Module 12

Modern Navigation Systems

Inertial Navigation, Doppler, and Atomic Clocks

Module 12D

Atomic Clocks

Summary of Module 12

- The Keplerian equations from earlier modules are re-examined in terms of conservation of angular momentum, thus yielding expressions for the acceleration versus time of moving bodies. The Schuler frequency is introduced and analyzed in the context of a feedback control system, using as an example problem the inertial navigation of an aircraft flying from Dulles Airport to Beijing. Angular momentum is then re-examined in the context of circus acrobatic performances. (12A)
- Sensors that can measure these accelerations are explored, including mechanical gyroscopes based on rotating flywheels, mechanical gyroscopes that use micro-electrical mechanical devices (MEMs), and optical gyroscopes that exploit the relativistic Sagnac effect (e.g., the ring-laser gyroscope). (12B)
- Doppler techniques are introduced using the Cospas-SarSat system as a prototypical example. Students will watch a video that describes Cospas-Sarsat in detail. (12C)
- **Atomic clocks and the Allan variance are introduced. A chip-scale atomic clock is described. (12D)**
- Students will begin submitting and/or presenting their final projects. (12E)

Reading

- Read about clocks in the primary text (Kayton and Fried, chapter 5.3.2 and throughout the text; consult the numerous entries in the index)
- Consult the numerous online resources on timing, particularly GPS.gov

The importance of clocks

- For GPS and celestial navigation, the importance of accurate time keeping and measurement should now be obvious
  - o GPS uses timing measurements to infer distance
  - o Celestial navigation requires precise knowledge of UTC/GMT in order to resolve ϕ_{se} into ϕ_s and ϕ_e, e.g. longitude (cf. local hour angle, as well)

Accurate clock measurements are essential for other systems

- Doppler measurements for Cospas-Sarsat
- Time difference of arrival using Distance Measuring Equipment (DME) or JTIDs relative navigation
    - Consult the text
- eLORAN
    - Even though Loran has been discontinued, it may be rejuvenated as a backup to GPS called enhanced-LORAN, or eLORAN
- Read the text to learn about these and other systems
    - With the knowledge that you have gained thus far, you should be well prepared to study these systems on your own as your interest dictate

Atomic clocks

- Atomic clocks use excitation of atomic energy transitions in Cesium or Rubidium as precision oscillators that are used to provide stable timing references for other systems
  - In particular, to define UTC and for timing of the GPS system
    - Each GPS satellite has multiple atomic clocks, each synchronized to UTC at the ground stations of the GPS "control segment"

Stability

- A typical high quality, temperature controlled crystal oscillator (TCXO) has a stability of 1 part in 10^7
- Atomic clocks have stability values of 1 part in 10^{11} or 1 part in 10^{12} or even better!

Locking one clock to another

- Phase-locking the TCXO in a GPS receiver on the ground to an atomic clock onboard a GPS satellite makes the performance of the TCXO almost as stable as that of the atomic clock!
- Timing, rather than navigation, is the primary product of the GPS constellation
  - see GPS.gov for more details

Definition of a second

- Top down
  - Divide the rotation of the earth into hours, minutes, and seconds
- Bottom up
  - A second is a given number of oscillations of the hyperfine transition in the Cesium 133 atom
- Compare this to the difference in definitions for nautical miles and statute miles!

From physics.nist.gov

Unit of time (second)

Abbreviations: CGPM, CIPM, BIPM

The unit of time, the second, was defined originally as the fraction 1/86 400 of the mean solar day. The exact definition of "mean solar day" was left to astronomical theories. However, measurement showed that irregularities in the rotation of the Earth could not be taken into account by the theory and have the effect that this definition does not allow the required accuracy to be achieved. In order to define the unit of time more precisely, the 11th CGPM (1960) adopted a definition given by the International Astronomical Union which was based on the tropical year. Experimental work had, however, already shown that an atomic standard of time-interval, based on a transition between two energy levels of an atom or a molecule, could be realized and reproduced much more precisely. Considering that a very precise definition of the unit of time is indispensable for the International System, the 13th CGPM (1967) decided to replace the definition of the second by the following (affirmed by the CIPM in 1997 that this definition refers to a cesium atom in its ground state at a temperature of 0 K):

> **The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.**

How atomic clocks work

Types of atomic clocks(*How things Work*)

- Cesium atomic clocks employ a beam of cesium atoms. The clock separates cesium atoms of different energy levels by magnetic field.
- Hydrogen atomic clocks maintain hydrogen atoms at the required energy level in a container with walls of a special material so that the atoms don't lose their higher energy state too quickly.
- Rubidium atomic clocks, the simplest and most compact of all, use a glass cell of rubidium gas that changes its absorption of light at the optical rubidium frequency when the surrounding microwave frequency is just right.

http://www.colorado.edu/ASEN/asen6090/atomic_clocks.html

Atomic Clocks on GPS satellites

In addition to longitude, latitude, and altitude, the Global Positioning System (GPS) provides a critical fourth dimension - time. Each GPS satellite contains multiple atomic clocks that contribute very precise time data to the GPS signals. GPS receivers decode these signals, effectively synchronizing each receiver to the atomic clocks. This enables users to determine the time to within 100 billionths of a second, without the cost of owning and operating atomic clocks.

http://www.gps.gov/applications/timing/

Cesium versus Rubidium

Rubidium Atomic Clock

The two most commonly used atomic clocks in recent years have been the cesium clock and the rubidium clock. Both involve the locking of an electronic oscillator to the atomic transition. The rubidium clock has had the advantage of portability, achieving an accuracy of about 1 in 10^12 in a transportable instrument. This has made it useful for carrying from one cesium clock to another to synchronize the clocks.

http://hyperphysics.phy-astr.gsu.edu/hbase/acloc.html

Rubidium

Rubidium standard

From Wikipedia, the free encyclopedia

> This article includes a list of references, related reading or external links, but **its sources remain unclear because it lacks inline citations**. Please improve this article by introducing more precise citations. *(August 2014)*

Rb oscillator

A **rubidium standard** or **rubidium atomic clock** is a frequency standard in which a specified hyperfine transition of electrons in rubidium-87 atoms is used to control the output frequency. It is the most inexpensive, compact, and widely used type of atomic clock, used to control the frequency of television stations, cell phone base stations, in test equipment, and global navigation satellite systems like GPS. Commercial rubidium clocks are less accurate than cesium atomic clocks, which serve as primary frequency standards, so the rubidium clock is a secondary frequency standard. However, rubidium fountains are currently being developed that are even more stable than caesium fountain clocks.

All commercial rubidium frequency standards operate by disciplining a crystal oscillator to the rubidium hyperfine transition of 6 834 682 610.904 Hz. The intensity of light from a rubidium discharge lamp that reaches a photodetector through a resonance cell will drop by about 0.1% when the rubidium vapor in the resonance cell is exposed to microwave power near the transition frequency. The crystal oscillator is stabilized to the rubidium transition by detecting the light dip while sweeping an RF synthesizer (referenced to the crystal) through the transition frequency.

Note the above discussion of how atomic clocks work.

Rubidium versus Cesium, cont'd

- Rb vs Cs is a tradeoff of short versus long-term stability
- GPS satellites use multiple rubidium and/or cesium clocks depending on the design block of the satellites (eg. BLK II-R versus BLK III, etc.)

Clock Errors

Clock errors are characterized in terms of a time offset, a frequency offset, aging (frequency drift), and measures of clock instability. Time offset, frequency offset, and sometimes frequency drift are part of the solution, whether it be the control segment's solution and prediction of the satellite clocks or the user's solution for his own time and frequency offset. Clock instability, on the other hand, hampers the capability to perform these functions. The most common measure of clock instability is in the form of the square root of an Allan variance [10], which is defined as

$$\sigma_y(\tau) = \sqrt{\frac{1}{2(M-1)} \sum_{k=1}^{M-1} (y_{k+1} - y_k)^2} \qquad (5.23)$$

where

$$y_k = \frac{\phi(t_k + \tau) - \phi(t_k)}{2\pi f_0 \tau} = \frac{\Delta\phi(t_k)}{2\pi f_0 \tau} \qquad (5.24)$$

From the text, chap. 5.

Clocks do not drift backward and forward relative to a mean, or average value.

Their stability is described in terms of a variance for which there is no mean!

The Allan Variance

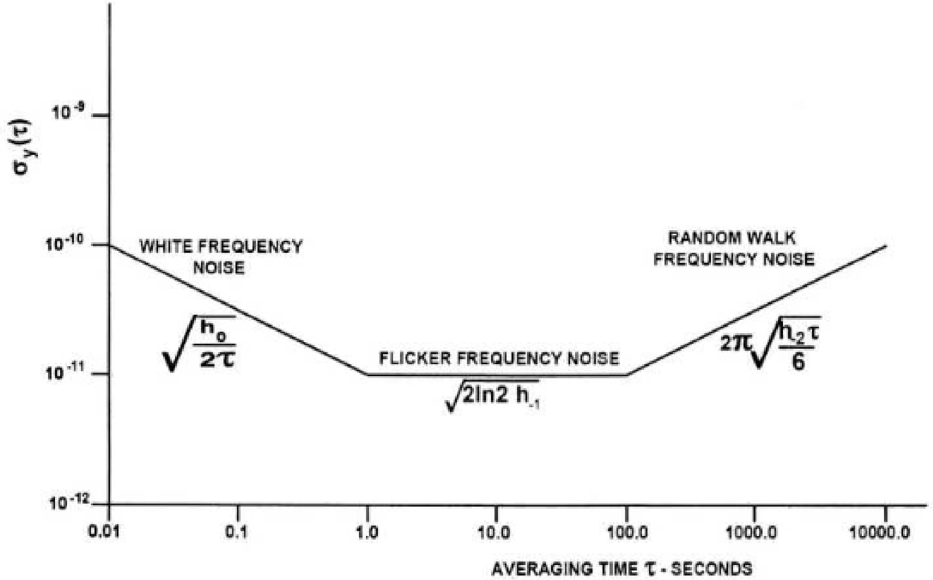


Figure 5.7 Typical square root of Allan variance.

This is called a bathtub curve. Note that the Allan variance is based on a running average, and the number of samples used for the average can be optimized in order to take advantage of the best part (the flat bottom) of the bathtub plot.

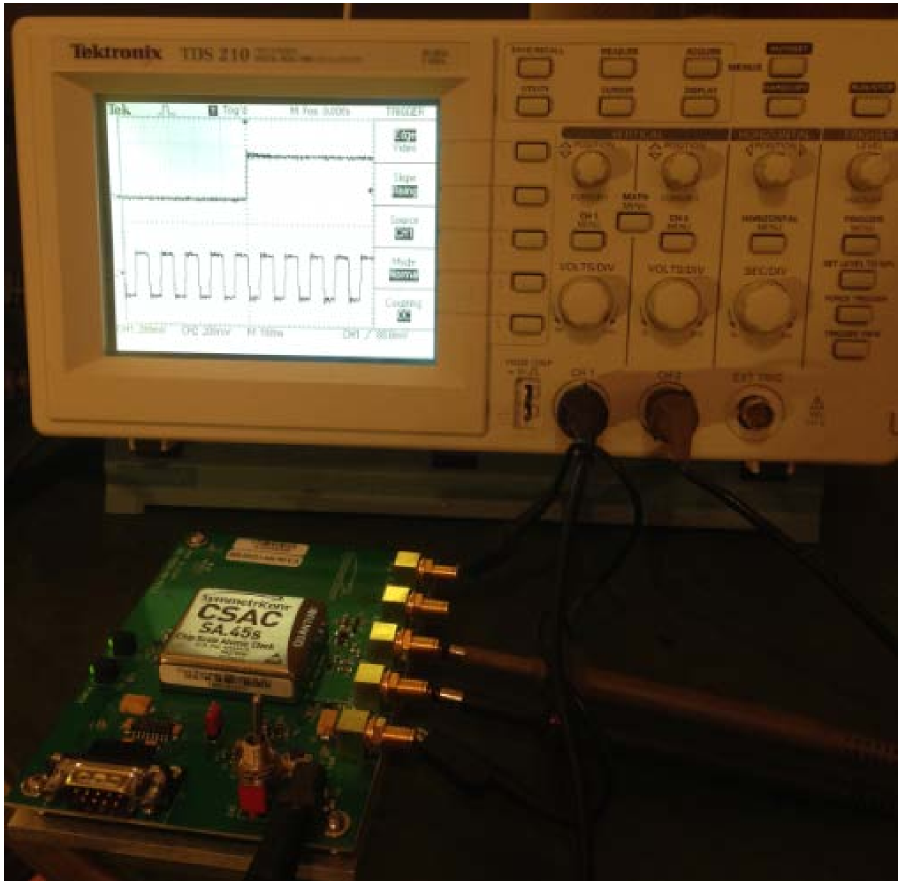
Recent developments

- It has been possible to purchase small rubidium based time standards (i.e., high stability oscillators) for many years
- It is now possible to purchase miniature cesium-based atomic clocks
    - Namely the Chip Scale Atomic Clock, or CSAC

Symmetricom Chip-scale Atomic Clock

This Cesium-based atomic clock produces 10 MHz and 1 PPS pulses with a stability as good as one part in 10^{12}.

CSAC showing 1 PPS and 10 MHz signals



The 10 MHz signal is useful as a general purpose oscillator of high stability.

The 1 pulse-per-second (1 PPS) signal is useful for clock Applications.

GPS receivers often provide both signals.

The CSAC provides these signals without the need for GPS.

Assignment 12-4

1. Explain what is meant by the "change in clock phase" by plotting $\sin(\omega t + \phi(t))$ for a randomly changing phase ϕ against the function $\sin(\omega t)$. Choose your parameters carefully so as to illustrate the relevance of "phase jitter" to precision timing. Think in terms of a *random walk* for which the mean position is not fixed.
2. What is a "cycle-slip"? Note that carrier phase GPS tracking is quite accurate unless there is a cycle slip.

End of Mod 12D