GATE

CRASH COURSE

ALL BRANCHES

Engineering Mathematics

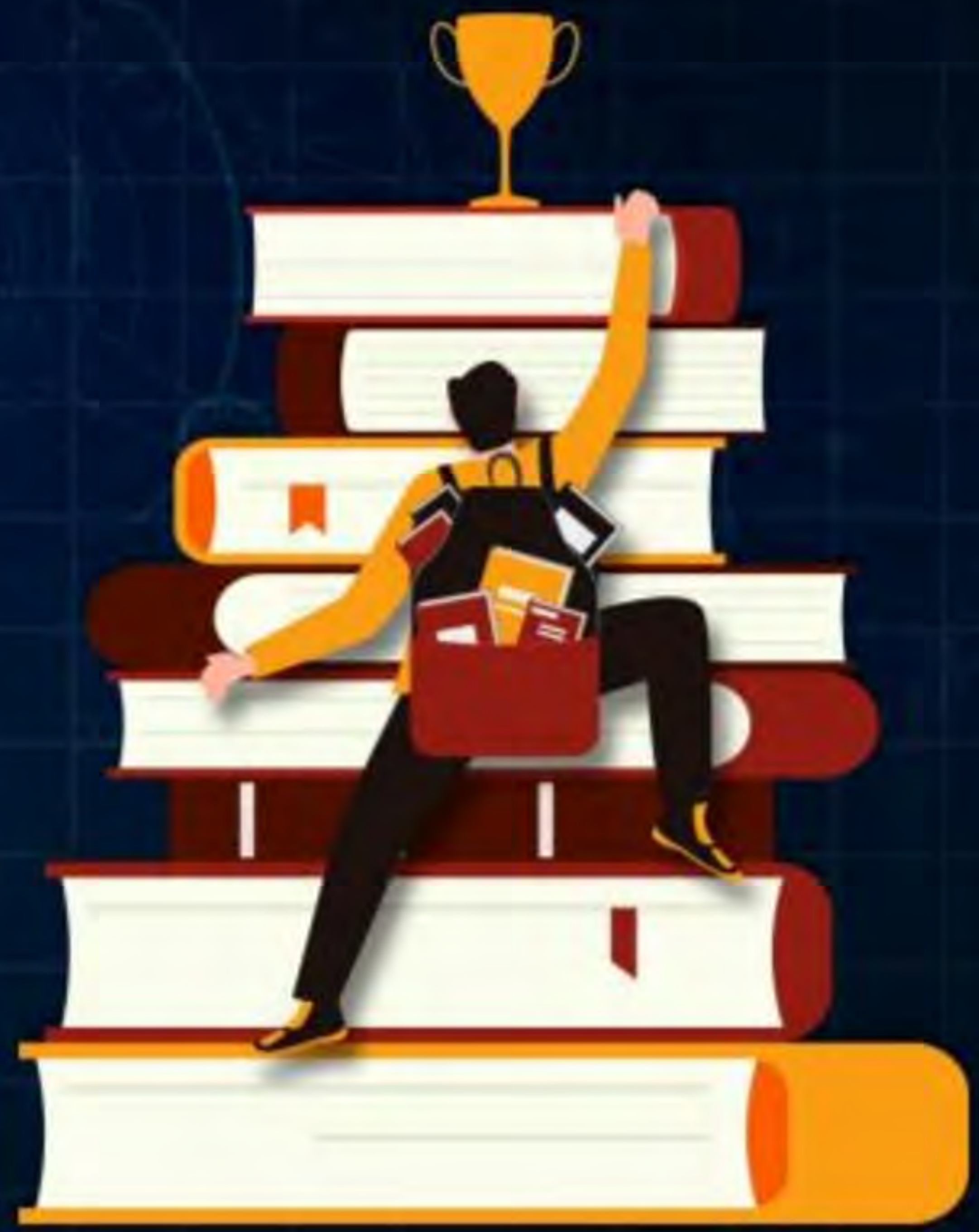
Linear Algebra (Part 03)
(Lec 03)

By - Dr. Puneet Sharma Sir

Topics to be covered

1. Homogeneous System
2. E Values & E. Vectors
3. Cayley Hamilton Th.

* In Case of underdetermined System unique Sol never exist.



Homog. Linear System of Eqn's : $\boxed{A_{m\times n} X_{n\times 1} = O_{m\times 1}}$

eg
$$\left.\begin{aligned} 2x-y+4z&=0\\ 3x+2y-z&=0\\ 7x+2y+5z&=0\\ -x-y+3z&=0 \end{aligned}\right\}$$

$X = \begin{bmatrix} x\\ y\\ z \end{bmatrix} = \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix}$ is the sol of this system.

and this type of sol always exist

(1) Homog system Never Inconsistent or always Consistent

$\because \rho(A) = \rho(A:O)$ always i.e there is No Need to write Aug Mat in this Chapter.

$A = \begin{bmatrix} 2 & -1 & 4\\ 3 & 2 & -1\\ 7 & 2 & 5\\ -1 & -1 & 3 \end{bmatrix}_{4\times 3}$, $O = \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} \Rightarrow [A:O] = \left[\begin{array}{ccc|c} 2 & -1 & 4 & 0\\ 3 & 2 & -1 & 0\\ 7 & 2 & 5 & 0\\ -1 & -1 & 3 & 0 \end{array}\right]_{4\times 4}$

(2) unique soln = Trivial soln = Zero soln. $\Rightarrow X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(3) ∞ solns exist = Non Trivial soln also exist = Non Zero soln also exist

$X = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{Zero soln.}}, \underbrace{\begin{bmatrix} - \\ - \\ - \end{bmatrix}, \begin{bmatrix} - \\ - \\ - \end{bmatrix}, \begin{bmatrix} - \\ - \\ - \end{bmatrix} \ldots\ldots \infty \text{ soln.}}_{\text{Non Trivial soln. / Non Zero solns.}}$

(4) underdetermined, Homog System always consist $\underline{\infty}$ soln.

(5) No soln $\Rightarrow$ Never exist in case of Homog System

(6) Zero soln $\neq$ No soln. (this concept is Basically applied in Homog System)

i.e $X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (X = No values of x, y, z exist)

Methods of Solving Homog System → Consider $\boxed{A_{m\times n} X_{n\times 1} = O_{m\times 1}}$

① **Rank Method** (always applicable)
(i.e. for $m>n$, $m=n$, $m<n$)

(i) if $\rho(A) =$ No. of Variables $\Rightarrow$ unique sol exist
(ii) if $\rho(A) <$ " " $\Rightarrow$ ∞ sol exist

② **Matrix Method**
(applicable only for $m=n$)

(i) if $|A| \neq 0 \Rightarrow$ unique sol exist
(ii) if $|A| = 0 \Rightarrow$ ∞ sol exist

Note ① unique sol exist $\simeq$ Trivial sol exist $\simeq$ Zero sol exist
② ∞ sol exist $\simeq$ Non T. sol also exist $\simeq$ Non Zero sol also exist

Q: The nature of soln of
$$\left.\begin{aligned}2x+y+z&=0\\ y-z&=0\\ x+y&=0\end{aligned}\right\} \Rightarrow \left.\begin{aligned}2x+y+z&=0\\ y&=z\\ x+y&=0\end{aligned}\right\} \Rightarrow \left.\begin{aligned}2x+2y&=0\\ x+y&=0\end{aligned}\right\} \Rightarrow x+y=0$$
will be ?

Soln:
$$\left.\begin{aligned}2x+y+z&=0\\ 0x+y-z&=0\\ x+y+0z&=0\end{aligned}\right\} \Rightarrow A=\begin{bmatrix}2&1&1\\0&1&-1\\1&1&0\end{bmatrix}_{3\times3}$$
$$\Rightarrow \rho(A)=2$$

Now, $|A| = 2[0+1]+1[-1-1] = 2-2 = 0$

$\because |A|=0 \Rightarrow \infty$ soln exist.

Let $z=y=k$ then $x=-k$, $k\in R$

$$X=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-k\\k\\k\end{bmatrix}=k\begin{bmatrix}-1\\1\\1\end{bmatrix}$$

Various soln are;

$$\underbrace{\begin{bmatrix}0\\0\\0\end{bmatrix}}_{\text{T. soln}},\ \underbrace{\begin{bmatrix}-1\\1\\1\end{bmatrix},\begin{bmatrix}-2\\2\\2\end{bmatrix},\begin{bmatrix}2\\-2\\-2\end{bmatrix},\begin{bmatrix}-1/3\\1/3\\1/3\end{bmatrix}}_{\text{Non Trivial soln}}\ldots\ldots \infty \text{ soln exist}$$

Analysis i.e out of non zero soln, how many are LI = ? = one

$\begin{bmatrix}-1\\1\\1\end{bmatrix}, \begin{bmatrix}-2\\2\\2\end{bmatrix}, \begin{bmatrix}5\\-5\\-5\end{bmatrix}, \begin{bmatrix}2/3\\-2/3\\-2/3\end{bmatrix}$ .... ∞ soln. i.e we have only one generator for this family of ∞ soln.

Shortcut i.e Number of LI solutions of $AX=0$ is $= \boxed{\text{No. of columns} - \rho(A)}$

If consider $A_{m\times n}$ then No of LI soln of Homog System $= n - \rho(A)$

$\boxed{\text{Nullity}(A) = \text{No of C} - \rho(A)}$

M-II $A_{3\times 3}$ so No of LI soln of $AX=0$ will be = ?

$N(A) = 3 - \rho(A) = 3 - 2 = 1$

The nullity of system of equations:

$$x_1 + x_2 - x_3 + x_4 = 0$$
$$2x_1 + 3x_2 + x_3 + 4x_4 = 0$$
$$3x_1 + 2x_2 - 6x_3 + x_4 = 0$$

(a) 1 (b) 2

(c) 3 (d) 4

it is underdetermined Homog system

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 1 & 4 \\ 3 & 2 & -6 & 1 \end{bmatrix}_{3\times 4}$$

$$\xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1}$$

$$A \sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -3 & -2 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 4} \Rightarrow \rho(A) = 2$$

So $N(A) = \text{No of } C - \rho(A) = 4 - 2 = 2$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ \alpha & 0 & 1 \end{bmatrix}_{3\times 3}$$

The value of α for which the system of equation

$$x + y + z = 0$$
$$y + 2z = 0$$
$$\alpha x + z = 0$$

has more than one solution is

(a) -1 (b) 0

(c) $\frac{1}{2}$ (d) 1

for ∞ soln

$\Downarrow$

$|A| = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ \alpha & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1(1-0) - 0 + \alpha(2-1) = 0$$

$1 + \alpha = 0$

$\alpha = -1$

Eigen Values & Eigen Vectors → for any Sq. Mat $A_{n\times n}$ if it is possible to find a relationship of the type $\boxed{AX = \lambda X}$ then scalar value λ is called E Value & Column Mat X is called E Vector

where λ may be $\underbrace{+ve, -ve, 0 \text{ or complex also}}$, $X \to$ Non Zero vector

i.e. Zero vector can't be taken as E Vector

Note → Value → any Constant & vector → Column Mat

Method to find E Value (λ) →

Consider $AX = \lambda X$

$AX - \lambda X = 0$

$AX - \lambda I X = 0$

$(A - \lambda I) X = 0$

or $\boxed{MX = 0}$

Non zero E Vector

Non zero soln.

∞ soln

& Condition for the existence of ∞ soln in Homog System, Det = 0

or $|M| = 0$

i.e $\boxed{|A - \lambda I| = 0}$ —— (1)

this is called C Eqn of A

& Values of λ solved by this eqn are called E Values.

eg If $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ then it's E Value $\lambda = 2$ & it's E Vector $X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\because AX = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2X$ Hence verified

Q. Find the E Values of $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

Sol: $A - \lambda I = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} (2-\lambda) & 1 \\ 0 & (2-\lambda) \end{bmatrix}$

= Char Matrix

So C.Eqn of A is $|A - \lambda I| = 0$

$\begin{vmatrix} (2-\lambda) & 1 \\ 0 & (2-\lambda) \end{vmatrix} = 0$

$(2-\lambda)^2 - 0 = 0 \Rightarrow \boxed{(\lambda-2)^2 = 0}$

& $\lambda = 2, 2$

Properties of E-Values → Let $A_{n\times n}$ & it's EValues are $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$

① Sum of EValues = Trace(A) i.e. $\boxed{\lambda_1+\lambda_2+\lambda_3+\cdots+\lambda_n = Tr(A)}$

② Product of EValues = Det(A) i.e. $\boxed{\lambda_1\cdot\lambda_2\cdot\lambda_3\cdots\cdots\lambda_n = |A|}$

③ $(\lambda=0) \Longleftrightarrow (|A|=0)$

eg $A=\begin{pmatrix}1&2&3\\4&5&6\\7&8&9\end{pmatrix} \Rightarrow |A|=0$

C.Eq of A is $|A-\lambda I|=0$

HW

$\lambda=\lambda_1, \lambda_2, 0$

eg $A=\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix} \Rightarrow \lambda=1,2,3$

$Tr=6$ & sum of EV $=6$

$|A|=2(4-1)=6$ & P of EV $=6$

eg $A=\begin{bmatrix}2&0&-1\\0&-3&2\\0&0&0\end{bmatrix} \Rightarrow \lambda=2,-3,0$

$Tr(A)=-1$ & $|A|=0$

(4) E Values of U.T.M, L.T.M, Diag Mat, Scalar Mat are just the Diag elements.

$$A = \begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix} = U.T.M \Rightarrow \lambda = 2, -1, 4, 3 \Rightarrow |A| = -24$$

(5) E Values of A^T are same as that of A

(6) " " A^m are $\lambda_1^m, \lambda_2^m, \ldots, \lambda_n^m$

(7) " " A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \ldots \frac{1}{\lambda_n}$

(8) " " (adj A) are $\frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}, \ldots, \frac{|A|}{\lambda_n}$

E Values of A^T are $2, -1, 4, 3$ ($\because A^T = L.T.M$)

" " A^3 are $(2)^3, (-1)^3, (4)^3, (3)^3$

$= 8, -1, 64, 27$

" " A^{-1} are $\frac{1}{2}, -1, \frac{1}{4}, \frac{1}{3}$

" " (adj A) are $\frac{-24}{2}, \frac{-24}{-1}, \frac{-24}{+4}, \frac{-24}{3}$

(9) E Values of Symm and Herm Mat are all Real Nos

(10) " " Skew Symm and Skew Herm are purely Img Nos

(11) " " orthogonal Mat & Unitary Mat are of unit Modulus.

(12) if $A^2 = I \Rightarrow \lambda = \pm 1$ ie E Values of Involutary Mat are ± 1

(13) if $A^2 = A \Rightarrow \lambda = 0$ or 1 ie " " Idempotent Mat are 0 or 1

Q. E Values of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

Sol: C Equⁿ of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} (2-\lambda) & 0 & 1 \\ 0 & (2-\lambda) & 0 \\ 1 & 0 & (2-\lambda) \end{vmatrix} = 0$$

Expanding along R_2

$$+(2-\lambda)\left[(2-\lambda)^2 - 1^2\right] = 0$$

$$(2-\lambda)\left[(2-\lambda-1)(2-\lambda+1)\right] = 0$$

$$(2-\lambda)(1-\lambda)(3-\lambda) = 0 \longrightarrow \text{C Equ}^n$$

ie E Values are $\lambda = 1, 2, 3$

Q. $A = \begin{bmatrix} 0 & +2i \\ 3i & 0 \end{bmatrix}$ then E Values are ?

C Equⁿ of A is $|A - \lambda I| = 0$ or $\begin{vmatrix} (0-\lambda) & +2i \\ 3i & (0-\lambda) \end{vmatrix} = 0$

$$\lambda^2 - (3i)(+2i) = 0$$

$$\lambda^2 - 6i^2 = 0$$

$$\lambda^2 + 6 = 0 \Rightarrow \lambda^2 = -6 \Rightarrow \lambda = \pm\sqrt{-6}$$

ie $\lambda = \pm\sqrt{6}.\sqrt{-1} = \pm\sqrt{6}\,i$

ie $\lambda = i\sqrt{6}$ & $-i\sqrt{6}$

If scalar λ is a characteristic root of the matrix A, then $(A - \lambda I)$ is ______.

(a) Singular matrix (b) Non-singular matrix

(c) Diagonal matrix (d) None of the above

$$Ax = \lambda x$$

$$(A - \lambda I)x = 0$$

$$\Rightarrow |A - \lambda I| = 0$$

i.e. $(A - \lambda I)$ is singular

We have a set of 3 linear equations in 3 unknowns. '$X \equiv Y$' means X and Y are equivalent statements and '$X \not\equiv Y$' means X and Y are not equivalent statements.

P : There is a unique solution.

Q : The equations are linearly independent.

R : All eigen values of the coefficient matrix are nonzero.

S : The determinant of the coefficient matrix is nonzero.

Which one of the following is TRUE?

(a) $P \equiv Q \equiv R \equiv S$ (b) $P \equiv R \not\equiv Q \equiv S$

(c) P Q R S (d) P Q R S

$A_{3\times3} \Rightarrow \boxed{AX=0}$

P unique sol $\Rightarrow |A| \neq 0$ Q

LI vector $\Rightarrow |A| \neq 0$

if $|A| \neq 0 \Leftrightarrow \lambda \neq 0$

S R

$R \equiv S \equiv P \equiv Q$

Let M be a skew-symmetric, orthogonal real matrix. Then only possible eigen values of M are ______.

(a) $-1, 1$

(b) $-i, i$

(c) 0

(d) $1, i$

$A \to$ S-Symm Mat then $\lambda =$ Purely Img

$A \to$ O-Mat $\Rightarrow |\lambda| = 1$

$|-i| = 1$ & $|i| = 1$

$z = x + iy \Rightarrow |z| = \sqrt{x^2 + y^2}$

The eigen values of $\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ are ______.

(a) –1, –1, 2
(b) –2, –1, 3
(c) 2, 2, 3
(d) None of the above

(ii) one of the E Value of A will be ? $\lambda = 0$

$A =$ S. Sym Mat of odd order

$\Rightarrow |A| = 0 \Rightarrow \lambda = 0$

Procedure of finding E Vector →

Q: Find the E Value and E Vectors of $A=\begin{bmatrix}2&1\\0&2\end{bmatrix}$

Sol: $\because$ A is U.T.M so $\lambda = 2, 2$

E. Vector for ($\lambda=2$) →

Consider $AX=\lambda X$

$$(A-\lambda I)X=0$$

$$(A-2I)X=0$$

$$\begin{bmatrix}(2-2)&1\\0&(2-2)\end{bmatrix}X=0$$

$$\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}x_2\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\Rightarrow x_2=0$$

Let $x_1=K$ so

$$X=\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}K\\0\end{bmatrix}=K\begin{bmatrix}1\\0\end{bmatrix}$$

i.e. E Vector for $\lambda=2$ is $X=\begin{bmatrix}1\\0\end{bmatrix}$

Cross check:

$$AX=\begin{bmatrix}2&1\\0&2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}=2\begin{bmatrix}1\\0\end{bmatrix}=2X$$

Q. $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ then E vectors of A are?

C.Equⁿ of A is $|A-\lambda I| = 0 \Rightarrow \lambda = \pm i$

E.Vector for $\lambda = i$ → consider $AX = \lambda X$

$$(A-\lambda I)X = 0 \Rightarrow (A - iI)X = 0$$

$$\begin{bmatrix} (0-i) & -1 \\ 1 & (0-i) \end{bmatrix} X = 0$$

$$\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix} X = 0$$

$$\begin{bmatrix} 1 & -i \\ -i & -1 \end{bmatrix} X = 0 \xrightarrow{R_2 \to R_2 + iR_1} \begin{bmatrix} 1 & -i \\ 0 & 0 \end{bmatrix} X = 0$$

i.e. $\begin{bmatrix} 1 & -i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} (x_1 - ix_2) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 = ix_2$$

Let $x_2 = K$ then $x_1 = iK$

So $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} iK \\ K \end{bmatrix} = K \begin{bmatrix} i \\ 1 \end{bmatrix}$

So Evector for $\lambda = i$ is $X = \begin{bmatrix} i \\ 1 \end{bmatrix}$

Similarly

Evector for $\lambda = -i$ is $X = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

The vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector of

$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then corresponding eigen value of A is

(a) 1
(b) 2
(c) 5
(d) −1

$$AX = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ -5 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = 5X$$

If $\{1, 0, -1\}^T$ is an eigen vector of the following matrix $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ then the corresponding eigen value is

(a) 1 (b) 2

(c) 3 (d) 5

$$AX = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = 1 \cdot X$$

EE 2014

The pair of eigen vectors corresponding to the two eigen values of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(a) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}$

Let us take (d); $AX = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} -i \\ 1 \end{bmatrix} = -i \begin{bmatrix} 1 \\ \frac{1}{-i} \end{bmatrix} = -i \begin{bmatrix} 1 \\ \frac{i^2}{i} \end{bmatrix} = -i \begin{bmatrix} 1 \\ i \end{bmatrix} = \lambda X$

$AX = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ i \end{bmatrix} = i \begin{bmatrix} \frac{-1}{i} \\ 1 \end{bmatrix} = i \begin{bmatrix} \frac{-i^2}{i} \\ 1 \end{bmatrix} = i \begin{bmatrix} i \\ 1 \end{bmatrix} = \lambda X$

Cayley Hamilton Theorem → "Every Sq Mat satisfies it's own C.Equ^n"

i.e we can Replace $\lambda \to A$ in C.Equ^n.

Consider $A_{4\times4}$ then it's C.Eq is $|A-\lambda I| = 0$

or $\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$

By C.H.Th, $\lambda \to A$

$$A^4 + a_1A^3 + a_2A^2 + a_3A + a_4I = 0$$

Application:-
① $\text{Trace}(A) = -a_1$
② $|A| = (-1)^4 a_4$

If Consider $A_{3\times3}$ then C.Eq is

$$|A-\lambda I| = 0$$

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

By C.H.Th, $\lambda \to A$

$$A^3 + a_1A^2 + a_2A + a_3I = 0$$

① $\text{Tr}(A) = -a_1$
② $|A| = (-1)^3 a_3$

for $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ C.Eqn in $\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

$\Downarrow$

$|A| = 3$

$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$

So $|A| = (-1)^3 a_3 = -(-3) = 3$

Shortcut C-Eqn of 2×2 Mat. i.e. $|A - \lambda I| = 0$

$\lambda^2 + a_1\lambda + a_2 = 0$

$\lambda^2 - (-a_1)\lambda + ((-1)^2 a_2) = 0$

$\boxed{\lambda^2 - (Tr(A))\lambda + (|A|) = 0}$

for eg $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

C.Eqn $\lambda^2 - (Tr)\lambda + (|A|) = 0$

$\lambda^2 - (0)\lambda + (1) = 0$

$\boxed{\lambda^2 + 1 = 0} \Rightarrow \lambda = \pm i$

Q: if $A_{3\times3}$ s.t it's CEq is $\lambda^3-4\lambda^2+2\lambda-5=0$ then find

it's $Tr(A)$, $|A|$ & $A^{-1}=$?

Sol: By C.H.Th, $\lambda \to A$ ie

$$\boxed{A^3-4A^2+2A-5I=0} \longrightarrow (1)$$

on comparision with standard form

$$A^3+a_1A^2+a_2A+a_3I=0$$

$$\Rightarrow a_1=-4,\ a_2=2,\ a_3=-5$$

(1) $Tr(A)=-a_1=-(-4)=+4$

(2) $|A|=(-1)^3 a_3=-(-5)=5$

(3) $\because |A|\neq 0 \Rightarrow A^{-1}$ exist

By (1), $A^{-1}\cdot(A^3-4A^2+2A-5I)=A^{-1}\cdot 0$

$$A^2-4A+2I-5A^{-1}=0$$

$$A^2-4A+2I=5A^{-1}$$

or $5A^{-1}=A^2-4A+2I$

$$\Rightarrow A^{-1}=\frac{1}{5}(A^2-4A+2I)$$

In matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ if $a + d = ad - bc = 1$, then $A^3 =$ ______

(a) $A - I$

(b) $A + I$

(c) $-I$

(d) 0

$A^3 = A^2 A$

$= (A - I) A$

$= A^2 - A$

$= (A - I) - A$

$= -I$

C-Eq is $\lambda^2 - (Tr(A))\lambda + (|A|) = 0$

$\lambda^2 - (a+d)\lambda + (ad-bc) = 0$

$\lambda^2 - \lambda + 1 = 0$

By C-H-Th, $A^2 - A + I = 0$

$\Rightarrow A^2 = A - I$ —— (1)

Thank

THANK

Keep Hustling!