GATE

CRASH COURSE

ALL BRANCHES

Engineering Mathematics

Probability and Statistics (Part 01)
(Lec 13)

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Topics *to be covered*

PROB & STATS

(Perm & Comb)

Counting Principle

① Addition Principle → Either or

② Multiplication Principle → AND

eg In a Restaurant there are 8 veg dishes & 5 Non Veg dishes then in how many ways you can order a dish ?

$$\text{Req ways} = \begin{cases} 8+5 = 13 \\ \cancel{8\times 5 = 40} \end{cases}$$

eg If there are 15 NITS & 20 IIITS in INDIA & you are selected in JEE then in how many ways student can choose a college ?

Sol:

$$\text{Req ways} = 15 + 20 = 35 \text{ ways} = {}^{35}C_1 \text{ ways}$$

Q: If there are 20 IITS, each having 7 Branches then in how many ways Topper can take admission?

$$\text{Req ways} = \underset{\text{Job 1}}{20} \times \underset{\text{Job 2}}{7} = 140 \text{ ways.}$$

Q: there are 8 Boys & 5 G in a class then in how many ways teacher can select

(1) either a Boy or a Girl = ? $= 8 + 5 = 13$

(2) A Boy & A Girl = ? $= 8 \times 5 = 40$

Q: How many 3 letter words can be formed (with/w/o meaning) using Eng. alphabets.

26

① if Repetition is Not allowed = ? = $\underline{26} \times \underline{25} \times \underline{24}$

② if " is allowed = ? = $\underline{26} \times \underline{26} \times \underline{26}$

Q How many 4 digit Nos can be formed using 1, 3, 5, 7, 9

① RNA = ? = $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2}$

② RA = ?

$= \underline{5} \times \underline{5} \times \underline{5} \times \underline{5}$

eg: How many 4 digit Nos can be formed with distinct digits?

(0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

Req four digit Nos (RNA) $= \underline{9 \text{ ways}} \times \underline{9 \text{ ways}} \times \underline{8 \text{ ways}} \times \underline{7 \text{ ways}}$

(ii) if RA then Total 4 digit Nos = ? $= \underline{9} \times \underline{10} \times \underline{10} \times \underline{10}$

In a test there are eight questions, in which four have three possible answers, three has two possible answers each and one question has five possible answers. The total number of possible answers will be?

(a) 2880 (b) 78 (c) 94 (d) 3240

Total Possible Ans =

(you have to ans all Questions)

$$\underset{Q_1}{\underline{3}} \times \underset{Q_2}{\underline{3}} \times \underset{Q_3}{\underline{3}} \times \underset{Q_4}{\underline{3}} \times \underset{Q_5}{\underline{2}} \times \underset{Q_6}{\underline{2}} \times \underset{Q_7}{\underline{2}} \times \underset{Q_8}{\underline{5}}$$

$$= 3^4 \times 2^3 \times 5$$

The number of five digits odd numbers greater than 40000 that can be made using the digits 0, 1, 2, 4, 5, 7 if digits can be repeated in the same number, is

(a) $3^2 \times 6^3$ (b) $2^3 \times 3^6$ (c) $2^3 + 3^6$ (d) $3^2 + 6^3$

Req 5 digit odd Nos (> 40000 & RA) = $\underset{(4\text{ or }5\text{ or }7)}{\underline{3}} \times \underline{6} \times \underline{6} \times \underline{6} \times \underset{(1\text{ or }5\text{ or }7)}{\underline{3}} = 3^2 \times 6^3$

given digits are 0, 1, 2, 4, 5, 7

In a test there are three multiple choice questions having four choices each. Number of sequences in which a student can fail to get all answers correct is

(a) 11 (b) 15 (c) 80 (d) 63

Total sequences of answers $= \underset{Q_1}{4} \times \underset{Q_2}{4} \times \underset{Q_3}{4} = 64$

Req Ans $= 64 - 1 = 63$

Q. In a test each student has to solve 5 T/F Questions. No two students have given same sequence of answers and none of the student has given all correct ans then find the max. number of students appearing in a test

(a) 4

~~(b)~~ 31

(c) 32

(d) 63

Total sequences of answers $= \underset{Q_1}{2} \times \underset{Q_2}{2} \times \underset{Q_3}{2} \times \underset{Q_4}{2} \times \underset{Q_5}{2} = 32$

Various seq. : $\underset{=1}{(TTTTT)}$, $\underset{=5}{(TTTTF)}$, $\underset{=10}{(TTTFF)}$, $\underset{=10}{(TTFFF)}$, $\underset{=5}{(TFFFF)}$, $\underset{=1}{(FFFFF)}$

But student knowing all correct ans is not there

So Req No. of Students $= 32 - 1 = 31$

Factorial → $\boxed{n! = n(n-1)(n-2)(n-3)\ldots\ldots 3.2.1} \Rightarrow \boxed{n! = n(n-1)!}$

eg $\lfloor 5 = 5\times4\times3\times2\times1$ (RNA), $\lfloor 4 = 4\times3\times2\times1$ (RNA) & $\lfloor 1 = 1$ & $\lfloor 0 = 1$

Q: How many 5 digit Nos can be formed using odd digits? (if RNA)

Req Nos $= \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 5!$

Q How many 4 letter words can be form using the letters "ROSE" if each letter is coming exactly once? (RNA)

$= \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 4!$

Q In how many ways 6 persons can be seated on 6 chairs?

Total seating arrangements (RNA) $= \underset{P_1}{6} \times \underset{P_2}{5} \times \underset{P_3}{4} \times \underset{P_4}{3} \times \underset{P_5}{2} \times \underset{P_6}{1} = 6! = {}^6P_6$

Q In how many ways we can arrange the letters of the word 'EQUATION' if

A, E, I, O, U, Q, T, N

① there is No Restriction = ? $= 8! = {}^8P_8$ (RNA)

② if words are starting with E = ? = $\underline{E}$ _ _ _ _ _ _ _ $= 1 \times 7! = 1 \times {}^7P_7$

③ words starts with E & ends with N = ? = E _ _ _ _ _ _ N (Arrangement) $= 1 \times 6! \times 1! = {}^6P_6$

Q. How many 4 letter words (w or w/o meaning) can be formed using the letters of the word 'EQUATION' = ?

1 2 3 4 5 6 7 8

(RNA)

$$= \frac{8\times}{P_1}\frac{7\times}{P_2}\frac{6\times}{P_3}\frac{5}{P_4} = {}^8C_4 \times 4! = {}^8P_4$$

Q. How many 5 letter words can be formed using the letters of the word 'LOGARITHMS'

1 2 3 4 5 6 7 8 9 10

(RNA)

$$= \frac{10\times}{P_1}\frac{9\times}{P_2}\frac{8\times}{P_3}\frac{7\times}{P_4}\frac{6}{P_5} = {}^{10}C_5 \times 5! = {}^{10}P_5$$

PERMUTATION & COMBINATION →

Combination (when our Counting is Based on selection only then use ^nC_r)

Permutation (when " " " " selection as well as arrangement then use ^nP_r)

$$^nC_r = \frac{n!}{r!(n-r)!} \quad \& \quad ^nP_r = \frac{n!}{(n-r)!} = \boxed{^nC_r \times r!} = \text{Selection} \times \text{Arrangement}$$

Note ① the concept of ^nP_r, ^nC_r & $r!$ is applicable when RNA

② If RA then only use Multiplication Rule

Q In how many ways cricket team can be selected from Batch of 15 players if

(1) there is No Restriction = ? = $^{15}C_{11}$

(2) A particular player is always selected = ? = $^{1}C_{1} \times {}^{14}C_{10}$

(3) " " " is never selected = ? = $^{14}C_{11}$

Q In a Batch of 10 Batsman, 8 Bowlers, 5 All rounders & 2 wicket keepers a cricket team has to be selected in which there must be 5 Batsman, 3 Bowlers, 2 all rounders and 1 W.K then in how many ways this can be done?

Total ways = $^{10}C_{5} \times {}^{8}C_{3} \times {}^{5}C_{2} \times {}^{2}C_{1}$

Q. How many four letters words can be formed using the letters of the word "FAILURE" if

= Arrangement

(1) there is no Restriction = ? $= \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} = {}^7C_4 \times 4! = {}^7P_4$

(RNA)

Best App

(2) f is included in each word = ? $= {}^1C_1 \times {}^6C_3 \times 4!$

(3) f is not included in any word = ? $= {}^6C_4 \times 4! = {}^6P_4$

Q. out of 7 Consonants and 5 Vowels, how many 5 letter words can be formed including 3 Consonants & 2 Vowels ? $= {}^7C_3 \times {}^5C_2 \times 5!$

In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls and in 190 games both the players were boy. The number of games in which one player was a boy and other was girl is

(a) 200 (b) 216 (c) 235 (d) 256

Boys $= m \Rightarrow {}^mC_2 = 190 \Rightarrow \frac{m(m-1)}{2} = 190 \Rightarrow m(m-1) = 380 = 20(20-1)$
$\Rightarrow m = 20$ Boys

Girls $= n \Rightarrow {}^nC_2 = 45 \Rightarrow \frac{n(n-1)}{2} = 45 \Rightarrow n(n-1) = 90 = 10(10-1) \Rightarrow n = 10$ Girls

Req. Games $= {}^mC_1 \times {}^nC_1$

$= {}^{20}C_1 \times {}^{10}C_1 = 200$

How many three letter computer passwords can be formed with at least one symmetric letter such that Repetition not allowed. It is given that symmetric letters are A, H, I, M, O, T, U, V, W, X, Z.

= Arrangement.

(a) 990 (b) 2730 (c) 12870 (d) 1560000

Symm letters = 11

Non Symm letters = 26 - 11 = 15

Trick: $\boxed{\text{At least one = Total - None}}$

ie At least one symm letter = Total - No symm letters

(RNA)

$= {}^{26}P_3 - {}^{15}P_3$

$= (26 \times 25 \times 24) - (15 \times 14 \times 13) = 12870$

Concept: (No symm letter) or (1 symm letter) or (2 symm letters) or (3 symm letters)

= Case I, = Case II, = Case III, = Case IV

$\underbrace{(1 \text{ symm letter}) \text{ or } (2 \text{ symm letters}) \text{ or } (3 \text{ symm letters})}_{\text{At least one}}$

= Total

M-II Req Numbers = we can follow either Case II or Case III or Case IV

= (?) + (?) + (?) = Ans

Q. No. of passwords in which there is at Most one symm letter

= Case I + Case II = ? + ? = Ans

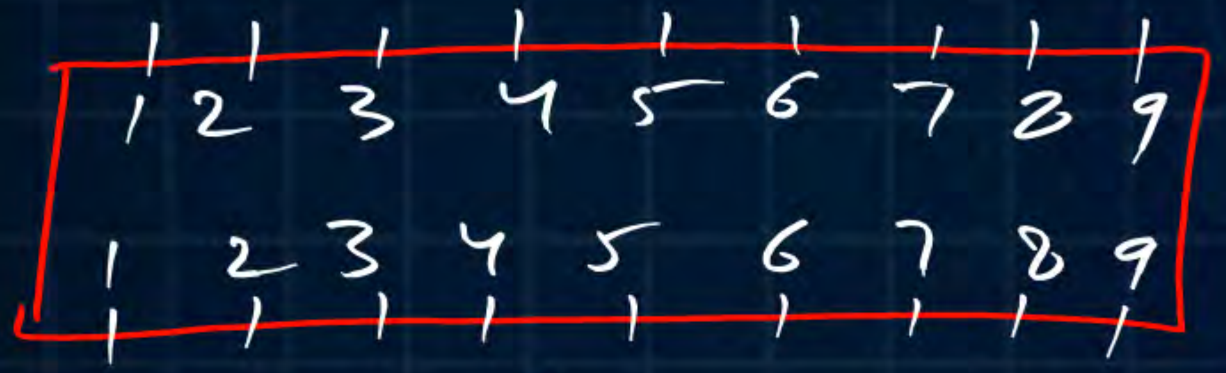
18 guests have to be seated, half on each side of a long table. Four particular guests desired to sit on one particular and three others on the other side, then how many seating arrangements can be made?

(a) $^{18}C_4 \cdot {}^{14}C_3 \cdot 9! \cdot 9!$

(b) $^{2}C_1 \cdot {}^{9}P_4 \cdot {}^{9}P_3 \cdot 11!$

(c) $^{9}P_4 \cdot {}^{9}P_3 \cdot 11!$

(d) $^{2}C_1 \cdot \frac{9!}{4!} \cdot \frac{9!}{3!}$



Req Arr (RNA) $= \underbrace{\left({}^4C_4 \times {}^9C_4 \times 4!\right)}_{J_1} \times \underbrace{\left({}^3C_3 \times {}^9C_3 \times 3!\right)}_{J_2} \times \underbrace{\left(11!\right)}_{J_3}$

$= {}^9P_4 \times {}^9P_3 \times 11!$

Circular Permutation →

(1) Number of $\underbrace{\text{linear arrangements}}_{\text{RNA}}$ of n different things $= n!$

(2) Number of $\underbrace{\text{Circular Arrangements}}_{\text{RNA}}$ of n different things $= (n-1)!$

Let $n=5$, then C. Arrangements $= (5-1)! = 4!$

(3) Number of $\underbrace{\text{linear arrangements}}_{\text{RNA}}$ of n things in which p are alike, q are alike, r are alike & Rest are different then $= \dfrac{n!}{p!\,q!\,r!}$

There are 5 gentlemen and four ladies to dine at a round table. In how many ways can they seat themselves so that no two ladies are together?

(a) 3280 (b) 2880 (c) 2080 (d) 2480

n = 9 persons

Total C. Arrangements $= (9-1)! = 8!$

fav. Number of C. Arrangements = No. of C.A in which No two Ladies are there

(first arrange Males circularly)

$= \underbrace{(5-1)!}_{\text{(Boys)} = \text{C.A}} \times \underbrace{({}^5C_4 \times 4!)}_{\text{(Girls)} = \text{L.A}} = 24 \times 120$

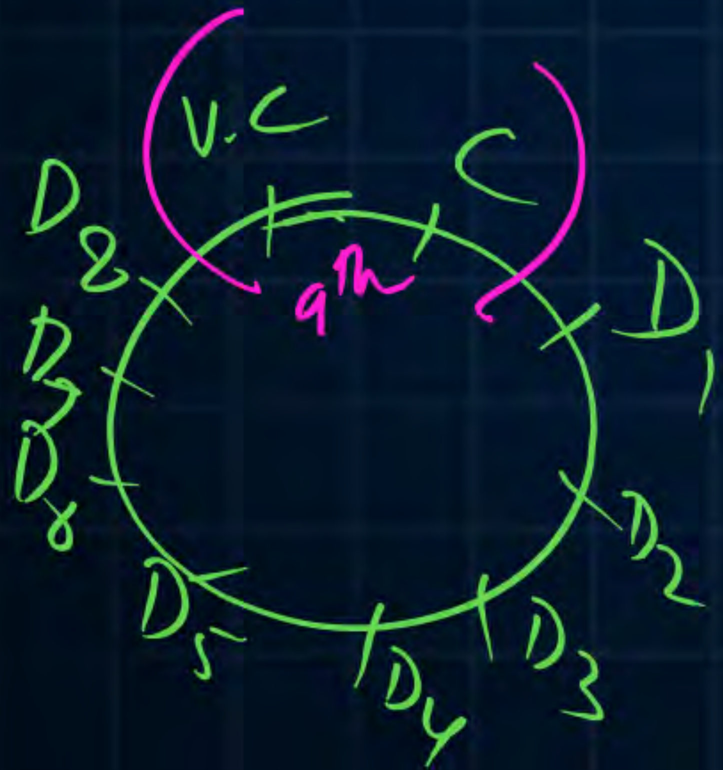
Req. Prob $= \frac{f}{T} = \frac{2880}{8!}$

In how many ways can 8 Directors, Vice-Chairman & Chairman of a firm be seated at a round table, if the Chairman has to sit between Vice-Chairman & Director

(a) $2 \times 9!$ (b) $2 \times 8!$ (c) $2 \times 7!$ (d) $3! \times 9!$

8 D, 1 V.C, 1 C i.e Total Persons = 10

Total C. Arrangement = $(10-1)! = 9!$



fav. C Arrangements = (which is Required) = $(9-1)! \times 2!$

Note - Req Prob = $\frac{\text{fav}}{\text{Total}} = \frac{8! \times 2!}{9!}$

Eg: 4 letter words using a, b, c, d = ? = 4! = 24 words
$\underbrace{a, b, c, d}_{RNA}$

Q 4 letter words using a, a, b, b = ? = $\frac{4!}{2!\,2!}$ = 6 words
RNA

M-II Various 4 letter words = (aabb), (abab), (abba), (bbaa), (baba), (baab) i.e. 6 words

Q five digit Nos using $\underbrace{4, 4, 4}_{p}, \underbrace{7, 7}_{q}$ = ? = $\frac{5!}{3!\,2!}$ = 10 Numbers
n = 5
RNA

Q How many 11 letter words can be formed using 'MISSISSIPPI'

$\underbrace{S, S, S, S}_{p=}, \underbrace{I, I, I, I}_{q=}, \underbrace{P, P}_{r=}, M \Rightarrow n = 11$ letters

Req 11 letter words $= \dfrac{11!}{4!\,4!\,2!}$

Q. Six identical coins are arranged in a row. The number of ways in which the number of tails is equal to number of heads is ?

(a) 20

(b) 120

(c) 9

(d) 40

Sol = $\left(\frac{H}{C_1}\ \frac{T}{C_2}\ \frac{H}{C_3}\ \frac{H}{C_4}\ \frac{T}{C_5}\ \frac{T}{C_6}\right) \longrightarrow = \frac{6!}{3!\,3!} = 20$

$n = 6$

$p = 3\ H$

$q = 3\ T$

only Selection/Rejection Based Questions →

if we have p alike items of 1^{st} kind, q alike items of 2^{nd} kind, r alike items of 3^{rd} kind & n different items then

Total Number of Selections & Rejection $= (p+1)(q+1)(r+1)2^n$

& No. of ways in which we can select at least one item

where $p+q+r+\text{Rest}(n) = \text{Total}$ (?)

$$= \boxed{(p+1)(q+1)(r+1)2^n - 1}$$

Note: while in case of Arrangements $= \dfrac{n!}{p!\,q!\,r!}$ where $p+q+r+\text{Rest} = \text{Total}(n)$ (?)

The total number of selections of fruits which can be made from 3 bananas, 4 apples, and 2 oranges is

(a) 39 (b) 315 (c) 512 (d) None

M-I $3B, 4A, 2O$

Total Selections or Rejections

$$\underset{B}{(4\text{ Ways})} \times \underset{A}{(5\text{ Ways})} \times \underset{O}{(3)} = 60\text{ ways}$$

So No. of Selection $= 60 - 1 = 59$ Ways

M-II $p = 3,\ q = 4,\ r = 2,\ \text{Diff } n = 0$

Req. Selection or Rejections

$$= (p+1)(q+1)(r+1)2^n$$

$$= (3+1)(4+1)(2+1)2^0 = 60$$

So Req. Selection $= 60 - 1 = 59$

Q. The Number of factors of 7875 will be?

5	7875
5	1575
5	315
3	63
3	21
7	7
	1

$$7875 = 3^2 \times 5^3 \times 7^1$$

Total factors = (3 ways) × (4 ways) × (2 ways)

= 24 factors

So Proper factors = 24 − 2 = 22 Ans

M-II Total Factor $= (p+1)(q+1)2^n = (2+1)(3+1)2^1$

$= 24$

$p=2,\ q=3,\ n=1$

Eg 36 → 1, 36 (Improper factors)

→ 2, 3, 4, 6, 9, 12, 18 (Proper factors)

& Total factors = 9

M-II $36 = 4 \times 9 = 2^2 \times 3^2$

So Total factors = (3 ways) × (3 ways) = 9 ways

The number of proper divisors of number 38808 can have?

(a) 70 (b) 71 (c) 72 (d) None

HW

$38808 = 2^a \times 3^b \times 5^c \times 7^d \times 11^e$ (Prime form)

Req. Ans $= (a+1)(b+1)(c+1)(d+1)(e+1) \ldots$

$= ?$

As per students $= 70$

2014 The numbr of factors of 2014 are ___?___

(a) 2

(b) 6

(c) 8

(d) 12

$$2014 = 2^1 \times 19^1 \times 53^1$$

So Total factors $= (2)(2)(2) = 8$

M-II 2014

1, 2014 (Improper)

2, 19, 53, 38, 106, 1007 (Proper)

Thank

THANK

Keep Hustling!