

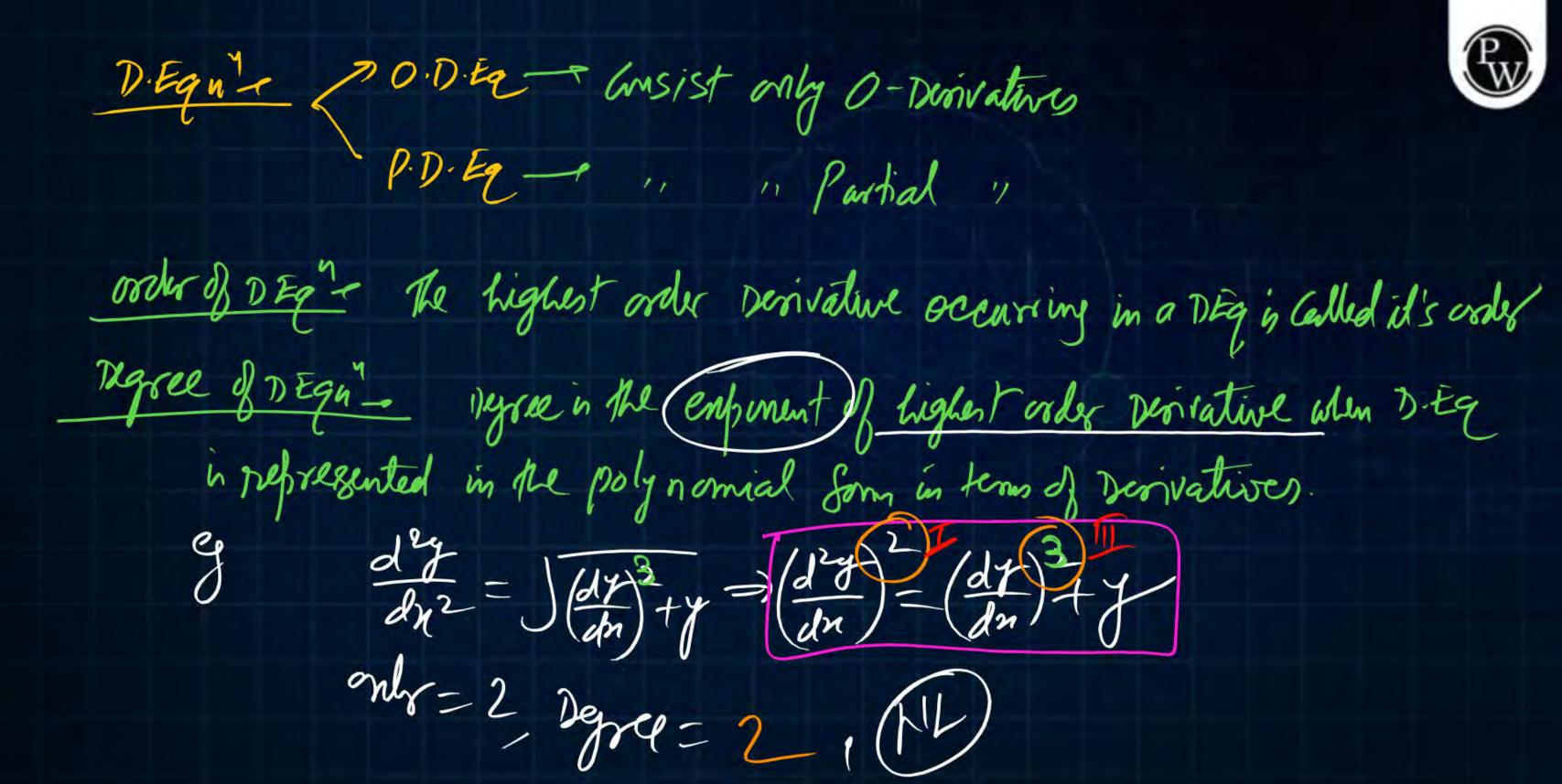


# ODICS to be covered

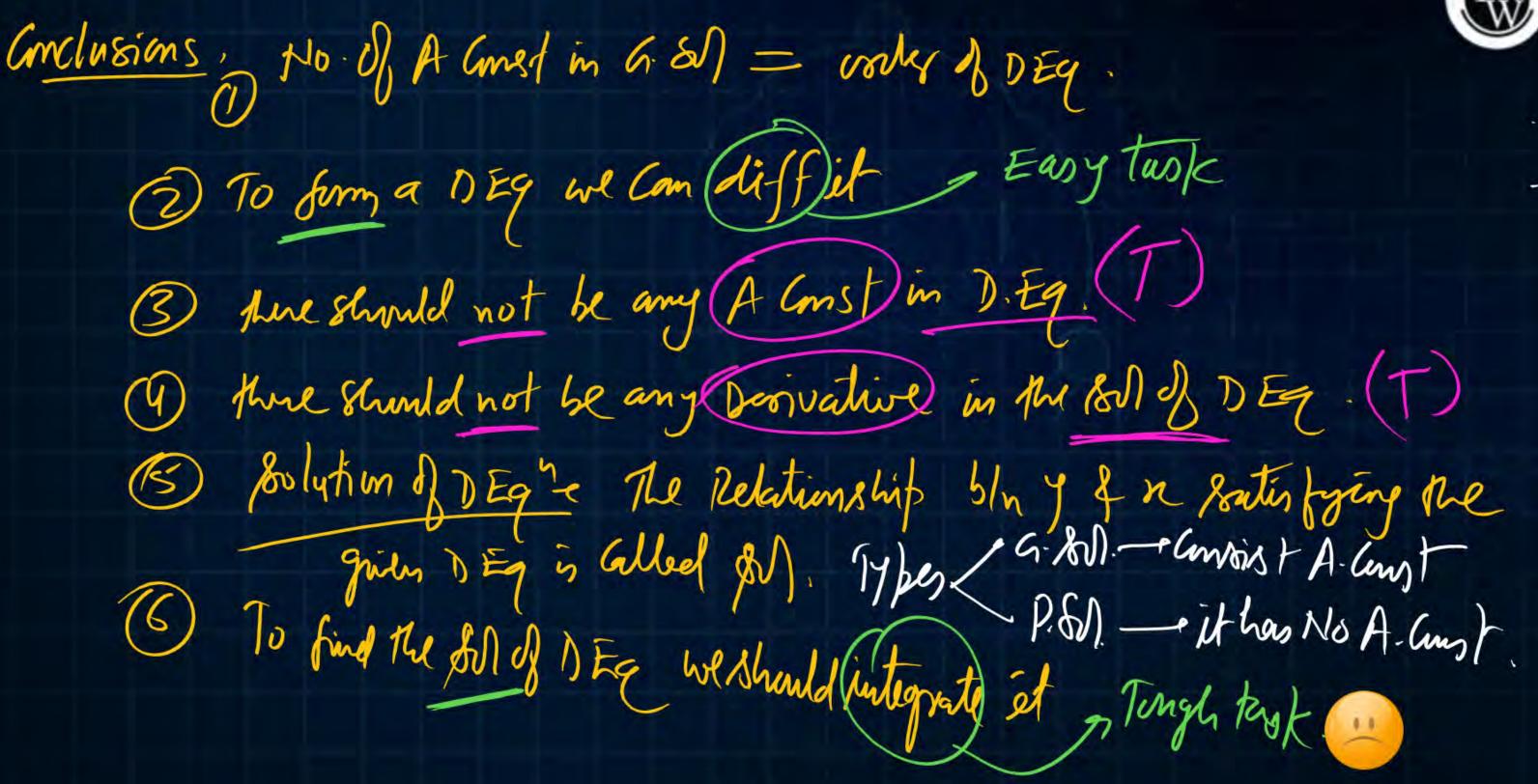
### DIFFERENTIAL EQUATIONS (Part 2)

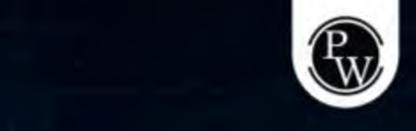
- (1) Enact DEq.
- 2) CF 4 PI Methods
- (3) Cauchy L.DE solving Method







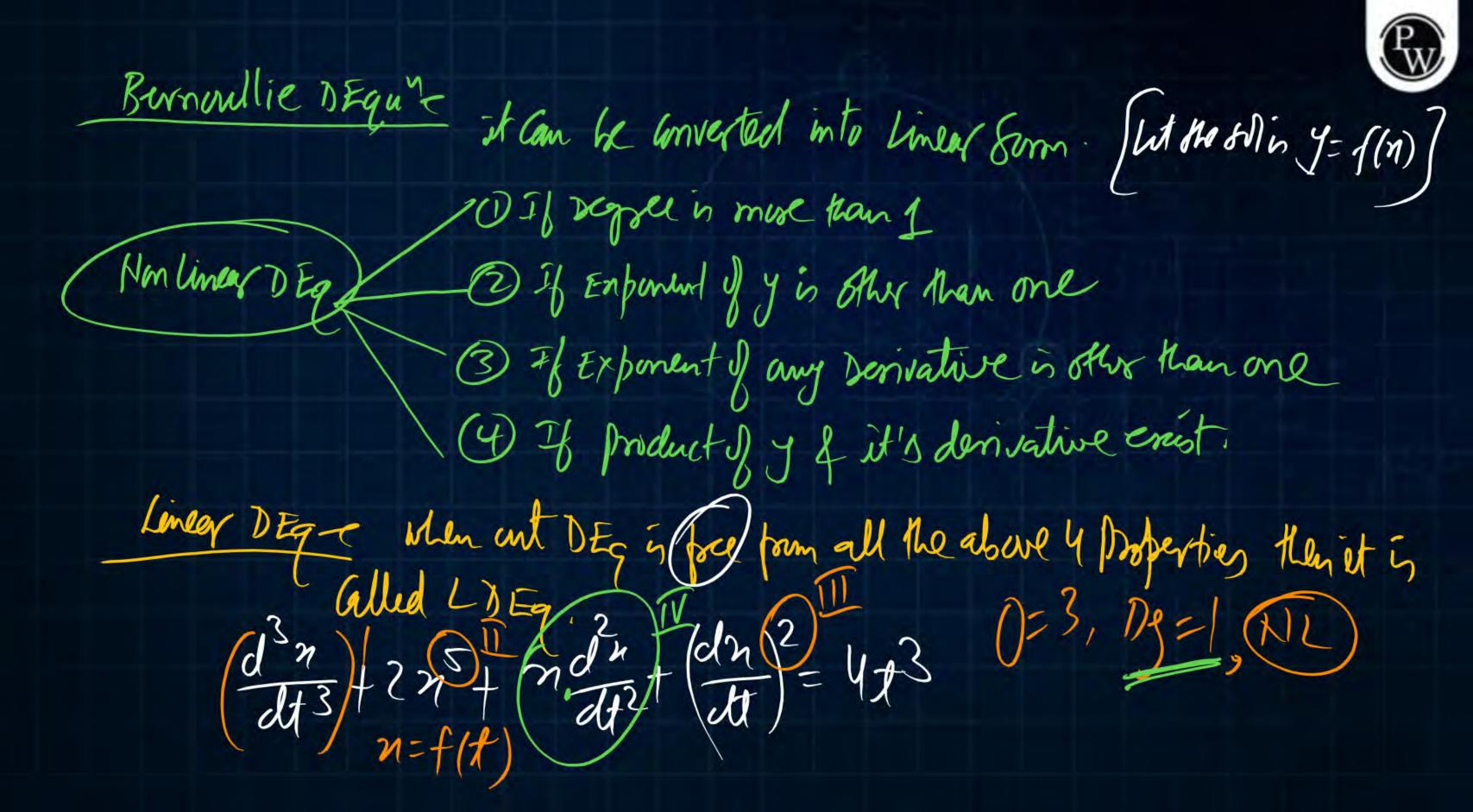




LINEAR D. Eg, of 1st order Green it's G. Formis of the py= 8 while page for of Malone 4 d/8 T.F =  $e^{\int P dn} e^{\int P dn} e^{\int P dn} = e^{\int P dn} e^{\int P dn} = e^{\int P dn}$ The I it is given as dn +pn=g where p& g one faired & alone

of It= (Spdy & Shis (n(It)= ) (It) dn+C

it is LD E is My



The solution of 
$$\frac{dy}{dx} = (4x + y + 1)^2$$
 is

(a) 
$$\frac{1}{2} \tan^{-1} \left( \frac{4x + y + 1}{2} \right) = c$$

(b) 
$$\frac{1}{2} \tan^{-1} \left( \frac{4x + y + 1}{2} \right) = x + c$$

(c) 
$$\frac{1}{2} \tan^{-1}(4x + 4y + 1) = x + c$$

(d) 
$$\frac{1}{2} \tan^{-1}(4x + y + 1) = c$$

Put 4n+y+1= t 4+ dt = dt In

$$\frac{dt}{dn} - 4 = 1^{2} = \int \frac{dt}{dt} = \int \frac{dt}{dt} = 1$$

$$= \int \frac{dt}{dt} = \int \frac{dt}{dt} = 1$$

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HOMOGENEOUS D. E9) it is of the type dy = f(a,y) where f(n,y) & g(n,y) and namely function of famely ie f(77,77)= 2. f(7,7) & g(7x,77)= 2.9(2,4) To solve (1) Put (y=vn)=) dy=d(vn)=(v+ndv) after that jam can use Variable separable Method

The solution of differential equation  $(x^2y - 2xy^2)$ 

$$dx - (x^3 - 3x^2y)dy = 0$$
, is

(a) 
$$\frac{x}{y} - 2\log x + 3\log y = c$$

(b) 
$$\frac{y}{x} - 2\log y + 3\log x = c$$

(c) 
$$\frac{x}{y} + 2\log x - 3\log y = c$$

(d) 
$$\frac{y}{x} + 2\log y - 3\log x = c$$

 $\frac{\partial N}{\partial y} = n^3 - 4ny \quad 1800 \text{ is No}$   $\frac{\partial N}{\partial n} = -3n^2 + 6ny$ 

(ny-zny2)dn - (n3 3n2y) dy=0  $-\frac{dy}{dn} = -\frac{n^2y - 2\pi y^2}{n^2y^2}$ 23 3 nzy  $V+n\frac{dV}{dn}=\frac{V-2V^2}{2}$  $\left(\frac{-3U}{V^2}\right)dV = \left(\frac{1}{2}\right)d\eta =$ 

$$\int (\frac{1}{2} - \frac{3}{7}) dV = \int (\frac{1}{2}) dn + C_1$$

$$-\frac{1}{7} - 3 \ln V = \ln x + \ln C$$

$$-\frac{3}{7} - 3 \ln (\frac{1}{2}) = \ln x + \ln C$$

$$-\frac{3}{7} - 3 (\frac{1}{7}) - \ln x = \ln C$$

$$\frac{3}{7} - 3 (\frac{1}{7}) - \frac{1}{7} \ln x = \ln C$$

$$\frac{3}{7} - 3 \ln (\frac{1}{7}) - \frac{1}{7} \ln x = \ln C$$



## ETACT D.Eg)

Consider a 1st DEG of the type [Montrody=0]—

This DEG is Called Exact (solvable) if [JM = JN]

H. Cond's

11. 20 +

& it's solution is given as

### The differential equation

$$(27x^2 + ky\cos x)dx + (2\sin x - 27y^3)dy = 0$$
  
is exact for  $k = ____.$ 

$$\frac{\partial m}{\partial y} = (Cosn)_{1} \frac{2M}{2N} = (2\cos n)$$

#### The differential equation

$$(\alpha xy^3 + y\cos x)dx + (x^2y^2 + \beta\sin x)dy = 0$$
  
is exact for

(a) 
$$\alpha = \frac{3}{2}, \beta = 1$$
 (b)  $\alpha = 1, \beta = \frac{3}{2}$   
(c)  $\alpha = \frac{2}{3}, \beta = 1$  (d)  $\alpha = 1, \beta = \frac{2}{3}$ 

M= 
$$\frac{dny^3}{f} = \frac{dN}{dy} = \frac{dN}{dy} = \frac{dN}{dy} = \frac{dN}{dy} + \frac{dN}{dy} + \frac{dN}{dy} = \frac{dN}{dy} + \frac{dN}{dy} + \frac{dN}{dy} = \frac{dN}{dy} + \frac{dN}{dy} + \frac{dN}{dy} + \frac{dN}{dy} = \frac{dN}{dy} + \frac{dN}{dy} +$$



For Enact, 
$$\frac{\partial r}{\partial y} = \frac{\partial r}{\partial n}$$

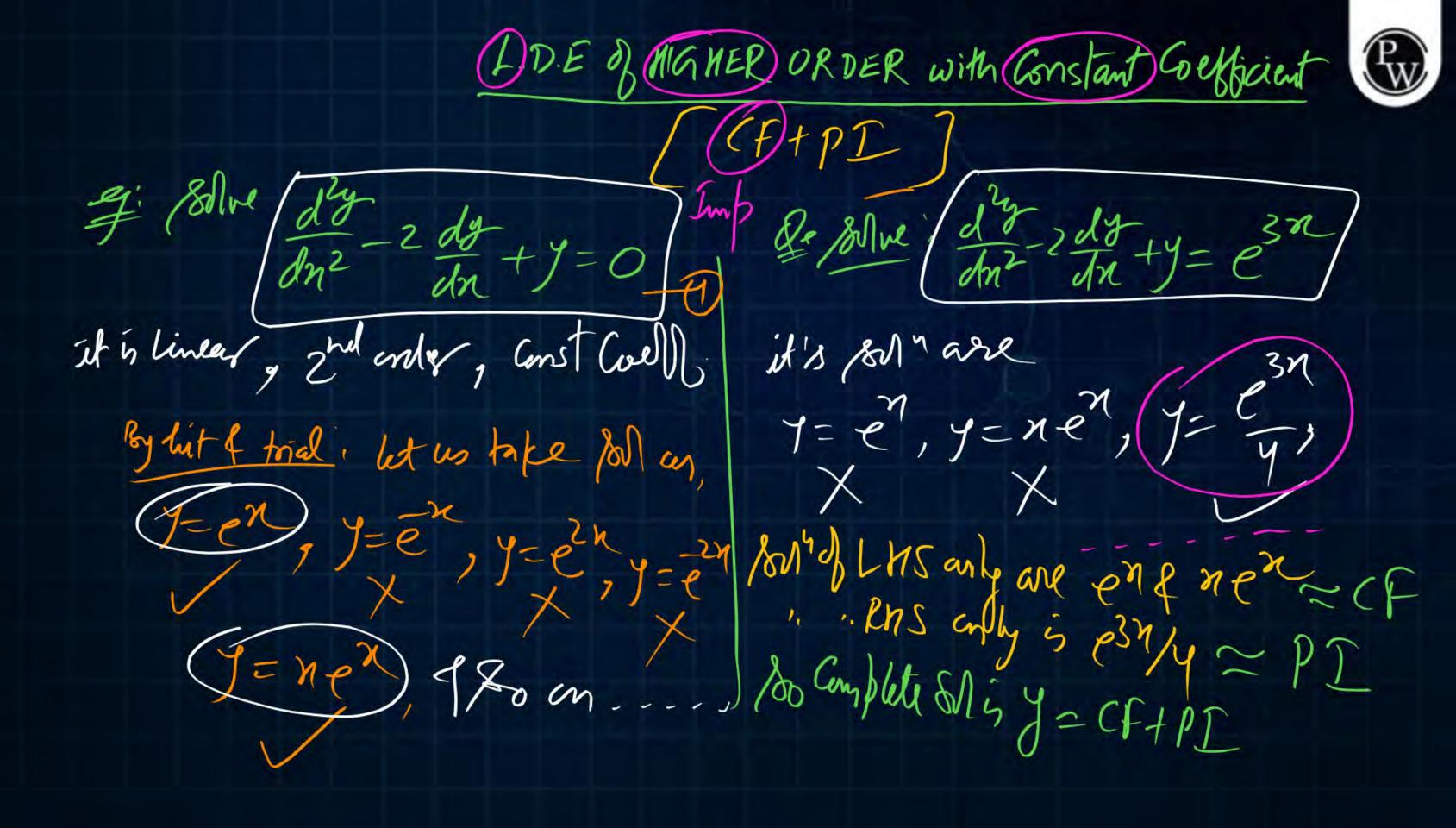
$$3x ny^2 + 6nx = 2ny^2 + \beta 6nx$$

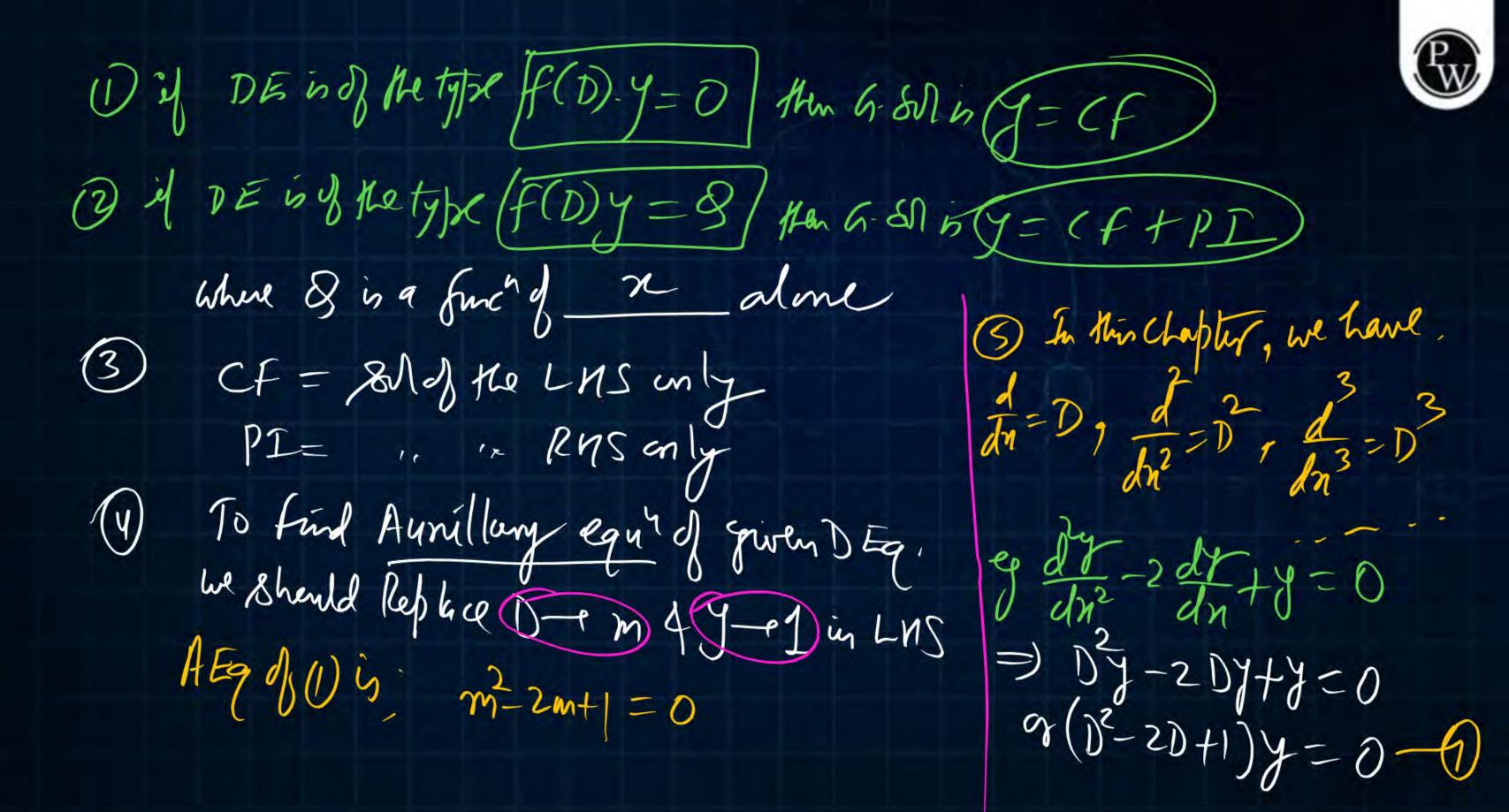
$$\Rightarrow 3d = 2$$

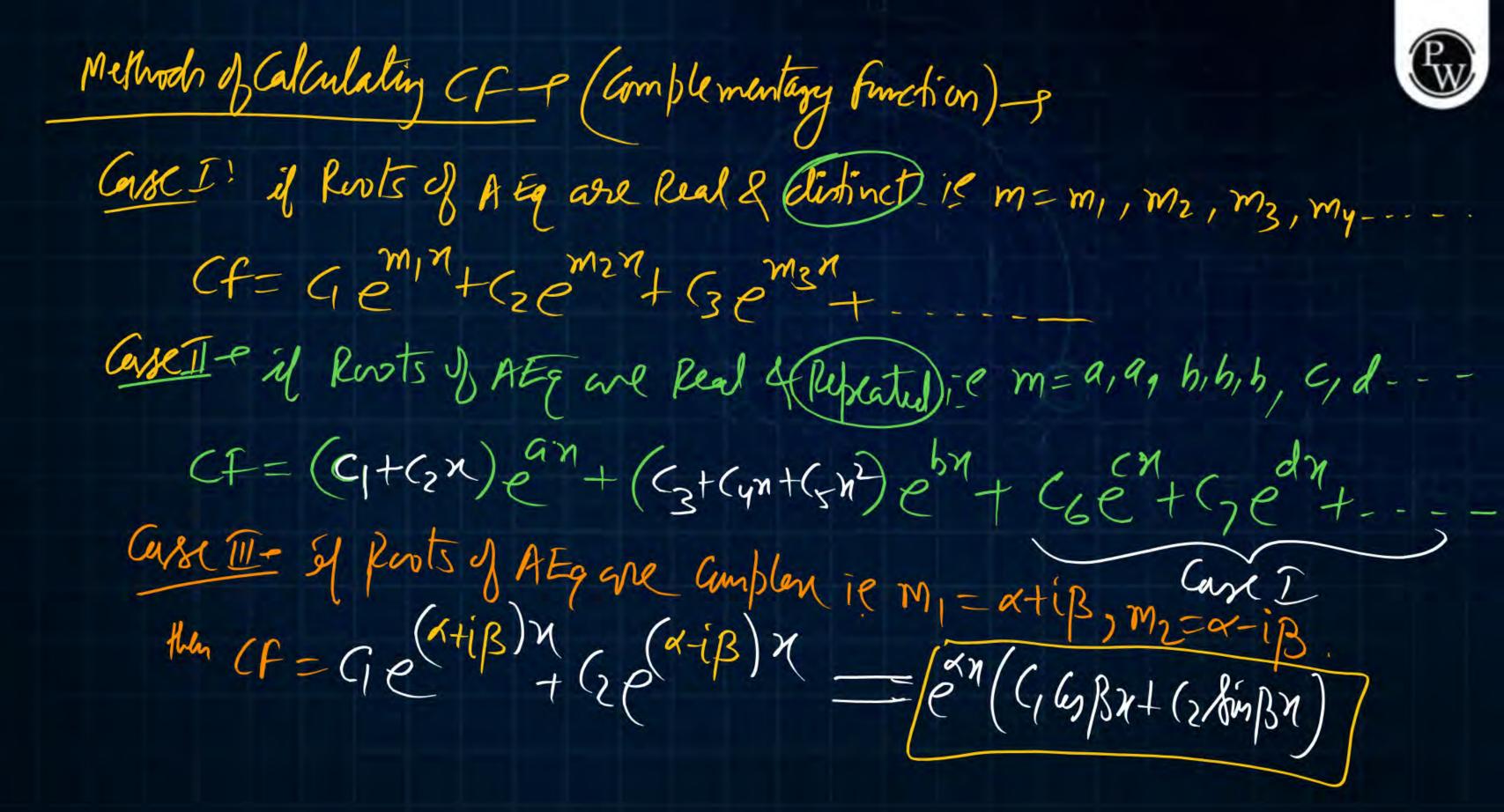
$$d = \frac{2}{3}$$

$$i \in \mathbb{Z}$$

De find the 1801 of (3n) + y(ann) dn + (n2y2+ pmn) dy=0 : it is Enect so it's solis (NL) Degree = [3nj3+y6n)dn+ (0)dy=c de ""  $y''=y \Rightarrow \int y'' dx = \int y dx$ or y' = 2? Not possible







De Bolve de 2 de 19 = 0 Ball: DJ-2DJ+J=0or 62-2D+1) y=0 -(1) AEq is  $m^2 - 2m + 1 = 0 = (m-1) = 0$ ~ m=1, sobylascil. CF= (G+C2X) e1X PITO So Completeson y= (f=(C1+(27)ex)

19 = 9(en)+6(nen) 10th is J=Gy,+GJ2 Whre y1=en 4 12= nen one also the sort of (1) (2) Here y, & y, a see (L.I) 800 J given DEG F No linear Relationship bhe the solutions.

For the differential equation  $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$ 

with initial conditions x(0) = 1 and  $\frac{dx}{dt}\Big|_{t=0} = 0$ , the

solution is

(a) 
$$x(t) = 2e^{-6t} - e^{-2t}$$
 (b)  $x(t) = 2e^{-2t} - e^{-4t}$ 

(c) 
$$x(t) = -e^{-6t} + 2e^{-4t}$$
 (d)  $x(t) = e^{-2t} + 2e^{-4t}$ 

5. Ferm is  $D^{2}n+6Dn+8n=0$   $(D^{2}+6D+8)n=0-1$ A Equ'  $m^{2}+6m+8=0=)n=2$  Equ'  $CF=(1e^{(2)})^{4}+(1e^{(2)$ 

= (n=f(t)) is it L.D. Ed 2 months with Const Coell in x & t Let d=D, d=-02 dx=D, d=-02

Models  $n = (f+f) = c_1 + c_2 + c_3 + c_4 + c_4 + c_5 + c_5$ 

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0 \text{ are}$$



(a) 
$$e^{-(1+i)x}$$
,  $e^{-(1-i)x}$  (b)  $e^{(1+i)x}$ ,  $e^{(1-i)x}$ 

(b) 
$$e^{(1+i)x}$$
,  $e^{(1-i)x}$ 

(c) 
$$e^{-(1+i)x}$$
,  $e^{(1-i)x}$  (d)  $e^{(1+i)x}$ ,  $e^{-(1-i)x}$ 

(d) 
$$e^{(1+i)x}$$
,  $e^{-(1-i)x}$ 

### 5. fam is (D2+2 D+2)y-9 AEqu m72m+2=0 $m = -2 \pm Jy - B$

$$M = -\frac{1}{1+1} \left( \frac{1}{1+1} \right) \left( \frac{1}{1+1}$$



### 1. Polation ship:

$$15 \, \text{m}_1 = -1 + i \, , \, \text{m}_2 = -1 - i \,$$

In CF = 
$$(e^{(1-i)}n + (2e^{(1-i)})n$$
  
and Nossible Ans will be?  
 $(F = e^{(1-i)}n + (2e^{(1-i)})n$   
 $(F = e^{(1)}n (C_i C_i 1.x + (2e^{(1-i)})n)$ 

42 What is the general solution of a homogeneous differential equation with the characteristic equation?

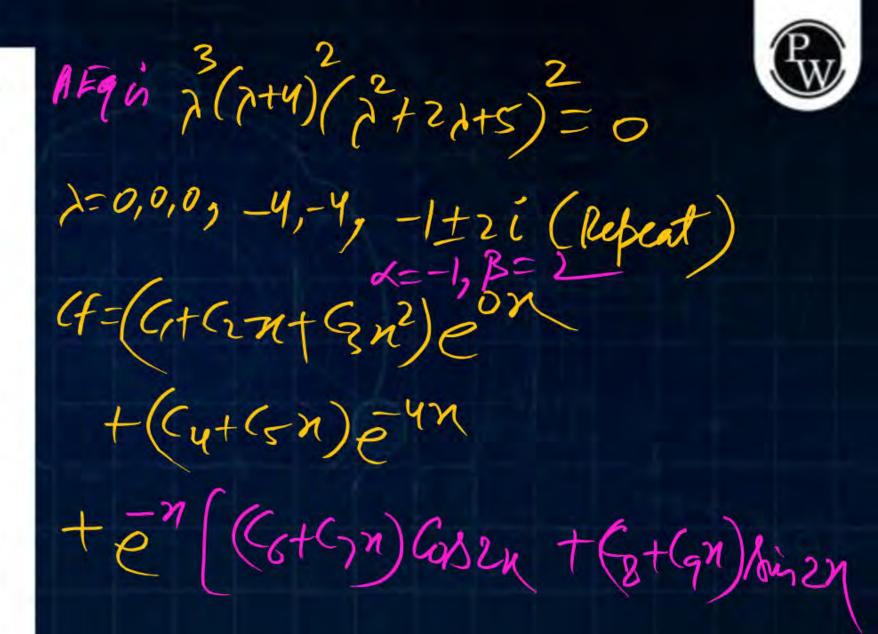
$$\lambda^{3}(\lambda + 4)^{2}(\lambda^{2} + 2\lambda + 5)^{2} \neq 0$$

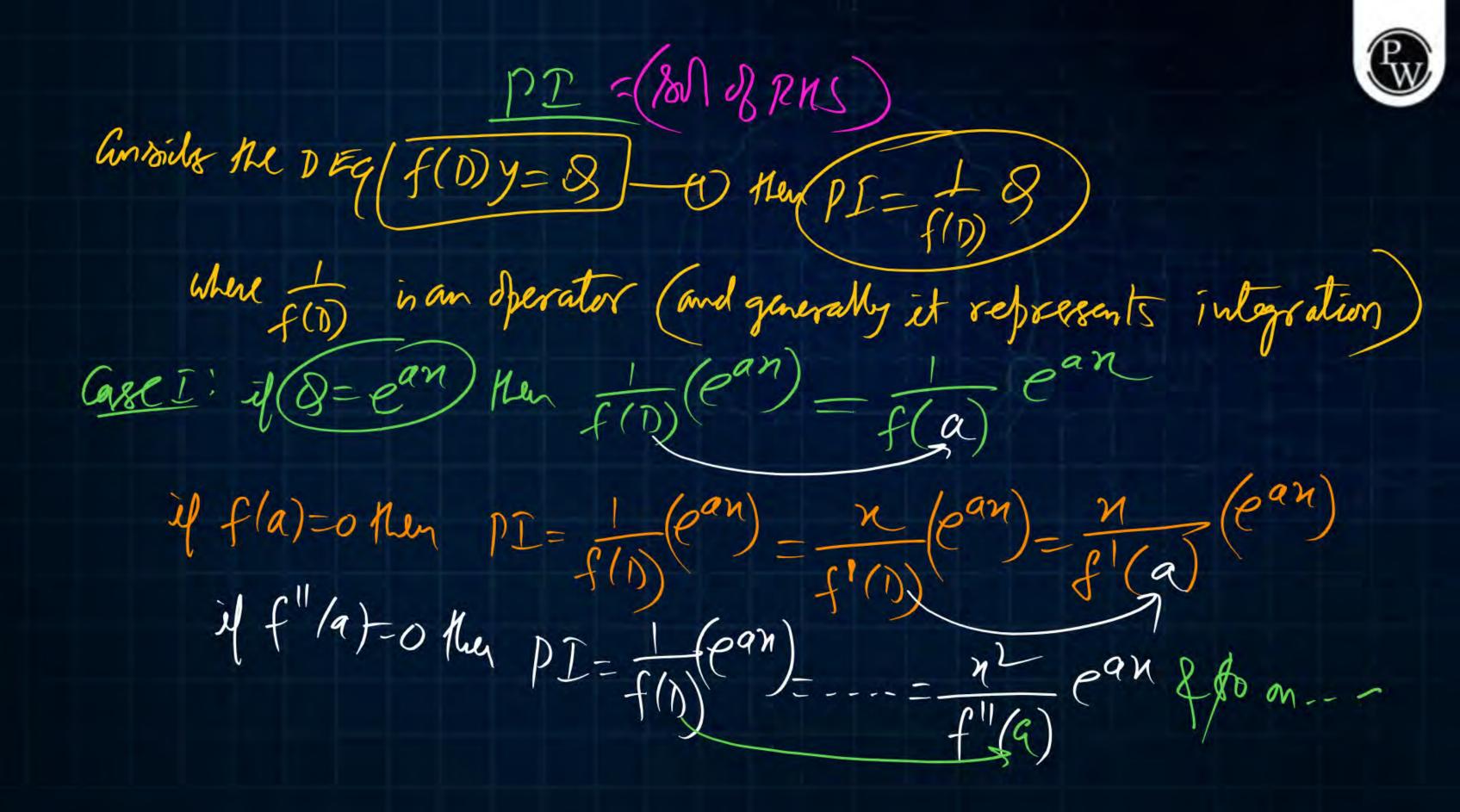
(a) 
$$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-4x} + c_5 x e^{4x} + e^x (c_6)$$
  
 $\cos 2x + c_7 \sin 2x + c_8 x \cos 2x + c_9 x \sin 2x$ 

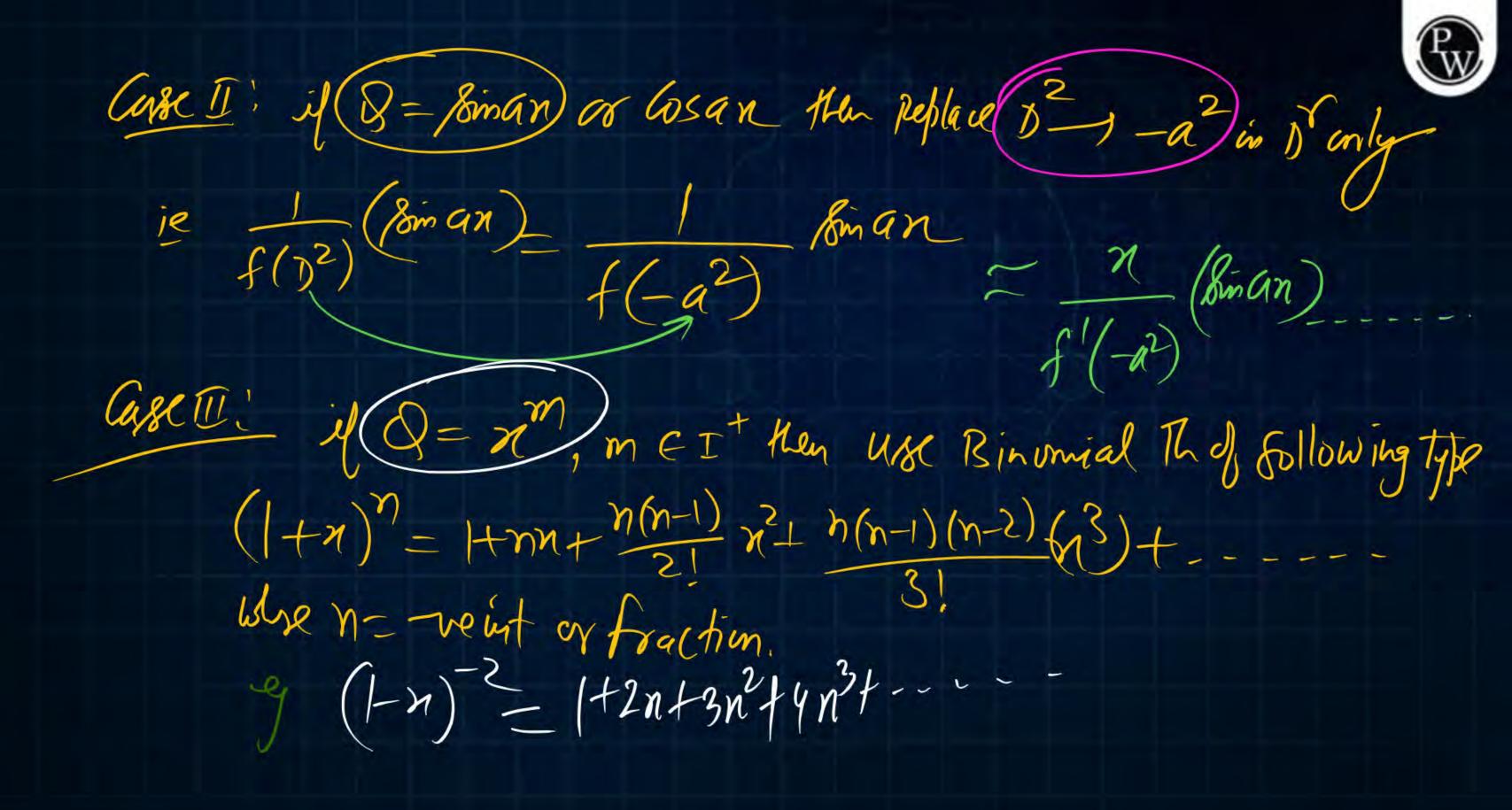
(b) 
$$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-4x} + c_5 x e^{-4x} + e^{-x} \{c_6 \cos 2x + c_7 \sin 2x\} + e^x \{c_8 x \cos 2x + c_9 x \sin 2x\}$$

(c) 
$$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-4x} + c_5 x e^{4x} + e^x \{c_6 \cos 2x + c_7 \sin 2x\} + e^{-x} \{c_8 x \cos 2x + c_9 x \sin 2x\}$$

(d) 
$$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-4x} + c_5 x e^{-4x} + e^{-x} \{c_6 \cos 2x + c_7 \sin 2x + c_8 x \cos 2x + c_9 x \sin 2x\}$$







What is the initial value if  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x}$ ,

with 
$$y(0) = 2$$
,  $\left(\frac{dy}{dx}\right)_{x=0} = 1$ ?

(a) 
$$y(x) = \left(\frac{13}{4} + \frac{1}{2}x\right)e^{-x} - \frac{5}{4}e^{-3x}$$
 (As per Students

(b) 
$$y(x) = \left(\frac{13}{4} + \frac{1}{2}x\right)e^{-3x} - \frac{5}{4}e^{-x}$$

(c) 
$$y(x) = \left(\frac{13}{4} + \frac{1}{2}x\right)e^{-x} + \frac{5}{4}e^{-3x}$$

(d) 
$$y(x) = \left(\frac{13}{4} - \frac{1}{2}x\right)e^{-x} - \frac{5}{4}e^{-3x}$$

AEG mtymt3 = u

m2+3m+m+3=0=1m=-1,-3

5. km (D+4D+3) y= = n PI= 1/8 2 1 (E/n)
2+40+3  $=\frac{\pi}{2D+4}\left(\frac{-m}{e^m}\right)\frac{\pi\left(\frac{-m}{e^m}\right)}{2(1)+4}$  $=\frac{\pi}{2}e^{\pi}$ 

Him use Bounday and ansig to Yarr suff

The particular integral of

$$(D^2 - 9)y = e^{3x} + \sin 2x$$
 is

(a) 
$$\frac{e^{3x}}{6} + \frac{\sin 2x}{13}$$
 (b)  $\frac{e^{3x}}{6} - \frac{1}{32}\sin 2x$ 

(c) 
$$\frac{e^{3x}}{6} - \frac{1}{13}\sin 2x$$
 (d)  $\frac{xe^{3x}}{6} - \frac{1}{13}\sin 2x$ 

$$PI = \frac{1}{D^2 q} (e^{3n}) = \frac{n}{2D} (e^{5n}) = \frac{n}{2(3)} e^{5n}$$

The particular integral of the differential equation

$$(D^2 - 4D + 3)y = \cos x$$
 is

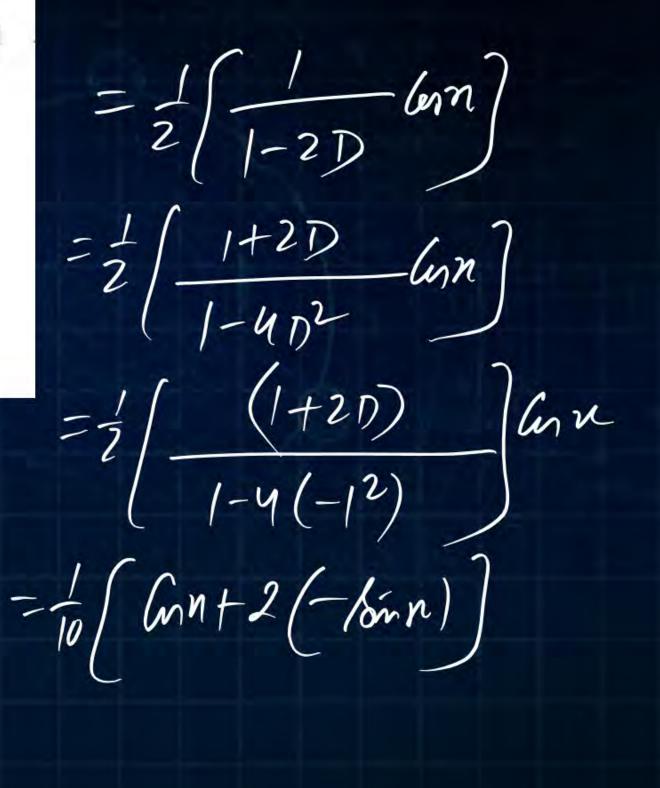
$$\frac{\cos x - 2\sin x}{10}$$
 (b) 
$$\frac{\cos x + 2\sin x}{10}$$

(c) 
$$\frac{2\cos x - 4\sin x}{5}$$
 (d)  $\frac{2\cos x + 4\sin x}{5}$ 

$$PI = \frac{1}{S(D)}S = \frac{1}{D^{2}4D+3}(GS1X)$$

$$= \frac{1}{(-1^{2})-4D+3}(GS1X)$$

$$= \frac{1}{-4D+2}(GS1X)$$





The respective expressions for complimentary function and particular integral part of the solution of the differential equation

$$\frac{d^4y}{dx^4} + 3\frac{d^2y}{dx^2} = 108x^2$$
 are

(a) 
$$\left[c_1 + c_2 x + c_3 \sin \sqrt{3x} + c_4 \cos \sqrt{3x}\right]$$
  
and  $\left[3x^4 - 12x^2 + c\right]$ 

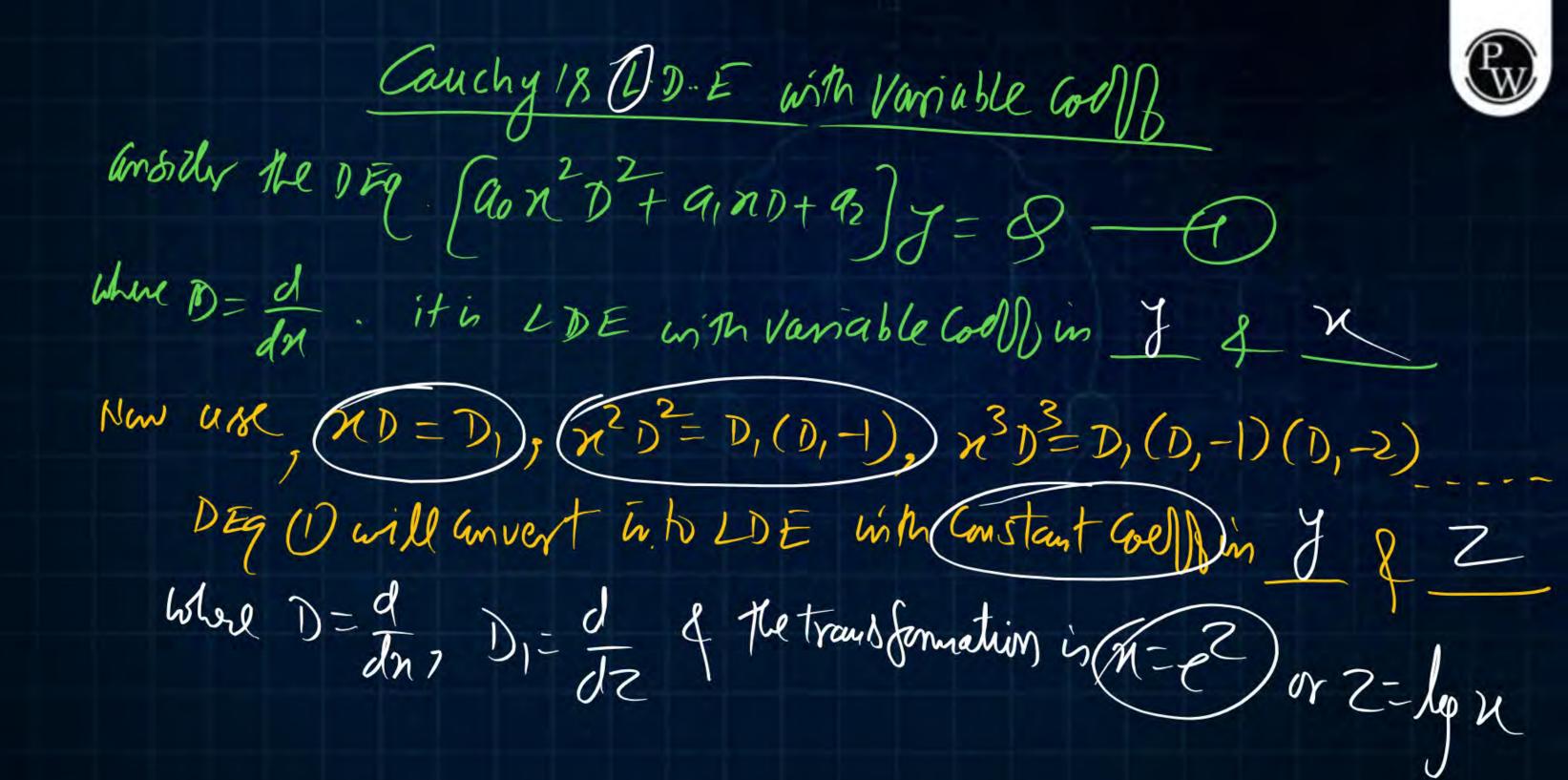
(b) 
$$\left[ c_2 + c_3 \sin \sqrt{3x} + c_4 \cos \sqrt{3x} \right]$$

and 
$$[5x^4 - 12x^2 + c]$$

(c) 
$$\left[c_1 + c_3 \sin \sqrt{3x} + c_4 \cos \sqrt{3x}\right]$$
  
and  $\left[3x^4 - 12x^2 + c\right]$ 

(d) 
$$\left[c_1 + c_2 x + c_3 \sin \sqrt{3x} + c_4 \cos \sqrt{3x}\right]$$
  
and  $\left[5x^4 - 12x^2 + c\right]$ 

S. Fam in 
$$(D^4 + 3D^2)y = 108n^2$$
 $PI = \int_{0}^{1} (D^2 + 3D^2)y = 108n^2$ 
 $= \int_{0}^{1} (D^2 + 3D^2)y = 108n^2$ 



$$n = e^{2} = 2 = lnn$$

$$\frac{dy}{dn} = \frac{dy}{dz} \frac{dz}{dn}$$

$$\frac{dy}{dz} = \frac{dy}{dz} \left(\frac{1}{n}\right)$$

$$\frac{dy}{dz} = \frac{dy}{dz}$$

$$\frac{dy}{dz} = \frac{dy}{dz}$$

$$\frac{dy}{dz} = \frac{dy}{dz}$$

$$\frac{dy}{dz} = \frac{dy}{dz}$$

$$\frac{dy}{dz} = \frac{dz}{dz}$$

$$\frac{dy}{dz} = \frac{dz}{dz}$$

$$\frac{dz}{dz}$$

$$P(D_1^2)^2 = D_1(D_1-1)$$

Proof; No Need

General solution of the Cauchy-Euler equation :

$$x^2 \frac{d^2y}{dx^2} - 7x \frac{dy}{dx} + 16y = 0$$
 is Gell 80 (F+PI)

(a) 
$$y = C_1 x^2 + C_2 x^4$$

(b) 
$$y = C_1 x^2 + C_2 x^{-4}$$

(c) 
$$y = (C_1 + C_2 \ln x)x^4$$

(d) 
$$y = C_1 x^4 + C_2 x^{-4} \ln x$$

AEQ g(2),  $m^2 Bm+16=0=)m=4,4$   $Cf=(1+C_2Z)e^{4Z}$ PL=0

Girly D Eg is ( n 1 - 7 n D + 16 ) 4 = 0 Using Cauchy's Teams fermations, D, (D,-1)-7 D, +16) J= 0 (37-8D/+16)y=0 f (with Constr 6.10100 by=CF+PI 9=(1+GZ)e42 y=(c1+G/n)(n)

The solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x \text{ is}$$

(a) 
$$y = (C_1 + C_2 x) \log x + 2 \log x + 3$$

(b) 
$$y = (C_1 + C_2 x^2) \log x + \log x + 2$$

$$(C)_{x}y = (C_{1} + C_{2}x)\log x + \log x + 2$$

(d) 
$$y = (C_1 + C_2 \log x)x + \log x$$

it 8115 y= (4+ (2/yn)n+ 2/yn)2

 $(n^2)^2 n D + 1) y = \mu n - t$  $(D_1(D_1-1)-D_1+1)y=e^{-1}$ (n-2n,+1) y=2-0) AEin m-2m+1=0=)m=1,1 G=(G+G2)e=(C+C2yn).n  $PI = f(\eta_1)(e^2) = \frac{1}{(\eta_1 - 1)^2}(e^2)$  $=\frac{2}{2}e^{2}=\frac{1}{2}n(4n)^{2}$ 

