GATE

DATA SCIENCE + CS & IT

Engineering Mathematics

SUPER 1500

Lec : 04 Linear - 1

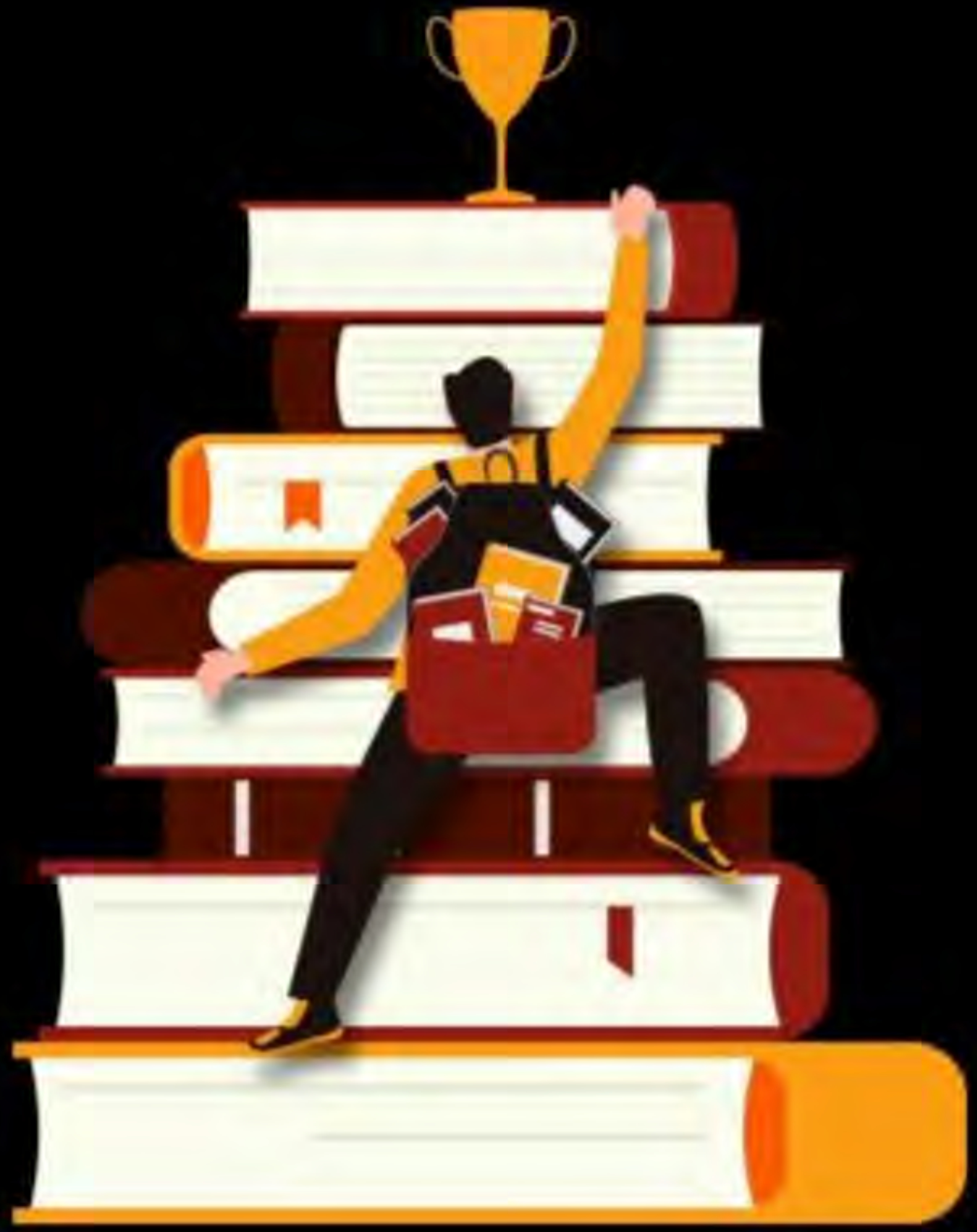
By - Dr. Puneet Sharma Sir

Topics to be covered

LINEAR ALGEBRA

1. Remaining Questions of E Values
2. E-Vectors
3. Cayley Hamilton Th
4. Diagonalisation
5. L-U Decomposition

Tomorrow No Class (12th Dec)
13th & 14th → only for DA students.



The eigen values of $A = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ are ______.

(a) Purely imaginary (b) Zero

(c) Real (d) None of the above

Hermitian Mat $\rightarrow$ E Values are Purely Real

If A is square symmetric real values matrix of dimensions 2n, then the eigen values of A are

(a) 2n distinct real values

(b) 2n real values not necessarily distinct

(c) n distinct paris of complex conjugate numbers

(d) n pairs of complex conjugate numbers, not necessarily distinct

given $A_{2n \times 2n}$ = Real Symmetric

$\Downarrow$

2n Real E Values exist

$\Downarrow$

Whether Repeated or Distinct

Let A be a 4 × 4 matrix with real entries such that –1, 1, 2, –2 are its Eigen values. If $B = A^4 - 5A^2 + 5I$ then trace of A + B is ______.

E Values of A are $1, -1, 2, -2$

$B = A^4 - 5A^2 + 5I$

$= (1)^4 - 5(1)^2 + 5(1) = 1$

$= (-1)^4 - 5(-1)^2 + 5(1) = 1$

$= (2)^4 - 5(2)^2 + 5(1) = 1$

$= (-2)^4 - 5(-2)^2 + 5(1) = 1$

$\text{Trace}(A+B) = ? = \text{Tr}(A) + \text{Tr}(B)$

$= [1 + (-1) + 2 + (-2)] + [1+1+1+1] = 4$

Let A be a 3 × 3 matrix with Eigen values –1, 1, 0. Then $|A^{100} + I|$ is ______.

$A_{3\times3}$ & it's E Values are $-1, 1, 0$

Let $B = (A^{100} + I)$

$= (-1)^{100} + 1 = 2$

$= (1)^{100} + 1 = 2$

$= (0)^{100} + 1 = 1$

then $|A^{100} + I| = ? =$ Product of E Values

$= (2)(2)(1) = 4$ Ans

The product of Eigen values of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & \frac{1}{2} & 0 & 0 & \dots & 0 \\ 1 & \frac{1}{2} & \frac{1}{3} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \dots & \frac{1}{n} \end{bmatrix} = L.T.M$$

is ____

(a) $n^2 + n + 1$

(b) $\frac{n(n+1)}{2}$

(c) $\frac{1}{n!}$

(d) $\frac{1}{n}$

Product of E Values $= |A|$

$$= 1 \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \dots \times \frac{1}{n}$$

$$= \frac{1}{1 \times 2 \times 3 \times 4 \times \dots \times n} = \frac{1}{n!}$$

ie (c)

The sum of Eigen values of the matrix

$$A = \begin{bmatrix} \frac{1}{1.2} & 0 & 0 & 0 & \cdots & 0 \\ \frac{1}{1.2} & \frac{1}{2.3} & 0 & 0 & \cdots & 0 \\ \frac{1}{1.2} & \frac{1}{2.3} & \frac{1}{3.4} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ \frac{1}{1.2} & \frac{1}{2.3} & \frac{1}{3.4} & \frac{1}{4.5} & \cdots & \frac{1}{n(n+1)} \end{bmatrix} = \text{L.T.M}$$

is ____

(a) $\frac{1}{n}$

(b) $1-\frac{1}{n+1}$

(c) $\frac{1}{n!}$

(d) $\frac{2}{n(n+1)}$

Sum of E Values $= Tr(A)$

$$= \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)}$$

$$= \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cancel{/} \cdots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1}$$

For the matrix A satisfying the equation given below, the eigen values are

$$[A]\begin{bmatrix}1&2&3\\7&8&9\\4&5&6\end{bmatrix}=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix} \quad \text{——— (1)}$$

$B=$ $\qquad$ $C=$

(a) (1, –j, j) $\qquad$ (b) (1, 1, 0) ✗

(c) (1, 1, –1) ✓ $\qquad$ (d) (1, 0, 0)

$|B| = |C| = 0$

In equⁿ (1) let $A = I_{23}$

let us cross check it i.e

$$\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&2&3\\7&8&9\\4&5&6\end{bmatrix}=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$$ Hence verified

So our assumption is correct, we can take $A = I_{23}$

E-Matrix → Mat obtained by applying single E-operation on Identity Mat is called E-Mat.

for eg; $I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix} = I_{23}$

i.e $A=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$

So it's E Values are

$|A-\lambda I| = 0$ - - - - $\lambda = 1, -1, 1$

Q: The product of Non Zero E.Values of $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{5\times 5}$ is ______

Sol: $A \xrightarrow[\substack{R_4 \to R_4 - R_2 \\ R_5 \to R_5 - R_1}]{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ i.e $\rho(A) = 2$

$\because$ No. of Non Zero E Values $\leq \rho(A) \Rightarrow$ i.e from 5 E Values, three E Values are 0, 0, 0

C.Equ^n is $|A - \lambda I| = 0$

$$\begin{vmatrix} (1-\lambda) & 0 & 0 & 0 & 1 \\ 0 & (1-\lambda) & 1 & 1 & 0 \\ 0 & 1 & (1-\lambda) & 1 & 0 \\ 0 & 1 & 1 & (1-\lambda) & 0 \\ 1 & 0 & 0 & 0 & (1-\lambda) \end{vmatrix} = 0 \xrightarrow[C_1 \to C_1 + C_5]{C_2 \to C_2 + C_3 + C_4} \begin{vmatrix} (2-\lambda) & 0 & 0 & 0 & 1 \\ 0 & (3-\lambda) & 1 & 1 & 0 \\ 0 & (3-\lambda) & (1-\lambda) & 1 & 0 \\ 0 & (3-\lambda) & 1 & (1-\lambda) & 0 \\ (2-\lambda) & 0 & 0 & 0 & (1-\lambda) \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda)\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & - & - & - \\ 0 & 1 & - & - & - \\ 0 & 1 & - & - & - \\ 1 & 0 & 0 & 0 & (1-\lambda) \end{vmatrix} = 0$$

$$\Rightarrow \lambda = 2, 3 \quad \& \quad \lambda = 0, 0, 0$$

So Req Product $= 2 \times 3 = 6$

Cross check :-

E Values of A are $\lambda = 2, 3, 0, 0, 0$

So Product of E Values $= 2 \times 3 \times 0 \times 0 \times 0$

$= 0$

$= |A|$

CAYLEY-HAMILTON THEOREM → Every Sq Mat satisfies it's CEq

i.e. $\lambda \to A$ in CEq.

for eq $A_{n\times n}$ then it's CEq is $|A-\lambda I|=0$

or $\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots\dots + a_{n-1}\lambda + a_n = 0$

① $Tr(A) = -a_1$ ② $\boxed{|A| = (-1)^n a_n}$, ③ $A^{-1} = ?$

④ Constant term $(a_n) = (-1)^n |A|$, ⑤ shortcut method of CEq for $A_{2\times 2}$

is $\lambda^2 - (Tr\,A)\lambda + (|A|) = 0$

If the constant term in the characteristic polynomial of a square matrix is other than zero Then the matrix is

(a) Necessarily singular

(b) Always non-singular

(c) Can't not say

(d) Data insufficient

$\because a_n \neq 0$

$\Rightarrow (-1)^n |A| = 0$

$\Rightarrow |A| \neq 0 \Rightarrow$ A is Non Sing.

If the characteristic equation of a matrix $x^3 + ax^2 + bx + c = 0$, then $|c|$ is equal to the absolute value of ____.

(a) one of the characteristic roots

(b) determinant of the matrix

(c) sum of all its characteristic roots

(d) sum of all the entriesin the matrix

entries in

$x^3 + ax^2 + bx + c = 0$. (C Eq^n)

Here $A_{3\times3}$ 100

Constant term $= (-1)^3 |A|$

$c = (-1)|A|$

$|c| = |(-1)|A|| = |A|$

PYQ

In matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ if $\underbrace{a + d}_{TrA}$ = $\underbrace{ad - bc}_{|A|}$ = 1, then A^3 = ______.

(a) A – I (b) A + I

(c) –I (d) 0

C. Equ^n $\boxed{\lambda^2 - (Tr)\lambda + (|A|) = 0}$

$\lambda^2 - \lambda + 1 = 0$

$\Rightarrow A^2 - A + I = 0$

$\boxed{A^2 = A - I}$ —— (1)

$A^3 = A^2 \cdot A$

$= (A - I)(A)$

$= A^2 - AI$

$= (A - I) - A$

$= -I$

The eigen values of square matrix A ~~re~~ are 1 and 3 then A^3 is _____.

(a) 12A – 13I (b) 13A – 12I

(c) 12A – 12I (d) 13A – 13I

$\because$ E Values of A are $\lambda = 1$ & $\lambda = 3$

so Consider $A_{2\times 2}$ & it's C Eq is

$$(\lambda-1)(\lambda-3)=0$$

$$\lambda^2-4\lambda+3=0$$

By $\lambda \to A$, $A^2-4A+3I=0$

$$\boxed{A^2=4A-3I} \text{ — (1)}$$

$$A^3 = A^2 \cdot A$$

$$=(4A-3I)A$$

$$= \ldots$$

$$= (b)$$

The eigen values of $P_{3\times3}$ are 1, –2, 3. Then P^{-1} = ______.

(a) $\frac{1}{6}[5I + 2P + P^2]$ (b) $\frac{1}{6}[5I + 2P - P^2]$

(c) $\frac{1}{6}[5I - 2P + P^2]$ (d) $\frac{1}{6}[5I - 2P - P^2]$

$$\boxed{\lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0}$$

$$(\lambda-1)(\lambda+2)(\lambda-3) = 0$$

C Eqn is $\boxed{(\lambda-1)(\lambda+2)(\lambda-3) = 0}$

$\lambda \to P$; $\boxed{P^3 - 2P^2 - 5P + 6I = 0}$

$$P^2 - 2P - 5I + 6P^{-1} = P^{-1} \cdot 0$$

$$P^{-1} = \frac{1}{6}\left(2P + 5I - P^2\right)$$

$$x^3 - 5x^2 + 7x - 3 = 0 \Rightarrow (x-1)^2(x-3) = 0$$

$$x^3 - 6x^2 + 11x - 6 = 0 \Rightarrow (x-1)(x-2)(x-3) = 0$$

EIGEN VECTORS

$$\boxed{AX = \lambda X} \Rightarrow (A - \lambda I)X = 0$$

① $X \longrightarrow$ Non Zero, ② $X \longrightarrow (kX)$

③ If (λ, X) is an E pair of A
then (λ^m, X) " " " of A^m } i.e. A & A^m has same E vector

④ For Repeated E Value 'λ'; $\boxed{GM \text{ of } \lambda = \text{No of Columns} - \rho(A - \lambda I)}$

(No. of L.I E Vectors for Repeated E Value λ)

For the matrix $A = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ = U.T.M one of the eigen values is equal to -2 which of the following is an eigen vector?

(a) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

✓ (d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

Let us take (d);

$$AX = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ -10 \\ 0 \end{bmatrix} = -2 \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} = -2X$$

Let us take (a)

$$AX = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ - \\ - \end{bmatrix} = 5 \begin{bmatrix} 3 \\ - \\ - \end{bmatrix}$$ ✗

The vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector of

$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then corresponding eigen value of A is

(a) 1 (b) 2

(c) 5 (d) –1

$$AX = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ -5 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$= 5X$$

MSQ

The pair of eigen vectors corresponding to the two eigen values of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is

2013

(a) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}$

EV of A are $\lambda^2-(0)\lambda+(1)=0$

$\Rightarrow \lambda = \pm i$

Similarly check (d)

$AX_1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} -i \\ 1 \end{bmatrix} = -i\begin{bmatrix} 1 \\ 1/-i \end{bmatrix} = -i\begin{bmatrix} 1 \\ i \end{bmatrix} = -iX_1$

$AX_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ i \end{bmatrix} = i\begin{bmatrix} -1/i \\ 1 \end{bmatrix} = i\begin{bmatrix} i \\ 1 \end{bmatrix} = iX_2$

ie (d) is also correct

let us take (a);

$AX_1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} i \\ 1 \end{bmatrix} = i\begin{bmatrix} 1 \\ \frac{1}{i} \end{bmatrix} = i\begin{bmatrix} 1 \\ \frac{i}{i^2} \end{bmatrix} = i\begin{bmatrix} 1 \\ -i \end{bmatrix} = iX_1$

$AX_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} = -i\begin{bmatrix} 1/-i \\ i/-i \end{bmatrix} = -i\begin{bmatrix} i^2/i \\ -1 \end{bmatrix} = -i\begin{bmatrix} i \\ -1 \end{bmatrix} = -iX_2$ ie (a) is correct

M-II (Conventional App) →

$$A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \Rightarrow \lambda^2+1=0 \Rightarrow \lambda=\pm i$$

E vector for $(\lambda=i)$: → $AX=\lambda X$

$$(A-\lambda I)X=0$$

$$(A-iI)X=0$$

$$\begin{bmatrix}-i & -1\\ 1 & -i\end{bmatrix}X=0$$

$$\xrightarrow{R_2\to R_2+\frac{1}{i}R_1}\begin{bmatrix}-i & -1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$-ix_1-x_2=0 \Rightarrow x_1=\frac{-1}{i}x_2=ix_2$$

Let $x_2=k$ then $x_1=ik$

$$X=\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}k\\ ik\end{bmatrix}=k\begin{bmatrix}1\\ i\end{bmatrix}$$

Similarly for $\lambda=-i$, $X=k\begin{bmatrix}1\\ -i\end{bmatrix}$

i.e. for A:

$\lambda=i$, $X=\begin{bmatrix}1\\ i\end{bmatrix}$ →

$\lambda=-i$, $X=\begin{bmatrix}1\\ -i\end{bmatrix} \longrightarrow \begin{bmatrix}i\\ 1\end{bmatrix}$

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\lambda = i, \quad x_1 = \begin{bmatrix} 1 \\ i \end{bmatrix} \longrightarrow \begin{bmatrix} i \\ -1 \end{bmatrix}$$

$$\lambda = -i, \quad x_2 = \begin{bmatrix} 1 \\ -i \end{bmatrix} \longrightarrow \begin{bmatrix} i \\ 1 \end{bmatrix}$$

one pair of LI E vectors are $\begin{bmatrix} 1 \\ i \end{bmatrix}$ & $\begin{bmatrix} i \\ 1 \end{bmatrix} \longrightarrow$ (d)

another pair of " " are $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ & $\begin{bmatrix} i \\ -1 \end{bmatrix} \longrightarrow$ (a)

HWB

Let $P = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ a & 2 & b \end{bmatrix}$ for some $a, b \in R$, suppose

1 and 2 are eigen values of P and $P\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$

then $P^4\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ is _____.

(a) $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 16 \\ 16 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 16 \\ -16 \\ 0 \end{bmatrix}$

CSIR

DIAGONALISATION

(1) $$P^{-1}AP = D \Longrightarrow \boxed{A = PDP^{-1}}$$

$$P = [x_1\ x_2\ x_2],\quad D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

(2) N. Cond^n for Diagonalisation:

$$\boxed{\text{No of LI EVectors of } A = \text{order of } A}$$

The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is $\Rightarrow$ U.T.M so $\lambda = 2, 2$

(a) 0

(b) 1

(c) 2

(d) Infinite

$$GM \text{ of } (\lambda = 2) = \text{No of } C - \rho(A - 2I)$$
$$= 2 - \rho\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$= 2 - 1$$
$$= 1$$

The number of linearly independent eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ is

(a) 1 (b) 2

(c) 3 (d) 4

$|A| = |A_1| \cdot |A_2|$

$= (-2)(12)$

$= -24$

$\Rightarrow$ A has different E Values

$|A| \neq 0 \Rightarrow$ A has Non Zero E Values

(whether distinct or Repeated)

C.Eqn is $|A - \lambda I| = 0$

time Consuming .... $\lambda = 3, 4, \frac{3 \pm \sqrt{17}}{2}$

ie A has all 4 different E Values

$\Rightarrow$ A has $\underline{4}$ L I E Vectors.

The number of linearly independent eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is ______.

(a) 1 (b) 2

(c) 3 (d) None of the above

$\because$ A is UTM so $\lambda = 1, 1, 1$

$GM \text{ of } (\lambda = 1) = \text{No of } C - \rho(A - 1.I)$

$= 3 - \rho \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$= 3 - 2 = 1$

For a given 2 × 2 matrix A, it is observed that

$A\begin{bmatrix}1\\-1\end{bmatrix}=-1\begin{bmatrix}1\\-1\end{bmatrix}$ and $A\begin{bmatrix}1\\-2\end{bmatrix}=-2\begin{bmatrix}1\\-2\end{bmatrix}$ then the matrix A is

(a) $A=\begin{bmatrix}2&1\\-1&-1\end{bmatrix}\begin{bmatrix}-1&0\\0&-2\end{bmatrix}\begin{bmatrix}1&1\\-1&-2\end{bmatrix}$ ✗

(b) $A=\begin{bmatrix}1&1\\-1&-2\end{bmatrix}\begin{bmatrix}1&0\\1&2\end{bmatrix}\begin{bmatrix}2&1\\-1&-1\end{bmatrix}$ ✗

(c) $A=\begin{bmatrix}1&1\\-1&-2\end{bmatrix}\begin{bmatrix}-1&0\\0&-2\end{bmatrix}\begin{bmatrix}2&1\\-1&-1\end{bmatrix}$ ✓

(d) $A=\begin{bmatrix}1&-2\\1&-3\end{bmatrix}$

$D=\begin{bmatrix}-1&0\\0&-2\end{bmatrix}, P=[x_1\ x_2]=\begin{bmatrix}1&1\\-1&-2\end{bmatrix}$

$P^{-1}=\frac{1}{(-1)}\begin{bmatrix}-2&-1\\1&1\end{bmatrix}=\begin{bmatrix}2&1\\-1&-1\end{bmatrix}$

$PAP^{-1}=D \Rightarrow A=PDP^{-1}=$ (c)

A matrix M has eigen values 1 and 4 with corresponding eigen vector

$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; ; $x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Respectively then M is

(a) $\begin{bmatrix} -4 & -8 \\ 5 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 9 & -8 \\ 5 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

$A = P\,D\,P^{-1}$

$= \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} P^{-1} \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \frac{1}{3} \begin{bmatrix} 9 & 6 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

L-U Decomposition →

Dolittle Method : $A = (\text{unit L.T.M}) \cdot (\text{U.T.M})$

Crout Method $A = (\text{L.T.M})(\text{unit U.T.M})$

Cholesky Method $A = LL^{\theta}$

The matrix $[A] = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of a lower triangular matrix x[L] and an upper triangular matrix [U]. The properly decomposed [L] and [U] matrices respectively are

(a) $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$ $\neq A$

(b) $\begin{bmatrix} 2 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\neq A$

(c) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 8 & 5 \end{bmatrix} \neq A$

(d) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$

In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is _____.

Crout Method: $A = LTM \times \text{unit } U.T.M$

$$A = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} \\ l_{21} & l_{21}u_{12} + l_{22} \end{bmatrix} \dashrightarrow l_{22} = 5$$

Thursday (12th Dec 2024) → No class.

Thank
THANK

Keep Hustling!