Module 5

Modern Navigation Systems

Position determination of satellites, planets, and stars in earth-centered coordinate systems

Module 5A

Satellite Orbital Parameters

Summary of Module 5

- Students will learn the features of, and similarities and differences between, the earth-centered inertial (ECI) and the earth-centered, earth fixed (ECEF) coordinate systems.
- Then, using actual orbital parameters from almanacs and ephemeris tables, students will learn to compute the positions in three dimensional space in both the ECI and ECEF frames of satellites, stars, planets, the sun, and the moon.
- The four equations that are used will form the basis for all the algorithms that follow.

Earth orbiting satellites

- The gravitational force between two bodies is given as $F = \frac{GM_1M_2}{r^2}$ where *G* is a universal constant, M_1 and M_2 are the masses of the respective objects, and *r* is the distance between them
- But *F*= *ma*, hence $a_2 = \frac{GM_1}{r^2}$
- When M_1 is the earth, at the surface of the earth, $a = g = 9.8\,{}^{m}/_{sec^2}$

To derive the orbital equations…

- For a satellite influenced by only the gravity of a *homogeneous* earth, break $\vec{a}$ into orthogonal components, a_θ, a_ϕ, and a_r.
- Integrate each component of acceleration once to get velocity as a function of time, twice to get position as a function of time
- Each integration yields a single constant of integration
    - There are 6 constants of integration
- There are three categories of solution
    - Parabolic trajectory
    - Hyperbolic trajectory
    - Elliptical trajectory
- If gravitational forces cannot be centered on a single point (the requirement of *homogeniety*), the differential equation given by $a = {}^{GM_{earth}}\!/_{r^2}$ does not have an analytic solution, and the trajectories must be solved for by using numerical techniques (discussed in Module 6).

Parabolic trajectories

- If the gravitational force is constant, the resulting trajectories are parabolas, such as when a batter hits a fly ball, or during the flight of a ballistic missile
  - The trajectory is a *ballistic* trajectory

Hyperbolic Trajectories

- These are a “non-periodic” solution to the differential equation for the acceleration due to gravity, and correspond to the deflection due to gravity as an object passes through the field of a large object, such as a comet passing by the sun

Halley's Comet

- Originally called Storer's comet, when Newton and Storer believed that comets followed open (non-orbiting) trajectories
  - o Storer lived in Huntingtown, Maryland in the 1600's
- Halley predicted the return of the comet, and it has since been known as Halley's comet.

Elliptical Orbits

- Periodic orbits are elliptical, and satisfy Kepler's three laws
  - o The orbits are elliptical
    - o A circle is the limiting case on an ellipse
  - o The orbits sweep out equal areas in equal times
    - o This is a direct consequence of conservation of angular momentum
  - o The period T^2 is proportional to the length of the semi-major axis r^3

Kepler's third law

- Took 16 years for Kepler to derive
- Consider the period of a pendulum whose length is equal to the radius of the earth r_e
  - $T = 2\pi\sqrt{{r_e}/{g}} = 2\pi\sqrt{{r_e}\Big/{\left({GM_e}/{{r_e}^2}\right)}} \rightarrow T^2 \propto {r_e}^3$
- T is known as the Shuler period, and is 84.4 minutes
- This is also the orbital period of a satellite orbiting a spherical earth at sea level

The Keplerian orbital parameters

- Length *a* of the semi-major axis of the orbital ellipse
- Eccentricity *e* of the ellipse (*e* = *0* is a circular orbit)
- Time t_0 at which the orbiting object is at a pre-defined position in the orbit (e.g., time of perigee passage)
- These three parameters define an orbit in the two dimensional orbital plane

The remaining three parameters

- The last three parameters relate to the position of the two dimension orbit in the three dimensional coordinate system of the planet (or other object) that is at the focal point of the orbit
- For example, pitch, roll, and yaw
- Known here as
  - Inclination
  - Argument of perigee
  - Right ascension

The two-dimensional orbit

- See figures 5.4 and 5.5 from the text (the next slides)

Figure 5.4 from the text

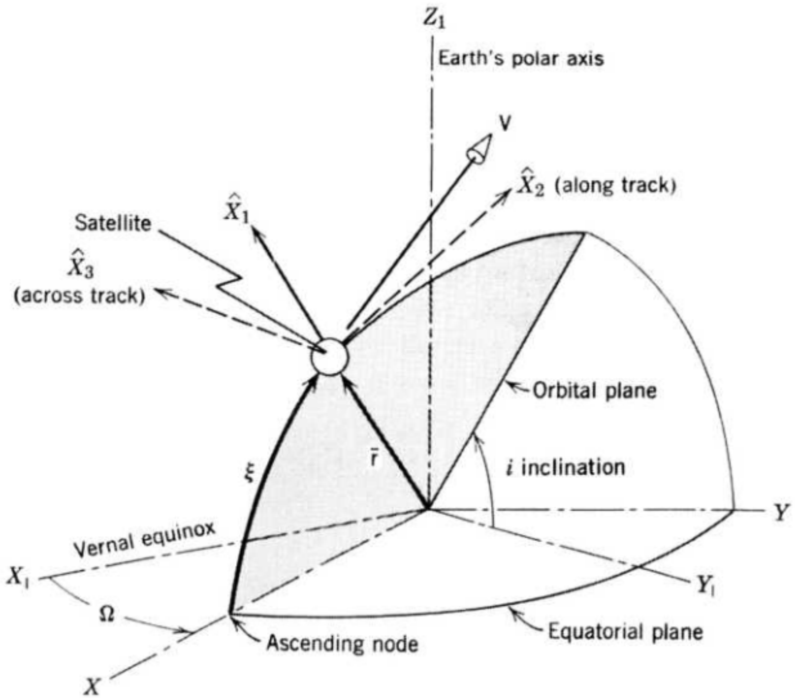

Figure 5.4 The orbital plane.

The two-dimensional orbit of the satellite around the Earth is shown separately In Figure 5.5 (the next slide).

The earth is at one of the focal points of the orbit.

Note: MatLab and other software packages get the angles wrong by 180 degrees, thus getting Kepler's 2nd Law "backwards".

Figure 5.5 from the text

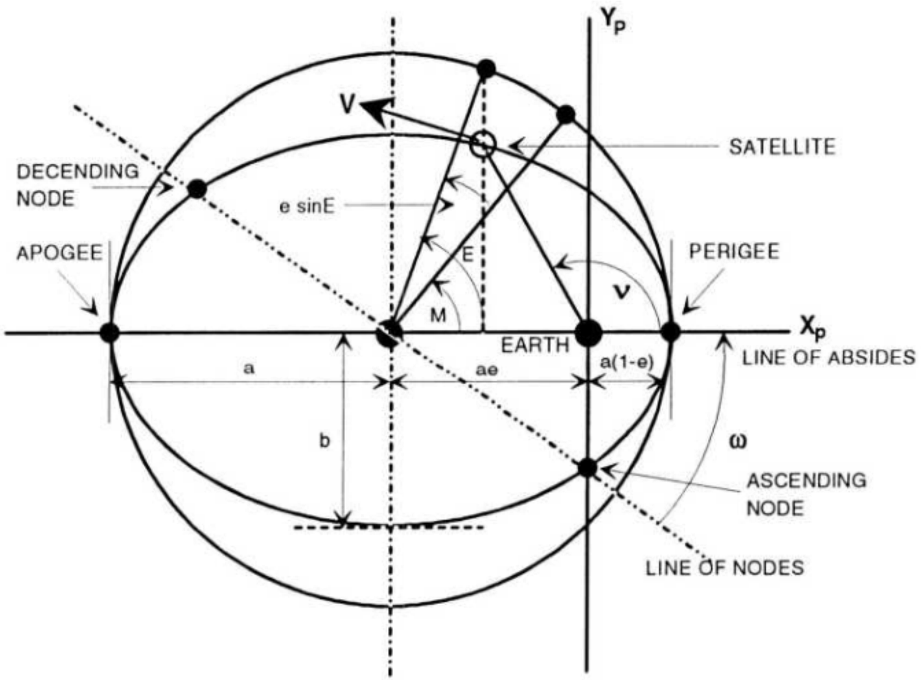

Figure 5.5 The elliptical orbit.

This rather cluttered diagram from the text depicts an elliptical orbit inscribed within a circle. The open circle shows the instantaneous position of a satellite within the elliptical orbit. Note that its position can be specified by the angle E, the *elliptic anomaly*.

The angle M is the *mean*, or average *anomaly*. It is easily computed. Then, E can be computed using the techniques developed in this module.

Orbital parameters as described in the text

1. Geocentric longitude of the ascending line of nodes, 0 (Figure 5.4).
This is the angle in the equatorial plane from some arbitrary reference to the ascending line of nodes. The reference X1 is conventionally taken as the direction of the vernal equinox, but, in practice, it is completely arbitrary as long as it isappropriately defined for the application. (See Equation 5.17 below.)
2. Inclination of the orbital plane with the equatorial plane, i (Figure 5.4).
3. The argument of perigee w, which is the angle between the direction of the ascending node and the direction of perigee measured in the plane of the orbit (Figure 5.5). The axis of the ellipse connecting the apogee and perigee is called the line of apsides.

Two of the remaining orbital elements are the dimensional parameters; namely the semimajor axis of the ellipse, a (Figure 5.5), and the eccentricity of the orbit, e, where (see equation 5.9) where b is the semiminor axis. Note that for a circular orbit, where the two axis are equal, the eccentricity is zero.

The sixth orbital element is the time of perigee passage, tp, measured with respect to some arbitrary time scale. tp establishes the phase of the satellite along the geometric path defined by the other elements.

Myron Kayton;Walter R. Fried. Avionics Navigation Systems (Kindle Locations 2311-2317). Kindle Edition.

Returning to M = E – e sinE

- The mean anomaly M, from Module 3, is related to the time (t-t_0) and the orbital period T by the simple equation $M = 2\pi(t - t_0)$/T.
- The eccentric anomaly E is then computed using the Newton's method or other iterative technique or approximation.

What is M?

- The mean anomaly is defined as an imaginary point, given as an angle, on a circle, and is a simple function of time
- E maps the position on the circle to a corresponding position on an ellipse
- This mapping is nonlinear, except in the case where e = 0, and the ellipse becomes the circle that is defined by M

Assignment 5.1

5.1.1 Derive the period of a pendulum by solving the differential equation *F = ma = -mg sin*θ where $a = l\frac{d^2\theta}{dt^2}$ and l is the length of the pendulum. Assume $sin\theta \approx \theta$. Draw a sketch the shows the angle θ and assume that l is the radius of the earth = 6358 km and that $g = 9.8\ m/s^2$. What is *T*?

5.1.2 Assume that *g* is a function of distance l that falls as $1/l^2$ (i.e., the inverse r^2 law for gravity). At what value of l is *T* = 24 hours?

End of Mod 5A