Module 3B

Modern Navigation Systems

Module 3

Spherical trigonometry and perturbation theory for solving great and small circle problems on or near the surface of the earth.

Module 3B

Spherical Trigonometry

Spherical trigonometry

- To consider properly the geometry of trajectories (i.e., segments of small and/or great circles) on the surface of a sphere, spherical trigonometry must be used.
- Of critical importance is the concept of a spherical right triangle. This is shown in the next slide, and is excerpted from the supplemental text, “Drawing the Line,” by Danson.

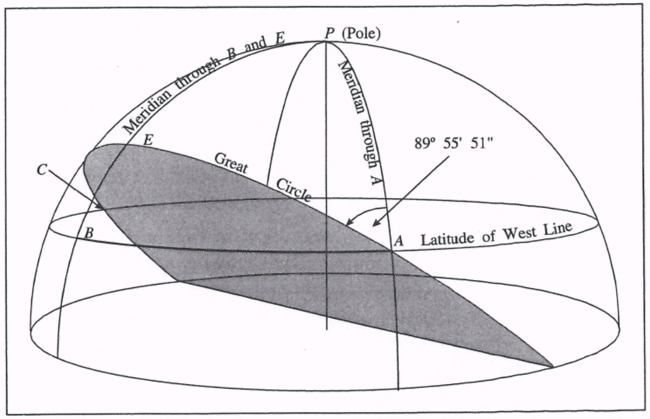
Summary of Module 3

- Students will learn to apply perturbation theory and spherical trigonometry to the problems of surveying and of comparing great circle and small circle routes for navigation on or near the surface of the earth.
- Students will repeat the spherical trigonometry computations performed by Charles Mason when surveying the Mason-Dixon line, and will compute the differences in miles flown between great circle and small circle routes from one city to another using Napier's rules.
- Students will apply perturbation theory to the simplification of the equations of small and great circles. Students will apply perturbation theory to the simplification of the equations of small and great circles.
- Students will also learn the nuances of Mercator and Lambert Conformal Conic map projections, with particular emphasis on the critical differences between aviation and nautical charts.

A spherical triangle

- A spherical triangle is defined when three planes pass through the surface of a sphere and through the sphere's center of volume (cf. http://www.rwgrayprojects.com/rbfnotes/trig/strig/strig.html)

Right Spherical Triangles



A 10-minute arc of the great circle. The great circle is the arc passing through *A, E,* and *C.* The distance *B* to *E* equals 1,714 feet.

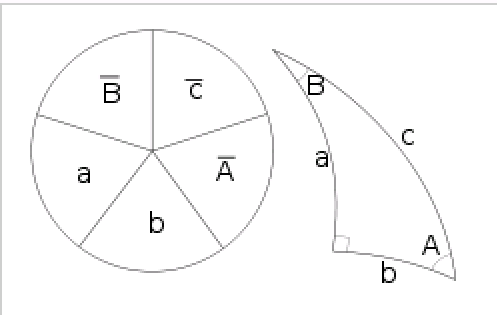
From Danson, the triangle AEP is a spherical right triangle and is formed by great circle segments. Since E is the midpoint of the arc AC, the included angle AEP is a right angle. The figure Illustrates the geometry used to survey the Mason-Dixon line, as will be discussed in Module 3B.

Note that AEB is a segment of a small circle.

Note also that the sum of the internal angles of a right spherical triangle is greater than 180 degrees.

Napier’s Pentagon

Napier's Pentagon



Napier's Circle shows the relations of parts of a right spherical triangle

Napier’s Pentagon, showing a mnemonic for the angular trigonometry of right spherical triangles (from Wikipedia).

A and B are angles, where C is a right angle.

a, b, and c are arc lengths, specified in radians for a sphere of radius 1.

Napier’s rules define how a, b, and c are explained on the next slide.

N.B., the complement bars mean that, for example, complement(B) = 90 degrees - B

Napier's rules

First write in a circle the six parts of the triangle (three vertex angles, three arc angles for the sides): for the triangle shown above left this gives *aCbAcB*. Next replace the parts which are not adjacent to C (that is *A, c, B*) by their complements and then delete the angle C from the list. The remaining parts are as shown in [previously]. For any choice of three contiguous parts, one (the *middle* part) will be adjacent to two parts and opposite the other two parts.

The ten Napier's Rules are given by

- sine of the middle part = the product of the tangents of the adjacent parts
- sine of the middle part = the product of the cosines of the opposite parts

From http://en.wikipedia.org/wiki/Spherical_trigonometry

Also from Wikipedia (ibid)

- The full set of rules for the right spherical triangle is (Todhunter,[1] Art.62)

$$
\begin{aligned}
&(\mathrm{R1}) \quad \cos c = \cos a \, \cos b, && (\mathrm{R6}) \quad \tan b = \cos A \, \tan c,\\
&(\mathrm{R2}) \quad \sin a = \sin A \, \sin c, && (\mathrm{R7}) \quad \tan a = \cos B \, \tan c,\\
&(\mathrm{R3}) \quad \sin b = \sin B \, \sin c, && (\mathrm{R8}) \quad \cos A = \sin B \, \cos a,\\
&(\mathrm{R4}) \quad \tan a = \tan A \, \sin b, && (\mathrm{R9}) \quad \cos B = \sin A \, \cos b,\\
&(\mathrm{R5}) \quad \tan b = \tan B \, \sin a, && (\mathrm{R10}) \quad \cos c = \cot A \, \cot B.
\end{aligned}
$$

[1]Todhunter, I. (1886). *Spherical Trigonometry* (5th ed.). MacMillan.

End of Mod 3B