GATE

CRASH COURSE

ALL BRANCHES

Engineering Mathematics

Calculus (Part 03)
(Lec 06)

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Topics *to be covered*

1. Derivatives
2. Integration

Some Standard Results :-

(1) $\boxed{\frac{d}{dx}(a^x) = a^x \log_e a}$

eg $\frac{d}{dx}(e^x) = e^x$

(2) Power Formula: $\boxed{\frac{d}{dx}(x^a) = a x^{a-1}}$

$\frac{d}{dx}(K) = 0$, $\frac{d}{dx}(x) = 1$, $\frac{d}{dx}(x^2) = 2x$

$\frac{d}{dx}(x^3) = 3x^2$, $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$, $\frac{d}{dx}|x| = \frac{x}{|x|} = \frac{|x|}{x}$

(3) $\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}}$ eg $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

(4) $\frac{d}{dx}(\sin x) = \cos x$, (6) $\frac{d}{dx}(\tan x) = \sec^2 x$

(5) $\frac{d}{dx}(\cos x) = -\sin x$, (7) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(8) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(9) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(10) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$

(11) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}, \quad -1 < x < 1$

(12) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \quad -\infty < x < \infty$

(13) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}, \quad -\infty < x < \infty$

(14) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}, \quad |x| > 1$

(15) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}, \quad |x| > 1$

(16) $\frac{d}{dx}|x| = \frac{x}{|x|} = \frac{|x|}{x}, \quad x \neq 0$

(17) Product formula $\rightarrow$

(i) $\frac{d}{dx}(fg) = fg' + gf'$

(ii) $\frac{d}{dx}(fgh) = fgh' + fg'h + f'gh$

(18) Quotient formula $\rightarrow$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{(g)^2}$$

(19) $\frac{d}{dx}(\text{const}) = 0$

(20) $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}(f(x))$

(21) $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$

Q. $y = e^x \log\sqrt{x}$ then $\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\underset{\mathrm{I}}{e^x} \cdot \underset{\mathrm{II}}{\log\sqrt{x}}\right]$$

$$= e^x\left(\frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\right) + \log\sqrt{x}\,(e^x)$$

$$= \frac{e^x}{2x} + \frac{1}{2} e^x \log x$$

$$\frac{d}{dx}|x| = \frac{d}{dx}\left(\sqrt{x^2}\right) = \frac{1}{2\sqrt{x^2}}(2x)$$

$$= \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$$

$$= \frac{x}{|x|}\,\frac{|x|}{|x|} = \frac{x|x|}{|x|^2}$$

$$= \frac{x|x|}{x^2} = \frac{|x|}{x},\ x \neq 0$$

Q. $y=(1+x)(1+x^2)(1+x^4)(1+x^8)$ then $\frac{dy}{dx}=?$

Sol: $y=\frac{(1+x)(1-x)(1+x^2)(1+x^4)(1+x^8)}{(1-x)}$

$=\frac{(1-x^2)(1+x^2)(1+x^4)(1+x^8)}{1-x}$

$=\frac{(1-x^4)(1+x^4)(1+x^8)}{1-x}$

$=\frac{(1-x^8)(1+x^8)}{1-x}=\frac{1-x^{16}}{1-x}$

$y=\frac{1-x^{16}}{1-x}$

$\frac{dy}{dx}=\frac{(1-x)\{-16x^{15}\}-(1-x^{16})\{-1\}}{(1-x)^2}$

$=$ Ans

Q. GATE: If $x\sqrt{1+y}+y\sqrt{1+x} = 0$ — (1); $x \neq 0$

then $\frac{dy}{dx} = ?$

(a) $\frac{1}{(1+x)^2}$ (b) $\frac{-1}{(1+x)^2}$ ✓

(c) $\frac{-x}{1+x}$ (d) 1

Sol:
$$x\sqrt{1+y} = -y\sqrt{1+x}$$
$$x^2(1+y) = y^2(1+x)$$
$$x^2 + x^2y - y^2 - y^2x = 0$$

$$(x-y)(x+y) + xy(x-y) = 0$$
$$(x-y)(x+y+xy) = 0$$

$x = y$ or $x+y+xy = 0$

Not possible & $y = \frac{-x}{1+x}$

$\because$ By (1)

$2x\sqrt{1+x} = 0$

$x = 0$ ✗

$$\frac{dy}{dx} = -\left[\frac{(1+x)(1) - x(1)}{(1+x)^2}\right]$$
$$= \frac{-1}{(x+1)^2}$$

Partial Derivative

$z = f(x, y)$

$\dfrac{\partial z}{\partial x} = ?$ [keeping y const.]

$\dfrac{\partial z}{\partial y} = ?$ [" x "]

for $y = f(x)$

$$f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

& for $z = f(x, y)$

$$\frac{\partial z}{\partial x} = \lim_{h \to 0} \left[\frac{f(x+h, y) - f(x, y)}{h} \right]_{y = \text{const.}}$$

$$\frac{\partial z}{\partial y} = \lim_{k \to 0} \left[\frac{f(x, y+k) - f(x, y)}{k} \right]_{x = \text{const.}}$$

$$\boxed{z = e^{ax+by} \cdot f(ax-by)}$$

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = ?$$

If $Z = e^{ax+by} F(ax - by)$; the value of $b \cdot \frac{\partial Z}{\partial x} + a\frac{\partial Z}{\partial y}$ is

(a) 2Z

(b) 2a

(c) 2b

(d) 2abZ ✓

$$\frac{\partial z}{\partial x} = e^{ax+by}\left[f'(ax-by) \cdot a\right] + f(ax-by)\left[e^{ax+by} \cdot (a)\right]$$

$$\frac{\partial z}{\partial y} = e^{ax+by}\left[f'(ax-by)(-b)\right] + f(ax-by)\left[e^{ax+by}(b)\right]$$

So $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abZ$

If $u = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$ then $u_x + u_y + u_z =$ ______.

$$u_x = \begin{vmatrix} 2x & y^2 & z^2 \\ 1 & y & z \\ 0 & 1 & 1 \end{vmatrix} + \begin{vmatrix} x^2 & 0 & z^2 \\ x & 0 & z \\ 1 & 0 & 1 \end{vmatrix} + \begin{vmatrix} x^2 & y^2 & 0 \\ x & y & 0 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 2x & y^2 & z^2 \\ 1 & y & z \\ 0 & 1 & 1 \end{vmatrix}$$

$$u_y = 0 + \begin{vmatrix} x^2 & 2y & z^2 \\ x & 1 & z \\ 1 & 0 & 1 \end{vmatrix} + 0 \quad \& \quad u_z = 0 + 0 + \begin{vmatrix} x^2 & y^2 & 2z \\ x & y & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$u_x + u_y + u_z = \left| \quad \right| + \left| \quad \right| + \left| \quad \right| = \cdots = 0$$

Euler Theorem for Partial Derivative →

Let $u = u(x, y)$ is $\underbrace{\text{Homogeneous func}^n}$ of degree n then

① $\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu}$ ② $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$

Note: u is called Homog func^n of degree n if $\boxed{u(\lambda x, \lambda y) = \lambda^n u(x, y)}$

Here n = Real No. (−ve, +ve, 0, Fraction)

eg: $u = x^7 + 3x^2y^5 + 2x^1y^6 + x^3y^4$

Here u is Homog func^n of degree $n = 7$

$\therefore u(\lambda x, \lambda y) = \lambda^7 u(x, y)$

Q. if $V = \dfrac{x+2y+3z}{x^8+y^8+z^8}$

then $x\dfrac{\partial V}{\partial x} + y\dfrac{\partial V}{\partial y} + z\dfrac{\partial V}{\partial z} = ?$

$V = V(x, y, z)$

& $V(\lambda x, \lambda y, \lambda z) = \dfrac{\lambda}{\lambda^8}\left[\dfrac{x+2y+3z}{x^8+y^8+z^8}\right] = \lambda^{-7} V(x, y, z)$

So V is Homog func^n of $n=-7$ so By E Th for v

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z} = nV = -7V$$

If $\sin u = \dfrac{x+2y+3z}{x^8+y^8+z^8}$ then

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} =$

(a) $\dfrac{1}{7}\tan u$ (b) $-7\tan u$

(c) $\dfrac{1}{7}\sec u$ (d) $-\dfrac{1}{7}\tan u$

Let $\sin u = v = \dfrac{x+2y+3z}{x^8+y^8+z^8}$

Hence

$$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) + z\frac{\partial}{\partial z}(\sin u) = -7\sin u$$

$$x(\cos u)\cdot\frac{\partial u}{\partial x} + y\cdot(\cos u)\frac{\partial u}{\partial y} + z(\cos u)\cdot\frac{\partial u}{\partial z} = -7\sin u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -7\tan u$$

PYQ

If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$, what is the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$?

(a) 0 (b) 1

(c) u (d) e^u

$$e^u = \left(\frac{x^2+y^2}{x+y}\right)$$

Let $v = e^u = \frac{x^2+y^2}{x+y}$ so v is Homo. fun^n of degree 1

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{v}{v'} = 1 \cdot \frac{e^u}{e^u} = 1$$

Chain Rule of Partial Derivatives $\rightarrow$ Let $z = f(x,y)$ & $x = x(r,s)$, $y = y(r,s)$

i.e. $z \longrightarrow (x,y) \longrightarrow (r,s)$

then $z_r = \dfrac{\partial z}{\partial r} = \dfrac{\partial z}{\partial x}\left(\dfrac{\partial x}{\partial r}\right) + \dfrac{\partial z}{\partial y}\left(\dfrac{\partial y}{\partial r}\right)$

$$z_s = \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial s}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial s}\right)$$

$z = f(x, y)$

$x = e^{u} + e^{-v}$

$y = e^{-u} - e^{+v}$

If $z = f(x, y)$ where $x = e^{u} + e^{-v}$, $y = e^{-u} - e^{v}$ then $z_u - z_v =$

(a) $xz_x - yz_y$ (b) $xz_x + yz_y$

(c) $xz_y + yz_x$ (d) $xz_y - yz_x$

$\because z \longrightarrow (x, y) \longrightarrow (u, v)$

$$\boxed{\begin{aligned} z_u &= \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial u}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial u}\right) \\ z_v &= \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial v}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial y}{\partial v}\right) \end{aligned}}$$

$= z_x\left(e^{u}\right) + z_y\left(-e^{-u}\right)$

$= z_x\left(-e^{-v}\right) + z_y\left(-e^{v}\right)$

$z_u - z_v = x z_x + z_y\left(-e^{-u} + e^{v}\right) = x z_x - y z_y$

JACOBIAN: if $u = u(x,y)$ & $v = v(x,y)$ i.e. $(u,v) \to (x,y)$

then Derivative of (u,v) with Respect to (x,y) is called Jacobian

& it is defined as $J(u,v) = \dfrac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$

Note: if $(u,v,w) \to (x,y,z)$ then

$$J = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$$

O.D: $y = f(x) \Rightarrow \frac{dy}{dx}$ = class 12^{th}

P.D: $z = f(x,y)$ $\begin{cases} z_x = ? \\ z_y = ? \end{cases}$ B.Tech

Chain Rule of P.D

$z \to (x,y) \to (r,s)$ $\begin{cases} \frac{\partial z}{\partial r} = ? \\ \frac{\partial z}{\partial s} = ? \end{cases}$

Jacobian if $(u,v) \to (x,y)$ then $\frac{\partial(u,v)}{\partial(x,y)} = ?$

Q. if $u = x^2 - y^2$ & $v = 2xy$

then $\frac{\partial(u,v)}{\partial(x,y)} = ?$

$$J = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} 2x & -2y \\ 2y & 2x \end{vmatrix}$$

$$= 4x^2 + 4y^2 = 4(x^2+y^2).$$

Q. if $u = x^2 - y^2$, $v = 2xy$ then $\frac{\partial(u,v)}{\partial(r,\theta)} = ?$ ∵ $x = r\cos\theta$, $y = r\sin\theta$

(a) $4r^3$ ✓

(b) $4r^2$

(c) $2r^2$

(d) $2r^3$

$(u,v) \to (x,y) \to (r,\theta)$

$$\boxed{\frac{\partial(u,v)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(r,\theta)}}$$

$$= 4(x^2+y^2)(r)$$

$$= 4(r^2)(r)$$

$$= 4r^3$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix}$$

$$= \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

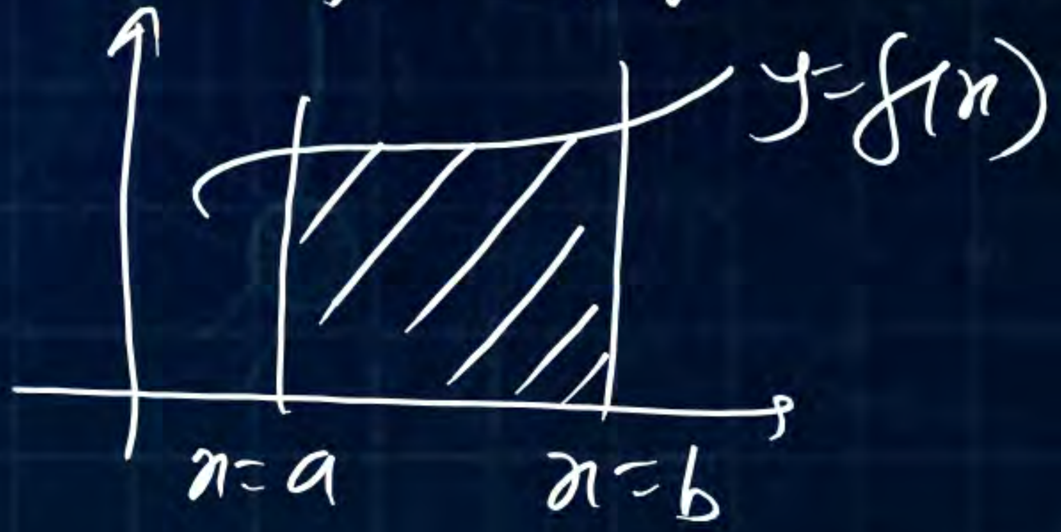
$$= r$$

where $x^2+y^2 = r^2$ & $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

INTEGRATION

- Indefinite Int → Collection of all the Antiderivatives is called Ind. Int.
- Definite Int → it represents the area under $f(x)$ b/w given limits

$$I = \int_a^b f(x)\,dx = \text{Shaded Area}$$



Methods of Solving Questions → (12^{th} Class) → There are four Methods →

(1) M-I → By using Standard Results.

(2) M-II → By using Substitution Method. (Imp)

(3) M-III → By using Integration By Parts

(4) M-IV → By using partial Fractions

Some Standard Results →

① (i) $\int x^a\,dx = \frac{x^{a+1}}{a+1} + c;\ a \neq -1$

① (ii) → $\int \left(\frac{1}{x}\right) dx = \log_e x + c$

② $\int a^x\,dx = \frac{a^x}{\log_e a} + c$

③ $\int e^x\,dx = e^x + c$

④ $\int \sin x\,dx = -\cos x + c$

⑤ $\int \cos x\,dx = \sin x + c$

⑥ $\int \tan x\,dx = \log \sec x + c$

⑦ $\int \cot x\,dx = \log \sin x + c$

⑧ $\int \operatorname{cosec} x\,dx = \log(\operatorname{cosec} x - \cot x) + c$

⑨ $\int \sec x\,dx = \log(\sec x + \tan x) + c$

⑩ $\int \sec^2 x\,dx = \tan x + c$

⑪ $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

⑫ $\int \sec x \tan x\,dx = \sec x + c$

⑬ $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

(14) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

(15) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+C$

(16) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+C$

(17) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left(x+\sqrt{x^2+a^2}\right)+C$

(18) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left(x+\sqrt{x^2-a^2}\right)+C$

(19) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C$

(20) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)+C$

(21) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)+C$

(22) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+C$

(23) $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right)+C$

(24) $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right)+C$

(25) Integration by parts $\rightarrow$ $\boxed{\int \underset{I}{u} \cdot \underset{II}{v}\, dx = u\int v\, dx - \int\left\{\frac{du}{dx}\cdot\int v\, dx\right\}dx + c}$

Log func^n

Trig func^n

I L A T E

Inverse Trig func^n

Algebraic func^n

Exp. func^n

the letter which is coming 1st in the word ILATE should be assumed as 1st func^n u

eg $I = \int \underset{u}{x^3}\, \underset{v}{e^x}\, dx = ?$ $= x^3(e^x) - \int (3x^2)(e^x)\, dx$

$= x^3e^x - 3\left[x^2(e^x) - \int (2x)(e^x)\, dx\right]$ = Do yourself.

Q. $\int \left(\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\right) dx = ?$

$= \int \left(\frac{x^5 - x^4}{x^3 - x^2}\right) dx$

$= \int \frac{x^4(x-1)}{x^2(x-1)} dx$

$= \int x^2 dx = \frac{x^3}{3} + C$

Q. $\int \frac{1}{\sin^2 x \cos^2 x} dx = ?$

$= \int \left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\right) dx$

$= \int (\sec^2 x + \operatorname{cosec}^2 x) dx$

$= \tan x - \cot x + C$

Questions Based on (M-II) →

Q. $\int x^2 \cdot \sin(x^3)\,dx = ?$

Put $x^3 = t$

$3x^2\,dx = dt \Rightarrow x^2\,dx = \frac{1}{3}dt$

$I = \int \sin(x^3)\cdot(x^2\,dx)$

$= \int \sin t \cdot \frac{1}{3}dt$

$= \frac{1}{3}(-\cos t) + C = -\frac{1}{3}\cos(x^3) + C$

Q. $I = \int \sec^3 x \cdot \tan x\,dx = ?$

$= \int \sec^2 x(\sec x \tan x)\,dx$

~~Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$~~

Let $\sec x = t \Rightarrow \underbrace{\sec x \tan x\,dx}_{} = dt$

$I = \int t^2\,dt = \frac{t^3}{3} = \frac{\sec^3 x}{3} + C$

(PQ) $I=\int \frac{x^2+x+1}{(x-1)^3}dx = ? = \int\left(\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}\right)dx$ = little bit lengthy.

(M-II) Put $x-1=t \Rightarrow x=(t+1)$

$dx=dt$

$$I=\int \frac{(t+1)^2+(t+1)+1}{t^3}dt$$

$$=\int \frac{t^2+1+2t+t+2}{t^3}dt$$

$$=\int \frac{t^2+3t+3}{t^3}dt$$

$$I=\int\left(\frac{1}{t}+\frac{3}{t^2}+\frac{3}{t^3}\right)dt$$

$$=\log t+3\left(\frac{t^{-2+1}}{-2+1}\right)+3\left(\frac{t^{-3+1}}{-3+1}\right)$$

$$=\log(x-1)-\frac{3}{(x-1)}-\frac{3}{2}\frac{1}{(x-1)^2}$$ Ans

Definite Integration →

(1) $$\int_a^b f(x)\,dx = \Big[\phi(x)\Big]_{x=a}^{b} = \boxed{\phi(b)-\phi(a)}$$

(2) $$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

(3) $$\int_{-a}^{a} f(x)\,dx = \int_0^a \left[f(x)+f(-x)\right]dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(-x)=f(x) \\ 0, & \text{if } f(-x)=-f(x) \end{cases}$$

(4) $$\int_0^{2a} f(x)\,dx = \int_0^a \left[f(x)+f(2a-x)\right]dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a-x)=f(x) \\ 0, & \text{if } f(2a-x)=-f(x) \end{cases}$$

Q. $I=\int_0^{\pi/4}(\tan^2 x)\,dx=?$

(a) $\pi/2$ (b) $1-\frac{\pi}{4}$

(c) $\frac{\pi}{4}-1$ (d) $\frac{\pi}{2}+1$

$$I=\int_0^{\pi/4}(\sec^2 x-1)\,dx$$

$$=\left(\tan x-x\right)_0^{\pi/4}$$

$$=\left(1-\frac{\pi}{4}\right)-(0-0)$$

$$=1-\pi/4$$

Q. $I=\int_0^{\pi/4}\sqrt{1+\sin 2x}\,dx=?$

Sol: $I=\int_0^{\pi/4}\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}\,dx$

$$=\int_0^{\pi/4}(\sin x+\cos x)\,dx$$

$$=\left(-\cos x+\sin x\right)_0^{\pi/4}$$

$$=\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-(-1+0)=1$$

Q: $I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx = ?$ —— (1)

$$= \int_0^{\pi} \frac{e^{\cos(\pi - x)}}{e^{\cos(\pi - x)} + e^{\cos(\pi - x)}} \cdot dx$$

W.K. that $\cos(\pi - x) = -\cos x$

By Prop (2)

$$= \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}} dx$$ —— (2)

Now (1) + (2) $\Rightarrow 2I = \int_0^{\pi} (1)\, dx = \pi \Rightarrow I = \frac{\pi}{2}$

Q. $I = \int_{-\pi/2}^{\pi/2} (\sin|x| + \cos|x|)\,dx = ?$

Even fn^n $\because f(-x) = f(x)$

$I = 2\int_0^{\pi/2} (\sin|x| + \cos|x|)\,dx$

$= 2\int_0^{\pi/2} (\sin x + \cos x)\,dx$

$= 2\left(-\cos x + \sin x\right)_0^{\pi/2}$

$= 2\left[(-0+1) - (-1+0)\right] = 4$

(18) $I = \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx = ?$

odd fn^n $\because f(-x) = -f(x)$

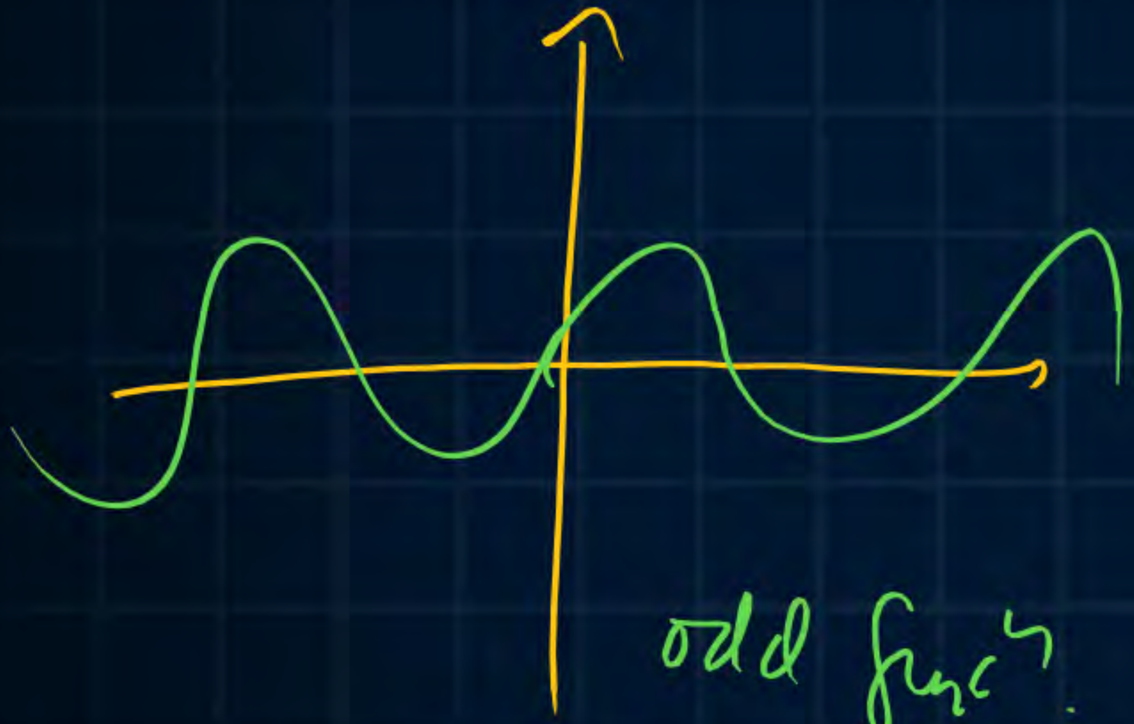
So By Prop (3), $I = 0$ Ans

$\because f(-x) = \log\left[\frac{2+x}{2-x}\right] = \log\left(\frac{2-x}{2+x}\right)^{-1}$

$= -\log\left(\frac{2-x}{2+x}\right) = -f(x)$

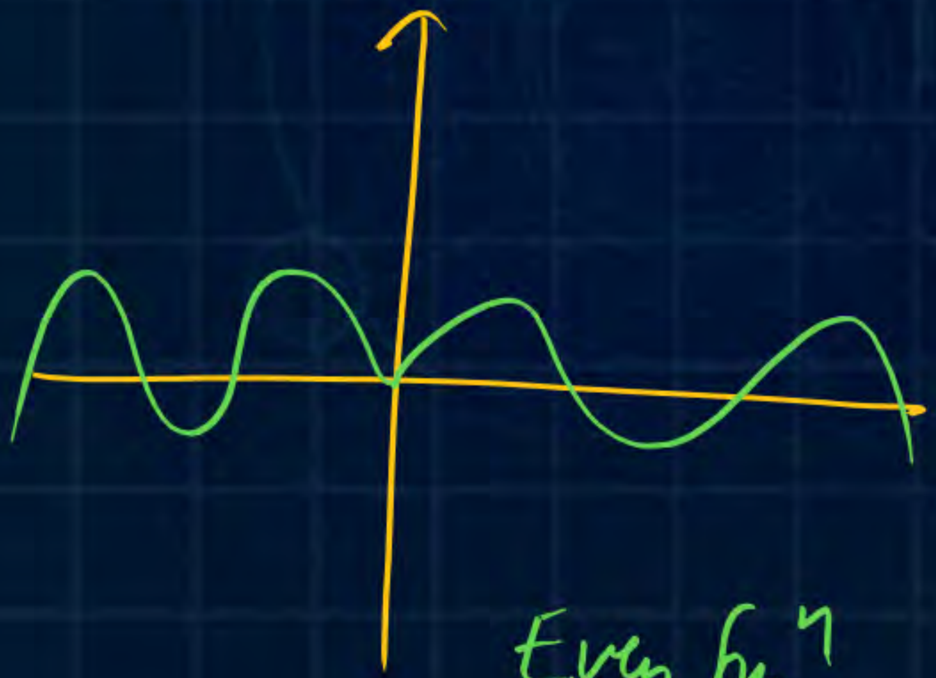
So $f(x)$ is an odd fun^n.

$y = \sin x$



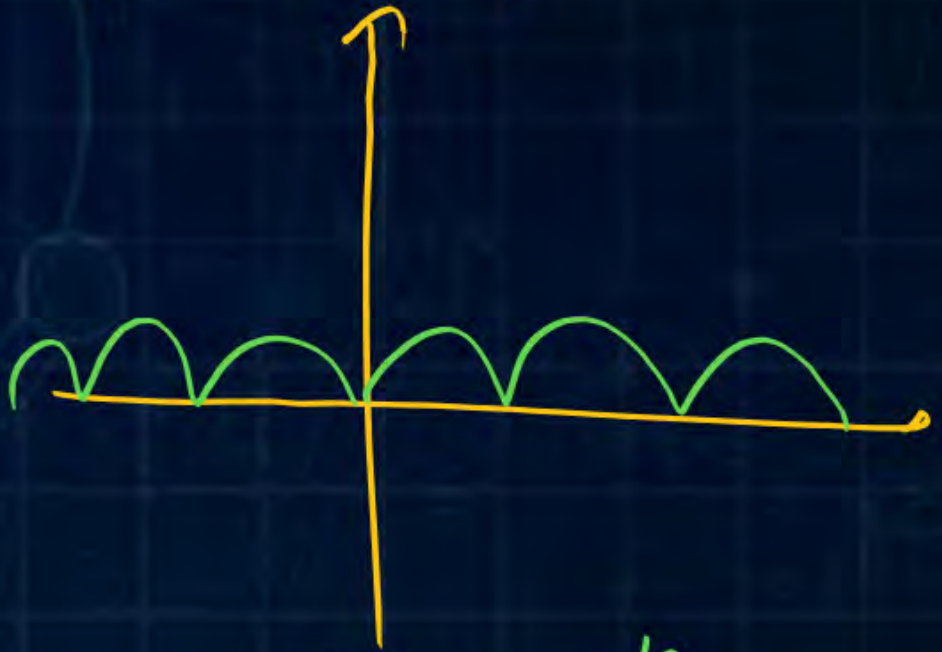
odd funcⁿ

$y = \sin|x|$



Even funⁿ

$y = |\sin x|$



Even fⁿ

If $f(n)=\int_0^{\pi/4}\tan^n x\,dx$ then $f(3)+f(1)=$ $\underline{0\cdot5}$.

$$f(3)+f(1)=\int_0^{\pi/4}\left(\tan^3 x+\tan x\right)dx$$

$$=\int_0^{\pi/4}\tan x\cdot\left(\tan^2 x+1\right)dx$$

$$I=\int_0^{\pi/4}\tan x\left(\sec^2 x\right)dx \quad (1)$$

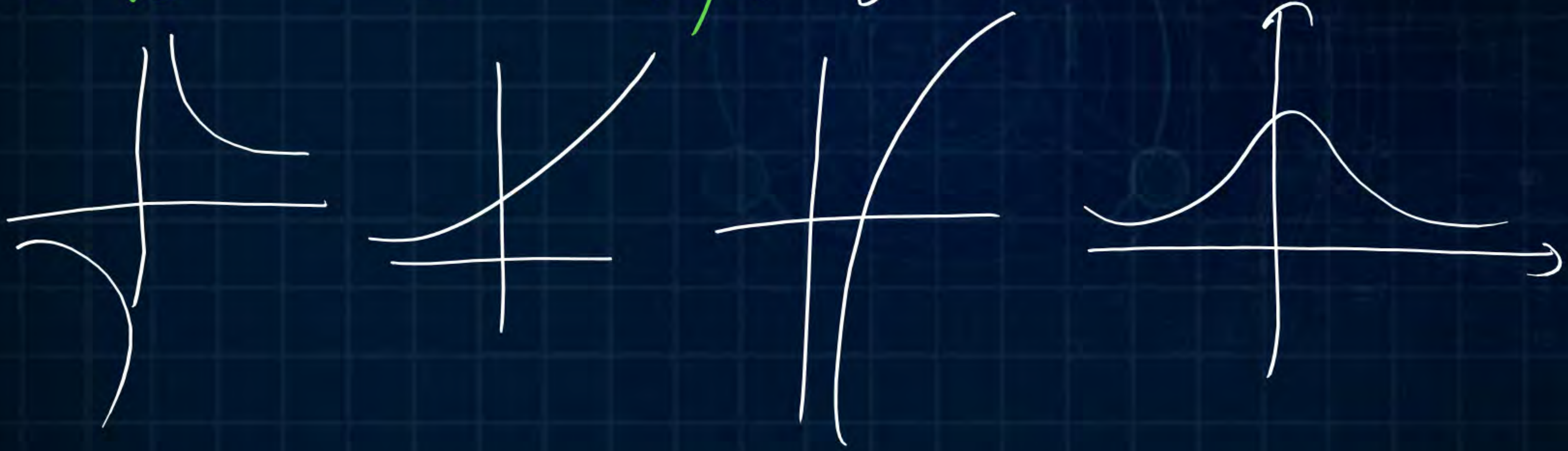
Put $\tan x=t$ → At $x=0,\ t=0$; At $x=\frac{\pi}{4},\ t=1$

$\sec^2 x\,dx=dt$

$$I=\int_0^1 t\,dt=\left(\frac{t^2}{2}\right)_0^1=\frac{1}{2}$$

Q: which of the following fnc^n is Bounded?

(a) $\frac{1}{x}$ (b) e^x (c) $\log x$ (d) e^{-x^2}



Which of the following integrals is unbounded?

(a) $\int_0^{\pi/4} \tan x dx$

(b) $\int_0^{\infty} \frac{1}{x^2+1} dx$

(c) $\int_0^{\infty} x e^{-x} dx$

(d) $\int_0^{1} \frac{1}{1-x} dx$

let us take (d),

$$I = \int_0^1 \left(\frac{1}{1-x}\right) dx = \left[\frac{\log(1-x)}{(-1)}\right]_0^1 = \left[-\log(1-x)\right]_0^1$$

$$= \left[\log(1-x)^{-1}\right]_0^1 = \left[\log\left(\frac{1}{1-x}\right)\right]_0^1 = \log\left(\frac{1}{0}\right) - \log\left(\frac{1}{1-0}\right)$$

$$= \log(\infty) - 0 = \infty$$

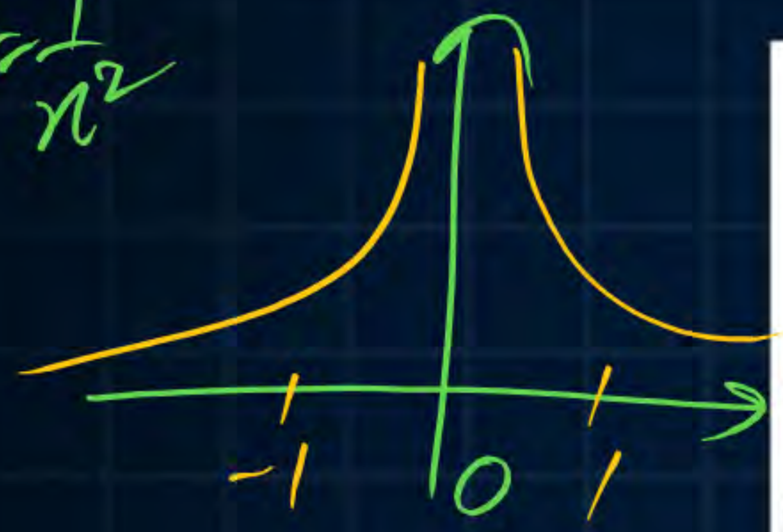
The integral $\int_0^1 \frac{1}{\sqrt{1-x^2}}dx$ converges to____.

$$= \left(\sin^{-1} x\right)_0^1 = \sin^{-1}(1) - \sin^{-1}(0)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{2}\right) - \sin^{-1}(\sin 0)$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2} \quad \underline{\text{Ans}}$$

$y = \frac{1}{x^2}$



The value of the integral $\int_{-1}^{1} \frac{1}{x^2} dx$ is

(a) 2 (b) does not exist

(c) –2 (d) ∞

$$I = \int_{-1}^{1} \left(\frac{1}{x^2}\right) dx = \int_{-1}^{0} \left(\frac{1}{x^2}\right) dx + \int_{0}^{1} \left(\frac{1}{x^2}\right) dx$$

$$= \left(-\frac{1}{x}\right)_{-1}^{0} + \left(-\frac{1}{x}\right)_{0}^{1}$$

$$= -\left(\frac{1}{0}+1\right) - \left(1 - \frac{1}{0}\right) = ?? = DNE$$

$\because$ At $x=0$, $f(x)$ is N.D

So we will split our integration at $\underline{0}$

Thank

THANK

Keep Hustling!