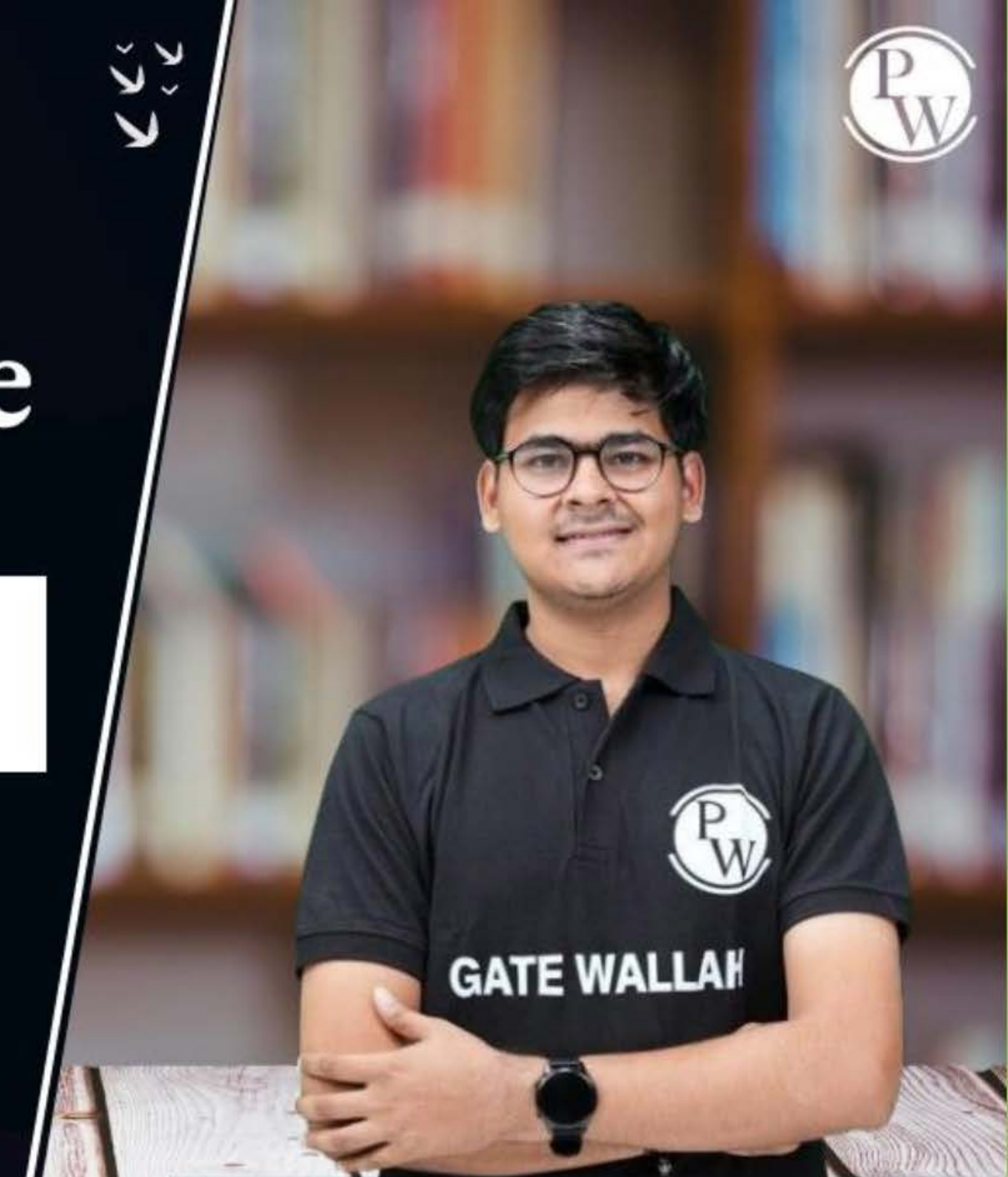
Data Science & Artificial Intelligence

Algorithms

Test Series 1500+

Lecture - 01



By- Aditya sir

ABOUT ME

Hello, I'm Aditya.

1. Represented college as the first Google DSC Ambassador.
2. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
3. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored working professions in field of Data Science and Analytics
11. Have been mentoring GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

1500+ Series: Practice

↳ Problem Solving (Concepts)

Algorithms: → 100+ questions

Treat this as a exam environment

1) Time & Space Complexity

2) Sorting

3) Divide and Conquer

4) Greedy Algos

5) Dynamic Programming

6) Heaps

7) Graph Algorithms

8) Miscilleneous

Questions Practice

Topic : Analysis of algorithm

#Q. Consider the following code.

```
main( )
    {
        i=1;
        while(i <= n)
        {
            i=10*i;
        }
    }
```

$i=1$ $i<=n$

$i=10*i$

Ans : D

What is the highest asymptotic worst case time complexity of above code fragment?

A $O(n^2)$

B $O(\sqrt{n})$ — 20 %

C $O(n)$

D $O(\log n)$ → 80 %

Soln :-

$i = 1 \longrightarrow \underline{10} \longrightarrow \underline{10^2} \longrightarrow \underline{10^3} \cdots\cdots$ i th iter 10^i

Assume loop ends after i iterations

$$10^i = n$$

$$i * \log_{10} 10 = \log_{10} n$$

$$\boxed{i = \log_{10} n} \longrightarrow O(\log n)$$

Topic : Analysis of algorithm

B

#Q. What is the time complexity of the following code ?

```
for (a = 0; a <= n ; a =a*2)
{
    for (b = 0; b < 100 ; b = b +2)
    {
        for (c = 1; c < 8*n; c ++)
        {
            print("AJ Sir")
        }
    }
}
```

Infinite loop

$a = 0;$ $a = a * 2$

$\frac{100}{2} \approx 50$

$\Rightarrow \approx 8n$

A $O(n^3)$ → 20%

B $O(n^2)$ → 20%

C $O(\log n)$

D None of These → 60%

Ans: D

Soln: $a = 0 \longrightarrow \; <= n \longleftrightarrow \; *2$

$0 \times 2 = 0 \longrightarrow 0 \times 2 \rightarrow 0 \longleftrightarrow 0 \times 2 \rightarrow$

$<= n$

$a = 1 \; ; \; a <= n ; \; a = a * 2 \longrightarrow \log_2 n$

$1 \rightarrow 2 \rightarrow 2^2 \rightarrow 2^3 \ldots$

$0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \ldots$ ∞ loop

Topic : Analysis of algorithm

#Q. Consider the following recurrence relation (T(n)):

$$T(n) = 9T\left(\frac{n}{3}\right) + C$$

What is the time complexity of above recurrence relation?

A. $\theta(\log n)$

B. $\theta(n^2 \log n)$

C. $\theta(n^2)$

D. $\theta(n^3)$

Ans : C

Time Complexity Recurrence :

1) Back - substitution $\longrightarrow$ value + TC

2) Masters method $\longrightarrow$ TC

3) Recurrence Tree $\longrightarrow$ TC

* i) Back - substitution :

$$T(n) = 9T(n/3) + c \quad \text{—— (1)}$$

$$T(n/3) = 9T(n/3^2) + c$$

$$T(n) = 9\left(\underbrace{9T(n/3^2) + c}_{T(n/3)}\right) + c$$

$$T(n) = 9^2 T(n/3^2) + 9^1 c + 9^0 \times c \quad \text{—— (2)}$$

$$T(n/3^2) = 9T(n/3^3) + c$$

$$T(n) = 9^2\left(9T(n/3^3) + c\right) + 9c + c$$

$$= 9^3 T(n/3^3) + 9^2 c + 9c + c$$

$$= 9^3 T(n/3^3) + \underline{\left(9^2 c + 9^1 c + 9^0 c\right)} \text{ --- } 3$$

$$\vdots$$

General term

$$T(n) = 9^K\, T(n/3^K) + C * \underbrace{(9^{K-1} + 9^{K-2} \dots 9^0)}$$

$$\underbrace{9^0 + 9^1 + \dots 9^{K-1}}$$

$$\left.\begin{aligned} a &= 9^0 \\ r &= 9 \\ 0 \longrightarrow K-1 \qquad n &= K \end{aligned}\right\}$$

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{1 \times (9^K - 1)}{9-1}$$

$$= \left(\frac{9^K - 1}{8}\right)$$

$$T(n) = 9^k \, T\left(\frac{n}{3^k}\right) + c * \left(\frac{9^k - 1}{8}\right)$$

For Base Condition

$$\frac{n}{3^k} = 1 \longrightarrow n = 3^k$$

$$\boxed{k = \log_3 n}$$

$$3^k = n$$

$$9^k = (3^2)^k$$

$$= (3^k)^2 = \underline{\underline{n^2}}$$

$$\left[(a^m)^n = (a^n)^m = a^{n*m} \right]$$

$$T(n) = 9^k \, T(n/3^k) + c * \frac{(9^k - 1)}{8}$$

$$9^k = n^2$$

$$3^k = n$$

$$= n^2 \times T(1) + c * \frac{(n^2 - 1)}{8}$$

$$T(n) = n^2 * b + \left(\frac{n^2 - 1}{8}\right) * c \longrightarrow \text{value of Recurrence.}$$

$$\boxed{T(n) = O(n^2)}$$

* Masters method:

$$\boxed{T(n) = a * T(n/b) + f(n)}$$ → Symmetric DnC

where $a \geq 1$, $b > 1$, $f(n) = +ve$

constants

no. of subproblems

Size of each subproblem

Case 1: If $F(n) = O(n^{\log_b a - \varepsilon})$, some $\varepsilon > 0$

then $T(n) = \Theta(n^{\log_b a})$

Case 2 : If $f(n) = \Theta\left(n^{\log_b a} * (\log n)^k\right)$, some k

a) $k \geq 0$, $T(n) = \Theta\left(n^{\log_b a} * (\log n)^{k+1}\right)$

b) $k = -1$, $T(n) = \Theta\left(n^{\log_b a} * \log \log n\right)$

Case 3 : If $f(n) = \Omega\left(n^{(\log_b a + \varepsilon)}\right)$, some $\varepsilon > 0$

and $a * f(n/b) \leq \delta * f(n)$, some $\delta < 1$

then $T(n) = \Theta(f(n))$

$$T(n) = 9T(n/3) + C$$

$$\left.\begin{aligned} a &= 9 \\ b &= 3 \\ f(n) &= C \end{aligned}\right\} \underline{\underline{\text{valid}}}$$

$$\log_b a = \log_3 9 = \underline{\underline{2}}$$

<u>Check case 1</u> : Is $C = O(n^{2-\varepsilon})$, some $\varepsilon > 0$? ✓

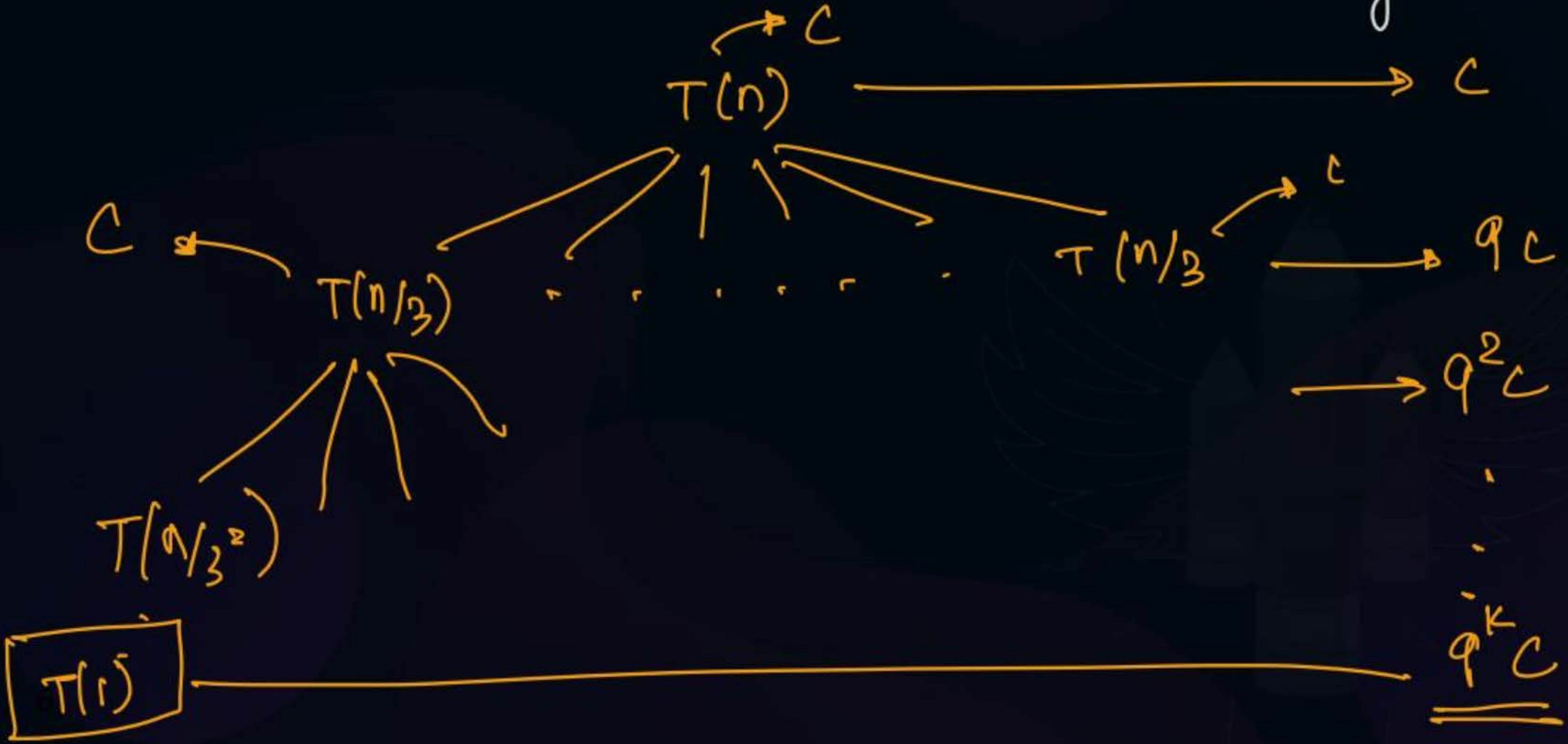
$\varepsilon = 1, 0.5 \ldots$

Here $\boxed{T(n) = \Theta(n^2)}$

3) Tree appr

$$T(n) = 9T(n/3) + c$$

Cost/Fee of Recurrce.



Last level $9^k c$

$$\text{Total Cost} = 9^0 c + 9^1 c + 9^2 \cdots 9^k c$$

$$= c\left(9^0 + 9^1 \cdots 9^k\right)$$

$$= \frac{c\left(9^{k+1} - 1\right)}{8}$$

$3^k = n$

$9^k = n^2$

$$= O\left(9^{k+1}\right)$$

$$= \boxed{O\left(n^2\right)}$$

Topic : Analysis of algorithm

#Q. Let f(n) = logloglog $\sqrt{n}$ and [illegible] then which one of the following is true?

(i) f(n) = θ(g(n)) · ×

(ii) f(n) = Ω(g(n)) → $f \geq g$

(iii) f(n) = O(g(n)) → $f \leq g$ ×

(iv) f(n) = ω(g(n))

↳ $f > g$

MSQ

$g(n) = 30^{(300)^{(20)}}$

A (i)

B (ii)

C (iv)

D (iii)

$f \geq_A g$

Ans: B, C

$f > g$ ✓

$f \geq g$ ×

Asymptotic Notations

O, Ω } Tight + loose Bound

O, o } UB

$\Theta \longrightarrow$ Tight Bound

Ω, ω } LB

o, ω } $\longrightarrow$ lose Bound

1) $f(n) = O(g(n)) \Longrightarrow \underline{\underline{a \leq_A b}}$

2) $f(n) = o(g(n)) \Longrightarrow \underline{\underline{a <_A b}}$

3) $f(n) = \Theta(g(n)) \Longrightarrow a =_A b$

4) $f(n) = \Omega(g(n)) \Longrightarrow a \geq_A b$

5) $f(n) = \omega(f(n)) \Longrightarrow a >_A b$

General Asymptotic Rate of Growth

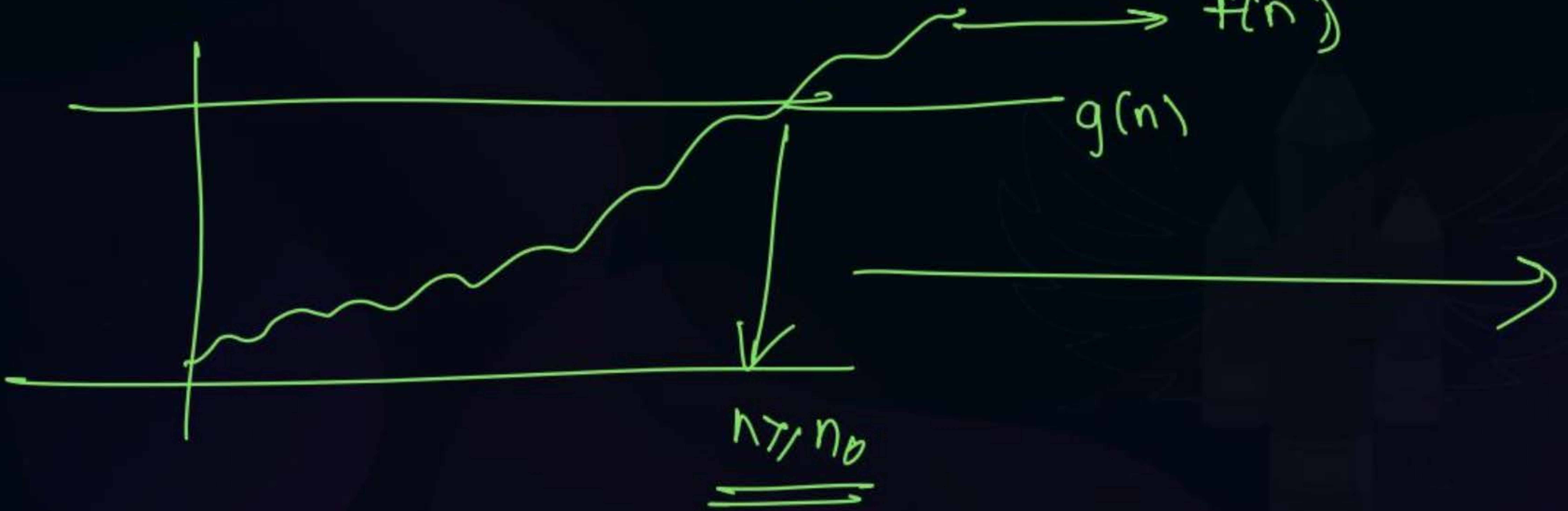
Decreasing Functn < Constant < log < Poly < Expo

$\frac{1}{n} < 10 < \log n < \sqrt{n} < n < n^2 \ldots < 2^n < n^n$

Soln :- $f(n) = \log \log \log(\sqrt{n}) \longrightarrow$ Increasing function of n

$g(n) = 30^{(300)^{20}}$

↳ Constant.



$$f(n) \underset{A}{\geq} g(n)$$

Topic : Analysis of algorithm

#Q. Sort the functions in increasing order of asymptotic (big - O) complexity f

$f_1(n) = n\log(n^*n)$

$f_2(n) = 500n$

$f_3(n) = (n)^{1.5}$

$f_4(n) = n^2$

Asymptotic Comparison

A. $f_1(n), f_2(n), f_4(n), f_3(n)$ ✗

B. $f_3(n), f_4(n), f_2(n), f_3(n)$ ✗

C. $f_2(n), f_4(n), f_1(n), f_3(n)$ ✗

D. $f_2(n), f_1(n), f_3(n), f_4(n)$ ✓

Ans: D

$f_1 \Rightarrow n\log(n \times n) \longrightarrow n\log(n^2) \longrightarrow$ Poly log

$f_2 \rightarrow 500n \longrightarrow$ Poly

$f_3 \longrightarrow n^{1.5} \longrightarrow$ Poly

$f_4 \longrightarrow n^2 \longrightarrow$ Poly

$$\boxed{n^{1.5} > \underline{n\log(n^{10})}}$$

(larger values of n)

$n\log n$

$$\underline{500n} < n^{1.5} < n^2$$

$n\sqrt{n}$ $\quad$ $500n$

$n = 1$

$1 < 500$

$n\log(n^2) \longrightarrow 2n\log n \longrightarrow O(n\log n)$

$500\cancel{n}$ $\quad \cancel{n}\log n$ $\quad \cancel{n}\sqrt{n}$ $\quad \cancel{n}\log n$

$500 < \log n$ $\qquad \sqrt{n} > \log n$

$$500n < n\log(n^2) < n^{1.5} < n^2$$

$f_2 \quad f_1 \quad f_3 \quad f_4$

[MCQ]

#Q17. What is the time complexity of insertion sort in best case, average case and worst case respectively is:

A $O(n), O(n), O(n^2)$ ✗

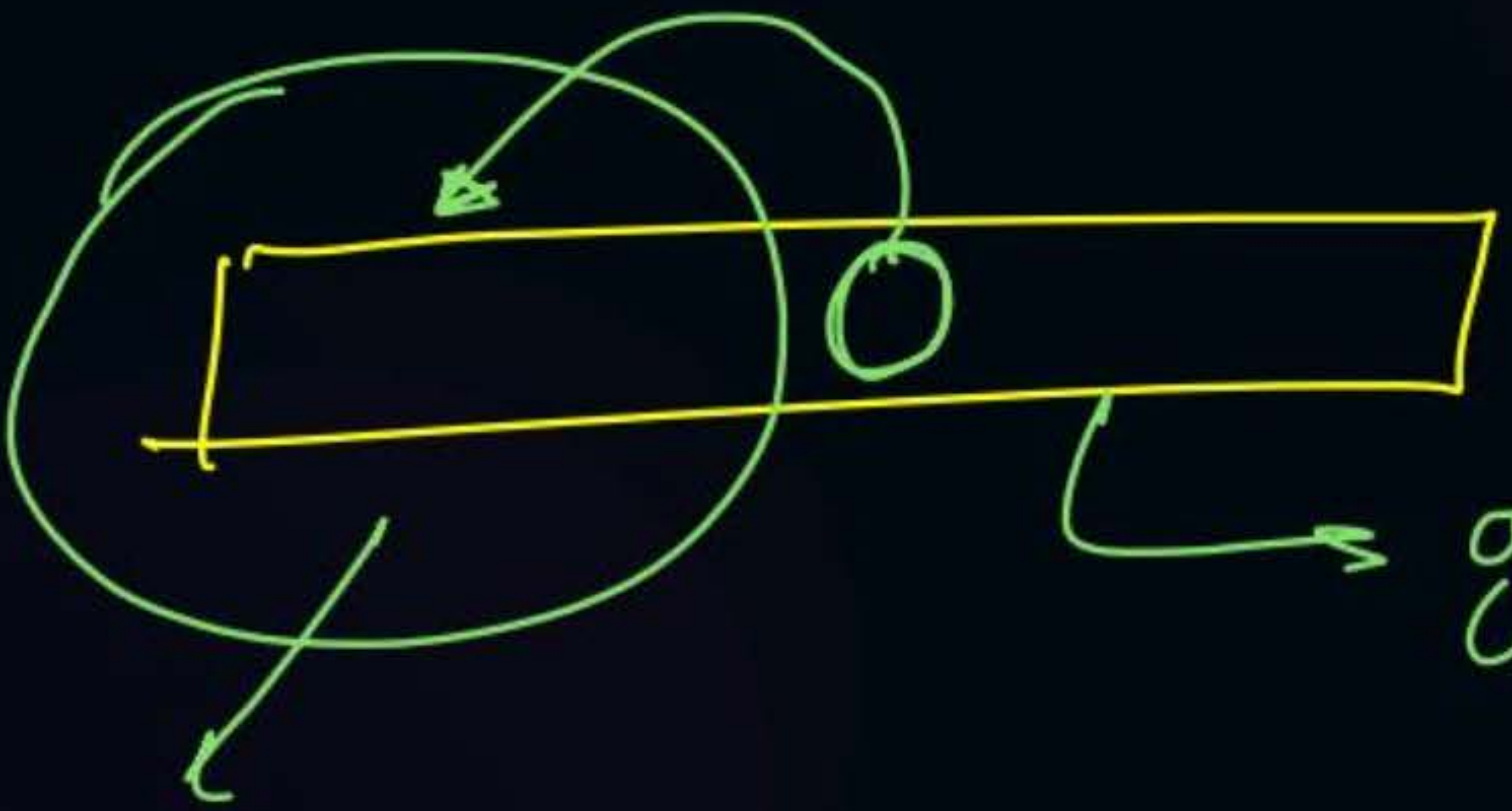
B $O(n), O(n^2), O(n^2)$ ✓

C $O(n^2), O(n^2), O(n^2)$ ✗

D $O(n), O(n\log n), O(n\log n)$ ✗

Ans: B

Insertion Sort



given unsorted

2	5	10	12

sorted

AC: $O(n^2)$

BC: $O(n)$

WC: $O(n^2)$

10	5	4	3

TC: no. of elem comp $(n-1)$

no. of swaps

0

[MCQ]

#Q18. How many swaps and comparisons are needed in selection sort to sort n element in worst case respectively?

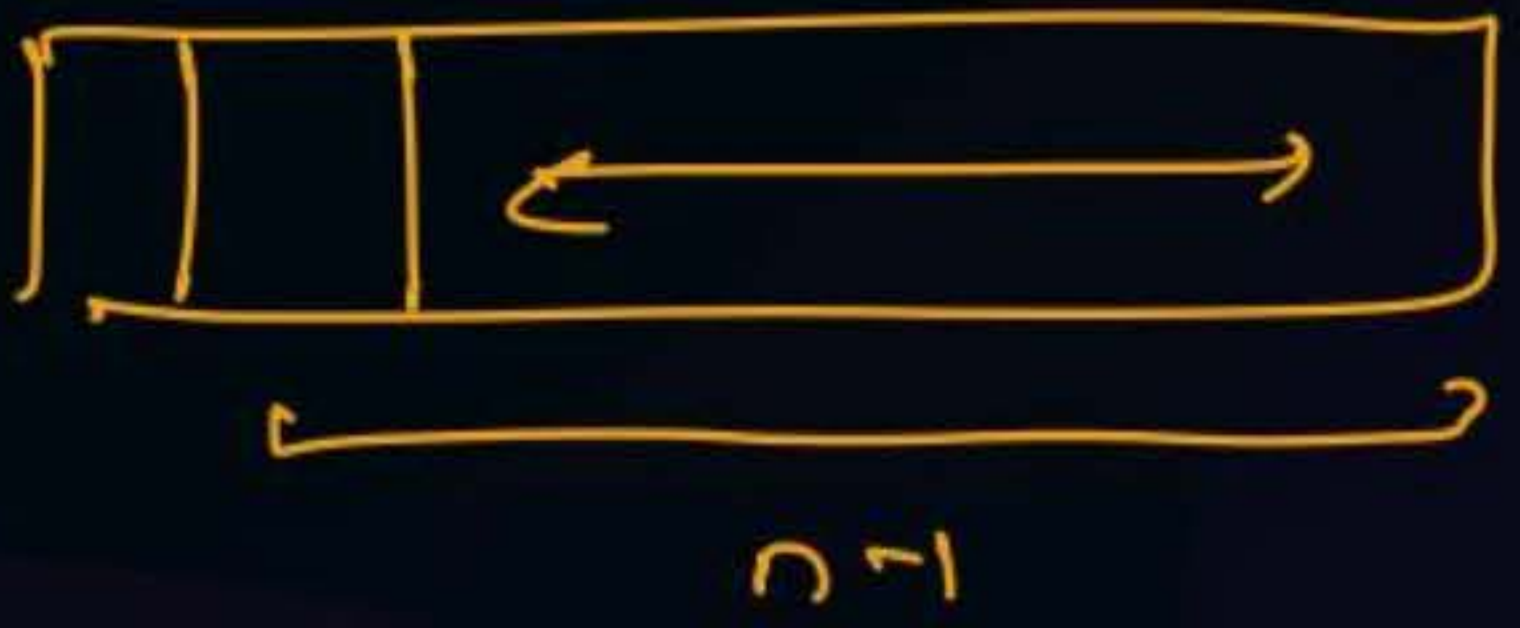
(A) $\underline{(n-1)}$, $\frac{n(n-1)}{2}$

(B) $\frac{n(n-1)}{2}$ $\frac{n(n-1)}{2}$

(C) (n), $\frac{n(n-1)}{2}$

(D) n^2, n^2

$$\left.\begin{matrix}(n-1)\\ n\end{matrix}\right\} \underline{O(n)}$$



Ans: A

Selection Sort:

passes

(n-1) always

exacty 1 swap in every pass

(n-1) swaps

$(n-1) + (n-2) \cdots\cdots 1$

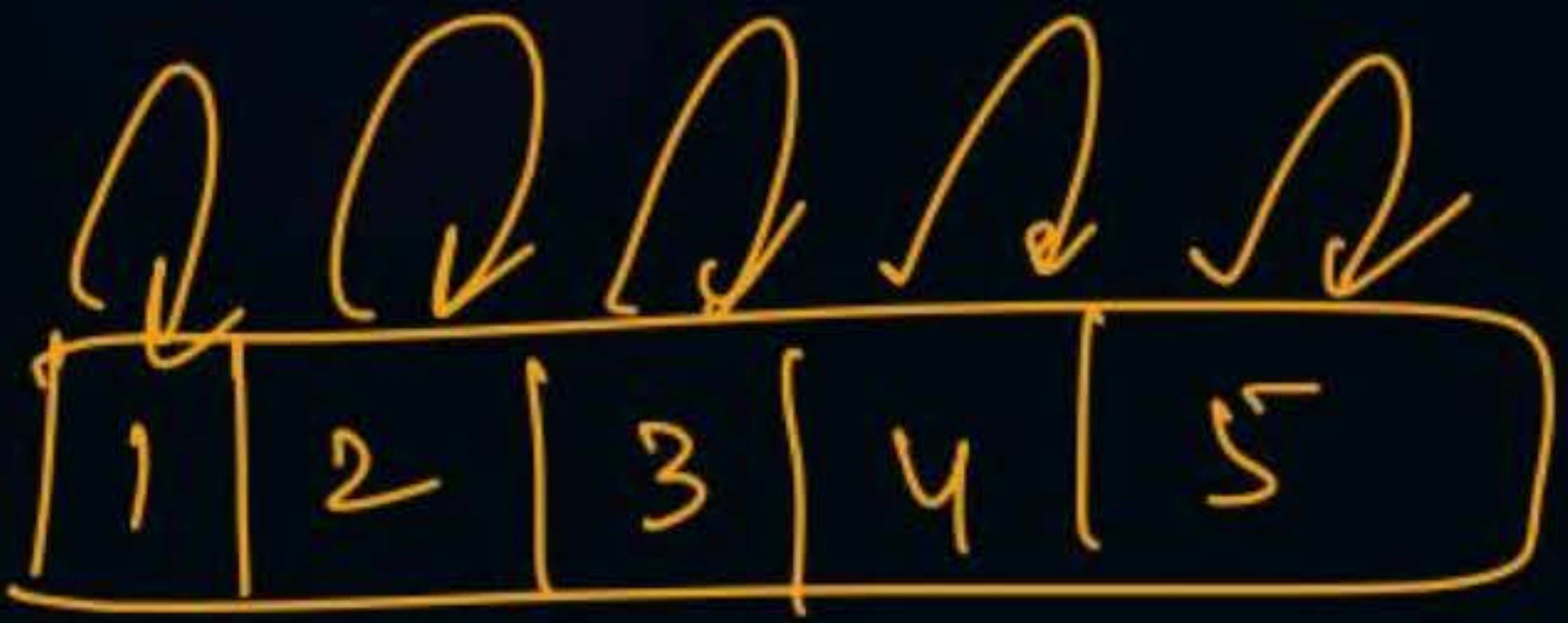
$$\left.\begin{array}{l} \left\{ \begin{array}{l} \dfrac{n(n-1)}{2} \longrightarrow \text{Comparison always} \\ + \\ (n-1) \text{ Swaps} \quad \text{always} \end{array} \right. \end{array}\right\}$$

$\Rightarrow \boxed{\Theta(n^2)}$ ✓

WC ✓

Selection Sort

$(n-1)$ Swaps



general: $\boxed{\text{not stable}}$

if (   )
{
}

Selection sort

1> NOT stable
2> inplace

[MCQ]

#Q19. Consider the following elements:

122	77	955	576	29	33	806	44	26	1020

What is the results after second pass of Radix sort?

4th pass

A 806, 1020, 122, 26, 29, 33, 44, 955, 576, 77

B 806, 29, 26, 122, 1020, 33, 44, 955, 77, 576

C 26, 29, 33, 44, 77, 122, 806, 576, 955, 1020

D 26, 29, 33, 44, 77, 122, 576, 806, 955, 1020

Ans: A

Soln:- 122 77 955 576 29 33 806 44 26 1020

max (digits) = 4

Sorted o/p : 4 passes

pass1: (units)

pass1 o/p: 1020 122 33 44 955 576 806 26 77 29

0	1	2	3	4	5	6	7	8	9
1020		122	33	44	955	576, 806, 26	77		29

pass 2 O/P = 806 1020 122 26 29 33 44 955 576 77

↳ Ans :-

0	1	2	3	4	5	6	7	8	9
806		1020, 122, 26, 29	33	44	955		576, 77		

I/P =

pass 3 O/P : [1020 26 29 33 44 77 122 576 806 955] 026

0	1	2	3	4	5	6	7	8	9
1020, 26, 29, 33, 44, 77	122				576			806	955

Pass 4 o/p:

26 29 33 44 77 122 576 806 955 1020

Sorted

0	1	2	3	4	5	6	7	8	9
26, 29, 33, 44, 77, 122, 576, 806, 955	1020								

[MCQ]

#Q20. Consider the following array

A	100	90	50	80	70	35	49	51

To sort this array using selection sort what is the result after 4th pass?

Ans: B

A 100, 90, 80, 70, 50, 35,49, 51

B 35,49, 50, 51, 70, 100, 90, 80

C 35, 50,49, 80, 70, 100, 90, 51

D 35,49, 50, 80, 70,100, 90, 50

Soln :-

i/p: 100 90 50 80 70 35 49 51

pass 1: 100 | 90 50 80 70 35 49 51

pass 2: 35 | 90 50 80 70 100 49 51 → o/p of pass 1

pass 3: 35 49 | 50 80 70 100 90 51 → pass 2 o/p:

pass 4: 35 49 50 | 80 70 100 90 51

34

pass 5: 35 49 50 51 | 70 100 90 80

↳ O/p of pass 4

pass 6: 35 49 50 51 70 | 100 90 80

pass 7: 35 49 50 51 70 80 | 90 100

↳ 35 49 50 51 70 80 90 | 100

automatically sorted.

n = 8 elems, 7 passes (n-1)

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THANK - YOU

Telegram Link for Aditya Jain sir:
https://t.me/AdityaSir_PW