Module 5

Modern Navigation Systems

Position determination of satellites, planets, and stars in earth-centered coordinate systems

Module 5E

A trip outdoors

Summary of Module 5

- Students will learn the features of, and similarities and differences between, the earth-centered inertial (ECI) and the earth-centered, earth fixed (ECEF) coordinate systems.
- Then, using actual orbital parameters from almanacs and ephemeris tables, students will learn to compute the positions in three dimensional space in both the ECI and ECEF frames of satellites, stars, planets, the sun, and the moon.
- The four equations that are used will form the basis for all the algorithms that follow.

Why include the stars here?

- There are separate modules later on in this course for celestial navigation and satellite navigation
- But, treating the topics independently camouflages the realization that the same algorithms apply to both situations
- It is easier to obtain the celestial equations from the satellite equations by setting r_e/r to zero. Adding the r_e/r terms after the fact to the celestial equations to obtain the satellite equations is difficult
- By focusing on the r_e/r connection, these same equations, but with $r_e/r = 1$, describe terrestrial navigation, which is typically treated as a third, separate topic, starting from scratch.
- You can see the stars, but cannot typically see satellites other than low earth orbit satellites and the International Space Station either post twilight or predawn, but not in the middle of the night.
- The stars move slowly through the sky, so that calculations of their position can be verified without the need to rush. Low earth orbit satellites move quickly (a few seconds to a few minutes of visibility).

The celestial sphere

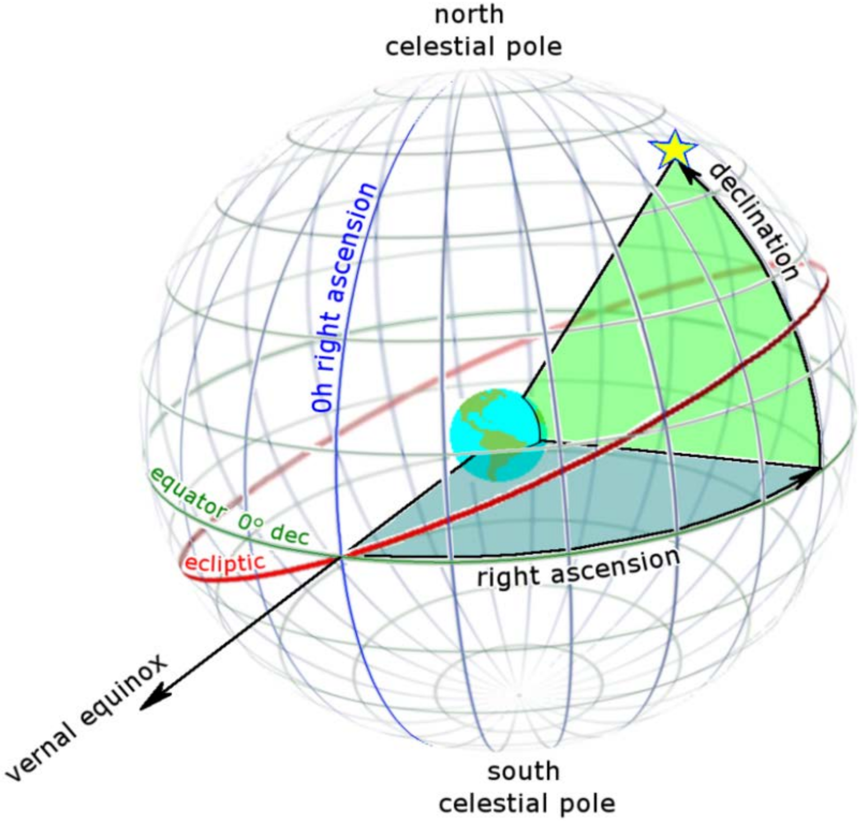
- The celestial sphere is a useful artifact that places all of the stars at a fixed, but enormous and unspecified distance from the earth on the surface of an imaginary sphere.
- This permits the stars to be identified by two angles, corresponding to an imaginary latitude (declination) and longitude (sidereal hour angle, or SHA) on the sphere, as measured from the earth, which is defined to be the center of the celestial sphere.
- Because of the extraordinary distance between the earth and even the closest star (other than the sun), the motion of the earth through its annual orbit has only a slight effect on these apparent “addresses” of the stars.
- For example, Polaris, the north star, has a declination (latitude) of almost 90 degrees, hence its usefulness for determining the approximate direction of north from anywhere in the earth’s northern hemisphere.
- Sirius, with a declination of -17 degrees and an SHA of 259, is viewed in the southern sky, rising in the southeast and setting in the southwest as the earth rotates.
- The star Spica has a declination of -11 degrees, and an SHA of 159 degrees.
- As it takes the earth almost 24 hours to rotate 360 degrees with respect to the celestial sphere, if Spica is viewed directly to the South at a given point in time from a location in the northern hemisphere, Sirius will appear in the same direction (albeit lower in elevation), at (259 – 159)*24/360 hours later.
- A more detailed explanation is given in the next slides, courtesy of Cornell University. Note that Cornell uses right ascension, the negative of the sidereal hour angle, to define celestial longitude.

The celestial sphere as defined by Cornell Univ. faculty

- The celestial sphere is an imaginary sphere of gigantic radius with the earth located at its center. The poles of the celestial sphere are aligned with the poles of the Earth. The celestial equator lies along the celestial sphere in the same plane that includes the Earth's equator.
- We can locate any object on the celestial sphere by giving it two coordinates, called the **Right Ascension** and the **Declination**. These are called *celestial coordinates*.
- Analogous to the longitude on Earth, the **Right Ascension** of an object on the celestial sphere is measured along the celestial equator, as the angular distance to some fiducial direction for with R.A. = 0 degrees. By convention, this fiducial direction is the point on the celestial where the Sun is found on the first day of spring (the vernal equinox).
- Analogous to the latitude on Earth, the **Declination** of an object on the celestial sphere is measured northward or southward from the plane containing the equator. The declination of the equator is 0 degrees, the North Celestial Pole, +90 degrees, the South Celestial Pole, -90 degrees.

http://www.astro.cornell.edu/academics/courses/astro201/cel_sphere.htm

Graphical Representation of the Celestial Sphere



The equator of the celestial sphere is aligned with the earth's equatorial plane. Since the earth's axis is tilted, with respect to its orbit around the sun, the sun's motion with respect to the celestial sphere follows the inclined plane known as the ecliptic. The orbits of many of the planets lie in or near the ecliptic, whereas the constellations of the Zodiac are in the equatorial plane of the celestial sphere.

http://en.wikipedia.org/wiki/Right_ascension

The celestial sphere, as viewed by Curious George

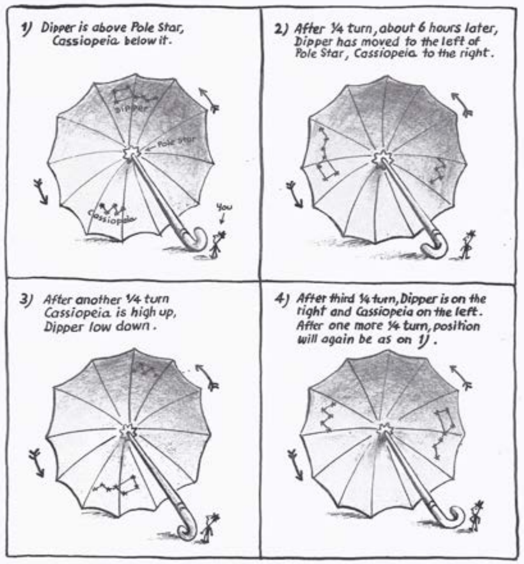


Figure 8: Umbrella Planetarium

UMBRELLA PLANETARIUM: The Pole Star is the only star which keeps its place in the sky. All the other stars and constellations wander around the pole *once daily*, counterclockwise, as though they were fixed to the inside of a vast hollow globe.

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From H. A. Rey, The Stars, 2nd Ed. Houghton Mifflin 2008.

The inside of the umbrella represents a portion of the celestial sphere. The handle of the umbrella represents the rotational axis of the earth.

Orientation of the Celestial Sphere

- The x-axis of the celestial sphere, in a right-handed coordinate system (east-north-up: ask in class for details) points in the “direction of the vernal equinox.”
- The vernal equinox is formed by a line from the earth to the sun on the first day of spring in the northern hemisphere.
- This line points to the constellation Pisces, but used to point at the constellation Aries. If you are interested, read about “the precession of the equinoxes” on line or in an appropriate text.

The ECI and ECEF definitions:

In the ECI frame, the stars are motionless.

Transformation from ECI to ECEF requires a precisely knowledge of Universal Time Coordinated (UTC), which will be discussed in later modules.

1. *Earth-centered, Earth -fixed (ECEF).* The basic coordinate frame for navigation near the Earth is ECEF, shown in Figure 2.3 as the y_i rectangular coordinates whose origin is at the mass center of the Earth, whose y_3-axis lies along the Earth's spin axis, whose y_1 axis lies in the Greenwich meridian, and which rotates with the Earth [10]. Satellite-based radio-navigation systems often use these ECEF coordinates to calculate satellite and aircraft positions.
2. *Earth-centered inertial (ECI).* ECI coordinates, x_i, can have their origin at the mass-center of any freely falling body (e.g., the Earth) and are nonrotating relative to the fixed stars. For centuries, astronomers have observed the small relative motions of stars ("proper motion") and have defined an "average" ECI reference frame [1 1]. To an accuracy of 10^{-5} deg /hr, an ECI frame can be chosen with its x_3-axis along the mean polar axis of the Earth and with its x_1- and x_2-axes pointing to convenient stars (as explained in Chapter 12). ECI coordinates have three navigational functions. First, Newton's laws are valid in any ECI coordinate frame. Second, the angular coordinates of stars are conventionally tabulated in ECI. Third, they are used in mechanizing inertial navigators, Section 7.5.1.

Definitions are from the text, chapter 2.

ECI versus ECEF, cont'd

- Earth centered earth fixed
  - This coordinate system rotates with the earth
  - The stars rotate through the sky; this is the coordinate system we live in: while stationary with respect to the earth, our positions in this system do not change
- Earth centered inertial
  - The earth rotates with respect to this coordinate system
  - To a first approximation, the stars do not

Greenwich Hour Angle of Aries: GHAϒ

- The SHA value for a star can be regarded, for this course, as being a constant.
- For celestial navigation, it is essential to know the apparent longitude, versus time, of a star as the earth rotates below the celestial sphere.
- Since the stars appear to rotate in unison (as it is the earth, not the stars, that is rotating), the entire celestial sphere can be oriented using a single reference point know as GHAϒ.
- GHAϒ is documented, by day and hour on a yearly basis, in the Nautical Almanac, one of the required texts for this course.

Relating the stars to earth-orbiting satellites

- As stated, it is not the tradition to study celestial and satellite navigation as a single subject.
- However, there is great utility in doing so, which is why the two topics are combined in this module.
- For an observer on the surface of the earth at a known longitude LONG, the following definitions apply:
  - GHA = Greenwich Hour Angle = SHA + GHA♈
  - GHA + LONG = Local Hour Angle, or LHA
  - LHA for a star is equivalent to ϕ_{es} for a satellite

Richharia's equations

- By replacing ϕ_{es} with the LHA of a star, replacing the elevation of the satellite with the elevation of the star, and setting r_e/r to zero, the satellite algorithms presented earlier can be used for celestial navigation.
- Among other things, this makes it possible to combine GPS and celestial measurements in a single navigation computation.

Location parameters

Satellites

- Right ascension
- Inclination
- Φ_{es}
- Elevation
- Azimuth
- Distance

Stars

- Right ascension
- Declination
- GHA♈
- Sidereal Hour Angle (SHA)
- Greenwich Hour Angle (GHA)
- Local Hour Angle (LHA)
- Elevation
- Azimuth

Of Particular Importance

- There is a one-to-one connection between the distance to a satellite/star and the elevation angle to a satellite/star.
- GPS measures distances, because this can be accomplished precisely via the use of radio receivers and atomic clocks
- Celestial techniques measure angles, as this can be accomplished precisely via the use of sextants and star trackers

Reprise of Richharia's equations

Finally, the following set of equations can be used to obtain satellite azimuth and elevation from a specified earth station:

$$\text{Right ascension, } \alpha = \arctan\left(y/x\right) \tag{B.43}$$

$$\text{Declination, } \delta = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \tag{B.44}$$

$$\text{Elevation, } \eta = \arctan\left(\frac{\sin\eta_s - \frac{R}{r}}{\cos\eta_s}\right) \tag{B.45}$$

where

$$\eta_s = \arcsin\left[\sin\delta\sin\theta_e + \cos\delta\cos\theta_e\cos\phi_{se}\right] \tag{B.46}$$

and R = Earth radius
r = satellite distance from Earth centre (use equation B.40c)
θ_e = earth station latitude
$\phi_{se} = \phi_s - \phi_e$
ϕ_s = satellite longitude
ϕ_e = earth station longitude.

x, y, and z, and hence right ascension, declination, and elevation, are determined by the orbital parameters.

Richharia's last 2 equations

$$\text{Azimuth, } A = \arctan\left[\frac{\sin\phi_{se}}{\cos\theta_e \tan\delta - \sin\theta_e \cos\phi_{se}}\right] \qquad \text{(B.47)}$$

This is important

Use the convention given in chapter 2, section 2.6 to obtain the azimuth quadrant.

Range

The distance ρ of a satellite from a given point on the Earth is given as

$$\rho = \sqrt{r^2 - R^2\cos^2\eta} - R\sin\eta \qquad \text{(B.48)}$$

Computing the look angles to a star from any point on earth

- This presumes that an observer at a known location on the surface of the earth (i.e., lat and long are known) also knows Greenwich time via use of an accurate chronometer or other source of Universal Time Coordinated (UTC).
- The detailed procedure is described on page 277 of the Nautical Almanac, under the topic computing *Hc*.

Computation of *Hc*, summarized

- Choose a star, and look up its SHA and declination (DEC)
  - o Either from the Nautical Almanac or online at http://www.usno.navy.mil/USNO/astronomical-applications
- For the time at which the star’s look angles are to be computed, look up GHAϒ in the Nautical Almanac.
- Compute the LHA from GHAϒ + SHA + LONG
- Use the equations that follow to compute the Hc and A, which are the elevation and azimuth to the star

From the Nautical Almanac

6. *The calculated altitude and azimuth.* The calculated altitude H_C and true azimuth Z are determined from the GHA and Dec interpolated to the time of observation and from the $Long$ and Lat estimated at the time of observation as follows:

Step 1. Calculate the local hour angle

$$LHA = GHA + Long$$

Add or subtract multiples of 360° to set LHA in the range 0° to 360°.

Step 2. Calculate S, C and the altitude H_C from

$$\begin{aligned} S &= \sin Dec \\ C &= \cos Dec \cos LHA \\ H_C &= \sin^{-1}(S \sin Lat + C \cos Lat) \end{aligned}$$

where $\sin^{-1}$ is the inverse function of sine.

Step 3. Calculate X and A from

$$\begin{aligned} &X = (S \cos Lat - C \sin Lat)/\cos H_C \\ &\text{If } X > +1 \quad \text{set} \quad X = +1 \\ &\text{If } X < -1 \quad \text{set} \quad X = -1 \\ &A = \cos^{-1} X \end{aligned}$$

where $\cos^{-1}$ is the inverse function of cosine.

Step 4. Determine the azimuth Z

$$\begin{aligned} &\text{If } LHA > 180° \quad \text{then} \quad Z = A \\ &\text{Otherwise} \quad Z = 360° - A \end{aligned}$$

Sample data page from a 2000 Almanac

124 2000 JUNE 20, 21, 22 (TUES., WED., THURS.)

UT		ARIES	VENUS −3.9		MARS +1.6		JUPITER −2.1		SATURN +0.2		STARS		
d	h	GHA ° ′	GHA ° ′	Dec ° ′	GHA ° ′	Dec ° ′	GHA ° ′	Dec ° ′	GHA ° ′	Dec ° ′	Name	SHA ° ′	Dec ° ′
20	00	268 30.6	177 04.1	N23 52.0	175 57.2	N24 12.8	212 47.0	N18 50.9	214 55.2	N17 10.1	Acamar	315 26.3	S40 18.1
	01	283 33.0	192 03.2	52.0	190 57.8	12.8	227 48.9	51.0	229 57.4	10.1	Achernar	335 34.6	S57 13.9
	02	298 35.5	207 02.3	52.1	205 58.4	12.8	242 50.8	51.2	244 59.6	10.2	Acrux	173 20.6	S63 06.3
	03	313 38.0	222 01.4	. . 52.2	220 59.0	. . 12.8	257 52.7	. . 51.3	260 01.7	. . 10.3	Adhara	255 20.9	S28 58.5
	04	328 40.4	237 00.5	52.2	235 59.7	12.8	272 54.6	51.4	275 03.9	10.3	Aldebaran	291 01.4	N16 30.5
	05	343 42.9	251 59.6	52.3	251 00.3	12.7	287 56.5	51.5	290 06.1	10.4			
	06	358 45.4	266 58.7	N23 52.3	266 00.9	N24 12.7	302 58.4	N18 51.6	305 08.3	N17 10.5	Alioth	166 29.4	N55 57.8
	07	13 47.8	281 57.8	52.4	281 01.6	12.7	318 00.3	51.8	320 10.4	10.5	Alkaid	153 06.6	N49 19.0
T	08	28 50.3	296 56.9	52.4	296 02.2	12.7	333 02.2	51.9	335 12.6	10.6	Al Na'ir	27 56.2	S46 57.4
U	09	43 52.8	311 56.0	. . 52.5	311 02.8	. . 12.7	348 04.1	. . 52.0	350 14.8	. . 10.7	Alnilam	275 57.0	S 1 12.2
	10	58 55.2	326 55.1	52.5	326 03.5	12.7	3 06.0	52.1	5 16.9	10.7	Alphard	218 06.3	S 8 39.6
E	11	73 57.7	341 54.2	52.6	341 04.1	12.6	18 07.9	52.3	20 19.1	10.8			
S	12	89 00.1	356 53.3	N23 52.6	356 04.7	N24 12.6	33 09.8	N18 52.4	35 21.3	N17 10.9	Alphecca	126 19.3	N26 43.0
D	13	104 02.6	11 52.4	52.7	11 05.3	12.6	48 11.7	52.5	50 23.5	10.9	Alpheratz	357 54.0	N29 05.3
A	14	119 05.1	26 51.5	52.7	26 06.0	12.6	63 13.6	52.6	65 25.6	11.0	Altair	62 17.8	N 8 52.2
Y	15	134 07.5	41 50.6	. . 52.8	41 06.6	. . 12.6	78 15.5	. . 52.7	80 27.8	. . 11.1	Ankaa	353 25.7	S42 18.1
	16	149 10.0	56 49.8	52.8	56 07.2	12.6	93 17.4	52.9	95 30.0	11.1	Antares	112 38.4	S26 25.9
	17	164 12.5	71 48.9	52.9	71 07.9	12.5	108 19.3	53.0	110 32.2	11.2			
	18	179 14.9	86 48.0	N23 52.9	86 08.5	N24 12.5	123 21.3	N18 53.1	125 34.3	N17 11.2	Arcturus	146 04.8	N19 11.0
	19	194 17.4	101 47.1	53.0	101 09.1	12.5	138 23.2	53.2	140 36.5	11.3	Atria	107 48.8	S69 01.7
	20	209 19.9	116 46.2	53.0	116 09.8	12.5	153 25.1	53.4	155 38.7	11.4	Avior	234 22.7	S59 30.8
	21	224 22.3	131 45.3	. . 53.0	131 10.4	. . 12.5	168 27.0	. . 53.5	170 40.8	. . 11.4	Bellatrix	278 43.2	N 6 20.9
	22	239 24.8	146 44.4	53.1	146 11.0	12.4	183 28.9	53.6	185 43.0	11.5	Betelgeuse	271 12.6	N 7 24.3
	23	254 27.2	161 43.5	53.1	161 11.7	12.4	198 30.8	53.7	200 45.2	11.6			
21	00	269 29.7	176 42.6	N23 53.1	176 12.3	N24 12.4	213 32.7	N18 53.8	215 47.4	N17 11.6	Canopus	264 01.2	S52 41.8
	01	284 32.2	191 41.7	53.2	191 12.9	12.4	228 34.6	54.0	230 49.5	11.7	Capella	280 49.9	N45 59.8
	02	299 34.6	206 40.8	53.2	206 13.6	12.4	243 36.5	54.1	245 51.7	11.8	Deneb	49 38.0	N45 16.8
	03	314 37.1	221 39.9	. . 53.2	221 14.2	. . 12.3	258 38.4	. . 54.2	260 53.9	. . 11.8	Denebola	182 44.0	N14 34.3
	04	329 39.6	236 39.0	53.3	236 14.8	12.3	273 40.3	54.3	275 56.1	11.9	Diphda	349 06.1	S17 59.1
	05	344 42.0	251 38.1	53.3	251 15.5	12.3	288 42.2	54.4	290 58.2	12.0			
	06	359 44.5	266 37.2	N23 53.3	266 16.1	N24 12.3	303 44.1	N18 54.6	306 00.4	N17 12.0	Dubhe	194 04.2	N61 45.3
W	07	14 47.0	281 36.3	53.4	281 16.7	12.2	318 46.0	54.7	321 02.6	12.1	Elnath	278 25.8	N28 36.4
E	08	29 49.4	296 35.4	53.4	296 17.4	12.2	333 47.9	54.8	336 04.8	12.1	Eltanin	90 50.3	N51 29.4
D	09	44 51.9	311 34.5	. . 53.4	311 18.0	. . 12.2	348 49.8	. . 54.9	351 06.9	. . 12.2	Enif	33 56.9	N 9 52.5
	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			

What to read?

- Read chapter 12 of the text by Kayton and Fried
- Read the tutorial section of the Nautical Almanac, starting on page 279.
- Note the table of declinations and SHAs for each of the 56 navigation stars that is located on each page of the Almanac.

Assignment 5.5

5.5.1 Go outside at night and identify Sirius, Polaris, Orion (early in the spring term, not in the summer), Jupiter (depending on the month), Venus, and the moon.

5.5.2a. Compute the look angles to a star of your choice based on your location.

5.5.2b Check your answers against the USNO Astronomical Observations web site at http://www.usno.navy.mil/USNO/astronomical-applications

5.5.2c. Go outside and look at the star, verifying that your az/el computations are correct. If you use a magnetic compass, correct for magnetic variation.

5.5.3 Using NASA's website for guidance on when and where to look, view the International Space Station.

5.5.4 If you can borrow a pair of binoculars, look at the moons of Jupiter.

5.5.5 Comment on your observations in a paragraph or two.

End of Mod 5E