Data Science & Artificial Intelligence

Algorithms

Test Series 1500+

Lecture - 02

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Theta (Θ) → Tight Bound

$f(n) = n + 5n^2$

$F(n) = O(g(n))$

iff $F(n) = O(g(n))$

and $F(n) = \Omega(g(n))$

① $n + 5n^2 \leq 10n^2$

$f(n) \leq c * g(n)$

$f(n) = O(g)$

$f(n) = O(n^2)$

$n + 5n^2 \geq 1n^2$

$f(n) = \Omega(n^2)$

$n + 5n^2 = \Theta(n^2)$

Topic : Analysis of algorithm

Dependent nested loop

#Q. Consider the following code

```
i = n;
while (i > 0)
{
    for(j = 1; j<=i; j = j+3)
    {
        print("1500 series")
    }

    i = i - 1
}
```

Time complexity of above code in terms of Big-Oh?

A. $(\log n)^2$

B. $\sqrt{\log n}$

C. n*n

D. loglogn

Soln: outer: while $\longrightarrow O(n)$

inner: For: $1 \longrightarrow i \quad +3 \longrightarrow O(i/3)$

$= O(i)$

$i = n \longrightarrow n/3$

$i = n-1 \longrightarrow (n-1)/3$

$i = n-2 \longrightarrow (n-2)/3$

$\vdots$

$i = 1 \longrightarrow (1)/3$

Total/overall TC: $\frac{n}{3} + \frac{(n-1)}{3} \ldots \frac{1}{3}$

$$= \frac{1}{3}\left[\sum_{i=1}^{n} i\right]$$

$$= \frac{1}{3}\left(\frac{n(n+1)}{2}\right)$$

$$= O(n^2)$$

Topic : Analysis of algorithm

#Q. Consider the following C-code

```
void foo (int n)
{
    for (a=1; a <= n; a=a*5)
        {
            for(b = n; b>0; b = b/3)
            {
                printf("1500 Series");
            }
        }
}
```

What is the worst time complexity of above program?

Independent nested loop.

A. O(1)

B. O(n)

C. O(logn*log n)

D. $O\sqrt{n}$

Ans: C

Soln:-

a: $1 \longrightarrow n$ $\underline{\underline{*5}}$ $\approx O(\log_5 n)$

b: $n \longrightarrow 1$ $/3$ $\approx O(\log_3 n)$

$1 \longrightarrow 5 \longrightarrow 5^2 \longrightarrow 5^3 \quad \ldots \quad 5^k \longrightarrow O(\log_5 n)$

$n \longrightarrow n/3 \longrightarrow n/3^2 \cdots \quad \cdots \quad 1 \longrightarrow O(\log_3 n)$

overall TC: $O(\log n * \log n)$

Topic : Analysis of algorithm

#Q. Consider the following asymptotic functions:

$f1 = 2^n$

$f2 = 1.001^n$

$f3 = e^n$

$f4 = 200$

$f5 = (0.8)^n$

Which of the following is correct increasing order of above functions?

A. f4,f5, f2, f1, f3

B. f2, f4, f5, f1, f3

C. f5, f4, f2, f1, f3

D. f5, f2, f1, f3, f4

Asymptotic Comparison

Ans: C

Soln :- $f_1 = 2^n$ ⟶ Expo

$f_2 = 1.001^n$ ⟶ Expo

$f_3 = e^n$ ⟶ Expo

} incr

2 1.001 2.7

$2.7 > 2 > 1.001$

$f_4 = 200$ ⟶ Const

Decr < Const < log < Poly < Expo

$F_5 = (0.8)^n$ ⟶ Expo. Decr

eg :- $\frac{1}{2} > \frac{1}{4} > \frac{1}{8}$ - - -> $\frac{1}{2} > \left(\frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)^3 \cdots \left(\frac{1}{2}\right)^n$

0.5 0.25

a^n	$a<1$ ⟶ decr
	$a>1$ ⟶ incr

$$F_5 < F_4 < F_2 < F_1 < F_3$$

[MCQ]

#Q17. What is the time complexity of insertion sort in best case, average case and worst case respectively is:

A $O(n), O(n), O(n^2)$

B $O(n), O(n^2), O(n^2)$

C $O(n^2), O(n^2), O(n^2)$

D $O(n), O(n\log n), O(n\log n)$

Ans: B

Topic : Analysis of algorithm

#Q. Arrange following function in the descending order growth rate.

$$f_1 = (e)^n, f_2 = \sqrt{n}^{\log n}, f_3 = (2)^n\ f_4 = (\log n)^n, f_5 = (n)^{\log n}$$

A f_2, f_5, f_3, f_1, f_4 ✗

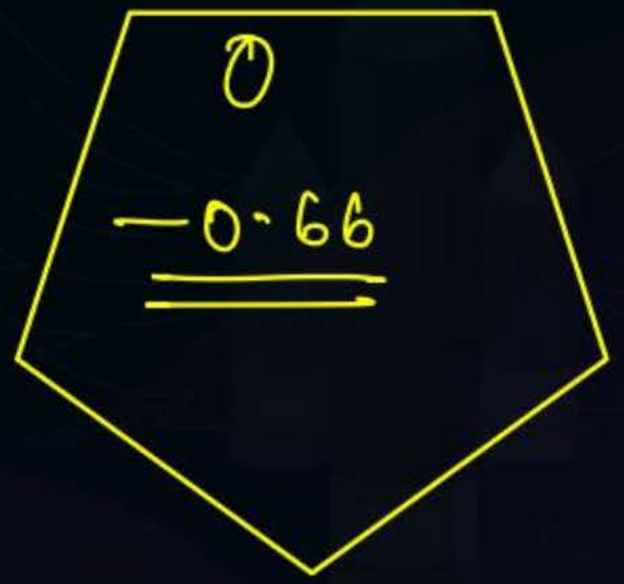
B f_3, f_4, f_2, f_5, f_1 ✗

C f_2, f_5, f_1, f_3, f_4 ✗

D f_4, f_1, f_3, f_5, f_2 ✓

42:r.

Ans : D



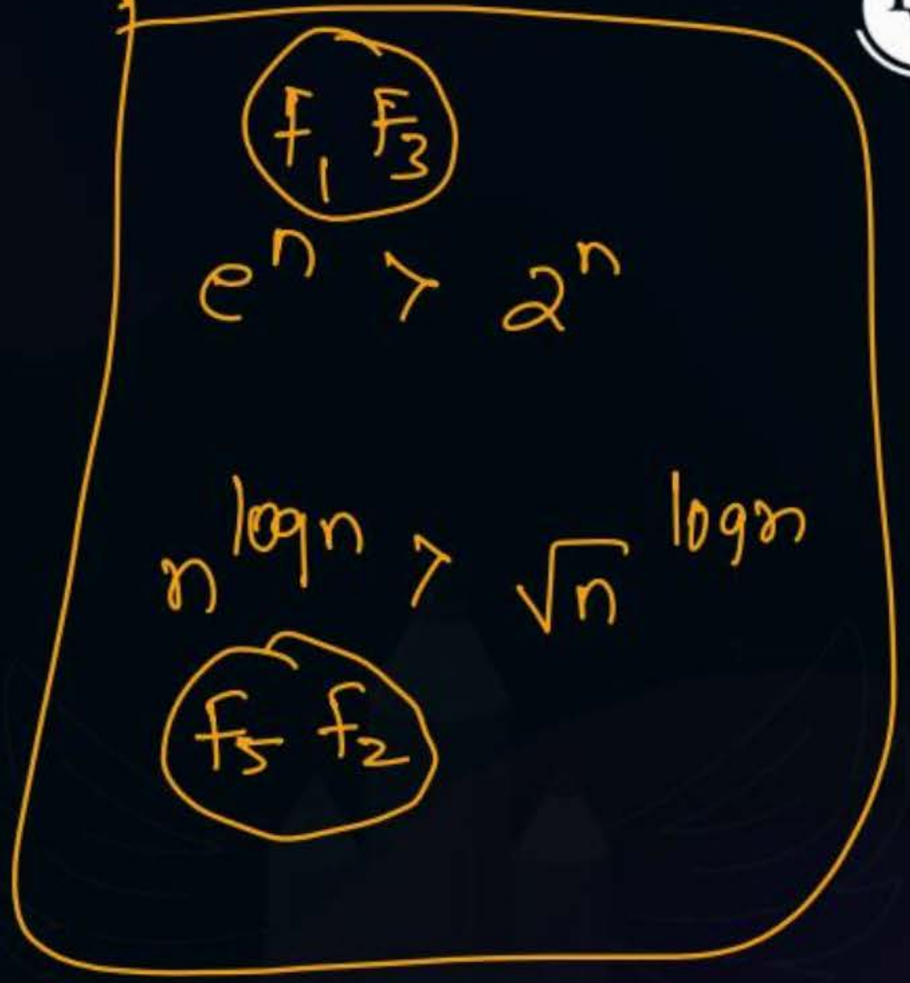
Soln :- $f_1 = e^n$ ⟶ expo

$f_2 = \sqrt{n}^{\log n}$ ⟶ expo

$f_3 = 2^n$ ⟶ expo

$f_4 = (\log n)^n$ ⟶ expo

$f_5 = n^{\log n}$ ⟶ expo



$n^{\log n}$ $\log(n)^n$

Taking log both sides

$\log n * \log n$ $n * \log(\log n)$

Topic : Analysis of algorithm

#Q. Consider the following recursion function

```
AJ(n)
{
    if (n < = 0)
        return 1;
else if (n% 2 ! = 0)
        return AJ(n -2);
else
        return AJ(n-1);
}
```

What is the time complexity of above code

odd - 2 → odd

even - 1 → odd

odd

even

Ans : C

A $\theta(\log n)$

B $\theta(2^n)$ ~~$\theta(2n)$~~

C $\theta(n)$

D None of these

n → even → odd
→ odd

12 → 11 → 9 → 7 → 5 → 3 → 1

13 → 11 → 9 → 7 → 5 → 3 → 1

$\approx (n/2)$

Topic : Analysis of algorithm

MSQ

33.3% ≈ :

#Q. $f(n) = 3^n$; $g(n) = 9^n$

Which of the following is correct?

A $f(n) = O(g(n))$ → Big oh

B $g(n) = \Omega(f(n))$

C $f(n) = o(g(n))$ → small oh.

D $g(n) = \omega(f(n))$

Ans:- A, B, C, D

= every $o \rightarrow O$

$O \rightarrow o$ ✗

Soln: $f = 3^n$ $g = 9^n$

$3^n = 9^n$ ✗

$f = 3^n$ $g = 9^n$

$3n \approx 9n$ ✓ $\longrightarrow O(n)$

$(a^m)^n = (a^n)^m = a^{m \times n}$

$9^n = (3^2)^n = (3^n)^2 = 3^n * 3^n$

$f = 3^n$ $g = 3^n * 3^n$

$f < g$ ✓

3^n $3^n * 3^n$

$1 < 3^n$

A) $f = O(g) \rightarrow f \leq g$ ✓ → True → less or equal.

B) $g = \Omega(f) \rightarrow g \geq f$ ✓

C) $f = o(g) \rightarrow f < g$ ✓

D) $g = \omega(f) \rightarrow g > f$ ✓

$5n^2 \rightarrow O(n^2)$

$\neq o(n^2)$

$5n \rightarrow o(n^2)$

$= O(n^2)$

Topic : Analysis of algorithm

MSQ

#Q. Consider the following functions $f(n) = n.3^n$ and $g(n) = 9^n$ then which of the following is correct?

A. $f(n) = \theta(g(n))$ ✗

B. $f(n) = \Omega(g(n))$ ✗

C. $f(n) = O(g(n))$ ✓

D. None of these ✗

Soln :

$n \times 3^n$

$9^n = 3^n \times 3^n$

$f \to g$ ✗

$f \leq g$ ✓

$n \leq A \leq 3^n$

$n\,3^n < 9^n$

Ans : C

$n * 3^n$ $\quad$ q^n

$\log(n * 3^n) \longrightarrow \log n + \log(3^n)$

$= \underline{\log n + n}$

$\underline{n * \log(3^n)}$ ✗

$\log(a \times b) = \underline{\log a + \log b}$

Topic : Analysis of algorithm

#Q. How many of the following statements is/are True?

A $10\sqrt{n} + \log n = O(n)$

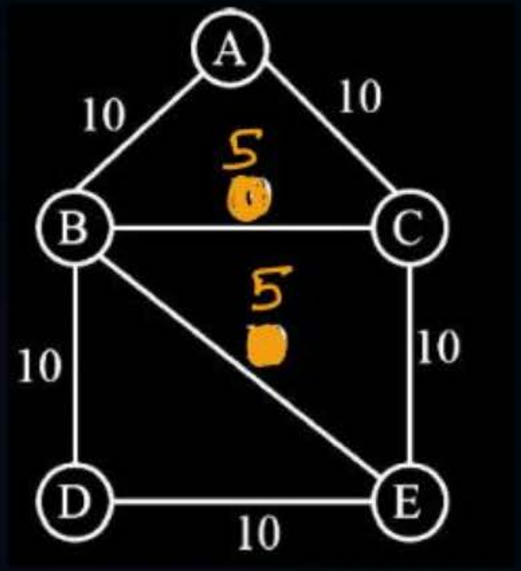
B $\sqrt{n} + \log n = O(\log n)$

C $\sqrt{n} + \log n = \theta(n)$

D $\sqrt{n} + \log n = \theta(\sqrt{n})$

[NAT]

#Q.6 Consider the following graph G:



H.W

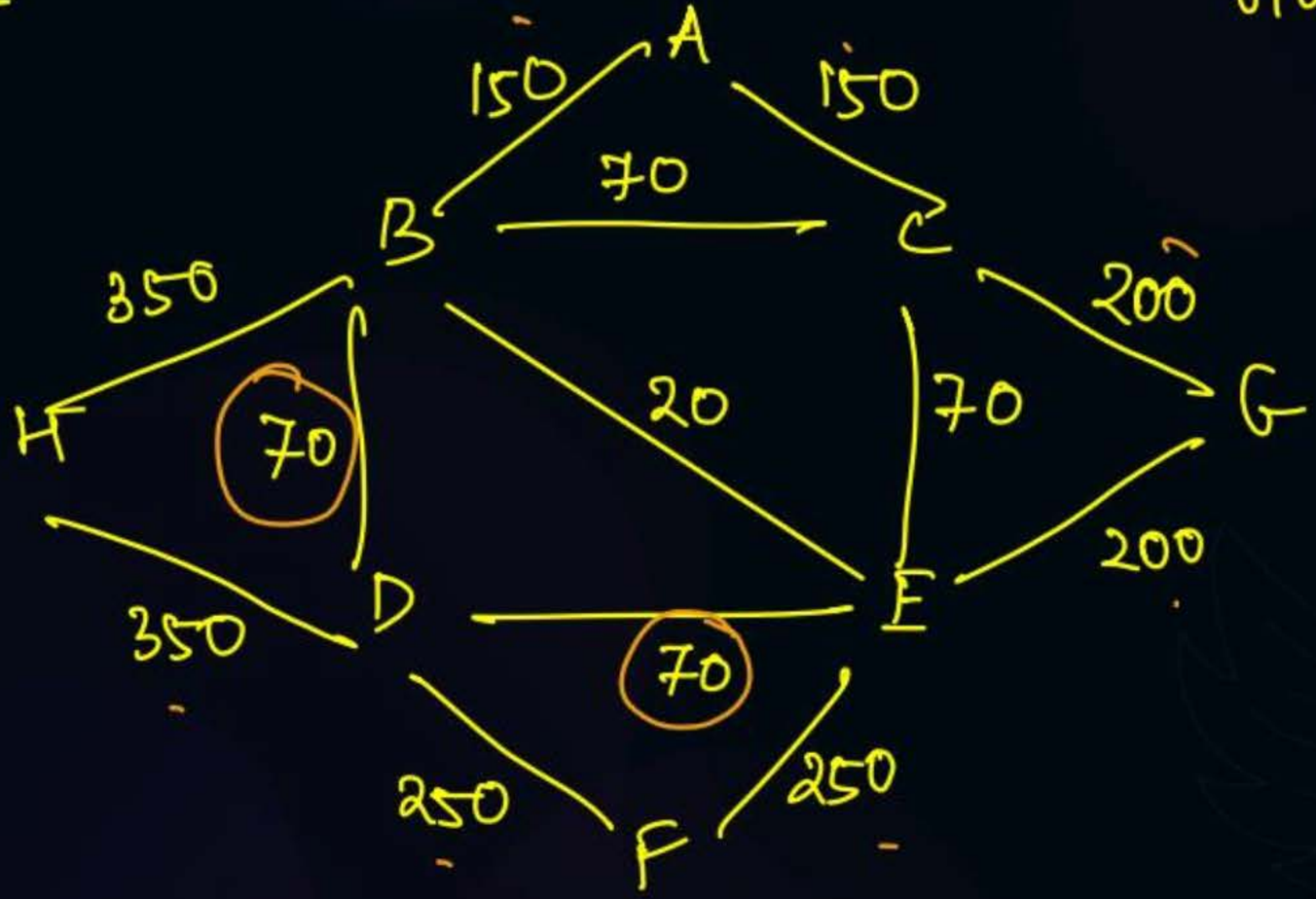
How many MST (minimum spanning tree) possible for above graph G?___

$n = 5 \longrightarrow$ edges in MCST $\rightarrow$ G

$(n-1)$

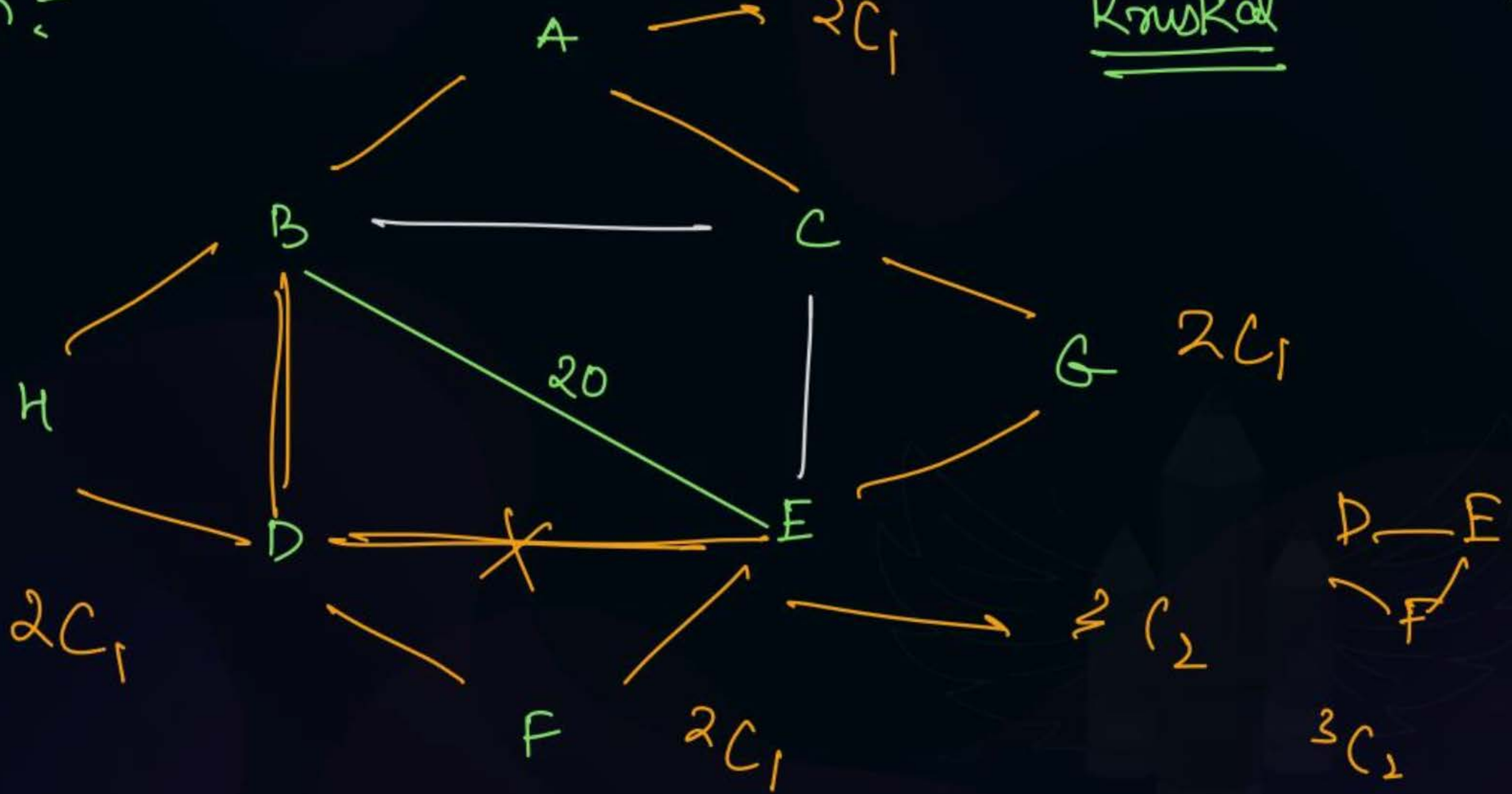
(9)

0%



How many distinct MCSTs are possible?

Soln :-



Kruskal

D — E
F
3C1

Left $\rightarrow {}^2C_1$

Right $\rightarrow {}^2C_1$

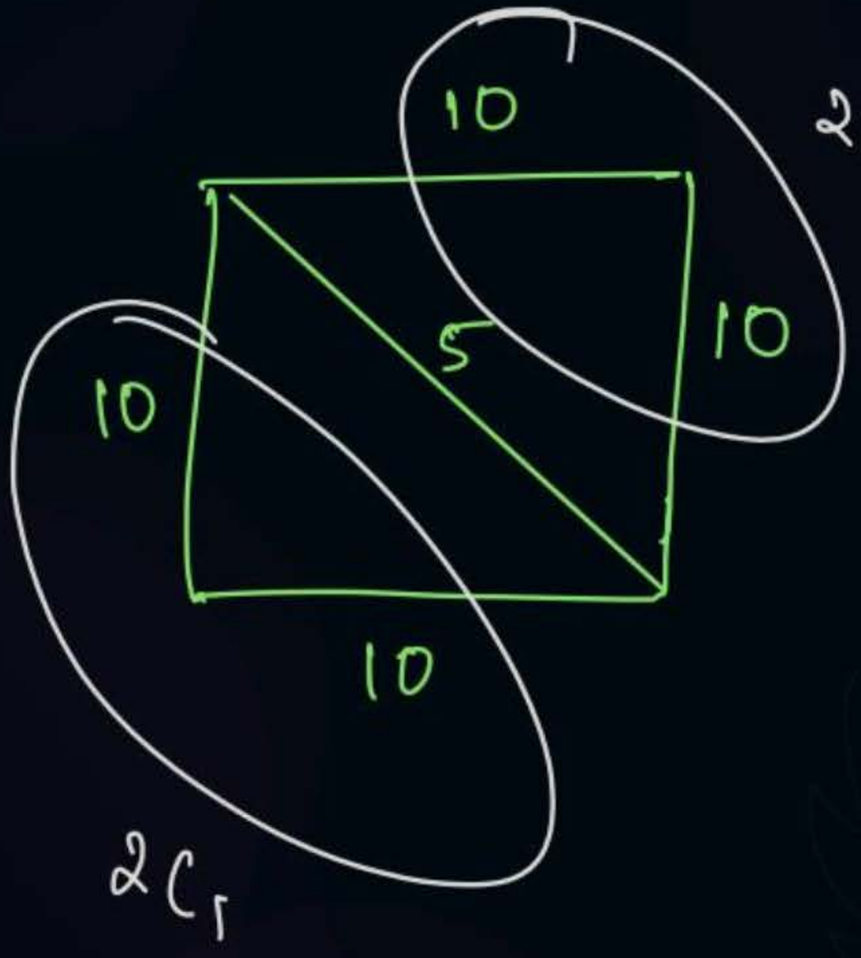
$${}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1$$

$= (2)^6$

$= \boxed{64}$

$3 \times 2 = 6$

$3 + 2 = 5$ X



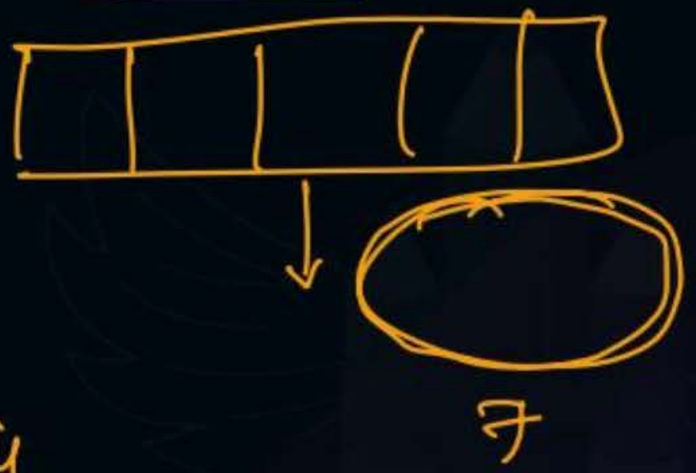
How many MCSTs ?

$^2C_1 * {}^2C_1 = 4$

Topic : Divide and Conquer

#Q4. Consider the number in the sequence 12, 15, 21, 27, 29, 31, 36, 46, 49, 60, 71, 75, 89, 98, 109 Using binary search, the number of comparisons required to search elements '109' is_

Ans : 4



$n = 15$ (odd)

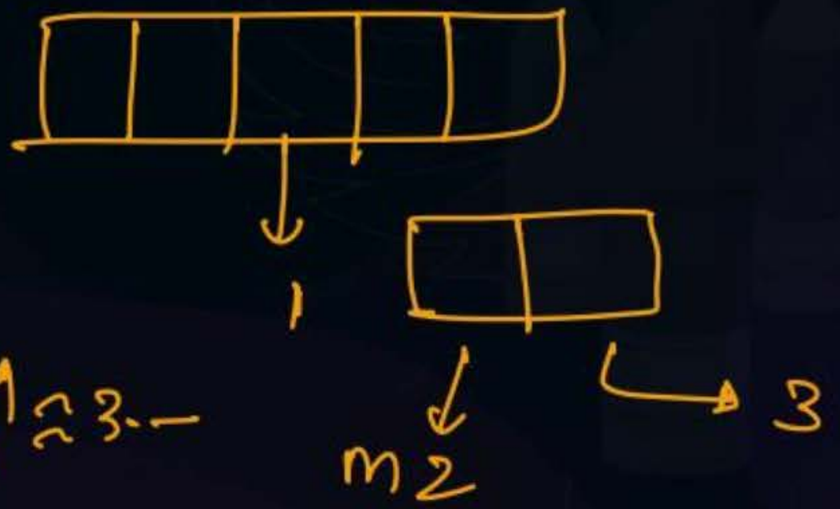
$\log_2 8 = 3$

$\log_2 16 \Rightarrow 4$

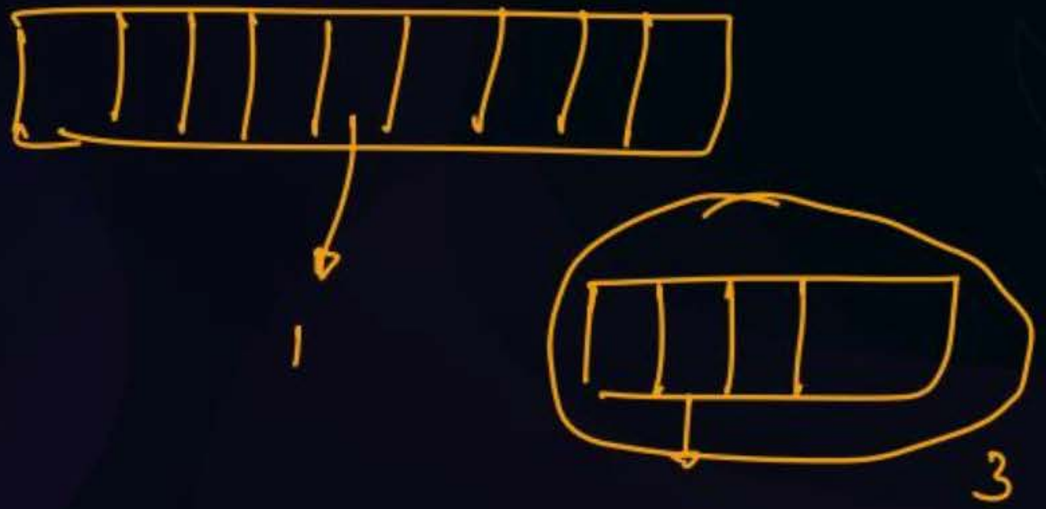
$\lceil \log_2 15 \rceil = 4$

no. of Comparisons in WC

n	Comparisons
1	1
3	2 ✓
5	3 ✓
7	3 ✓
9	4 ✓



$\log_2 1 \simeq 3$ —



$$\lceil \log_2(n) \rceil$$

ceil(3.1) = 4

Topic : Analysis of algorithm

#Q. Consider two function $f(n) = 10n + 2\log n$ and $g(n) = 2(\log(n^3)) + 5n$, then which of the following is correct option?

A. $f(n) = \theta(g(n))$ ✓ → equal rate of growth

B. $f(n) = O(g(n))$

C. $g(n) = O(f(n))$

D. $g(n) = O(\log n)$ ✗

$f = \theta(g)$

$f = O(g)$

$f = \Omega(g)$

$g(n) = O(f(n))$

$f = O(g)$

$g = O(f)$

Ans:- A, B, C

Soln :-

$f(n) = 10n + 2\log n$

$g(n) = 2(\log(n^3)) + 5n$

$\log_a n^x$

$= x \log_a n$

$= 3 \times 2 \times \log n + 5n$

$g(n) = 6\log n + 5n$

$f(n) = 2\log n + 10n$

$f(n) \underset{A}{=} g(n)$

$6\log n + 5n \rightarrow O(n)$

Topic : Analysis of algorithm

#Q. Consider the following code:

```
main( )
i = 1;
while (i ≤ n)
{
    i = i * 5;
}
```

what is the time complexity of above code?

$1 \to 5 \to 5^2 \longrightarrow 5^K$

$5^K = n$

$K = \log_5 n$

A $\theta(n)$

B $\theta(\sqrt{n})$

C $\theta(\log n)$

Ans : C

D None of these

THANK - YOU

Telegram Link for Aditya Jain sir:
https://t.me/AdityaSir_PW