Data Science & Artificial Intelligence

Algorithms

Test Series 1500+

Lecture - 10

By- Aditya sir

Recap of Previous Lecture

Topic

Misc Questions

Topic

Topics to be Covered

Topic — Questions

Topic — PYQs

Doubts

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#Q. Suppose K_n is a complete graph with 'V' vertices. How many edge disjointness spanning trees are possible ?

A) V

B) $\lceil \frac{V}{3} \rceil$

C) $\lceil \frac{V}{2} \rceil$

D) $\lceil \frac{V}{4} \rceil$

n = 4 → 4

B) n = 4 → 3

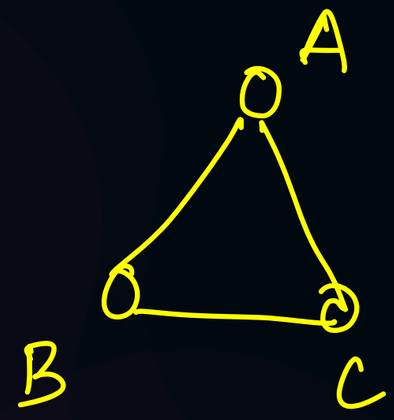
C) n = 4 → 2

D) n = 4 → 1

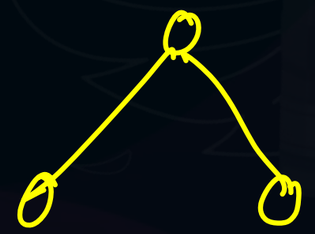
Soln :- K_n → Complete graph

edge disjointness → No edge in Common

eg1) K_3 → V = 3



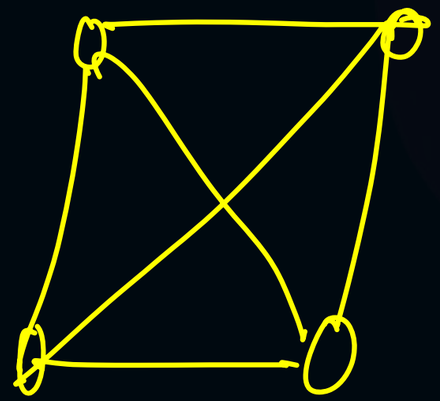
or



or

→ 1

eg2:) $K_4 \rightarrow V = 4$

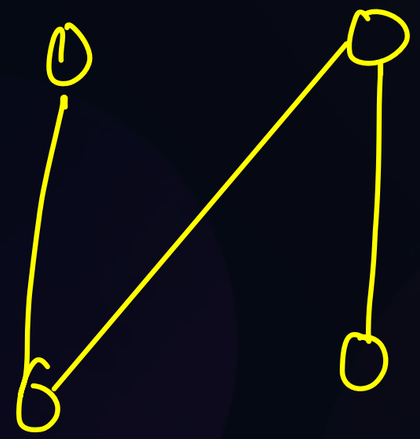


MST

$n = 4$

3 edges

(2)



edge disjoint MCSTs

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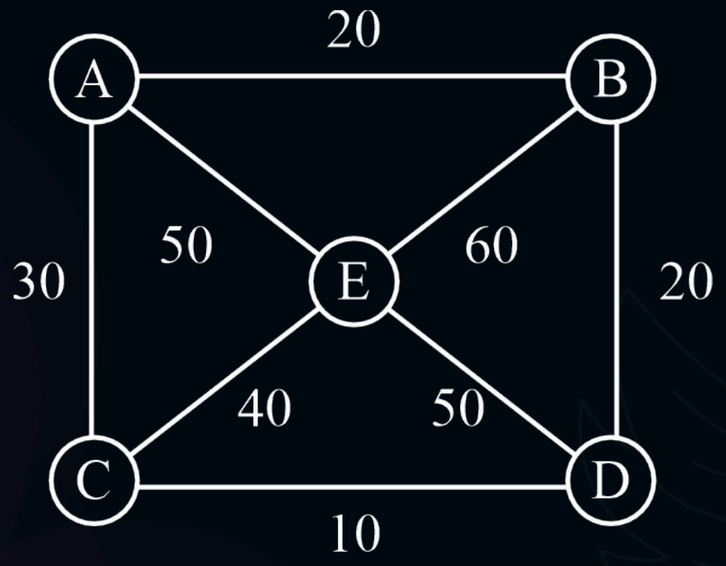
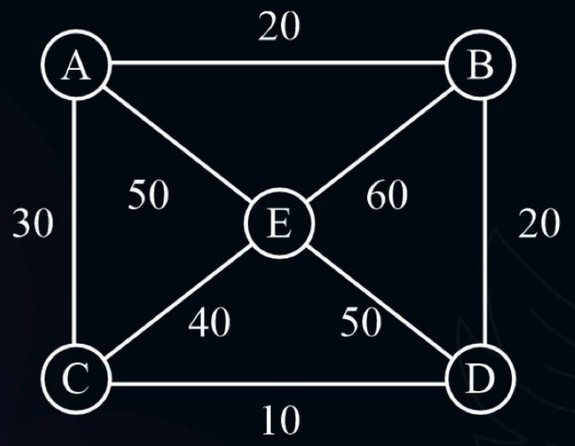
#Q. Assume Dijkstra's algorithm is used to find the shortest path from node 'A' in the following graph G :



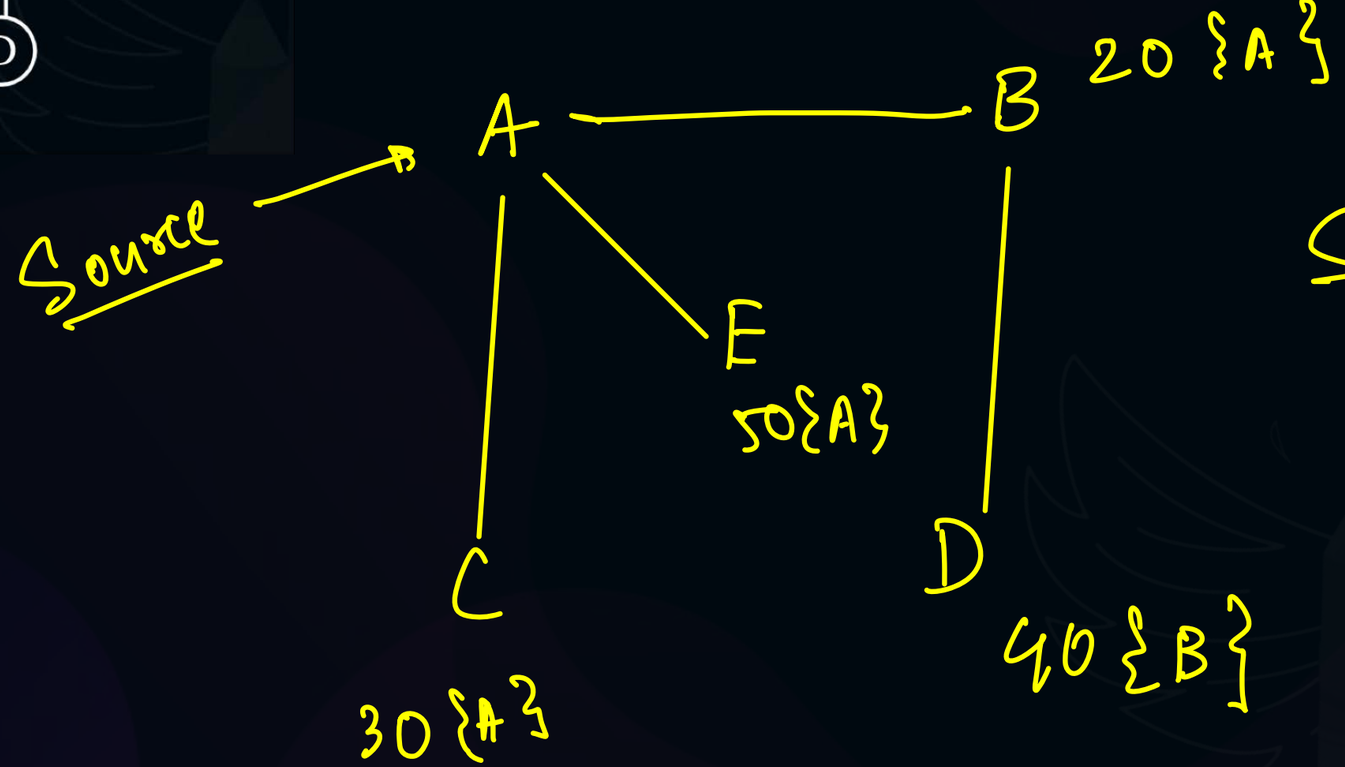
Ans : 4

A) The number of edges are not included in any of the shortest path from node A is _____.

B) Sum of cost of these edges.



Dijkstra SSSP : Spanning Tree :



SSSP

Spanning Tree

CDX

EBX

CEX EDX

B) Cost of missing edges = 10 + 60 + 40 + 50

= 160

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(-ve cycle may be present)

#Q. Consider the following statements: which are True ?

S_1: If graph contain positive and negative edge weight then, Bellman ford always give the correct answer. to SSSP.

False

S_2: Bellman ford algorithms find out all negative edges weight cycle in the given graph if they are reachable form source.

True

A S_1 only ✗

B S_2 only ✓

C Both ✗

D None ✗

Ans: B

Bellman Ford → SSSP

DP

Soln :- Algos to Solve SSSP :

	+ve wt. edges	-ve wt edges but no -ve wt cycle	-ve wt Cycle
✓ 1) Dijkstra's SSSP (Greedy)	✓	✗	✗
2) Bellman Ford : (Dynamic Programming)	✓	✓	✗

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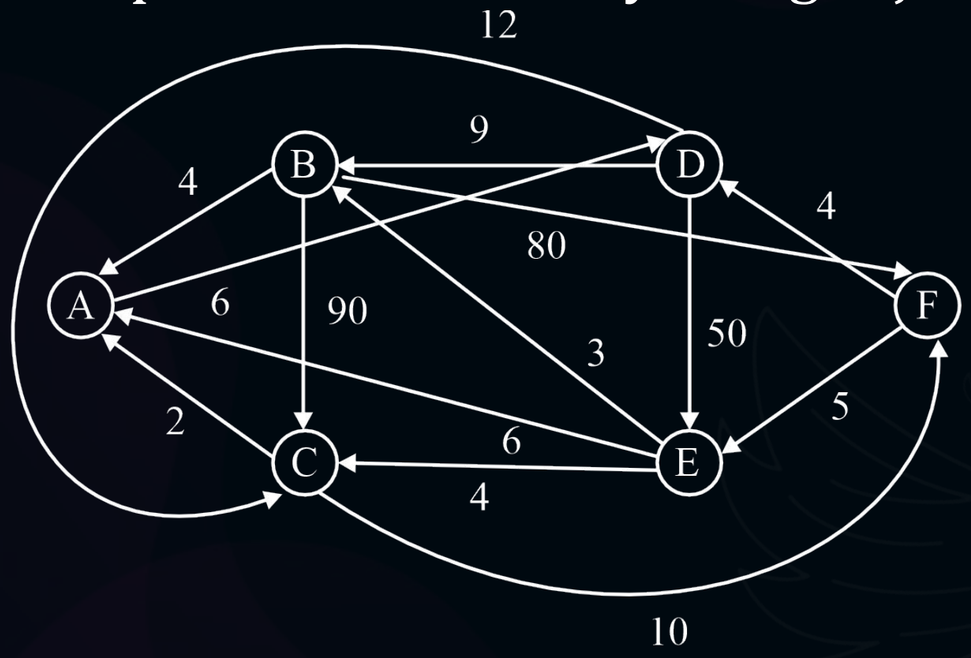
#Q. Consider the following directed graph G:

What will be shortest path from B to E by using Dijkstra's algorithms?



A B A D C F E

B B A D E ✗

C B A D F E ✗

D B D E ✗

Soln: A) BADCFE

B → A → D → C → F → E

4 + 6 + 12 + 10 + 5 = 37

B) BADE

B → A → D → E

4 + 6 + 50 = 60 ✗

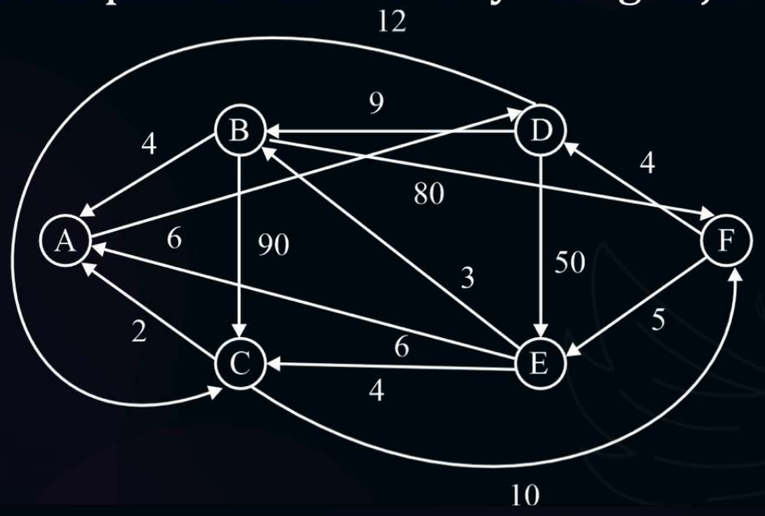
C) BADFE

B → A → D → F → E

4 + 6 + ✗

D) BDE

$B \rightarrow D \rightarrow E$



Dijkstra SSSP: Spanning Tree appr

source

10 {A}

B

D

A

4 {B}

C

96 {B}

22 {D}

E

90 60 {D}

37 {F}

F

80 {B}

32 {C}

[B → A → D → C → F → E]

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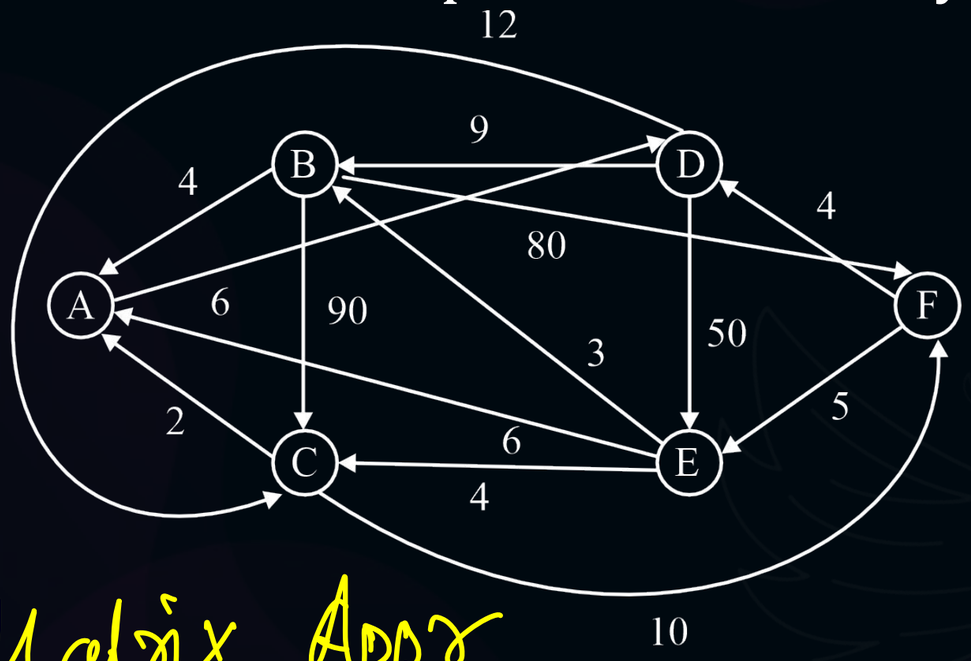
#Q. Consider the following directed graph G:

What will be the cost of shortest path from B to E by using Dijkstra's algorithms?



Solve using Matrix Appr:

Soln :-

Vertex set	A	B	C	D	E	F
{B}	(4)	0	90	∞	∞	80
{B, A}	4	0	90	(10)	∞	80
{B, A, D}	4	0	(22)	10	60	80
{B, A, D, C}	4	0	22	10	60	(32)
{B A D C F}	4	0	22	10	(37)	32

{B A D C F E} [4 0 22 10 37 32]

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#Q. Suppose, there are 8 sorted list of $\frac{n}{8}$ elements each if we merge them into single sorted list of n elements, n is 1000 elements then , what is the difference between key comparisons in worst case and best case ?

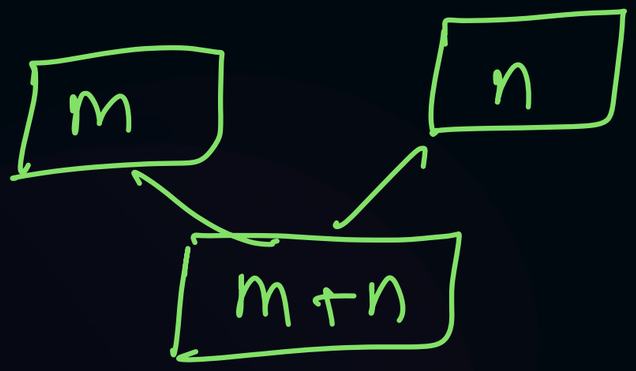
HAT

elem-wise comparisons

merging Algo

default : 2-way merging.

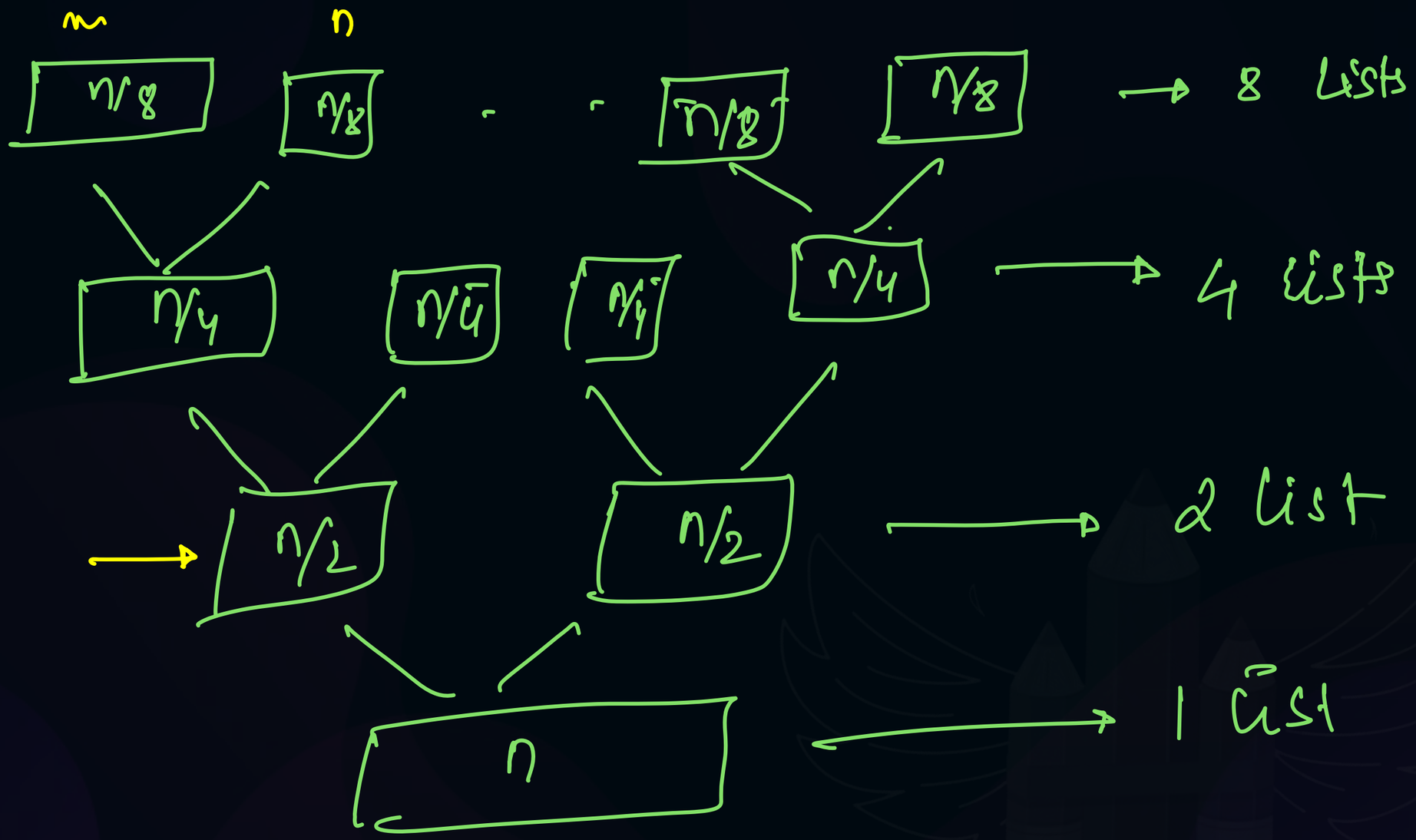
In general, Merging Algo:-



Best Case: $\min(m, n)$

Worst Case: $m+n-1$

Eg:



BC : n/8

WC n/4 - 1

BC : n/4

WC : n/2 - 1

BC : n/2

WC : n - 1

Overall :

Best Case : $4 \times (n/8) + 2 \times (n/4) + 1 \times n/2$

(min comp at every merge)

$= n/2 + n/2 + n/2$

$= \lceil 3n/2 \rceil$ ✓

Worst Case = $4(n/4 - 1) + 2(n/2 - 1) + 1 \times (n-1)$

$= n - 4 + n - 2 + n - 1$

$= \boxed{3n - 7}$ ✓

Diff = WC − BC

= $3n-7-\left(\frac{3n}{2}\right)$

= $3n-\frac{3n}{2}-7$

= $\boxed{\frac{3n}{2}-7}$

Given $n = 1000$

→ $\frac{3\times 1000}{2} - 7$

= $3\times 500 - 7$

= $1500 - 7$

= $\boxed{1493}$

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#Q. Consider an array which contain n indexes [1 to n] Number of inversions in this array are atmost n then which algorithm is best suitable to sort the above arrays.

A Insertion sort

B Bubble sort

C Selection sort

D Merge sort

Ans : A

Soln : At max no. of inversions :

TC of insertion sort : $O(n + d)$

n = total no. of elems

d = max no. of inversions

$d = O(n)$

$TC : O(n + n) = O(n)$

WC: Insertion Sort : $O(n^2)$

| 50 | 40 | 25 | 10 | 2 |

overall WC

not a valid case for given question.

at most n invasions → almost sorted array

| 10 | 20 | 30 | 40 | 50 | 5 |

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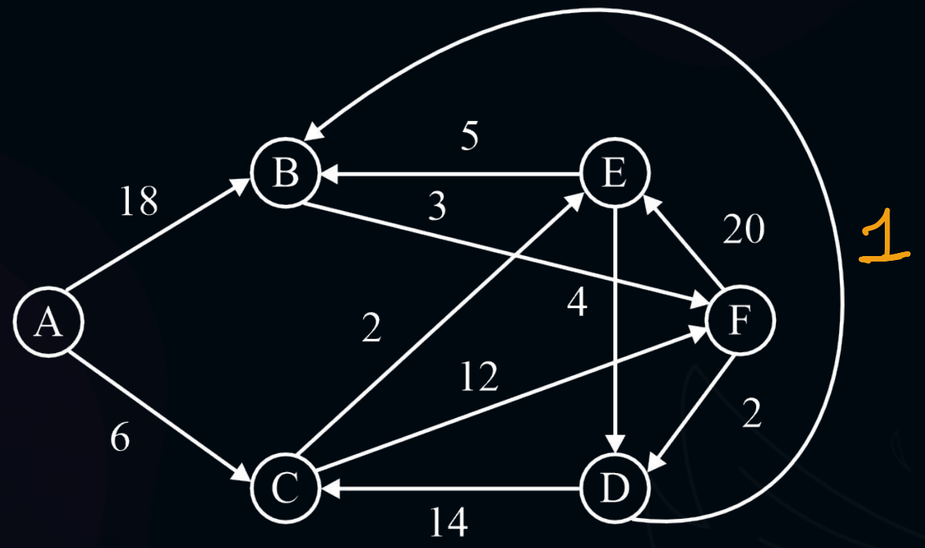
#Q. Let G be the directed , weighted graph shown below:



If the cost of the shortest path from A to F is X and number of same cost of A to F is Y then the value of X * Y is_____.

HW → Solve using Dijkstra: Min cost A → F

Appr 2:

All possible paths from A → F

1) A → B → F : 18 + 4 = 22

2) A → C → E → B → F ⇒ 6 + 2 + 5 + 3 ⇒ 16

A → C → E → D → B → F ⇒ 6 + 2 + 4 + 1 + 3 = 16

A → C → F ↳ 6 + 12 = 18

X = min Cost $A \rightarrow F$ = 16

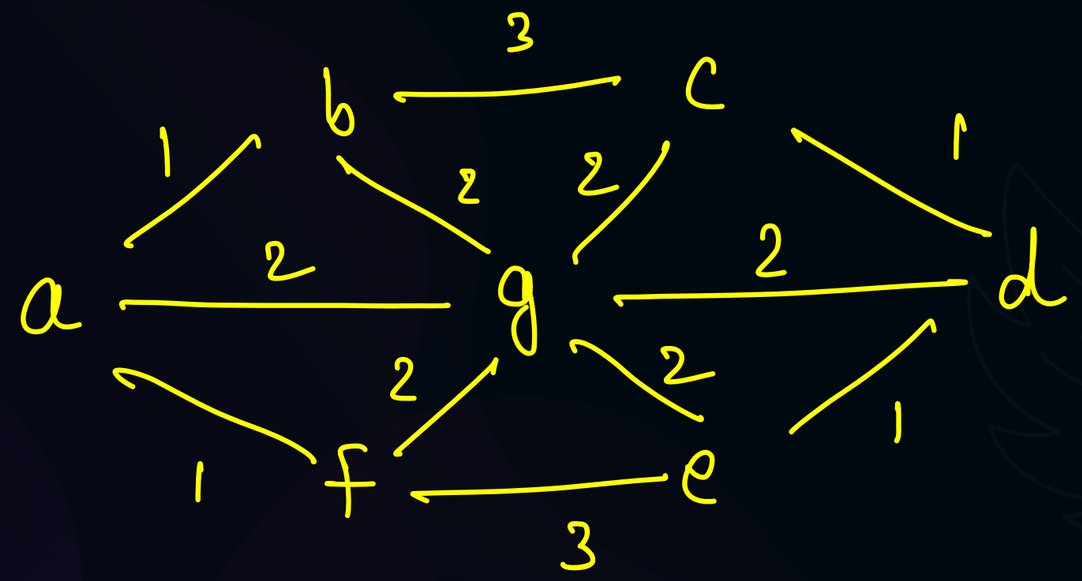
Y = Such paths = 2

$X * Y$ = 16×2

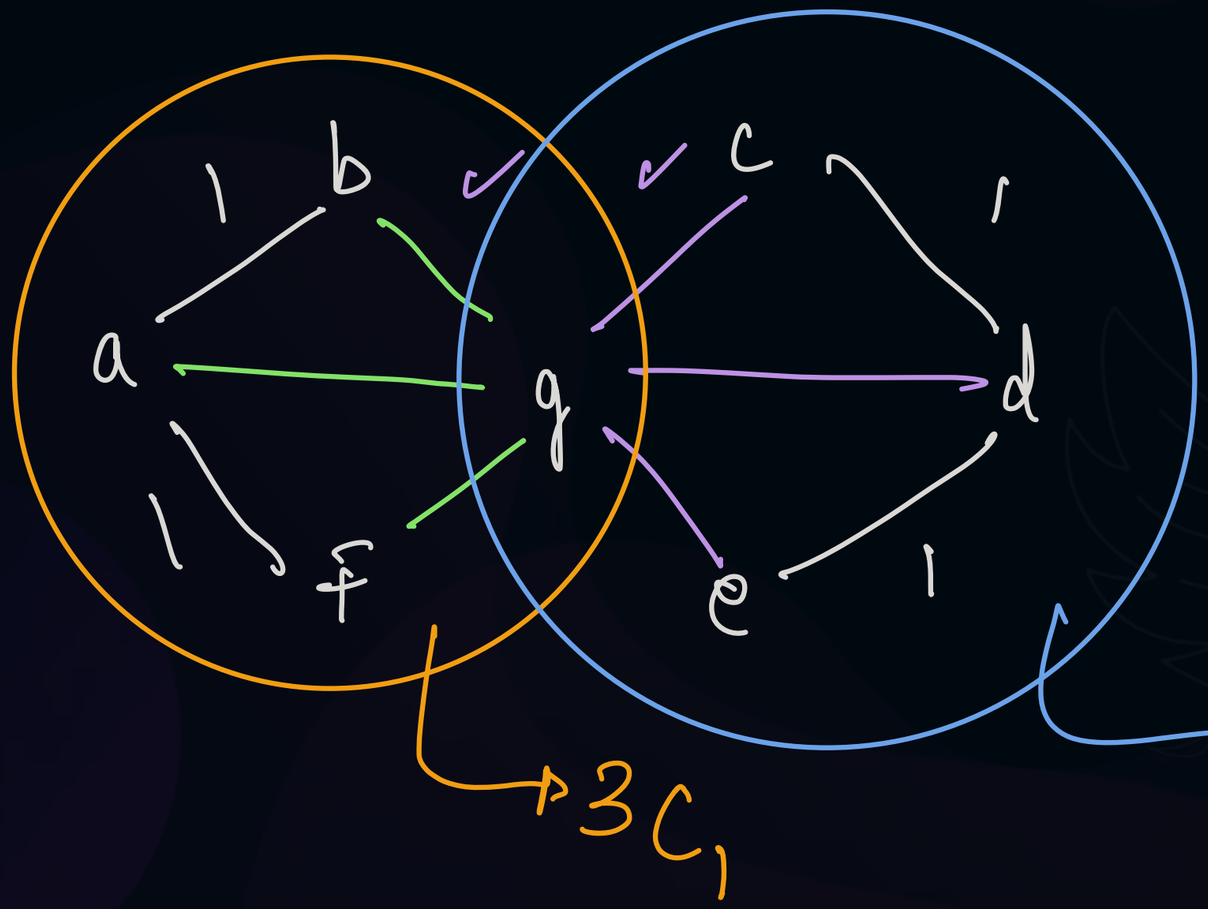
= $\boxed{32}$

PYQ :- 2024 : 2m

The no. of distinct MCST of the below graph?



Soln : Kruskal Algo :



n = 7

edges in MCST

= n - 1

= 7 - 1

= 6

Total MSTs

$= {}^{3}C_1 \times {}^{3}C_1$

$= 3 \times 3$

$= \boxed{9}$

$3+3X$

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#Q. Which of the following algorithm can be used to sort n integers in the range $[1,\ldots 10^3]$ in O(n) time?

wc: $O(n^2)$

wc: $O(n^2)$

A Selection sort

B Bubble sort

C Radix sort

D Quick sort

wc: $O(n^2)$

Ans: C

Sol n :

[1 ⟶ 1000]

n elems

TC: O(n) : Radix Sort : TC: O(n * d)

n → no. of elems

d → max no. of digits in any elem of i/p arr

[1 ⟶ 10^3 ⟶ max digits → d = 4]

⌈1000⌉

TC: O(n * 4) = O(n)

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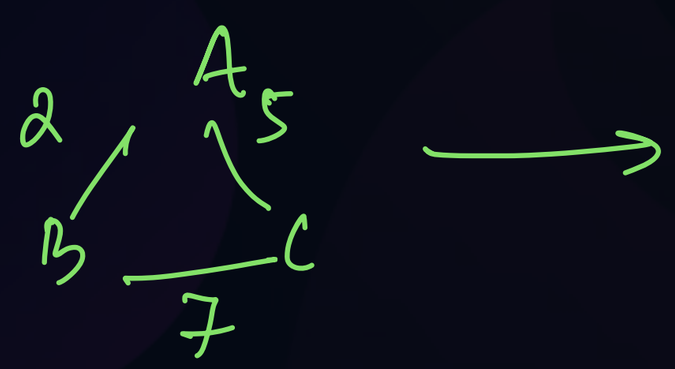
#Q. Suppose, a graph contain 50 vertices and 120 edges the weight of MST is 300. If the weight of each edge of G is increased by 6, then the weight of MST becomes_____.

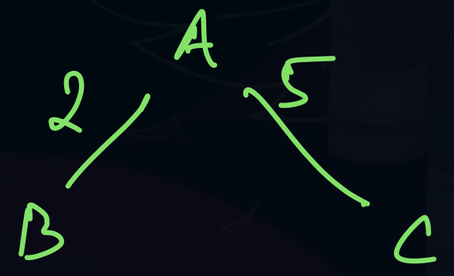
Given :- $n = 50$, $e = 120$ ⟶ $+6$

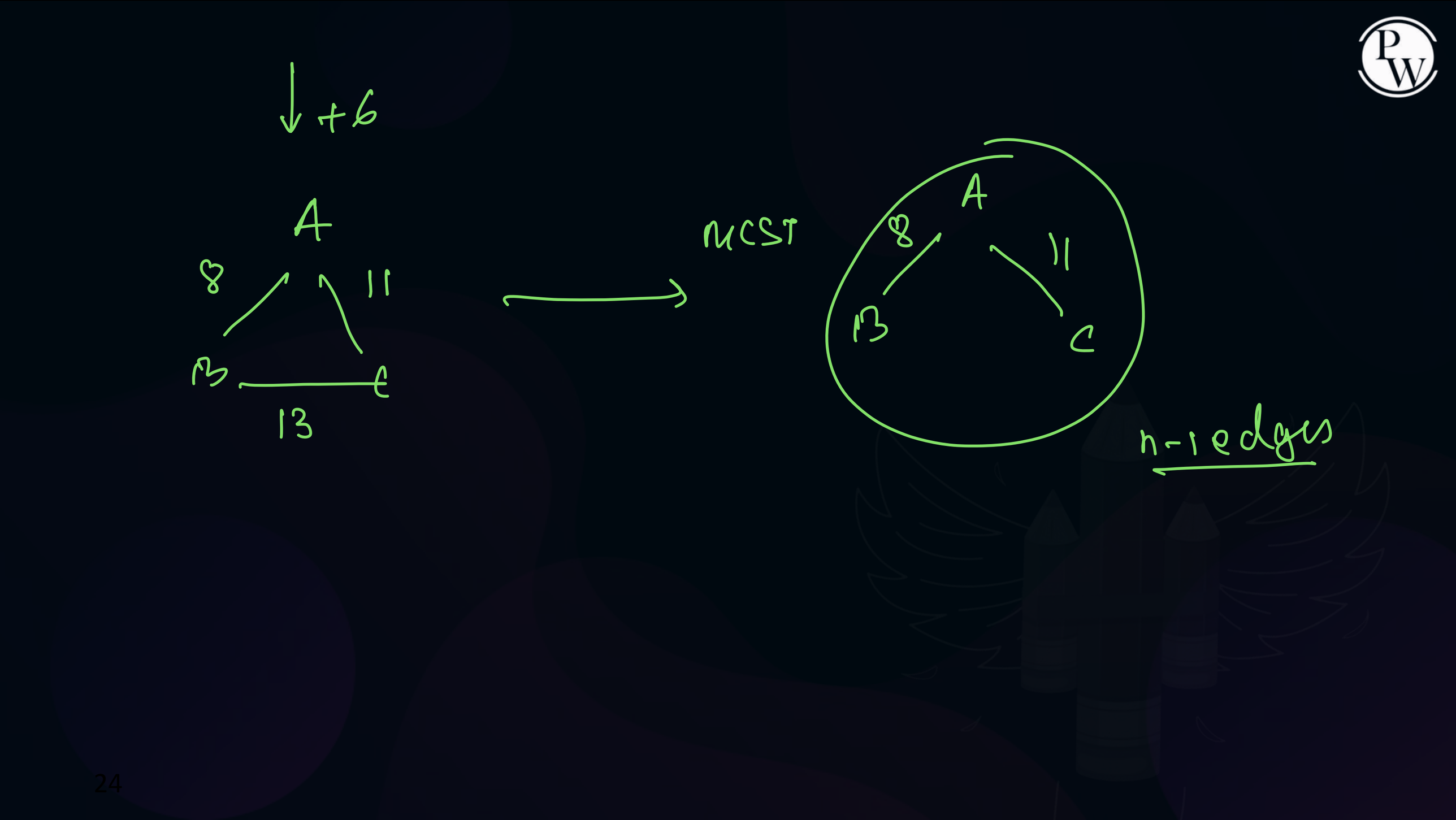
Cost MCST ⟶ 300

Cost ⇒ MCST ?



MCST




↓+6
A
8
11
B
C
13
MCST
A
8
11
B
C
n-1 edges

In general n vertices ⟶ (n-1) edges in MCST

MCST
↓
300

$\xrightarrow[+6]{e}$

$300 + (n-1) * 6$

$300 + (50-1) * 6$

$300 + 49 \times 6$

$300 + (300 - 6)$

$300 + 294 = \boxed{594}$

2 mins Summary

Topic

Topic

Topic

Topic

Misc Questions → Graphs
↳ Sorting
↳ MCST
↳ Shortest paths

THANK - YOU

Telegram Link for Aditya Jain sir:
https://t.me/AdityaSir_PW