# Data Science & Artificial Intelligence

## Algorithms

**Test Series 1500+**

**Lecture - 03**


By- Aditya sir

# Topic : Dynamic Programming

#Q. The Flyod-Warshall algorithm for all pairs shortest paths computation is based on

APSP

A. Greedy method

B. Divide and Conquer

C. Dynamic Programming (DP)

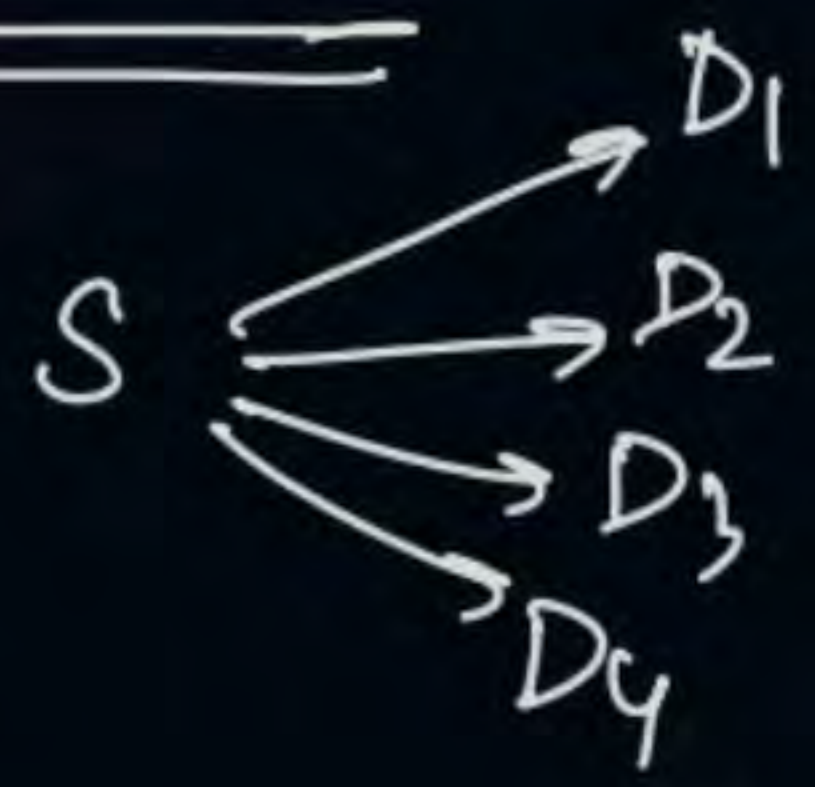
D. Heap algorithm

Dynamic Programming

# ★ Shortest Path Algos :

1) Single Pair Shortest Path $A \longrightarrow B$

2) Single Source Shortest Paths : (SSSP)



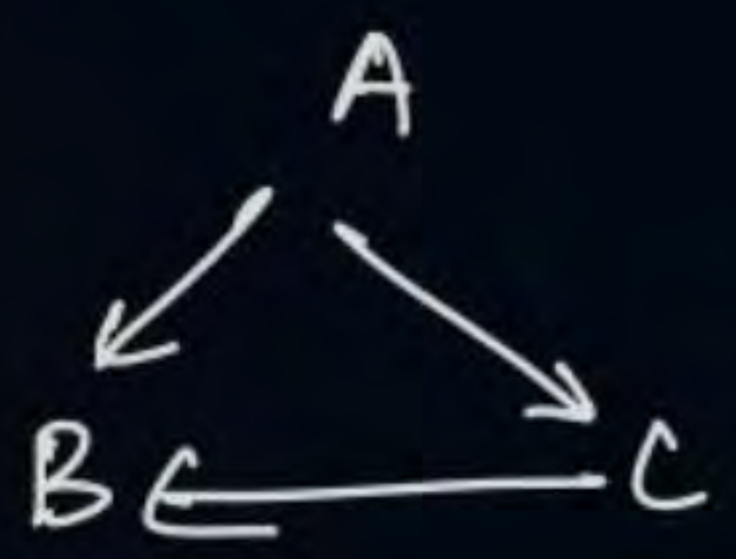

→ Dijkstra SSSP ( Greedy )

↳ a) Matrix based ⟶ only min cost

b) Spanning Tree based ⟶ gives min cost + path.

→ Bellman Ford ⟶ SSSP ⟶ Dynamic Programming

# 3) All pairs Shortest Paths




$A \to B$
$A \to C$

$B \to A$
$B \to C$

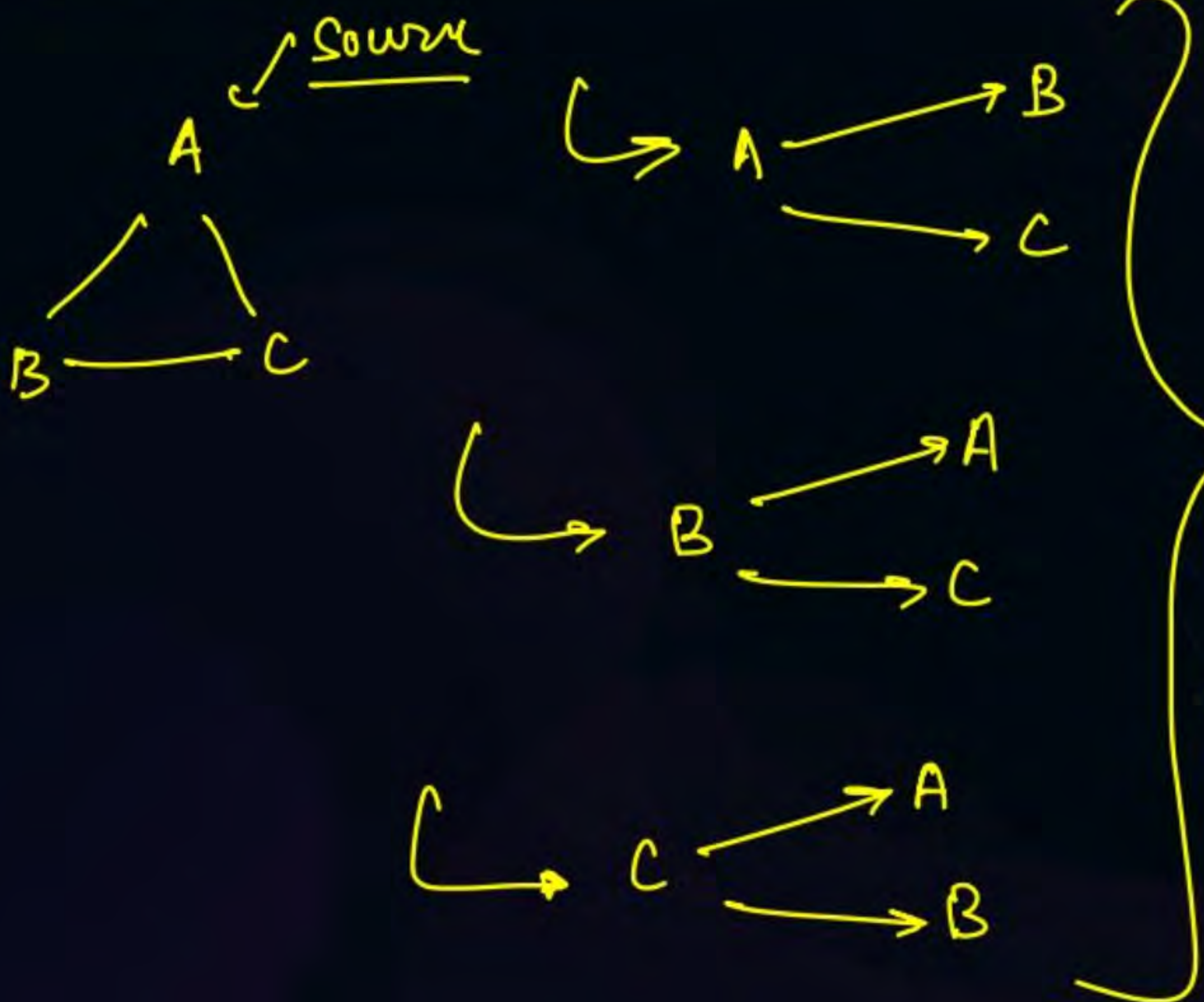
$C \to A$
$C \to B$

APSP

Floyd Warshall

(DP)

Can we solve APSP using Dijkstra SSSP? Yes

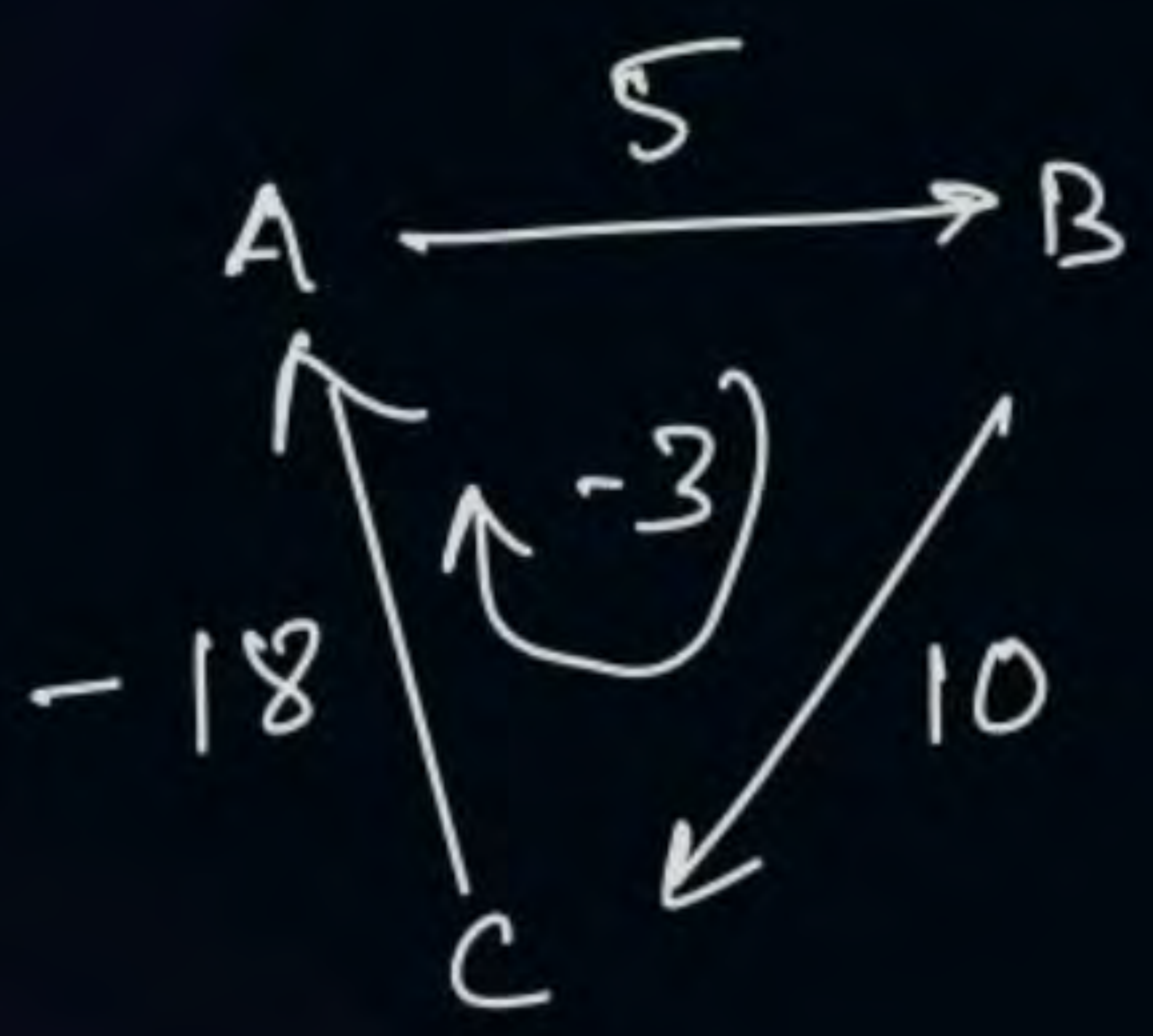



APSP

* Dijkstra starting at every node.

# Dijkstra vs Bellman-Ford

| | (G) Dijkstra | (DP) Bellman Ford |
|---|---|---|
| 1) when all pos wt edges | ✓ | ✓ |
| 2) when there are -ve wt edges but no -ve wt cycle. | Dijkstra × (may or may not work) | ✓ |
| 3) when there are -ve wt edges as well as -ve wt cycle reachable from source | × | × |




$5 + 10 - 18$

$A \to B : 5$

$A \to C : 15$

$5 \to 2 \to -1 \to -4 \to -7 \; \ldots \ldots \; -\infty$

$15 \to 12 \to 9 \to 6 \; \ldots \ldots \; -\infty$

[MCQ]

#Q11. Consider a connected weighted graph $G = (V, E)$, where $|V| = n$, $|E| = m$, if all the edges have distinct positive integer weights, then the maximum number of minimum weight spanning trees in the graph is ?

MCSTs

max count of MCST

A) n

B) m

C) $\boxed{1}$ ✓

D) $n^{n-2}$ ✗

Distinct pos wts

MCST unique (structure + cost)

## * Minimum Cost Spanning Tree (MCSTs)

1) n - vertices MCST ⟶ (n-1) edges always

2) Cost of the MCST by both Prims and Kruskal will always be same.

Min Cost → unique

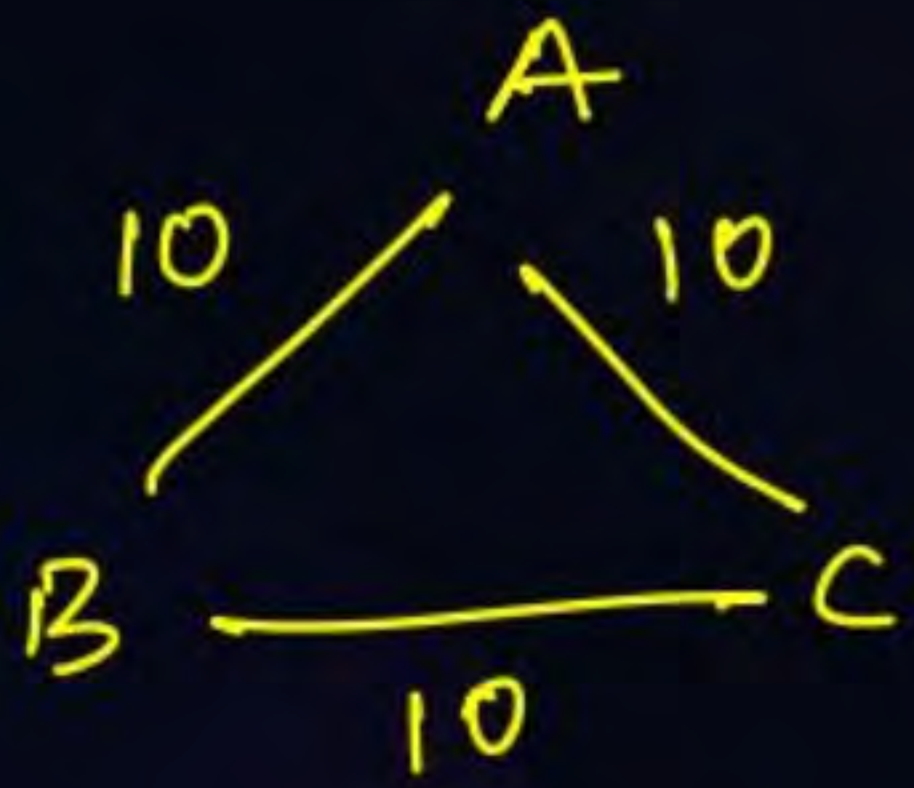
3) Structure of MCST by both Prims and Kruskal

1) Same — all edges are of distinct cost

2) Can be diff → when duplicate wt edges are present.

# Size of Soln Space for MCST

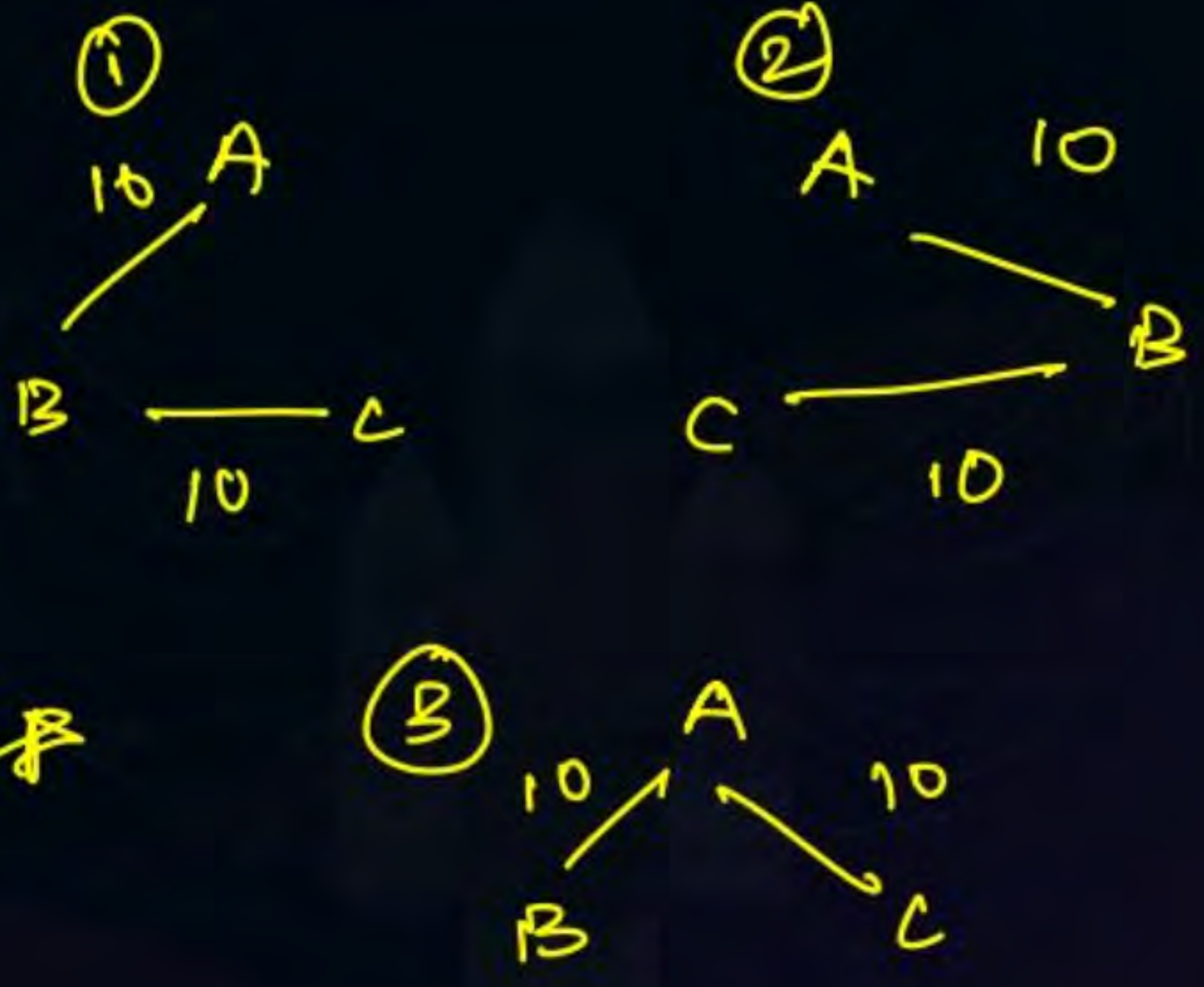
Max no. of Spanning Trees $= \boxed{n^{(n-2)}}$




MCSTs

3 MCSTs

$n=3, \quad 3^{(3-2)} = 3^1 = 3$



## ★ Complete graph with all edges of equal wt

$$\left\{\begin{array}{c}\text{every Spanning Tree} \\ \text{is a} \\ \text{MCST in} \\ \text{this Case}\end{array}\right\} \Rightarrow n^{(n-2)} \quad \text{MCSTs}$$

$$n=4 \qquad 4^{(4-2)} = \boxed{16}$$

[NAT]

#Q12. What is the cost the minimum spanning tree for the graph shown below using prim's algorithms?

MCST


(Q-2) Count of MCST


MCSTs (2) = $2C_1$

Soln :- 1) Prim's Algo :


Cost = 1 + 2 + 1 + 1 + 2 + 3

= 3 + 4 + 3 = $\boxed{10}$

## 2) Appr 2 : Kruskal Algo

$n = 7$

$MCST \Rightarrow e = n-1 = 6$


$Cost = 1+1+1+2+2+3 = \boxed{10}$

* 3) Dijkstra MCST (Bonus)



cost = 10

# Topic : Analysis of algorithm

#Q. Consider the following functions:

$f(n) = 3^\wedge(n)$

$g(n) = n^{\sqrt{n}}$

(Asymptotic notations Comparison)

Which of the following is correct?

A. $f(n) = O(g(n))$ ✗

B. $f(n) = \Omega(g(n))$ ✓

C. $f(n) = \theta(g(n))$ ✗

D. None of these ✗

Ans: B

Soln: $f(n) = 3^n$

$g(n) = n^{(\sqrt{n})}$

$$3^n > n^{\sqrt{n}}$$

Taking log both sides.

$$n \cancel{\log} 3 \qquad \sqrt{n} \log n$$

$$\sqrt{n} * \cancel{\sqrt{n}} \qquad \cancel{\sqrt{n}} \log n$$

$$\sqrt{n} > \log n$$

$$\sqrt{64} = 8$$

$$\log_2 64 = \underline{\underline{6}}$$

$\uparrow$

$$f > g$$

$$f = \Omega(g)$$

$$f = \omega(g)$$

$$g = O(f)$$

$$g = o(f)$$

## Topic : Analysis of algorithm

#Q. Sort the functions in ascending order of asymptotic(big-O) complexity.
$f_1(n) = n$, $f_2(n) = (0.5)^n$, $f_3(n) = n^{\log n}$, $f_4(n) = 5000$, $f_5(n) = (\log n)^{\log n}$

A. $f_4(n), f_2(n), f_1(n), f_5(n), f_3(n)$ ✗

B. $f_2(n), f_1(n), f_4(n), f_5(n), f_3(n)$ ✗

C. $f_2(n), f_4(n), f_1(n), f_5(n), f_3(n)$ ✓

D. $f_1(n), f_5(n), f_4(n), f_3(n), f_2(n)$ ✗

Asymptotic growth

$f_1 < f_5$

$f_1 < f_3$

$f_5 < f_3$

$f_1 < f_5 < f_3$

$n > 5000$ $f_1 > f_4$ $n^{\log n} > \log n^{\log n}$

Soln :-

$F_1(n) = n \longrightarrow$ Poly

$F_2(n) = \boxed{(0.5)^n} \longrightarrow$ Expo

$F_3(n) = n^{(\log n)} \longrightarrow$ Expo

$F_4(n) = \underline{5000} \longrightarrow$ Const

$F_5(n) = (\log n)^{\log n} \longrightarrow$ Expo

$a^n \longrightarrow$ inc $a > 1$

dec $a < 1$

$(1/2)^2 > \left(\frac{1}{2}\right)^3 > \cdots 1/2^{\infty}$

$F_2 < F_4$

$f_1$ vs $f_5$

$$n < (\log n)^{\log n}$$

Taking log

$\log n \qquad \log n * \log\log n.$

Let $\log n = x.$

$x \qquad x * \log x$

$1 < \log x$

# Topic : Dynamic Programming

12.5 %

#Q. How many binary search trees are possible for 3 elements?

→ Catalan number

$$\boxed{\frac{1}{(n+1)} * {}^{2n}C_n}$$

$n = 3$

$$\Rightarrow \frac{1}{4} * {}^{6}C_3$$

$$= \frac{1}{4} \times \frac{6!}{3! \times 3!}$$

BT (unlabelled Trees)

$$= \frac{1}{\cancel{4}} \times \frac{\cancel{6} \times 5 \times \cancel{4} \times \cancel{3!}}{\cancel{3!} \times \cancel{3!}}$$

$$= \boxed{5}$$

Manually:-

BSTs ?

$n = 4$

$$^{2n}C_n * \frac{1}{(n+1)}$$

$$= \frac{1}{5} \times {}^{8}C_4$$

$$\frac{1}{\cancel{5}} \times \frac{8 \times 7 \times 6 \times \cancel{5} \times \cancel{4!}}{4! * \cancel{4!}}$$

$$= \frac{\cancel{8}^{2} \times 7 \times \cancel{6}}{\cancel{4} \times \cancel{6}}$$

$$= \boxed{14}$$

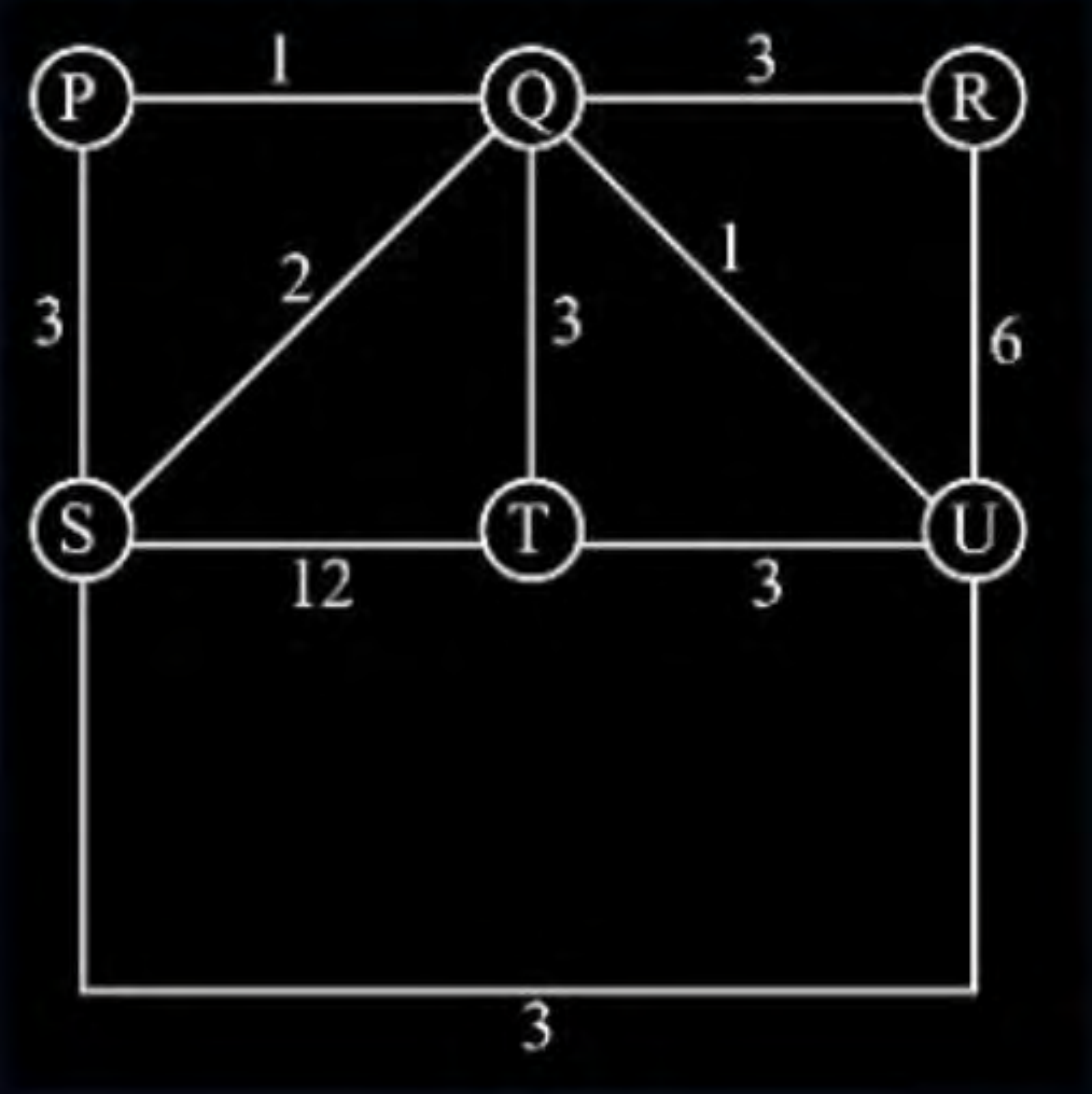
[NAT]

#Q13. Consider the following graph G

If the cost of MST is P and number of such spanning trees are Y then the value of P+Y is____.

count

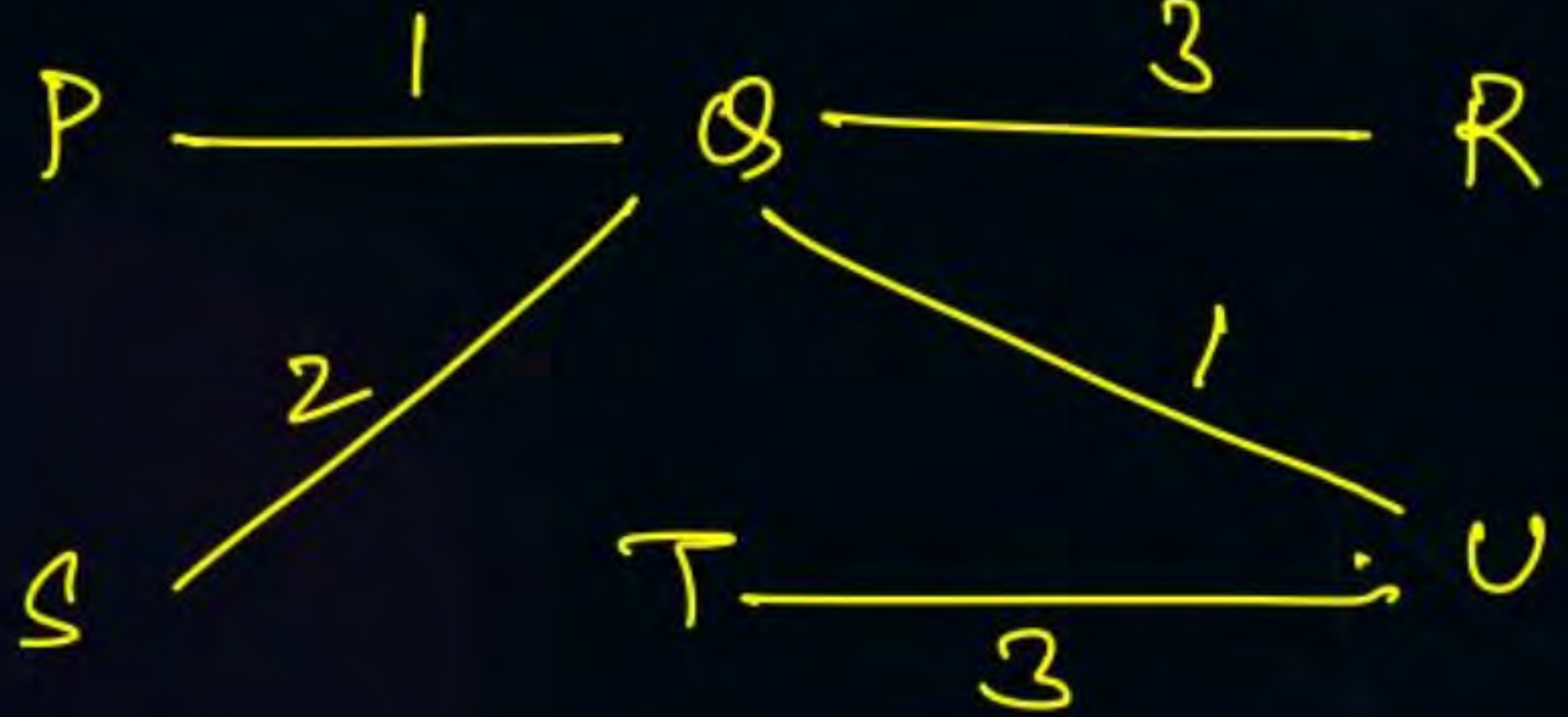
MCST

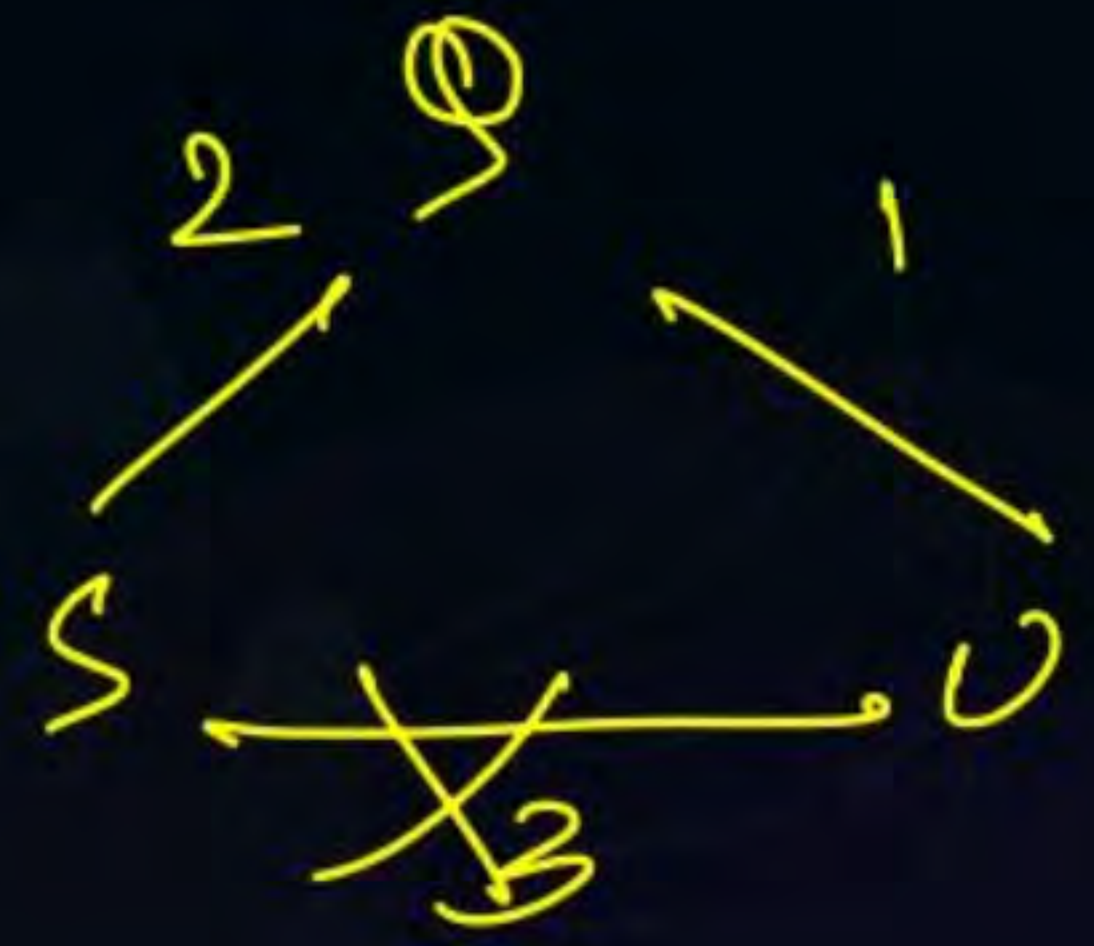


Soln:- Cost of MCST (P)

$n = 6$

$MCST = e = n - 1 = \boxed{5}$

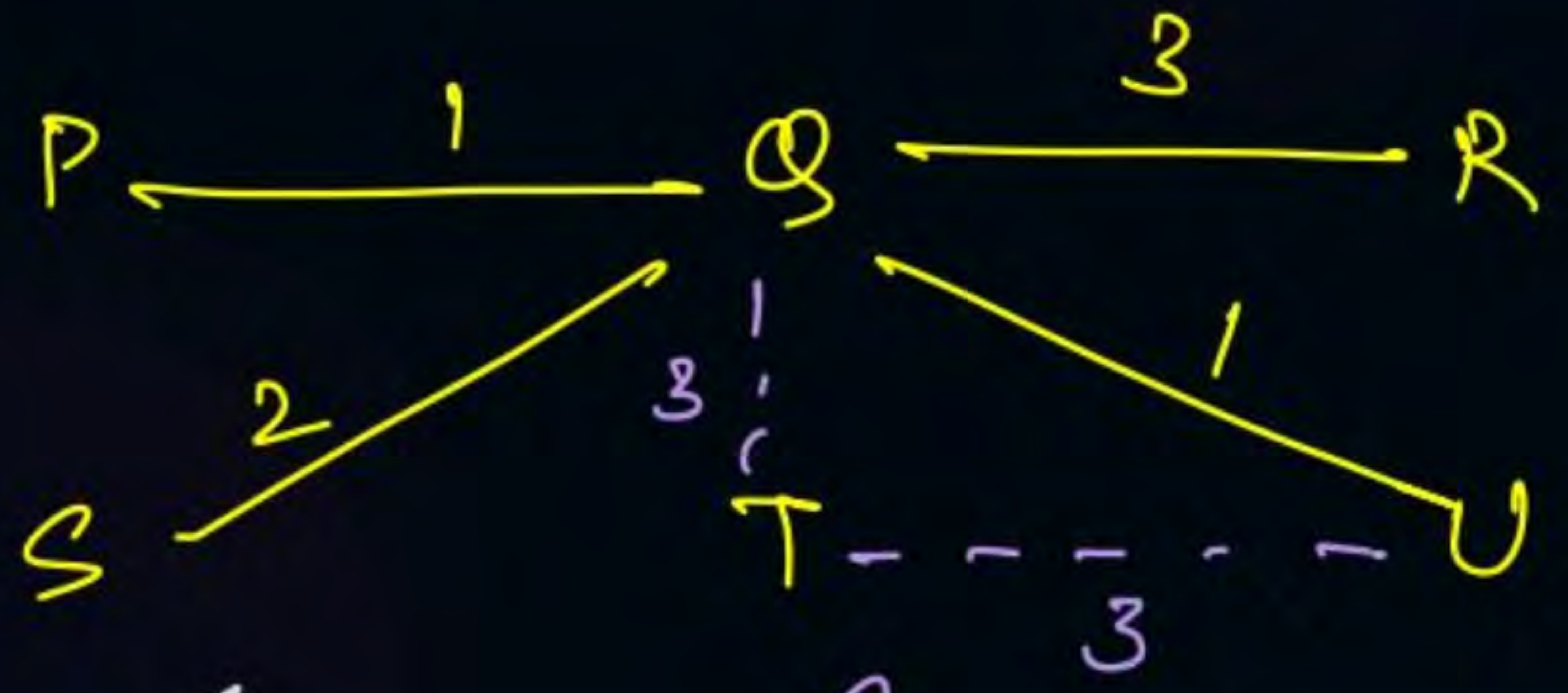


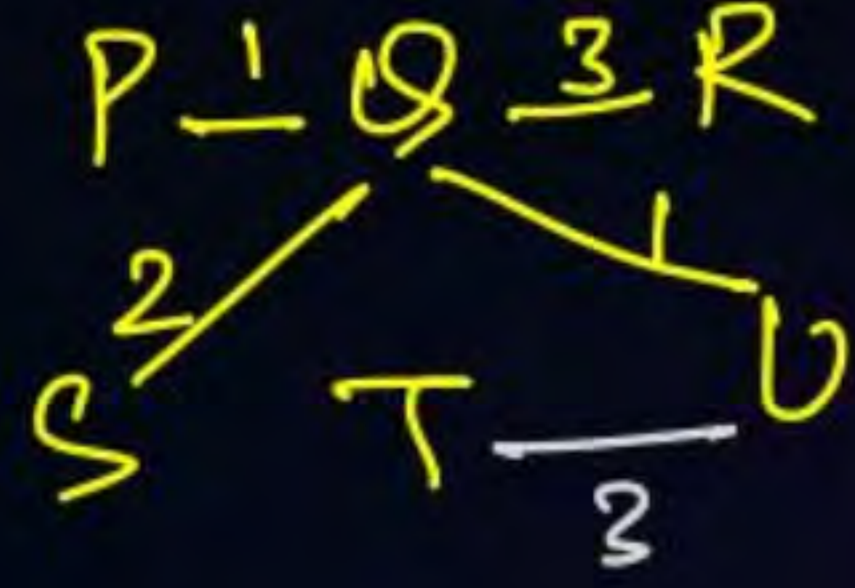




$$\text{Cost of MST} = P = 1 + 1 + 2 + 3 + 3$$
$$= 4 + 6$$
$$= \boxed{10}$$

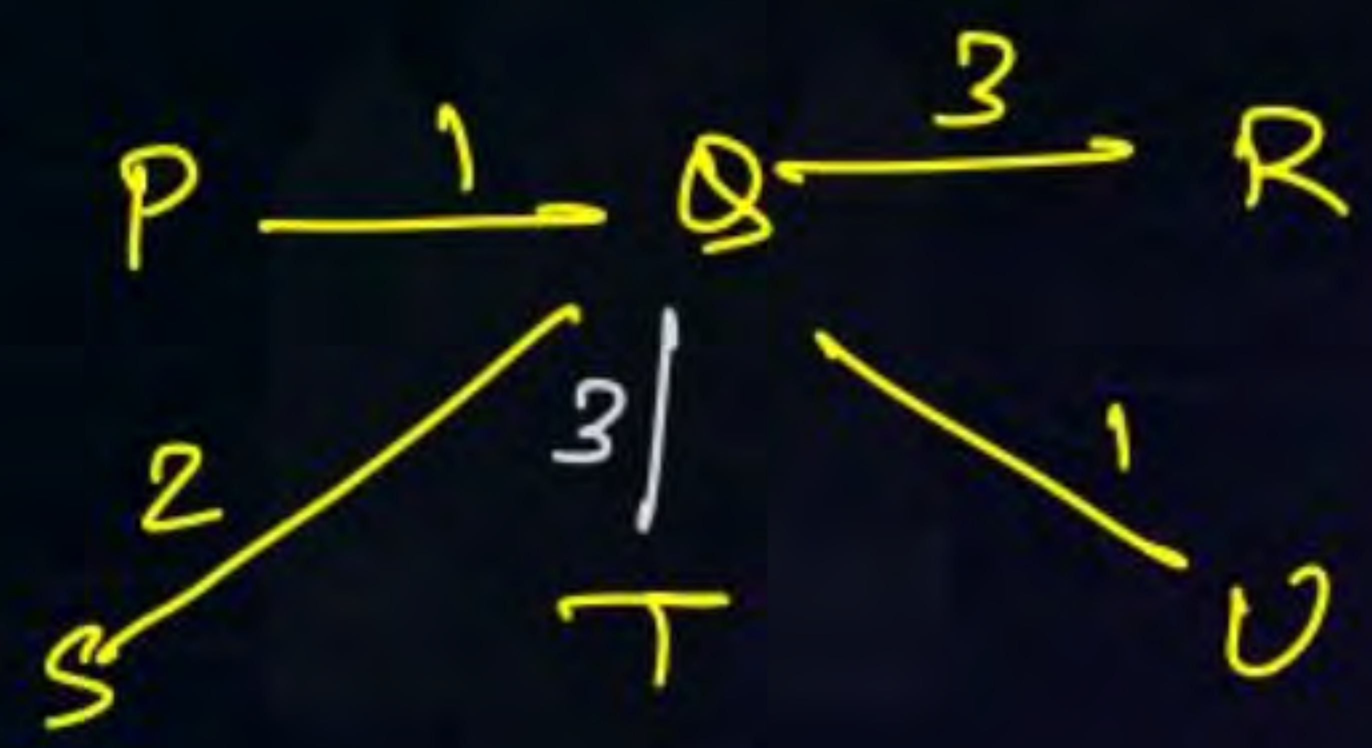
## Count of MCST:

$y = 2$




$^2C_1 = 2$




2 MCSTs

$P = 10$

$Y = 2$

40%

ans = $P + Y = 10 + 2$

$= \boxed{12}$

#Q5. Merging 4 sorted files having 200,100, 250, 150 records will take how many comparisons to be merged into a single sorted file, if 2 files are merged at a time?

→ V. Imp :- Greedy

↓

(Optimal Merge Pattern + Merging Algo)

default

Ans: 1397

50%

Soln :-

$F_1 = 200$ — 3 (no. of records in file $f_1$)

~~$F_2$~~ $= 100$ — 1 " $F_2$

$F_3 = 250$ — 4 " $F_3$

~~$F_4$~~ $= 150$ — 2 " $F_4$

OPM

~~200~~ ~~250~~ 250


optimal 2-way Binary merge tree.


Misconcept

2 Types of Questns:

a) Total no. of Record Movements

b) No. of elem Comparisons (Merging Algo)

Sln: a) Total Record Movmts

$= 700 + 450 + 250$

$= 700 + 700$

$= \underline{1400}$

b) Elem Comparisons

$\Rightarrow (700-1) + (450-1) + (250-1)$

$= 1400 - 3$

$= \boxed{1397}$

a) Record Movements

$\Rightarrow$

$$\boxed{m} \qquad \boxed{n}$$

$$\boxed{m+n}$$

m+n Rec movements

b) Elem Comparisons

$$\boxed{m} \qquad \boxed{n}$$

$$\boxed{m+n}$$

$BC \rightarrow \min(m, n)$

☆ $\boxed{WC} \longrightarrow (m+n-1)$

# Topic : Analysis of algorithm

#Q. Consider the following functions:

$4^n = 2^n * 2^n$

$n! \approx n^n$

$f_1 = 2^{2^n}$

$f_2 = n!$

$f_3 = 4^n$

$f_4 = 2^n$

$f_1 > f_2 > f_3 > f_4$

What is the correct Decreasing order of above functions?

A. $f_1\ f_4\ f_3\ f_2$

B. $f_4\ f_2\ f_3\ f_1$ ×

C. $f_1\ f_2\ f_3\ f_4$ ✓

D. $f_4\ f_3\ f_2\ f_1$ (incr)

$2^{2^n}$ vs $n^n$

Taking log.

$2^n > n \log n$

## Topic : Dynamic Programming

#Q. How many distinct function calls are exists in Fibonacci of $n^3$ elements?

A. $O(n)$

B. $O(n^2)$

C. $O(2^{n^2})$

D. $O(n^3)$

O(input size)

Ans: D

Soln :-

$$Fibo(n) = Fibo(n-1) + Fibo(n-2)$$

$fibo(5) \longrightarrow \underline{6}$

$fibo(n) \longrightarrow \underline{n+1}$

$Fibo(n^2) \longrightarrow \underline{n^2+1}$

$Fibo(n^3) \longrightarrow \underline{n^3+1}$

- $\boxed{Fibo(5)}$
  - $\boxed{Fibo(4)}$
    - $\boxed{Fibo(3)}$
      - $\boxed{Fibo(2)}$
        - $\boxed{F(1)}$
        - $\boxed{F(0)}$
      - Fibo(1)
    - Fibo(2)
      - F(1)
      - F(0)
  - Fibo(3)
    - Fibo(2)
      - F(1)
      - F(0)
    - Fibo(1)

Distinct Function calls

= F(5).
F(4).
F(3).
F(2).
F(1).
F(0).

# THANK - YOU

**Telegram Link for Aditya Jain sir:**
**https://t.me/AdityaSir_PW**