GATE

CRASH COURSE

ALL BRANCHES

Engineering Mathematics

Linear Algebra (Part 02)
(Lec 02)

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Topics *to be covered*

LINEAR ALGERA (LEC-2)

1. RANK
2. VECTORS (LD/LI)
3. Non Homogeneous system

(*) Singular Mat : $|A| = 0$ & Non Sing Mat : $|A| \neq 0$

RANK : It is the order of Non Singular $\boxed{\text{submatrix}}$ that can exist in a given Mat

Eg $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & -1 \end{bmatrix}_{3\times 3} \Rightarrow |A| \neq 0$ (Non Sing) so $\rho(A) = 3$

Submatrix → Matrix obtained by deleting some Rows, some Columns or both is called submatrix.

Eg $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ -1 & 2 & -3 \end{bmatrix}_{3\times 3}$ $\because |A| = 0$ i.e $\rho(A) \neq 3$

$\rho(A) \neq 2$ also $\because$ all the submatrices of 2×2 are singular

Hence $\rho(A) = 1$

eg $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & -6 \\ -1 & 2 & 7 \end{bmatrix}_{3\times 3}$ $\Rightarrow \because |A| = 0 \Rightarrow \rho(A) \neq 3$

2×2

$\exists$ a submatrix of the type $A_1 = \begin{bmatrix} 4 & -6 \\ 2 & 7 \end{bmatrix}_{2\times 2}$ By deleting R_1 & C_1

which is Non sing $\because |A_1| \neq 0$ so $\rho(A)$ = order of A_1 = Two

eg $A = \begin{bmatrix} 2 & -1 & 3 & 4 & 1 \\ -2 & 1 & -3 & -4 & -1 \\ 0 & 0 & 7 & 0 & 0 \end{bmatrix}_{3\times 5}$

2×2

$\rho(A) \neq 5$ or 4 $\because$ it is not possible to obtain a submatrix of 5×5 & 4×4

$\rho(A) \neq 3$ $\because$ all the submatrices of 3×3 are singular

Most of the Matrices of order 2×2 are also, But $\exists$ submatrix of the type

$A_1 = \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix}_{2\times 2}$ By deleting C_1, C_4, C_5 & R_1 | i.e. A_1 is Non sing so $\rho(A)$ = two

$\because |A_1| \neq 0$

Properties of RANK :-

① $\boxed{\rho(A_{m\times n}) \le \min\{m,n\}}$ eg $\rho(A_{3\times 5}) \le 3$

eg $\rho(A_{m\times n}) = n$ & $\rho(B_{n\times p}) = p$ then $\rho(AB) = ?$

$\Rightarrow$ $n < m$ — ① $\quad p < n$ — ② $\quad$ So $\underset{m\times p}{\rho(AB)} \le p$ ~~Ans~~

$\Rightarrow \boxed{p < n < m}$

② if $A_{n\times n}$ st A is Non Sing $\Rightarrow \rho(A) = n$ & $\rho(\text{Null Mat}) = 0$ (defined)

③ So $\boxed{1 \le \rho(A_{m\times n}) \le \min\{m,n\}}$ $\quad$ eg $O = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(O) = 0$

④ $\rho(A) = \rho(A^T) = \rho(A^\theta) = \rho(A^{-1}) = \rho(AA^T) = \rho(AA^\theta)$

(5) $\boxed{\rho(AB) \le \min\{\rho(A), \rho(B)\}}$ i.e. Rank of the product can never exceed their Individual Rank.

eg $\rho(A_{5\times7}) = 4$, $\rho(B_{7\times6}) = 3$ then $\rho(AB)_{5\times6} = ?$ (≤ 5) (≤ 3)

(*) $\rho(\underset{=A}{\text{Row Mat}})_{1\times n} = 1$

$\rho(\underset{=B}{\text{Column Mat}})_{n\times1} = 1$

$\rho(AB)_{1\times1} \Rightarrow \rho(AB) \le 1$?

$(BA)_{n\times n} \Rightarrow \rho(BA) \le n$? (≤ 1)

(*) $\boxed{\text{E-operations do not alter the Rank of Matrix}}$ i.e. we are free to apply all three E-operations while calculating Rank.

But take care, if we are calculating Determinant, we can only apply 3rd E-operations

E-Operations:

① $R_i \leftrightarrow R_j$

② $R_i \to K R_i$

③ $R_i \to R_i + K R_j$

Equivalent Mat :- Matrix obtained by applying E-operations are equivalent to each other

(*) Equivalent Matrices have same Rank.

Echelon form :- Mat $A_{m\times n}$ is said to be in an E-form if

① No. of zeros before the 1st the Non zero element in a Row should be in an Increasing order in the subsequent Rows.

② All zero Rows (if exist) should occur at a bottom of a Matrix

(*) ρ (Echelon form) = No. of Non zero Rows.

eg

$$\begin{bmatrix}1&-2&3&4\\0&1&2&-1\\0&0&4&2\\0&0&0&7\end{bmatrix}, \begin{bmatrix}1&-2&3&0\\0&-3&2&1\\0&0&0&7\\0&0&0&0\\0&0&0&0\end{bmatrix}, \begin{bmatrix}2&-1&4&3\\0&3&0&0\\0&0&5&0\\0&0&0&-1\end{bmatrix}, \begin{bmatrix}2&-1&4&3\\0&0&5&7\\0&0&0&2\end{bmatrix}$$

$\rho(A)=4$ ✓ $\rho(A)=3$ ✓ $\rho(A)=4$ ✓ $\rho(A)=3$ ✓

$$\begin{bmatrix}-2&4&0&1\\0&0&2&1\\0&0&-3&0\\0&0&0&0\end{bmatrix}, \begin{bmatrix}1&2&-3&4\\0&0&0&0\\0&-1&2&1\\0&0&3&2\end{bmatrix}$$

✗ $\rho(A)=\,?$ ✗ $\rho(A)=\,?$

it is too early to find Rank of Q5 & Q6 ∵ these Matrices are not in E Form.

(*) By Using E-Row operations, we can convert any Mat into an E-Form.

L-D & L-I vectors

vector → Any Column Mat is considered as vectors i.e $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$

OR

Any Row Mat can also be considered as vector. $X = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix}_{1\times n}$

eg $\vec{AB} = 2\hat{i} - 3\hat{j} + 4\hat{k} \Rightarrow X = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} = 3\text{-vector}$

ie $A_{3\times 4} = \begin{array}{c} \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \\ \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \end{array}$ has 3 Row vectors & 4 Column Vectors

Linearly Dependent Vectors → If ∃ linear relationship b/n the vectors then vectors are called (LD)

Linearly Independent Vectors → If we have no Linear relationship b/n the vectors then vectors are called (LI)

eg $X_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $X_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, $X_3 = \begin{bmatrix} -2 \\ 6 \\ -10 \end{bmatrix}$ then X_1, X_2, X_3 are (LD)

(M-I) By observation, $X_1 - X_2 = -\frac{1}{2} X_3$

or $\boxed{2X_1 - 2X_2 + X_3 = 0}$

∴ linear Relationship exist so vectors are (LD)

(M-II) $A = [X_1\, X_2\, X_3] = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -10 \end{bmatrix}_{3\times3}$

∵ $|A| = \ldots\ldots = 0$

So vectors are (LD)

Methods to check the Nature of Vectors → Let the given vectors are $x_1, x_2, x_3, \dots, x_r$

then Consider a Mat $A = [x_1\ x_2\ x_3 \dots x_r]$

① **General Method** → (always applicable)

(i) if $\rho(A) =$ No. of Vectors then vectors are LI

(ii) if $\rho(A) <$ " " " " " LD

② **Matrix Method** → (applicable only when A is sq Mat)

(i) if $|A| \neq 0 \Rightarrow$ vectors are LI

(ii) if $|A| = 0$ " " LD

Another Def^n of Rank → $\rho(A) =$ (No. of LI Row vectors) or (No of LI Column vectors)

eg Def 1 → $\rho(A_{5\times6}) = 3$ → there exist at Most 3 LI Row vectors
→ " " " " 3 LI Column vectors

Def 2 → $\rho(A_{5\times6}) = 3$ → ∃ at least one Non sing Submatrix of 3×3
→ all the Submatrices of 4×4 & 5×5 are sing.

Q. The vectors $(1\ 2\ -1)'$, $(2\ 3\ 4)'$, $(4\ -3\ 2)'$, $(1\ 1\ -2)'$ are ?

Sol: $X_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, X_3 = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}, X_4 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ So $A = [X_1\ X_2\ X_3\ X_4]$

i.e. $A = \begin{bmatrix} 1 & 2 & 4 & 1 \\ 2 & 3 & -3 & 1 \\ -1 & 4 & 2 & -2 \end{bmatrix}_{3\times 4}$

$\rho(A) \le 3$ ($\because$ we have no submatrix of 4×4)

i.e. $\rho(A) < 4$ i.e. $\rho(A) <$ No. of vectors $\Rightarrow$ vectors are LD

Q. $A = [X_1\ X_2\ X_3] = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & -1 & 1 \\ 3 & 2 & -1 \\ & & 0 \end{bmatrix}_{4\times 3} \Rightarrow \rho(A) \le 3$

$\rho(A) = 3 \Rightarrow$ LI

$\rho(A) < 3 \Rightarrow$ LD

If $\rho(A_{6\times7}) = 4$ → $\exists$ at least one Non Sing Submatrix of $\underline{4\times4}$

→ Every Sq Submatrix of 5×5 & 6×6 are Sing.

$\rho(A_{6\times7}) = 4$ → A will have at Most 4 LI Row vectors

→ " " " " " 4 LI Column vectors

(Learn By ♡)

$|A| \neq 0 \Rightarrow$ LI

$|A| = 0 \Rightarrow \underline{LD}$

If A and B are matrices of same order then

(a) $\rho(A+B) \leq \rho(A)+\rho(B)$

(b) $\rho(A+B) \geq \rho(A)+\rho(B)$

(c) $\rho(A+B)=\rho(A)+\rho(B)$

(d) None of the above

$$\rho(A+B) \leq \rho(A)+\rho(B)$$

Applying elementary transform to a matrix its rank ________.

(a) increases

(b) decreases

(c) does not change

(d) None of the above

If for a matrix, rank equals both the number of rows and number of columns, then the matrix is called

(a) non-singular (b) singular

(c) transpose (d) minor

if $A_{n\times n}$ & $\boxed{\rho(A) = n}$ i.e $\rho(A) = \text{No of Rows}$

$\rho(A) = \text{No of Columns}$

so $|A| \neq 0$

MCQ

The rank of a 3 × 3 matrix C (= AB), found by multiplying a non-zero column matrix A of size 3 × 1 and a non-zero row matrix B of size 1 × 3, is

(a) 0 (b) 1

(c) 2 (d) 3

$$\left.\begin{array}{l} A_{3\times1} \Rightarrow \rho(A)=1 \\ B_{1\times3} \Rightarrow \rho(B)=1 \end{array}\right\} \Rightarrow C = AB = A_{3\times1}B_{1\times3} = (AB)_{3\times3}$$

$\rho(C) \leq 3$ But $\rho(C) \neq 3$ or 2

so $\rho(C) \leq 1$

(*) if it is MSQ then Ans are (a) & (b) both

The rank of the matrix $\begin{bmatrix} 0 & i & -i \\ -i & 0 & i \\ i & -i & 0 \end{bmatrix}$ is ______ 3×3

(a) 1 ✓(b) 2

(c) 3 (d) 4

A is skew symm Mat of odd order i.e of 3×3 so $|A| = 0$

i.e $\rho(A) \neq 3$

$$A_1 = \begin{bmatrix} i & -i \\ 0 & i \end{bmatrix}_{2\times 2} \Rightarrow |A_1| = i^2 - 0 = -1$$

i.e A_1 is Non sing $\Rightarrow \rho(A) = 2$

The rank of 4 × 4 skew-symmetric matrix

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{bmatrix}$$ is ____.

4×4

$|A| =$ perfect square

$|A| = 4 = (2)^2$ i.e $|A| \neq 0$ i.e A is Non sing.

$\rho(A) = 4$

The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$ is ______.

(a) 1 (b) 2

(c) 3 (d) 4

$$\xrightarrow[R_3 \to R_3 - R_2]{R_4 \to R_4 - R_3} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$\xrightarrow[R_4 \to R_4 - R_2]{R_3 \to R_3 - R_2} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{4\times 4} = A_1$$

2×2

$\rho(A_1) = \text{two}$

$\because \exists$ Non Sing Submatrix of 2×2 in A_1

$\because A \sim A_1 \Rightarrow \rho(A) = 2$

The rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 3 & 6 & 9 & 12 \\ 0 & 5 & 10 & 15 & 20 \end{bmatrix}$ is

________.

(a) 0 (b) 1

(c) 2 (d) 3

$$\xrightarrow{R_3 \to R_3 - R_2} \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 3 & 6 & 9 & 12 \\ 0 & 2 & 4 & 6 & 8 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 3R_1} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = A_1 \quad \because \rho(A_1) = \text{one}$$

$$\because A \sim A_1 \Rightarrow \rho(A) = \text{one}$$

Let $P = \begin{bmatrix}1 & 1 & -1\\2 & -3 & 4\\3 & -2 & 3\end{bmatrix}$ and $Q = \begin{bmatrix}-1 & -2 & -1\\6 & 12 & 6\\5 & 10 & 5\end{bmatrix}$ be two matrices. Then the rank of P + Q is______.

$$P+Q = \begin{bmatrix}0 & -1 & -2\\8 & 9 & 10\\8 & 8 & 8\end{bmatrix}_{3\times 3} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix}0 & -1 & -2\\8 & 9 & 10\\0 & -1 & -2\end{bmatrix} \Rightarrow |P+Q| = 0$$

2×2

ie $P+Q$ is sing so $\rho(P+Q) \neq 3$

so $\rho(P+Q) = 2$ $\because \exists$ Non sing submatrix of 2×2

The rank of a matrix A is defined as ______.

(a) The number of zero rows in A.

(b) The number of linearly dependent rows (or columns) in A.

(c) The number of linearly independents rows (columns) in A.

(d) The number of such columns which has been obtained by linear combination of some other columns in A.

$q_1, q_2, q_3 \cdots q_m$ are n-dimensional vecors with $m < n$. This set of vectors is linearly dependent. Q is the matrix with $q_1, q_2, q_3 \cdots q_m$ as the columns. the rank of Q is

(a) less than m (b) m

(c) between m and n (d) n

$$Q = \begin{bmatrix} q_1 & q_2 & q_3 & \cdots & q_m \end{bmatrix}_{n\times m} \quad \text{where } q_i = \begin{bmatrix} \vdots \end{bmatrix}_{n\times 1}$$

$\rho(Q) <$ No of vectors ($\because$ vectors are LD)

$$\boxed{\rho(Q) < m}$$

Consider the following two statements:

I. The maximum number of linearly independent column vectors of a matrix A is called the rank of A. (T)

II. If A is an $n \times n$ square matrix, it will be nonsingular if rank $A = n$. (T)

With reference to the above statements, which of the following applies?

(a) Both the statements are false

(b) Both the statements are true

(c) I is true but II is false

(d) I is false but II is true

System of Equations

Non Homogeneous system $(AX = B)$

Homogeneous system $(AX = 0)$

eg
$$
\left.\begin{aligned}
2x - y + 4z &= 3\\
x + 2y - 2z &= -1\\
3x - y + 4z &= 0\\
3x + 2y + 3z &= 2
\end{aligned}\right\}
\begin{bmatrix} 2 & -1 & 4 \\ 1 & 2 & -2 \\ 3 & -1 & 4 \\ 3 & 2 & 3 \end{bmatrix}_{4\times 3}
\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1}
=
\begin{bmatrix} 3 \\ -1 \\ 0 \\ 2 \end{bmatrix}_{4\times 1}
$$

$$\boxed{A_{4\times 3} X_{3\times 1} = B_{4\times 1}}$$

Consider $\boxed{A_{m\times n} X_{n\times 1} = B_{m\times 1}}$

- if $m > n \Rightarrow$ System is overdetermined
- if $m = n \Rightarrow$ " is equally "
- if $m < n \Rightarrow$ " is under "

$A_{m\times n}$ → Coeff Mat

$X_{n\times 1}$ → Solution of System / Variable Mat / unknown vector

& $[A : B]$ = Augmented Mat

Methods of Solving $(AX = B)$

① RANK method (applicable for $m > n, m = n, m < n$)

(i) if $\rho(A) = \rho(A:B)$ = No of variables $\Rightarrow$ (unique sol exist)

(ii) if $\rho(A) = \rho(A:B)$ < " " $\Rightarrow$ (∞ sol ")

(iii) if $\rho(A) \neq \rho(A:B) \Rightarrow$ No sol exist.

Matrix Method (applicable only for $m = n$)

(i) if $|A| \neq 0 \Rightarrow$ unique sol exist

(ii) if $|A| = 0$ & $(\mathrm{adj}A)B = 0$ then ∞ sol.

(iii) if $|A| = 0$ & $(\mathrm{adj}A)B \neq 0$ then No sol.

(1) Consistent System $\rightarrow$ If there exist sol. (whether unique or ∞)

Inconsistent " $\rightarrow$ If there exist No sol.

(2) N. Condⁿ for a system to be consistent is $\boxed{\rho(A) = \rho(A \vdots B)}$

If the system of n linear equations in n unknowns has more than one solution, then its associated matrix ______.

(a) has rank $< n$ (b) has rank $= n$

(c) has rank $> n$ (d) has rank one

given, $A_{n\times n}$ & $AX = B$ has ∞ sol^n $\Rightarrow \rho(A) = \rho(A:B) < n$

$A_{3\times 4} \Rightarrow \rho(A) = 3$

$[A \vdots B]_{3\times 5} \Rightarrow \rho(A \vdots B) = 3$

$\Rightarrow \infty$ sol.

Let M be an $m \times n$ $(m < n)$ matrix with rank m. Then

(a) for every b in R^m, $Mx = b$ has unique solutions.

(b) for every b in R^m, $Mx = b$ has a solution but it is not unique.

(c) there exists $b \in R^m$ for which $Mx = b$ has no solution.

(d) None of the above

$Mx = B \Rightarrow$ unique ✗

$Mx = B \Rightarrow \infty$ sol. ✗

$\rho(M) = \rho(M \vdots B) <$ No of V.

$\rho(M) < \rho(M \vdots B) \Rightarrow$ No sol.

given $(M)_{m\times n}$

& $\rho(M) = m \quad (m < n)$

$\Rightarrow \rho(M) <$ No of Variable (No of equ^n < No of Variables)

Eg: $x - 2y + 4z = 3$
$4x - 8y + 16z = 12$ has ∞ sol

$$[A : B] = \left[\begin{array}{ccc|c} 1 & -2 & 4 & 3 \\ 4 & -8 & 16 & 12 \end{array}\right]_{2\times 4}$$ (2×3)

$$\xrightarrow{R_2 \to R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & -2 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]_{2\times 4}$$ (2×3)

$\rho(A) = \text{one}$ & $\rho(A:B) = \text{one}$

$\because \rho(A) = \rho(A:B) <$ No of Variables (3)

$\Rightarrow \infty$ sol exist.

Eg: $x - 2y + 4z = 3$
$4x - 8y + 16z = 10$ has No. sol.

Sol: $$[A : B] = \left[\begin{array}{ccc|c} 1 & -2 & 4 & 3 \\ 4 & -8 & 16 & 10 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & -2 & 4 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right]_{2\times 4}$$

$\rho(A) = 1$ & $\rho(A:B) = 2$

$\because \rho(A) \neq \rho(A:B) \Rightarrow$ No sol.

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & l & m \end{array}\right]_{3\times 4}$$

$$\xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1}$$

The system of equations

$x + y + z = 6$

$x + 4y + 6z = 20$

$x + 4y + lz = m$

has NO solution for values of l and m given by

(a) $l = 6,\ m = 20$ (b) $l = 6,\ m \neq 20$

(c) $l \neq 6,\ m = 20$ (d) $l \neq 6,\ m \neq 20$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & l-1 & m-6 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 0 & l-6 & m-20 \end{array}\right]_{3\times 4}$$

for No Sol: $\rho(A) \neq \rho(A:B)$

let $\rho(A) = 2 \Rightarrow l = 6$

& $\rho(A:B) = 3 \Rightarrow m \neq 20$

Q: the system $x+2y+3z=4$, $2x+y-2z=-2$, $x-4y+z=3$ has ______ soln

Method

Sol: $A_{3\times3}$ i.e. it is equally determined system so we can follow Matn

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{vmatrix} = \cdots\cdots \neq 0$$ so unique soln exist

Thank

THANK

Keep Hustling!