GATE

CRASH COURSE

Data Science & AI

Subject

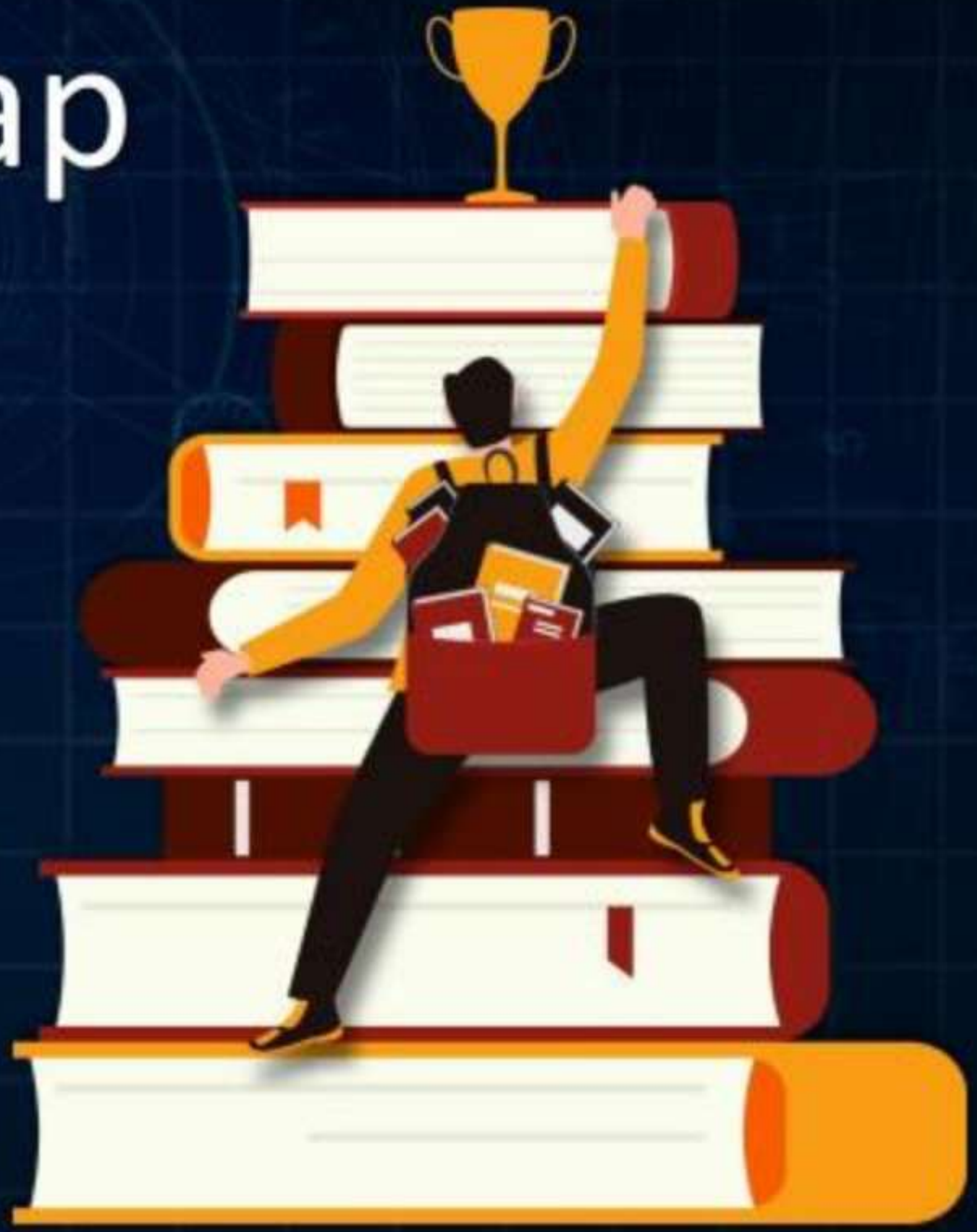
Data Structure & Algorithms
Lec No. 04 : Tree Fundamentals

By – Satya Sir

Last Class

Quick Recap

1. Linked Lists - Types
2. Singly Linked Lists
3. Doubly Linked List
4. Examples



Topics to be covered

40% of weightage

1. Trees Terminology
2. Types Of Binary Trees
3. Tree Traversals
4. Formulae of Binary Trees

Trees

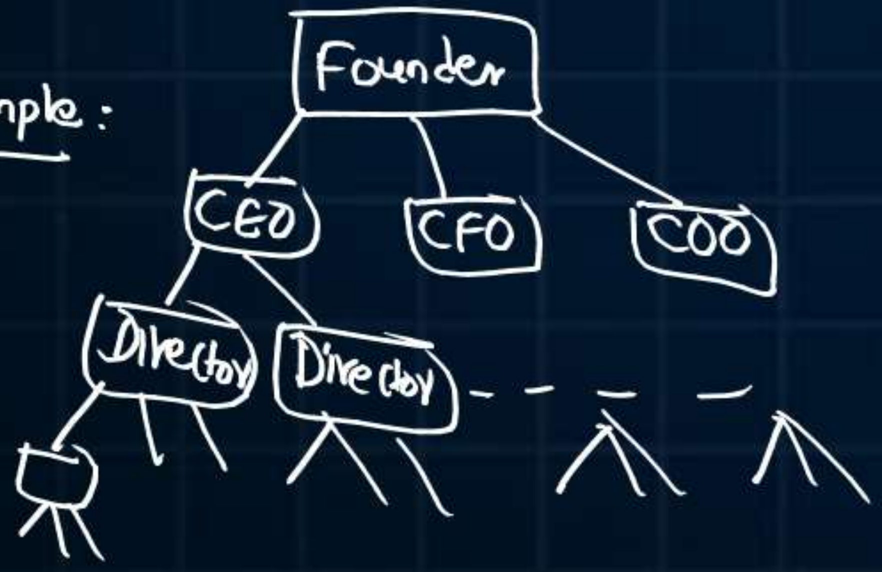
- Non-linear data Structure

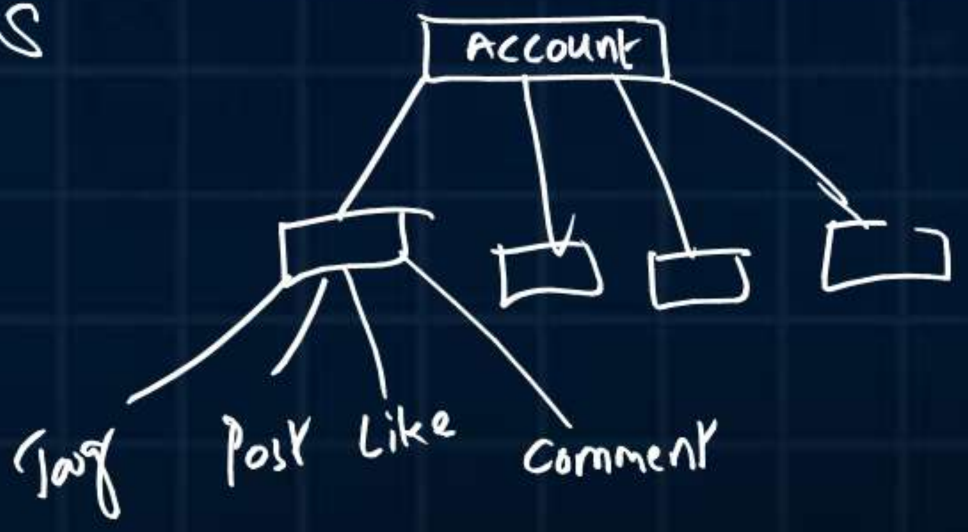
- Acyclic - Graph

- Elements (Nodes) are arranged / represented / Organized into multiple levels.

- It is also known as Hierarchical DS

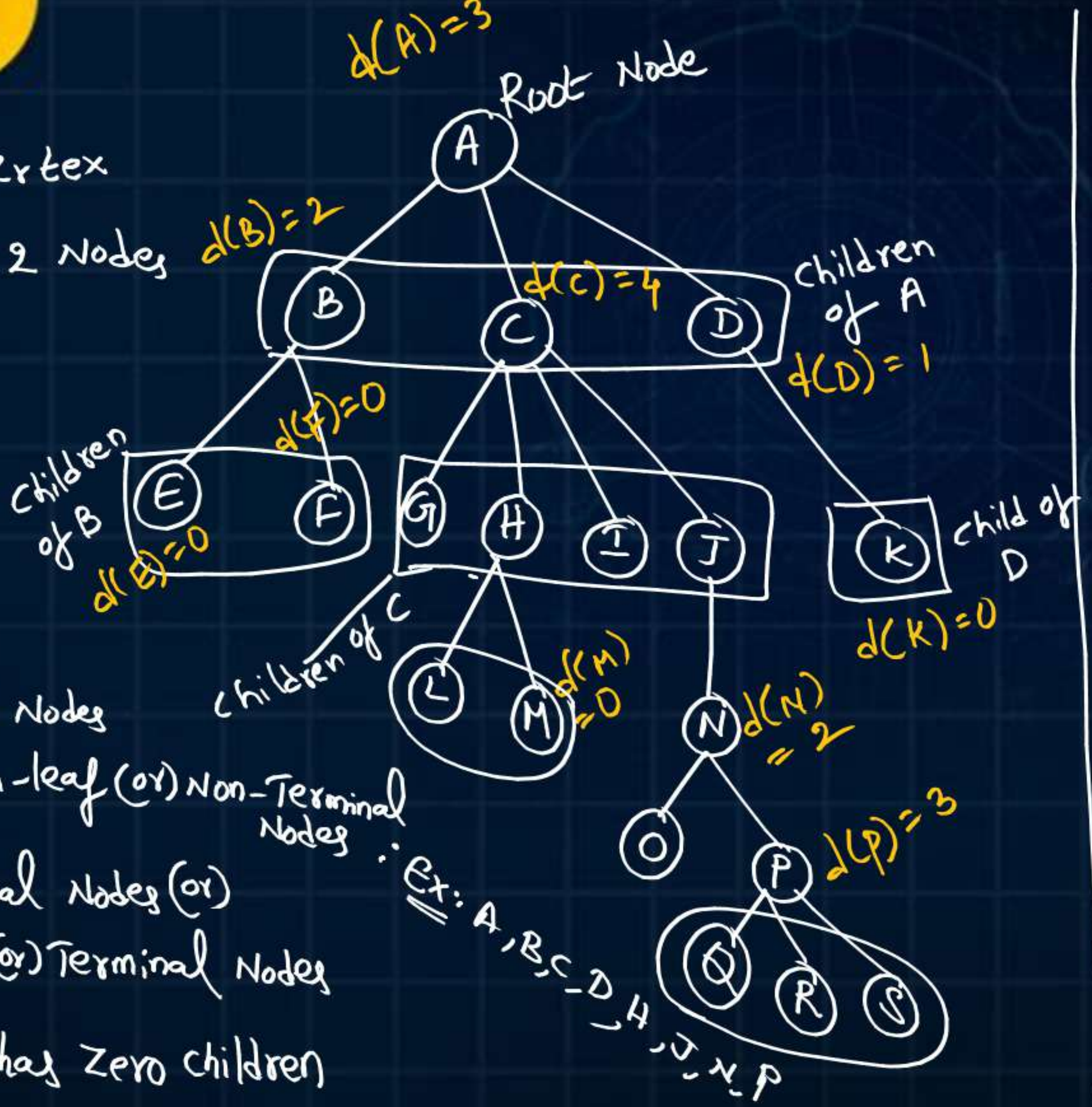
- Example:





Trees

- Each Node | Element == Vertex
- Each link Connecting any 2 Nodes == Edge
- A Node with out Parent == Root Node
- Nodes of Tree
  - → Internal Nodes (or) Non-leaf (or) Non-Terminal Nodes : Ex: A, B, C, D, H, J, N, P
  - → External Nodes (or) Leaf (or) Terminal Nodes
- Leaf Node : A Node that has Zero children
  Ex: E, F, G, L, M, I, O, Q, R, S, K



- The Number of children Per Node == degree of a Node.
- Degree of a Tree == Max {degrees}
  $= \{3, 0, 2, 4, 1\}$
  $= 4$
- The Nodes with common Parent == Siblings

Trees

** Depth of a Node == Level of a Node

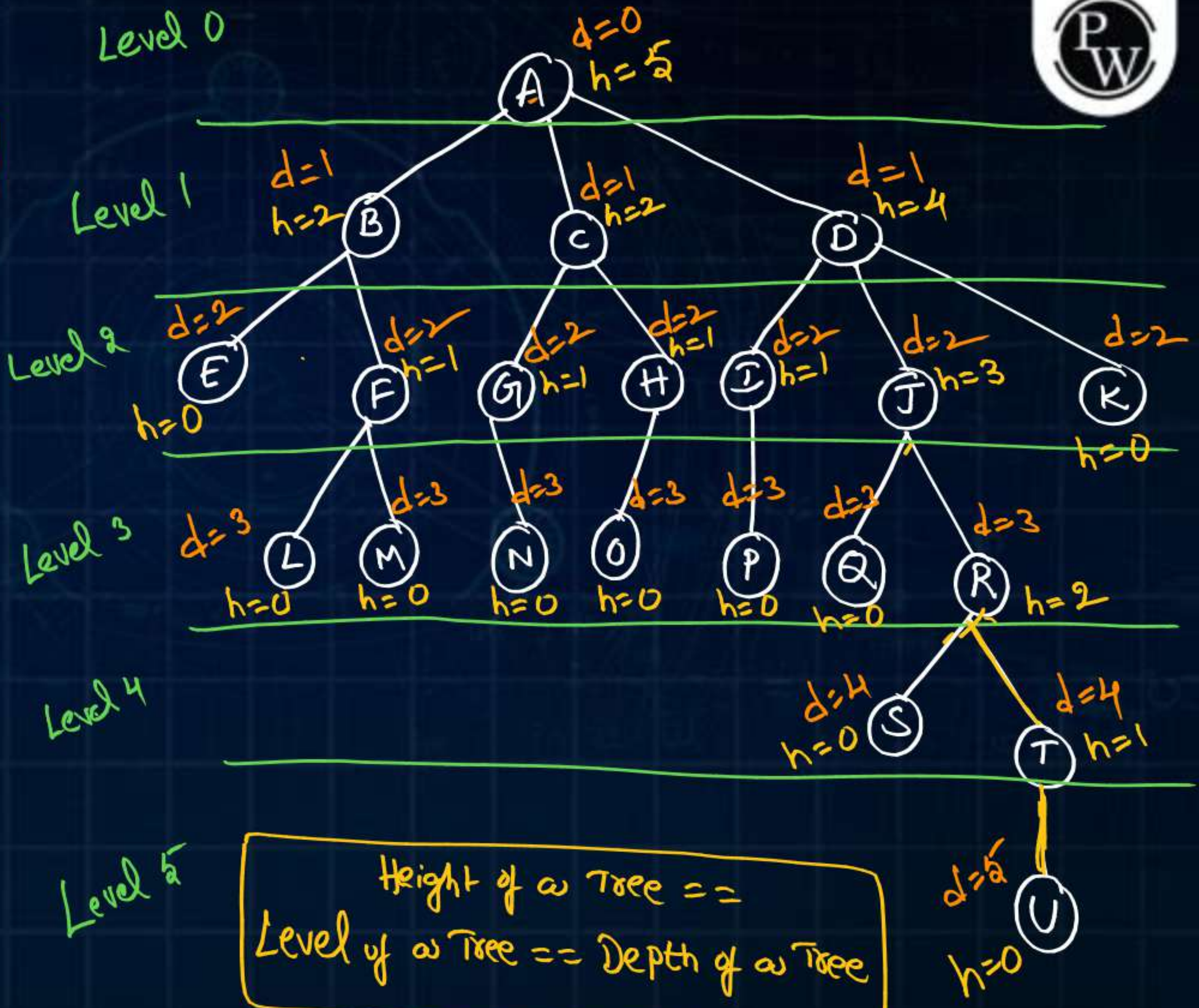
Level, Depth, Height

(Any Numbering by default Starts from '0').

- Level numbering starts with 0 from Root and Continue till Leaf level.

- Depth of a Node : The Number of Edges from Root Node to respective Node.

- Height of a Node : The Number of Edges from respective Node to Leaf Node in the Longest Path.



Height of a Tree == Level of a Tree == Depth of a Tree

Trees

Binary Tree : A Tree, whose degree <= 2

⇒ A Tree in which, Each Node can have Maximum 2 children.

[zero child | 1 child | 2 child]

Types of Binary Trees

1) Full Binary Tree == A Binary Tree in which Each Node can have either 0 child (or) 2 children.

2) Complete Binary Tree == A Binary Tree in which Nodes are filled from top to bottom, left to right.

3) Perfect Binary Tree == A FBT in which all leaf Nodes at same level.

4) Skewed Binary Tree == A Binary in which all nodes (Except leaf) are having either only left child == Left-skewed or only right child == Right-skewed

5) Binary Search Tree == A Binary Tree, $\boxed{\text{Left subtree} < \text{Parent value} < \text{Right subtree}}$ at each level.

6) Binary Heap == A CBT in which, Parent value > All Children == Max-Heap (OR) Parent Value < All children == Min-Heap

7) AVL Tree == A Height balanced BST

i Balance factor = {-1, 0, +1} of Each Node

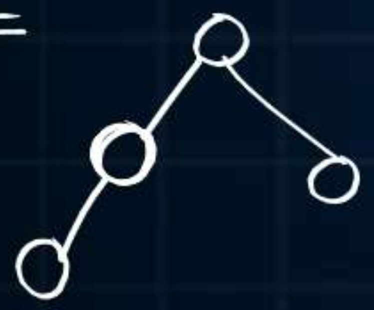
Trees

Examples

Ex:1

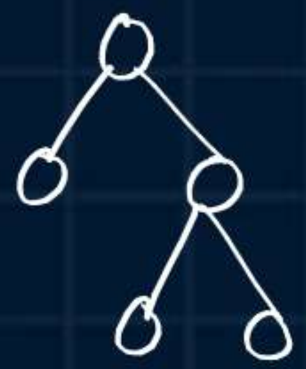
CBT, FBT, PBT, SBT, BST, Heap, AVL . . .

Ex:2



CBT, Not FBT, Not PBT

Ex:3



FBT, Not PBT, Not CBT

Ex:4

CBT, FBT, Not PBT

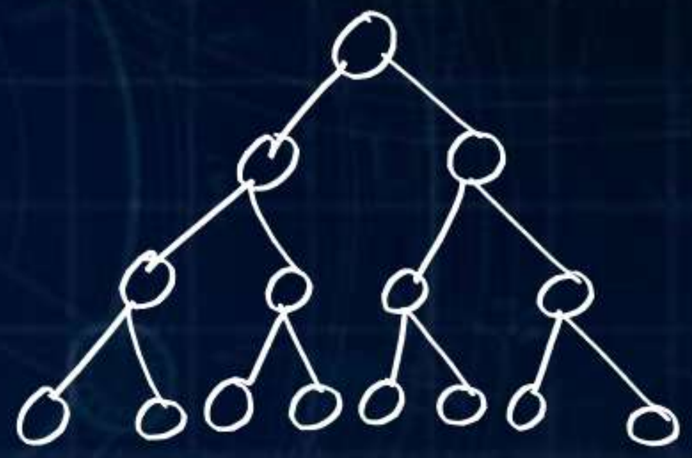
Ex:5

CBT, not FBT, Not PBT

Ex:6

Not CBT, Not FBT, Not PBT

Ex:7



CBT, FBT, PBT.

- → Every PBT is FBT and CBT.
- → Every CBT need not to be FBT, PBT
- → Every FBT need not to be CBT, PBT

Trees

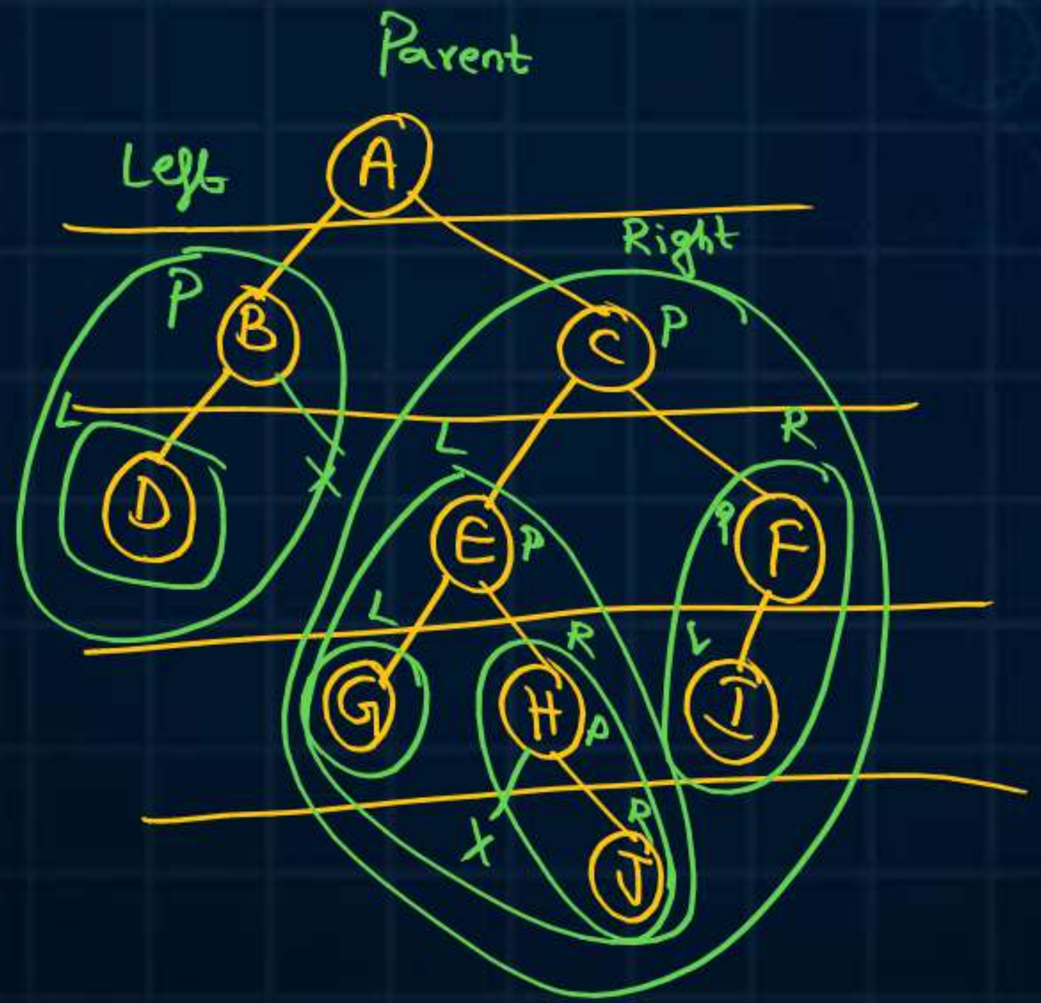
Traversal == The order, in which nodes are accessed (or) visited.

Tree-Traversals

2 Traversals : 1) Breadth-First Traversal (Level-order Traversal)

2) Depth-First Traversal
- In-order : Left Parent Right (LPR)
- Pre-order : Parent Left Right (PLR)
- Post-order : Left Right Parent (LRP)

Example



Level order : A, B, C, D, E, F, G, H, I, J

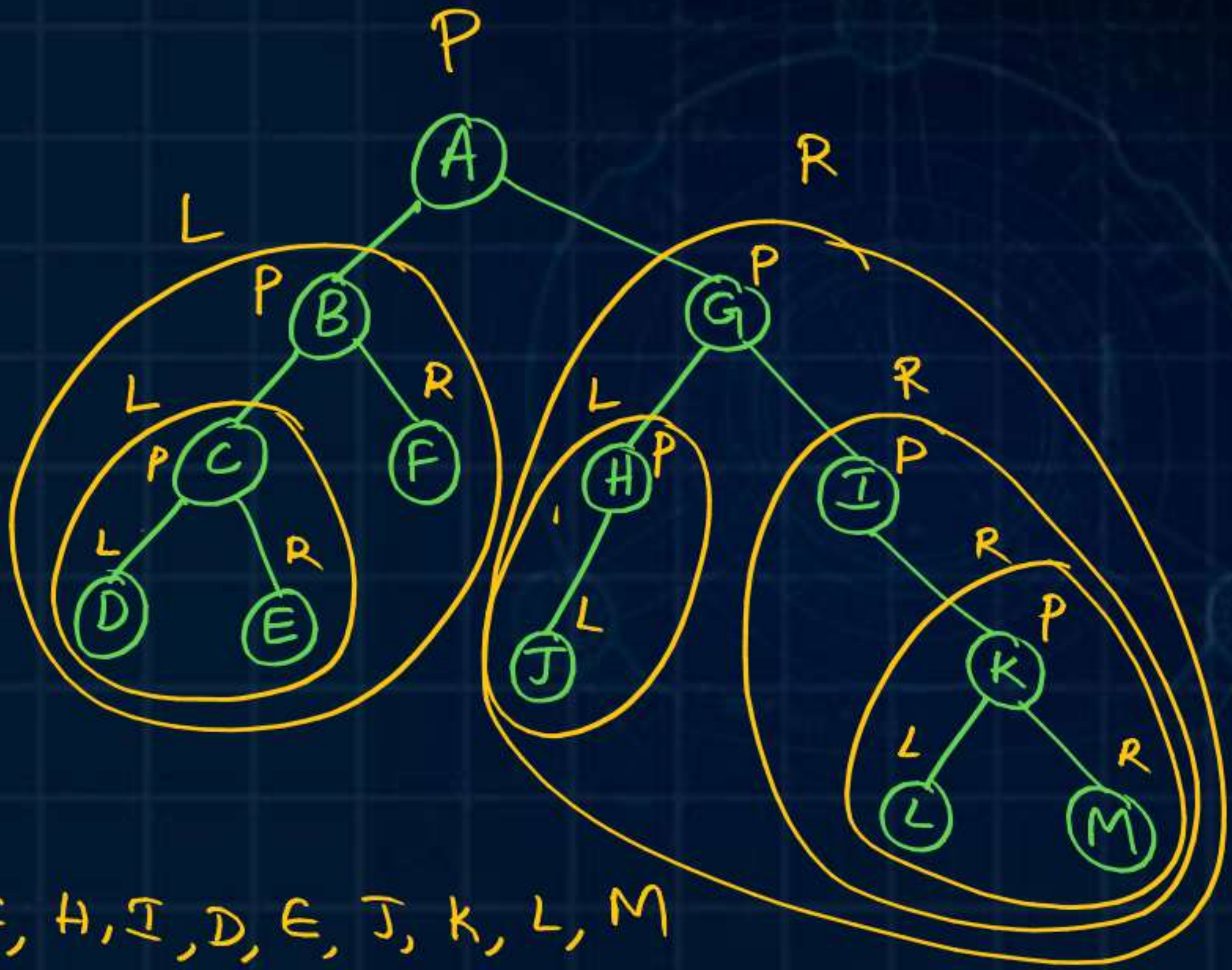
In order : D, B, A, G, E, H, J, C, I, F

Pre order : A, B, D, C, E, G, H, J, F, I

Post order : D, B, G, J, H, E, I, F, C, A

Trees

Example-2



Level order : A, B, G, C, F, H, I, D, E, J, K, L, M

In-order (LPR) : D, C, E, B, F, A, J, H, G, I, L, K, M

Pre-order (PLR) : A, B, C, D, E, F, G, H, J, I, K, L, M

Post-order (LRP) : D, E, C, F, B, J, H, L, M, K, I, G, A

Trees

Numbering starts from '0'

1) In PBT, No. of Nodes at Level 'L' == 2^L

2) In a Binary Tree, If 'x' leaf Nodes, Then No. of Nodes with Exactly 2 children == $x-1$

3) In a Binary Tree with 'N' Nodes, Max height = $N-1$

min height = $\lceil \log_2 (N+1) \rceil$

4) In a Binary Tree, with height 'H' to form, min Nodes = $H+1$

Max. Nodes = $2^{H+1}-1$

5) The No. of Unlabelled Binary Trees with 'n' nodes == $\frac{{}^{2n}C_n}{n+1}$

6) The No. of Labelled Binary Trees with 'n' nodes == $\frac{{}^{2n}C_n}{n+1} * n!$

Question

In order : Left | Right child Identification

Pre | Post order : Parent Identification

#Q. Consider the In-order traversal of a binary tree is C B G E D F H A J I L K (LPR) and Pre order traversal is A B C D E G F H I J K L. The Post Order Traversal is ________ (PLR)

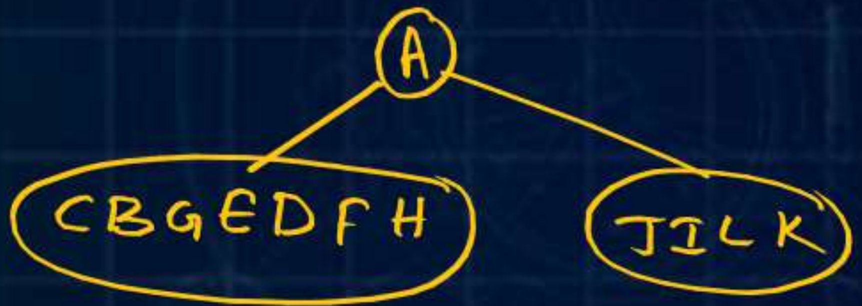
a) C G E H F B D J L K I A

b) C G E F H D B J L K I A

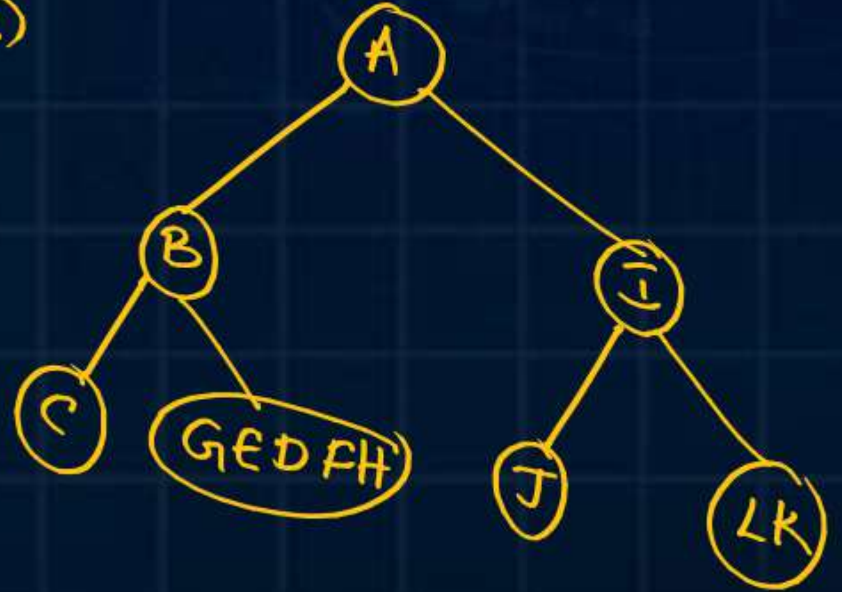
c) C G E H F D B J L K I A

d) C G E H F D B L J K I A

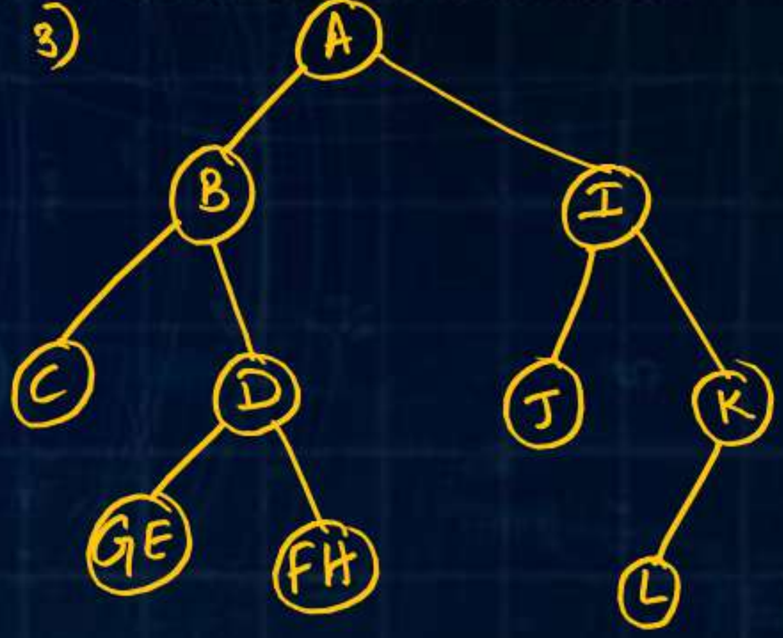
1)



2)



3)



4)

Post-order Traversal (LRP) : C, G, E, H, F, D, B, J, L, K, I, A

Question

H/W

#Q. Consider the In-order traversal of a binary tree is 12 8 11 15 27 10 18 23 35 30 38 41 46 and Post order traversal is 12 11 27 15 8 23 30 38 46 41 35 18 10. The Pre Order Traversal is ____________

a. 10 8 12 15 11 27 18 35 41 23 38 30 46
b. 10 8 12 15 11 27 18 35 23 41 30 38 46
c. 10 8 12 15 11 18 27 35 23 41 38 30 46
d. 10 8 12 15 11 27 18 35 23 41 38 30 46

Summary

Tree Fundamentals

- Tree Terminology
- Types of Binary Trees
- Tree Traversals
  - BFT
  - DFT
    - In order
    - Pre order
    - Post order
- Tree Formulae

To be Contd ... :)

Thank

THANK

Keep Hustling!