# Module 10

## Modern Navigation Systems

## Solving for Time

## Module 10A
## Fundamental concepts

# Summary of Module 10

- **Students will learn how to determine time from celestial measurements, and about the relationship between index and time offset errors for sextant and GPS observations, respectively. (10A)**
- Students will learn how to read and use Nevil Maskelyne's 1804 Nautical Almanac, the predecessor to today's modern Nautical and Air Almanacs. (10B)
- Students will use Newton's method to determine latitude, learn how to relate the lunar distance to GHAϒ in order to determine GMT/UTC time and longitude, and learn how to compute the corrections for parallax using Richharia's equations. (10C)
- Students will use these new skills to analyze actual celestial observation data from the Lewis and Clark expedition. (10D)

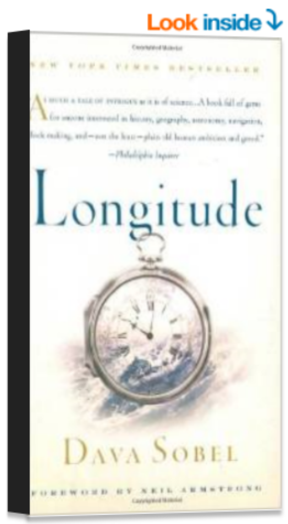
# Reading/viewing

- Review, as needed, relevant material from previous modules.
  - In particular, review
    - Module 2A, in which you determined the look angles to both Geostationary and GPS satellites including the parallax terms
    - Module 8, where you inverted the formulas from Module 5 in order to determine position from measurement of look angles, rather than determining look angles from position.
    - Module 7, in which you explored least-squares problems
- Re-read, as needed, relevant portions of the primary text, by Kayton and Fried
  - Chapter 2, with particular attention to equation 2.20
  - Chapter 5, with particular attention to sections regarding clock errors, clock characteristics and GPS.
  - Chapter 5 will be the main topic of Module 11.

# Additional Reading

- Review the lists of supplemental material from Module 1C
- In particular, consider purchasing or downloading *Longitude*, by Dava Sobel and *Drawing the Line*, by Danson.
    - These are not text books, and are fun to read.
- Also, the Lewis and Clark Journals contain unedited celestial data with supporting text written by Lewis and Clark, themselves, and are available for free download.
- River Horse is a travelogue of crossing America by boat, and describes in detail the modern day features of the route taken by Lewis and Clark from St. Louis to the Pacific ocean
- Maskelyne's Nautical Almanac, available from Amazon, provides for a remarkable comparison with the modern day Nautical Almanac jointly published by the US and United Kingdom.
- Descriptions of these books are repeated in the following slides from Module 1.

# Longitude, by Dava Sobel

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**Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time** Paperback – October 30, 2007

by Dava Sobel (Author)

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# Drawing the Line, by Danson

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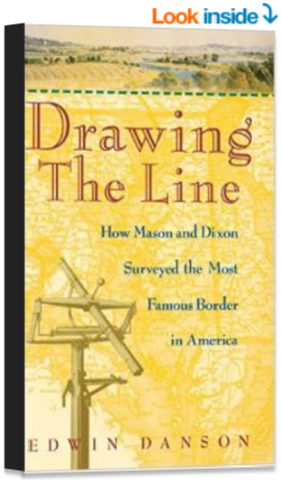


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**Drawing the Line : How Mason and Dixon Surveyed the Most Famous Border in America** Hardcover – December 8, 2000

by Edwin Danson (Author)

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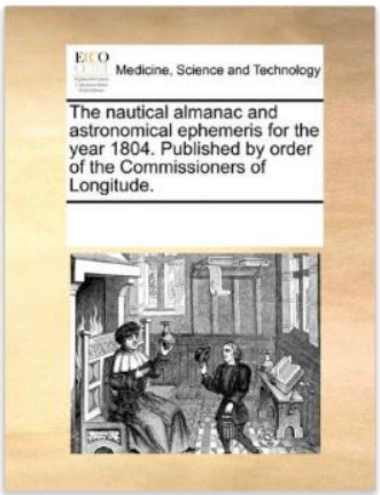
# The 1804 Nautical Almanac, by Maskelyne





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**The nautical almanac and astronomical ephemeris for the year 1804. Published by order of the Commissioners of Longitude.** Paperback – June 1, 2010

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# Lewis and Clark Journals


Online – University of Nebraska, or Amazon (incl. Kindle)

# River Horse, by William Least-Heat Moon

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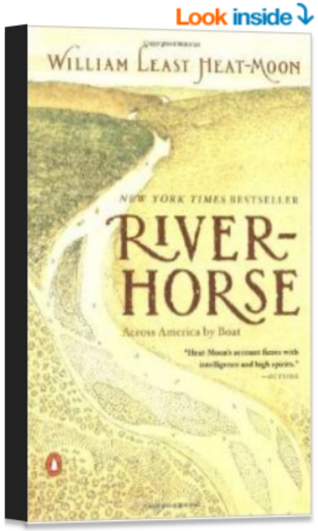


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**River-Horse: The Logbook of a Boat Across America** Paperback – April 1, 2001

by William Least Heat-Moon (Author)

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The author of **Blue Highways** and **PrairyErth** "takes us on a lifetime voyage full of imagery, insight and appreciation." --**Cleveland Plain Dealer**

In his most ambitious journey ever, **William Least Heat-Moon** sets off aboard a small boat named Nikawa ("river horse" in Osage) from the Atlantic at New York Harbor in hopes of entering the Pacific near Astoria, Oregon. He and his companion, Pilotis, struggle to cover some 5,000 watery miles, often

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# Online computational assistance

- Go to USNO's web site, specifically
  - o Celestial Navigation Data for Assumed Position and Time at http://aa.usno.navy.mil/data/docs/celnavtable.php
- This page will serve as your custom "calculator app" for performing the assignments with a minimum of tedium

# Background to the problem

- In the previous module, we deliberately chose to analyze geostationary satellites for two reasons
  - They are close enough to earth that the parallax term (cf. Richharia's equations) cannot be ignored
  - Longitude $\phi_e$ of an "earth station" is easily computed from determination of $\phi_{es}$, because $\phi_s$ , the sub-satellite longitude, is known in advance and <u>**does not vary with time**</u>

# The general problem of *time*

- We have meticulously investigated celestial and satellite navigation techniques under the assumption that **we know the time**, relative to Greenwich Mean Time and Universal Time Coordinated, that our observations of stars and satellites were made
- However, this generous and critical assumption is seldom true

# Knowing the time

- It is possible, by observing the sun, to determine local apparent noon, the instant at which the sun is at its highest point in the sky
  - Meaning that its azimuth angle is ±180°
- At this instant, the elevation of the sun in the sky can be used to compute latitude
  - Provided that one knows the declination of the sun, which varies between ±23 ½ degrees due to the relative tilt of the earth's axis.
- But, to adjust local noon to <u>**mean**</u> time, one must know the **equation of time** correction to for clock errors due to the elliptical orbit of the earth around the sun
  - This correction varies slowly enough that knowledge of the daily calendar date is adequate to estimate this correction, which does not apply to GPS navigation

# Knowing GMT/UTC

- However, knowledge of local apparent noon is not adequate for determining longitude
  - Knowledge of Greenwich Mean Time / Universal Time Coordinated (GMT/UTC) is required

# This is because…

- Without knowledge of time, it is impossible to separate $\phi_{es} = \phi_e - \phi_s$ into knowledge of one's longitude $\phi_e$ because
  - for stars, the rotation of the earth
  - and for satellites, because of the relative rotations of the earth and of the satellite(s).

# There are three ways to determine GMT/UTC

- Observe the moons of Jupiter
  - Because they are sufficiently far away, this can be done without correcting for parallax
- Measure the lunar distance
  - This is the angle between the moon and the sun, or the moon and a star
    - It varies with time, and hence can be used to solve for time
    - But, it requires corrections for parallax, because the moon is close to earth and $r_e/r$ cannot be ignored
- Possess an accurate chronometer that maintains GMT, and compare it with measurement of local apparent noon to determine longitude
  - Until Harrison built his famous chronometer, this was not thought to be possible
  - GPS provides the chronometer via its ensemble of atomic clocks on board GPS satellites that are slaved to UTC
  - Chronometers and atomic clocks will be discussed in Module 12

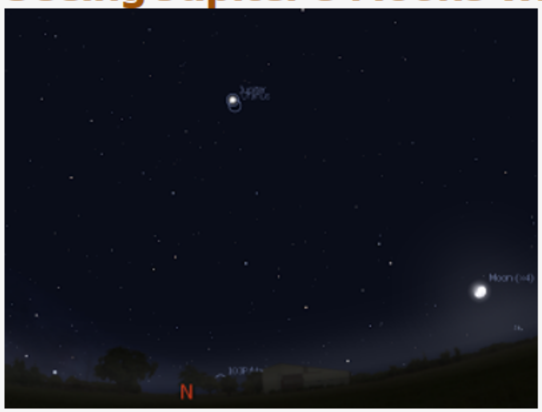
# Jupiter's moons

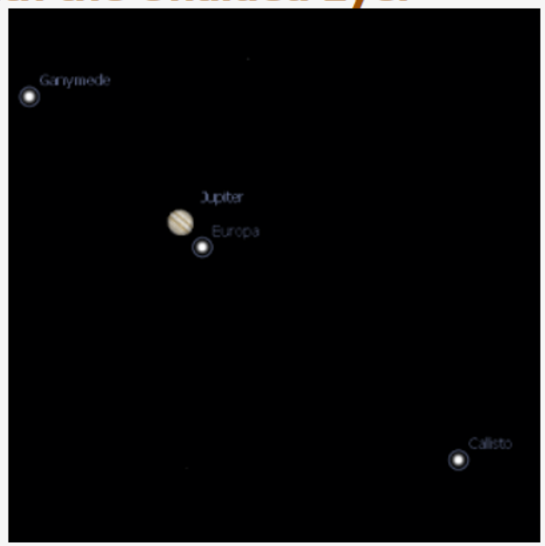
- Jupiter has over 70 moons
- Four of these are readily visible through binoculars
  - Europa, Callisto, Io, and Ganymede
- They orbit Jupiter at approximately the same inclination, so the moons appear to be collinear when viewed from earth
- Because their orbital radii differ, their orbital periods differ (because of Kepler's third law)
- This makes it possible to use their relative positions as a clock
- Since Jupiter is far away from earth, there is no need to make corrections for parallax, and viewers at any point on earth from which Jupiter can be seen will deduce the same value for GMT/UTC

# Viewing Jupiter’s largest moons

Sunday, September 26, 2010

## Seeing Jupiter's Moons with the Unaided Eye.




Left image, Jupiter at 11:00 pm on Monday 27 September; Right image, Orientation of the Moons, Callisto will be to the right and below Jupiter, Ganymede will ne up and to the left.

It is not commonly known, but Jupiter's moons are bright enough to be seen with the unaided eye. However, they are so close to bright Jupiter that the intensity of it's light (and the optical imperfections of our eyes), makes it impossible to see them except under special circumstances).

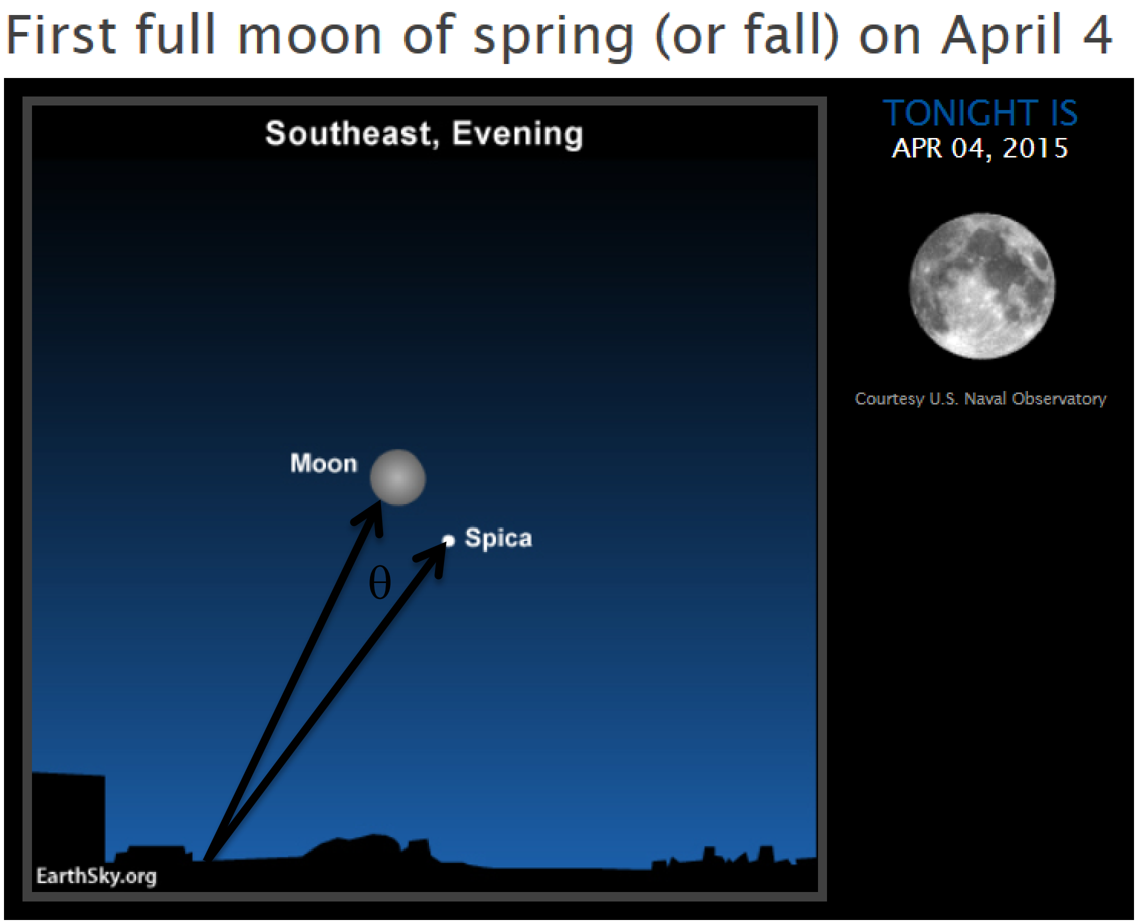
At opposition bright Ganymede (mag 4.6) and Callisto (mag 5.7) can be far enough away from Jupiter to see when they are at their maximum distance from Jupiter in their orbits. Jupiter's light will still probably obscure them for all but those with the most sensitive vision. However, if you use a wall or post to just block out Jupiter's light, you should see them pop into view.

http://astroblogger.blogspot.com/2010/09/seeing-jupiters-moons-with-unaided-eye.html

# The lunar distance

- All of the stars appear to rotate the earth in unison
  - What is really being observed is not the motion of the stars, but the rotation of the earth "beneath" them
- Since the moon orbits the earth, with an orbital period different from that of earth's daily rotation, its motion relative to the sun or stars can be used to determine time
- This is done by measuring, with a sextant, the angle between the moon and the sun and/or stars
  - This angle is called the *lunar distance*
- But, because the moon is close to earth, the parallax error of this measurement is significant, and must be corrected as part of the navigation algorithm
- The lunar distance must also be corrected for the large size of the moon (cf. semi-diameter, as explained in the Buckley video), and correction for atmospheric refraction via corrections from a look-up table in the Nautical Almanac is essential.

# Lunar distance θ between the moon and the star Spica



# Computing the lunar distance

- The "lunar distance" is the angle, measured with a sextant, between the sun and moon. When measured from the surface of the earth, the angle will be different from that computed using vectors from the center of the earth. This is the topic of the next sub-module.
- To compute the geocentric lunar distance, define vectors using the right ascension and declination values for the sun and moon, transform to Cartesian coordinates, and compute the dot product as shown in Module 2 and in the spreadsheet *moon angles rev 5a*, which is included with this module.

# Even if time is known, another problem remains

- For sextant measurements, there is an “offset” error that causes the sextant to indicate a non-zero value of angle even when the the angle between two objects being viewed is zero
  - This error, called the “index” error remains constant for a given sextant, and is easily measured and corrected for, as shown in the Buckley video (cf. Module 8)

# Sextant index error, cont'd

- In principle, one can also "solve for" the index error
  - By making an additional measurement of a third or fourth star during celestial observations

# For GPS

- The crystal oscillator used for a local clock in every GPS receiver has an offset error that changes with time and temperature and as a result of mechanical shocks to the crystal over time
- It is solved for as part of the GPS navigation solution by using four, rather than a minimum of three, satellites for determining *x, y, z*, and *t*.
- Since the GPS satellites broadcast "Almanac" information (cf. Module 11) that contains UTC data, the *t* described here is the time error, namely the clock offset error of the GPS receiver
- The time offset error is a GPS receiver is directly equivalent to the index error of a sextant.

# What next?

- In preparation for the sub-modules that follow, you will use the USNO site at http://aa.usno.navy.mil/data/docs/celnavtable.php.
- Using the web site as a calculator, you will be able to answer several questions, such as what time is the sun directly south of Greenwich, England, and what is the observed angle between the sun and moon at local apparent noon 38° 36′ / 91° 57′ (cf. assignment 10.1).

# Algorithmic points

- The approach of using the USNO web page as a celestial calculator is equivalent to solving the equation $M = E - e \sin E$ for numerous values of $E$ in order to create a lookup table.
- The sight reduction tables at the end of the modern Nautical Almanac, which Buckley demonstrates in his video, are based on this same approach.

# Assignment 10.1

1. Go to the USNO web site and determine the times of sunrise, noon, and sunset on June 2, 1804, at lat/long 38° 36′ / 91° 57′. Noon is when the azimuth at which the sun is observed to be 180 degrees.
2. Determine an estimate for local noon by finding the mid-point between sunrise and sunset. What is the difference between your two values for noon? This difference should be equivalent to the equation of time. (Refer to Module 8 for a discussion of the equation of time.)
3. Using the az/el predictions of the USNO calculator, compute vectors to the sun and moon and use the vector dot product to compute the "lunar distance" between the sun and moon at noon GMT on June 2$^{nd}$ 1804 (cf. equation 2.20 from the primary text as explained in Module 2 and the attached spread sheet *moon angles rev 5a*).

# End of Mod 10A