Module 12

Modern Navigation Systems

Inertial Navigation, Doppler, and Atomic Clocks

Module 12A

Inertial Navigation Principles

Summary of Module 12

- **The Keplerian equations from earlier modules are re-examined in terms of conservation of angular momentum, thus yielding expressions for the acceleration versus time of moving bodies. The Schuler frequency is introduced and analyzed in the context of a feedback control system, using as an example problem the inertial navigation of an aircraft flying from Dulles Airport to Beijing. Angular momentum is then re-examined in the context of circus acrobatic performances. (12A)**
- Sensors that can measure these accelerations are explored, including mechanical gyroscopes based on rotating flywheels, mechanical gyroscopes that use micro-electrical mechanical devices (MEMs), and optical gyroscopes that exploit the relativistic Sagnac effect (e.g., the ring-laser gyroscope). (12B)
- Doppler techniques are introduced using the Cospas-SarSat system as a prototypical example. Students will watch a video that describes Cospas-Sarsat in detail. (12C)
- Atomic clocks and the Allan variance are introduced. A chip-scale atomic clock is described. (12D)
- Students will begin submitting and/or presenting their final projects. (12E)

The videos

- "Science of the Circus" (12A)
  - o This is a special feature on a Ringling Brothers Circus DVD. It deals with inertia, ballistic trajectories, center of gravity, and conservation of angular momentum
- Gyro precess (12B)
  - o This shows how a gyroscope precession changes direction when the direction of rotation of the gyroscope changes
- Gimble1 (12B)
  - o This shows the effects of torque on the behavior of a gimbled gyroscope
- iNEMO demos 1, 2 and 3 (12B)
  - o These show the operation of the ST Micro-electronics inertial measurement system via the use of a rotation platform and PC-based demo software.
- Cospas-Sarsat
  - o This describes the operation of the Cospas-Sarsat satellite search and rescue system. (12C)

Reading and Video

- Read Chapters 3 and 7, “Multisensor Navigation Systems” and “Inertial Navigation,” of the primary text (Kayton & Fried).
- View the video, “Science of the Circus”.

Angular Momentum

- Linear momentum is mass times velocity
  - $\vec{p} = m\vec{v}$
- Angular momentum is
  - $\vec{l} = \vec{r} \times \vec{p} = \vec{r} \times m\omega\vec{r} = mr^2\omega = I\omega$
    - *Where I is the moment of inertia and* ω *is the rotational angular velocity of the mass* m

Torque

- Torque $T = \vec{r} \times \vec{F} = {d\vec{L}}/{dt}$
- When *torque = 0*, angular momentum is conserved
- This is the case for orbits where the force is an inverse square force such as
  - $F = \frac{GM_1M_2}{r^2}$
- This yields Kepler's laws

The Schuler Frequency

- Consider the period of a pendulum $T = 2\pi\sqrt{{}^{l}/_{g}}$
- The gravitational force *g* is given by
  - $g = {}^{GM}/_{r^2}$
- If *M* is the mass of the earth, and *l* = *r* equals the radius of the earth, one gets Kepler's third law, and *T* is the Schuler period of 84.4 minutes

Effect of the Schuler Frequency on inertial systems

- The differential equations for inertial systems, if subjected to impulsive forces, exhibit homogeneous solutions that oscillate at the Schuler frequency
- “Straight and level” flight over a curved earth is actually a great circle in which the aircraft is always accelerating (cf. the orbit propagator from the earlier module), and its inertial systems are subject to Schuler oscillations caused by unwanted disturbance signals
- A nonstop flight from Washington to Beijing experiences numerous Schuler cycles that must be compensated for using feedback control loops

Schuler oscillations in an inertial navigation system

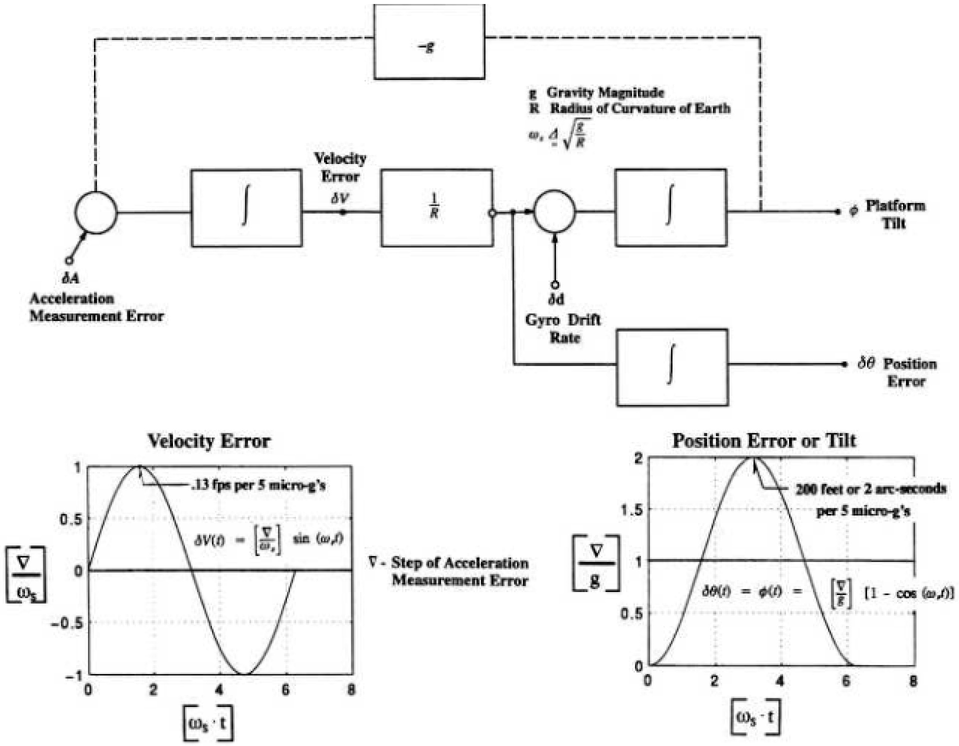

Figure 3.2 Inertial system error response to a step of acceleration measurement error. Constant acceleration measurement error induces a zero-mean Schuler oscillation in velocity error and an identical nonzero mean oscillation in tilt and position error.

This figure, from the primary text, shows the Schuler Oscillations induced by a “step” function disturbance to an inertial system.

Because the system is deliberately undamped, the oscillations persist and and must be addressed using feedback control techniques.

Cumulative error

- Inertial systems measure forces, and hence accelerations.
- To determine position, they integrate acceleration twice, first to obtain velocity, then to estimate position.
- As a consequence, error growth is proportional to the square of time since the system was initialized (i.e., set to a known position value).
- This limits the accuracy of inertial-only systems during long-duration journeys (e.g., flights from Washington to Beijing).

TABLE 7.1 Typical inertial navigator specification (1996)

Parameter	Value
Navigation accuracy	0.8 nmi/hr
Velocity accuracy	2.5 ft/sec RMS
Pitch-and-roll accuracy	0.05 deg rms
Azimuth accuracy	0.05 deg rms
Alignment time gyrocompass	3–8 min
stored heading	30–90 sec
Size	500–1000 in.^{3}
Weight	20–30 lb
Power	30–150 w
Acceleration capability	30 g
Angular rate capability	400 deg/sec
Mean time between failures in a fighter environment	3500 hr

Integrated navigation systems

- To address the t^2 error growth, integration of inertial systems with other forms of navigation is common
- The use of combined GPS/INS systems is particularly effective
  - o GPS yields excellent long term navigation
  - o INS gives excellent short-time response to rapid accelerations and maneuvers
- This yields “aided” systems, in which the INS and the GPS subsystems “aid” each other

Aided systems

- The coupling between components of aided systems can be either “tight” or “loose”
    - Tight: a single Kalman filter combines acceleration measurements from an inertial measurement unit (IMU) with the pseudorange measurements of a GPS receiver to produce a navigation solution
    - Loose: the inertial and GPS systems each produce a navigation solution using separate Kalman filters, and the two solutions are combined using weighting techniques (e.g., weighted least squares)
- Review material in earlier modules on Kalman filters and the many references to Kalman filters in the primary text.

Angular Momentum in “everyday” life

- Watch the “circus” video to see how circus performers utilize conservation of angular momentum to predict, control, and stabilize their motion during their stunts
- Consider how figure skaters control the rotational rate of their spins by moving their arms, thus changing their moment of inertia.
- Note that for circular orbits, ω is fixed, and hence angular momentum is conserved.
- For elliptical orbits, as r increases, ω decreases, and angular momentum is conserved (Kepler’s second and third laws)

Assignment 12-1

1. Compute the Schuler period and compare it to the orbital period of a satellite orbiting a smooth, spherical earth at sea level.
2. Explain why figure skaters spin faster when they draw their arms in.
3. Explain why tightrope walkers throw their balance pole up in the air in a “torque-free maneuver” , then rotate quickly underneath before catching the pole, in order to change directions on the tightrope.

End of Mod 12A