Module 3

Modern Navigation Systems

Spherical trigonometry and perturbation theory for solving great and small circle problems on or near the surface of the earth.

Module 3A

Flat Earth Approximations

Summary of Module 3

- Students will learn to apply perturbation theory and spherical trigonometry to the problems of surveying and of comparing great circle and small circle routes for navigation on or near the surface of the earth.
- Students will repeat the spherical trigonometry computations performed by Charles Mason when surveying the Mason-Dixon line, and will compute the differences in miles flown between great circle and small circle routes from one city to another using Napier’s rules.
- Students will apply perturbation theory to the simplification of the equations of small and great circles. Students will apply perturbation theory to the simplification of the equations of small and great circles.
- Students will also learn the nuances of Mercator and Lambert Conformal Conic map projections, with particular emphasis on the critical differences between aviation and nautical charts.

Geometry of navigation on a spherical earth

- The next slide, excerpted from chapter 2 of the text, shows the definitions of angles and distances for an aircraft's path from one point above the spherical earth to another.
- Read the accompanying material in the text.
- Note the relationship between spherical and Cartesian coordinates.
- The diagram in the next slide will form the basis for an assignment later in this module. For the present, it is meant to show the relationships between the various paths that can be used to define an aircraft's journey from one point to another.

Geometry for navigation above a spherical earth

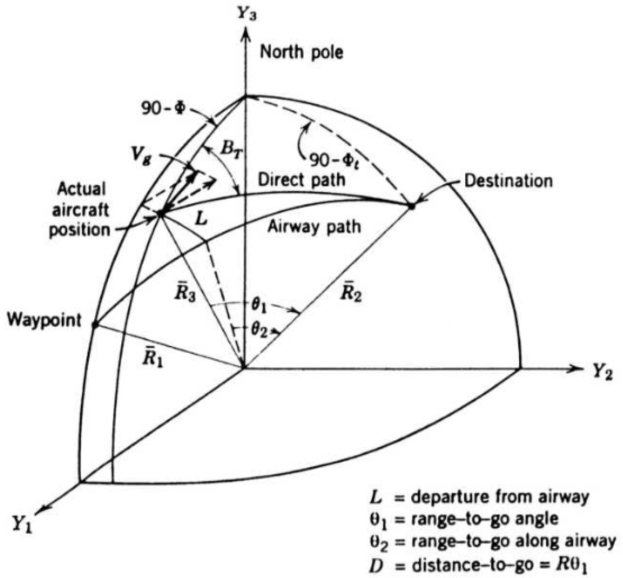

Figure 2.8 Course-line calculations.

From the text.

Computation of arc lengths

- From the previous module, you have learned how to compute great circle distances by using the vector dot product
- An earlier assignment (2.1) was to derive equations 2.22 for the great circle distance between two points and the corresponding bearing angle. These equations are shown on the next slide.

Equations for spherical trigonometry

The first equation is the result of a simple dot-product evaluation of the angle between vectors from the center to the surface of the earth

$$\cos \frac{D}{R_G} = \sin \Phi \sin \Phi_t + \cos \Phi \cos \Phi_t \cos(\lambda - \lambda_t)$$

$$\sin B_T = \frac{\cos \Phi_t}{\sin(D/R_G)} \sin(\lambda - \lambda_t) \tag{2.22}$$

The second equation is the bearing angle from the tip of one vector to the other, computed using the equations of spherical trigonometry.

Comments on equations 2.22

- Note that the vector dot product yields a value for the angle, in radians, of D/r, where D is the great circle distance between two points and r is the radius of the earth
- This means that to compute D, and inverse cosine needs to be applied.

The flat earth approximations

- When two points on the surface of the earth are close
  - Where “close” is defined as the case where D << r
- A flat earth model can be used. This results in the Cartesian equations shown on the next slide.

Range D and bearing B_T for a flat earth

$$D = [(x - x_t)^2 + (y - y_t)^2]^{1/2}$$

$$B_T = \tan^{-1} \frac{x - x_t}{y - y_t}$$

From text, chapter 2

Derivation of the flat earth equations

- To derive the flat earth equations, one approximates cos θ by its small angle Taylor series approximation
  - $\cos\theta \sim 1 - (\theta^2)/2$
- This yields the well-known Pythagorean results based on the flat earth definition that $x^2 + y^2 = d^2$

The Pythagorean theorem in 3 dimensions

- Using the equations from module 2a, slide 9, it can be shown that in three dimensions,
  - $x^2 + y^2 + z^2 = d^2$ (1)
- Indeed, in a Euclidean *hyperspace*, the Pythagorean equation can be extended to an arbitrary number of dimensions.
- Note that in a Euclidean space, the triangle inequality holds, namely that the hypotenuse of a triangle is shorter than the sum of the lengths of the sides of the triangle.
- Note that in a non-Euclidean space, the distance from point A to point B need not equal the distance from point B to point A; the distances are *non-symmetric*.
- For this course, equation (1), above, is sufficient.

Assignment 3.1

3.1.1 Show, using the equations from Mod 2a, slide 9, that $x^2 + y^2 + z^2 = r^2$

3.1.2 Derive, for the circle x2 + y2 = 1000 the tangent line that approximates the circle at the angle of 30 degrees from the origin

3.1.3 Plot the line segment and the circle segment on the same graph using scales that demonstrate the quality of the approximation

End of Mod 3A