DS & AI

Artificial Intelligence

Propositional logic & Reasoning under uncertainty
1500+ series

Lecture - 7

By - Siddharth Sabharwal Sir

Topic : Proposition and predicate & Bayesian network

#Q. Consider a Bayesian network with three binary variables A B and C. The join probability distribution P(A, B, C) is given in the table below:

A	B	c	P(A, B, C)
0	0	0	0.3
0	0	1	0.7
0	1	0	0.3
0	1	1	0.3
1	0	0	0.3
1	0	1	0.7
1	1	0	0.3
1	1	1	0.3

$$\boxed{\frac{P(A=1, B=1)}{P(B=1)} = \frac{.6}{.6+.6} = 1/2}$$

Using variable elimination, calculate the conditional probability P(A = 1| B = 1).

Topic : Proposition and predicate & Bayesian network

#Q. Consider a Bayesian network representing climates patterns, there are two variables: W(weather) and R(rain). The conditional probability tables are follows:

Prior Probab of w

W(weather)	Probability P(W)
Sunny	0.6
Cloudy	0.4

Posterior

Rain(R)	W(weather)	Probability (R =1|W)
1	Sunny	0.3
1	Cloudy	0.7

$W \rightarrow R$

$P(R=1) = P(R=1/W=S)\,P(W=S) + P(R=1/W=C)\,P(W=C)$

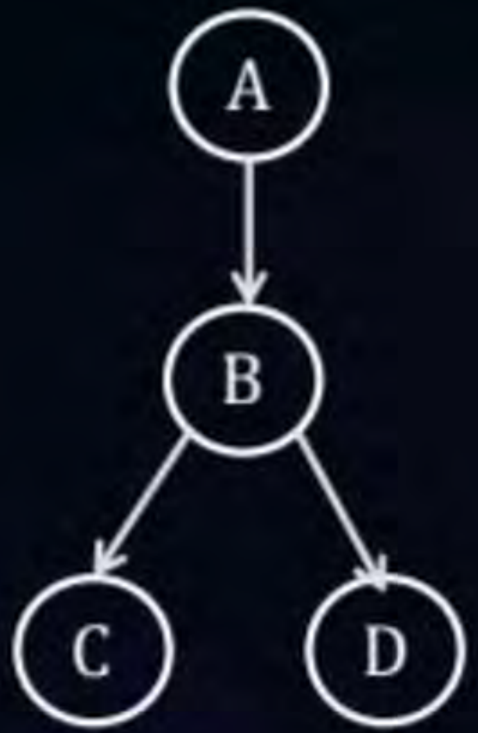
$.3 \times .6 + .7 \times .4$

$\Rightarrow .46$

Compute the marginal probability P(R= 1)__________. (Upto to 2 decimal)

Topic : Proposition and predicate & Bayesian network

#Q. Given the following Bayesian Network consisting of four Bernoulli random variables and the associated conditional probability tables:



	P(·)
A = 0	0.6
A =1	0.4

	P (B = 0|.)	P (B = 1|.)
A = 0	0.6	0.6
A =1	0.4	0.4

	P (C = 0|.)	P (C = 1|.)
B = 0	1	0
B =1	0	1

	P (D = 0|.)	P (D = 1|.)
B = 0	0.6	0.6
B =1	1	1

$P(D=1/B=0)\, P(C=0/B=0)$
$P(B=0/A=1)\, P(A=1)$

$.6 \times 1 \times .4 \times .4 \Rightarrow .096$

The value of P(A=1, B=0, C=0, D = 1)
___(Rounded off to three decimal places)

Prior → P(variable)

Posterior → P(variable/evidence)

Topic : Proposition and predicate & Bayesian network

#Q. Consider the process of likelihood weighting in approximate Bayesian inference.
Which of the above statements is/are correct?

A. Likelihood weighting samples from the ~~prior~~ Posterior distribution of variables.

B. Likelihood weighting assigns weights to samples based on how well they match the observed evidence.

C. Likelihood weighting always guarantees exact posterior probabilities.

D. Likelihood weighting is computationally efficient for large Bayesian networks.

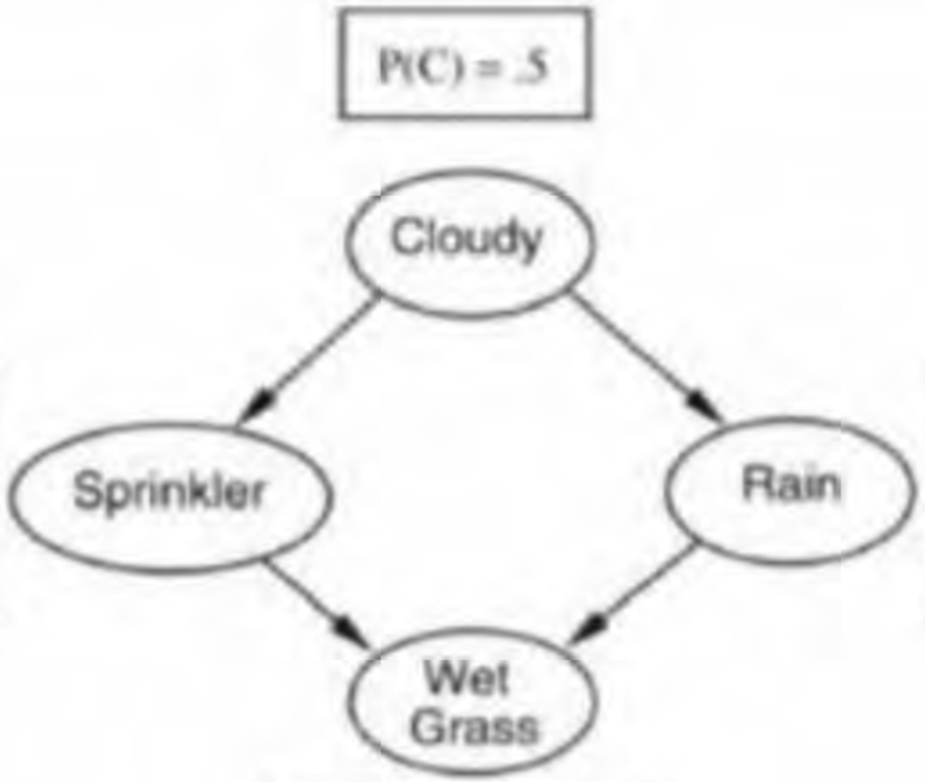
Topic : Proposition and predicate & Bayesian network

#Q. Consider a Bayesian network with two variables: A and B. The joint probability distribution is given as follows:

Using $\boxed{\text{variable elimination,}}$ calculate the marginal probability P(B = 1).

A	B	P(A,B)
0	0	0.2
0	1	0.3
1	0	0.1
1	1	0.4

3) Consider the following Bayesian Network. Suppose you are doing $\boxed{\text{likelihood sampling to determine } P(S \mid \neg C, W).}$



C	P(S)
T	.10
F	.50

C	P(R)
T	.80
F	.20

S	R	P(W)
T	T	.99
T	F	.90
F	T	.90
F	F	.00

weight $P(\neg C)\, P(W/S,R)$

$P(\neg C)\, P(W/S\neg R)$

$(.5 \times .9) \rightarrow .45$ ✓

$P[S/\neg C, W]$

45 ✓

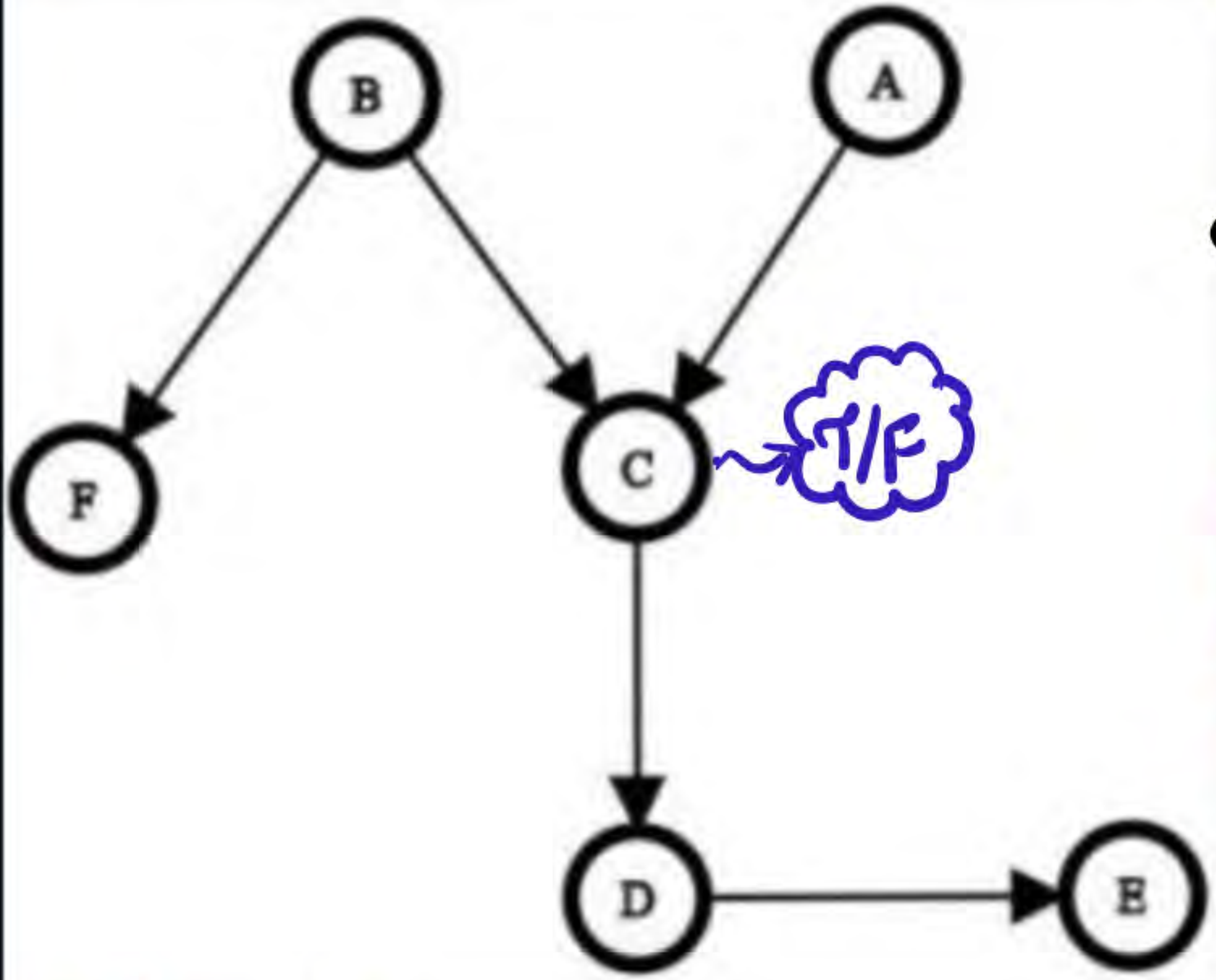
Let the weight for the sample $\boxed{(\neg C, S, \neg R, W) \text{ be w. What is 100w?}}$ (Round off your answer to the closest integer)

$$B \rightarrow C \leftarrow A$$

$$P[C|AB)P(A) + P(C|\sim AB)\,P(\sim A) \rightarrow$$

$$P[C|B]$$

2) Consider the following Bayesian Network. Given evidence about C which of the following pair of variables are conditionally independent.



- A and B ✓
- A and D ✓
- D and E ✓
- A and F ✓

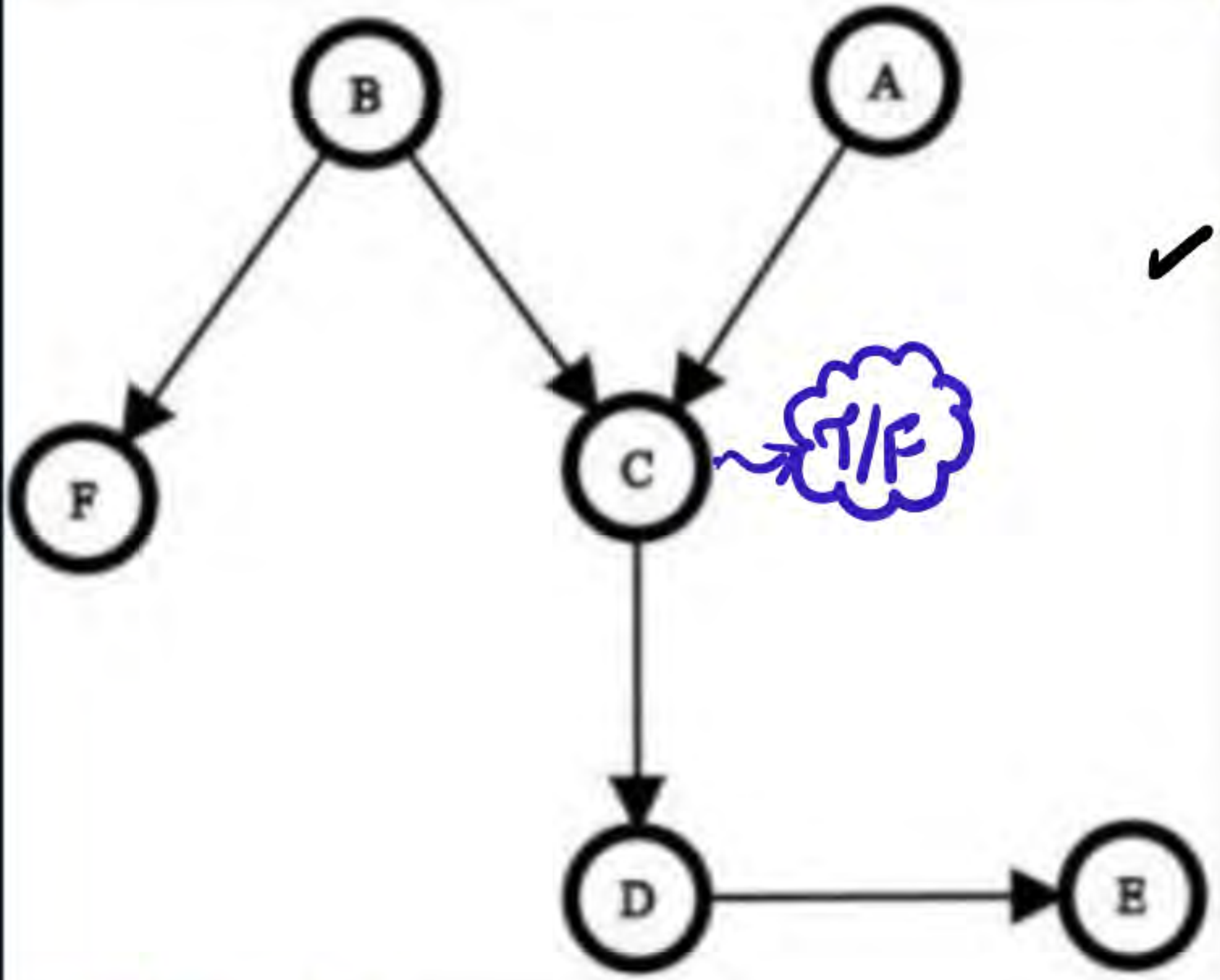
$$\bullet\ P[A/C] \neq P[A/CB]$$

$$\frac{P[A,C]}{P[C]} \neq \frac{P[A,C,B]}{P[C,B]}$$

$$\frac{P[C/A)P(A)}{P(C)} \Leftrightarrow \frac{P[A,C,B]}{P(A,C,B)+P(\sim A,C,B)} = \frac{P[C/A,B)\,P(A)}{P[C/B)}$$

$$\frac{P[C/A,B]\,P(A)\cancel{P(B)}}{P[C/AB)\,P(A)\cancel{P(B)} + P(C/\sim A,B)\cancel{P(B)}\,P(\sim A)}$$

2) Consider the following Bayesian Network. Given evidence about C which of the following pair of variables are conditionally independent.



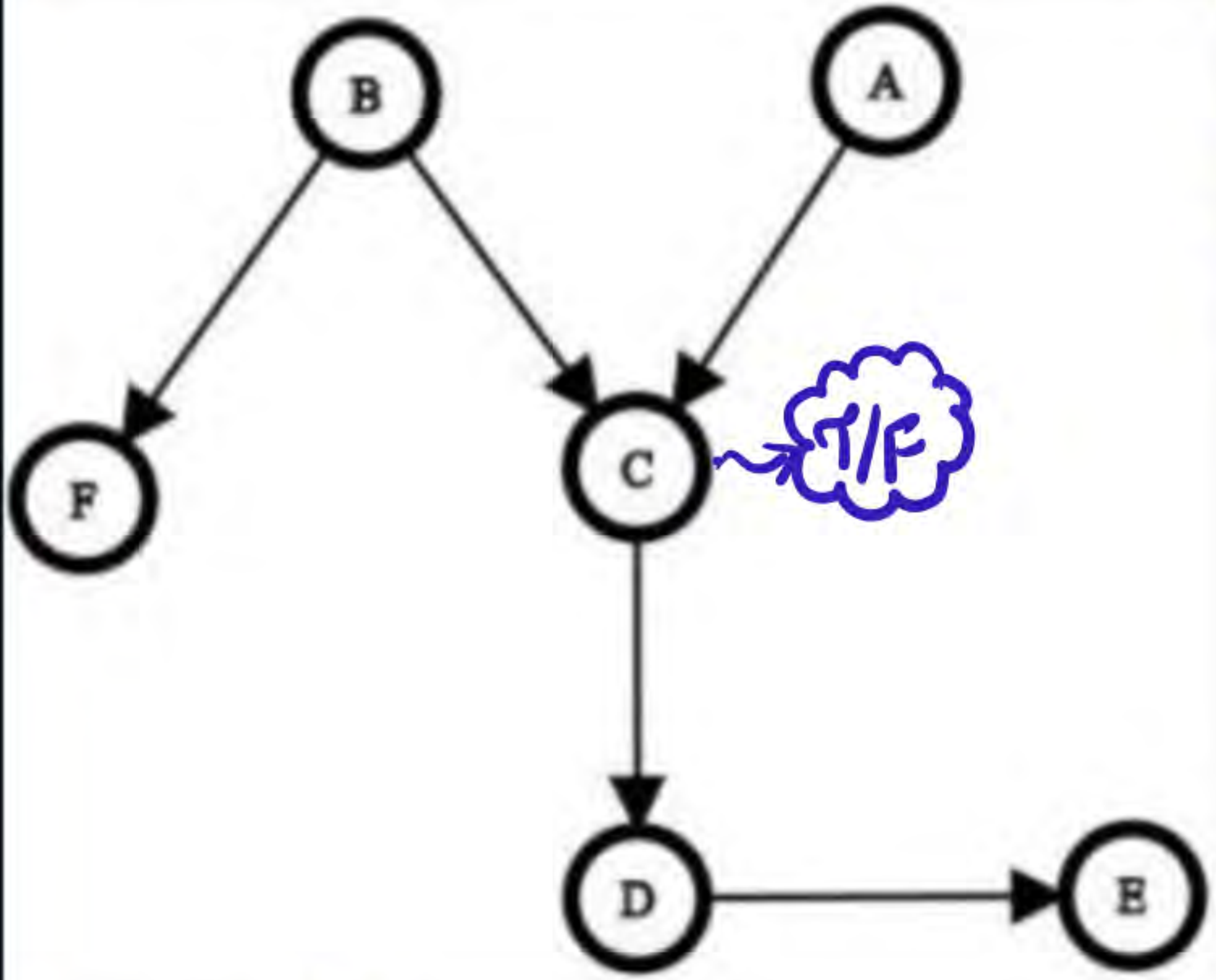
$$\boxed{P[A/C] = P[A/C,D]}$$

$$\frac{P[A,C]}{P[C]} , \quad \frac{P[A,C,D]}{P[C,D]}$$

$$\frac{\cancel{P[D/C]}\, P[C/A]P(A)}{\cancel{P[D/C]}\, P[C]}$$

- A and B
- A and D
- D and E
- A and F

2) Consider the following $\boxed{\text{Bayesian Network}}$. Given evidence about C which of the following pair of variables are conditionally independent.



$P[A/C] \quad \cdot \quad P[A/C,F]$

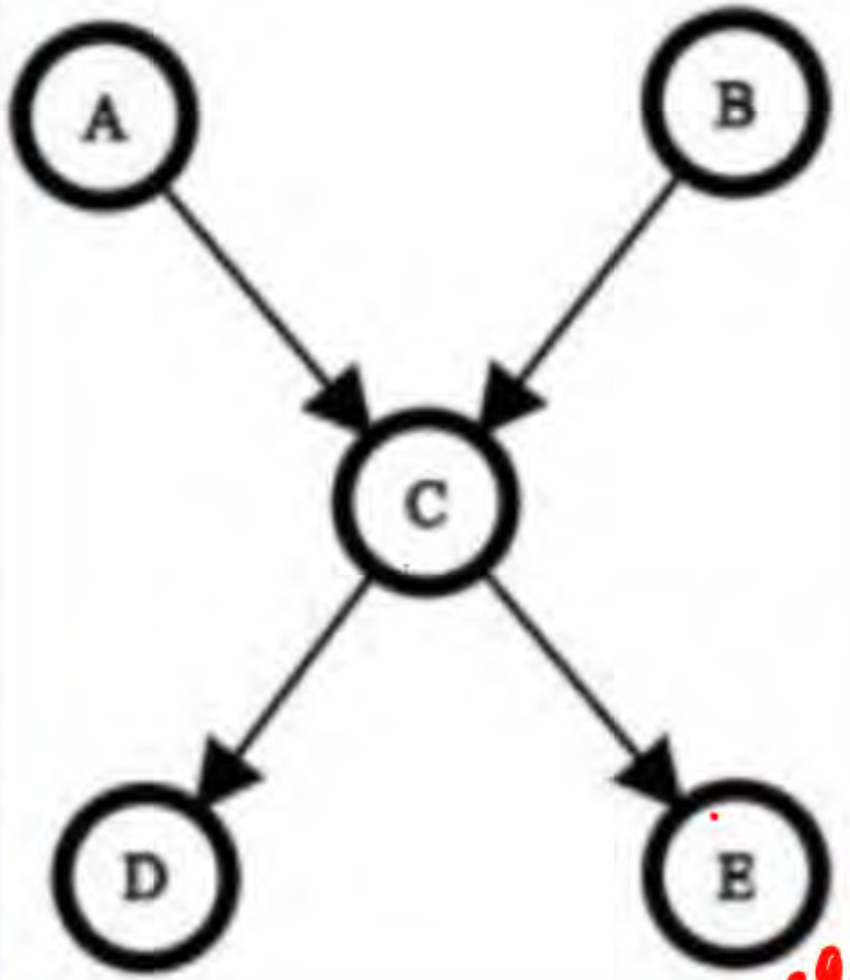
$\dfrac{P[A,C]}{P(C)} \quad , \quad \dfrac{P[A,C,F]}{P(C,F)}$

$P[A,C,B,F] + P(A,C,\bar{B},F)$

$P(C,B,F) + P(C,\bar{B},F)$

- A and B ✓
- ✔ A and D ✓
- ✗ D and E ✓
- A and F ✓

9) Consider the following Bayesian Network with the given information. What is the value of P(¬a | d, e) up to 3 decimal places?



H.W

$$P[\neg a / d, e] = \frac{P[\neg a, d, e]}{P[d, e]}$$

$.9 \times .7 \times .95 \times .001 \times .002$

- $P[a, b, c, d, e]$ — $P[\sim a, b, c, d, e]$

$.05 \times .01 \times .05 \times .001 \times .002$

- $P[a, b, \sim c, d, e]$ — $P[\sim a, b, \sim c, d, e]$

$.9 \times .7 \times .94 \times .001 \times .998$

- $P[a, \sim b, c, d, e]$ — $P[\sim a, \sim b, c, d, e]$
- $P[a, \sim b, \sim c, d, e]$ — $P[\sim a, \sim b, \sim c, d, e]$

$.05 \times .01 \times .06 \times .001 \times .998$

P(a) = 0.001, P(b) = 0.002

P(c | a, b) = 0.95, P(c | a, ¬b) = 0.94, P(c | ¬a, b) = 0.29, P(c | ¬a, ¬b) = 0.001

P(d | c) = 0.9, P(d | ¬c) = 0.05

P(e | c) = 0.7, P(e | ¬c) = 0.01

Recall that during Gibbs Sampling, samples are generated through an iterative process.

Assume that the only evidence that is available is A = +a. Clearly fill in the circle(s) of the sequence(s) below that could have been generated by Gibbs Sampling.

Sequence 1

1 :	+a	−b	−c	+d
2 :	+a	−b	−c	+d
3 :	+a	−b	+c	+d

Sequence 2

1 :	+a	−b	−c	+d
2 :	+a	−b	−c	−d
3 :	−a	−b	−c	+d

Sequence 3

1 :	+a	−b	−c	+d
2 :	+a	−b	−c	−d
3 :	+a	+b	−c	−d

Sequence 4

1 :	+a	−b	−c	+d
2 :	+a	−b	−c	−d
3 :	+a	+b	−c	+d

Bidirectional Search

from start node ⇝ (BFS)

⇜ goal node

- The Bidirectional Search Algorithm is an efficient graph traversal and search technique used to find the shortest path between an initial state (source node) and a goal state (target node). Unlike traditional search algorithms like BFS or DFS, it simultaneously searches from the source and the goal, meeting in the middle. This reduces the search space significantly, making it faster in many cases.
- The search is conducted in two simultaneous phases:
- Forward Search: Starts from the initial state and moves towards the goal state. BFS
- Backward Search: Starts from the goal state and moves towards the initial state. BFS
- The algorithm stops when the two searches meet at a common node, forming the shortest path.

Bidirectional Search

- $\boxed{\text{Steps of the Algorithm}}$
- Initialization:
- Create two frontiers: one for the forward search (Frontier1) and one for the backward search (Frontier2).
- Start Frontier1 from the initial node and Frontier2 from the goal node.
- Maintain two visited sets: Visited1 for forward search and Visited2 for backward search.

Bidirectional Search

- Steps of the Algorithm
- Alternate Search:
- Expand nodes alternately from Frontier1 and Frontier2.
- For each node expanded, generate its neighbors and add unvisited ones to the respective frontier.
- Check for Intersection:
- After expanding nodes in both searches, check if any node is present in both Visited1 and Visited2. If so, the searches have met, and the path can be reconstructed.

Bidirectional Search

- Path Reconstruction:

- Trace back from the meeting node to the source using the forward search tree and to the goal using the backward search tree.
- Combine the two paths to get the complete shortest path.

- Termination:
- The algorithm terminates when a common node is found or when one of the frontiers is empty, indicating no path exists.

Bidirectional Search

BFS ⇒ Time Comp $O(b^d)$

- Advantages

- Reduced Search Space: The search space is reduced to approximately $\boxed{O(b^{d/2})}$ compared to $O(b^d)$ for BFS, where b is the branching factor and d is the depth of the solution.
- Efficiency: It is faster for problems with large search spaces.

Bidirectional Search

uninformed graph BFS $\rightarrow O(b^{d/2})$

Informed Search A* $\rightarrow O(b^{d/2})$

- Disadvantages

- Complexity in Implementation: Managing two simultaneous searches and checking for intersection can be complex.
- Memory Usage: It requires storing two frontiers and visited sets, which can increase memory usage.
- Heuristic Requirement: It may not work well without additional information (heuristics) for some problems.

Bidirectional Search

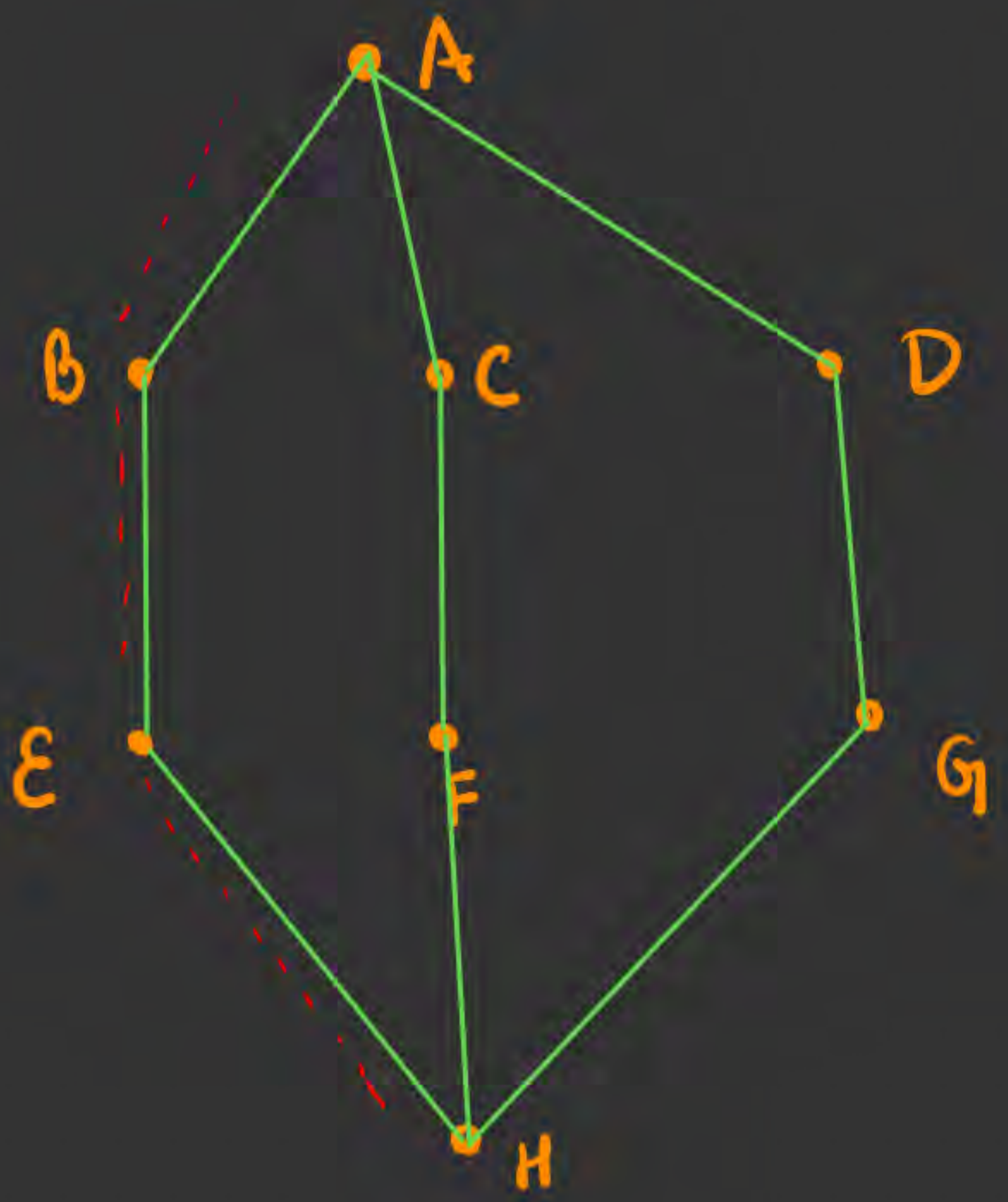
- Applications

- Shortest Path Finding: In navigation systems, robot motion planning, and network routing.
- Artificial Intelligence: Used in games and AI-based pathfinding tasks.
- State-Space Search: Problems like the 8-puzzle or finding transformations between words.

Bidirectional Search

Comparison with Other Algorithms

Feature	BFS	DFS	Bidirectional Search
Search Space	$O(b^d)$	$O(b \cdot m)$	$O(b^{d/2})$
Memory Usage	High	Low	Moderate
Optimality	Yes (for BFS)	No	Yes (if path is found)
Speed	Moderate	Fast (not optimal)	Faster for large graphs



Search 1

1)
3)

Close

A B C D E

open

~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~ F G

Search 2

2)
4)

Close

H E F G

open

~~H~~ ~~E~~ ~~F~~ ~~G~~ B C D

```
    A --- B --- C
   / \    |    / \
  D   E   F   G   H
       \  |    \
        I J     K
```

Close

A D E B

open

~~A~~ ~~D~~ ~~E~~ ~~B~~ J F

Close

K G C B

open

~~K~~ ~~G~~ ~~C~~ ~~B~~ H

Topic : Bayesian

#Q. Consider five random variables U, V, W, X, and Y whose joint distribution satisfies:

$P(U, V, W, X, Y) = P(U)P(V)P(W \mid U, V)\, P(X \mid W)\, P(Y \mid W)$

Which ONE of the following statements is FALSE?

$P(Y/\omega) = P(Y/\omega, v) = \dfrac{P(Y,\omega,v)}{P(\omega,v)} = \dfrac{P(Y/\omega)\cancel{P(\omega/v)}\,\cancel{P(v)}}{\cancel{P(\omega,v)}}$

A. Y is conditionally independent of V given W

B. X is conditionally independent of U given W

C. U and V are conditionally independent given W

D. Y and X are conditionally independent given W

A and B are Independent

$$P[A] = P[B]$$

$$P(A/B) = P(A)$$

A and B are Independent given C

$$P[A/C] = P[A/B, C]$$

$$P[V/W] \neq P[V/W, U)$$

$$\neq \frac{P[V, W, U]}{P[W, U]}$$

$$P[V/W] \neq \frac{P[W/V, U]\,P(V)\,P(\cancel{U})}{P[W/U)\,P(\cancel{U})}$$

whose joint distribution

| W)

C. U and V are conditionally independent given W

D. Y and X are conditionally independent given W

Topic : Bayesian

#Q. Consider five random variables U, V, W, X, and Y whose joint distribution satisfies:

$$P(U, V, W, X, Y) = P(U)P(V)P(W \mid U, V)\, P(X \mid W)\, P(Y \mid W)$$

Which ONE of the following statements is FALSE?

$$P[Y/\omega] = P[Y/\omega, x] = \frac{P[Y, \omega, x]}{P[\omega, x]}$$

$$= \frac{P[Y/\omega]\,P[X/\omega]\,P[\omega]}{P[\omega, x]}$$

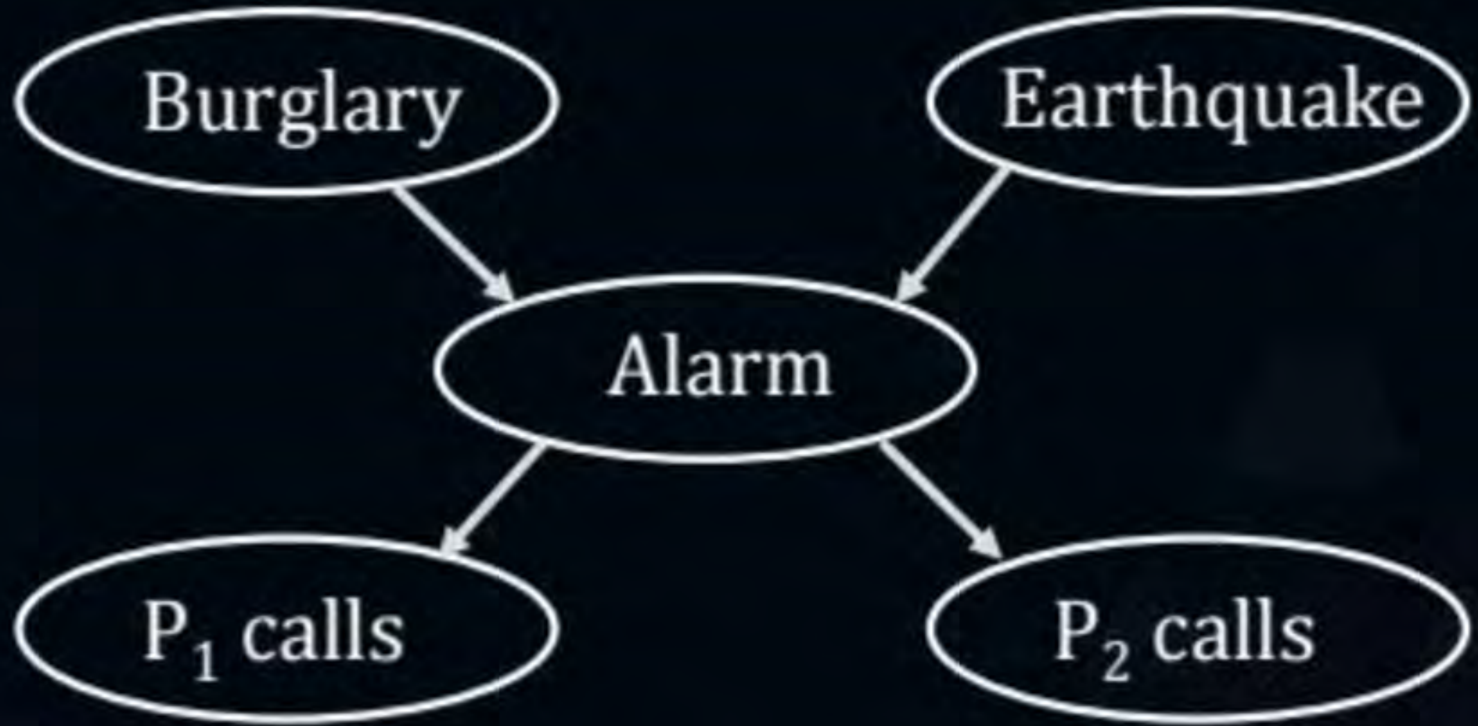
(D) Y and X are conditionally independent given W

Topic : Bayesian

#Q. Calculate following $P(P_1, P_2, A \sim B, \sim E)$

P(B = True)	P(B = False)
0.001	0.999

P(E = True)	P(E = False)
0.002	0.998



Burglary	Earthquake	Alarm True	Alarm False
B	E	P(A =T)	P(A=F)
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

Alarm	P_1 Calls (True)	P_1 Calls (False)
A	$P(P_1 = T)$	$P(P_1 = F)$
T	0.90	0.10
F	0.05	0.95

Alarm	P_2 Calls (True)	P_2 Calls (False)
A	$P(P_2 = T)$	$P(P_2 = F)$
T	0.70	0.30
F	0.01	0.99

#Q. Let x and y be two propositions. Which of the following statements is a tautology /are tautologies?

A $(\neg x \wedge y) \Rightarrow (y \Rightarrow x)$

B $(x \wedge \neg y) \Rightarrow (\neg x \Rightarrow x)$

C $(\neg x \wedge y) \Rightarrow (\neg x \Rightarrow y)$

D $(x \wedge \neg y) \Rightarrow (x \Rightarrow x)$

Topic : Bayesian

#Q. P and Q are two proposition. Which of the following logical expression are-

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

A. Only I and II

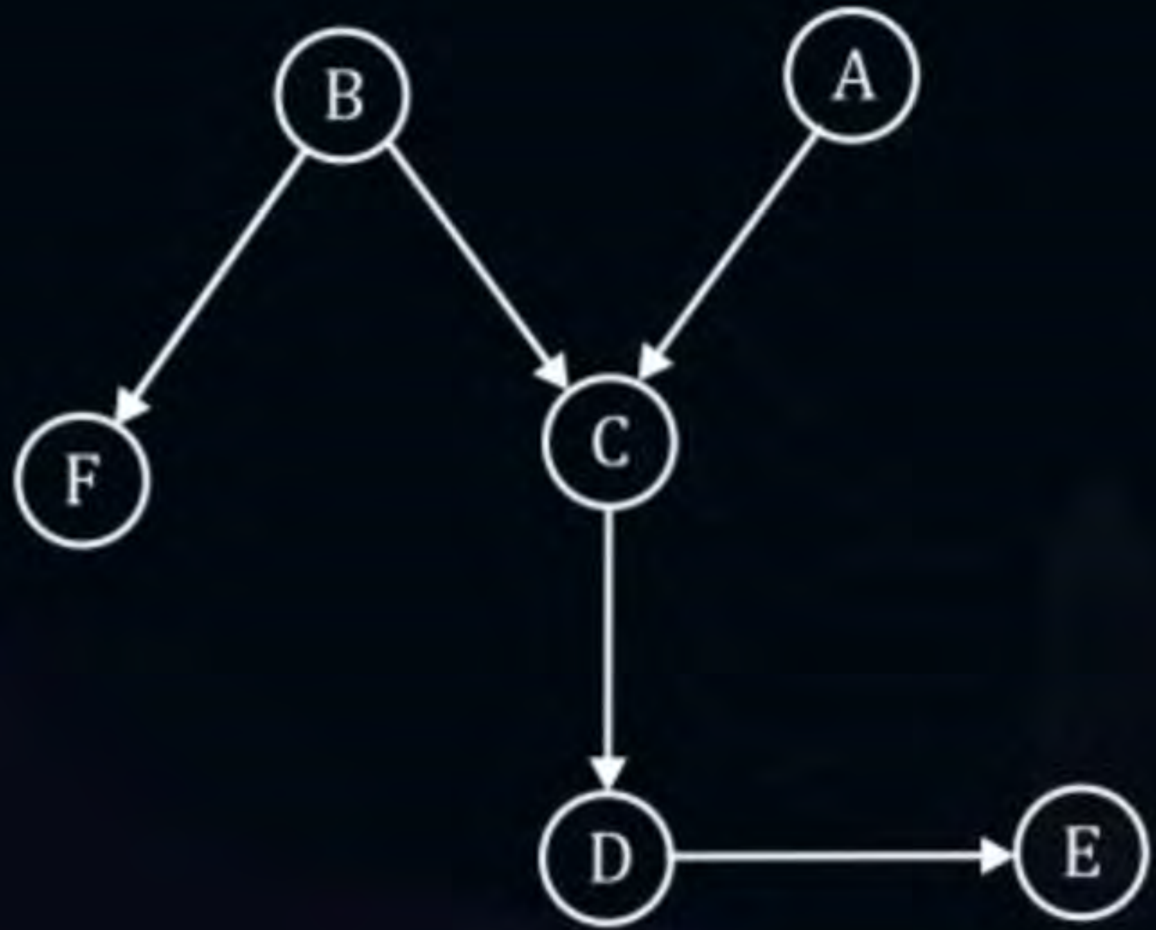
B. Only I, II and III

C. Only I, II and IV

D. All of I, II, III and IV

Topic : Bayesian

#Q. Consider the following Bayesian Network. Given evidence about C which of the following pair of variables are conditionally independent.



A A and B

B A and D

C D and E

D A and F

Topic : Bayesian

#Q. Suppose A and B are conditionally independent given C. Then which of the following are true:

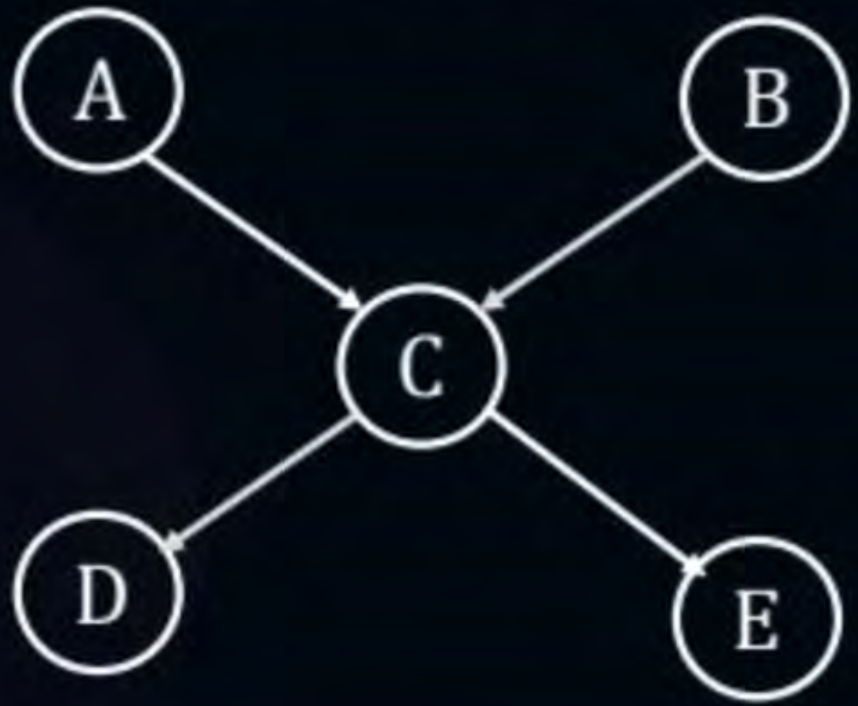
A. $P(A,B) = P(A).P(B)$

B. $P(A,B,C) = P(A|B,C).P(B|C).P(C)$

C. $P(A,B,C) = P(A|C).P(B|C).P(C)$

D. $P(A,B|C) = P(A|C).P(B|C)$

#Q. Consider the following Bayesian Network with the given information. What is the value of $P(\neg a \mid d, e)$ up to 3 decimal places?



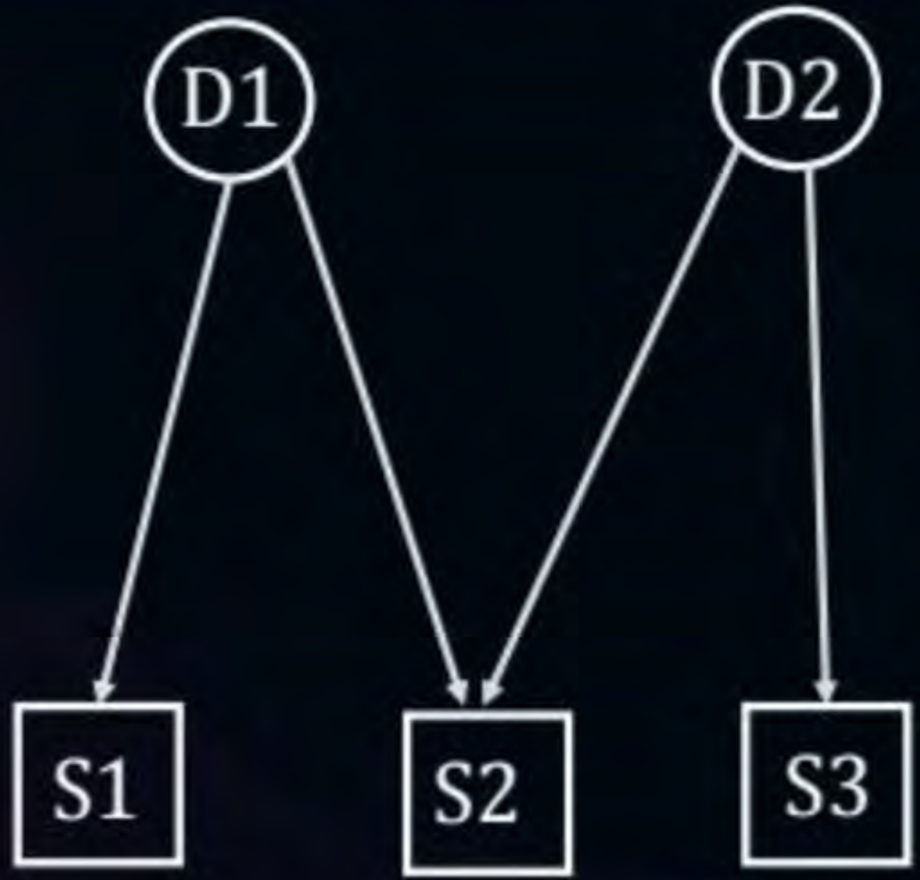
$P(a) = 0.001, P(b) = 0.002$

$P(c \mid a, b) = 0.95, P(c \mid a. \neg b) = 0.94. P(c \mid \neg a, b) = 0.29, P(c \mid \neg a, -b) = 0.001$

$P(d \mid c) = 0.9, P(d \mid \neg c) = 0.05$

$P(e \mid c) = 0.7, P(e \mid \neg c) = 0.01$

#Q. A patient goes to a doctor with symptoms S1, S2 and S3. The doctor suspects disease D1 and D2 and constructs a Bayesian network for the relation among the disease and symptoms as the following:



What is the joint probability distribution in terms of conditional probabilities?

A. P(D1) * P(D2|D1) * P(S1|D1) * P(S2|D1) * P(S3|D2)

B. P(D1) * P(D2) * P(S1|D1) * P(S2|D1) * P(S3|D1, D2)

C. P(D1) * P(D2) * P(S1|D2) * P(S2|D2) * P(S3|D2)

D. P(D1) * P(D2) * P(S1|D1) * P(S2|D1,D2) * P(S3|D2)

Topic : Bayesian

#Q.

P(A)
0.6

A	P(B |A)
T	0.8
F	0.3

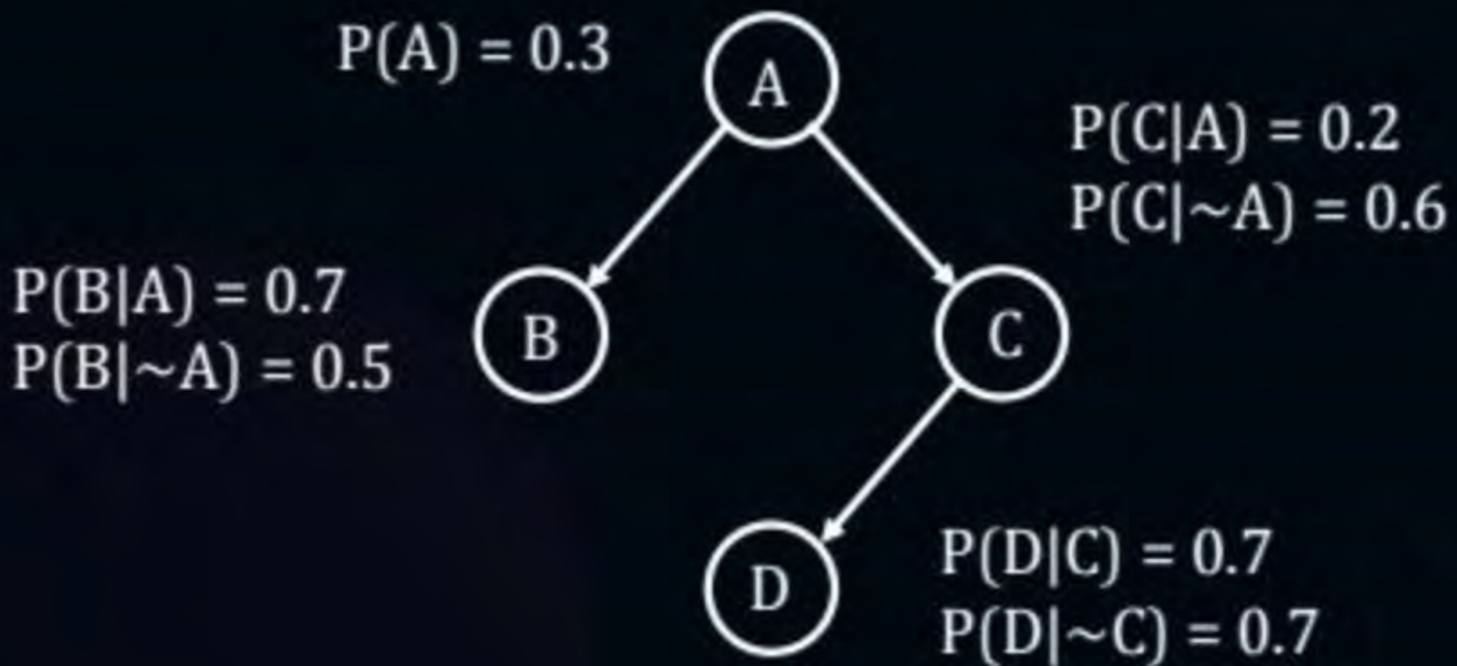
B	P(C |B)
T	0.2
F	0.9

$$A \longrightarrow B \longrightarrow C$$

A. Given the above Bayesian network, compute P((B-teue) AND (C-false)).
B Compute P(B)

Topic : Bayesian

#Q. What is $P(C \mid B)$



A $\frac{5}{11}$

B $\frac{1}{4}$

C $\frac{1}{2}$

D $\frac{6}{11}$

#Q. Which of the following is the correct syntax for expressing that "not all humans are immortal" in first-order logic?

A. $\neg\exists x(\text{Human}(x) \wedge \text{Immortal}(x))$

B. $\forall x(\text{Human}(x) \wedge \neg \text{Immortal}(x))$

C. $\neg \forall x (\text{Human}(x) \wedge \text{Immortal}(x))$

D. $\exists x(\text{Human}(x) \wedge \neg \text{Immortal}(x))$

Topic : Bayesian

#Q. Identity the equivalent mathematical logic for the following statements:
"Some man are player"

A $\exists x(Man(x) \vee Player(x))$

B $\forall x(Man(x) \rightarrow Player(x))$

C $\exists x(Man(x) \rightarrow Player(x))$

D $\exists x(Man(x) \wedge Player(x))$

Topic : Bayesian

#Q. Consider the following logical inferences:

I_1: If Socrates is human, then Socrates is mortal. Socrates is human.
Inference: Socrates is mortal.

I_2: If it rains today, school will close.
School is not closed today. Inference: It will not rain today.

Which of the following is true?

A. Both I_1, and I_2, are correct inferences.

B. I_1 is correct but I_2 is not a correct inference.

C. I_1 is not correct but I_2 is not a correct inference.

D. Both I_1, and I_2 are not correct inference.

Topic : Bayesian

#Q. Which of the following statements is/are correct?

A. Exact inference always uses variable elimination while approximate inference always uses sampling methods.

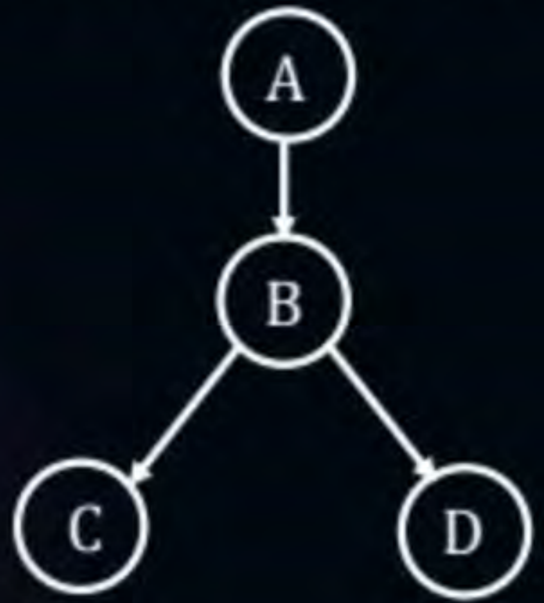
B. Exact inference compute the exact marginal probabilities, while approximate inference estimate them.

C. Exact inference always faster than approximate inference.

D. Exact inference computes the exact probabilities in graphical model.

Topic : Bayesian

#Q. Given the following Bayesian Network consisting of four Bernoulli random variables and the associated conditional probability tables:



	$P(\cdot)$
$A=0$	0.6
$A=1$	0.4

	$P(B=0\,|\,\cdot)$	$P(B=1\,|\,\cdot)$
$A=0$	0.6	0.6
$A=1$	0.4	0.4

	$P(C=0\,|\,\cdot)$	$P(C=1\,|\,\cdot)$
$B=0$	1	0
$B=1$	0	1

	$P(D=0\,|\,\cdot)$	$P(D=1\,|\,\cdot)$
$B=0$	0.6	0.6
$B=1$	1	1

The value of P(A = 1, B = 0, C= 0 D = 1)______.(Rounded off to three decimal places)

Topic : Bayesian

#Q. Consider the process of likelihood weighting in approximate Bayesian inference.
Which of the above statements is/are correct?

A Likelihood weighting samples from the prior distribution of variables.

B Likelihood weighting assigns weights to samples based on how well they match the observed evidence

C Likelihood weighting always guarantees exact posterior probabilities.

D Likelihood weighting is computationally efficient for large Bayesian networks.

#Q. Consider the following two statements:

S_1 : All clear explanations are satisfactory.

S_2: Some excuses are unsatisfactory.

Which of the following statements follows from S_1 and S_2 as per inference rules of logic?

A. Every excuses are not clear explanations.

B. Some excuses are clear explanations.

C. Some excuses are not clear explanations.

D. Some explanations are clear excuses.

Topic : Bayesian

#Q. Consider a Bayesian network representing transportation modes with two variables: T(Transportation) and C(Car). The conditional probability tables are as follows:

P(Transportation)	P(A)
Bus	0.5
Trian	0.5

C (Car)	P(Transportation)	P(C =1|T)
1	Bus	0.2
1	Train	0.8

Based on this Bayesian network, which of the following statements is/are correct?

S_1: The probability of using a car given that the transportation mode is bus is 0.2.

S_2: The probability that the transportation mode is bus given that a car is used is greater than the probability that the transportation mode is train given that a car is used.

S_3: The marginal probability of using a car $P(C = 1)$ increases when the conditional probability $P(C = 1 \mid Train)$ increases.

A. Only S_1 and S_3

B. Only S_1 and S_3

C. Only S_2 and S_3

D. All S_1, S_2 and S_3

#Q. Consider the following sentence:
"Every student who studies hard passes the exam".
Definitions:

- S(x): X is a student
- H(x): X studies hard
- P(x): X passes the exam

Which of the following first-order statements correctly translates the sentence?

A. $\forall x(S(x) \wedge H(x) \rightarrow P(x))$

B. $\forall x(S(x) \rightarrow (H(x) \rightarrow P(x))$

C. $\forall x(H(x) \rightarrow (S(x) \rightarrow P(x))$

D. $\forall x(P(x) \rightarrow (S(x) \wedge (H(x))$

Topic : Bayesian

#Q. Consider the following predicate statements:

P_1: $\neg\forall x \neg(P(x) \rightarrow \exists y\, Q(y))$

P_2: $\exists x(\neg P(x) \vee \exists y\, Q(y))$

P_2: $\exists x(\neg \exists y Q(y) \rightarrow \neg P(x))$

P_4: $\neg \forall x(P(x)\ A \neg \exists\, y Q(y))$

Which of the above predicates are equivalent to the predicate statements:

$\exists x(P(x) \rightarrow \exists y Q(y))$

A. P_1, P_2, P_3

B. P_1, P_3, P_2

C. P_2, P_3, P_4

D. All of these

Topic : Bayesian

#Q. When performing variable elimination, what is a factor?

A. A variable in the Bayesian network

B. A conditional probability table (CPT)

C. A subset of variables and associated probabilities

D. An observed evidence variable

Topic : Bayesian

#Q. In variable elimination, what does "eliminating" a variable mean?

A. Removing the variable from the network

B. Setting the variable to its most likely value

C. Summing over the variable in factors

D. Multiplying the variable's probabilities

Topic : Bayesian

#Q. In Gibbs sampling, if we want to estimate the probability $P(X|Y, Z)$, how many variables are sampled in each iteration?

A. All variables

B. Only variable X

C. Only variables Y and Z

D. Variable X, Y, and Z

Topic : Bayesian

#Q. What is the primary goal of approximate inference through sampling?

A. To guarantee exact inference results

B. To find the most likely hypothesis

C. To provide an approximation of the true distribution

D. To sample from a uniform distribution

Topic : Bayesian

#Q. In Gibbs Sampling, how are variables updated in each iteration?

A. All variables are updated simultaneously.

B. Only one variable is updated, and the rest remain fixed.

C. Variables are updated in pairs.

D. Variables are updated in a random order.

THANK - YOU