Module 6

Modern Navigation Systems

Orbit Propagators

Module 6B
Using an orbit propagator

Summary of Module 6

- Students will combine the perturbation analysis and position determination skills from previous modules into a numerical orbit propagator that can be used to predict the effects of thrust and delta-velocity (delta-V) on the position and velocity of an object orbiting the earth or other body, including the effects of inhomogeneous gravitational fields, drag, and the gravitational forces of multiple moons and planets.
- The study of the ocean tides will be used as an example of a three-body problem that requires a numerical orbit propagator. The world's largest mechanical tide computer, located 12 miles from JHU/APL at the NOAA building in Silver Spring, MD will be used as an example of a mechanical computer for performing orbital computations.

The Excel Spreadsheet Orbits525_445

- An Excel spreadsheet entitled Orbits525_445.xls is posted in this module.
- It implements the equations developed in Module 6A for a satellite orbiting the earth.
- Note how the various units have been scaled, particularly with respect to how the gravitational constant g is used when the distance from the satellite to the earth is changed.
- To use the spreadsheet, change any of the entries in row 2, which are identified using easily understood variable names from earlier slides and modules.
- Save the spreadsheet under different names after making changes. Keep the original file intact in case you make entry errors in subsequent versions of the file.

Key orbital features that can be simulated with the Excel propagator

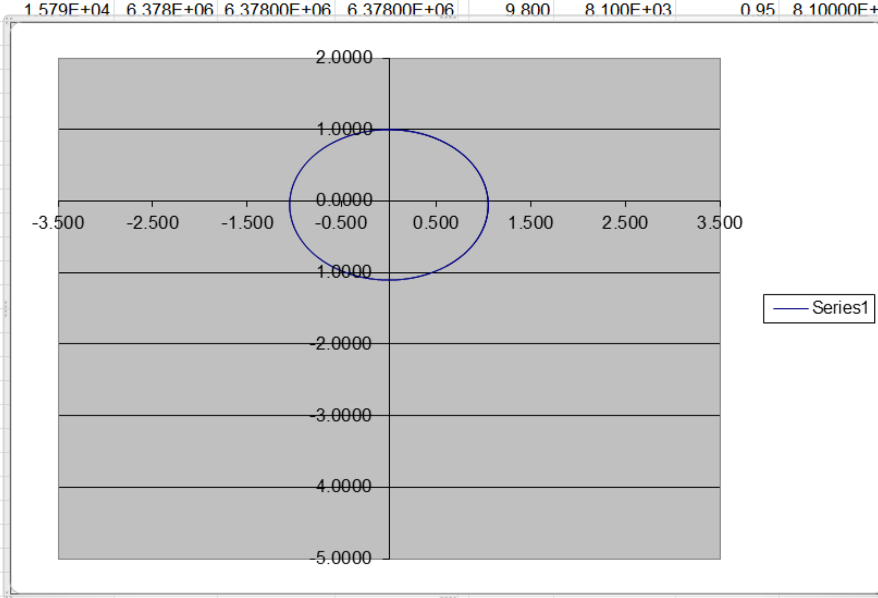
- A circular orbit can be made elliptical by increasing the velocity at perigee to boost the altitude at apogee.
- A second boost at apogee can re-circularize the orbit, but its apogee radius.
  - This two stage operation is used to place satellites in geostationary orbit when they had been released by the space shuttle in low earth orbit
  - This is called a geostationary transfer orbit.
- A velocity decrease using “retro-rockets” permits an orbiting vehicle to de-orbit
  - This is done for manned flight to permit re-entry, or to dump an unused satellite (such as NASA’s Skylab module) into the Ocean or the Australian outback. See http://www.spaceref.com/iss/skylab.deorbit.html
- Note that the de-orbiting process is an “inexact” science, due to atmospheric drag and friction
  - Mercury 7, Gemini VIII, Apollo 13, and STS Columbia are the classic re-entry emergency stories from NASA history
- For moving a satellite to a different “slot” in the same orbit, one can change the altitude using thrusters or a brief engine burn to change the orbital period until the spacecraft aligns with the proper value of anomaly in the orbit, at which time the maneuver is reversed.
- All of Kepler’s laws can be demonstrated;
  - Ellipticity of an orbit
  - Equal areas
  - T^2 versus r^3

To make changes:

- Because of the manner in which the cells are coded, it is best to make changes only to the top row of the spread sheet, and dovetail graphs from several versions of the spread sheet together
- For example, to boost altitude at a particular point in an orbit, adjust the starting point and the new altitude at the beginning of a new simulation
- To compensate for Kepler's laws, adjust the time interval per row, rather than the number of rows
    - Note, however, that the accuracy off the simulator changes as the time increment is changed
        - This is why the orbits have a tendency to "spiral" in with time
- Cut and paste the graphs using snippet or paint.

Using the Orbit Propagator

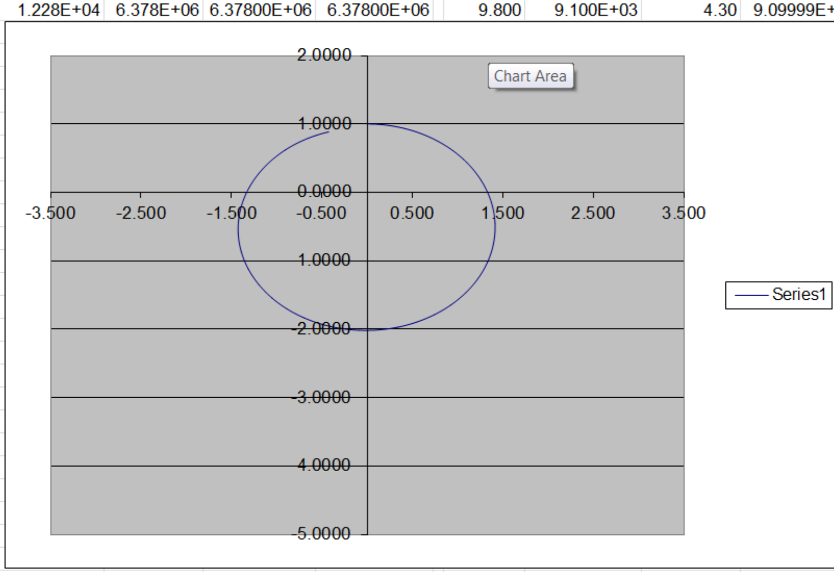
	A	B	C	D	E	F	G	H	I	J	K	L
1		Δt	x_0	y_0	r_0			g_{earth}	u_0	v_0	r_e	
2		0.65	0.000E+00	6.378E+06	6.378E+06			9.800	8.100E+03	0.00	6.378E+06	
3	n	time	x_n	y_n	r_n			g_n	u_n	v_n	$u_{(n+1)}$	$v_{(n+1)}$
4	0	0	0.000E+00	6.378E+06	6.37800E+06	6.37800E+06		9.800	8.100E+03	0.00	8.10000E+03	-6.370E+00
5	1	0.65	5.265E+03	6.378E+06	6.37800E+06	6.37800E+06		9.800	8.100E+03	0.32	8.10000E+03	-6.054E+00
6	2	1.3	1.053E+04	6.378E+06	6.37800E+06	6.37800E+06		9.800	8.100E+03	0.63	8.10000E+03	-5.738E+00
7	3	1.95	1.579E+04	6.378E+06	6.37800E+06	6.37800E+06		9.800	8.100E+03	0.95	8.10000E+03	-5.422E+00
8	4	2.6									03	-5.105E+00
9	5	3.25									03	-4.789E+00
10	6	3.9									03	-4.473E+00
11	7	4.55									03	-4.157E+00
12	8	5.2									03	-3.841E+00
13	9	5.85									03	-3.524E+00
14	10	6.5									03	-3.208E+00
15	11	7.15									03	-2.892E+00
16	12	7.8									03	-2.576E+00
17	13	8.45									03	-2.260E+00
18	14	9.1									03	-1.943E+00
19	15	9.75									03	-1.627E+00
20	16	10.4									03	-1.311E+00
21	17	11.05									03	-9.948E-01
22	18	11.7									03	-6.786E-01
23	19	12.35									03	-3.624E-01
24	20	13									03	-4.625E-02
25	21	13.65									03	2.699E-01
26	22	14.3									03	5.861E-01
27	23	14.95									03	9.023E-01
28	24	15.6									03	1.218E+00
29	25	16.25									03	1.535E+00
30	26	16.9									03	1.851E+00
31	27	17.55									03	2.167E+00
32	28	18.2	1.474E+05	6.376E+06	6.37808E+06	6.37808E+06		9.800	8.100E+03	8.85	8.09990E+03	2.483E+00
33	29	18.85	1.527E+05	6.376E+06	6.37809E+06	6.37809E+06		9.800	8.100E+03	9.17	8.09989E+03	2.799E+00



Note the almost circular orbit when $u_0 = v_{x0} =$ 8.1 km/sec

Increasing the velocity

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Δt	x_0	y_0	r_0			g_{earth}	u_0	v_0	r_e		
2		0.45	0.000E+00	6.378E+06	6.378E+06			9.800	9.100E+03	0.00	6.378E+06		
3	n	time	x_n	y_n	r_n			g_n	u_n	v_n	$u_{(n+1)}$	$v_{(n+1)}$	L4/k4
4	0	0	0.000E+00	6.378E+06	6.37800E+06	6.37800E+06		9.800	9.100E+03	0.00	9.10000E+03	-4.410E+00	0.000
5	1	0.45	4.095E+03	6.378E+06	6.37800E+06	6.37800E+06		9.800	9.100E+03	1.43	9.10000E+03	-2.978E+00	0.000
6	2	0.9	8.190E+03	6.378E+06	6.37800E+06	6.37800E+06		9.800	9.100E+03	2.86	9.10000E+03	-1.545E+00	0.000
7	3	1.35	1.228E+04	6.378E+06	6.37800E+06	6.37800E+06		9.800	9.100E+03	4.30	9.09999E+03	-1.128E-01	0.000
8	4	1.8									03	1.320E+00	0.000
9	5	2.25									03	2.752E+00	0.000
10	6	2.7									03	4.185E+00	0.000
11	7	3.15									03	5.617E+00	0.001
12	8	3.6									03	7.049E+00	0.001
13	9	4.05									03	8.482E+00	0.001
14	10	4.5									03	9.914E+00	0.001
15	11	4.95									03	1.135E+01	0.001
16	12	5.4									03	1.278E+01	0.001
17	13	5.85									03	1.421E+01	0.002
18	14	6.3									03	1.564E+01	0.002
19	15	6.75									03	1.708E+01	0.002
20	16	7.2									03	1.851E+01	0.002
21	17	7.65									03	1.994E+01	0.002
22	18	8.1									03	2.137E+01	0.002
23	19	8.55									03	2.281E+01	0.003
24	20	9									03	2.424E+01	0.003
25	21	9.45									03	2.567E+01	0.003
26	22	9.9									03	2.710E+01	0.003
27	23	10.35									03	2.853E+01	0.003
28	24	10.8									03	2.997E+01	0.003
29	25	11.25									03	3.140E+01	0.003
30	26	11.7									03	3.283E+01	0.004
31	27	12.15									03	3.426E+01	0.004
32	28	12.6	1.147E+05	6.377E+06	6.37825E+06	6.37825E+06		9.800	9.100E+03	40.11	9.09964E+03	3.570E+01	0.004
33	29	13.05	1.187E+05	6.377E+06	6.37827E+06	6.37827E+06		9.800	9.100E+03	41.54	9.09961E+03	3.713E+01	0.004
34	30	13.5	1.228E+05	6.377E+06	6.37829E+06	6.37829E+06		9.800	9.100E+03	42.97	9.09958E+03	3.856E+01	0.004
35	31	13.95	1.269E+05	6.377E+06	6.37831E+06	6.37831E+06		9.799	9.100E+03	44.40	9.09956E+03	3.999E+01	0.004



Note how the orbit becomes elliptical when u_0 is increased.

What about the tides?

- An orbit propagator permits one to have multiple sources of gravity
- The earth's ocean tides are dominated by the gravity of the sun and the moon
- There is a repeat cycle of 19.2 years
- Tidal heights versus time look sinusoidal, with two high and two low tides per day
- Tidal effects aren't limited to the oceans
  - Cracks in rock formations can be traced to tidal stresses

Tide simulators

- Tides can be predicted using a variety of input data based on actual measurements to set a large number (~30) parameters
- The world’s largest mechanical tide computer is at NOAA in Silver Spring

The tides…

UNDERSTANDING TIDES

by

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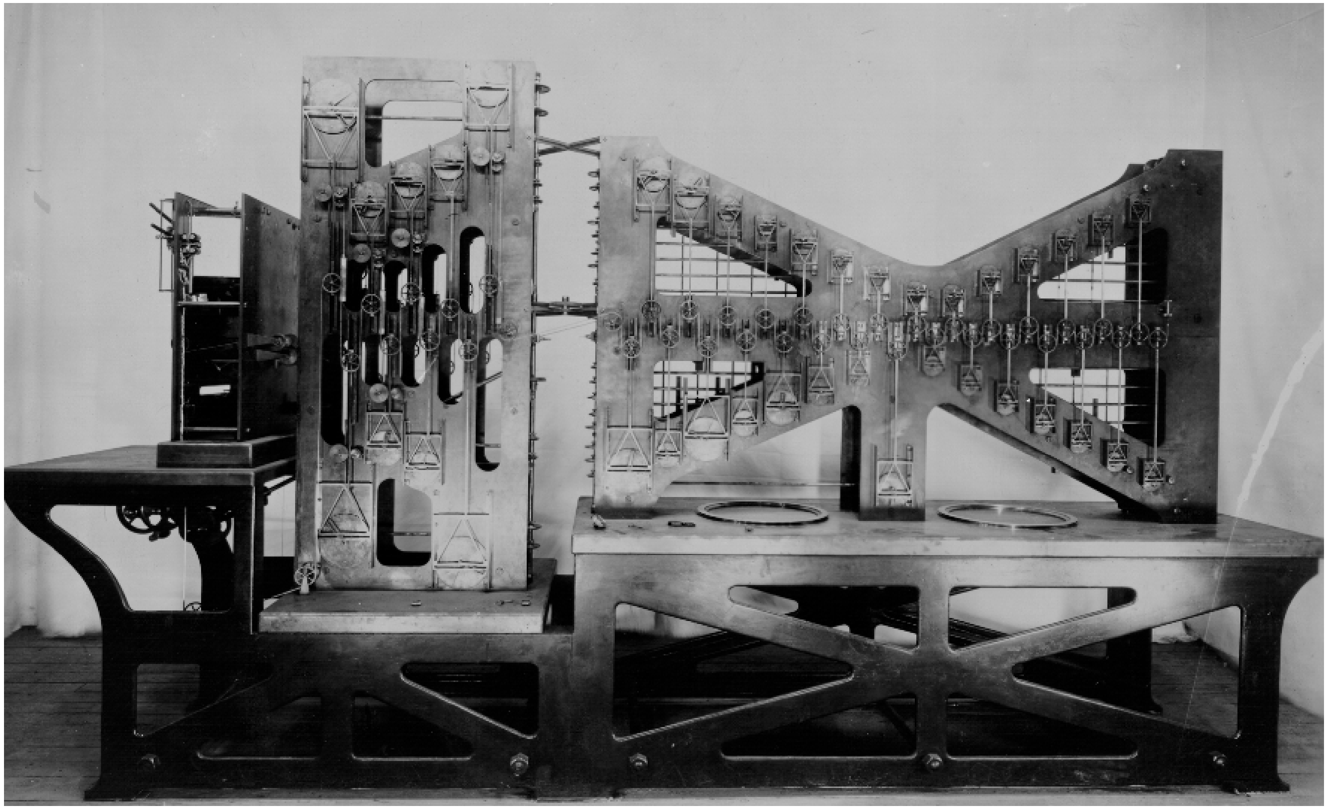
U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Ocean Service

NOAA's role in tide prediction

Tide predictions by this organization began in 1844 using the lunitidal interval method. This method involved the time interval from the transit of the moon over the local meridian to the first high tide. Time adjustments were made for the phases of the moon in order to take into account some of the solar effects. In 1867 the harmonic method was introduced. The tedious additions of the cosine curves encouraged the invention of the mechanical Maxima and Minima Tide Predictor of William Ferrel in 1885. This machine summed 19 constituents. It is now in the National Museum of American History of the Smithsonian Institution. A second machine, developed by Rollin A. Harris and E. G. Fischer, summed 37 constituents and was used from 1912 through 1965. This one is now located in the NOS facilities in Silver Spring, MD, along with the first Tide Tables (1867) and other items of interest in the history of tides and tidal currents.

NOAA Tide Predicting Machine #2



Assignment 6.2

6.2.1 Use the Excel spread sheet to plot graphs, which should be pasted into a single Word or pdf file, that illustrate all of the features listed in slide 4 of this sub-module

6.2.2 Explain why the moon's orbital period is 28 days, in apparent violation of Kepler's third law.

End of Mod 6B

End of Module 6