Module 2A

Vector mathematics, great circles, and latitude/longitude

In this module

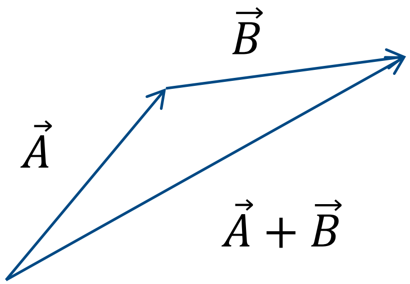
- We review
  - Vector sums
  - The vector dot product
  - ECEF and ECI coordinate systems
  - The use of the vector dot product to compute great circle distances on the surface of a sphere

Vector sums

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{x} + \left(A_y + B_y\right)\hat{y} + (A_z + B_z)\hat{z}$$



Vector dot product

Vector dot product, also known as scalar product, or inner product

$$\overrightarrow{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

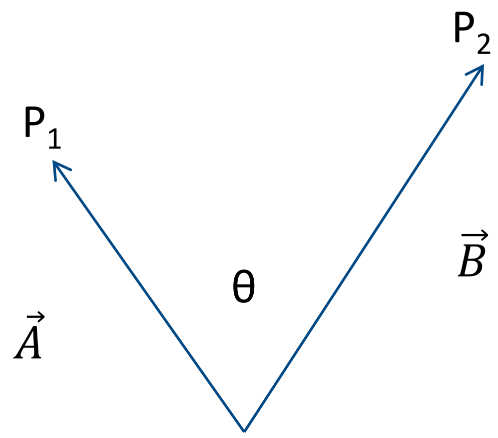
Meaning of the vector dot product:

$$\overrightarrow{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |A||B|cos\theta$$

where θ is the angle between the two vectors.

Note that the vector dot product produces a *scalar*, not a *vector*.

The vector dot product, cont'd.



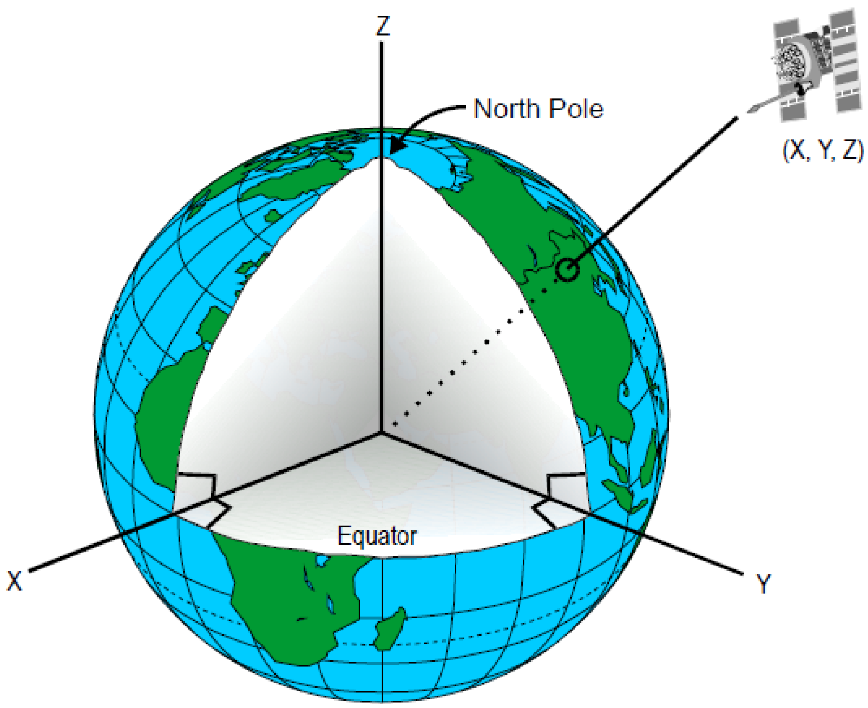
The dot product is particularly useful for computing great circle distances on the surface of a sphere. If the vectors **A** and **B** point from the center of the earth to its surface, and each have magnitude r_e, the radius of the earth, then the dot product divided by r_e^2 yields a value for *cos* θ. Taking the inverse cosine to obtain a value for θ, then Multiplying $r_e\theta$ yields the great circle distance between points P_1 and P_2 on the surface of a sphere, in this case the earth. The angle must be specified in *radians*.

The lat/long/alt ECEF coordinate system

- ECEF means *earth centered earth fixed*. It is the coordinate system in which the earth does not appear to be moving with respect to a position on the surface of the earth.
- In the ECEF frame, the concepts of latitude, longitude, and altitude are well-defined.

ECI, or Earth Centered Inertial Frame

- The ECI frame rotates with respect to the ECEF frame
- The rotation period is called a *sidereal* day, and is 23 hours, 56 minutes, 4.0916 seconds
  - This is the time it takes the earth to rotate 360 degrees
- The solar day
  - 24 hours is the times it takes from noon to noon
  - The solar day is longer than a sidereal day because of the earth's motion in its orbit around the sun
- This topic will be discussed in more detail in a later module



Latitude and longitude

This Wikipedia page excerpt shows the earth with respect to the Cartesian ECEF and/or ECI coordinate systems. In the ECEF system, the Cartesian axes rotate in synchrony with the earth. From: http://upload.wikimedia.org/wikipedia/commons/3/32/Earth_Centered_Inertial_Coordinate_System.png (accessed Oct 16, 2014).

Vector representation of Latitude and Longitude

- A position x, y, z on the surface of the earth can be written as the vector
  - $A_x = r_e \, cos\ (lat)\ cos\ (long)$
  - $A_y = r_e \, cos\ (lat)\ sin\ (long)$
  - $A_z = r_e \, sin\ (lat)$

Two points on the surface of the earth

- Two points on the surface of the earth can be represented as vectors **A** and **B**.
- The great circle distance can be determined from the dot product of **A** and **B**, as discussed earlier.

Radio waves, light, and great circles

- Electromagnetic waves travel between two points by the shortest possible path
  - This is *Fermat's principle* or the *principle of least time*
- This means that radio signals and light waves travel in great circles at the surface of the earth

Assignment 2.1

2.1.1 Look up the latitude and longitude of San Francisco and Washington, D.C., and compute the great circle distance between the two cities.

2.1.2 Derive equation 2.20 in the text.

End of Mod 2A