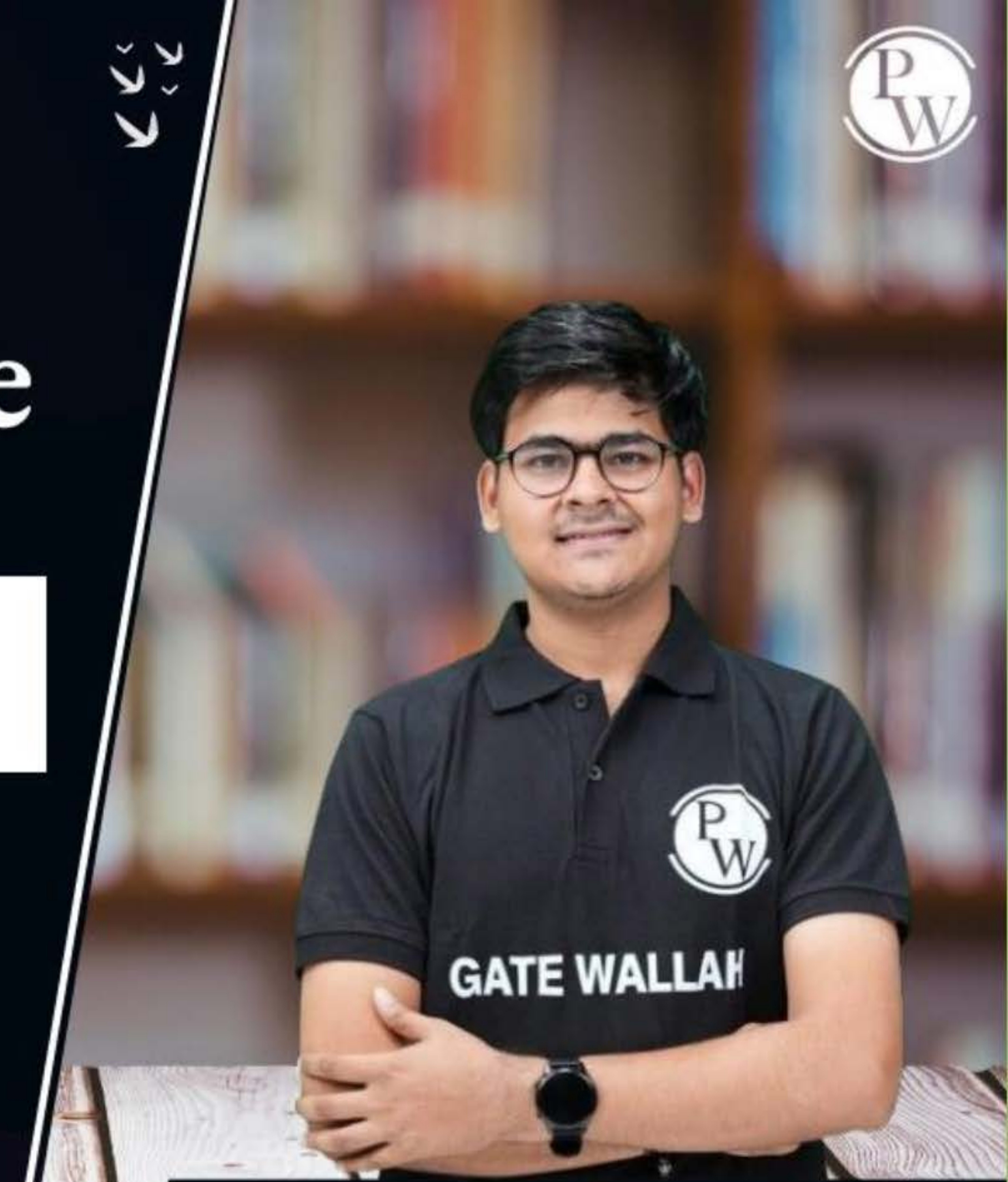
Data Science & Artificial Intelligence

Algorithms

Test Series 1500+



Lecture - 09

By- Aditya sir

Recap of Previous Lecture

Topic Questions $\longrightarrow$ Misc

Topic

Topics to be Covered

Topic

Topic

Questions

- Sorting
- Time Complexity
- Graphs
- Divide & Conquer
- Misc

Topic : Test Series 1500+

#Q. What is the worst case time complexity of Quick sort for n elements when the median is selected as the pivot:

A $\theta(n)$

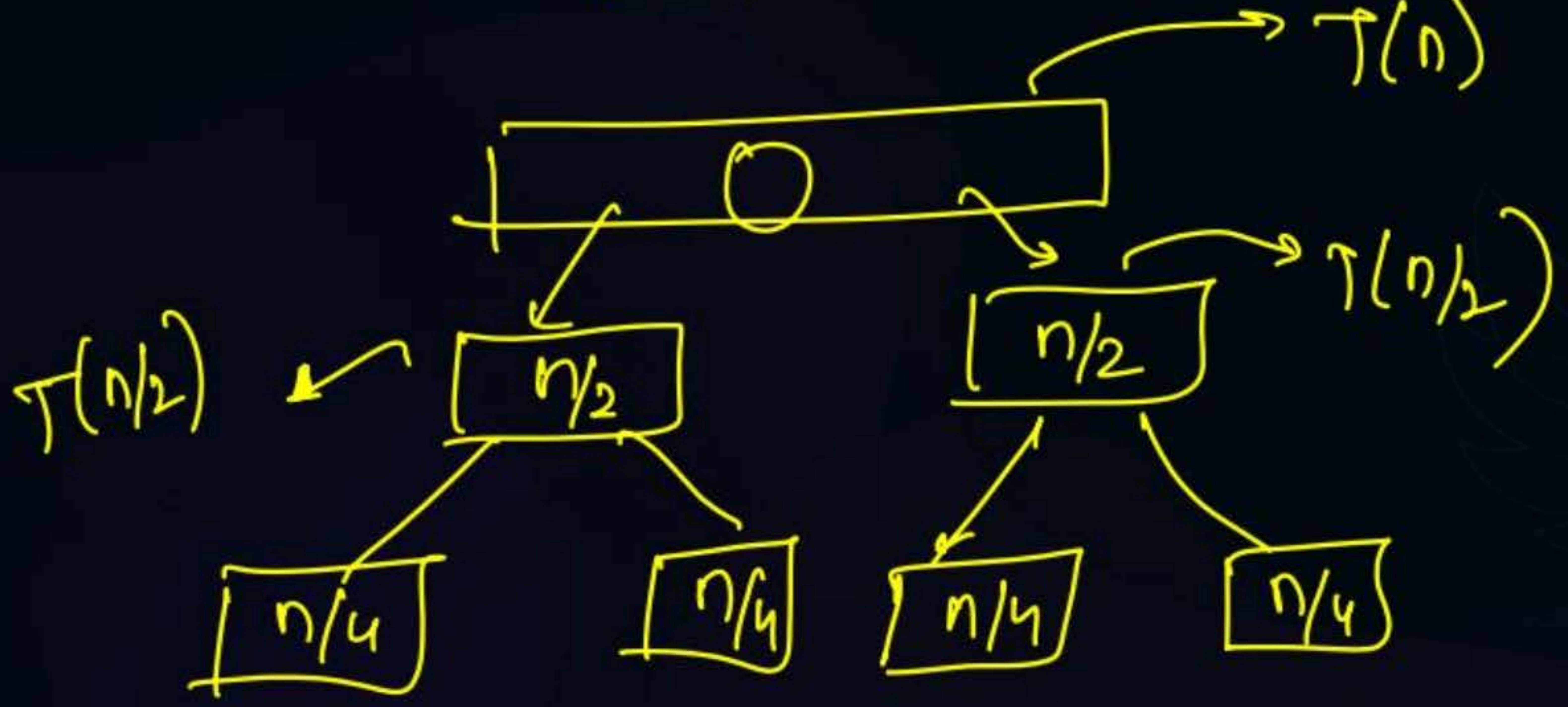
B $\theta(n \log n)$

C $\theta(n^2)$

D $\theta(n^2 \log n)$

Soln: For standard/default variant of Quick Sort.

Best Case : $\Omega(n \log n)$



$$T(n) = T(n/2) + T(n/2) + \underline{O(n)}$$

partition algo

B.C : $T(n) = 2T(n/2) + O(n)$

$$\boxed{T(n) = 2T(n/2) + n}$$

Back - substitution:

what is the value of the recurrence ?

$$T(n) = 2T(n/2) + n \quad \text{—— (1)}$$

$$T(n/2) = 2T(n/2^2) + \frac{n}{2}$$

$$T(n) = 2\left(2T(n/2^2) + n/2\right) + n$$

$$= 2^2 T(n/2^2) + n + n$$

$$T(n) = 2^2 T(n/2^2) + 2n \quad \text{——— } 2$$

$$T(n/2^2) = 2T(n/2^3) + n/2^2$$

$$T(n) = 2^2\left[2T(n/2^3) + n/2^2\right] + 2n$$

$$= 2^3 T(n/2^3) + n + 2n$$

$$T(n) = 2^3 T(n/2^3) + 3n \quad \text{——— } 3$$

⋮

Generel term

$$\boxed{T(n) = 2^K T(n/2^K) + K*n}$$

Base Condition : $n/2^K = 1 \Rightarrow 2^K = n$

$2^K = n$

$$\boxed{K = \log_2 n}$$

$T(n) = n\,T(1) + n*\log_2 n$

$$T(1) = c$$

$$\boxed{T(n) = n * c + n * \log_2 n}$$ → value of Recurrence.

For TC: $O(n \log_2 n)$

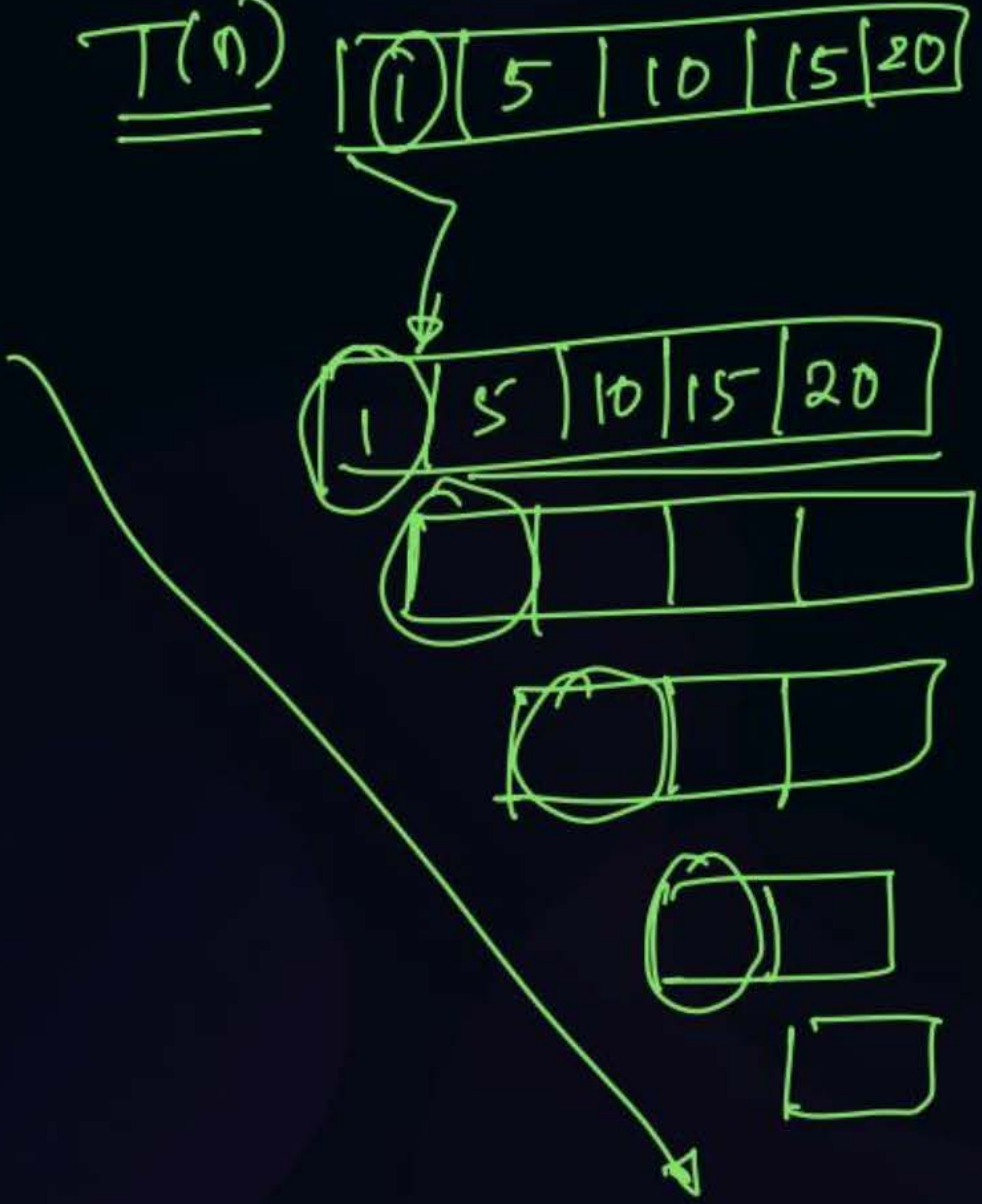
Worst Case TC of Quick sort :

i/p

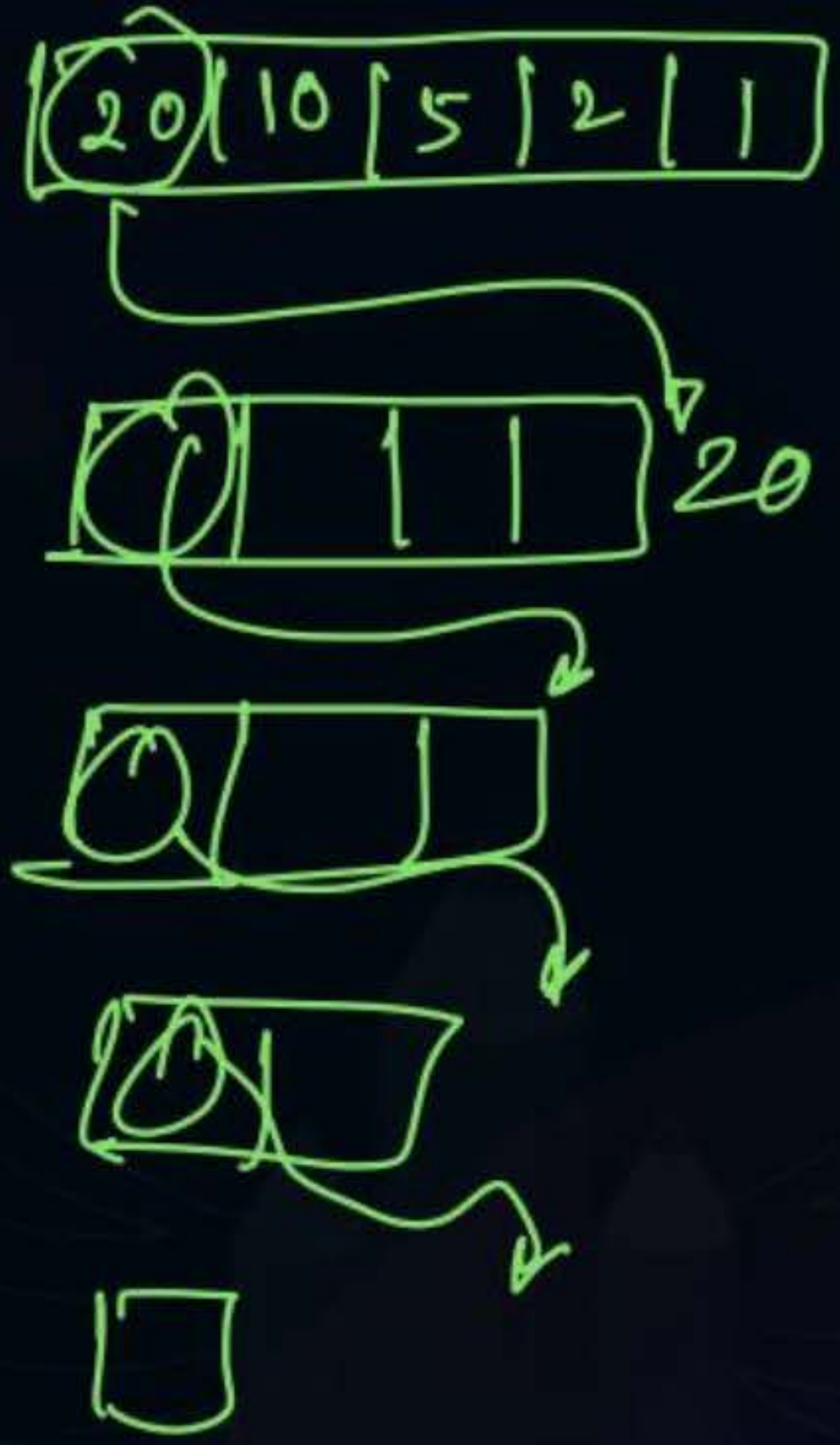
↳ sorted : ascending or descending order.

(1st elem as pivot)

$\underline{\underline{T(n)}}$



OR



$$T(n) = T(n-1) + n$$ partition

Solve using Back substitution and give the value of Recurrence ?

$$T(n) = T(n-1) + n \quad \text{--- } ①$$

$$T(n-1) = [T(n-2) + (n-1)]$$

$$T(n) = T(n-2) + (n-1) + n$$

$$T(n) = T(n-2) + (n + n-1) \quad \text{--- } ②$$

$$T(n-2) = T(n-3) + (n-2)$$

$$T(n) = T(n-3) + [n + (n-1) + (n-2)] \quad \text{--- } ③$$

$$\vdots$$

General term

$$T(n) = T(n-k) + [n + (n-1) \ldots (n-(k-1))]$$

For Base Condition

$$n-k = 1$$

$$\boxed{k = n-1}$$

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$$T(n) = T(n-K) + [n + (n-1) \ldots n-(n-1-1)]$$

$$= T(n-K) + [n + (n-1) \ldots \cancel{n} - \cancel{n} + 2]$$

$$= T(n-K) + [n + (n-1) \ldots + 2]$$

$$c + \frac{n(n-1)}{2}$$

$$= T(1) + \sum_{i=2}^{n} i$$

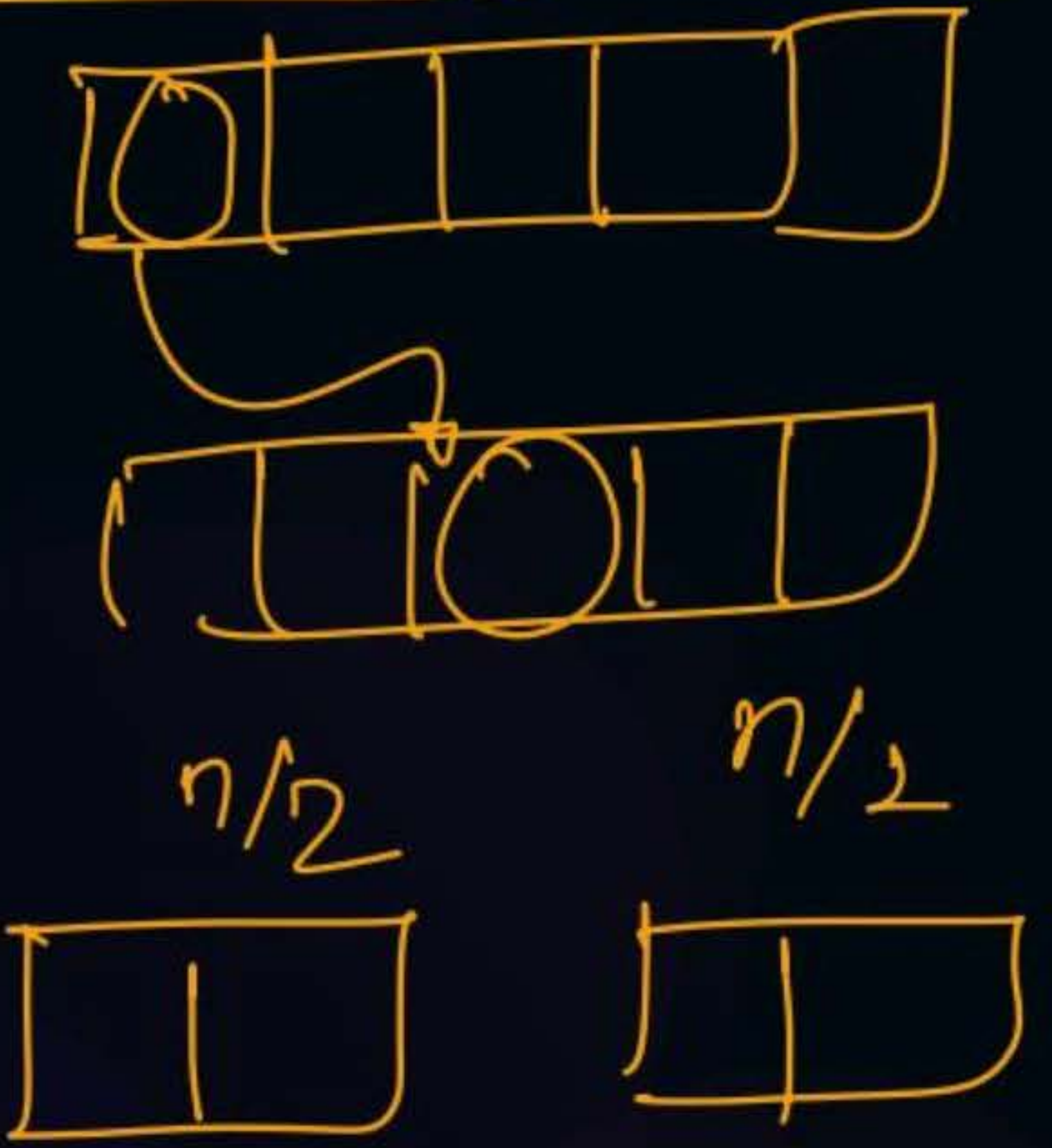
$$= C + \frac{n(n+1)}{2} - 1 = \boxed{\frac{n^2+n-2}{2} + C}$$

Value of Recurrence

$$TC = \underline{O(n^2)}$$

median as Pivot:

Before partition



Best Case of Quick Sort

★ Master's method:

$$\boxed{T(n) = a * T(n/b) + F(n)}$$

where $a \geq 1$

$b > 1$

$F(n) =$ +ve function

Case 1 :- If $F(n) = O\left(n^{\log_b a - \varepsilon}\right)$ for some $\varepsilon > 0$

then $\boxed{T(n) = \Theta\left(n^{\log_b a}\right)}$

Case 2 :- If $F(n) = \Theta\left(n^{\log_b a} * (\log n)^k\right)$, for some k

a) $k \geq 0$, $T(n) = \Theta\left(n^{\log_b a} * (\log n)^{k+1}\right)$

b) $k = -1$, $T(n) = \Theta\left(n^{\log_b a} * \log(\log n)\right)$

Case 3 :- If $f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right)$, for some $\varepsilon > 0$

and $a * f(n/b) \le \delta * f(n)$, some $\delta < 1$

then $\boxed{T(n) = \Theta(f(n))}$

(Q) Solve this using Master's method:

$$T(n) = 16T(n/4) + n^{2.5}$$

$n^{(a+b)} = n^a * n^b$

A) $\Theta(n^2)$

B) $\Theta(n^2\sqrt{n})$

C) $\Theta(\sqrt{n})$

D) $\Theta(n^2 \log n)$

$$n^{2.5} = n^{(2+0.5)}$$
$$= n^2 * n^{0.5}$$
$$n^{2.5} = n^2 * \sqrt{n}$$

Ans: B

Soln:-

$$T(n) = 16T(n/4) + n^{2.5}$$

$a = 16$

$b = 4$

$f(n) = n^{2.5}$

} valid ✓

$\log_b a = \log_4 16$

$= \boxed{2}$

<u>Case 1:</u> Is $F(n) = O\left(n^{\log_b a - \varepsilon}\right)$, for some $\varepsilon > 0$?

Is $n^{2.5} = O\left(n^{2-\varepsilon}\right)$, for some $\varepsilon > 0$?

$\hookrightarrow$ Fails

Case 2 :- Is $f(n) = \Theta\left(n^{\log_b a} * (\log n)^k\right)$, some k

$$n^{2.5} = \Theta\left(n^2 * (\log n)^k\right), \text{ for some } k$$

a) $k \geq 0$? ✗

b) $\underline{k = -1}$ ✗

$$n^{2.5} \qquad n^2 (\log n)^k$$

$$\cancel{n^2}\, n^{0.5} \qquad \cancel{n^2} (\log n)^k$$

$$\underbrace{n^{0.5} = (\log n)^k}_{} \;?\; ✗$$

Case 3 :- If $f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right)$, for some $\varepsilon > 0$

$$n^{2.5} = \Omega\left(n^{2+\varepsilon}\right), \text{ for } \varepsilon > 0 \text{ ?}$$

$$\varepsilon = 0.1,\ 0.001$$

True

and $\quad a * f(n/b) \le \delta * f(n)$

$$16 * \left(\frac{n}{4}\right)^{2.5} \le \delta * n^{2.5}$$

$$16 \times \frac{\cancel{n^{2.5}}}{4^{2.5}} \le \delta \times \cancel{n^{2.5}}$$

$$\frac{\cancel{16}}{\cancel{4^2} \times \sqrt{4}} \le \delta$$

$$\frac{1}{\sqrt{4}} \le \delta \qquad \text{True}$$

$$\frac{1}{2} \le \delta \quad , \text{ for some } \delta < 1$$

$$\frac{1}{2} \longleftrightarrow 1$$

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Here $T(n) = \theta(f(n))$

$$T(n) = \theta(n^{2.5})$$

(Q) $T(n) = T(\sqrt{n}) + 10$

Solve using master's method.

Soln : $T(n) = T(\sqrt{n}) + 10$ (using Back substitution)

$$T(n) = T(n^{1/2}) + 10 \quad \text{--- (1)}$$

$$T(n^{1/2}) = T(n^{1/2^2}) + 10$$

$$T(n) = T(n^{1/2^2}) + (10 + 10) \quad \text{--- (2)}$$

$$T(n^{1/2^2}) = T(n^{1/2^3}) + 10$$

$$T(n) = T\left(n^{1/2^3}\right) + (10+10+10)$$

$$T(n) = T\left(n^{1/2^3}\right) + 3 * 10 \longleftarrow 3$$

$$\vdots$$

General Term

$$\boxed{T(n) = T\left(n^{1/2^k}\right) + k * 10}$$

For Base Condition,

$$n^{1/2^k} = 2$$

Taking $\log_2(\,)$ both sides

$$\log_2\left(n^{1/2^k}\right) = \log_2 2$$

$$\frac{1}{2^k} * \log_2 n = 1$$

$$2^k = \log_2 n$$

$$\boxed{K = \log_2 \log_2 n}$$

$$T(n) = T\left(n^{1/2^K}\right) + K * 10$$

$$= T(2) + \log_2 \log_2 n * 10$$

$$\boxed{T(n) = c + 10 * \log_2 \log_2 n} \Rightarrow TC : \Theta(\log \log n)$$

(2) using Master's method

$$T(n) = T(\sqrt{n}) + 10 \longleftrightarrow$$ not a valid form of masters mtd.

Using change of variable mtd

Let $n = 2^k$

$n^{1/2} = 2^{k/2}$

$$T(n) = T(n^{1/2}) + 10$$

$$T(2^k) = T(2^{k/2}) + 10 \quad \text{——— } 1$$

Let $T(2^k) \longrightarrow P(k)$

then $T(2^{k/2}) \longrightarrow P(k/2)$

eqn (1) becomes.

$$T(2^K) = T(2^{K/2}) + 10$$

$$\boxed{P(K) = P(K/2) + 10} \text{ —— (2)}$$

$$\left.\begin{array}{l} a=1 \\ b=2 \\ f(n)=10 \end{array}\right\} \checkmark$$

$$\log_b a = \log_2 1 = \underline{\underline{0}}$$

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Case 1 : $10 = O(k^{0-\varepsilon})$, some $\varepsilon > 0$?

$10 = O(k^{-\varepsilon})$, some $\varepsilon > 0$

decr < Const

decr

Fails

Case 2 : Is $10 = \Theta(k^0 * (\log k)^{k'})$, some k'

a) $k' \geq 0$? for $k' = 0$ $10 = \Theta(1)$ ✓

True.

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Here $P(K) = \Theta(K^0 * (\log K)^{K'+1})$

$$P(K) = \Theta((\log K)^1)$$

$$\Theta(\log K)$$

$2^K = n$

$K = \log_2 n$

$$T(n) = \Theta(\log(\log n))$$

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[MCQ]

#Q16. The Flyod-Warshall algorithm for all pairs shortest paths computation is based on

A Greedy method X

B Divide and Conquer X

C Dynamic Programming ✓

D Heap algorithm X

Floyd - warshall APSP

→ $\boxed{DP}$

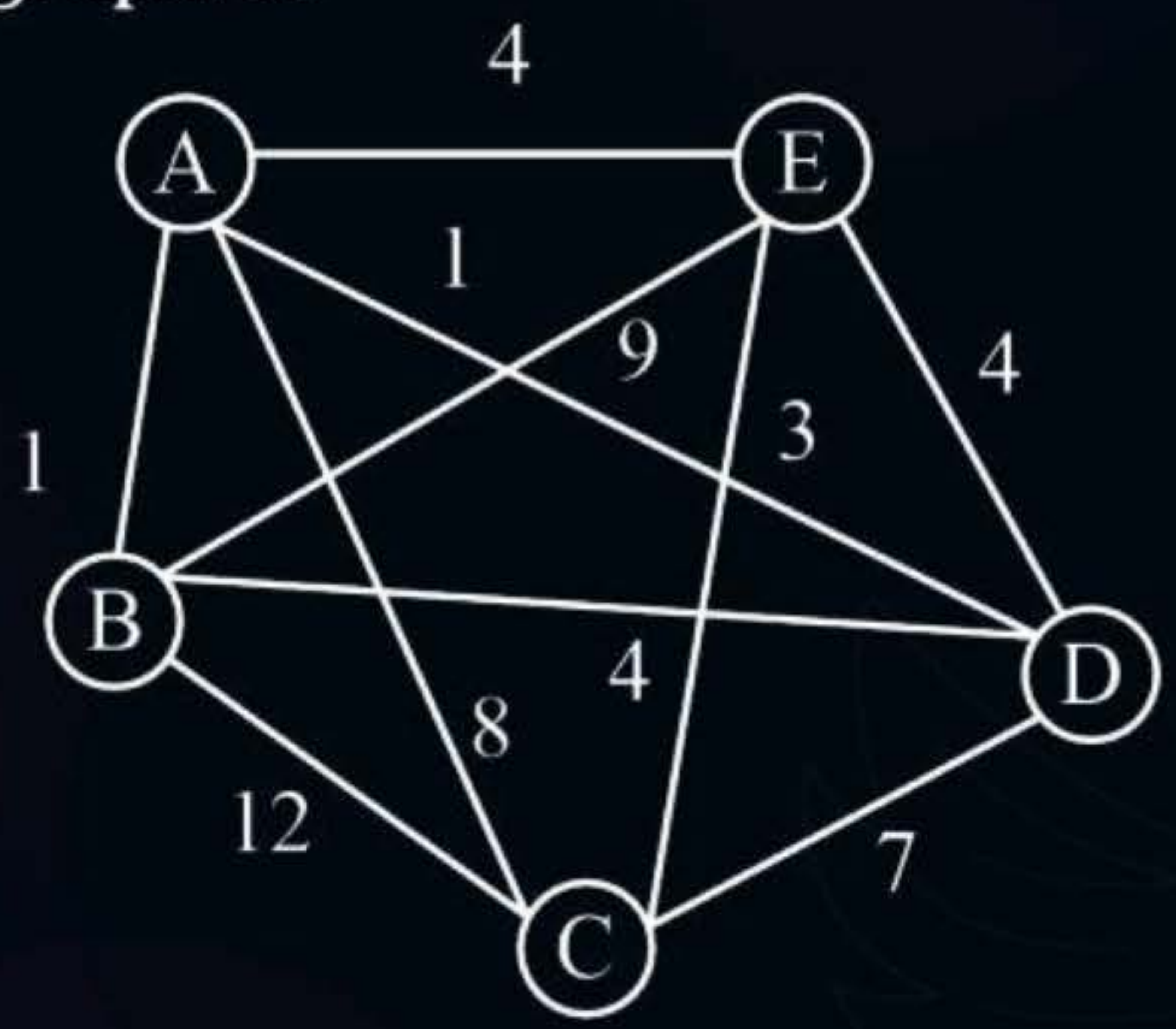
→ Optimization Problem

n - vertices

Time Complexity : $O(n^3)$ ✓

Space Complexity: $O(n^2)$ ✓

#Q. Consider the following graph G:

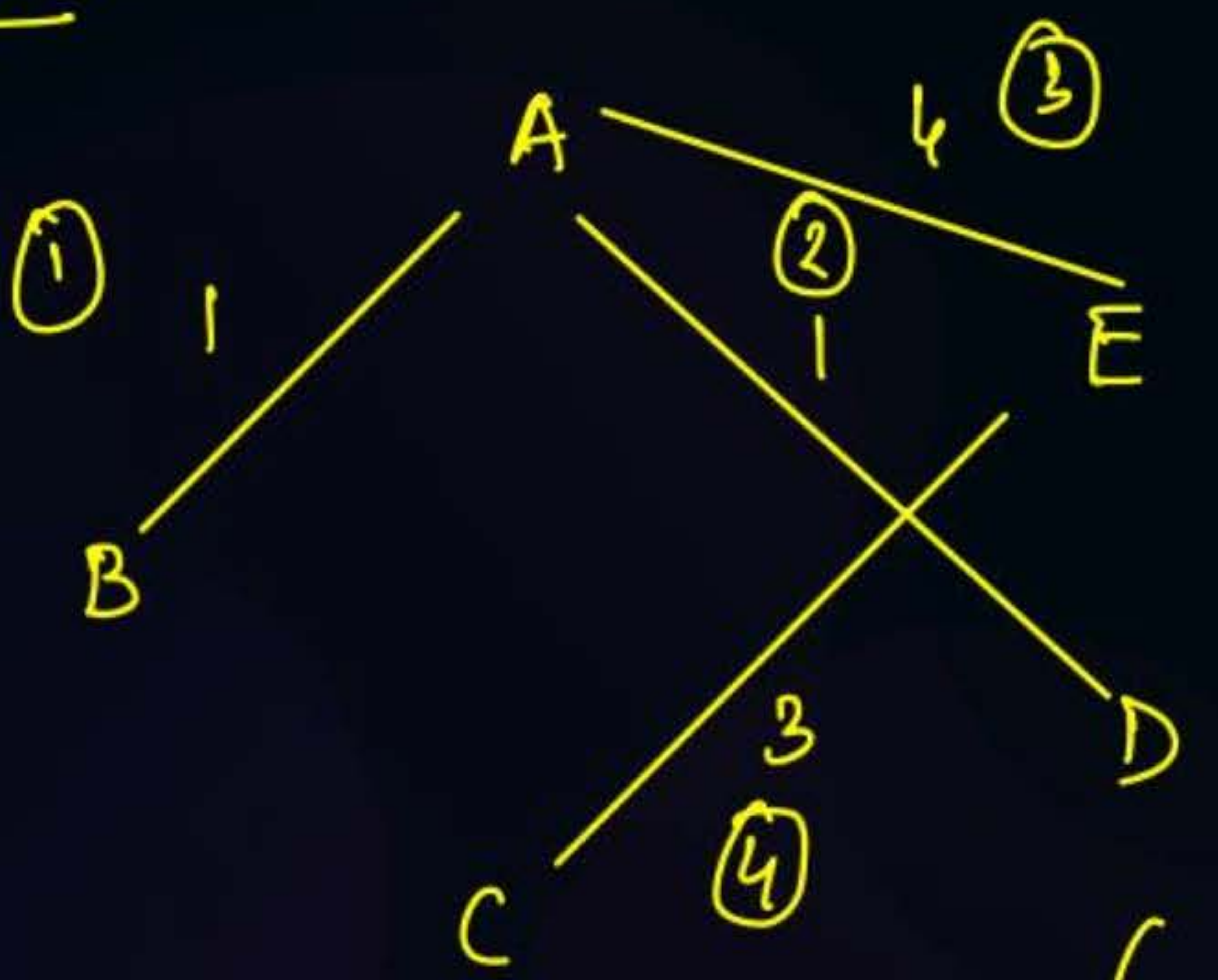


What is the minimum possible weight of a spanning tree such that vertex A is a leaf node ?

Soln:- If standard MCST Question.

(vertex A need not be leaf)

i) **Prims**



$n = 5$

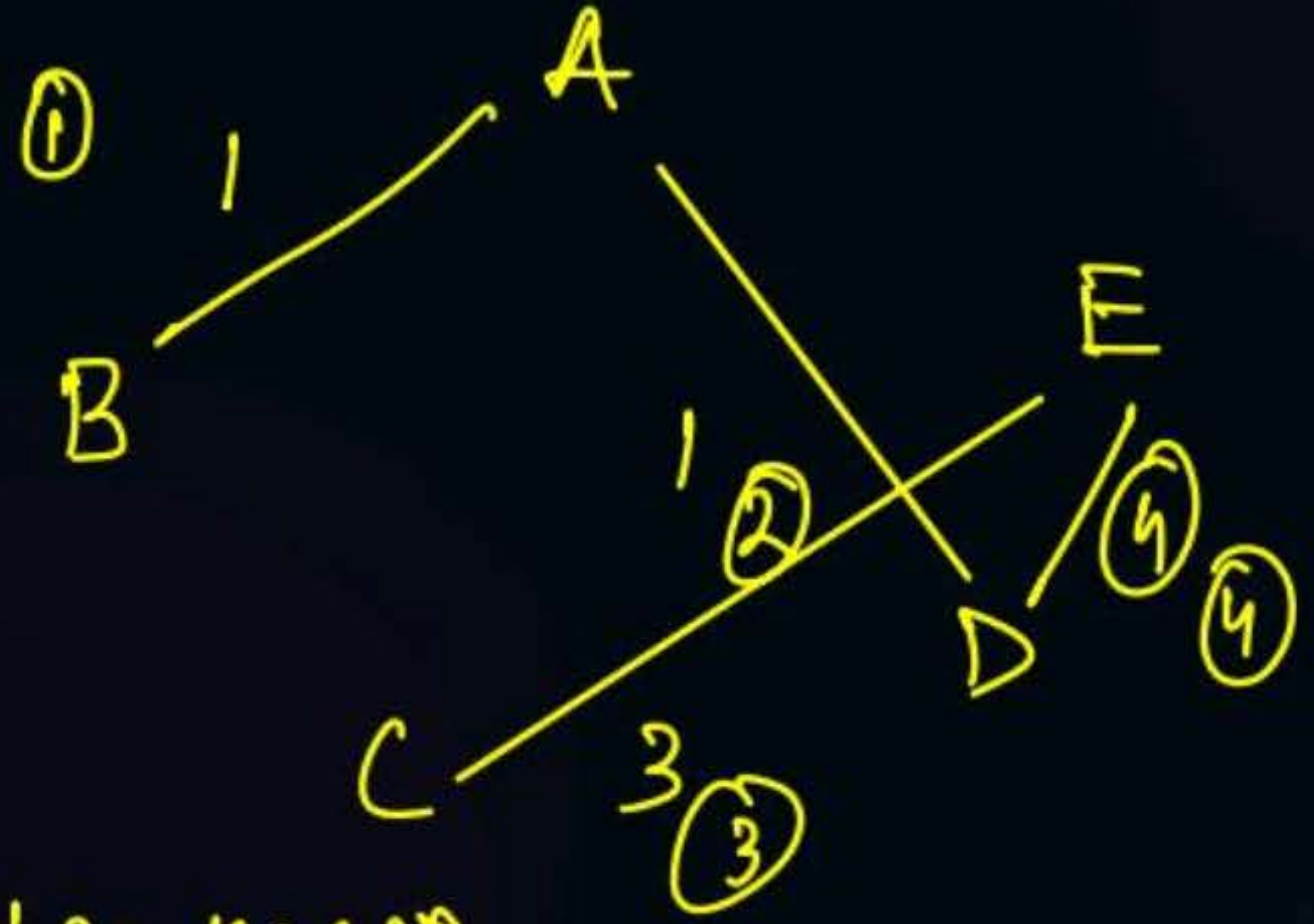
edges in MCST $= n-1$

$= 5-1 = \boxed{4}$ ✓

Cost of MCST $= 1+1+4+3$

$= 7+2 = 9$

(2) Kruskal's



Cost of MCST

= 1+1+4+3

= 9

(3) Bonus : Dijkstra MCST

↳ H.W

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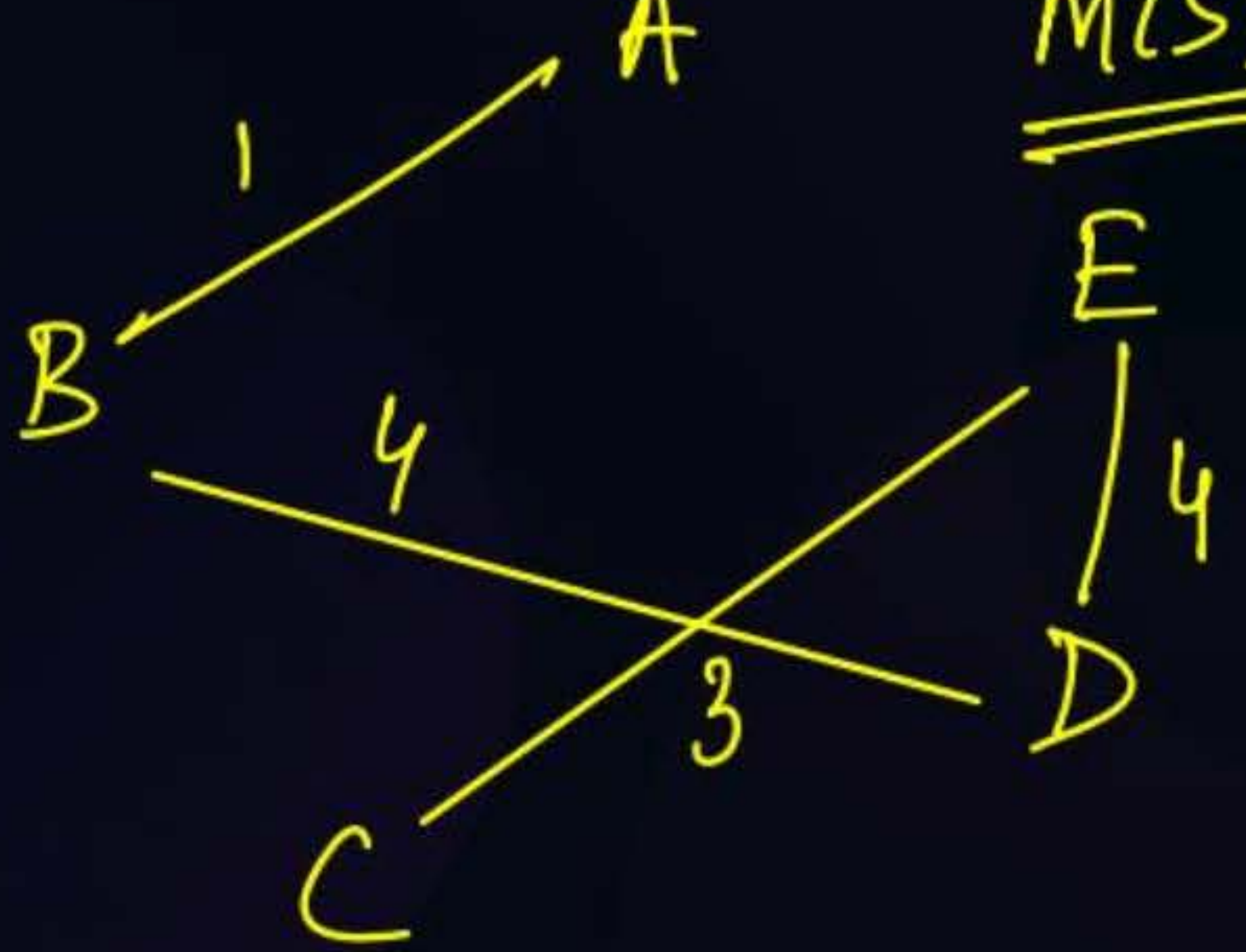
Soln for this questn :-

leaf node $\longrightarrow$ [only 1 edge should be connected]

MCST



Min Cost of such an MCST where A is leaf

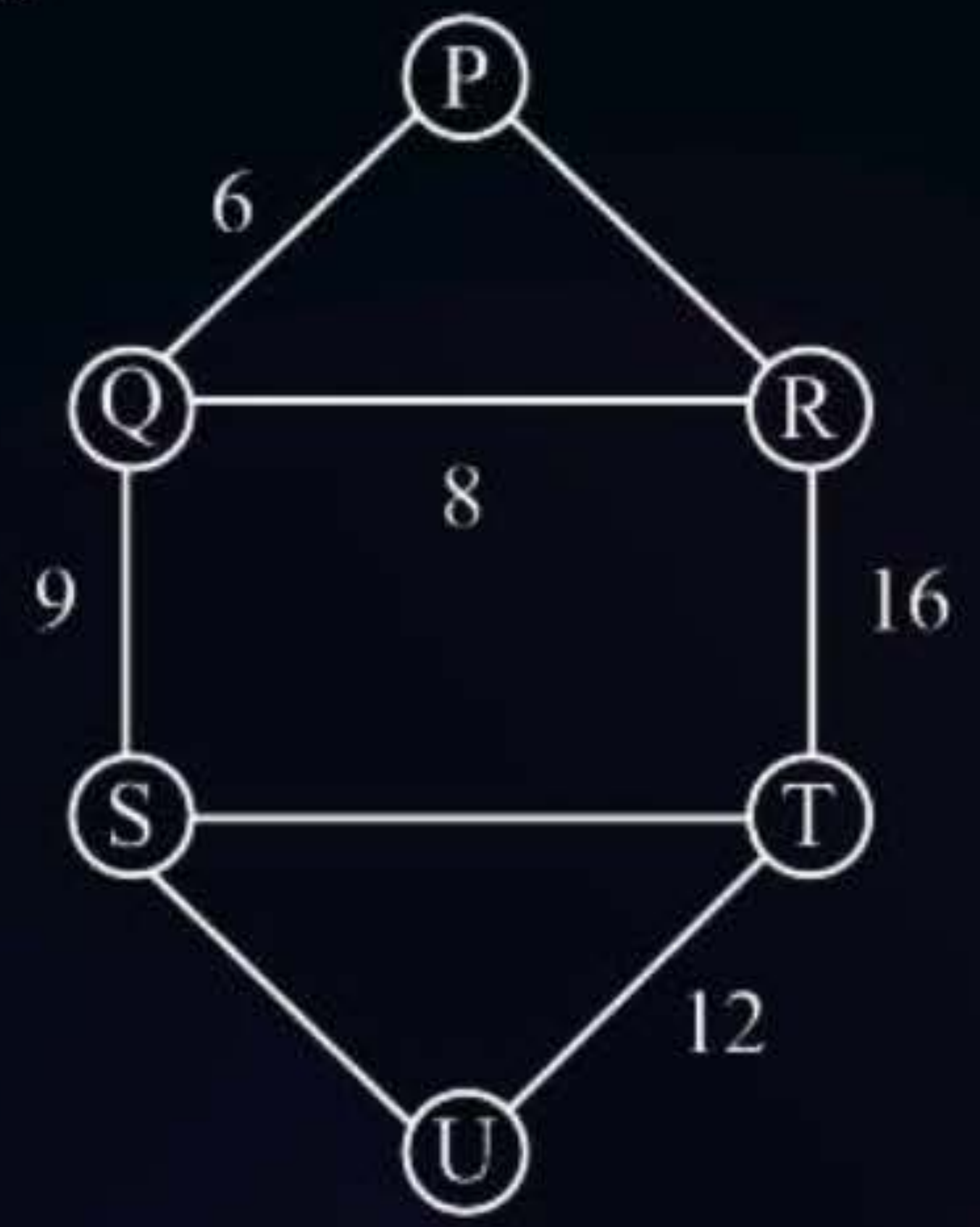
$= 1 + 4 + 4 + 3$

$= \boxed{12}$ ✓

Topic : Test Series 1500+

#Q. Consider the following graph:

G:



Part A) when duplicate edge cuts are allowed

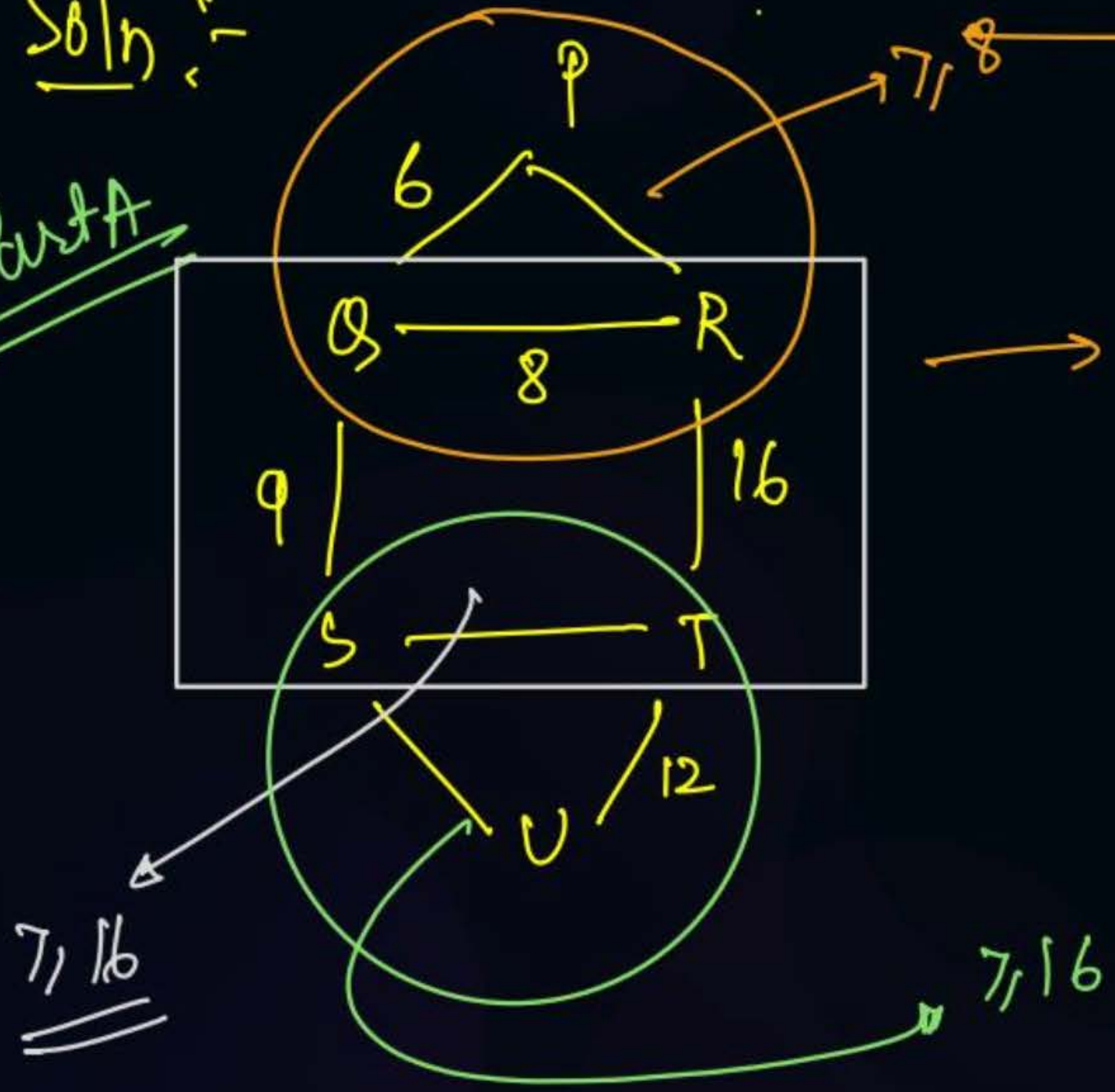
Part B) when duplicate " " not allowed.

MCST marked with edge weight of 51.

What is the ~~sum of~~ minimum sum of weight of all edges of graph G:

Soln :-

Part A



Given MST

$PR \longrightarrow 7 \,//\, 8 \longrightarrow 8$

$ST \longrightarrow 7 \,//\, 16 \longrightarrow 16$

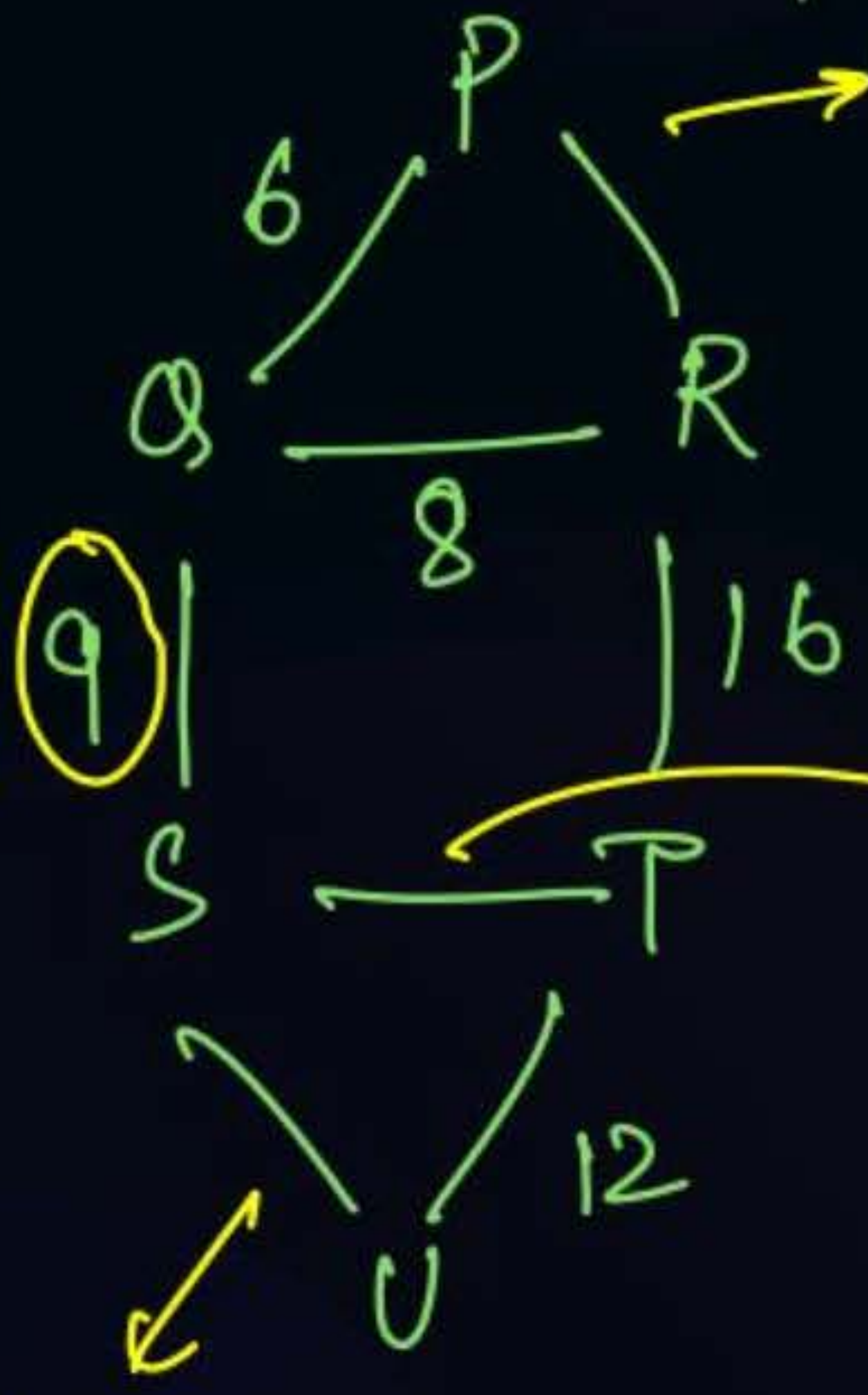
$SV \longrightarrow 7 \,/\, 16 \longrightarrow 16$

$51 + 8 + 16 + 16$

$= 51 + 40$

$= \boxed{91}$

Case 2:



$> 8 \rightarrow 9, 10, 11 \ldots$

$\times \downarrow \boxed{10}$

$> 16 \rightarrow 17, 18, 19$

$\downarrow$ ✓ $\boxed{17}$

> 16

$> 16 \rightarrow 17, 18, 19 \ldots$

$\times \downarrow$ ✓ $\boxed{18}$

$(51 + 10 + 17 + 18)$

$51 + 45$

$\boxed{96}$ ✓

2 mins Summary

Topic

Topic

Topic

New variety Questions

Misc

Topic

THANK - YOU

Telegram Link for Aditya Jain sir:
https://t.me/AdityaSir_PW