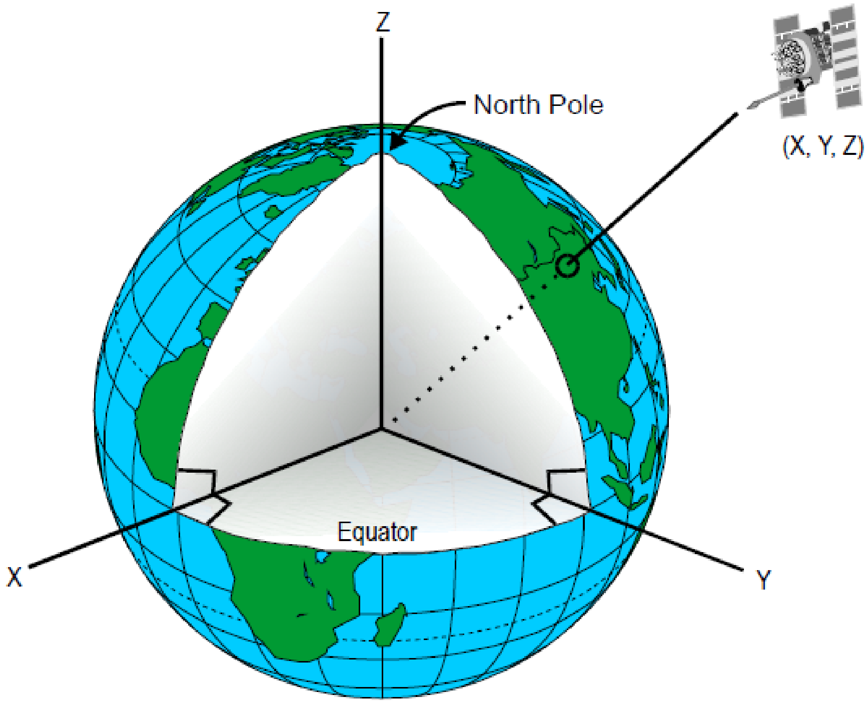
Module 2B

Small Circles on the Surface of the Earth

In this module

- We analyze
  - The difference between small and great circles
  - Using the vector notation and equations from module 2a



Latitude and longitude

- Lines of longitude are great circles
  - The center of each “meridian” cicle is at the center of the earth
- Lines of latitude are small circles
  - The center of each small circle is at a point on the *z* axis
  - Where the value of z is given by r_e sin(latitude)
  - And the radius of the small circle is given by r_{small} = r_e cos(latitude)
  - The circumference of the small circle is given by 2π x rsmall
  - The arc length of a portion of a small circle is given by r_{small} times the included angle (in radians) of the beginning and end of the arc, in the usual manner taught in high school geometry

From: http://upload.wikimedia.org/wikipedia/commons/3/32/Earth_Centered_Inertial_Coordinate_System.png (accessed Oct 16, 2014).

Rotation of Axes

- A small circle need not be limited to lines of constant latitude
- By rotating the axes of the coordinate system such that a new z-axis is defined that, for example
  - Points to a particular star
  - A new small circle is defined that represents the “locus” of all points on the surface of the earth to which the angle above the earth’s horizon to the star is constant
  - This defines “circles of position”
    - Which is relevant to stars <u>and</u> satellites (e.g. GPS)

Two small circles representing circles of position on the surface of the earth

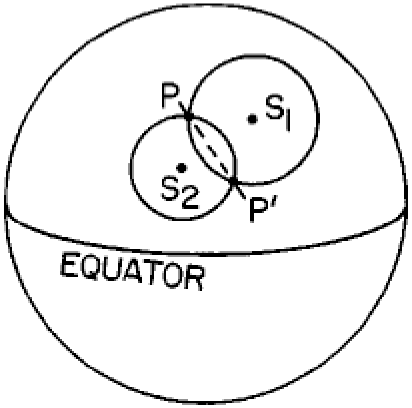


Fig. 6. In this sketch of the terrestrial sphere, $S1$ and $S2$ are the simultaneous geographic positions of the respective substellar points of star 1 and star 2. The two circles of position correspond to the observed zenith distances of the two stars and intersect at points P and P', the two possible geographic positions of the observer.

Basic principles of celestial navigation

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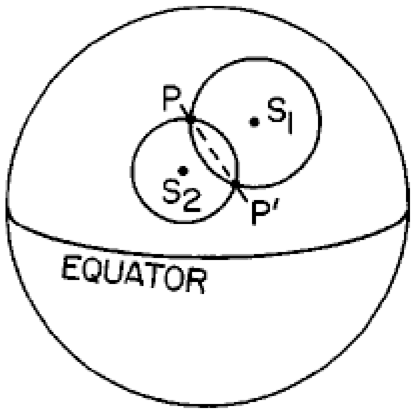
An observer on any point on one of the *small circles* will measure the same value for the angular elevation of the star S1 or S2 above the earth's horizon.

Van Allen, for whom the Radiation belts are Named, had a long Association with JHU/APL.

Why this is important

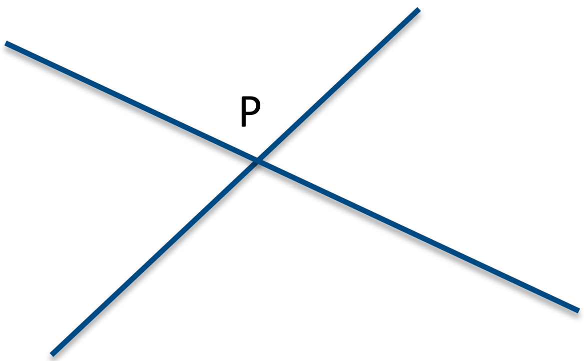


If an observer on the surface of the earth measures the angle to each of two stars, or the distance to each of two GPS satellites, the observer's position is at either the point P or the point P'.

This forms the basis of both GPS and celestial navigation, as you will learn in upcoming modules.

When viewed “up-close”…

- Small circles look like lines!
- Thus, the equations of linear algebra (e.g., MatLab) can be used to advantage.



Determination of the coordinates of the point P becomes simply the solution of two linear equations in two unknowns.

In module 3, you will learn how to apply perturbation theory in order To approximate segments of small circles as straight lines.

Assignment 2.2

2.2.1 Apply an appropriate rotation of axes to determine the small circle and great circle distances between Washington D.C. and Dallas, TX. Consult an algebra book or Wikipedia for guidance if the rotation of axes computation proves to be challenging.

2.2.2 Verify that the great circle distance is shorter.

End of Mod 2B