Module 9

Modern Navigation Systems

Satellite navigation using geostationary satellites

Module 9A
Fundamental concepts

Summary of Module 9

- **Students will extend the algorithms used with sextants to the case of near-earth satellites, in particular geostationary satellites, thus introducing parallax (i.e, the ratio of the earth's radius *R* to the distance *r* from the center of the earth to a satellite or the moon). (9A)**
- Students will compute their location on earth using plotting techniques along with an "assumed position" and values for elevation measurements to each of the satellites that correspond to the "actual position." Students will re-compute their location on earth using the sextant algorithm from Module 8 with the same information. (9B)
- Students will compute their location on earth using multivariable Newton's method using angle measurements to the two satellites. Then, they will combine angle and azimuth "measurements" for only a single satellite, thus developing a "mixed-mode" algorithm that would correspond to, for example the use of a gravity based inclinometer and a magnetic compass. (9C)

Reading/viewing

- Review, as needed, relevant material from previous modules.
  - o In particular, review the assignments from Module 5, in which you determined the look angles to both Geostationary and GPS satellites.
- Re-read relevant portions of the primary text, Chapter 5
  - o With particular emphasis on 5.7.3 regarding the Wide Area Augmentation System (WAAS) which uses geostationary satellites

Why study geostationary satellites?

- They are close enough to earth that parallax must be taken into account
  - This is captured in the R/r terms in Riccharia's equations (repeated here; cf. Module 5), and is a feature that must be addressed when using satellites for navigation
- But, ϕ_{es} (i.e., the LHA of the satellite) is independent of time
  - Thus postponing the need to either know or "solve for" time
- And, it isn't necessary to solve for time-dependent orbital parameters
  - ϕ_s, the sub-satellite longitude, is all that is needed for specifying the location in orbit of a geostationary satellite

Review of earlier material

- Previously, it was noted that
  - $M = E - e \sin E$, where e is a known constant
- and that
  - If E is known, computation of M is easy
  - If M is known, computation of E must be done iteratively

For navigation

- Computation of M is analogous to computing the look angles to a satellite
- Iterative computation of E is analogous to
  - Measuring the look angles to a satellite
  - Comparing the measurements with the predicted look angles computed based on an “assumed position”
  - Adjusting the “assumed position” iteratively until the difference between the measured and predicted look angles becomes acceptably small

Pseudorange techniques

- The primary text refers to pseudorange techniques, which are what GPS uses (cf. Module 11)
  - Pseudorange is inferred from measurements of the relative transit times of radio signals from satellites whose positions at any instant in time are known to high levels of accuracy and precision

What is “pseudorange”?

- If a satellite is a distance *r* from a ground station, it takes a time $t = r/c$ for the signal to travel this distance
  - Where *c* is the speed of light
- But, if the range measurement is subject to errors due to uncertainties in the knowledge of time, due to clock errors on, for example, a GPS satellite, then
  - The inferred distance is referred to as a “pseudorange”, as defined in section 5.2 of the primary text
- When corrected for timing errors (cf. Module 10), the pseudorange becomes
  - The “corrected pseudorange”

Range versus angle

- Examination of Riccharia's final equation (below) shows a direct correspondence between the range to a satellite and the elevation angle η to the satellite
  - where R is the radius of the earth and r is the range to the satellite:

Range

The distance ρ of a satellite from a given point on the Earth is given as

$$\rho = \sqrt{r^2 - R^2 \cos^2 \eta} - R \sin \eta \qquad \text{(B.48)}$$

Range versus angle, cont'd

- GPS uses range because its precision is based on the use of atomic clocks on each satellite, and is extraordinarily good with respect to the best angle measurement techniques
- Range measurements can be accomplished in moving GPS receivers by using low cost omnidirectional antennas that can track all of the satellites in view at once
- Whereas precise angular measurements of the angles of arrival of radio signals require physically large antennas that can typically point to only one satellite at a time

Recap of Riccharia's equations

Finally, the following set of equations can be used to obtain satellite azimuth and elevation from a specified earth station:

$$\text{Right ascension, } \alpha = \arctan\left(y/x\right) \tag{B.43}$$

$$\text{Declination, } \delta = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \tag{B.44}$$

$$\text{Elevation, } \eta = \arctan\left(\frac{\sin\eta_s - \dfrac{R}{r}}{\cos\eta_s}\right) \tag{B.45}$$

where

$$\eta_s = \arcsin\left[\sin\delta\sin\theta_e + \cos\delta\cos\theta_e\cos\phi_{se}\right] \tag{B.46}$$

and R = Earth radius
r = satellite distance from Earth centre (use equation B.40c)
θ_e = earth station latitude
$\phi_{se} = \phi_s - \phi_e$
ϕ_s = satellite longitude
ϕ_e = earth station longitude.

Richharia's final equations

$$\text{Azimuth, } A = \arctan\left[\frac{\sin\phi_{se}}{\cos\theta_e \tan\delta - \sin\theta_e \cos\phi_{se}}\right] \quad \text{(B.47)}$$

This is important

Use the convention given in chapter 2, section 2.6 to obtain the azimuth quadrant. *(see the next slide)*

Range

The distance ρ of a satellite from a given point on the Earth is given as

$$\rho = \sqrt{r^2 - R^2\cos^2\eta} - R\sin\eta \quad \text{(B.48)}$$

The azimuth rules, from Module 5

- True azimuth is given from A in the previous slide by:
- In the northern hemisphere:
  - A_{true} = 180 + A when the satellite is west of the earth station
  - A_{true} = 180 - A when the satellite is east of the earth station
- In the southern hemisphere:
  - A_{true} = 360 - A when the satellite is west of the earth station
  - A_{true} = 180 - A when the satellite is east of the earth station

For geostationary satellites

- The declination of a GEO satellite is 0.
- The suborbital longitude is a constant
- The putative look angles and range for an “assumed position” are easily computed using Riccharia’s equations

What's next?

- In the next sub-modules, you will simulate navigation on land by using one and/or two geostationary satellites and by using several different algorithms, based on measurements of elevation, azimuth, and/or range.
- In preparation, you will compute the look angles and ranges to the two XM satellites at 27W and 115W, using http://www.dishpointer.com to check your results.

Graphical solution using a plotting table

The position line of an observation is plotted on a chart using the intercept

$$p = H_O - H_C$$

and azimuth Z with origin at the calculated position ($Long$, Lat) at the time of observation, where H_C and Z are calculated using the method in section 6, page 279. Starting from this calculated position a line is drawn on the chart along the direction of the azimuth to the body. Convert p to nautical miles by multiplying by 60. The position line is drawn at right angles to the azimuth line, distance p from ($Long, Lat$) towards the body if p is positive and distance p away from the body if p is negative. Provided there are no gross errors the navigator should be somewhere on or near the position line at the time of observation. Two or more position lines are required to determine a fix.

from the Nautical Almanac

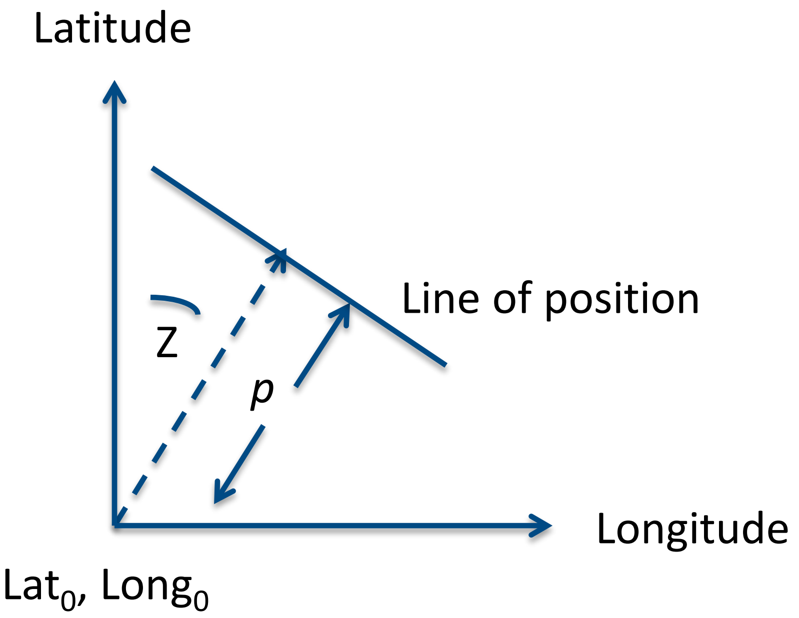
This technique is also known as the “Marc-St. Hilaire” Algorithm**

Z = computed azimuth
to a measured star
based on the initial
position guess Lat_0, $Long_0$

p is the distance
between Ho and Hc (based
on turning the angle into
a distance by scaling the arc-length
by the radius of the earth)

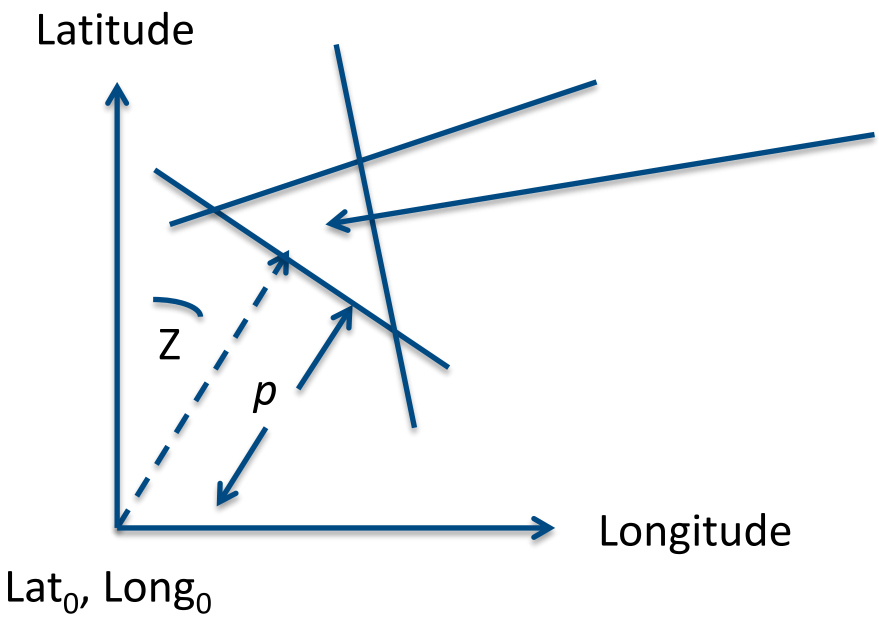
p and Z are used to compute a line of position
The intersection of multiple lines
of position is used to compute Lat_1, $Long_1$

The process is iterated until the
incremental change in estimated
position falls below the desired margin of error.



**Admiral Marc St. Hilaire of the French Navy, late 1800’s.

Multiple lines of position



Lat$_1$, Long$_1$ based on least Squares intercept of the Multiple lines of position

Note that a chart table is ideal for solving this problem graphically! Parallel rulers (covered in a future module) referenced to a “compass rose” on a chart make this Straightforward and easy.

Numerical Method Using Least Squares

11. *Position from intercept and azimuth by calculation.* The position of the fix may be calculated from two or more sextant observations as follows.

If p_1, Z_1, are the intercept and azimuth of the first observation, p_2, Z_2, of the second observation and so on, form the summations

$$A = \cos^2 Z_1 + \cos^2 Z_2 + \cdots$$
$$B = \cos Z_1 \sin Z_1 + \cos Z_2 \sin Z_2 + \cdots$$
$$C = \sin^2 Z_1 + \sin^2 Z_2 + \cdots$$
$$D = p_1 \cos Z_1 + p_2 \cos Z_2 + \cdots$$
$$E = p_1 \sin Z_1 + p_2 \sin Z_2 + \cdots$$

The least squares equations

where the number of terms in each summation is equal to the number of observations.

With $G = AC - B^2$, an improved estimate of the position at the time of fix (L_I, B_I) is given by

$$L_I = L_F + (AE - BD)/(G \cos B_F), \qquad B_I = B_F + (CD - BE)/G$$

Calculate the distance d between the initial estimated position (L_F, B_F) at the time of fix and the improved estimated position (L_I, B_I) in nautical miles from

$$d = 60\sqrt{((L_I - L_F)^2 \cos^2 B_F + (B_I - B_F)^2)}$$

If d exceeds about 20 nautical miles set $L_F = L_I$, $B_F = B_I$ and repeat the calculation until d, the distance between the position at the previous estimate and the improved estimate, is less than about 20 nautical miles.

Assignment 9.1

1. Calculate the range, elevation angle, and azimuth to the XM satellites at 27W and 115W from a presumed location of Latitude: 39.1427°N and Longitude: 76.8606°W, which corresponds to zip code 20723.
2. Check your results at http://www.dishpointer.com

End of Mod 9A