Module 11

Modern Navigation Systems

Navigating with GPS

Module 11C

Using the sextant computations with GPS instead of celestial data

Summary of Module 11

- The module begins with a description of the GPS space segment (i.e., the satellite constellation). (11A)
- This is followed by a description of the signal structure that provides the “ranging codes” used by GPS receivers in order to determine position, velocity, etc. (11B)
- **Students will use actual GPS almanac data to compute azimuth (i.e., values for Hc) and elevation angles to GPS satellites from an assumed position on earth, as was done in Module 5. (11C)**

Module 11C, cont'd

- Measurements from actual GPS satellites will be used to compute the "observed" values of elevation angles (i.e., values for Ho).
- Using these results, students will apply the least-squares sextant algorithms and compare their results with the position fix generated by the GPS receiver.

How this module is arranged

- There are three parts to this sub-module, but they are captured in this single file and the associated .MP4 recording of a GPS receiver display from Module 11B. The three activities of this sub-module are:
  - Computing look angles to GPS satellites from a knowledge of time and an "assumed position". These provide the values Hc used in the Nautical Almanac.
  - Comparing these look angles with those computed by the GPS receiver. The latter provide the observed values Ho used in the sextant algorithms.
  - Using the values of Hc and Ho recursively to generate a position fix using the least-squares version of the Marc St. Hilaire algorithm from the Nautical Almanac

Reading/Viewing

- View the GPS MP4 again, if needed.
- Examine the pdf almanac for week 519, and study the data provided in the context of your understanding of the GPS constellation and of the orbital models from Module 5.
- Examine the spreadsheet provided and outline the equations that it contains in order to understand the structure and purpose of the spreadsheet.

Solving for position versus computing look angles

- Note the distinction from the assignment in Module 5, which addressed computation of look angles from a known position.
- In the present case, this is carried forward using the Marc St. Hilaire and least-squares algorithms (from the Nautical Almanac) to solve for position via measurement of look angles.
- In both situations, prior, independent knowledge of universal time (GMT/UTC) is required.
- But, "solving for time" in the present context is accomplished in an Ad Hoc manner by guessing a value for what is identified as the "right ascension offset" in the spreadsheet that follows.

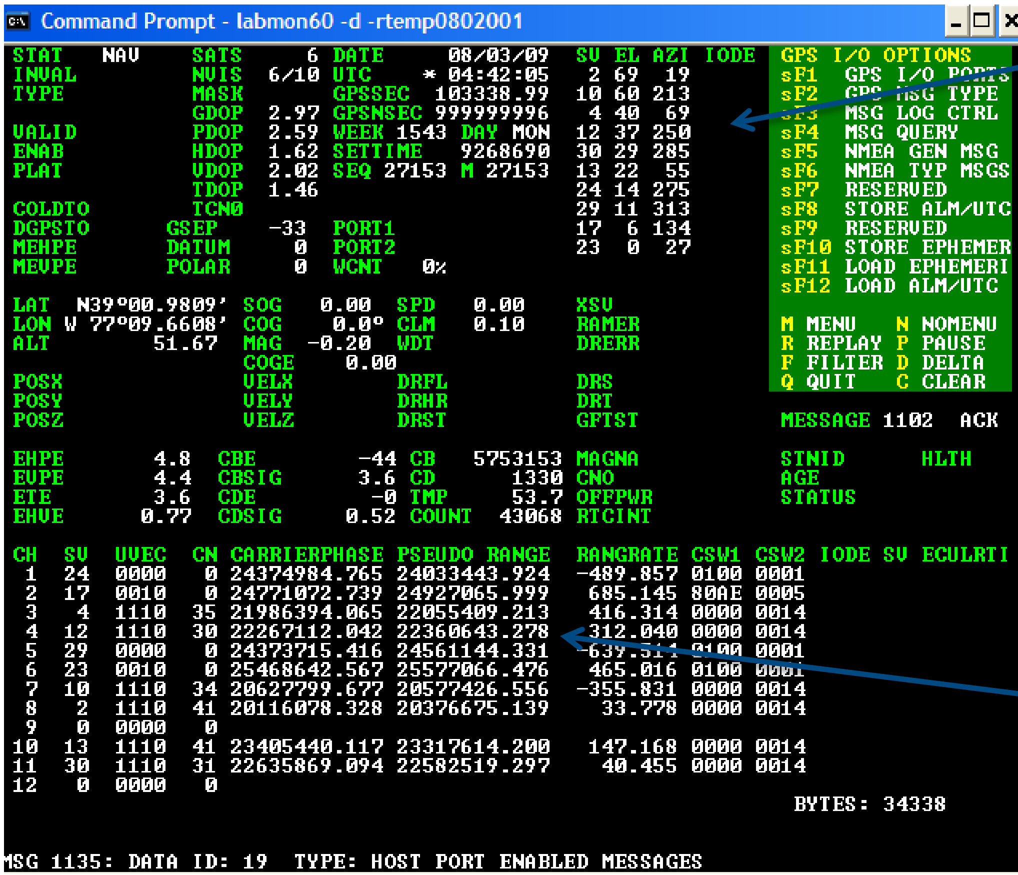
RA offset

- The use of GPS using the spread sheet here is ambiguous with respect to UTC time.
- An offset value for the time of right ascension is used to lock the GPS computations to the observed angles to the GPS satellites (as computed from the measured pseudoranges).
- Choosing a value for the RA offset that makes the measured and computed values of the angles to the GPS satellites is the GPS equivalent of the method of lunars
- This works because the angular positions of the GPS satellites on the “GPS sphere” are not constant, in contrast with the stars as placed on the “celestial sphere”.

GPS example

- The spreadsheet that follows applies the Celestial algorithms to GPS data
- This spreadsheet is included with this module as "*GPS computations4 highlighted.xlsx*"
    - You can use this spreadsheet to complete the assignment questions.
- With reference to the next slide
    - The image of an actual GPS receiver display gives the measured ranges to satellites
    - The GPS receiver also computes Az/El, ***based on these ranges***, to each satellite being tracked by the receiver, and adds this info to the display

GPS readout for Week 1543 (aka week 519)



Az/El angles calculated
From the measured ranges

This is real data from actual satellites using a Rockwell Zodiac 12 channel C/A code GPS receiver.

This display was obtained at a lat/long of 39 -77 on August 3, 2009.

Measured ranges to satellites

Using the GPS Almanac

- Almanac gives orbital parameters
  - Week 1543 rolls over after 1024 counts to become week 519 almanac, which is obtained from the AGI website (AGI developed and sells the Satellite Toolkit software)
  - The week 519 almanac is included as a file uploaded to this module.
- Using this data and an assumed position, it is possible to calculate values for Hc and Azimuth, as was done for the celestial example
  - But here, the Hc and Az are the directions from a presumed location on earth to the GPS satellites of interest!
  - Instead of the equations from the Nautical Almanac, the equations from Richharia are used instead

GPS week 1543 = week 519

GPS Date Calendar

- GPS Satellite Performance
- GPS Satellite Outage Calendar
- AGI Blogs
- The Nog
- Technical Questions

This calendar presents dates specific to the GPS community. The data for each day are as follows:

Row	Example	Definition
First	1	Calendar day of the month
Second	1485:2	Full GPS week since 1st epoch : day of week number
Third	462:172800	GPS Week since latest epoch : seconds of week at midnight for that day
Fourth	183	Julian Day Number

August | 2009 | Go To Month | Today

July			August 2009			September
Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1 1542:6 518:518400 213
2 1543:0 519:0 214	3 1543:1 519:86400 215	4 1543:2 519:172800 216	5 1543:3 519:259200 217	6 1543:4 519:345600 218	7 1543:5 519:432000 219	8 1543:6 519:518400 220
9 1544:0 520:0 221	10 1544:1 520:86400 222	11 1544:2 520:172800 223	12 1544:3 520:259200 224	13 1544:4 520:345600 225	14 1544:5 520:432000 226	15 1544:6 520:518400 227

http://adn.agi.com/GNSSWeb/

Review/Refer to Assignment 5.3 from Module 5:

1. Derive the look angles from 39N 77W to the GPS satellite given on slide 10 at GPS time week 801 and time of week 500,000 seconds. Note the distinction between time of almanac applicability, time of week, and the time for which the az/el to the satellite is desired.

Orbital parameters for a typical GPS satellite

******** Week 801 almanac for PRN-01 ********

ID: 01

Health: 000

Eccentricity: 0.3765106201E-002

Time of Applicability(s): 503808.0000

Orbital Inclination(rad): 0.9617064849

Rate of Right Ascen(r/s): -0.7817468486E-008

SQRT(A) (m 1/2): 5153.614258

Right Ascen at Week(rad): 0.7017688714E+000

Argument of Perigee(rad): 0.434909394

Mean Anom(rad): 0.4480223834E+000

Af0(s): -0.1049041748E-004

Af1(s/s): 0.0000000000E+000

week: 801

Note that the right ascension varies with time! $\Omega = \Omega_0 + d\Omega/dt$ (rate of right ascension)

Note that sqrt(a), rather Than a, is provided!

These terms refer to drift of the atomic clocks onboard each GPS satellite.

Orbital parameters for a typical GPS satellite

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ID: 01
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Eccentricity: 0.3765106201E-002
Time of Applicability(s): 503808.0000
Orbital Inclination(rad): 0.9617064849
Rate of Right Ascen(r/s): -0.7817468486E-008
SQRT(A) (m 1/2): 5153.614258
Right Ascen at Week(rad): 0.7017688714E+000
Argument of Perigee(rad): 0.434909394
Mean Anom(rad): 0.4480223834E+000
Af0(s): -0.1049041748E-004
Af1(s/s): 0.0000000000E+000
week: 801

Note that the mean anomaly Is specified at time of applicability

Right ascension is specified at t = 0 (“at week”)

Combining GPS and celestial techniques

- Guess a lat/long
- Use the GPS almanac data with an assumed position on earth and knowledge of time to estimate the look angles to each of several satellites
    - This provides values of Hc
- Treat the elevation data from the GPS display as the observed elevation angle values Ho
    - The GPS receiver serves as the sextant, and provides the values of Ho for the same satellites

Solving for position

1. Use the Hc values obtained using an *assumed position* and the GPS almanac data
2. Use the Ho values provided by the GPS receiver
3. Run the least squares Marc St. Hilaire algorithm from the Nautical Almanac to produce a new estimate of position
4. Recompute the Hc values using this new estimate of position
5. Repeat steps 2 - 5 until further repetitions of the algorithm yield no significant improvements in the estimate of position

Consider the attached spreadsheet

- The spreadsheet described in the next few slides combines the Richharia equations and the Marc St. Hilaire algorithm.
- The Excel spreadsheet is included as part of this module.
- Original, fixed data from the GPS receiver is highlighted in yellow.
- Other cells contain results computed within the spreadsheet from this fixed data.
- By using the relevant equations from Richharia and the Nautical Almanac and testing them against the provided values and Excel equations, it is possible to reproduce fully the results of the spread sheet.

The “GPS” spreadsheet, top of page

SV	Az	El	carrier phase	range rate	toa	i	e	dW/dt	sqrt a	W at week	W at t	ω	M
2	19	69	20116078.328	33.778	319488	0.9405422211	9.066581726E-03	-8.236384019E-09	5153.621582	-1.663788676E+00	-1.664639816E+00	2.875033498	-1.083742499E+00
4	69	40	21986394.065	416.314	319488	0.9397392273	8.646488190E-03	-8.250935934E-09	5153.711426	-1.645491719E+00	-1.646344362E+00	0.499282241	1.780311346E+00
12	250	37	22267112.042	312.040	319448	0.9678840637	3.174304962E-03	-7.861672202E-09	5153.561523	2.545034051E+00	2.544221634E+00	-0.622095108	-2.770511270E+00
10	213	60	20627799.677	-355.831	319448	0.9573497772	8.568286896E-03	-7.807102520E-09	5153.594238	-5.787190199E-01	-5.795257980E-01	0.624537706	-8.134663105E-02
13	55	22	23405440.117	147.168	319448	0.9948787689	4.104137421E-03	-7.450580597E-09	5153.634766	5.038890839E-01	5.031191484E-01	1.640797257	-4.869393110E-01
30	285	29	24373715.416	40.455	319448	0.9529457092	1.174354553E-02	-8.029019227E-09	5153.514648	2.496153712E+00	2.495324001E+00	1.435623646	1.020320773E+00

lat	minutes	long	minutes	week	t - seconds	t - toa	sec per week	µ	sqrt m	lat - rad	long - rad	Rearth
39.15	0	-77.53	0.000	519.000	103338.990	-216149.01	604800.00	3.986004418E+14	1.99650E+07	0.683296	-1.353154	6378000.00

SV	Az	El	T period of sat	T - hours	M at t	guess for E	M = E - e sin E	x0	y0	recalc sqrt a	M
2	19	69	43077.30836	11.965919	-32.611	-32.619	-32.611	9.322929017E+06	-2.478322992E+07	5145.753039	-1.0837424990
4	69	40	43079.56132	11.966545	-29.745	-29.736	-29.745	-2.664433058E+06	2.640175011E+07	5151.296460	1.7803113460
12	250	37	43075.80234	11.965501	-34.299	-34.300	-34.299	-2.570474288E+07	-6.764124012E+06	5155.562682	-2.7705112700
10	213	60	43076.62269	11.965729	-31.609	-31.611	-31.609	2.601072267E+07	-5.148073984E+06	5149.299505	-0.0813466311
13	55	22	43077.63896	11.966011	-32.014	-32.016	-32.014	2.185803166E+07	-1.499836064E+07	5148.684188	-0.4869393110
30	285	29	43074.62695	11.965174	-30.509	-30.500	-30.509	1.642199313E+07	2.106298635E+07	5168.003664	1.0203207730

SV	Az	El	Px	Py	Pz	Qx	Qy	Qz	x	y	z	r
2	19	69	0.245	0.946	0.213	-0.541	0.316	-0.779	15700134.951	998629.290	21298685.235	26478774.342
4	69	40	0.215	-0.897	0.387	0.553	0.438	0.709	14016572.836	13962123.961	17684637.198	26535855.223
12	250	37	-0.486	0.730	-0.480	-0.741	-0.053	0.669	17506316.513	-18412326.956	7810395.534	26579826.564
10	213	60	0.863	-0.163	0.478	-0.233	0.711	0.663	23652404.816	-7889429.143	9021142.605	26515285.397
13	55	22	-0.323	0.442	0.837	-0.856	-0.514	-0.059	5767203.468	17380616.818	19167104.451	26508948.864
30	285	29	-0.453	-0.377	0.808	0.744	-0.659	0.110	8228407.238	-20073941.574	15577594.043	26708261.871

Az corrections:

Middle of page

SV	Az	El	RA	declination	RA offset	fs	ϕε	hs	Az - computed	El - computed	Az - corrected	El - computed	numerator	denom	flag 1	flag 2
2	19	69	0.064	0.935	1.2524	-1.189	-1.353154	1.2956	20.953791	69.37	20.95	69.37	0.16	0.427057	0	0
4	69	40	0.783	0.729	1.2524	-0.469	-1.353154	0.9064	69.251983	41.57	69.25	41.57	0.77	0.292989	0	0
12	250	37	-0.811	0.298	1.2524	-2.063	-1.353154	0.8446	69.749018	37.41	249.75	37.41	-0.65	-0.24045	1	1
10	213	60	-0.322	0.347	1.2524	-1.574	-1.353154	1.1844	33.191165	61.21	213.19	61.21	-0.22	-0.33539	1	1
13	55	22	1.250	0.808	1.2524	-0.002	-1.353154	0.6104	55.366100	22.09	55.37	22.09	0.98	0.674134	0	0
30	285	29	-1.182	0.623	1.2524	-2.434	-1.353154	0.7269	-73.593456	29.67	286.41	29.67	-0.88	0.259826	1	0

SV	Az	El	carrier phase	RA off. deg	pseudorange	computed range
2	19	69	20116078.328	71.76252	20376675.139	20414373.706
4	69	40	21986394.065	71.76252	22055409.213	21871144.183
12	250	37	22267112.042	71.76252	22360643.278	22218156.063
10	213	60	20627799.677	71.76252	20577426.556	20747262.663
13	55	22	23405440.117	71.76252	23317614.200	23442967.805
30	285	29	22635869.094	71.76252	22582519.297	22969491.255

sidereal day hrs	t - seconds	t - hours
23.9344696	103338.99	28.705275000
twice sat period		
23.9318380		
time mod 23.9344	fractional day	long offset
4.7708	0.1993	71.758011466
		offset - rads
		1.2524

Computation	degrees
P1	-0.37
P2	-1.57
P3	-0.41
P4	-1.21
P5	-0.09
P6	-0.67

A	2.22
B	1.64
C	3.78
D	0.01
E	0.01
G	5.69

Bottom of page

	degrees	rads	cos^2 Z	AE - BD	0.01
Z1	20.95	0.365713	0.872112	CD - BE	0.02
Z2	69.25	1.208675	0.125499	cos BF	0.78
Z3	249.75	4.358943	0.119808	BF	39.15
Z4	213.19	3.720888	0.700316	LF	-77.53
Z5	55.37	0.966321	0.323000	**Lat new**	**39.15**
Z6	286.41	4.998737	0.079779	**Long new**	**-77.53**
	P COS Z	P SIN Z	cosZsinZ		
	-0.34	-0.13	0.33		
	-0.56	-1.47	0.33		
	0.14	0.38	0.32		
	1.01	0.66	0.46		
	-0.05	-0.08	0.47		
	-0.19	0.65	-0.27		
			sin^2		
			0.13		
			0.87		
			0.88		
			0.30		
			0.68		
			0.92		

Assignment 11.3

1. Using the attached spreadsheet "*GPS computations4 highlighted*," and the attached week 519 GPS almanac files, guess an assumed position (i.e., 0 degrees lat, 0 degrees long) and use the spreadsheet to iterate until the actual position of 39 -77, for which your Hc values will agree with the Ho values from the GPS receiver, is derived. To accomplish this, you will need to examine the equations used in the spreadsheet and reverse engineer their function. Produce a block diagram that describes the algorithms represented in the spreadsheet.
2. Experiment with the value of RA offset to see its effect on the accuracy of the solutions. Comment on your conclusions.
3. Comment on the relationship of your "exploration" of this problem to the method of lunars from Module 10.

End of Mod 11C