Module 5

Modern Navigation Systems

Position determination of satellites, planets, and stars in earth-centered coordinate systems

Module 5B

Satellite Position Determination from Satellite Orbital Parameters

Summary of Module 5

- Students will learn the features of, and similarities and differences between, the earth-centered inertial (ECI) and the earth-centered, earth fixed (ECEF) coordinate systems.
- Then, using actual orbital parameters from almanacs and ephemeris tables, students will learn to compute the positions in three dimensional space in both the ECI and ECEF frames of satellites, stars, planets, the sun, and the moon.
- The four equations that are used will form the basis for all the algorithms that follow.

An important consideration

- In this module, one's position on earth is known. Using orbital parameters, the relative position of the satellite, star, or other object is computed in terms of angles and distances.
- In future modules, the problem is reversed. Knowing the positions of satellites and stars, one's own position is computed.
    - When doing this using recursive techniques, the "forward" and "reverse" computations are often alternated in an iterative fashion.

The discussion that follows is based on the text by Richharia

Satellite Communication Systems

Design Principles

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(2nd ed. McGraw-Hill)

This text is out of print, but contains a particular nice appendix that details equations found in many texts, including in various parts of the course text.

From Richharia:

(9) **Satellite position from orbital parameters**

"…In the first step, satellite position is estimated in the orbital plane; the second step involves transforming the satellite coordinates to the three-dimensional earth-centred coordinate sys tem; finally, the earth-centred coordinates of the satellite are transformed to an earth-station-centred coordinate system for obtaining the look angle of the satellite from the earth station.

The following orbital parameters are assumed known: eccentricity , ascending node, inclination , mean anomaly at a reference time called epoch (mean anomaly = 0 if epoch is taken at perigee pass), and argument of perigee."

Orbital parameters for a typical GPS satellite

******** Week 801 almanac for PRN-01 ********
ID: 01
Health: 000
Eccentricity: 0.3765106201E-002
Time of Applicability(s): 503808.0000
Orbital Inclination(rad): 0.9617064849
Rate of Right Ascen(r/s): -0.7817468486E-008
SQRT(A) (m 1/2): 5153.614258
Right Ascen at Week(rad): 0.7017688714E+000
Argument of Perigee(rad): 0.434909394
Mean Anom(rad): 0.4480223834E+000
Af0(s): -0.1049041748E-004
Af1(s/s): 0.0000000000E+000
week: 801

These will be covered
In more detail in a future
Module.

Richharia, cont'd

Some useful relationships involving eccentric anomaly E, true anomaly ν and mean anomaly M are:

$$\cos E = \frac{\cos \nu + e}{1 + e \cos \nu} \tag{B.34}$$

$$\cos \nu = \frac{\cos E - e}{1 - e \cos E} \tag{B.35}$$

where e is the orbit eccentricity.

The mean anomaly M at time t is given by

$$M = M_0 + \dot{\omega}(t - t_0) \tag{B.36}$$

where M_0 is the mean anomaly at a reference time t_0 (epoch) and $\dot{\omega}$ is the angular velocity of the satellite.

Equations B.34 – 36 are in the course text, as well as in many other texts.

Note: ω *with a dot over it is used by Richharia to distinguish it from* ω *without a dot, which is the argument of perigee*

Equations, cont'd

Step 1

(a) The mean anomaly at the specified time is determined from equation (B.36).

(b) The eccentric anomaly is determined by solving Kepler's equation

$$M = E - e\sin E \tag{B.37}$$

At this point, use Newton's method to determine E from knowledge of M and e.

Finding the position of the satellite in the two-dimensional x_0, y_0 plane

(c) The position of the satellite in the orbital plane is given by

$$x_o = a(\cos E - e) \tag{B.40a}$$

$$y_o = a(1 - e^2)^{\frac{1}{2}} \sin E \tag{B.40b}$$

$$\text{radius, } r = (x_0^2 + y_0^2)^{\frac{1}{2}} \tag{B.40c}$$

Where a is the length of the semi-major axis of the orbit.

Transforming from the orbital plane into the earth centered coordinate system

Step 2

The inclination of the satellite, the right ascension of the ascending node and the argument of perigee are used to transform the perifocal coordinate system to the geocentric equatorial coordinate system. The following equation set can be used for this transformation:

$$P_x = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i \tag{B.41a}$$

$$P_y = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i \tag{B.41b}$$

$$P_z = \sin\omega\sin i \tag{B.41c}$$

$$Q_x = -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i \tag{B.41d}$$

$$Q_y = -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i \tag{B.41e}$$

$$Q_z = \cos\omega\sin i \tag{B.41f}$$

These equations are also in chapter 2 of your text.

Using the direction cosines…

Satellite position in the geocentric coordinate system is given by

$$x = P_x x_o + Q_x y_o \tag{B.42a}$$

$$y = P_y x_o + Q_y y_o \tag{B.42b}$$

$$z = P_z x_o + Q_z y_o \tag{B.42c}$$

The (almost) final results

Finally, the following set of equations can be used to obtain satellite azimuth and elevation from a specified earth station:

$$\text{Right ascension, } \alpha = \arctan\left(y/x\right) \tag{B.43}$$

$$\text{Declination, } \delta = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \tag{B.44}$$

$$\text{Elevation, } \eta = \arctan\left(\frac{\sin\eta_s - \dfrac{R}{r}}{\cos\eta_s}\right) \tag{B.45}$$

where

$$\eta_s = \arcsin\left[\sin\delta\sin\theta_e + \cos\delta\cos\theta_e\cos\phi_{se}\right] \tag{B.46}$$

and R = Earth radius
r = satellite distance from Earth centre (use equation B.40c)
θ_e = earth station latitude
$\phi_{se} = \phi_s - \phi_e$
ϕ_s = satellite longitude
ϕ_e = earth station longitude.

Richharia's final equations

$$\text{Azimuth, } A = \arctan\left[\frac{\sin\phi_{se}}{\cos\theta_e \tan\delta - \sin\theta_e \cos\phi_{se}}\right] \qquad \text{(B.47)}$$

This is important

Use the convention given in chapter 2, section 2.6 to obtain the azimuth quadrant.

Range

The distance ρ of a satellite from a given point on the Earth is given as

$$\rho = \sqrt{r^2 - R^2\cos^2\eta} - R\sin\eta \qquad \text{(B.48)}$$

The inverse tangent rules

- True azimuth is given from A in the previous slide by:
- In the northern hemisphere:
  - $A_{true} = 180 + A$ when the satellite is west of the earth station
  - $A_{true} = 180 - A$ when the satellite is east of the earth station
- In the southern hemisphere:
  - $A_{true} = 360 - A$ when the satellite is west of the earth station
  - $A_{true} = 180 - A$ when the satellite is east of the earth station

The two-dimensional orbit

- Figures 5.4 and 5.5 from the text (the next slides) are repeated from the previous module.

Figure 5.4 from the text

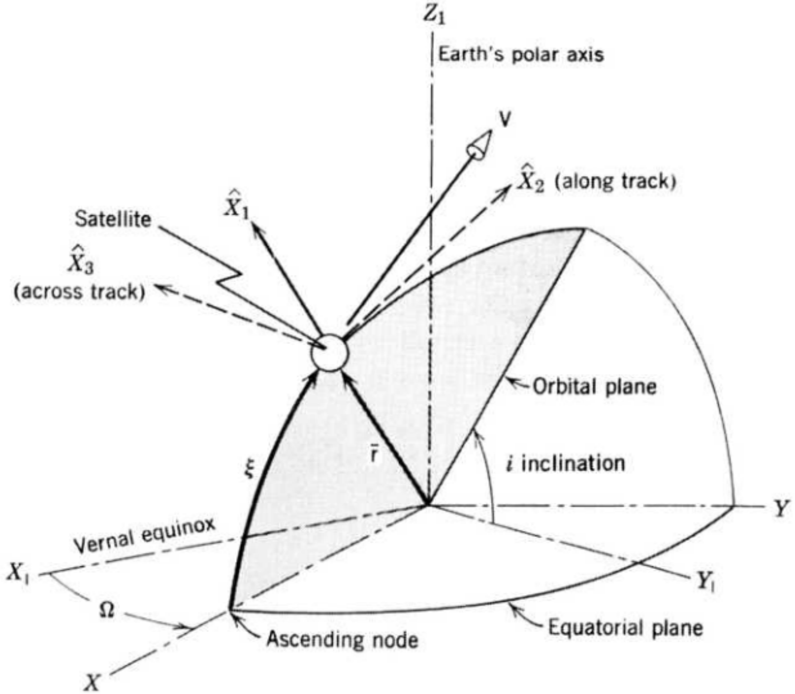

Figure 5.4 The orbital plane.

The two-dimensional orbit of the satellite around the Earth is shown separately In Figure 5.5 (the next slide).

The earth is at one of the focal points of the orbit.

Note: MatLab and other software packages get the angles wrong by 180 degrees, thus getting Kepler's 2nd Law "backwards".

Figure 5.5 from the text

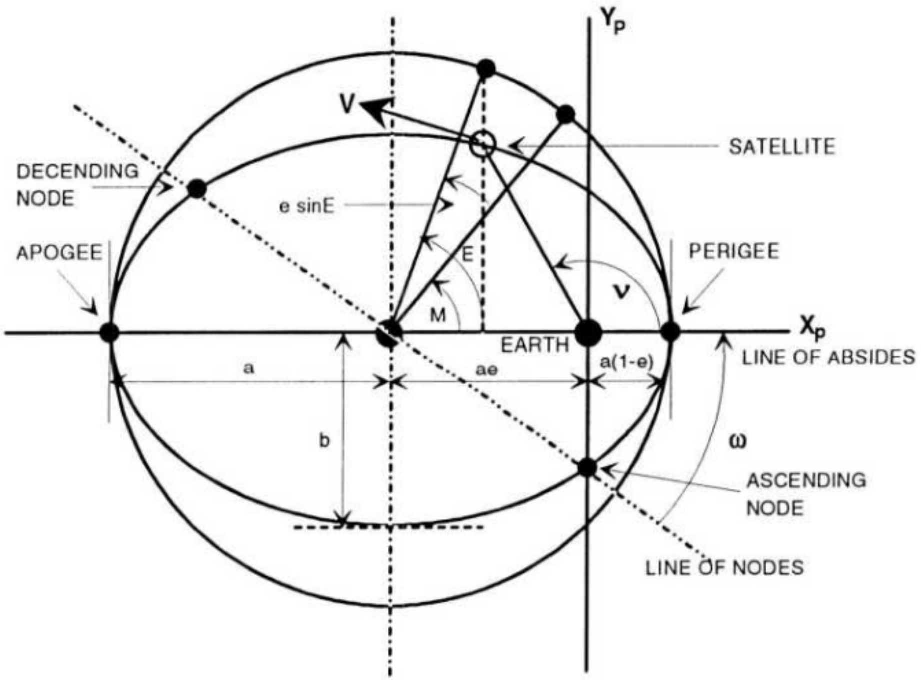

Figure 5.5 The elliptical orbit.

This rather cluttered diagram from the text depicts an elliptical orbit inscribed within a circle. The open circle shows the instantaneous position of a satellite within the elliptical orbit. Note that its position can be specified by the angle E, the *elliptic anomaly*.

The angle M is the *mean*, or average *anomaly*. It is easily computed. Then, E can be computed using the techniques developed in this module.

Assignment 5.2

5.2.1 Derive the look angles from 39N 77W to the geostationary DirectTV satellite at 119W. Check your results using http://www.directv.com/DTVAPP/customer/dishPointer.jsp

End of Mod 5B