SIGHT REDUCTION PROCEDURES

METHODS AND FORMULAE FOR DIRECT COMPUTATION

1. *Introduction.* In this section formulae and methods are provided for *calculating* position at sea from observed altitudes taken with a marine sextant using a computer or programmable calculator.

The method uses analogous concepts and similar terminology as that used in *manual* methods of astro-navigation, where position is found by plotting position lines from their intercept and azimuth on a marine chart.

The algorithms are presented in standard algebra suitable for translating into the programming language of the user's computer. The basic ephemeris data may be taken directly from the main tabular pages of a current version of *The Nautical Almanac*. Formulae are given for calculating altitude and azimuth from the *GHA* and *Dec* of a body, and the estimated position of the observer. Formulae are also given for reducing sextant observations to observed altitudes by applying the corrections for dip, refraction, parallax and semi-diameter.

The intercept and azimuth obtained from each observation determines a position line, and the observer should lie on or close to each position line. The method of least squares is used to calculate the fix by finding the position where the sum of the squares of the distances from the position lines is a minimum. The use of least squares has other advantages. For example it is possible to improve the estimated position at the time of fix by repeating the calculation. It is also possible to include more observations in the solution and to reject doubtful ones.

2. *Notation.*

GHA = Greenwich hour angle. The range of *GHA* is from 0° to 360° starting at 0° on the Greenwich meridian increasing to the west, back to 360° on the Greenwich meridian.

SHA = sidereal hour angle. The range is 0° to 360°.

Dec = declination. The sign convention for declination is north is positive, south is negative. The range is from −90° at the south celestial pole to +90° at the north celestial pole.

Long = longitude. The sign convention is east is positive, west is negative. The range is −180° to +180°.

Lat = latitude. The sign convention is north is positive, south is negative. The range is from −90° to +90°.

LHA = *GHA* + *Long* = local hour angle. The *LHA* increases to the west from 0° on the local meridian to 360°.

H_C = calculated altitude. Above the horizon is positive, below the horizon is negative. The range is from −90° in the nadir to +90° in the zenith.

H_S = sextant altitude.

H = apparent altitude = sextant altitude corrected for instrumental error and dip.

H_O = observed altitude = apparent altitude corrected for refraction and, in appropriate cases, corrected for parallax and semi-diameter.

Z = Z_n = true azimuth. Z is measured from true north through east, south, west and back to north. The range is from 0° to 360°.

I = sextant index error.

D = dip of horizon.

R = atmospheric refraction.

HP = horizontal parallax of the Sun, Moon, Venus or Mars.
PA = parallax in altitude of the Sun, Moon, Venus or Mars.
S = semi-diameter of the Sun or Moon.
p = intercept $= H_O - H_C$. Towards is positive, away is negative.
T = course or track, measured as for azimuth from the north.
V = speed in knots.

3. *Entering Basic Data.* When quantities such as GHA are entered, which in *The Nautical Almanac* are given in degrees and minutes, convert them to degrees and decimals of a degree by dividing the minutes by 60 and adding to the degrees; for example, if $GHA = 123° 45'6$, enter the two numbers 123 and 45·6 into the memory and set $GHA = 123 + 45{\cdot}6/60 = 123°7600$. Although four decimal places of a degree are shown in the examples, it is assumed that full precision is maintained in the calculations.

When using a computer or programmable calculator, write a subroutine to convert degrees and minutes to degrees and decimals. Scientific calculators usually have a special key for this purpose. For quantities like Dec which require a minus sign for southern declination, change the sign from plus to minus after the value has been converted to degrees and decimals, *e.g.* $Dec = \mathrm{S}\,0° 12'3 = \mathrm{S}\,0°2050 = -0°2050$. Other quantities which require conversion are semi-diameter, horizontal parallax, longitude and latitude.

4. *Interpolation of GHA and Dec.* The GHA and Dec of the Sun, Moon and planets are interpolated to the time of observation by direct calculation as follows: If the universal time is $a^h\ b^m\ c^s$, form the interpolation factor $x = b/60 + c/3600$. Enter the tabular value GHA_0 for the preceeding hour (a) and the tabular value GHA_1 for the following hour ($a+1$) then the interpolated value GHA is given by

$$GHA = GHA_0 + x(GHA_1 - GHA_0)$$

If the GHA passes through 360° between tabular values add 360° to GHA_1 before interpolation. If the interpolated value exceeds 360°, subtract 360° from GHA.

Similarly for declination, enter the tabular value Dec_0 for the preceeding hour (a) and the tabular value Dec_1 for the following hour ($a+1$), then the interpolated value Dec is given by

$$Dec = Dec_0 + x(Dec_1 - Dec_0)$$

5. *Example.* (a) Find the GHA and Dec of the Sun on 2000 December 3 at $19^h\ 03^m\ 25^s$ UT.

The interpolation factor $x = 03/60 + 25/3600 = 0^h0569$

page 235 $19^h\ GHA_0 = 107° 28'9 = 107°4817$

$20^h\ GHA_1 = 122° 28'6 = 122°4767$

$19^h0569\ GHA = 107{\cdot}4817 + 0{\cdot}0569(122{\cdot}4767 - 107{\cdot}4817) = 108°3355$

$19^h\ Dec_0 = \mathrm{S}\,22° 13'1 = -22°2183$

$20^h\ Dec_1 = \mathrm{S}\,22° 13'5 = -22°2250$

$19^h0569\ Dec = -22{\cdot}2183 + 0{\cdot}0569(-22{\cdot}2250 + 22{\cdot}2183) = -22°2187$

GHA Aries is interpolated in the same way as GHA of a body. For a star the SHA and Dec are taken from the tabular page and do not require interpolation, then

$$GHA = GHA \text{ Aries} + SHA$$

where GHA Aries is interpolated to the time of observation.

(b) Find the *GHA* and *Dec* of *Vega* on 2000 December 3 at $19^h\ 03^m\ 25^s$ UT.

The interpolation factor $x = 0^h\!.0569$ as in the previous example

page 234 $\quad 19^h\ GHA\ \text{Aries}_0 = 357°\ 54'\!.4 = 357°\!.9067$

$20^h\ GHA\ \text{Aries}_1 = 12°\ 56'\!.9 = 372°\!.9483$ (360° added)

$19^h\!.0569\ GHA\ \text{Aries} = 357{\cdot}9067 + 0{\cdot}0569(372{\cdot}9483 - 357{\cdot}9067) = 358°\!.7632$

$SHA = 80°\ 46'\!.0 = 80°\!.7667$

$GHA = GHA\ \text{Aries} + SHA = 79°\!.5299$ (multiple of 360° removed)

$Dec = \text{N}\,38°\ 47'\!.2 = +38°\!.7867$

6. *The calculated altitude and azimuth.* The calculated altitude H_C and true azimuth Z are determined from the *GHA* and *Dec* interpolated to the time of observation and from the *Long* and *Lat* estimated at the time of observation as follows:

Step 1. Calculate the local hour angle

$$LHA = GHA + Long$$

Add or subtract multiples of 360° to set *LHA* in the range 0° to 360°.

Step 2. Calculate S, C and the altitude H_C from

$$S = \sin Dec$$
$$C = \cos Dec \cos LHA$$
$$H_C = \sin^{-1}(S \sin Lat + C \cos Lat)$$

where $\sin^{-1}$ is the inverse function of sine.

Step 3. Calculate X and A from

$$X = (S \cos Lat - C \sin Lat)/ \cos H_C$$
$$\text{If } X > +1 \text{ set } X = +1$$
$$\text{If } X < -1 \text{ set } X = -1$$
$$A = \cos^{-1} X$$

where $\cos^{-1}$ is the inverse function of cosine.

Step 4. Determine the azimuth Z

$$\text{If } LHA > 180° \text{ then } Z = A$$
$$\text{Otherwise } Z = 360° - A$$

7. *Example.* Find the calculated altitude H_C and azimuth Z when

$$GHA = 53° \quad Dec = \text{S}\,15° \quad Lat = \text{N}\,32° \quad Long = \text{W}\,16°$$

For the calculation

$$GHA = 53°\!.0000 \quad Dec = -15°\!.0000 \quad Lat = +32°\!.0000 \quad Long = -16°\!.0000$$

Step 1. $\quad LHA = 53{\cdot}0000 - 16{\cdot}0000 = 37{\cdot}0000$

Step 2. $\quad S = -0{\cdot}2588$

$C = +0{\cdot}9659 \times 0{\cdot}7986 = 0{\cdot}7714$

$\sin H_C = -0{\cdot}2588 \times 0{\cdot}5299 + 0{\cdot}7714 \times 0{\cdot}8480 = 0{\cdot}5171$

$H_C = 31°\!.1346$

Step 3. $X = (-0{\cdot}2588 \times 0{\cdot}8480 - 0{\cdot}7714 \times 0{\cdot}5299)/0{\cdot}8560 = -0{\cdot}7340$

$A = 137^\circ\!.2239$

Step 4. Since $LHA \leq 180^\circ$ then $Z = 360^\circ - A = 222^\circ\!.7761$

8. *Reduction from sextant altitude to observed altitude.* The sextant altitude H_S is corrected for both dip and index error to produce the apparent altitude. The observed altitude H_O is calculated by applying a correction for refraction. For the Sun, Moon, Venus and Mars a correction for parallax is also applied to H, and for the Sun and Moon a further correction for semi-diameter is required. The corrections are calculated as follows:

Step 1. Calculate dip

$$D = 0^\circ\!.0293\sqrt{h}$$

where h is the height of eye above the horizon in metres.

Step 2. Calculate apparent altitude

$$H = H_S + I - D$$

where I is the sextant index error.

Step 3. Calculate refraction (R) at a standard temperature of 10° Celsius (C) and pressure of 1010 millibars (mb)

$$R_0 = 0^\circ\!.0167/\tan(H + 7{\cdot}31/(H + 4{\cdot}4))$$

If the temperature T° C and pressure P mb are known calculate the refraction from

$$R = fR_0 \quad \text{where} \quad f = 0{\cdot}28P/(T + 273)$$

otherwise set $R = R_0$

Step 4. Calculate the parallax in altitude (PA) from the horizontal parallax (HP) and the apparent altitude (H) for the Sun, Moon, Venus and Mars as follows:

$$PA = HP\cos H$$

For the Sun $HP = 0^\circ\!.0024$. This correction is very small and could be ignored.

For the Moon HP is taken for the nearest hour from the main tabular page and converted to degrees.

For Venus and Mars the HP is taken from the critical table at the bottom of page 259 and converted to degrees.

For the navigational stars and the remaining planets, Jupiter and Saturn set $PA = 0$.

If an error of 0'.2 is significant the expression for the parallax in altitude for the Moon should include a small correction OB for the oblateness of the Earth as follows:

$$PA = HP\cos H + OB$$

where $OB = -0^\circ\!.0032\sin^2 Lat\cos H + 0^\circ\!.0032\sin(2Lat)\cos Z\sin H$

At mid-latitudes and for altitudes of the Moon below 60° a simple approximation to OB is

$$OB = -0^\circ\!.0017\cos H$$

Step 5. Calculate the semi-diameter for the Sun and Moon as follows:

Sun: S is taken from the main tabular page and converted to degrees.

Moon: $S = 0^\circ.2724HP$ where HP is taken for the nearest hour from the main tabular page and converted to degrees.

Step 6. Calculate the observed altitude

$$H_O = H - R + PA \pm S$$

where the plus sign is used if the lower limb of the Sun or Moon was observed and the minus sign if the upper limb was observed.

9. *Example*. The following example illustrates how to use a calculator to reduce the sextant altitude (H_S) to observed altitude (H_O); the sextant altitudes given are assumed to be taken on 2000 December 3 with a marine sextant, zero index error, at height 5·4 m, temperature −3° C and pressure 982 mb, the Moon sights are assumed to be taken at 10^h UT.

Body limb	Sun lower	Sun upper	Moon lower	Moon upper	Venus —	*Polaris* —
Sextant altitude: H_S	21·3283	3·3367	33·4600	26·1117	4·5433	49·6083
Step 1. Dip: $D = 0{\cdot}0293\sqrt{h}$	0·0681	0·0681	0·0681	0·0681	0·0681	0·0681
Step 2. Apparent altitude: $H = H_S + I - D$	21·2602	3·2686	33·3919	26·0436	4·4752	49·5402
Step 3. Refraction: R_0 f $R = fR_0$	0·0423 1·0184 0·0431	0·2262 1·0184 0·2304	0·0251 1·0184 0·0256	0·0338 1·0184 0·0344	0·1801 1·0184 0·1834	0·0142 1·0184 0·0144
Step 4. Parallax: HP	0·0024	0·0024	(54′.6) 0·9100	(54′.6) 0·9100	(0′.1) 0·0017	—
Parallax in altitude: $PA = HP \cos H$	0·0022	0·0024	0·7598	0·8176	0·0017	—
Step 5. Semi-diameter: Sun : $S = 16{\cdot}3/60$ Moon : $S = 0{\cdot}2724HP$	0·2717 —	0·2717 —	— 0·2479	— 0·2479	— —	— —
Step 6. Observed altitude: $H_O = H - R + PA \pm S$	21·4910	2·7690	34·3740	26·5789	4·2935	49·5258

Note that for the Moon the correction for the oblateness of the Earth of about $-0^\circ.0017 \cos H$, which equals $-0^\circ.0014$ for the lower limb and $-0^\circ.0015$ for the upper limb, has been ignored in the above calculation.

10. *Position from intercept and azimuth using a chart*. An estimate is made of the position at the adopted time of fix. The position at the time of observation is then calculated by dead reckoning from the time of fix. For example if the course (track) T and the speed V (in knots) of the observer are constant then *Long* and *Lat* at the time of observation are calculated from

$$Long = L_F + t\,(V/60)\sin T/\cos B_F$$
$$Lat = B_F + t\,(V/60)\cos T$$

where L_F and B_F are the estimated longitude and latitude at the time of fix and t is the time interval in hours from the time of fix to the time of observation, t is positive if the time of observation is after the time of fix and negative if it was before.

The position line of an observation is plotted on a chart using the intercept

$$p = H_O - H_C$$

and azimuth Z with origin at the calculated position ($Long$, Lat) at the time of observation, where H_C and Z are calculated using the method in section 6, page 279. Starting from this calculated position a line is drawn on the chart along the direction of the azimuth to the body. Convert p to nautical miles by multiplying by 60. The position line is drawn at right angles to the azimuth line, distance p from ($Long, Lat$) towards the body if p is positive and distance p away from the body if p is negative. Provided there are no gross errors the navigator should be somewhere on or near the position line at the time of observation. Two or more position lines are required to determine a fix.

11. *Position from intercept and azimuth by calculation.* The position of the fix may be calculated from two or more sextant observations as follows.

If p_1, Z_1, are the intercept and azimuth of the first observation, p_2, Z_2, of the second observation and so on, form the summations

$$A = \cos^2 Z_1 + \cos^2 Z_2 + \cdots$$
$$B = \cos Z_1 \sin Z_1 + \cos Z_2 \sin Z_2 + \cdots$$
$$C = \sin^2 Z_1 + \sin^2 Z_2 + \cdots$$
$$D = p_1 \cos Z_1 + p_2 \cos Z_2 + \cdots$$
$$E = p_1 \sin Z_1 + p_2 \sin Z_2 + \cdots$$

where the number of terms in each summation is equal to the number of observations.

With $G = AC - B^2$, an improved estimate of the position at the time of fix (L_I, B_I) is given by

$$L_I = L_F + (AE - BD)/(G\cos B_F), \qquad B_I = B_F + (CD - BE)/G$$

Calculate the distance d between the initial estimated position (L_F, B_F) at the time of fix and the improved estimated position (L_I, B_I) in nautical miles from

$$d = 60\sqrt{((L_I - L_F)^2 \cos^2 B_F + (B_I - B_F)^2)}$$

If d exceeds about 20 nautical miles set $L_F = L_I$, $B_F = B_I$ and repeat the calculation until d, the distance between the position at the previous estimate and the improved estimate, is less than about 20 nautical miles.

12. *Example of direct computation.* Using the method described above, calculate the position of a ship on 2000 June 21 at $21^h\,00^m\,00^s$ UT from the marine sextant observations of the three stars *Regulus* (No. 26) at $20^h\,39^m\,23^s$ UT, *Antares* (No. 42) at $20^h\,45^m\,47^s$ UT and *Kochab* (No. 40) at $21^h\,10^m\,34^s$ UT, where the observed altitudes of the three stars corrected for the effects of refraction, dip and instrumental error, are 37°4204, 20°3226 and 47°2050 respectively. The ship was travelling at a constant speed of 20 knots on a course of 325° during the period of observation, and the position of the ship at the time of fix $21^h\,00^m\,00^s$ UT is only known to the nearest whole degree W 15°, N 32°.

SOLVE THIS!

	UT	SUN GHA	SUN Dec	MOON GHA	MOON v	MOON Dec	MOON d	MOON HP
	d h	° ′	° ′	° ′	′	° ′	′	′
	17 00	179 47.3	N23 22.8	358 57.5	11.8	S20 57.0	3.0	54.1
	01	194 47.1	22.9	13 28.3	11.7	21 00.0	2.9	54.1
	02	209 47.0	23.0	27 59.0	11.8	21 02.9	2.9	54.1
	03	224 46.9	. . 23.1	42 29.8	11.7	21 05.8	2.7	54.1
	04	239 46.7	23.1	57 00.5	11.7	21 08.5	2.7	54.1
	05	254 46.6	23.2	71 31.2	11.7	21 11.2	2.5	54.1
	06	269 46.5	N23 23.3	86 01.9	11.7	S21 13.7	2.5	54.1
	07	284 46.3	23.3	100 32.6	11.7	21 16.2	2.3	54.1
S	08	299 46.2	23.4	115 03.3	11.7	21 18.5	2.3	54.1
A	09	314 46.1	. . 23.4	129 34.0	11.7	21 20.8	2.2	54.1
T	10	329 45.9	23.5	144 04.7	11.6	21 23.0	2.0	54.1
U	11	344 45.8	23.6	158 35.3	11.7	21 25.0	2.0	54.0
R	12	359 45.7	N23 23.6	173 06.0	11.6	S21 27.0	1.8	54.0
D	13	14 45.5	23.7	187 36.6	11.6	21 28.8	1.8	54.0
A	14	29 45.4	23.8	202 07.2	11.7	21 30.6	1.6	54.0
Y	15	44 45.3	. . 23.8	216 37.9	11.6	21 32.2	1.6	54.0
	16	59 45.1	23.9	231 08.5	11.6	21 33.8	1.4	54.0
	17	74 45.0	23.9	245 39.1	11.6	21 35.2	1.4	54.0
	18	89 44.9	N23 24.0	260 09.7	11.6	S21 36.6	1.2	54.0
	19	104 44.7	24.0	274 40.3	11.6	21 37.8	1.2	54.0
	20	119 44.6	24.1	289 10.9	11.6	21 39.0	1.0	54.0
	21	134 44.4	. . 24.2	303 41.5	11.5	21 40.0	0.9	54.0
	22	149 44.3	24.2	318 12.0	11.6	21 40.9	0.9	54.0
	23	164 44.2	24.3	332 42.6	11.6	21 41.8	0.7	54.0
	18 00	179 44.0	N23 24.3	347 13.2	11.6	S21 42.5	0.7	54.0
	01	194 43.9	24.4	1 43.8	11.6	21 43.2	0.5	54.0
	02	209 43.8	24.4	16 14.4	11.5	21 43.7	0.4	54.0
	03	224 43.6	. . 24.5	30 44.9	11.6	21 44.1	0.4	54.0
	04	239 43.5	24.5	45 15.5	11.6	21 44.5	0.2	54.0
	05	254 43.4	24.6	59 46.1	11.6	21 44.7	0.1	54.0
	06	269 43.2	N23 24.6	74 16.7	11.6	S21 44.8	0.1	54.0
	07	284 43.1	24.7	88 47.3	11.6	21 44.9	0.1	54.0
	08	299 43.0	24.7	103 17.9	11.5	21 44.8	0.2	54.0
S	09	314 42.8	. . 24.8	117 48.4	11.6	21 44.6	0.3	54.0
U	10	329 42.7	24.8	132 19.0	11.6	21 44.3	0.3	54.0
N	11	344 42.6	24.9	146 49.6	11.6	21 44.0	0.5	54.0
D	12	359 42.4	N23 24.9	161 20.2	11.6	S21 43.5	0.6	54.0
A	13	14 42.3	24.9	175 50.8	11.7	21 42.9	0.7	54.0
Y	14	29 42.2	25.0	190 21.5	11.6	21 42.2	0.7	54.0
	15	44 42.0	. . 25.0	204 52.1	11.6	21 41.5	0.9	54.0
	16	59 41.9	25.1	219 22.7	11.6	21 40.6	1.0	54.0
	17	74 41.8	25.1	233 53.3	11.7	21 39.6	1.1	54.0
	18	89 41.6	N23 25.2	248 24.0	11.6	S21 38.5	1.2	54.0
	19	104 41.5	25.2	262 54.6	11.7	21 37.3	1.3	54.0
	20	119 41.3	25.2	277 25.3	11.7	21 36.0	1.4	54.0
	21	134 41.2	. . 25.3	291 56.0	11.7	21 34.6	1.4	54.0
	22	149 41.1	25.3	306 26.7	11.7	21 33.2	1.6	54.0
	23	164 40.9	25.3	320 57.4	11.7	21 31.6	1.7	54.0
	19 00	179 40.8	N23 25.4	335 28.1	11.7	S21 29.9	1.8	54.0
	01	194 40.7	25.4	349 58.8	11.7	21 28.1	1.9	54.0
	02	209 40.5	25.5	4 29.5	11.8	21 26.2	2.0	54.0
	03	224 40.4	. . 25.5	19 00.3	11.7	21 24.2	2.1	54.0
	04	239 40.3	25.5	33 31.0	11.8	21 22.1	2.1	54.0
	05	254 40.1	25.6	48 01.8	11.8	21 20.0	2.3	54.0
	06	269 40.0	N23 25.6	62 32.6	11.8	S21 17.7	2.4	54.0
	07	284 39.9	25.6	77 03.4	11.8	21 15.3	2.5	54.0
	08	299 39.7	25.6	91 34.2	11.8	21 12.8	2.6	54.0
M	09	314 39.6	. . 25.7	106 05.0	11.9	21 10.2	2.6	54.0
O	10	329 39.5	25.7	120 35.9	11.9	21 07.6	2.8	54.0
N	11	344 39.3	25.7	135 06.8	11.8	21 04.8	2.9	54.0
D	12	359 39.2	N23 25.8	149 37.6	11.9	S21 01.9	2.9	54.0
A	13	14 39.0	25.8	164 08.5	12.0	20 59.0	3.1	54.0
Y	14	29 38.9	25.8	178 39.5	11.9	20 55.9	3.2	54.0
	15	44 38.8	. . 25.8	193 10.4	12.0	20 52.7	3.2	54.1
	16	59 38.6	25.9	207 41.4	12.0	20 49.5	3.4	54.1
	17	74 38.5	25.9	222 12.4	12.0	20 46.1	3.4	54.1
	18	89 38.4	N23 25.9	236 43.4	12.0	S20 42.7	3.6	54.1
	19	104 38.2	25.9	251 14.4	12.0	20 39.1	3.6	54.1
	20	119 38.1	26.0	265 45.4	12.1	20 35.5	3.8	54.1
	21	134 38.0	. . 26.0	280 16.5	12.1	20 31.7	3.8	54.1
	22	149 37.8	26.0	294 47.6	12.1	20 27.9	3.9	54.1
	23	164 37.7	26.0	309 18.7	12.1	S20 24.0	4.0	54.1
		SD 15.8	d 0.0	SD 14.7		14.7		14.7

Lat.	Twilight Naut.	Twilight Civil	Sunrise	Moonrise 17	Moonrise 18	Moonrise 19	Moonrise 20
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	■	■	■	■
N 70	□	□	□	■	■	■	■
68	□	□	□	24 19	00 19	00 52	00 53
66	□	□	□	23 05	23 49	24 12	00 12
64	////	////	01 31	22 28	23 14	23 44	24 02
62	////	////	02 09	22 02	22 49	23 22	23 45
60	////	00 50	02 35	21 42	22 29	23 04	23 30
N 58	////	01 40	02 56	21 25	22 12	22 50	23 18
56	////	02 10	03 13	21 11	21 58	22 37	23 07
54	00 46	02 32	03 27	20 59	21 46	22 26	22 58
52	01 32	02 50	03 39	20 48	21 36	22 16	22 50
50	02 00	03 06	03 50	20 38	21 26	22 07	22 42
45	02 46	03 35	04 13	20 18	21 06	21 49	22 26
N 40	03 16	03 58	04 31	20 02	20 50	21 34	22 13
35	03 39	04 16	04 46	19 48	20 36	21 21	22 01
30	03 58	04 31	04 59	19 36	20 24	21 10	21 51
20	04 27	04 56	05 21	19 15	20 04	20 50	21 34
N 10	04 50	05 17	05 40	18 58	19 46	20 34	21 19
0	05 09	05 35	05 57	18 41	19 30	20 18	21 05
S 10	05 26	05 52	06 15	18 25	19 13	20 02	20 51
20	05 42	06 09	06 33	18 07	18 55	19 45	20 36
30	05 58	06 28	06 55	17 46	18 35	19 26	20 19
35	06 07	06 39	07 07	17 35	18 23	19 15	20 09
40	06 17	06 51	07 21	17 21	18 09	19 02	19 57
45	06 27	07 04	07 38	17 05	17 53	18 47	19 44
S 50	06 39	07 20	07 59	16 45	17 33	18 28	19 27
52	06 44	07 28	08 09	16 36	17 24	18 19	19 19
54	06 50	07 36	08 20	16 25	17 13	18 09	19 11
56	06 56	07 45	08 33	16 13	17 01	17 57	19 01
58	07 03	07 55	08 47	15 59	16 47	17 44	18 50
S 60	07 10	08 07	09 05	15 43	16 31	17 29	18 37

Lat.	Sunset	Twilight Civil	Twilight Naut.	Moonset 17	Moonset 18	Moonset 19	Moonset 20
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	■	■	■	■
N 70	□	□	□	■	■	■	■
68	□	□	□	01 15	01 25	02 34	04 12
66	□	□	□	02 07	02 40	03 37	04 53
64	22 32	////	////	02 39	03 17	04 12	05 21
62	21 53	////	////	03 03	03 43	04 37	05 43
60	21 27	23 13	////	03 22	04 03	04 56	06 00
N 58	21 07	22 22	////	03 38	04 20	05 13	06 14
56	20 50	21 52	////	03 51	04 34	05 27	06 27
54	20 35	21 30	23 17	04 03	04 47	05 38	06 37
52	20 23	21 12	22 30	04 13	04 57	05 49	06 47
50	20 12	20 57	22 03	04 22	05 07	05 58	06 56
45	19 50	20 27	21 17	04 42	05 27	06 18	07 14
N 40	19 32	20 04	20 46	04 57	05 44	06 34	07 28
35	19 17	19 46	20 23	05 11	05 57	06 48	07 41
30	19 04	19 31	20 04	05 22	06 09	06 59	07 51
20	18 41	19 06	19 35	05 42	06 30	07 19	08 10
N 10	18 22	18 46	19 13	06 00	06 48	07 37	08 26
0	18 05	18 27	18 54	06 16	07 05	07 53	08 41
S 10	17 47	18 10	18 37	06 32	07 21	08 09	08 56
20	17 29	17 53	18 20	06 50	07 39	08 27	09 12
30	17 08	17 34	18 04	07 10	08 00	08 46	09 30
35	16 55	17 23	17 55	07 21	08 12	08 58	09 41
40	16 41	17 12	17 46	07 35	08 25	09 11	09 53
45	16 24	16 58	17 35	07 51	08 42	09 27	10 07
S 50	16 03	16 42	17 24	08 10	09 02	09 46	10 24
52	15 54	16 35	17 18	08 20	09 11	09 55	10 32
54	15 42	16 26	17 13	08 30	09 22	10 06	10 41
56	15 30	16 17	17 06	08 42	09 34	10 17	10 51
58	15 15	16 07	17 00	08 56	09 48	10 30	11 03
S 60	14 57	15 55	16 52	09 12	10 05	10 46	11 16

Day	SUN Eqn. of Time 00h	SUN Eqn. of Time 12h	SUN Mer. Pass.	MOON Mer. Pass. Upper	MOON Mer. Pass. Lower	MOON Age	MOON Phase
d	m s	m s	h m	h m	h m	d %	
17	00 51	00 57	12 01	00 04	12 29	15 100	○
18	01 04	01 10	12 01	00 53	13 17	16 98	
19	01 17	01 23	12 01	01 41	14 06	17 94	

UT	ARIES	VENUS −3.9		MARS +1.6		JUPITER −2.1		SATURN +0.2		STARS		
	GHA	GHA	Dec	GHA	Dec	GHA	Dec	GHA	Dec	Name	SHA	Dec
d h	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		° ′	° ′
20 00	268 30.6	177 04.1	N23 52.0	175 57.2	N24 12.8	212 47.0	N18 50.9	214 55.2	N17 10.1	Acamar	315 26.3	S40 18.1
01	283 33.0	192 03.2	52.0	190 57.8	12.8	227 48.9	51.0	229 57.4	10.1	Achernar	335 34.6	S57 13.9
02	298 35.5	207 02.3	52.1	205 58.4	12.8	242 50.8	51.2	244 59.6	10.2	Acrux	173 20.6	S63 06.3
03	313 38.0	222 01.4	. . 52.2	220 59.0	. . 12.8	257 52.7	. . 51.3	260 01.7	. . 10.3	Adhara	255 20.9	S28 58.5
04	328 40.4	237 00.5	52.2	235 59.7	12.8	272 54.6	51.4	275 03.9	10.3	Aldebaran	291 01.4	N16 30.5
05	343 42.9	251 59.6	52.3	251 00.3	12.7	287 56.5	51.5	290 06.1	10.4			
06	358 45.4	266 58.7	N23 52.3	266 00.9	N24 12.7	302 58.4	N18 51.6	305 08.3	N17 10.5	Alioth	166 29.4	N55 57.8
07	13 47.8	281 57.8	52.4	281 01.6	12.7	318 00.3	51.8	320 10.4	10.5	Alkaid	153 06.6	N49 19.0
T 08	28 50.3	296 56.9	52.4	296 02.2	12.7	333 02.2	51.9	335 12.6	10.6	Al Na'ir	27 56.2	S46 57.4
U 09	43 52.8	311 56.0	. . 52.5	311 02.8	. . 12.7	348 04.1	. . 52.0	350 14.8	. . 10.7	Alnilam	275 57.0	S 1 12.2
E 10	58 55.2	326 55.1	52.5	326 03.5	12.7	3 06.0	52.1	5 16.9	10.7	Alphard	218 06.3	S 8 39.6
S 11	73 57.7	341 54.2	52.6	341 04.1	12.6	18 07.9	52.3	20 19.1	10.8			
D 12	89 00.1	356 53.3	N23 52.6	356 04.7	N24 12.6	33 09.8	N18 52.4	35 21.3	N17 10.9	Alphecca	126 19.3	N26 43.0
A 13	104 02.6	11 52.4	52.7	11 05.3	12.6	48 11.7	52.5	50 23.5	10.9	Alpheratz	357 54.0	N29 05.3
Y 14	119 05.1	26 51.5	52.7	26 06.0	12.6	63 13.6	52.6	65 25.6	11.0	Altair	62 17.8	N 8 52.2
15	134 07.5	41 50.6	. . 52.8	41 06.6	. . 12.6	78 15.5	. . 52.7	80 27.8	. . 11.1	Ankaa	353 25.7	S42 18.1
16	149 10.0	56 49.8	52.8	56 07.2	12.6	93 17.4	52.9	95 30.0	11.1	Antares	112 38.4	S26 25.9
17	164 12.5	71 48.9	52.9	71 07.9	12.5	108 19.3	53.0	110 32.2	11.2			
18	179 14.9	86 48.0	N23 52.9	86 08.5	N24 12.5	123 21.3	N18 53.1	125 34.3	N17 11.2	Arcturus	146 04.8	N19 11.0
19	194 17.4	101 47.1	53.0	101 09.1	12.5	138 23.2	53.2	140 36.5	11.3	Atria	107 48.8	S69 01.7
20	209 19.9	116 46.2	53.0	116 09.8	12.5	153 25.1	53.4	155 38.7	11.4	Avior	234 22.7	S59 30.8
21	224 22.3	131 45.3	. . 53.0	131 10.4	. . 12.5	168 27.0	. . 53.5	170 40.8	. . 11.4	Bellatrix	278 43.2	N 6 20.9
22	239 24.8	146 44.4	53.1	146 11.0	12.4	183 28.9	53.6	185 43.0	11.5	Betelgeuse	271 12.6	N 7 24.3
23	254 27.2	161 43.5	53.1	161 11.7	12.4	198 30.8	53.7	200 45.2	11.6			
21 00	269 29.7	176 42.6	N23 53.1	176 12.3	N24 12.4	213 32.7	N18 53.8	215 47.4	N17 11.6	Canopus	264 01.2	S52 41.8
01	284 32.2	191 41.7	53.2	191 12.9	12.4	228 34.6	54.0	230 49.5	11.7	Capella	280 49.9	N45 59.8
02	299 34.6	206 40.8	53.2	206 13.6	12.4	243 36.5	54.1	245 51.7	11.8	Deneb	49 38.0	N45 16.8
03	314 37.1	221 39.9	. . 53.2	221 14.2	. . 12.3	258 38.4	. . 54.2	260 53.9	. . 11.8	Denebola	182 44.0	N14 34.3
04	329 39.6	236 39.0	53.3	236 14.8	12.3	273 40.3	54.3	275 56.1	11.9	Diphda	349 06.1	S17 59.1
05	344 42.0	251 38.1	53.3	251 15.5	12.3	288 42.2	54.4	290 58.2	12.0			
06	359 44.5	266 37.2	N23 53.3	266 16.1	N24 12.3	303 44.1	N18 54.6	306 00.4	N17 12.0	Dubhe	194 04.2	N61 45.3
W 07	14 47.0	281 36.3	53.4	281 16.7	12.2	318 46.0	54.7	321 02.6	12.1	Elnath	278 25.8	N28 36.4
E 08	29 49.4	296 35.4	53.4	296 17.4	12.2	333 47.9	54.8	336 04.8	12.1	Eltanin	90 50.3	N51 29.4
D 09	44 51.9	311 34.5	. . 53.4	311 18.0	. . 12.2	348 49.8	. . 54.9	351 06.9	. . 12.2	Enif	33 56.9	N 9 52.5
N 10	59 54.4	326 33.6	53.4	326 18.6	12.2	3 51.7	55.1	6 09.1	12.3	Fomalhaut	15 35.0	S29 37.1
E 11	74 56.8	341 32.7	53.5	341 19.2	12.1	18 53.6	55.2	21 11.3	12.3			
12	89 59.3	356 31.8	N23 53.5	356 19.9	N24 12.1	33 55.5	N18 55.3	36 13.5	N17 12.4	Gacrux	172 12.2	S57 07.1
S 13	105 01.7	11 30.9	53.5	11 20.5	12.1	48 57.4	55.4	51 15.6	12.5	Gienah	176 02.7	S17 32.7
D 14	120 04.2	26 30.0	53.5	26 21.1	12.1	63 59.3	55.5	66 17.8	12.5	Hadar	149 02.0	S60 22.6
A 15	135 06.7	41 29.1	. . 53.5	41 21.8	. . 12.0	79 01.2	. . 55.7	81 20.0	. . 12.6	Hamal	328 12.4	N23 27.6
Y 16	150 09.1	56 28.2	53.5	56 22.4	12.0	94 03.1	55.8	96 22.2	12.7	Kaus Aust.	83 56.9	S34 23.0
17	165 11.6	71 27.3	53.6	71 23.0	12.0	109 05.0	55.9	111 24.3	12.7			
18	180 14.1	86 26.4	N23 53.6	86 23.7	N24 12.0	124 06.9	N18 56.0	126 26.5	N17 12.8	Kochab	137 18.8	N74 09.6
19	195 16.5	101 25.6	53.6	101 24.3	11.9	139 08.8	56.1	141 28.7	12.9	Markab	13 48.4	N15 12.3
20	210 19.0	116 24.7	53.6	116 24.9	11.9	154 10.7	56.3	156 30.9	12.9	Menkar	314 25.9	N 4 05.3
21	225 21.5	131 23.8	. . 53.6	131 25.6	. . 11.9	169 12.6	. . 56.4	171 33.0	. . 13.0	Menkent	148 19.4	S36 22.4
22	240 23.9	146 22.9	53.6	146 26.2	11.8	184 14.5	56.5	186 35.2	13.0	Miaplacidus	221 42.5	S69 43.4
23	255 26.4	161 22.0	53.6	161 26.8	11.8	199 16.4	56.6	201 37.4	13.1			
22 00	270 28.9	176 21.1	N23 53.6	176 27.5	N24 11.8	214 18.3	N18 56.7	216 39.6	N17 13.2	Mirfak	308 55.3	N49 51.5
01	285 31.3	191 20.2	53.6	191 28.1	11.8	229 20.2	56.9	231 41.7	13.2	Nunki	76 10.5	S26 17.7
02	300 33.8	206 19.3	53.6	206 28.7	11.7	244 22.2	57.0	246 43.9	13.3	Peacock	53 34.7	S56 43.8
03	315 36.2	221 18.4	. . 53.6	221 29.4	. . 11.7	259 24.1	. . 57.1	261 46.1	. . 13.4	Pollux	243 40.5	N28 01.5
04	330 38.7	236 17.5	53.6	236 30.0	11.7	274 26.0	57.2	276 48.3	13.4	Procyon	245 10.7	N 5 13.4
05	345 41.2	251 16.6	53.6	251 30.6	11.6	289 27.9	57.3	291 50.4	13.5			
06	0 43.6	266 15.7	N23 53.6	266 31.3	N24 11.6	304 29.8	N18 57.5	306 52.6	N17 13.6	Rasalhague	96 15.5	N12 33.7
07	15 46.1	281 14.8	53.6	281 31.9	11.6	319 31.7	57.6	321 54.8	13.6	Regulus	207 54.5	N11 58.0
T 08	30 48.6	296 13.9	53.6	296 32.5	11.5	334 33.6	57.7	336 57.0	13.7	Rigel	281 22.1	S 8 12.2
H 09	45 51.0	311 13.0	. . 53.6	311 33.2	. . 11.5	349 35.5	. . 57.8	351 59.1	. . 13.7	Rigil Kent.	140 05.3	S60 50.3
U 10	60 53.5	326 12.1	53.6	326 33.8	11.5	4 37.4	57.9	7 01.3	13.8	Sabik	102 23.8	S15 43.4
R 11	75 56.0	341 11.2	53.6	341 34.5	11.4	19 39.3	58.1	22 03.5	13.9			
S 12	90 58.4	356 10.3	N23 53.6	356 35.1	N24 11.4	34 41.2	N18 58.2	37 05.7	N17 13.9	Schedar	349 52.2	N56 32.0
D 13	106 00.9	11 09.4	53.6	11 35.7	11.4	49 43.1	58.3	52 07.8	14.0	Shaula	96 35.3	S37 06.2
14	121 03.3	26 08.5	53.6	26 36.4	11.3	64 45.0	58.4	67 10.0	14.1	Sirius	258 43.0	S16 43.1
A 15	136 05.8	41 07.6	. . 53.6	41 37.0	. . 11.3	79 46.9	. . 58.5	82 12.2	. . 14.1	Spica	158 41.8	S11 09.8
Y 16	151 08.3	56 06.7	53.6	56 37.6	11.3	94 48.8	58.7	97 14.4	14.2	Suhail	223 00.2	S43 26.2
17	166 10.7	71 05.8	53.5	71 38.3	11.2	109 50.7	58.8	112 16.5	14.2			
18	181 13.2	86 04.9	N23 53.5	86 38.9	N24 11.2	124 52.6	N18 58.9	127 18.7	N17 14.3	Vega	80 45.4	N38 47.1
19	196 15.7	101 04.0	53.5	101 39.5	11.2	139 54.5	59.0	142 20.9	14.4	Zuben'ubi	137 16.4	S16 02.6
20	211 18.1	116 03.1	53.5	116 40.2	11.1	154 56.4	59.1	157 23.1	14.4		SHA	Mer. Pass.
21	226 20.6	131 02.2	. . 53.5	131 40.8	. . 11.1	169 58.4	. . 59.3	172 25.2	. . 14.5		° ′	h m
22	241 23.1	146 01.3	53.4	146 41.4	11.0	185 00.3	59.4	187 27.4	14.6	Venus	267 12.9	12 14
23	256 25.5	161 00.4	53.4	161 42.1	11.0	200 02.2	59.5	202 29.6	14.6	Mars	266 42.6	12 15
Mer. Pass	h m 6 01.0	v −0.9	d 0.0	v 0.6	d 0.0	v 1.9	d 0.1	v 2.2	d 0.1	Jupiter	304 02.9	9 45
										Saturn	306 17.7	9 35