GATE

CRASH COURSE

DS & AI

Database Management System

ER Model

&

Relational Model

(Part-3)

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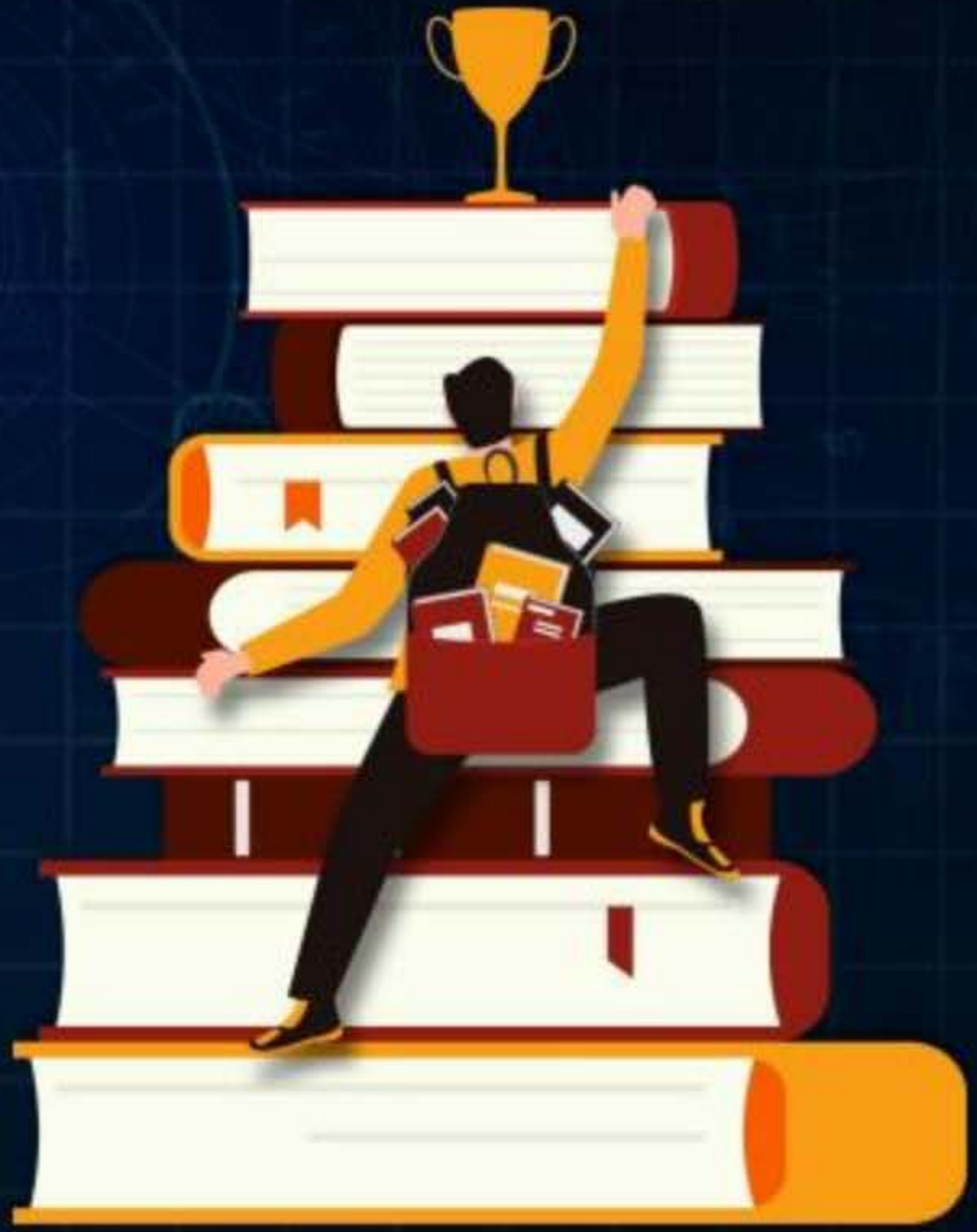
Lec No. 10

Topics *to be covered*

1. Functional dependency
2. Closure of an attribute set
3. Super key and candidate key
4. Relationship between two FD sets



Topic : Functional dependency (FD)

- Functional dependency defines the relationship between two sets of attributes in a relational table.

$R(A, B, C, D, E)$

let $X = \{A, B\}$

$Y = \{C, E\}$

if $X \longrightarrow Y$

i.e. if $\{A, B\} \longrightarrow \{C, E\}$.

then if we know values of A & B, then we can determine values of "C & E" if $X \rightarrow Y$ exists in rel^n R.

then whenever values w.r.t. attribute set 'X' are same in any pair of tuples, then values w.r.t. attribute set Y will also be equal in those tuple

* if two tuples $t_1, t_2 \in R$ s.t. $t_1.X = t_2.X$, but $t_1.Y \neq t_2.Y$, then we can conclude that "$X \rightarrow Y$" does not hold in rel^n R

#Q. From the following instance of a relation schema R (A,B,C), we can conclude that:

$A \rightarrow B$ may hold in relⁿ 'R' — can not conclude

A	B	C
1	1	1
1	1	0
2	3	2
2	3	2

"$B \rightarrow C$" does not hold in relⁿ R

Student

Sid	Sname
1	A
2	B
3	A
4	C

Student

Sid	Sname
1	A
2	B
3	C
4	D

(A) A functionally determines B, and B functionally determines C

(B) A functionally determines B, and B does not functionally determine C

(C) B does not functionally determine C

(D) A does not functionally determine B, and B does not functionally determine C

Topic : Types of Functional Dependency

A	B	C
1	2	1
1	3	3
1	4	5
1	4	7

$AB \rightarrow B$

① **Trivial FD** : if $X \supseteq Y$, then FD $X \rightarrow Y$ is called a trivial FD

eg: $AB \rightarrow B$

- Every trivial functional dep. will always hold in Relⁿ

{X & Y are non-Empty set}

② **Non-trivial FD** : if $X \rightarrow Y$ holds in relⁿ R, and $X \cap Y = \phi$, then $X \rightarrow Y$ is a non-trivial FD.

$AB \rightarrow CD$

③ **Semi-non-trivial FD** : if $X \rightarrow Y$ holds in relⁿ R and (i) $X \not\supseteq Y$ and (ii) $X \cap Y \neq \phi$, then $X \rightarrow Y$ is called a semi-non trivial FD.

$AB \rightarrow BC \equiv AB \rightarrow B \;\&\; AB \rightarrow C$

Q:- Let R(A, B, C) be a relation,
how many different non-trivial FDs can be defined w.r.t. attributes of relⁿ R.

$A \rightarrow B$
$A \rightarrow C$
$B \rightarrow A$
$B \rightarrow C$
$C \rightarrow A$
$C \rightarrow B$

$A \rightarrow BC$
$B \rightarrow AC$
$C \rightarrow AB$

$AB \rightarrow C$
$AC \rightarrow B$
$BC \rightarrow A$

Ans = 12

$X \longrightarrow Y$

$^3C_1 * (^2C_1 + ^2C_2) = 3*3 = 9$
$+$
$^3C_2 * (^1C_1) = 3*1 = 3$

eg: R(A, B, C, D, E)

$X \longrightarrow Y$

$|X| = 1$ $\quad ^5C_1 * (^4C_1 + ^4C_2 + ^4C_3 + ^4C_4)$
$+$
$|X| = 2$ $\quad ^5C_2 * (^3C_1 + ^3C_2 + ^3C_3)$
$+$
$|X| = 3$ $\quad ^5C_3 * (^2C_1 + ^2C_2)$
$+$
$|X| = 4$ $\quad ^5C_4 * (^1C_1)$

Topic : Properties of Functional Dependencies

① Reflexivity :- In FD $X \rightarrow Y$, if $X \supseteq Y$ then $X \rightarrow Y$ is reflexive FD, and Every reflexive FD always holds true in relⁿ.

② Augmentation: If $X \rightarrow Y$ exists in the relation then $XZ \rightarrow YZ$ will also exist in the relⁿ.

③ Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$ exist in the relⁿ then $X \rightarrow Z$ will also exist in Relⁿ

Armstrong's Axioms

Topic : Properties of Functional Dependencies

④ Decomposition :.
(Splitting Rule)

If $X \rightarrow YZ$ exists in the relation
then $X \rightarrow Y$ and $X \rightarrow Z$ will also hold in Relⁿ

but if $XY \rightarrow Z$ exists in Relⁿ
then ($X \rightarrow Z$ & $Y \rightarrow Z$) need not hold in relation

⑤ Composition :

if $X \rightarrow Y$ & $P \rightarrow Q$ exists in Relⁿ
then $XP \longrightarrow YQ$ will also exists in Relⁿ

⑥ Union :- if $X \rightarrow Y$ & $X \rightarrow Z$ exists in relⁿ
then $X \rightarrow YZ$ will also exist in the relⁿ

⑦ Pseudo transitivity :-

If $AB \rightarrow C$ & $BC \rightarrow D$ exists in the relⁿ

then $AB \rightarrow CD$ will also exist in Relⁿ

$(AB)^+ = \{A, B, C, D\}$

$AB \rightarrow \underbrace{AB}_{\text{trivial}}CD \Rightarrow \boxed{AB \rightarrow CD}$

Topic : Closure of an attribute set

Closure of an attribute set X (i.e., X^+) can be defined as set of all the attributes which can be functionally determined from attribute of set X.

#e.g. Consider the following FD set

$F = \{ AB \rightarrow CD, AF \rightarrow D, DE \rightarrow F, C \rightarrow G, F \rightarrow E, G \rightarrow A\}$

find the closure of following set of attributes.

(i) $\{C,F\}^+ = \{C, F, G, E, A, D\}$

(ii) $\{B,G\}^+ = \{B, G, A, C, D\}$

(iii) $\{A,F\}^+ = \{A, F, D, E\}$

(iv) $\{A,B\}^+ = \{A, B, C, D, G\}$

Key Concept :-

A set of attribute that can determine all the attributes of a relation is called a Key of that relⁿ.

* R(A, B, C, D, E)

$(ABCDE)^{+} = \{A, B, C, D, E\}$

$\therefore$ (ABCDE) is a Key of relⁿ R

Topic : Super key

Let R be the relational schema, and let X be some attribute set over relation R. If X^{+} determines all attributes of relation R, then X is called super key of relation R.

#e.g. Assume a relation R (A,B,C,D) that has the following functional dependencies:

A→B

B→C

C→D

Find all superkeys of relation R

$(A)^+ = \{A, B, C, D\}$ — All attributes

$(AB)^+ = \_, \_, \_$

$(AC)^+ = \_, \_$

$(AD)^+ = \_, \_$

$(ABC)^+ = \_, \_$

$(ABD)^+ = \_, \_$

$(ACD)^+ = \_, \_$

$(ABCD)^+ = \_, \_$

All of them are S.K

$(B)^+ = \{B, C, D\}$ — Not all attributes

$(BC)^+ = \{B, C, D\}$

$(BD)^+ = \{B, C, D\}$

$(BCD)^+ = \{B, C, D\}$

$(C)^+ = \{C, D\}$

$(CD)^+ = \{C, D\}$

$(D)^+ = \{D\}$

∴ None of them are S.K

Topic : Candidate key (Minimal Super key)

No attribute can be removed without destroying its property of being a key

- Let R be the relational schema, and let X be the super key of relation R.
  If no proper subset of X is a super key the X is minimal super key i.e., X is Candidate key

$R(A,B,C,D,E)$

Proper Subsets of $\{A,B,C\}$

$(ABC)^+ = \{\underbrace{A,B,C,D,E}_{\text{All attributes}}\}$

$\therefore (ABC)$ is S.K

if, ϕ^+ = Not all attribute

$\{A\}^+ = \text{—}, \text{———}$

$\{B\}^+ = \text{—}, \text{———}$

$\{C\}^+ = \text{—}, \text{———}$

$\{A,B\}^+ = \text{—}, \text{———}$

$\{A,C\}^+ = \text{—}, \text{———}$

$\{B,C\}^+ = \text{—}, \text{———}$

$\therefore$ None of them are S.K

$\Downarrow$

$\therefore (ABC)$ is Minimal S.K

i.e., (ABC) is C.K

1. A relation may have more than one C.K.
2. Attributes that belongs to any of the C.K. are called Prime attributes, and attributes that does not belong to any of the C.K. are called non-prime attributes
3. A Super key with a single attribute is always a C.K.
4. Every C.K is a S.K, but Every S.K. need not be a C.K.
5. A C.K which is formed of a single attribute is called a simple C.K, if C.K. is formed by Combining two or more attributes, then it is called Composite C.K.

#Q. Assume a relation R (A, B, C, D, E) that has the following functional dependencies:

AB→C,

B→E,

C→D

Find the Candidate key of R.

A & B are not present in R.H.S. of any FD,

∴ A & B are essential attributes

Every essential attribute must be present in Every Key.

(ABD) is C.K

$$(AB)^+ = \{\underbrace{A, B, C, E, D}_{\text{all attributes}}\}$$

∴ (AB) is S.K

Proper subsets of {A,B}

$\{A\}^+ = \{A\}$ Not a S.K

$\{B\}^+ = \{B, E\}$ Not a S.K

No proper subset of AB is a S.K

Hence (AB) is C.K

C.K = (AB)

∴ Prime attributes = {A,B}

None of the P.A is Present in R.H.S of any FD

∴ Relation will have Only one C.K i.e. (AB)

Hence {C,D,E} are non-prime attributes

Note:

Let R be the relational schema and a non-trivial FD $X \to Y$ exists in reln R such that, "Y" is a prime attribute, then relation R will have more than one C.K.

#Q. Assume a relation R (A, B, C, D) that has the following functional dependencies:

AB→CD, $\equiv$ AB → C, AB → D

D→A

Find all the Candidate keys of R.

$(AB)^+ = \{A, B, C, D\}$

$\therefore$ (AB) is a S.K

$(A)^+ = \{A\}$

$(B)^+ = \{B\}$

No proper subset of (AB) is a S.K

$\therefore$ (AB) is a C.K

AB → D

$\therefore$ we can replace "D" by "AB"

$\boxed{AB}$ is a C.K

Prime attributes = {A, B}

in, D → A RHS is a Prime attribute

$\therefore$ we may replace "RHS" by "LHS" to form a new Superkey

Replace A by D

DB is a S.K

$(D)^+ = \{D, A\}$

$(B)^+ = \{B\}$

No proper subset of (DB) is a key

$\therefore$ $\boxed{DB}$ is also a C.K

Prime Attribute = {A, B, D}

Ans:-

Two Candidate Keys for Relⁿ (AB) & (DB)

Prime Attributes = {A, B, D}, Non-prime Attribute = {C}

Topic : Membership test

- Membership test is used to check whether a given FD is a member of given FD set or not.

- To check whether $X \rightarrow Y$ is a member of FD set F or not (i.e., $F \vDash X \rightarrow Y$ or not)

  We first obtain X^+ (closure of X) w.r.t. FD set F.

  If $Y \in X^+$, then $X \rightarrow Y$ is a member of FD set F
  otherwise not a member of FD set F

#Q. Let FD set $F = \{AB \to C, BC \to D\}$

Check whether $AB \to D$ is a member of F or not?

#Q. Let FD set $F = \{ AB \to C, C \to A \}$

Check whether $C \to B$ is a member of F or not?

Topic : Relationship between two FD sets

- Let F and G are any two FD sets.

- If all the FDs of FD set F are member of FD set G, then $F \subseteq G$

- If all the FDs of FD set G are member of FD set F, then $G \subseteq F$

- If both $F \subseteq G$ and $G \subseteq F$ are true, then $F = G$

#Q. Consider the following FD sets

$F1 = \{A \to B, B \to C, AB \to D\}$ and $F2 = \{A \to B, B \to C, A \to C, A \to D\}$

Find the relationship between FD sets F1 and F2

#Q. Consider the following FD sets

$F1 = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$ and $F2 = \{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$

Find the relationship between FD sets F1 and F2

Thank

THANK

Keep Hustling!