Module 3

Modern Navigation Systems

Spherical trigonometry and perturbation theory for solving great and small circle problems on or near the surface of the earth.

Module 3D

Perturbation Theory

Summary of Module 3

- Students will learn to apply perturbation theory and spherical trigonometry to the problems of surveying and of comparing great circle and small circle routes for navigation on or near the surface of the earth.
- Students will repeat the spherical trigonometry computations performed by Charles Mason when surveying the Mason-Dixon line, and will compute the differences in miles flown between great circle and small circle routes from one city to another using Napier’s rules.
- Students will apply perturbation theory to the simplification of the equations of small and great circles.
- Students will also learn the nuances of Mercator and Lambert Conformal Conic map projections, with particular emphasis on the critical differences between aviation and nautical charts.

This sub-module

- In this sub-module, you will learn to derive equation 2.21 from the text
- The task is given as problem 2.4 at the end of chapter 2.

Perturbation Theory

- Navigation problems often involve computing the point at which 2 or more small circles intersect
- As stated in Module 2, it is convenient to model portions of these circles as straight lines, so that the computation of the point at which the circles intersect can be computed using the techniques of linear algebra.
- In situations that involve measurement and/or model errors, the linear algebra techniques can be augments by least squares, weighted least squares, and eventually Kalman filter techniques.

Small angle approximations

- As stated before, Taylor series approximations are useful for deriving flat earth approximations
- In general, however, the situation of interest involves perturbations to an angle, such as $\cos(\theta + \Delta\theta)$, where the latter term represents a small perturbation to the value of θ.

Expansion of $\cos(\theta + \Delta\theta)$

- Using trigonometric identities
  - $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
- If $b \ll 1$, then
  - $\sin(b) \sim b - b^3/3!$
  - $\cos(b) \sim 1 - b^2/2!$
- $\cos(a+b)$ becomes $\cos(a)[1 - b^2/2!] - \sin(a)[b - b^3/3!]$
- Extending this technique to the evaluation of complicated equations is called perturbation analysis, or perturbation theory

Geometry for navigation above a spherical earth

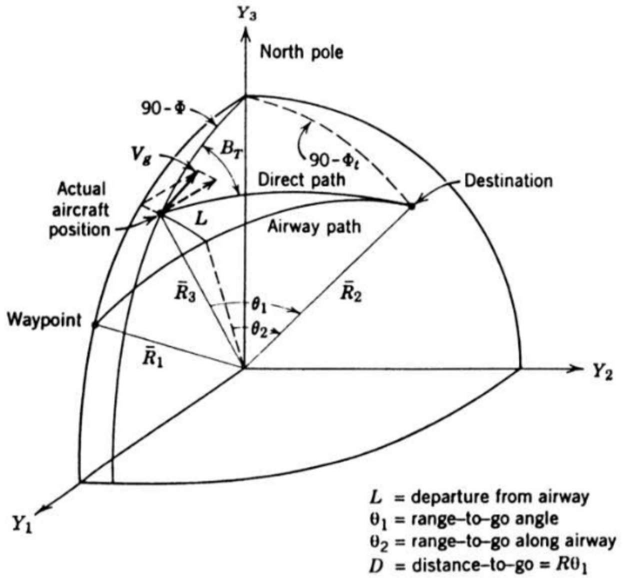

Figure 2.8 Course-line calculations.

This figure from the text defines the parameters used in equation 2.21.

Equation 2.21 from the text

$$\Delta D = \frac{(x - x_t)^2 (y - y_t)}{2RD} \tan \Phi = \frac{D^2}{4R} \sin B_T \sin 2B_T \tan \Phi$$

2.4. Derive Equation 2.21. *Hint:* solve the spherical triangle for the cosine of the range angle and express the coordinates of the waypoint as the present position plus a small increment. Expand the sines to third order and the cosines to second order.

- Solution of problem 2.4 is a classic example of the application of perturbation theory to navigation problems.

- The assignment for this sub-module is to solve problem 2.4, namely to derive equation 2.21.

To derive 2.21…

- It is necessary to combine perturbation techniques, the flat earth model, and the conversion from spherical to Cartesian coordinates
- It is also necessary to use the equations from slide 9 of module 2, repeated here (see next slide) to relate $(x - x_t)$ to $r\cos(\Delta\theta)$, for example.

Slide 9 from Module 2: Vector representation of Latitude and Longitude

- A position x, y, z on the surface of the earth can be written as the vector
  - $A_x = r_e \cos(lat) \cos(long)$
  - $A_y = r_e \cos(lat) \sin(long)$
  - $A_z = r_e \sin(lat)$
- The goal here is to convert arc lengths $r\theta$ into Cartesian parameters $(x - x_t)$

Deriving 2.21

- The instructions given in problem 2.4 of the text for deriving eq. 2.21 are explicit
- Follow them precisely
  - o Expand the sin(a+b) and cos(a+b) terms as shown in the previous slides
  - o Define $x - x_t$ and $y - y_t$ in terms of their spherical counterparts (e.g., $r\theta - r\theta_t$)
  - o Redefine D as $D_0 + \Delta D$, and expand as shown in the previous slides.

Assignment 3.3

3.3.1 Solve problem 2.4 from the text.

End of Mod 3D