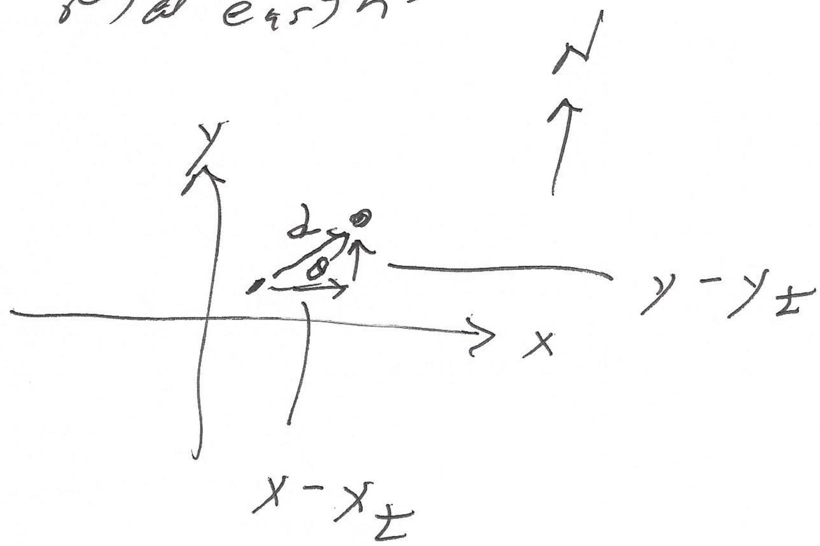
525, 445, 31
Fall 2016
9/27/16

2.20 Flat earth:




$$d^2 = (x - x_t)^2 + (y - y_t)^2$$

$$\tan \theta = \frac{y - y_t}{x - x_t}$$

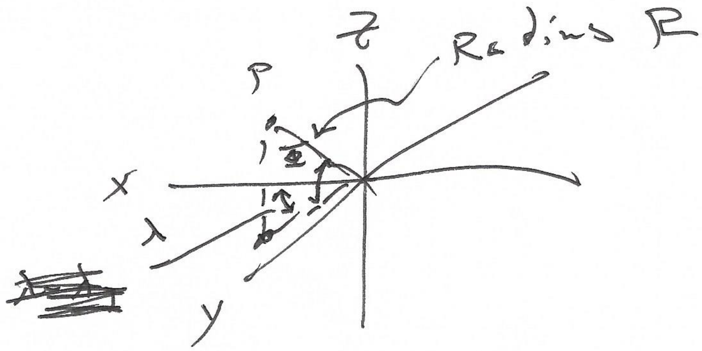
but, bearing is 90 − θ, from North in a clockwise direction:

$$\beta = 90 - \theta, \quad \tan \beta = \frac{x - x_t}{y - y_t}$$

2.22a

$$\cos\frac{D}{R} = \sin\Phi \sin\Phi_T + \cos\Phi \cos\Phi_T \cos(\lambda - \lambda_T)$$

This is an <u>exact</u> equation.




In $x, y, z$, $\vec{P} = R\cos\Phi\cos\lambda\,\hat{x} + R\cos\Phi\sin\lambda\,\hat{y} + R\sin\Phi\,\hat{z}$

$$\vec{P}_T = R\cos\Phi_T\cos\lambda_T\,\hat{x} + R\cos\Phi_T\sin\lambda_T\,\hat{y} + R\sin\Phi_T\,\hat{z}$$

$$\vec{P}\cdot\vec{P}_T = |\vec{P}||\vec{P}_T|\cos\theta = R^2\cos D/R \quad (\text{since } R\theta = D)$$

$$\vec{P}\cdot\vec{P}_T = R^2\cos\Phi\cos\lambda\cos\Phi_T\cos\lambda_T + R^2\cos\Phi\sin\lambda\cos\Phi_T \overset{\sin}{\cancel{\cos}}\lambda_T + R^2\sin\Phi\sin\Phi_T$$

Note that $\cos(\lambda - \lambda_T) = \cos\lambda \cos\lambda_T + \sin\lambda \sin\lambda_T$

$$\vec{P} \cdot \vec{P}_T = R^2 \left[ \cos\Phi \cos\Phi_T \left( \cos\lambda \cos\lambda_T + \sin\lambda \sin\lambda_T \right) \right]$$
$$+ R^2 \sin\Phi \sin\Phi_t$$

$$\cancel{R^2} \cos\frac{D}{R} = \cancel{R^2} \left[ \sin\Phi \sin\Phi_t + \cos\Phi \cos\Phi_T \cos(\lambda - \lambda_T) \right.$$

QED

For 2.22b,* consult "spherical law of cosines," which can be proved using vector dot products, to compute the bearing from north in spherical geometry.

* optional

For 2.21, start with 2.22a

$\Longrightarrow \quad \Phi_t = \Phi + \Delta\Phi$

$\lambda_t = \lambda + \Delta\lambda$

$\sin\Phi_t = \sin(\Phi + \Delta\Phi)$

$= \sin\Phi \cos\Delta\Phi + \cos\Phi \sin\Delta\Phi$

$= \sin\Phi\left(1 - \frac{(\Delta\Phi)^2}{2}\right) + \cos\Phi(\Delta\Phi)$

etc.

write $\cos\left(\frac{D+\Delta D}{R}\right)$ for $\cos\frac{D}{R}$ to get

$$1 - \frac{(D+\Delta D)^2}{2R} \simeq 1 - \frac{D^2}{2R} + \frac{-2(\Delta D)D}{2R}$$

(from memory - this may need further refining)