general

i.e. $\boxed{AB \neq BA}$

Let us take (c): $(A+B)^2 + (A-B)^2$

$$= (A+B)(A+B) + (A-B)(A-B)$$
$$= A^2 + \cancel{AB} + \cancel{BA} + B^2 + A^2 - \cancel{AB} - \cancel{BA} + B^2 = 2A^2 + 2B^2$$

The matrix $A = (a_{ij})_{m \times n}$ is defined as $a_{ij} = i + j$ for all i, j then the sum of all the elements of matrix A = ____.

(a) $\frac{mn}{2}(m+n+1)$

(b) $\frac{mn}{2}(m+n+2)$

(c) $\frac{m}{2}\left(\frac{n(n+1)}{2}\right)$

(d) $\frac{n}{2}\left(\frac{m(m+1)}{2}\right)$

Let $A = [a_{ij}]_{2\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

$\sum a_{ij} = 21$

for $m = 2, n = 3$

Let us check (b);

$\frac{mn}{2}(m+n+2) = \frac{2\times 3}{2}(2+3+2)$

$= 3(7) = 21$

Hence verified

The value of $\begin{bmatrix}1\\2\\3\end{bmatrix} \times \begin{bmatrix}3 & 2 & 4\end{bmatrix} \times \begin{bmatrix}4\\6\\7\end{bmatrix}$ equals

3×1 $\quad$ 1×3 $\quad$ 3×1

(a) $\begin{bmatrix}52\\-104\\156\end{bmatrix}$

(b) $[52 \quad -104 \quad 15]$ 1×3

(c) $\begin{bmatrix}52\\104\\156\end{bmatrix}$

(d) None of these

$\underset{3\times1}{\begin{bmatrix}1\\2\\3\end{bmatrix}} \underset{1\times1}{\left[(12+12+28)\right]}$

$\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}52\end{bmatrix} = \begin{bmatrix}52\\104\\156\end{bmatrix}$

(c)

If ω be the cube root of unity then

$$\begin{bmatrix} \omega & \omega^2 \\ 1 & \omega \\ \omega^1 & 1 \end{bmatrix}_{3\times 2} \times \begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix}_{2\times 3} \times \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}_{3\times 1}$$ equals

(a) $\begin{bmatrix} \omega-\omega^2 \\ \omega-\omega^2 \\ \omega-2\omega^2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}[\omega-\omega^2]$ (d) None

$$\begin{pmatrix} 1 & \omega^2 \\ 1 & \omega \\ \omega & 1 \end{pmatrix}_{3\times 2} \begin{bmatrix} (1+\omega^2+\omega^4) \\ (\omega+\omega^3+\omega^2) \end{bmatrix}_{2\times 1}$$

$$\begin{bmatrix} 1 & \omega^2 \\ 1 & \omega \\ \omega & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (\because \omega^3 = 1)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$$

Thank

THANK

Keep Hustling!