Module 8

Modern Navigation Systems

Sextant Navigation

Module 8A
Sextant Observations and Computations

Summary of Module 8

- **Students will use simulated and/or actual measurements of the angles of three or more stars above the horizon to determine their position on the surface of the earth at a known instant in time. (8A)**
- The Equation of Time will be Introduced (8B)
- The concept of dilution of precision will be introduced and linked to the least squares algorithms developed earlier. (8C)

Reading/viewing

- Read the mathematical techniques section of the Nautical Almanac
- Read the primary text, Chapter 12
- Read the pdf version of the article *The Circle Game*, which has been posted in Module 8.
- For the assignment, consult the pdf sextant exercise file the is included with Module 8. When solving the problem, use the enclosed pdf almanac pages for the year 2000.
- View two videos using Amazon Prime
    - You can access these videos fro free using the Amazon Prime 30 day free offer
        - Watch William Buckley's "Celestial Navigation Simplified"
        - Watch Robert Redford in "All is Lost"
            - If you don't want to watch the entire movie, scroll forward to where he uses a sextant

A pdf of this article is posted in Module 8.

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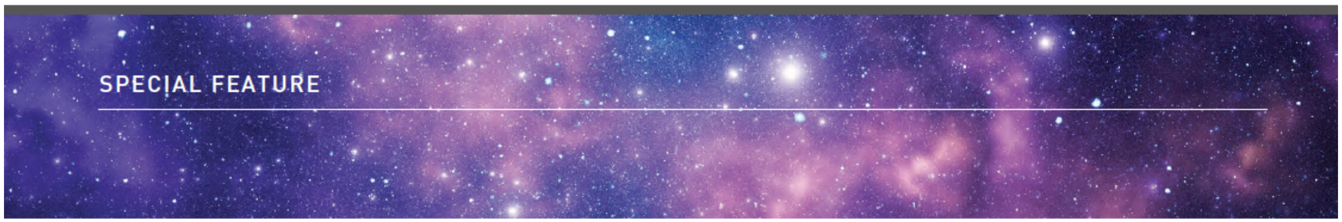


The Circle Game:
The use of the Lunar Distance and Related Measurements for Celestial and Satellite-Based Navigation and Timekeeping

Daniel G. Jablonski[†]

Abstract

The determination of a position on the Earth's surface depends critically on the mathematics and measurement of circles as functions of time, distance, and angle. The multi-century history of this topic has led us to the world's significant dependence on the Global Positioning System (GPS) and other satellite navigation systems. Based on a simple paradigm, the problem of solving three equations in three unknowns, we revisit the history of Maskelyne and Harrison and the Longitude prize; the survey of the Maryland-Pennsylvania border by Mason and Dixon; the vicissitudes of Lewis and Clark; and the development of GPS. In particular, we reanalyze the measurements made by Lewis and Clark on June 2, 1804 in light of the recent (2000) analysis of these data by Bergantino and Preston. We show that this analysis, when viewed retrospectively, provides insight into the manner in which GPS combines the techniques of Maskelyne and Harrison, Mason and Dixon, Ellicott and Patterson, and Lewis and Clark. We then re-evaluate the celestial measurements of Lewis and Clark and provide some new analyses of their data.

To use a sextant with the sun

- It is essential to be use the sun shields to protect your eyes
- Accurate time keeping is needed in order to perform a running fix
  - Since there is only one sun, to make the requisite number of observations, it is necessary to observe the sun at different points in time
  - From a moving vessel, a motion model is needed. A simple model is given in the Nautical Almanac.
  - But, this is an ideal application of a Kalman filter (cf. Module 7).
- Other important features include the need to understand the motion of the sun during the various seasons
  - Measurement of the sun angle at different times of day permits navigation during daylight hours, based on the concept of a “running fix”
  - This leads to the “equation of time”, which describes the relationship between the sun angles and the seasons, and which is discussed in Mod 8B.

Correction of sextant measurements

- The Buckley video shows in detail, and the Nautical Almanac explains in detail, how to correct actual sextant measurements for
  - Dip
  - Refraction
  - Index error
  - Semi-diameter
- These measurement issues are well explained and straightforward to account for

To use a sextant with the stars

- It is essential to be able to identify the stars that you will be measuring
- This is best accomplished by going outside and looking at the stars
- Other important features include the need to understand the motion of the sun during the various seasons
  - Measurement of the sun angle at different times of day permits navigation during daylight hours, based on the concept of a "running fix"
  - This leads to the "equation of time", which describes the relationship between the sun angles and the seasons

A "moderate cost" sextant

Astra III Professional

The Astra III Professional, with its low cost, high accuracy, and excellent handling qualities, is destined to be the standard for professional seamen. It has a bronze arc fused to an aluminum frame allowing a higher accuracy along the arc while maintaining a lighter weight than a solid bronze or brass sextant. The handle is a new more comfortable design and has an electrical outlet suitable for powering accessories.

The additional weight of the bronze arc adds about 8 oz. and gives the instrument an excellent balance and heft. All of the accessories shown for the Astra IIIB are applicable for this model; ie. LED lighting of arc and drum, oil, adjusting wrench, and instructions, all in a varnished wooden case. A 3.5 x 40 scope is standard. SW 16 lbs.

http://www.landfallnavigation.com

Astra III Professional

#NA3P(-) List Price $1,100.00 Only **$875.00**

The ASTRA III

- Is a moderate cost sextant that is suitable for navigation at sea
- Was the subject of a lawsuit with respect to import duties
  - The item is treated as a pair of binoculars or other optical instrument
- The ASTRA III is the sextant used by Robert Redford in the film "All is Lost"

An ASTRA III

Figure 1. A sextant, showing the protractor gauge, adjustable and fixed mirrors, and solar filters. This particular instrument is fitted with a "bubble horizon", permitting its use at night in aircraft when no horizon is visible. Many modern aircraft still have "sextant ports", windows that facilitate observations of the night sky.

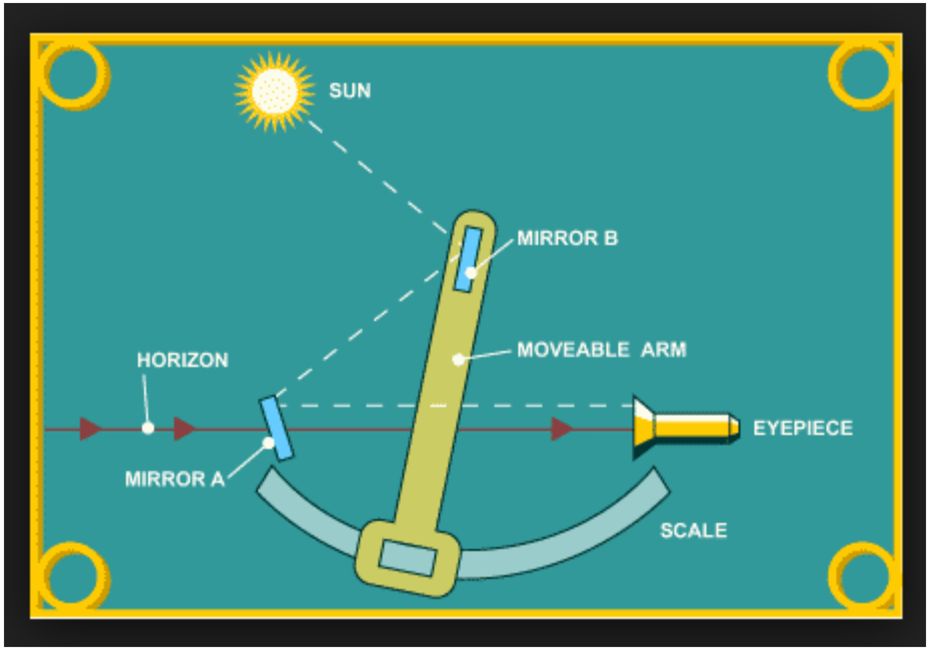
View of the sun through the ASTRA III

Figure 2c. Side-by-side views of the direct and reflected Sun images seen through a sextant. The tree branches are reflected from extra solar filters, which rotate out of the way when not being used.

Note the use of a birdbath as an artificial horizon.

The elevation angle measurement to be used for navigation is one-half of the angle between the sun and its image in the birdbath.

How a sextant works



NOVA Online | Shackleton's Voyage of Endur…
www.pbs.org - 436 × 293 - Search by image
Animation of sextant in use

A sextant is an angle measurement device that eliminates "common-mode" errors.

Swinging the measurement arm lines up a visual image of the sun (or a star) with the horizon. The angle between the two is identified on the "scale".

If the sextant moves during the measurement (it is handheld), the measurement accuracy is unaffected.

To "learn" the stars and satellites, go outside…

- Go outside and look at the stars and planets.
- Find out from NASA's website when the ISS is going to be visible. Go outside and watch that, as well.
- View two or more planets. Venus, Mars, and Jupiter are the best candidates. Mercury is sometimes visible, and Saturn can sometimes be seen. Binoculars are needed to see the most visible 4 moons of Jupiter (it has over 70 moons), and a telescope is needed to see the rings of Jupiter.
- Read on the web about Iridium flares. You may occasionally see one.

To identify stars

- Look at the star charts in the Nautical Almanac (which is not a trivial endeavor)
- Look at the cardboard insert in the Almanac that lists the SHA and declination values for the primary navigation stars. Identify one of the stars on the list, and then identify others by their positions relative to this first star.
    - This is done by computing the delta values of declination and SHA, and then viewing the sky with these coarse difference angles as guidance.

For example,

- Sirius has declination of -17 and SHA of 259.
- Rigel, one of the stars in the constellation Orion, has declination of -8 and SHA of 281.
- Thus, the stars are separated by 3 degrees of declination (latitude), are both below the earth’s equatorial plane, and are separated by 22 degrees of longitude.
- This is easily seen using a celestial globe.

A celestial globe

- A celestial globe shows all of the stars on the surface of a sphere, independent of the distance the stars are from the earth.
- The sun orbits the earth with a tilt angle of 23 ½ degrees, which corresponds to the 23 ½ degree tilt of the earth's rotational axis with respect to the earth's orbital plane around the sun.

A celestial globe

Each star has a lat/long on the celestial sphere; these are called GHA and Declination.

The sun

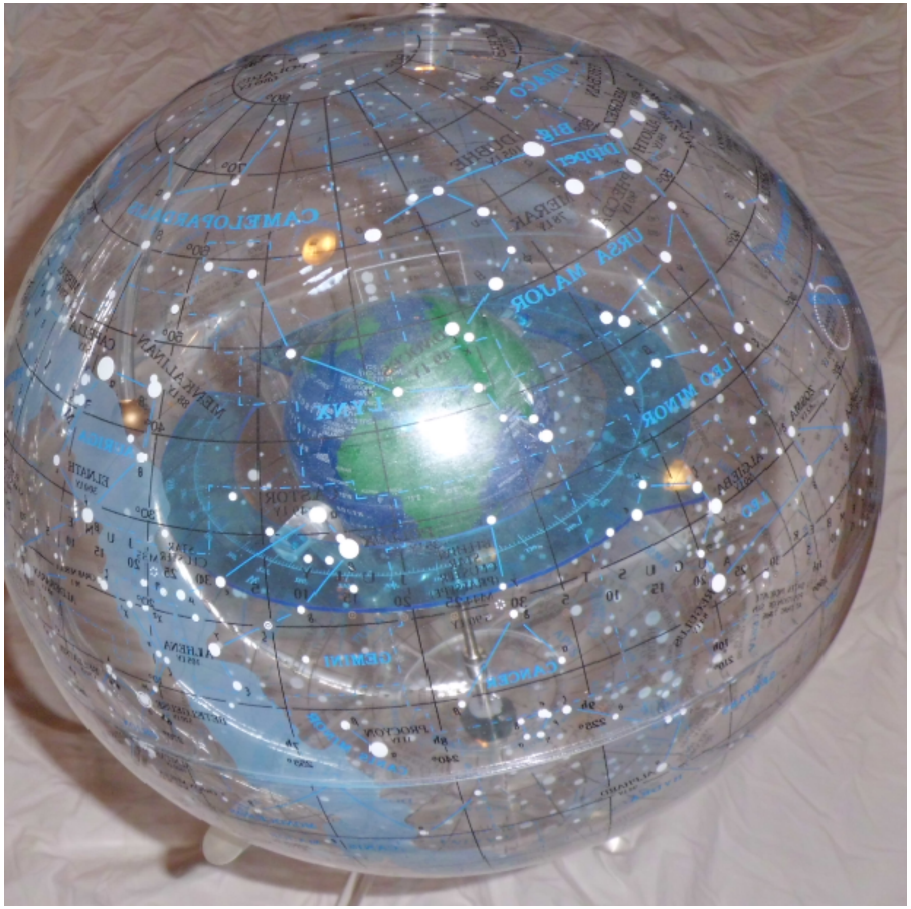
SHA versus GHA♈ versus LHA (cf. Module 5)

- The sidereal hour angle, or SHA of a star, is to a good approximation independent of the motion of the earth (subject to the precession of the equinoxes).
- The Greenwich hour angle of Aries (GHA♈) provides the relative angle between the earth and the celestial sphere, thus linking a star's SHA to the position of the earth, via the Greenwich Hour Angle, or GHA.
- The local hour angle, or LHA, accounts for the rotation of the earth about its axis.
- Knowledge of universal time coordinated (equivalent to Greenwich Mean Time) is needed to determine GHA♈ and to compute the LHA.

Relationships between the various hour angles

- GHA = SHA + GHAϒ
- LHA = GHA + longitude
  - For an earth orbiting satellite, LHA corresponds to ϕ_{es}, the angle between the sub-earth longitude of the satellite and the longitude of a satellite earth station (cf. the satellite pointing equations of Riccharia in Module 5).
  - The GHA of a star is analogous to the sub-orbital longitude of a satellite, and LHA is the difference between this longitude and the local longitude of the navigator who is using a sextant.
- Due to the earth's rotation, GHA is a function of time (as is GHAϒ, but not SHA). Except for a geostationary satellite, LHA is also a function of time.

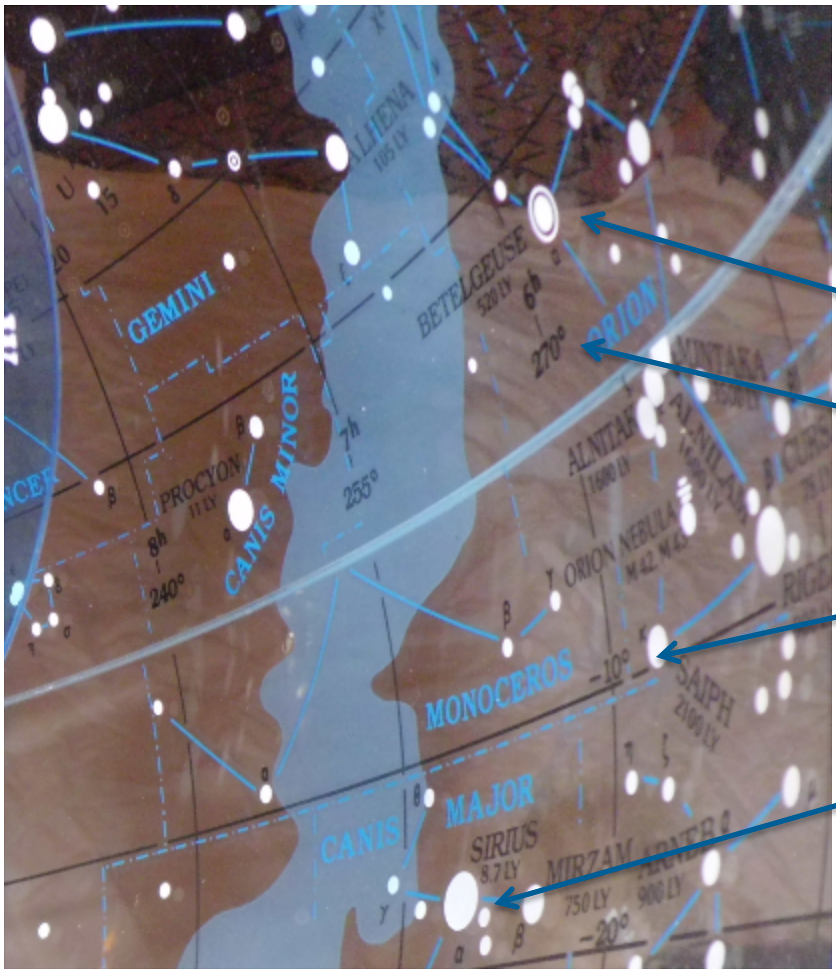
A close-up of the celestial sphere



The stars are printed inside the sphere so that an erasable marker can be used to write on the outside of the sphere.

Inside the celestial sphere

SHA and Declination



Betelgeuse: SHA 271 Dec +7

270 degree SHA reference

-10 degree declination reference

Sirius: SHA 259 Dec -17

Celestial Measurements

- Choose stars to measure
- Measure the elevation angles to the stars at morning or evening twilight (so that you can see both the stars and the horizon), making an accurate recording of the time of each measurement (GMT or universal time coordinated – UTC)
- Or, measure the sun angle above the horizon at multiple points in time during daylight hours.

The celestial navigation algorithm

- Look up the SHA's and declination values of the stars that you measured using the Nautical Almanac
- With knowledge of the time (GMT or universal time coordinated – UTC) of each of your measurements, look up the corresponding GHA♈ values.
- Using the equations from the Nautical Almanac, compute the expected elevation angles to the and azimuths to the chosen stars (or the sun) for the points in time at which their elevation angles were measured.

The celestial navigation algorithm, cont'd

- Guess a lat/long for your location.
  - The accuracy of this estimate is unimportant; it simply serves as the initial guess (similar to the initial guess in Newton's algorithm)
- Compare your measured (observed) elevation angles Ho with the computed values Hc for each star.
- Using the procedure in the Nautical Almanac, use the difference between Hc and Ho, along with the predicted azimuth angle, to compute a new estimate of your position.

Graphical solution using a plotting table

The position line of an observation is plotted on a chart using the intercept

$$p = H_O - H_C$$

and azimuth Z with origin at the calculated position ($Long$, Lat) at the time of observation, where H_C and Z are calculated using the method in section 6, page 279. Starting from this calculated position a line is drawn on the chart along the direction of the azimuth to the body. Convert p to nautical miles by multiplying by 60. The position line is drawn at right angles to the azimuth line, distance p from ($Long, Lat$) towards the body if p is positive and distance p away from the body if p is negative. Provided there are no gross errors the navigator should be somewhere on or near the position line at the time of observation. Two or more position lines are required to determine a fix.

from the Nautical Almanac

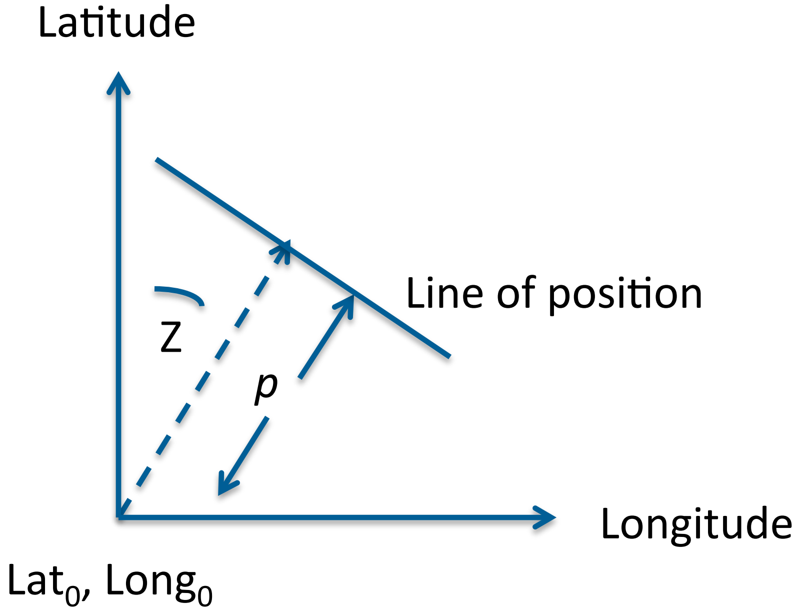
This technique is also known as the "Marc-St. Hilaire" Algorithm**

Z = computed azimuth to a measured star based on the initial position guess Lat_0, $Long_0$

p is the distance between Ho and Hc (based on turning the angle into a distance by scaling the arc-length by the radius of the earth)

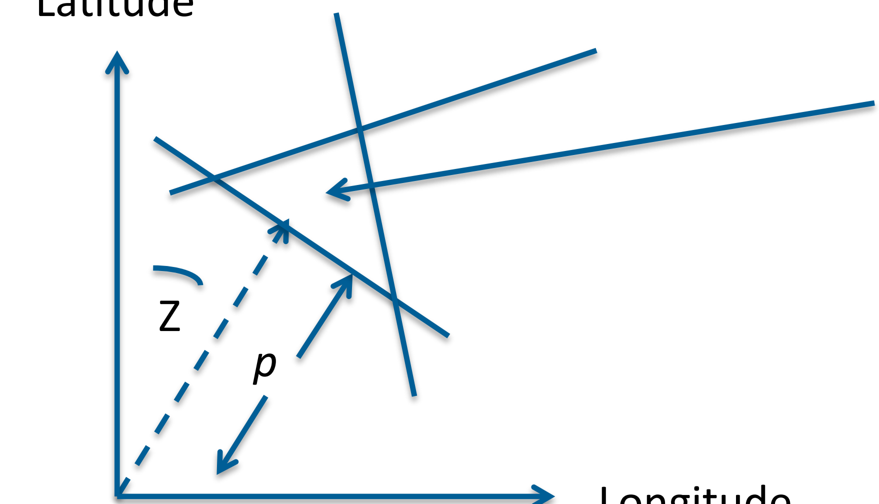
p and Z are used to compute a line of position
The intersection of multiple lines
Of position is used to compute Lat_1, $Long_1$

The process is iterated until the incremental change in estimated position falls below the desired margin of error.



**Admiral Marc St. Hilaire of the French Navy, late 1800's.

Multiple lines of position



Lat_1, $Long_1$ based on least Squares intercept of the Multiple lines of position

Note that a chart table is ideal for solving this problem graphically! Parallel rulers (covered in a future module) referenced to a "compass rose" on a chart make this Straightforward and easy.

Numerical Method Using Least Squares

11. *Position from intercept and azimuth by calculation.* The position of the fix may be calculated from two or more sextant observations as follows.

If p_1, Z_1, are the intercept and azimuth of the first observation, p_2, Z_2, of the second observation and so on, form the summations

$$A = \cos^2 Z_1 + \cos^2 Z_2 + \cdots$$
$$B = \cos Z_1 \sin Z_1 + \cos Z_2 \sin Z_2 + \cdots$$
$$C = \sin^2 Z_1 + \sin^2 Z_2 + \cdots$$
$$D = p_1 \cos Z_1 + p_2 \cos Z_2 + \cdots$$
$$E = p_1 \sin Z_1 + p_2 \sin Z_2 + \cdots$$

where the number of terms in each summation is equal to the number of observations.

With $G = A\,C - B^2$, an improved estimate of the position at the time of fix (L_I, B_I) is given by

$$L_I = L_F + (A\,E - B\,D)/(G \cos B_F), \qquad B_I = B_F + (C\,D - B\,E)/G$$

Calculate the distance d between the initial estimated position (L_F, B_F) at the time of fix and the improved estimated position (L_I, B_I) in nautical miles from

$$d = 60\sqrt{((L_I - L_F)^2 \cos^2 B_F + (B_I - B_F)^2)}$$

If d exceeds about 20 nautical miles set $L_F = L_I$, $B_F = B_I$ and repeat the calculation until d, the distance between the position at the previous estimate and the improved estimate, is less than about 20 nautical miles.

The least squares equations

Assignment 8.1

1. Reproduce the calculations and results of the sextant example in the Nautical Almanac. (See the pdf file included with this module, and use the year 2000 data provided with the file. To check your work, use the 2015 Almanac and its example, which includes the solution.)
2. Next, identify the latitude and longitude of your location using Google Earth or other application. Then, go outside and pick a few stars that by now you have learned to name. (The available stars will depend on the season.) Estimate the azimuth and elevation to each star. Record the time.
3. Calculate the Hc and azimuth to each star for this actual location and time. Compare, in tabular form, to your estimates from above. These calculated values of Hc are to be used as the values of Ho in your computations.
4. Calculate new values of Hc for each star under the assumption that you are at 0 degrees longitude and 0 degrees latitude for the same instant in time. These incorrect values are then used, with your Ho values, to drive the algorithm in the Nautical Almanac (from the pdf file referenced above). Using several iterations of this algorithm, show that the algorithm eventually converges to your correct position.

End of Mod 8A