Module 5

Modern Navigation Systems

Position determination of satellites, planets, and stars in earth-centered coordinate systems

Module 5D

ECI versus ECEF coordinates and the role of ϕ_{es}

Summary of Module 5

- Students will learn the features of, and similarities and differences between, the earth-centered inertial (ECI) and the earth-centered, earth fixed (ECEF) coordinate systems.
- Then, using actual orbital parameters from almanacs and ephemeris tables, students will learn to compute the positions in three dimensional space in both the ECI and ECEF frames of satellites, stars, planets, the sun, and the moon.
- The four equations that are used will form the basis for all the algorithms that follow.

As shown before:

1. *Earth-centered, Earth -fixed (ECEF)*. The basic coordinate frame for navigation near the Earth is ECEF, shown in Figure 2.3 as the y_i rectangular coordinates whose origin is at the mass center of the Earth, whose y_3-axis lies along the Earth's spin axis, whose y_1 axis lies in the Greenwich meridian, and which rotates with the Earth [10]. Satellite-based radio-navigation systems often use these ECEF coordinates to calculate satellite and aircraft positions.
2. *Earth-centered inertial (ECI)*. ECI coordinates, x_i, can have their origin at the mass-center of any freely falling body (e.g., the Earth) and are nonrotating relative to the fixed stars. For centuries, astronomers have observed the small relative motions of stars ("proper motion") and have defined an "average" ECI reference frame [1 1]. To an accuracy of 10^{-5} deg /hr, an ECI frame can be chosen with its x_3-axis along the mean polar axis of the Earth and with its x_1- and x_2-axes pointing to convenient stars (as explained in Chapter 12). ECI coordinates have three navigational functions. First, Newton's laws are valid in any ECI coordinate frame. Second, the angular coordinates of stars are conventionally tabulated in ECI. Third, they are used in mechanizing inertial navigators, Section 7.5.1.

From the text, chapter 2.

ECI versus ECEF

- Earth centered earth fixed
  - This coordinate system rotates with the earth
  - The stars rotate through the sky; this is the coordinate system we live in: while stationary with respect to the earth, our positions in this system do not change
- Earth centered inertial
  - The earth rotates with respect to this coordinate system
  - To a first approximation, the stars do not

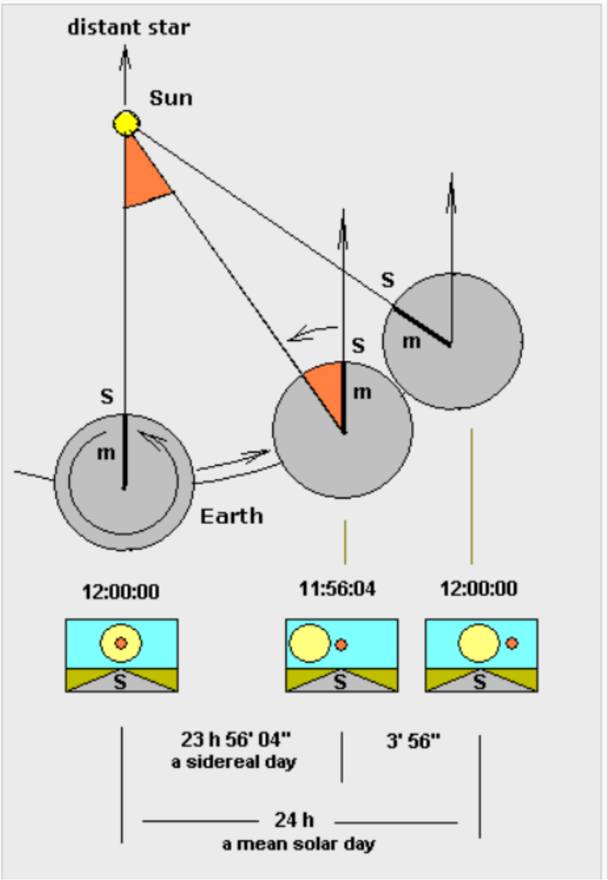
The role of ϕ_{es}

- To understand ϕ_{es} from Richharia's equations, pretend that the sun is orbiting the earth. At noon, the sun is not "directly overhead", but is at its highest point in the sky. It thus appears to be at the same longitude as the observer on the surface of the earth, but not at the same latitude.
- At noon (as defined by a sundial, not by a clock), the time is called *local apparent noon*, and ϕ_{es} is zero.

Geostationary satellites

- In the ECI frame, a geostationary satellite appears to be moving through the sky with a period equal to the time it takes the earth to rotate 360 degrees.
- Note that this time, called a sidereal day, is 23 hours 56 minutes, and about 4 seconds long.
- The time interval between noon and noon on two consecutive days is 24 hours. The earth rotates more than 360 degrees from noon to noon as it travels through its orbit around the sun. This is shown in the next slide.

Picture of a sidereal day



A sidereal day is the time that it takes a meridian *m* on the earth to rotate so that it faces the same distant star.

A solar day is the time it takes *m* to face the sun on two consecutive days.

As stated earlier, the earth's orbit is elliptical. Because of Kepler's second law, solar days vary in length over the course of a year; hence the term *mean solar day*.

http://sl.wikipedia.org/wiki/Siderski_%C4%8Das

For geostationary and geosynchronous satellites

- ϕ_{es} is independent of time for geo satellites
  - o The inclination of geostationary sats is 0
  - o The inclination of geosynchronous sats is not zero
  - o The orbital periods of each are the same
- Note that in order for their antennas to always point at the earth, geo satellites must rotate as they orbit!

Looking at the stars…

- The stars appear to rotate in unison above the earth in the ECEF frame but are stationary in the ECI frame.
- The longitude angle (i.e., right ascension) between a point on the earth and a star is called the local hour angle, or *LHA*, and changes as the earth rotates.

A final point

- In Richharia's equations, the term $R/r = r_e/r$, where r_e is the radius of the earth and r the distance to a star or satellite, appears
  - For satellites, this "parallax" term is critical
  - For stars, where the distance to the star r is essentially infinite (with respect to terrestrial distance scales), this term is 0.
  - For terrestrial sources, such as radio navigation beacons used by aircraft, $r_e/r = 1$.

The (almost) final results (from Richharia)

Finally, the following set of equations can be used to obtain satellite azimuth and elevation from a specified earth station:

$$\text{Right ascension, } \alpha = \arctan\left(y/x\right) \tag{B.43}$$

$$\text{Declination, } \delta = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \tag{B.44}$$

$$\text{Elevation, } \eta = \arctan\left(\frac{\sin\eta_s - \dfrac{R}{r}}{\cos\eta_s}\right) \tag{B.45}$$

where

$$\eta_s = \arcsin\left[\sin\delta\sin\theta_e + \cos\delta\cos\theta_e\cos\phi_{se}\right] \tag{B.46}$$

and R = Earth radius
r = satellite distance from Earth centre (use equation B.40c)
θ_e = earth station latitude
$\phi_{se} = \phi_s - \phi_e$
ϕ_s = satellite longitude
ϕ_e = earth station longitude.

Richharia's final equations

$$\text{Azimuth, } A = \arctan\left[\frac{\sin\phi_{se}}{\cos\theta_e \tan\delta - \sin\theta_e \cos\phi_{se}}\right] \qquad \text{(B.47)}$$

This is important

Use the convention given in chapter 2, section 2.6 to obtain the azimuth quadrant.

Range

The distance ρ of a satellite from a given point on the Earth is given as

$$\rho = \sqrt{r^2 - R^2\cos^2\eta} - R\sin\eta \qquad \text{(B.48)}$$

Assignment 5.4

5.4.1 With reference to the figure on slide 7 of this sub-module, sketch the earth and the north star in two dimensions, showing that the line from the center of the earth to the north star is parallel to the line from a point of the earth to the same star (because the star is so far away). Then show that the angle to the star with respect to the local horizon at a point on the surface of the earth, is equal to the latitude of that point.

End of Mod 5D