GATE

CRASH COURSE

ALL BRANCHES

Engineering Mathematics

Linear Algebra (Part 01)
(Lec 01)

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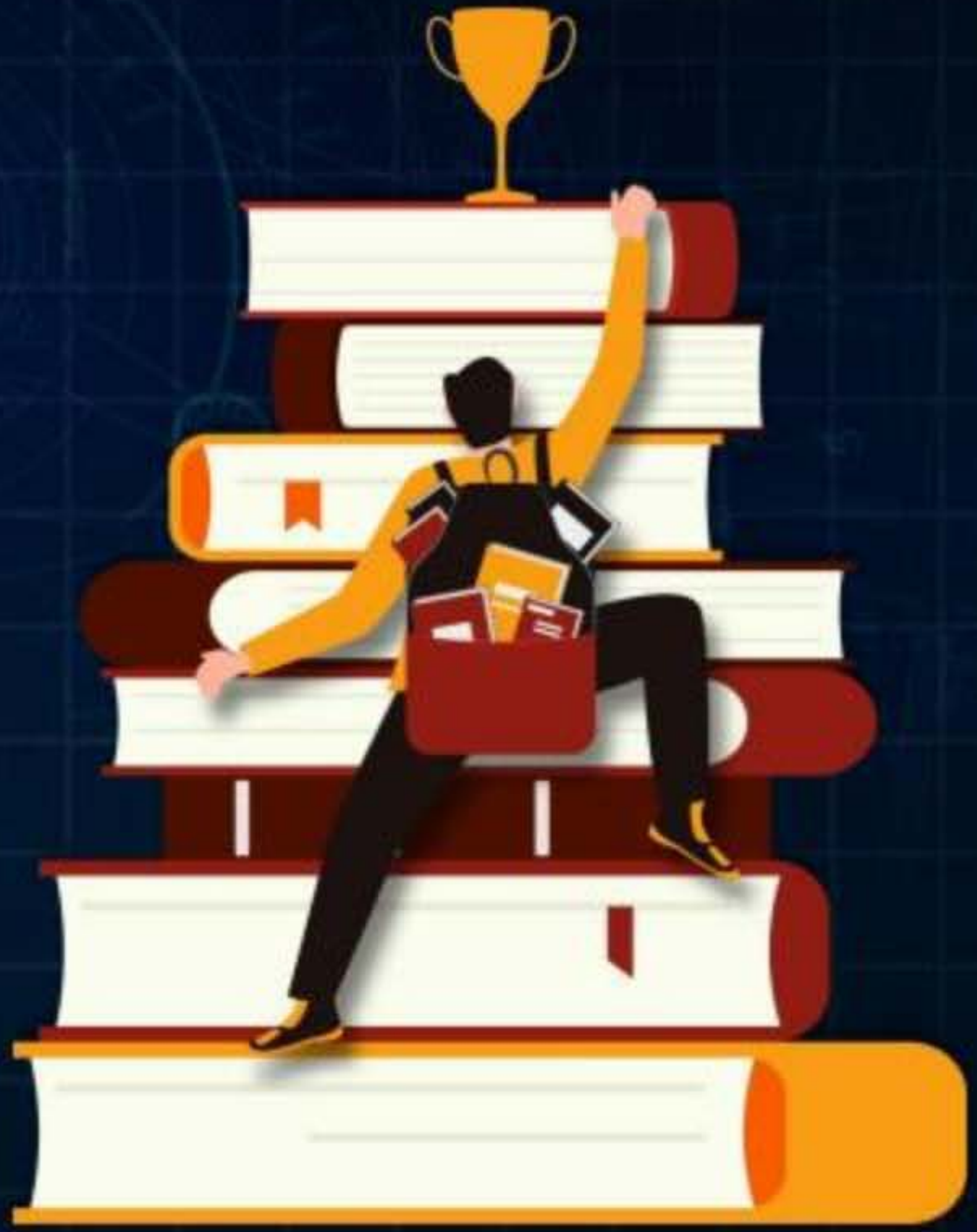
Topics *to be covered*

Linear Algebra (part 1)

→ Determinant
→ Matrices
→ Algebra Matrices
→ Types of Matrices



1. Linear Algebra → 3 days
2. Calculus → 3 "

CS/DA & All Branches

3. Vector Cal → 1 "
4. Complex → 2 " → CE X
5. D Eqⁿ → 2 "
6. N. Tech → 1 " → EC/EE ×
7. TR & FS → 1 "
8. Prob & Stats → 3 " → CS/DA & All Branches

(*) The concept of Trace, Determinant & Inverse exist only for Square Matrices

(*) " " " Transpose, Tranjugate Mat also defined for Rectangular Matrices

(*) Matrix:

$$A = [a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$ = Rectangular Arrangement

?!

m = No of Rows

n = " " Columns

(*) Sq Matx if $m = n$ then Mat is called Sq Mat

$$A = [a_{ij}]_{n\times n} \quad \text{eg } A = [a_{ij}]_{4\times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4\times 4}$$

Note: if $i = j \Rightarrow$ Diag elements

if $i > j \Rightarrow$ Lower diag elements

if $i < j \Rightarrow$ upper " "

(1) Determinant i.e. $A_{4\times4}$ then $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$ = Numerical Value

(may be Real or may be complex)

i.e. $|A| \in R$ or C

(2) $\text{Trace}(A) = \sum a_{ii} \; \forall i$ = Sum of Diag. elements

(3) To find the value of Det we will follow; $\begin{vmatrix} + & - \\ - & + \end{vmatrix}$, $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$, $\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix}$

Sign of $a_{ij} = (-1)^{i+j}$

eg Sign of $a_{31} = (-1)^{3+1} = +ve$

Sign of $a_{34} = (-1)^{3+4} = -ve$

eg $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

eg $\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} = ? = (2)(-1) - (4)(-3) = 10$

eg $|A| = \begin{vmatrix} 2 & 0 \\ 3 & -i \end{vmatrix} = ? = (2)(-i) - (3)(0)$

$= -2i$

Eg $|A| = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 0 & 2 \\ -1 & 2 & 3 \end{vmatrix} = ?$

$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$, Expanding along R_2

$= -1\begin{vmatrix} -3 & 1 \\ 2 & 3 \end{vmatrix} + (0)\begin{vmatrix} ? \end{vmatrix} - 2\begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix}$

$= -[-9-2] + 0 - 2[4-3]$

$= +11 - 2 = 9$

M-II

$\begin{matrix} 2 & -3 & 1 & 2 & -3 \\ 1 & 0 & 2 & 1 & 0 \\ -1 & 2 & 3 & -1 & 2 \end{matrix}$

$|A| = (0+6+2) - (0+8-9)$

$= 9$

Eg: $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{matrix}$

$|A| = (45+56+96) - (105+48+72)$

$= 0$

Eg $|A| = \begin{vmatrix} 1 & 2 & 4 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$\begin{matrix} 1 & 2 & 4 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{matrix}$

$|A| = (45+84+128) - (140+48+72)$

$=$

Q: $|A| = \begin{vmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{vmatrix}_{4\times 4} = ?$

$\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix}$, Expanding along R_1

$$|A| = +(0)\,|\;?\;| - (1)\begin{vmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \end{vmatrix}$$

$$+\,0\,|\;?\;| - (1)\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{vmatrix}$$

$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

$$|A| = 0 - 1\left\{-(1)\begin{vmatrix} -1 & 1 \\ -1 & -1 \end{vmatrix}\right\} + 0 - 1\left\{+(-1)\begin{vmatrix} -1 & 1 \\ -1 & -1 \end{vmatrix}\right\}$$

$$= +\{+1-(-1)\} + \{1-(-1)\}$$

$$= 2+2 = 4 = (2)^2 = \text{Perfect sq.}$$

(M-II): A is skew symm Mat of Even order

so $|A|$ = perfect square

Ex

To find Determinant of U.T.M, L.T.M, Diagonal Mat, Scalar Mat, Identity Mat we can multiply Diagonal Elements.

$$\begin{pmatrix} 2 & -1 & 4 & 3 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 3 \end{pmatrix}, \begin{pmatrix} -2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & -1 & 3 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 7 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 7 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

U.T.M	L.T.M	Diag Mat	Scalar Mat	Identity Mat
Det = −24	Det = 0	Det = −12	Det = 7^4	Det = 1
Trace = 8	Tr = 2	Tr = 7	Tr = 28	Tr = 4
A^{-1} = exist	A^{-1} = DNE	A^{-1} = exist	A^{-1} = exist	A^{-1} = exist

Singular Mat → if $|A| = 0$ $\Longleftrightarrow$ then A is called singular

Non singular Mat → if $|A| \neq 0$ $\Longleftrightarrow$ then A is called Non singular

Invertible Mat → if A^{-1} exist then A is called Invertible & $\boxed{A^{-1} = \frac{\text{adj} A}{|A|}} = \frac{(\text{Cof} A)^T}{|A|}$

Q: Necessary Cond^n for a Mat to be Invertible is $|A| \neq 0$

ie Matrix must be Non singular

Shortcut Method to find Inverse of 2×2 Mat → $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

eg $A = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix}$ then $A^{-1} = ?$ $= \frac{1}{(-6)}\begin{bmatrix} -3 & -0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ \frac{1}{6} & -\frac{1}{3} \end{bmatrix}$

eg $M = \begin{bmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{bmatrix}$ then $M^{-1} = ?$ $= \dfrac{\text{adj } M}{|M|} = \dfrac{1}{(1)}\begin{bmatrix} 3/5 & -4/5 \\ +4/5 & 3/5 \end{bmatrix}$

$\because |M| = \left(\frac{3}{5}\right)\left(3/5\right) - \left(-\frac{4}{5}\right)\left(\frac{4}{5}\right) = \frac{9}{25} + \frac{16}{25} = 1$ ie $|M| \neq 0$ so M^{-1} = exist

Shortcut Method of finding Inverse of 3×3 Mat →

$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 0 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ then $A^{-1} = ?$ $= \dfrac{\text{adj } A}{|A|} = \dfrac{(\text{cof } A)^T}{|A|} = \dfrac{1}{9}\begin{bmatrix} -4 & 11 & -6 \\ -5 & 7 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

Sol: $\because |A| = 9$ ie $|A| \neq 0 \Rightarrow A^{-1}$ = exist.

$$\begin{array}{c|cccc} 2 & -3 & 1 & 2 & -3 \\ \hline 1 & 0 & 2 & 1 & 0 \\ -1 & 2 & 3 & -1 & 2 \\ 2 & -3 & 1 & 2 & -3 \\ 1 & 0 & 2 & 1 & 0 \end{array}$$

(ii) $\text{adj } A = ? = \begin{bmatrix} -4 & 11 & -6 \\ -5 & 7 & -3 \\ 2 & 1 & 3 \end{bmatrix}$

(iii) $\text{cof } A = ? = (\text{adj } A)^T = \begin{bmatrix} -4 & -5 & 2 \\ 11 & 7 & 1 \\ -6 & -3 & 3 \end{bmatrix}$

(*) if $A = [a_{ij}]_{m\times n}$

→ Transpose of A : $A^T = [a_{ji}]_{n\times m}$

→ Conjugate of A : $\overline{A} = [\overline{a_{ij}}]_{m\times n}$

→ Tranjugate of A : $A^\theta = \overline{(A^T)}$ or $(\overline{A})^T = [\overline{a_{ji}}]_{n\times m}$

(Transposed conjugate)

eg $A = \begin{pmatrix} 2 & 4-i & 7i \\ 3+2i & 0 & -5 \end{pmatrix}_{2\times 3}$

$\Rightarrow \overline{A} = \begin{bmatrix} 2 & 4+i & -7i \\ 3-2i & 0 & -5 \end{bmatrix}_{2\times 3}$

& $A^T = \begin{bmatrix} 2 & 3+2i \\ 4-i & 0 \\ 7i & -5 \end{bmatrix}_{3\times 2}$

& $A^\theta = \overline{(A^T)} = \begin{bmatrix} 2 & 3-2i \\ 4+i & 0 \\ -7i & -5 \end{bmatrix}_{3\times 2}$

Note → $z = x+iy$ then $\overline{z} = x-iy$

Replace $i \to (-i)$ to find conjugate

(1) Reversal Law → $(ABC)^T = C^T B^T A^T$

$(ABC)^\theta = C^\theta B^\theta A^\theta$

$(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

e.g. $(AB)^{-1} = B^{-1} A^{-1}$.

(2) $AB = A_{m \times n} B_{n \times p}$ = defined

i.e. N. Cond^n to find Matrix Product is

$\boxed{\text{No. of Columns in } 1^{st} \text{ Mat} = \text{No. of Rows in } 2^{nd} \text{ Mat}}$

(3) Condition to find $A+B$ & $A-B$ is

$\boxed{\text{“ Matrices must be of same order ”}}$

e.g. $A_{2\times3}$, $B_{2\times3}$, $C_{3\times3}$ then

$A \pm B$ defined But $A \pm C$ & $B \pm C$ N.D

(4) Cond^n to find $\frac{A}{B}$ = ? senseless Quest.

∵ Concept of Division in Matrix Theory is N.D

(5) Mat. Multiplication is Not Commutative in general

i.e. $\boxed{AB \neq BA}$ in general.

(*) If $AB = C$ where A & B are Non Singular Matrices of same order, then $B = ?$

$\Rightarrow A^{-1}$ & B^{-1} exist

WRONG APP $\rightarrow$ $\cancel{B = \frac{C}{A} = CA^{-1}}$

$A^{-1}(AB) = A^{-1}C$

$IB = A^{-1}C$

$\boxed{B = A^{-1}C}$ Ans.

$\cancel{B = CA^{-1}}$

(*) if $\boxed{AB = BA = I} \Rightarrow A^{-1} = B$, $B^{-1} = A$

Q. Multiplication of E & F is G then $F = ?$

where $E = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$, $G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(a) E^{-1}, (b) E (c) F^{-1} (d) G

A.T.Q. $EF = G$

i.e. $EF = I$ $\rightarrow$ $F = E^{-1} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$, $E = F^{-1}$

Symmetric Mat → if $A^T = A$

eg $A = \begin{pmatrix} 2 & -3 & 4 \\ -3 & 0 & 5 \\ 4 & 5 & 1 \end{pmatrix} \Rightarrow A$ is symm $(\because A^T = A)$

Skew symm Mat → if $A^T = -A$

eg $\begin{pmatrix} 0 & -3 & 5 \\ 3 & 0 & 1 \\ -5 & -1 & 0 \end{pmatrix}_{3\times 3} \Rightarrow |A| = 0$

$\Rightarrow A$ is skew sym $(\because A^T = -A)$

Hermitian Mat → if $A^\theta = A$

Skew Hermitian Mat → if $A^\theta = -A$

eg $\begin{pmatrix} 2 & 5+i & -7 \\ 5-i & -2 & 3i \\ -7 & -3i & 5 \end{pmatrix}$ (Herm $\because A^\theta = A$)

eg $\begin{pmatrix} 2i & 5+i & -7 \\ -5+i & -2i & 3i \\ 7 & 3i & 5i \end{pmatrix}$ (skew Herm $\because A^\theta = -A$)

Orthogonal Mat → if $AA^T = I$ (or $A^{-1} = A^T$)

Unitary Mat → if $AA^\theta = I$ (or $A^{-1} = A^\theta$)

Nilpotent Mat → if $A^k = 0$ then A is called N. Mat of power k (where k is least +ve integer)

Idempotent Mat → if $A^2 = A$ eg $\because I^2 = I$ so I is an Idempotent Mat

Involutary Mat → if $A^2 = I$

Eg: $A = \begin{bmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{bmatrix} \Rightarrow A$ is O-Mat ($\because AA^T = I$) or $A^{-1} = A^T$

where $A^{-1} = \frac{1}{(1)}\begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}$ & $A^T = \begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}$

Eg $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \Rightarrow A$ is U-Mat ($\because AA^\theta = I$)

$A^T = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$

Note: $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ & $\omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Also $\boxed{\bar{\omega} = \omega^2 \ \& \ \overline{\omega^2} = \omega, \ \omega^3 = 1, \ 1+\omega+\omega^2 = 0}$

$A^\theta = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{\omega} & \overline{\omega^2} \\ 1 & \overline{\omega^2} & \bar{\omega} \end{bmatrix}$

$= \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$

Now $AA^\theta = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Q. $A = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$ then A is ?

Sol: $A^2 = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

ie A is Nilpotent Mat of index 2

(*) $A_{n \times n}$ st A is skew symm then

① $|A| = \begin{cases} 0, & n = \text{odd} \\ \text{perfect sq.}, & n = \text{even} \end{cases}$

② $\sum a_{ij} = 0$ whether $n = \text{odd}$ or $n = \text{even}$

eg $A = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 4 \\ -1 & -4 & 0 \end{bmatrix}_{3\times 3}$

$\Sigma a_{ij} = 0$

$|A| = 0[0+4] - (-2)[0+4] + 1[-8+0]$
$= 0 + 8 - 8 = 0$

$A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}_{2\times 2}$

$\sum a_{ij} = 0$

$|A| = 9 = (3)^2$

(*) Max no. of Different elements that will be Required to construct a Symm Mat of order $n \times n$ will be? $= \dfrac{n(n+1)}{2}$ learn

$$A_{2\times2} = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \longrightarrow \text{max } 3 \text{ elements}$$

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \longrightarrow \text{Max } 6 \text{ ''}$$

$$= \begin{bmatrix} a & b & c & d \\ - & e & f & g \\ - & - & h & i \\ - & - & - & j \end{bmatrix}_{4\times4} \longrightarrow \text{Max } 10 \text{ elements}$$

. . . . . .

(*) Max no. of terms that can be obtained in the General Expansion of $|A|_{n\times n}$ = ? = $n!$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \rightarrow 2 \text{ terms} = 2!$$

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a[ei - hf] - b[gi - gf] + c[dh - eg]$$

$$= 6 \text{ terms} = 3!$$

& so on.....

Q. If A is Hermitian Mat then (iA) will be?

(a) Symm (b) Skew Symm Mat (c) Herm (d) S.H. Mat

Sol: given $A^\theta = A$ —— (1)

Now, $(iA)^\theta = i^\theta A^\theta$

$= (\bar{i})^T (A)$

$= (-i)^T . A$

$= -iA$

$= -(iA)$

For the given orthogonal matrix Q.

$$Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$$

The inverse is

(a) $\begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$

(b) $\begin{bmatrix} -3/7 & -2/7 & -6/7 \\ 6/7 & -3/7 & -2/7 \\ -2/7 & -6/7 & 3/7 \end{bmatrix}$

(c) $\begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

(d) $\begin{bmatrix} -3/7 & -6/7 & -2/7 \\ -2/7 & -3/7 & -6/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$

$$Q = \begin{pmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 3 & 6 & 6 \\ -6 & 3 & 2 \\ 2 & 6 & -3 \end{pmatrix} = \text{O-Mat}$$

then $Q^{-1} = ? = Q^T =$ option (c)

Let $\alpha = e^{2\pi/5}$ and matrix

$$M = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & 0 & \alpha^4 \end{bmatrix}_{5\times 5}$$

then trace of the matrix $I + M + M^2 =$ _____.

$\alpha = e^{\frac{2\pi i}{5}}$ = 5^{th} Root of unity

$\alpha^5 = 1$, $\boxed{1+\alpha+\alpha^2+\alpha^3+\alpha^4=0}$

$w = e^{\frac{2\pi i}{3}}$ = cube root of unity

$w^3 = 1$, $1+w+w^2=0$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow Tr(I) = 5$

$$, M = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & 0 & \alpha^4 \end{bmatrix}$$

$Tr(M) = 1+\alpha+\alpha^2+\alpha^3+\alpha^4$

$= 0$

$$, M^2 = \begin{bmatrix} 1 & - & - & - & - \\ 0 & \alpha^2 & - & - & - \\ 0 & 0 & \alpha^4 & - & - \\ 0 & 0 & 0 & \alpha^6 & - \\ 0 & 0 & 0 & 0 & \alpha^8 \end{bmatrix}$$

$Tr(M^2) = 1+\alpha^2+\alpha^4+\alpha^6+\alpha^8$

$= 1+\alpha^2+\alpha^4+\alpha+\alpha^3$ $(\because \alpha^5=1)$

$= 0$

$Tr(I+M+M^2) = 5+0+0 = 5$

The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication.

(a) If $a = b$ or $\theta = n\pi$, n is an integer

(b) always

(c) never

(d) If $a \cos\theta \neq b \sin\theta$

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$AB = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a\cos\theta & -b\sin\theta \\ a\sin\theta & b\cos\theta \end{bmatrix}$$

$2\times 2 \quad 2\times 2 \quad 2\times 2$

$$\|$$

$$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} a\cos\theta & -a\sin\theta \\ b\sin\theta & b\cos\theta \end{bmatrix}$$

A.T.Q, $\boxed{AB = BA}$

so $a\sin\theta = b\sin\theta$

$(a-b)\sin\theta = 0$

$a-b=0$ or $\sin\theta = 0$

$a=b$ or $\theta = n\pi$, $n \in I$

Summary

Telegram: drpuneet sir pw

Tel: drpuneet sir pw

Thank

THANK

Keep Hustling!