Module 2 Assignment

Modern Navigation Systems – EN.525.645.81

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# 2.1 Vector Mathematics, Great Circles, and Latitude/Longitude

1. Look up the latitude and longitude of San Francisco and Washington, D.C., and compute the great circle distance between the two cities. Compare this with an airline’s frequent flier miles and Mapquest driving miles between the two cities.

Let *re =* 3959 miles

Lat/Long of San Francisco: [α,β] = [37.77, -122.42]

Lat/Long of Washington D.C.: γ,ω = [38.91, -77.04]

Taking the dot product yields:

We obtain the value of θ as such:

Thus the great circle distance is computed as such:

According to Google Maps, the driving distance is approximately 2,812 miles.

According to <http://webflyer.com> the frequent flyer miles between Washington Dulles and San Francisco International Airport is 2,410 miles, a much closer approximation to the great circle distance calculated above.

1. Derive equation 2.20 in the text for the great circle distance between two points.

Use hint doc?

# 2.2 Small Circles on the Surface of the Earth

1. Apply an appropriate rotation of axes to determine the small circle and great circle distances between Washington D.C. and Dallas, TX. Consult an algebra book or Wikipedia for guidance if the rotation of axes computation proves to be challenging.

Let *re =* 3959 miles

Lat/Long of Dallas, TX: [α,β] = [32.78, -96.80]

Lat/Long of Washington D.C.: γ,ω = [38.91, -77.04]

1. Verify that the great circle distance is shorter.

# 2.3 The Vector Cross Product, Angular Momentum, and Kepler’s Laws

1. Compute the cross product of the vectors 2**i** + 3**j** and 5**i** – 7**j**. Show that the cross product points in the **k** direction. **i**, **j**, and **k** are unit vectors in the x, y, and z directions.

**u =** 2**i** + 3**j**

**v =** 5**i** – 7**j**

The cross product points in the **k** direction since it is the only direction with a non-zero scalar value in the computer cross product.

1. Explain, mathematically, the change in rotation rate of an ice skater as he/she brings his/her arms close to their body during a spin.

According to Kepler’s first law, rotational momentum is always conserved in a system (when there is no drag).

In the definition for angular momentum we have:

Where **L** is the angular momentum vector, m is a scalar for the mass, and r is the scalar distance between the center of the ice skater’s body to the end of his/her arms.

If the skater pulls in their arms, they decrease the value of r. In order to conserve angular momentum either mass or angular velocity must increase by the square of r. Since the mass in the system is constant the angular velocity must increase to conserve the momentum. Thus, the skater begins to spin faster.

1. Explain how tightrope walkers reverse direction on a tightrope while holding a balancing pole. Why do such poles bend?