



# Module 13

## Modern Navigation Systems

### Terrestrial and Marine Navigation Systems

#### Module 13A

#### Terrestrial Equations and Hyperbolic Navigation Systems

# Summary of Module 13

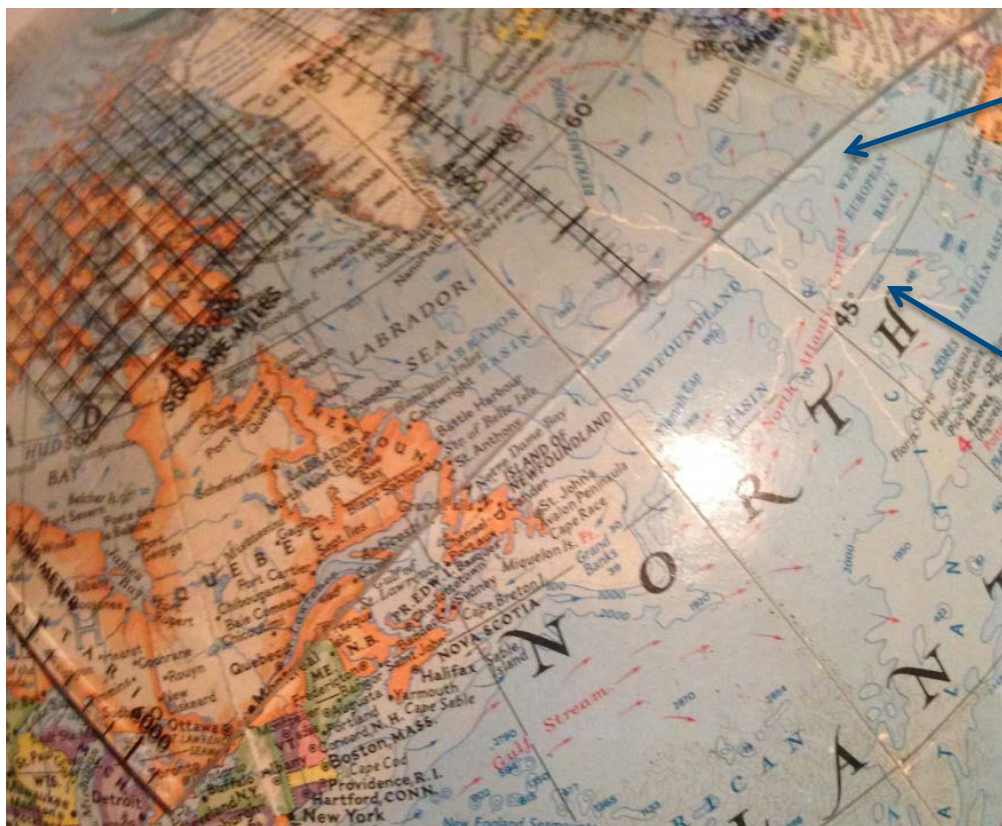
- **The simplification of the GPS and celestial algorithms to the 2-dimensional case is presented. But first, a few catch-up slides are included to elaborate further on concepts explored in earlier modules. The sub-module concludes with a brief introduction to hyperbolic radio-navigation systems. (13A)**
- The basics of practical terrestrial and marine navigation will be introduced via a combination of slides, charts, and photographs that illustrate various important features. (13B)
- The module includes a photo-documentary of the voyage of a commercial freighter from Bremerhaven, Germany to Newark, New Jersey in 1978, pre-GPS. (13C)
- Students will continue presenting their final projects.



# Reading

- Read chapter 4 of the primary text by Kayton and Fried.

# Great versus small circles



The great circle from Spain to Montreal

The small circle at 45° N from Spain to Montreal



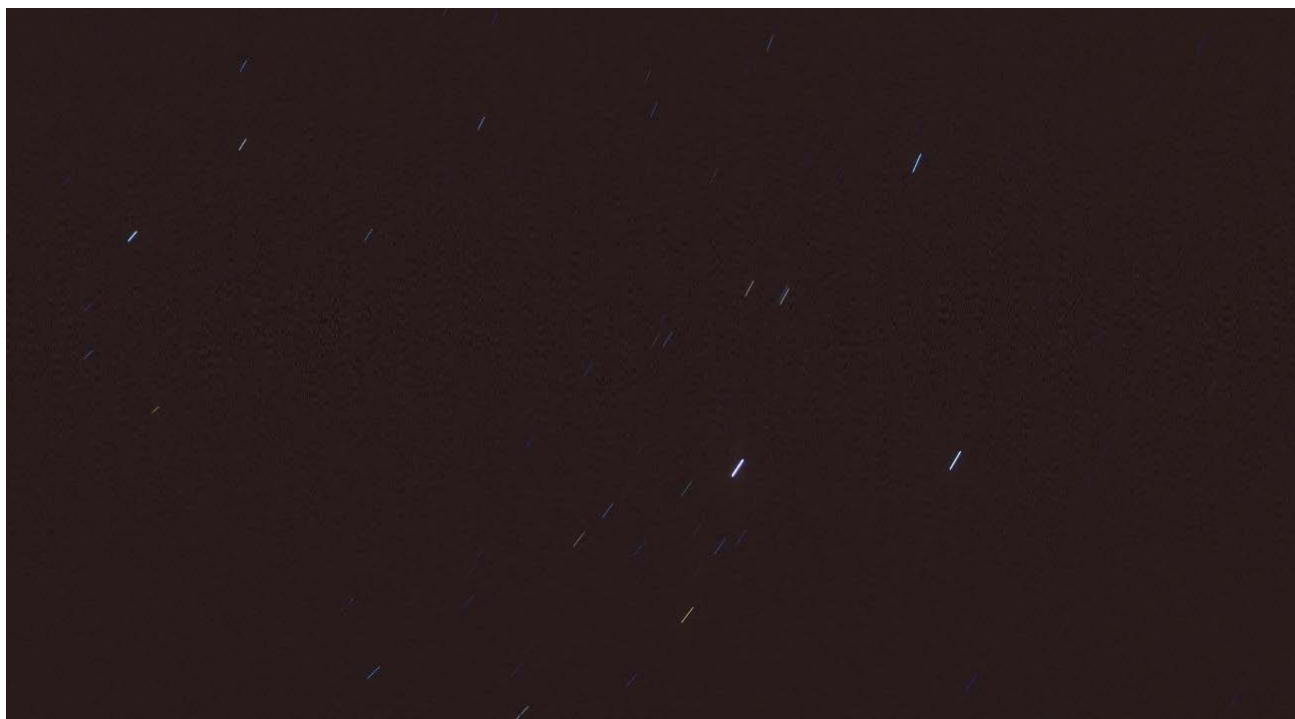
The equation of time, shown on a plotting tool that comes with the National Geographic globe shown on the previous slide.

The vertical axis represents the sun's declination over the course of a solar year, while the horizontal axis represents the extent to which sidereal time precedes or lags behind Greenwich Mean Time as the earth travels through its non-circular orbit and speeds up or slows down in accordance with Kepler's laws.

The months illustrate the direction that the sun travels through the analemma.



# Two minute exposure of the stars rotating through the night sky



Note that the stars are clearly rotating through the night sky, and that their colors vary.

The pole star, Polaris, if visible in the field of view of this photograph, would be at the center of rotation of the stars.

# Sun Dogs



“Sun dogs” are false images of the sun at  $\pm 22.5^\circ$  on either side of the actual sun. The false images are due to refraction of light by planar hexagonal ice crystals suspended in the atmosphere. The photo on the left was taken on the Arctic ice pack in spring 1986. The two photos on the right were taken on I-66 outside the Washington Beltway in 2015.

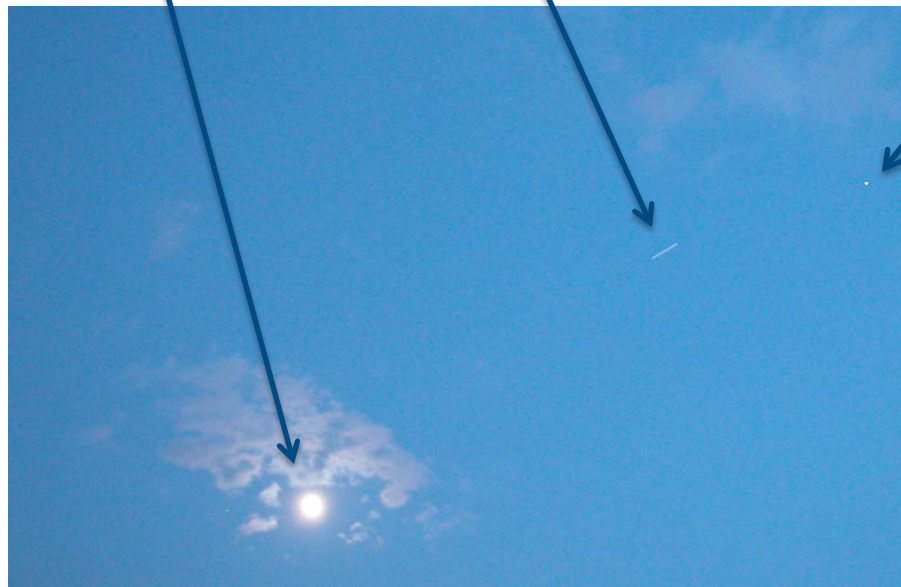
This illustrates dramatically how refraction can affect observation of both optical and radio-navigation signals.

# “Celestial Trio”

The international space station

The moon

Venus (or Jupiter?)



This time-lapse video frame is from NASA/JPL and aired on WTTG TV On or about July 10 2006.

Note that all three objects are visible in daylight, and that the space station is clearly moving rapidly. It is about 250 miles away and is moving at approximately 16,000 mph.



# Richharia's equations

$$\text{Right ascension, } \alpha = \arctan(y/x) \quad (\text{B.43})$$

$$\text{Declination, } \delta = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \quad (\text{B.44})$$

$$\text{Elevation, } \eta = \arctan\left(\frac{\sin \eta_s - \frac{R}{r}}{\cos \eta_s}\right) \quad (\text{B.45})$$

where

$$\eta_s = \arcsin[\sin \delta \sin \theta_e + \cos \delta \cos \theta_e \cos \phi_{se}] \quad (\text{B.46})$$

and  $R$  = Earth radius

$r$  = satellite distance from Earth centre (use equation B.40c)

$\theta_e$  = earth station latitude

$\phi_{se} = \phi_s - \phi_e$

$\phi_s$  = satellite longitude

$\phi_e$  = earth station longitude.

$$\text{Azimuth, } A = \arctan\left[\frac{\sin \phi_{se}}{\cos \theta_e \tan \delta - \sin \theta_e \cos \phi_{se}}\right] \quad (\text{B.47})$$

The distance  $\rho$  of a satellite from a given point on the Earth is given as

$$\rho = \sqrt{r^2 - R^2 \cos^2 \eta} - R \sin \eta \quad (\text{B.48})$$

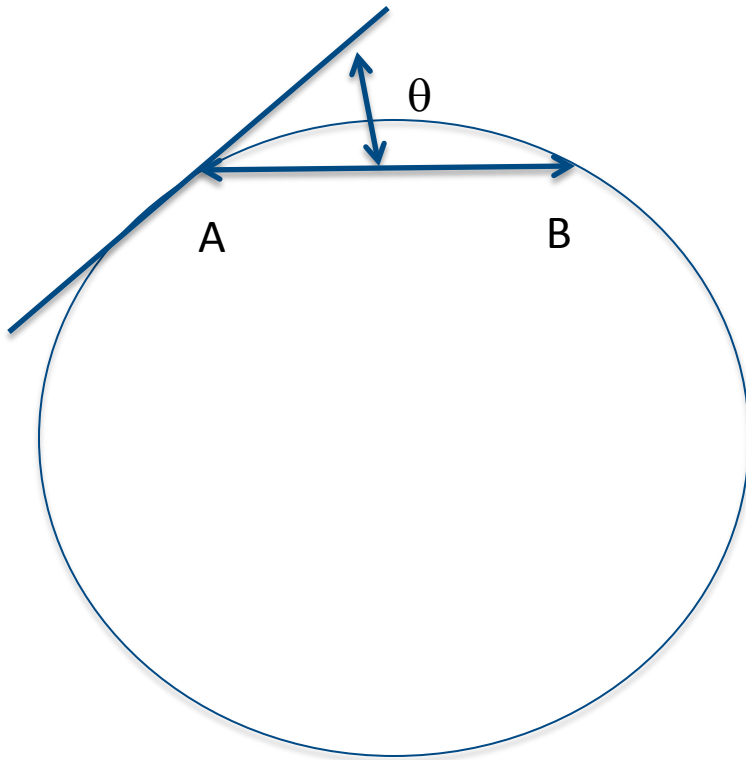
Set  $R = r$  for terrestrial navigation on the surface of the Earth (i.e.  $-R/r = 1$ )

This is in contrast to celestial navigation, where  $R/r = 0$ ;

lunar navigation, where  $R/r = (4,000 \text{ miles}/240,000) \text{ miles}$ ,

and satellite navigation, where  $R/r$  varies between 4,000/100,000 and 4000/4200 (HEO and LEO, with GEO yielding  $R/r$  of 4,000/26,000).

# The implication or $R/r = 1$



Richharia's equations give the distance between A and B and the corresponding elevation angle  $\theta$ . Note that this yields a negative value for  $\theta$  with respect to the tangent line (the horizon line) drawn in the figure.

The great circle distance is the arc-length between A and B.

For short distances (with respect to the radius of the earth), the flat earth model is accurate. This was demonstrated by problem 2.4 from the text in the early weeks of the course.



# The $R/r$ terrestrial limit

- All of the equations of the previous modules remain accurate.
- But, in the limit of  $R/r = 1$ , two dimensional Cartesian coordinates and the corresponding equations can be used as shown in previous modules for
  - computing lines of position
  - drawing lines of position on charts using a compass rose, dividers, and a parallel rule.



# Charts

- Charts are of tremendous importance for marine and terrestrial navigation
  - But, a straight line between points A and B on a nautical chart, where straight lines represent a constant bearing, might go through an intermediate point C.
  - On an aviation chart, in which straight lines represent great circles, the same line from point A to point B might not go through point C.



# Hyperbolic navigation systems

- Radio signals at low frequency, which can propagate over the earth's horizon, are used to set up standing waves across the surface of the earth.
- This yields *hyperbolic* navigation systems, which are covered in detail in section 4.5 of the primary text by Kayton and Fried.



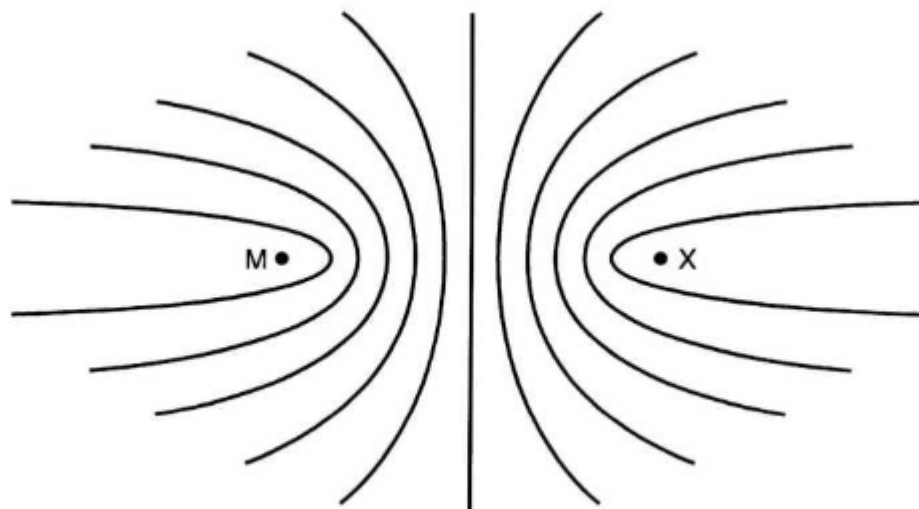


# Hyperbolic Nav Systems

- Hyperbolic Navigation Systems are no longer active
  - GPS has replaced them
- The following systems existed for many years
  - Loran-C, for navigation in the *littoral* (near-land) regions
  - DECCA, a private commercial system used in shipping lanes near Europe
  - OMEGA, a world-wide system

# Lines of constant path delay

Fig. 4.30 is from  
Kayton & Fried.



**Figure 4.30** Hyperbolic lines of constant TD for a typical master-secondary pair.

Two radio beacons M and X have lines of constant path difference in their combined standing wave pattern that form hyperbolas. When nodes and anti-nodes of this standing wave pattern are traversed, a radio receiver keeps count of the number of traversals.

# Standing waves from three beacons

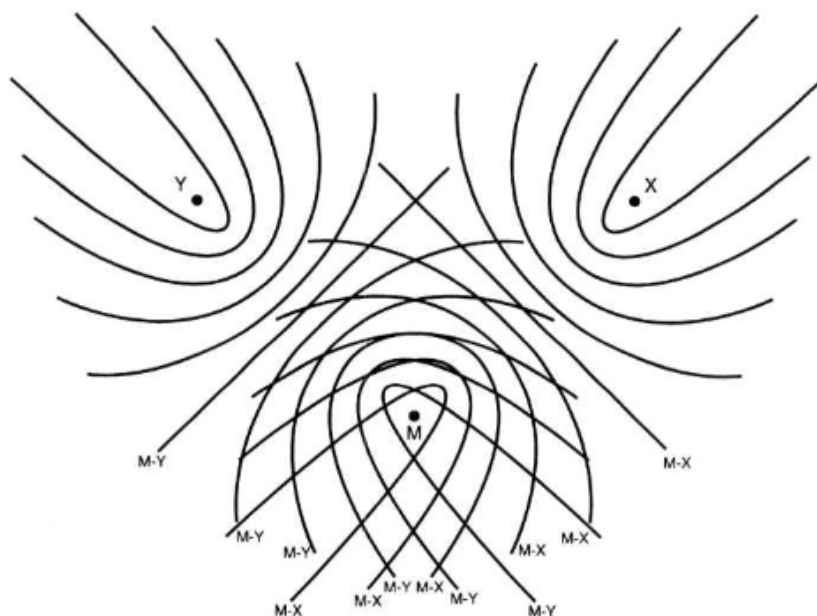


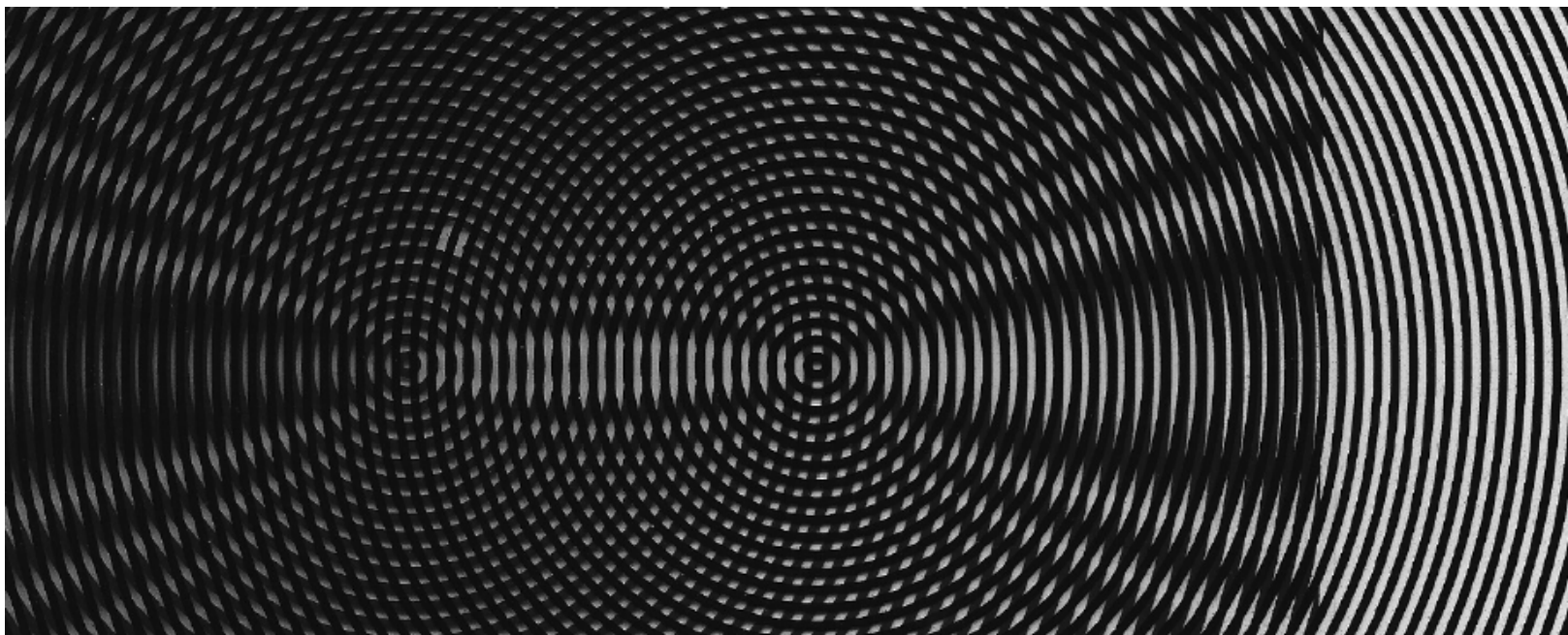
Figure 4.31 Hyperbolic lines of constant TD for a typical triad.

Adding a third beacon permits navigation in two-dimensions by aircraft and ships that are equipped with appropriate receivers.



Figure 4.29 Airborne Loran receiver.

# A Moire plot



This illustration, called a Moire plot, is formed by the overlap of two identical vu-graphs that contain nothing more than concentric circles. The circles represent phase fronts of an omni-directional radio wave in two-dimensions. Note that the clearly-visible hyperbolas, which hyperbolic systems measure and count, are nothing more than an optical illusion.

This is also the defining principle of directional and phased-array antennas.



# The future of Loran-C

- e-Loran is a proposed upgrade to Loran-C
- like Loran-C, it would be essentially un-jammable, and has been proposed as a backup for the GPS system.
- For further reading on hyperbolic systems, consult chapter 4 of the primary text.





# Assignment 13-1

1. For an arc-length of 45 degrees on the surface of the earth, verify that setting  $R/r = 1$  yields the negative elevation values and straight line distances illustrated earlier in this sub-module.
2. What is the difference between the straight line distance and the arc-length.
3. For a tower or aircraft of height  $h$  above the surface of the earth, estimate how far it is to the visible horizon. This problem is hugely important with respect to line of sight communication and navigation.
4. Use your Nautical Almanac skills and the online tools at USNO to determine whether the planet on slide 8 is Venus or is it Jupiter.



# End of Mod 13A