Module 10

Modern Navigation Systems

Solving for Time

Module 10C
Using the Lunars

Summary of Module 10

- Students will learn how to determine time from celestial measurements, and about the relationship between index and time offset errors for sextant and GPS observations, respectively. (10A)
- Students will learn how to read and use Nevil Maskelyne's 1804 Nautical Almanac, the predecessor to today's modern Nautical and Air Almanacs. (10B)
- **Students will use Newton's method to determine latitude, learn how to relate the lunar distance to GHA♈ in order to determine GMT/UTC time and longitude, and learn how to compute the corrections for parallax using Richharia's equations. (10C)**
- Students will use these new skills to analyze actual celestial observation data from the Lewis and Clark expedition. (10D)

Clearing the lunars

- The lunar distance is no longer included in the modern Nautical Almanac
  - Although it was explicitly provided in the 1804 and earlier versions of the almanac
- It can be calculated from modern almanacs using equation 2.20 from the primary text or the equation coded into the attached spreadsheet entitled .
- But, this is the geocentric lunar distance, which differs from that measured from a point on the surface of the earth.
- Reconciling the measured and theoretical values, as well as adjusting for refraction and semidiameter of the moon, is referred to as “clearing the lunars”

"Solving for time"

- For a given value of ϕ_{es}, matching the lunar distance measurement, cleared of parallax, refraction, and semi-diameter, to a table of lunar distance versus time yields the GMT/UTC time and GHA♈. This also yields a value for longitude.
- First, begin to solve this problem by inverting Richharia's equation for the elevation to a star using Newton's method and angle observations of any two stars, independent of time.
  - Repeated use of the calculator at USNO to explore "search bins" of elevation angle versus latitude can be used in lieu of Newton's method. GPS receivers use this technique during signal acquisition by searching multivariable (e.g., range and Doppler) bins (cf. Mod 11).
- This implementation of Newton's method requires knowledge only of the stars' SHA and declination values. Use of these parameters does not require knowledge of GMT/UTC time.

Recap of Richharia's equations

Finally, the following set of equations can be used to obtain satellite azimuth and elevation from a specified earth station:

$$\text{Right ascension, } \alpha = \arctan(y/x) \tag{B.43}$$

$$\text{Declination, } \delta = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \tag{B.44}$$

$$\text{Elevation, } \eta = \arctan\left(\frac{\sin\eta_s - \dfrac{R}{r}}{\cos\eta_s}\right) \tag{B.45}$$

where

$$\eta_s = \arcsin\left[\sin\delta\sin\theta_e + \cos\delta\cos\theta_e\cos\phi_{se}\right] \tag{B.46}$$

and R = Earth radius
r = satellite distance from Earth centre (use equation B.40c)
θ_e = earth station latitude
$\phi_{se} = \phi_s - \phi_e$
ϕ_s = satellite longitude
ϕ_e = earth station longitude.

For a star, R/r = 0.

Thus $\eta = \eta_s$ and eqn. B.46 Becomes:

$\sin\eta = \sin\delta\sin\theta_e + \cos\delta\cos\theta_e\cos\phi_{se}$

Measuring the values of η for two stars whose declinations are known is sufficient to solve for earth station latitude θ_e and the quantity ϕ_{se}

Once ϕ_{se} is known, the lunar distance can be used to find time and longitude.

Richharia's final equations

$$\text{Azimuth, } A = \arctan\left[\frac{\sin\phi_{se}}{\cos\theta_e \tan\delta - \sin\theta_e \cos\phi_{se}}\right] \qquad \text{(B.47)}$$

This is important

Use the convention given in chapter 2, section 2.6 to obtain the azimuth quadrant. *(see the next slide)*

Range

The distance ρ of a satellite from a given point on the Earth is given as

$$\rho = \sqrt{r^2 - R^2\cos^2\eta} - R\sin\eta \qquad \text{(B.48)}$$

The azimuth rules, from Module 5

- True azimuth is given from A in the previous slide by:
- In the northern hemisphere:
  - A_{true} = 180 + A when the satellite is west of the earth station
  - A_{true} = 180 - A when the satellite is east of the earth station
- In the southern hemisphere:
  - A_{true} = 360 - A when the satellite is west of the earth station
  - A_{true} = 180 - A when the satellite is east of the earth station

To make this approach work…

- It is necessary to rewrite ϕ_{es} as

  $\phi_{es} = [\phi_e + GHA\Upsilon] + SHA$ and

  $\cos\phi_{es} = [(\cos SHA)(\cos(GHA\Upsilon + \phi_e) - (\sin SHA)(\sin(GHA\Upsilon + \phi_e))]$

- Then, use Newton's method to solve for θ_e and $[GHA\Upsilon + \phi_e]$
  - The objective functions $f(\theta_e, [GHA\Upsilon + \phi_e])$ are written in terms of sin(elevation), so as to avoid the need to use arcsines and their derivatives! (This can be seen by inspection of the attached spreadsheet.)

- Next, use the lunar distance to determine GHAϒ by table lookup from the Nautical Almanac

- This yields GMT/UTC time and the earth station longitude ϕ_e

Example: Celestial data from USNO

http://aa.usno.navy.mil/data/docs/celnavtable.php

Celestial Navigation Data for 2015 Apr 14 at 12:00:00 UT

For Assumed Position: Latitude N 39 00.0
Longitude W 77 00.0

Object	GHA	Dec	Hc	Zn	Refr	SD	PA	Sum
	Almanac Data				Altitude Corrections			
	° '	° '	° '	°	'	'	'	'
SUN	359 54.9	N 9 23.5	+15 54.3	90.7	-3.5	15.9	0.1	12.6
MOON	56 17.4	S 9 38.9	+37 40.4	153.9	-1.3	16.5	47.9	63.1
MARS	344 36.1	N14 53.2	+ 7 29.0	76.9	-6.9	0.0	0.1	-6.8
SATURN	139 44.8	S18 50.7	+ 7 40.4	238.1	-6.8	0.2	0.0	-6.6
ALIOTH	188 39.9	N55 52.6	+21 06.1	326.0	-2.5	0.0	0.0	-2.5
ALKAID	175 18.2	N49 14.2	+23 47.4	315.1	-2.2	0.0	0.0	-2.2
ALPHECCA	148 30.4	N26 39.8	+30 10.7	281.4	-1.7	0.0	0.0	-1.7
ALPHERAT	20 03.2	N29 10.3	+42 35.9	83.8	-1.1	0.0	0.0	-1.1
ALTAIR	84 27.7	N 8 54.5	+59 10.4	194.5	-0.6	0.0	0.0	-0.6
ANTARES	134 45.2	S26 27.8	+ 5 12.5	229.5	-9.3	0.0	0.0	-9.3
ARCTURUS	168 15.0	N19 06.1	+10 56.9	285.8	-4.9	0.0	0.0	-4.9
CAPELLA	302 53.6	N46 00.7	+ 4 25.4	30.0	-10.5	0.0	0.0	-10.5
DENEB	71 51.4	N45 19.9	+82 36.7	29.3	-0.1	0.0	0.0	-0.1
DIPHDA	11 15.7	S17 54.3	+ 6 20.3	119.2	-8.0	0.0	0.0	-8.0
DUBHE	216 10.5	N61 40.2	+15 57.2	341.2	-3.4	0.0	0.0	-3.4
ELTANIN	113 06.0	N51 29.1	+62 03.5	308.5	-0.5	0.0	0.0	-0.5
ENIF	56 06.7	N 9 56.7	+55 28.3	141.7	-0.7	0.0	0.0	-0.7
FOMALHAU	37 43.6	S29 32.4	+12 18.4	145.7	-4.4	0.0	0.0	-4.4
HAMAL	350 20.4	N23 31.9	+17 01.4	73.2	-3.2	0.0	0.0	-3.2
KAUS AUS	106 02.7	S34 22.3	+11 51.4	204.2	-4.6	0.0	0.0	-4.6
KOCHAB	159 39.8	N74 05.5	+39 13.7	339.5	-1.2	0.0	0.0	-1.2
MARKAB	35 58.0	N15 17.1	+47 00.2	111.8	-0.9	0.0	0.0	-0.9
MIRFAK	330 59.7	N49 54.8	+20 05.4	41.2	-2.7	0.0	0.0	-2.7
NUNKI	98 17.4	S26 16.4	+21 45.6	200.5	-2.5	0.0	0.0	-2.5
RASALHAG	118 25.8	N12 33.0	+44 52.2	245.7	-1.0	0.0	0.0	-1.0
SABIK	124 31.6	S15 44.5	+19 32.0	228.9	-2.8	0.0	0.0	-2.8
SCHEDAR	12 00.2	N56 37.1	+44 55.8	44.8	-1.0	0.0	0.0	-1.0
SHAULA	118 40.7	S37 06.6	+ 4 46.3	212.1	-10.0	0.0	0.0	-10.0
VEGA	102 58.7	N38 47.8	+69 51.0	277.7	-0.4	0.0	0.0	-0.4
POLARIS	339 57.8	N89 19.7	+38 54.9	0.9	-1.2	0.0	0.0	-1.2
ARIES	22 20.7							

Moon phase is waning crescent, 25% illuminated

Consider the stars Altair and Arcturus, both of which are visible (on paper, for algorithm testing purposes) while both the sun and moon are also visible.

From the 2015 Nautical Almanac, the declination and SHA of Altair are N08° 54.5’ and 62° 07.0’, and for Arcturus are N19° 06.1’ and 145° 54.3’.

Newton's Method

- Using the spreadsheet *April 6 2015 Lat1 using 2nd order Newton-Raphson for Altair Arcturus.xlsx* (appended to this module), the following results are obtained for the celestial data from the preceding slide:

1	θ_e	ϕ	phi se	eta sub s	f1(x,y)	f2(x,y)	J11(x,y)	J12(x,y)	J21(x,y)	J22(x,y)	Coeff Det	RHS 1	RHS 2	Xnum	Ynum	s1	s2		R/r	0.151	0.151266	
2	0	0	0	0	-0.9724295	-0.39669	0.327250884	-0.529700519	0.15484833	-0.87324	-0.20375	0.97243	0.396692	-0.63904	-0.02076	3.136447	0.101898	Arcturus	SHA_1	145.905	δ_1	19.102
3	3.136447	0.101898	0	0	-0.9542916	-0.39046	0.36962648	-0.509611451	0.12942207	-0.86428	-0.25351	0.954292	0.390463	-0.62579	0.020819	2.468546	-0.08212	Altair	SHA_2	62.117	δ_2	8.908
4	5.604993	0.019774	0	0	-0.9369076	-0.38408	0.402131714	-0.494936698	0.10901145	-0.8541	-0.28951	0.936908	0.384077	-0.61012	0.052316	2.107443	-0.18071	Arcturus	η_1	10.948		
5	7.712436	-0.16093	0	0	-0.9199561	-0.37766	0.429104556	-0.483167377	0.0911141	-0.84327	-0.31783	0.919956	0.377662	-0.5933	0.078235	1.866729	-0.24616	Altair	η_2	59.173		

1	θ_e	ϕ	phi se	eta sub s	f1(x,y)	f2(x,y)
2564	38.99998	-54.6544	0	0	-2.47E-15	1.55E-15
2565	38.99998	-54.6544	0	0	-2.47E-15	1.55E-15

Latitude

GHAϒ + Longitude

From the Almanac, GHAϒ = 22.345°; hence Long = -77

What about parallax?

- Richharia’s equations B.45 and B.46 can be used to compute the elevation to the moon from the celestial data with and without the parallax term R/r.
- Read about PA on page 280 of the 2015 Nautical Almanac to investigate how this relates to the parallax correction term that is handled as an “add-on” correction when using the USNO web-based tool or data from the Nautical Almanac.

Assignment 10.3

1. Compute the lunar parallax correction given in the celestial data from the USNO website using Richharia's equations as described in the sub-module 10C on slide 11. How do the results from the Richharia equations relate to the PA correction from the USNO website tool and explained in the Nautical Almanac?
2. Using the included spreadsheet with the USNO data for two stars that are different from Altair and Arcturus, repeat the Newton's method exercise demonstrated in the sub-module. Compute a value for latitude. Then, look up the GHAϒ in the Nautical Almanac that corresponds to the time used with the USNO app in order to derive a value for longitude and to verify the assumed value of GMT/UTC. Pay attention to GDOP.
3. Using the spread sheet *moon angles rev 5a* or your own equations, compute the lunar distance between the sun and moon for this value of GMT/UTC, and show that this value changes when the value of GMT/UTC changes. This demonstrates how the lunar distance can be used to determine GMT/UTC and longitude, hence validates the method of lunars. Note that we have dodged the question of "clearing the lunars," a necessary step when relating actual measured lunar distances with the geocentric values deduced from Almanac data (cf. problem 1, above).

End of Mod 10C