GATE

CRASH COURSE

ALL BRANCHES

Engineering Mathematics

Probability and Statistics (Part 02)
(Lec 14)

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Topics to be covered

Prob & Stats (Part 2)

(BASICS of PROBABILITY)

R.Exp → Not sure about the outcome

Dipuneet sir PW

S.Space → Total possible outcomes of R.Exp in set form

Event :→ Any subset of S.Space is called Event.

eg $S_{Die} = \{1, 2, 3, 4, 5, 6\}$, Pair of Dice $S = \begin{Bmatrix} (11)(12)(13) \dots (16) \\ (21)(22), \quad ,(26) \\ \vdots \\ (61),(62) \dots ,(66) \end{Bmatrix}$

$n(S) = 6$

$n(S) = \underset{D_1}{6} \times \underset{D_2}{6} = 36$ pair

eg $S_{coin} = \{H, T\} \Rightarrow n(S) = 2$,

eg Three coins are tossed simultaneously then $\Rightarrow n(S) = \underset{C_1}{2} \times \underset{C_2}{2} \times \underset{C_3}{2} = 8$ Triplets

$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$

$S = \{1, 2, 3, 4, 5, 6\}$

Event: eg $E_1 = \{\text{odd No.}\} = \{1, 3, 5\} \Rightarrow n(E) = 3$

$E_2 = \{\text{No} \geq 3 \text{ occurs}\} = \{3, 4, 5, 6\} \Rightarrow n(E) = 4$ & so on - - -

eg $S = \left\{ \begin{matrix} (11) & (12) & (13) & \cdots & (16) \\ (21) & (22) & \cdots & & (66) \end{matrix} \right\} \Rightarrow E_1 = \{\text{odd no. of } 1^{st} \text{ Die}\}$

$n(S) = 36$

$= \left\{ \begin{matrix} (12) & (13) & \cdots & (16) \\ (31) & (32) & \cdots & (36) \\ (51) & (52) & \cdots & (56) \end{matrix} \right\} \Rightarrow n(E_1) = 18$

Impossible Event: $\because \phi \subset S \Rightarrow \phi$ is also an Event & $P(\phi) = 0$

Sure Event: $\because S \subseteq S \Rightarrow S$ " " " Event & $P(S) = 1$

$$0 \leq P(E) \leq 1$$

(M-I) $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{\text{No. of elements in Fav Event}}{\text{No. of elements in S. Space}}$ (By counting the elements in E & S)

(M-II) $\text{Req Prob} = \dfrac{\text{Fav No of Cases}}{\text{Total No. of Cases}}$ (Using concept of P & C)

(M-III) Using some standard Results & standard Def^n.

Note- A coin is tossed 4 times
4 coins are tossed simultaneously
$\Rightarrow$ Both have same sample space

$S = \{\underbrace{(HHHH)}_{=1} \underbrace{(HHHT)}_{=4} \underbrace{(HHTT)}_{=6} \underbrace{(HTTT)}_{=4} \underbrace{(TTTT)}_{=1}\}$

$n(S) = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} = 16$ Quadruples

(1) $P(B) = ? = \frac{50B}{100B} = \frac{1}{2}$

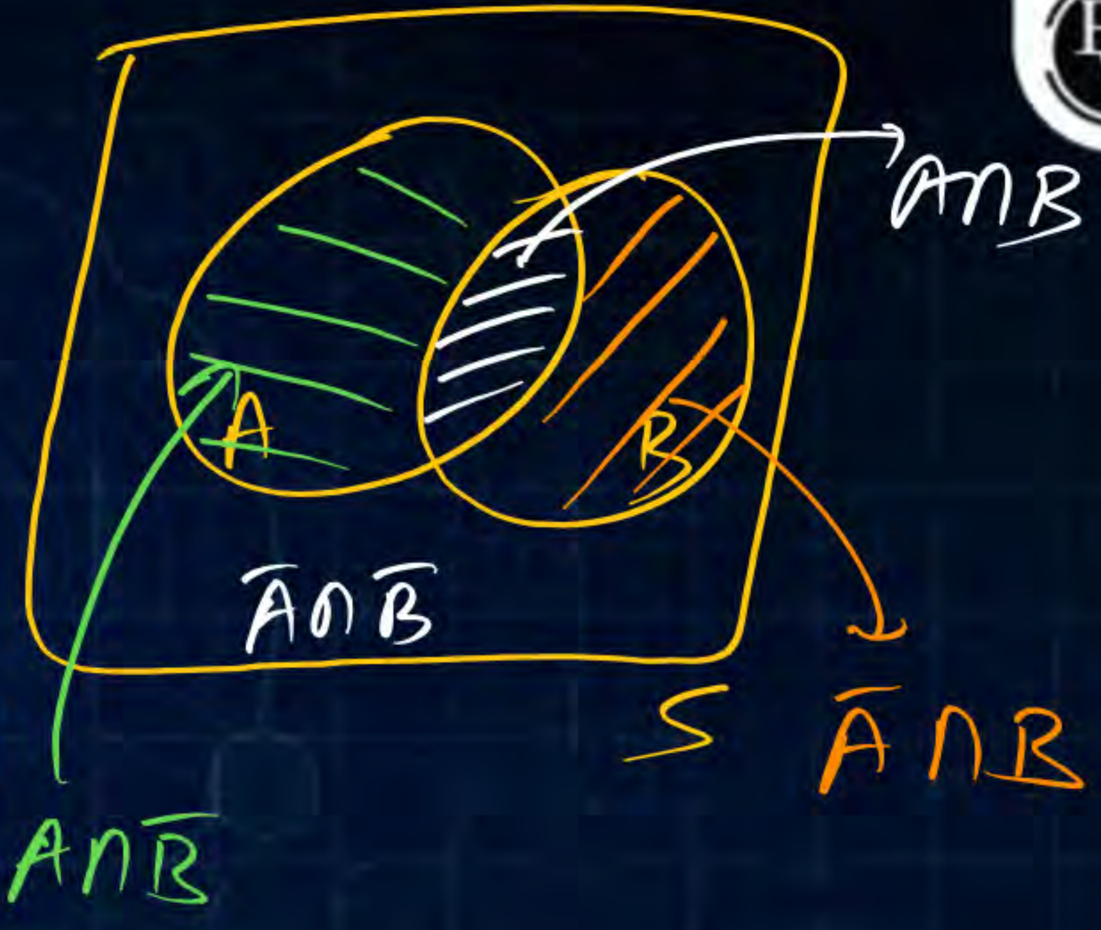
(2) A family has 5 kids in which 2 are Boys then

$P(\text{selecting one Boy}) = \frac{fav}{total} = \frac{{}^2C_1}{{}^5C_1} = \frac{2}{5} \neq \frac{1}{2}$

Note:

1. Either A or B or Both = ? = $A \cup B$
2. Both A & B = ? = $A \cap B$
3. Neither A Nor B = ? = $\bar{A} \cap \bar{B}$

$$\boxed{(A \cap \bar{B}) \cup (\bar{A} \cap B) \cup (A \cap B) = A \cup B}$$



* Addition theorem of Prob. → $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

* Multi theorem of Prob → $P(A \cap B) = P(A/B) \cdot P(B)$

*

$$\boxed{P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)}$$

Mutually Exclusive Events → if $\boxed{A \cap B = \phi}$ then $\Longleftrightarrow$ (A & B are called M.E)

i.e. for ME Events A & B we have $P(A \cap B) = 0$

& $P(A \cup B) = P(A) + P(B) - 0$

Independent Events → if $\boxed{P(A \cap B) = P(A) \cdot P(B)}$ then $\Longleftrightarrow$ (A & B are Independent)

& $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

ME

eg $S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1, 3, 5\}$, $E_2 = \{2, 4, 6\}$

$\because E_1 \cap E_2 = \phi \Rightarrow P(E_1 \cap E_2) = 0$

Ind → $S = \{H, T\}$, $D = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{H\}$, $E_2 = \{2, 4, 6\}$

$P(E_1 \cap E_2) = \frac{1}{2} \times \frac{3}{6} = \frac{1}{4}$

A fair dice is tossed two times. The probability that the second toss results in a value that is higher than the first toss is ____.

(a) 2/36 (b) 2/6

(c) 5/12 (d) 1/2

$$S = \begin{Bmatrix} (11) & (12) & - & - & (16) \\ (21) & (22) & - & - & (26) \\ \vdots & & & & \\ (61) & (62) & - & - & (66) \end{Bmatrix}$$

$= 15$

$= 15$

$= 6$

$n(S) = 36$ pair

fav. Cases $A = \begin{Bmatrix} (12), (13), (14), (15), (16) \\ (23) (24) (25) (26) \\ (34) (35) (36), (45), (46), (56) \end{Bmatrix} \Rightarrow n(A) = 15$ pair

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}$

Four fair coins are tossed simultaneously. The probability that at least one heads and at least one tails turn up is

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{15}{16}$

$S = \{(HHHH), (HHHT), \ldots, (TTTT)\}$

$n(S) = 16$ Quadruples

M-I

$S = \{\underbrace{(HHHH)}_{=1}, \underbrace{(HHHT), (HHTT), (HTTT)}_{fav = 14}, \underbrace{(TTTT)}_{=1}\}$

So Req Prob $= \frac{14}{16} = \frac{7}{8}$

Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number on the faces showing up is (up to third decimal place) ____________

$P(A) = \frac{6}{216} = 0.027$

M-I

$S = \{(111), (112) \ldots (116), (211), (212) \ldots (666)\}$

$n(S) = \underset{D_1}{6} \times \underset{D_2}{6} \times \underset{D_3}{6} = 216$ Triplets

Same Triplets $= \{(111), (222), (333), (444), (555), (666)\} \Rightarrow n(A) = 6$

A

Two dice each numbered from 1 to 6 are thrown together. Let A and B be two events given by

A : Even number on the first dice

B : Number on the second dice is greater than 4

$A \cap B = \{(25),(26),(45),(46),(65),(66)\} \Rightarrow n(A \cap B) = 6$

(i) What is the value of $P(A \cap B)$ and $P(A \cup B)$ respectively?

(a) 1/2, 1/6 (b) 1/4, 2/3 (c) 2/3, 1/6 (d) 1/6, 2/3

$n(S) = 6 \times 6 = 36$ pair

M-(I)

$A = \{(\underline{\text{Even No}}, \underline{\text{Something}})\}$

$= 3 \times 6 = 18$

$P(A) = \frac{18}{36} = \frac{1}{2}$

$B = \{(\underline{\text{Something}}, \underline{\text{No} \geq 5})\} = 6 \times 2 = 12$

$P(B) = \frac{12}{36} = \frac{1}{3}$

$A \cap B = \{(\underline{\text{Even}}, \underline{\geq 5})\} = \underline{3\text{ way}} \times \underline{2\text{ way}} = 6$ way

$P(A \cap B) = \frac{6}{36} = \frac{1}{6}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{3}$

What is the probability that a leap year selected at random will contain 53 Sundays?

$= 366$ days $= 52$ weeks & 2 days

($= 52$ Sunday) ($= ?$)

$S = \{(MT), (TW), (WTh), (ThF), (F\,Sat), (Sat\,Sun), (Sund\,Mon)\}$

$n(S) = 7$ pair

fav Pair $A = \{(Sat\,Sund), (Sund\,Mon)\} = 2$

Req Prob $= \dfrac{\text{fav}}{\text{Total}} = \dfrac{2}{7}$

Consider the following statements :

I. The probability that there are 53 Sundays in a leap year is twice the probability that there are 53 Sundays in a non-leap year.

II. The probability that there are 5 Mondays in the month of March is thrice the probability that there are 5 Mondays in the month of April.

Which of the statements given above is/are correct?

(a) Only I ✓ (b) Only II × (c) Both I and II (d) Neither I nor II ×

(I) $P(53 \text{ Sund in Lp yr}) = \frac{2}{7}$

$P(53 \text{ '' '' N.Lp yr}) = \frac{1}{7}$

ie (I) is True

March: 31 days ⇒ 4 weeks & 3 days

= 4M & (?)

$S = \{(MTW), (TWTh), (WThF), (ThFSat), (FSatS), (SatSM), (SMT)\} \Rightarrow P(\text{Mond}) = \frac{3}{7}$

April: 30 days = 4 weeks & 2 days ⇒ $P(\text{Mon}) = \frac{2}{7}$ So (ii) is false

Mutually Exclusive Events →

if A & B are ME then $A \cap B = \phi$ → $P(A \cap B) = 0$

→ $P(A \cup B) = P(A) + P(B)$

Ind Events →

if A & B are Ind then $\boxed{P(A \cap B) = P(A) \cdot P(B)}$

Assertion (A) : The probability of drawing either an ace or a king from a deck of card in a single draw is $\frac{2}{13}$. True

A K 52 Cards

Reason (R) : For two events E_1 and E_2 which are not mutually exclusive, the probability is given by

$$P(E_1 + E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$ (True)

(a) A and R are true, R is the correct explanation of A

(b) A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

$A = \{Ace\} \Rightarrow P(A) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

$K = \{king\} \Rightarrow P(K) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

$\because A \cap K = \phi \Rightarrow P(A \cap K) = 0$

$P(A \cup K) = P(A) + P(K) - 0$

$= \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$

A bag contains 5 red and 7 black balls and a second contains 4 blue and 3 green balls. A ball is taken out from each bag. Find the probability that

(i) one ball is red and other blue

(ii) one ball is black and other green

① $P\left[(R \text{ from } 1^{st} \text{ Bag}) \cap (\text{Blue from } 2^{nd} \text{ Bag})\right] = \frac{5}{12} \times \frac{4}{7}$

② $P\left[(\text{Black from } 1^{st}) \cap (\text{Green from } 2^{nd})\right] = \frac{7}{12} \times \frac{3}{7} = \frac{1}{4}$

A fair dice is rolled twice. The probability that an odd number will follow an even number is

(JHANDBAM QUEST)

(a) $\frac{1}{2}$

(b) $\frac{1}{6}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

Both Rollings are Ind

P[odd No ∩ Even No]

$= \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

Two dice are tossed. One dice is regular and the other is biased with probabilities $P(1) = P(6) = 1/6$, $P(2) = P(4) = 0$ and $P(3) = P(5) = 1/3$. The probability of obtaining a sum of 4 is

(a) 1/9 (b) 1/12 (c) 1/18 (d) 1/24

Both Die are Independent

ME

$$\text{Req Prob} = P(\text{Sum is } 4) = P[(1 \cap 3) \text{ or } (2 \cap 2) \text{ or } (3 \cap 1)]$$

$$= \frac{1}{6} \times \left(\frac{1}{3}\right) + \frac{1}{6} \times (0) + \frac{1}{6} \times \left(\frac{1}{6}\right) = \frac{1}{12}$$

A dart is thrown at a dart board whose dimensions are 5 m × 5 m. If the probability of missing the dart board is 0.25, find the probability of hitting the board at a point that is at a maximum distance of 2 m from the centre of the board.

(a) $\frac{3\pi}{25}$ (b) $\frac{4\pi}{25}$ (c) $\frac{5\pi}{5}$ (d) $\frac{6\pi}{25}$

Q1 → $P(\text{Dart will hit Dart Board}) = 1 - 0.25 = 0.75 = 3/4$

Q2 → Shaded area $= \pi r^2 = \pi(2)^2 = 4\pi$

Total area $= 5 \times 5 = 25$

$P(\text{Shaded area}) = \frac{F}{T} = \frac{4\pi}{25}$

Req Ans $= P(Q1) \times P(Q2)$

$= \frac{3}{4} \times \frac{4\pi}{25} = \frac{3\pi}{25}$

Conditional Prob

$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$ = it is the prob of A when B has already occurred

ie " " " A where B is the condⁿ.

or $\boxed{P(A\cap B) = P(A/B)\cdot P(B)}$

Note: If A & B are Ind. Events then condⁿ has No meaning

$\therefore P(A/B) = ? = \frac{P(A\cap B)}{P(B)} = \frac{P(A)\cdot P(B)}{P(B)} = P(A)$

If $P(A) = 1/3$, $P(B) = 1/4$, $P(A/B) = 1/6$, then what is $P(B/A)$ equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

$\because P(A/B) = \frac{1}{6}$

$\frac{P(A \cap B)}{P(B)} = \frac{1}{6}$

$P(A \cap B) = \frac{1}{6} \times \frac{1}{4}$

So $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1/24}{1/3}$

$= \frac{1}{8}$

If A and B are events such that

$$P(A \cup B) = 0.5, P(\bar{B}) = 0.8 \text{ and } P(A/B) = 0.4,$$

What is $P(A \cap B)$ equal to?

(a) 0.08 (b) 0.02 (c) 0.8 (d) 0.2

$P(\bar{B}) = 0.8$

$P(B) = 0.2$

$P(A/B) = 0.4$

$$\frac{P(A \cap B)}{P(B)} = 0.4$$

$P(A \cap B) = 0.4 \times 0.2 = 0.08$

Thank

THANK

Keep Hustling!