Module 5

Modern Navigation Systems

Position determination of satellites, planets, and stars in earth-centered coordinate systems

Module 5C

The practical details of determining Satellite Position from Satellite Orbital Parameters

Summary of Module 5

- Students will learn the features of, and similarities and differences between, the earth-centered inertial (ECI) and the earth-centered, earth fixed (ECEF) coordinate systems.
- Then, using actual orbital parameters from almanacs and ephemeris tables, students will learn to compute the positions in three dimensional space in both the ECI and ECEF frames of satellites, stars, planets, the sun, and the moon.
- The four equations that are used will form the basis for all the algorithms that follow.

An important consideration

- As stated previously, we are assuming (for the time being) that one's position on earth is known. Using orbital parameters, the relative position of the satellite, star, or other object is then computed in terms of angles and distances.
- Getting this right involves several nuances that are not always obvious
- In this sub-module, we will review and elaborate on these "details"

Some useful nomenclature

- *Perigee* and *apogee* refer to orbits around the earth (cf. geoid)
- For orbits around the sun, the corresponding terms are *perihelion* and *aphelion* (cf. helios)
- For orbits around the moon, the terms are *pericynthion* and *apcynthion*

First, orbits are two-dimensional ellipses

- An ellipse is a two-dimensional geometric shape. It lies entirely within a two-dimensional plane.
    - The orientation of the ellipse is given by its semi-major and semi-minor axes
        - The x-axis is typically the semi-major axis
        - For a circle, this can be ambiguous.
    - The center of this x-y coordinate system is the center of the ellipse, not either of the focal points.
    - The mean and eccentric anomalies are referenced to the origin, whereas the true anomaly is referenced to the focal point around which the object of interest (i.e., the satellite) orbits

Second, there are three orbital parameters associated with the ellipse

- The length *a* of the semi-major axis
  - Sometimes expressed as $\sqrt{a}$ so as to require less storage space in a radiation-hardened memory chip onboard a satellite
- The eccentricity *e* of the orbit relates the semi-major axis *a* to the semi-minor length *b* by equation 5.9 from the text:

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2} \qquad (5.9)$$

- The time of perigee passage t_p, given by equation 5.12

$$M = n(t - t_p) \qquad (5.12)$$

The analysis begins with the Mean anomaly, or mean motion

The mean motion is the average angular rate of the satellite radius vector $\bar{\mathbf{r}}$. It is defined as

$$n = \sqrt{\frac{\mu}{a^3}} \tag{5.10}$$

in radians per second, where $\mu = 3.98605 \times 10^{14}$ meters^3/sec^2 is the WGS 84 value of the Earth's universal gravitational parameter [7, 8]. The period of the orbit is then

$$T_p = \frac{2\pi}{n} \tag{5.11}$$

in seconds. The mean motion is used to compute the mean anomaly

$$M = n(t - t_p) \tag{5.12}$$

Some definitions

- WGS-84 means World Geodetic Survey, 1984, which defines the ECEF coordinate system to account for the oblateness of the earth
  - o Since the earth is neither spherical nor homogeneous in terms of its mass distribution, its gravitational field is non-uniform.
  - o WGS provides the single, agreed coordinate frame for defining the local perturbations to the uniform gravitational field in detail

Why gravity matters

- Perturbations to a uniform field cause satellites to drift in their orbits
  - o Specifically, this causes some of the orbital parameters to vary with time
  - o These variations must be modelled accurately in order to predict, for example, the instantaneous location of GPS satellites
    - o Without accurate knowledge of the location of each satellite, determination of the position of a GPS receiver is not possible
    - o Read the text material on GPS to understand this in detail

Orbital parameters for a typical GPS satellite

******** Week 801 almanac for PRN-01 ********
ID: 01
Health: 000
Eccentricity: 0.3765106201E-002
Time of Applicability(s): 503808.0000
Orbital Inclination(rad): 0.9617064849
Rate of Right Ascen(r/s): -0.7817468486E-008
SQRT(A) (m 1/2): 5153.614258
Right Ascen at Week(rad): 0.7017688714E+000
Argument of Perigee(rad): 0.434909394
Mean Anom(rad): 0.4480223834E+000
Af0(s): -0.1049041748E-004
Af1(s/s): 0.0000000000E+000
week: 801

Note that the right ascension varies with time!

Note that sqrt(a), rather Than a, is provided!

These terms refer to drift of the atomic clocks onboard each GPS satellite.

Fourier Theory

- Most applications of Fourier theory involve sines and cosines
- Bessel functions are used for the same purpose in cylindrical geometries
  - e.g., for modelling waveguide modes in cylindrical optical fibers
- Legendre polynomials are used for spherically symmetric systems, such as atoms (s, p, d, and f orbitals), biconical antennas, and perturbations to the earth’s gravitational field.
  - Satellite Toolkit, a commercial product, includes highly accurate gravitational models
  - This will be covered in more detail in the module on orbit propagators

Returning to the mean motion…

- Computation of *M* from knowledge of the orbital period *T* is straightforward
- *E* is then computed
- $x_0(t)$ and $y_0(t)$, the instantaneous positions of the satellite in the orbital ellipse, are computed next

Transforming to the ECEF coordinate system…

- Right ascension Ω, inclination i, and argument of perigee ω (e.g., yaw, pitch, and roll) are used to transform the position x_0, y_0 into an earth-centered three dimensional coordinate system
- Note that inclination is always positive, from 0 – 90 degrees
  - o “negative” inclination is accomplished by adding 180 degrees to the value of right ascension

Coordinate system definitions

1. *Earth-centered, Earth -fixed (ECEF)*. The basic coordinate frame for navigation near the Earth is ECEF, shown in Figure 2.3 as the y_i rectangular coordinates whose origin is at the mass center of the Earth, whose y_3-axis lies along the Earth's spin axis, whose y_1 axis lies in the Greenwich meridian, and which rotates with the Earth [10]. Satellite-based radio-navigation systems often use these ECEF coordinates to calculate satellite and aircraft positions.
2. *Earth-centered inertial (ECI)*. ECI coordinates, x_i, can have their origin at the mass-center of any freely falling body (e.g., the Earth) and are nonrotating relative to the fixed stars. For centuries, astronomers have observed the small relative motions of stars ("proper motion") and have defined an "average" ECI reference frame [1 1]. To an accuracy of 10^{-5} deg /hr, an ECI frame can be chosen with its x_3-axis along the mean polar axis of the Earth and with its x_1- and x_2-axes pointing to convenient stars (as explained in Chapter 12). ECI coordinates have three navigational functions. First, Newton's laws are valid in any ECI coordinate frame. Second, the angular coordinates of stars are conventionally tabulated in ECI. Third, they are used in mechanizing inertial navigators, Section 7.5.1.

From the text, chapter 2.

Surface-based coordinate systems

3. *Geodetic spherical coordinates*. These are the spherical coordinates of the normal to the reference ellipsoid (Figure 2.2). The symbol z_1 represents longitude l; z_2 is geodetic latitude F $_T$, and z_3 is altitude h above the reference ellipsoid. Geodetic coordinates are used on maps and in the mechanization of dead-reckoning and radio-navigation systems. Transformations from ECEF to geodetic spherical coordinates are given in [9] and [23].

4. *Geodetic wander azimuth*. These coordinates are locally level to the reference ellipsoid. $\hat{z}_3$ is vertically up and $\hat{z}_2$ points at an angle, a, west of true north (Figure 2.3). The wander-azimuth unit vectors, z_1 and z_2, are in the level plane but do not point east and north. Wander azimuth is the most commonly used coordinate frame for worldwide inertial navigation and is discussed below and in Section 7.5.1.

5. *Direction cosines*. The orientation of any z-coordinate frame (e.g., navigation coordinates or body axes) can be described by its direction cosines relative to ECEF y-axes. Any vector **V** can be resolved into either the y or z-coordinate frame. The y and z components of **V** are related by the equation

From the text, chapter 2.

Transforming to a coordinate system centered on the earth's surface

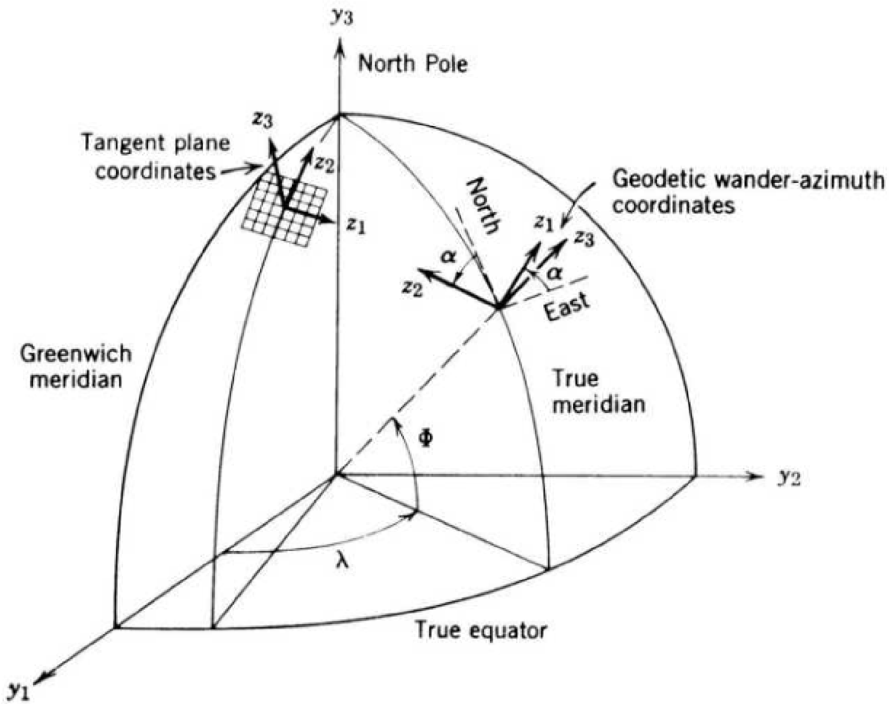

Figure 2.3 Navigation coordinate frames.

Tangent and wander-azimuth coordinates
on the surface of the earth,
from the text, chapter 2.

Direction cosine transformations

$$[\mathbf{V}]_z = [C_{zy}][\mathbf{V}]_y \tag{2.7}$$

where

$$
\begin{aligned}
C_{11} &= -\cos a \sin l - \sin a \sin F \cos l \\
C_{12} &= \cos a \cos l - \sin a \sin F \sin l \\
C_{13} &= \cos F \sin a \\
C_{21} &= \sin a \sin l - \sin F \cos a \cos l \\
C_{22} &= -\sin a \cos l - \cos a \sin F \sin l \\
C_{23} &= \cos F \cos a \\
C_{31} &= \cos F \cos l \\
C_{32} &= \cos F \sin l \\
C_{33} &= \sin F
\end{aligned} \tag{2.8}
$$

The navigation computer calculates in terms of the C_{ij}, which are usable everywhere on Earth. The familiar geographic coordinates can be found from the relations

$$\sin F = C_{33},$$

F is the latitude of the “observer” position on earth. See the text for more details.

The (almost) final results (from Richharia)

Finally, the following set of equations can be used to obtain satellite azimuth and elevation from a specified earth station:

$$\text{Right ascension, } \alpha = \arctan\left(y/x\right) \tag{B.43}$$

$$\text{Declination, } \delta = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \tag{B.44}$$

$$\text{Elevation, } \eta = \arctan\left(\frac{\sin\eta_s - \dfrac{R}{r}}{\cos\eta_s}\right) \tag{B.45}$$

where

$$\eta_s = \arcsin\left[\sin\delta\sin\theta_e + \cos\delta\cos\theta_e\cos\phi_{se}\right] \tag{B.46}$$

and R = Earth radius
r = satellite distance from Earth centre (use equation B.40c)
θ_e = earth station latitude
$\phi_{se} = \phi_s - \phi_e$
ϕ_s = satellite longitude
ϕ_e = earth station longitude.

Richharia's final equations

$$\text{Azimuth, } A = \arctan\left[\frac{\sin\phi_{se}}{\cos\theta_e \tan\delta - \sin\theta_e \cos\phi_{se}}\right] \tag{B.47}$$

This is important

Use the convention given in chapter 2, section 2.6 to obtain the azimuth quadrant.

Range

The distance ρ of a satellite from a given point on the Earth is given as

$$\rho = \sqrt{r^2 - R^2\cos^2\eta} - R\sin\eta \tag{B.48}$$

The inverse tangent rules

- True azimuth is given from A in the previous slide by:
- In the northern hemisphere:
  - A_{true} = 180 + A when the satellite is west of the earth station
  - A_{true} = 180 - A when the satellite is east of the earth station
- In the southern hemisphere:
  - A_{true} = 360 - A when the satellite is west of the earth station
  - A_{true} = 180 - A when the satellite is east of the earth station

This is the most important slide of all!!!!

- The earth rotates under the satellite, independently and with no direct effect on satellite motion other than time dependent gravitational variations that we will ignore in this course.
- The parameter $\phi_{es} = -\phi_{se}$ is a function of time
  - o If time is known, everything else is easy
  - o If time is not known, nothing is easy!
- For the time being, assume that time is known!

Assignment 5.3

5.3.1 Derive the look angles from 39N 77W to the GPS satellite given on slide 10 at GPS time week 801 and time of week 500,000 seconds. Note the distinction between time of almanac applicability, time of week, and the time for which the az/el to the satellite is desired.

End of Mod 5C