Module 10 – Assignment

Modern Navigation Systems – EN.525.645.81

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1. Using material from earlier modules, compute the orbital radius and height above the earth for a GPS satellite based on its orbital period of 12 hours.

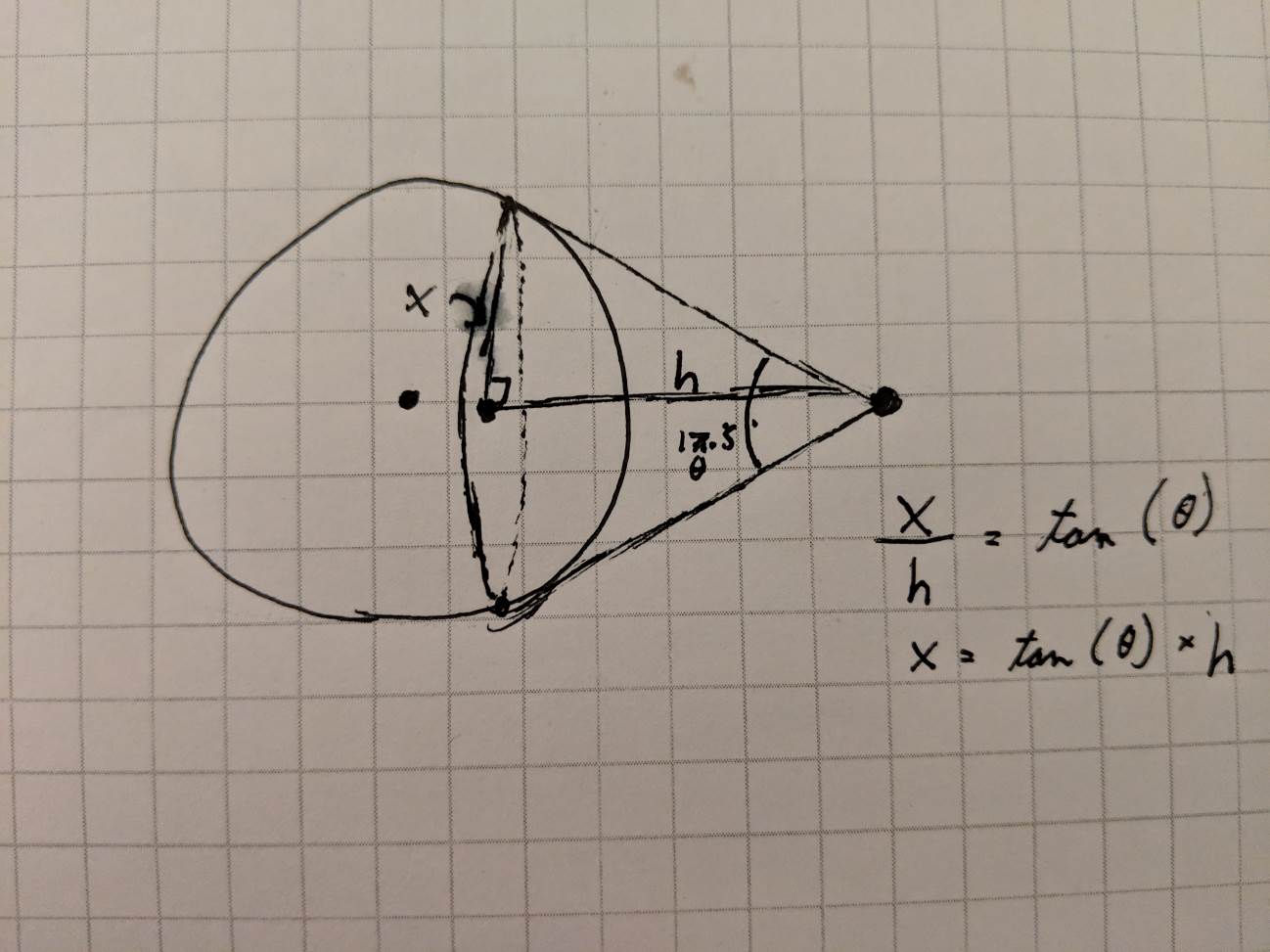
Looking back at Module 5, we have the equation for the orbital radius L in terms of the orbital period, T:

For T = 12 hrs = 43,200 seconds:

The height above the Earth is thus:

1. Estimate the ground footprint, in square miles, of a single satellite using simple trigonometry to determine what percentage of the earth’s surface can be “seen” from a single satellite.

See diagram for a rough sketch of system being estimated.



For this problem, we assume a 17.3° boresight angle view of the satellite to the Earth. We also assume that h is somewhere roughly between 20,238km and 26,609km as found in the previous problem, say the average.

We first find the radius of the cone’s base, x:

Thus the surface area of the imaginary cone projected onto the sphere is estimated by:

This value is slightly larger in reality since the cone’s base is not taking into account, the curvature of the Earth.

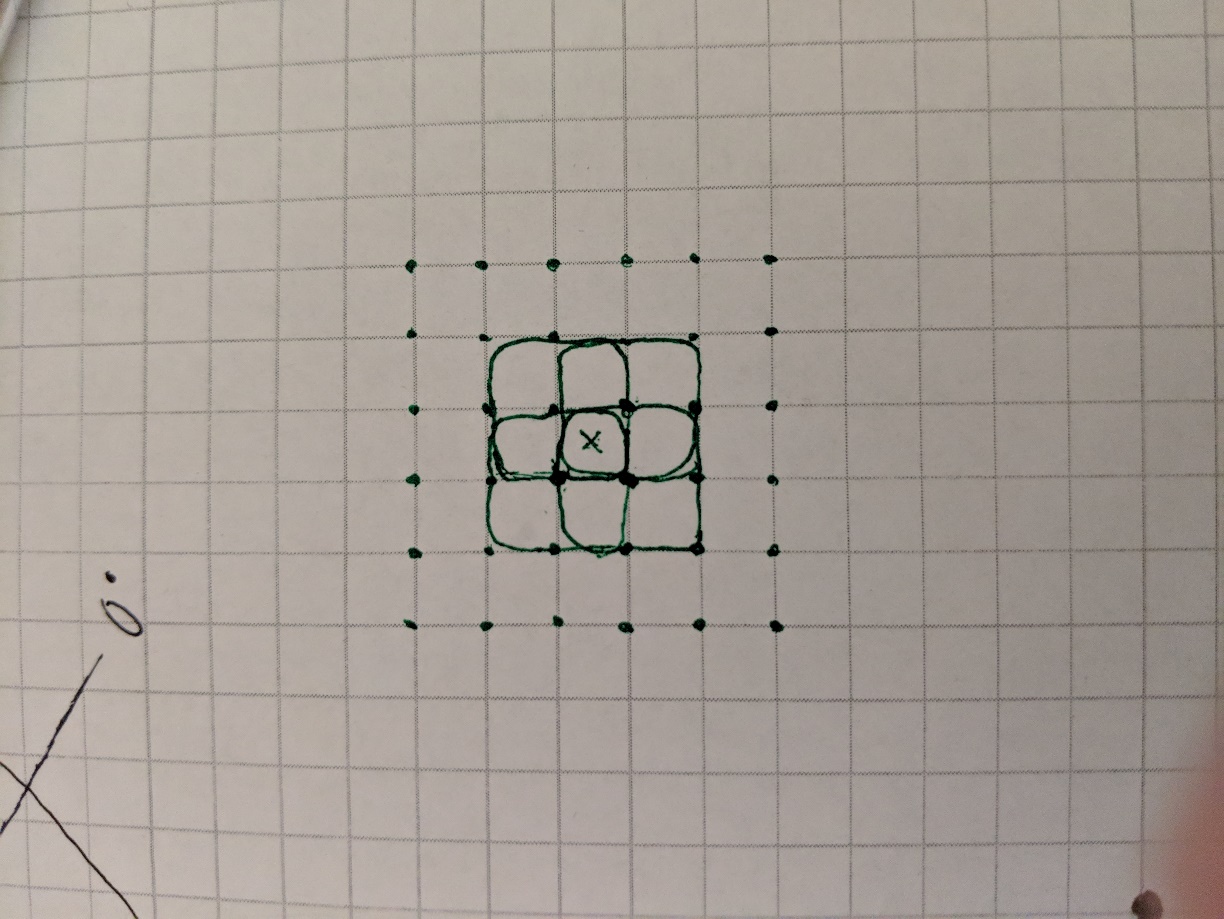
The total surface area of the Earth is given as:

Thus the rough percentage coverage of the Earth that can be seen by the satellite is:

1. Continue this “back of the envelope” analysis to estimate the total number of GPS satellites that are needed in order for a GPS receiver to have visibility of at least four satellite simultaneously for “almost” 100% of the time from “almost” anywhere on earth.

Assuming an SA that is more realistic for a non-flat earth model, let SA = 45e6 km2.

If we visualize the Earth projected on a flat surface the coverage map of a single GPS satellite can be approximated to a round square shape.



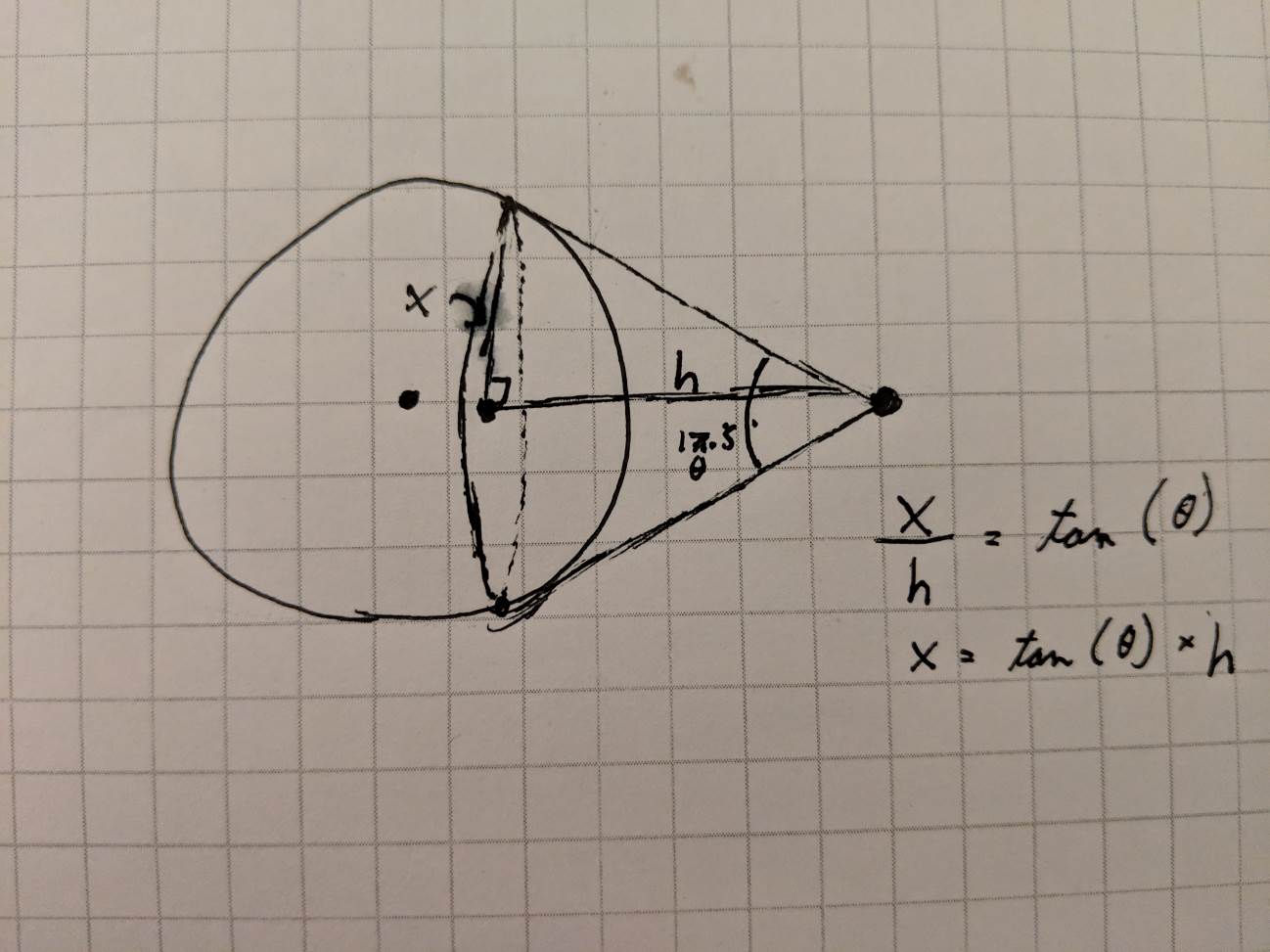
If we imagine the Earth map on projected on a grid system like shown above then we can see how four overlapping squares each placed one unit apart cover a square unit of surface area marked with the “x” in the above figure. Thus, you require, at a minimum, a GPS satellite per each grid unit until the entire Earth map is covered.

The Earth’s circumference is 40,075 km. If we treat this as the length of our projected Earth map and assume that the projection is a square then we can loosely estimate the number of units (ie. the number of GSP satellites) needed along one row of the map as:

Since our map is a square projection, the very edge of the map wraps back to the beginning of map, so we may subtract the last unit out of our calculation and obtain the approximate number of units (ie. dots or GPS satellites required) as:

In reality, this value is most likely smaller because of the distortions from the square map projection.

1. Compute the distance from a point on the earth to a GPS satellite that is located at the horizon. Express your answer in meters, and compare it to the distance in meters to a satellite that is directly overhead.



Using our diagram from the first problem as a reference we may solve for the hypotenuse (y) of the right triangle in order to determine the distance to a GPS satellite that is just above the horizon as:

This is compared to the distance to a GPS satellite that is directly overhead which is (calculated in problem 1 above):

1. Compute the travel time, at the speed of light, for each of these two signals.

|  |  |  |
| --- | --- | --- |
| Distance (d) | Speed of Light (C) | Signal Travel Time (T) |
| 23,693,000 m | 2,99,792,000 m/s | 79.03 ms |
| 20,238,000 m | 2,99,792,000 m/s | 67.51 ms |

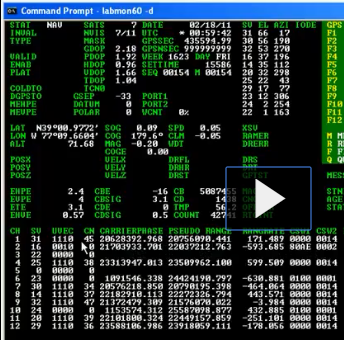
1. The chip time of a PRN is about 1 microsecond, which corresponds to a travel distance of 1000 feet when the speed of light is approximated as 1 foot per nanosecond. A typical tracking loop in a radio receiver can track a signal to a precision of about 5% of a code chip. Given this, estimate the accuracy of a C/A code receiver, before consideration of GDOP. Compare this to the position error numbers presented in the MP4 video.

A PRN transmission period consists of 1023 chips at 1 microsecond each. This means that the transmission period of the GPS satellite for C/A codes is:

If a typical receiver can track a signal to a precision of ≈5% then at 1000 feet for a single chip the accuracy of the receiver is approximately:

Looking at the Space Vehicle (SV) number 31 which has an high elevation angle (unobstructed view) the range values for the carrier and the pseudo range is calculated as:

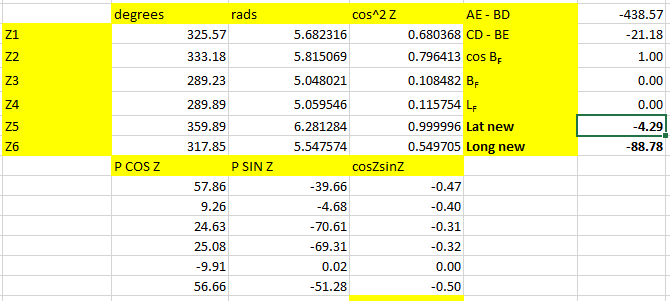
This is a much larger error than the theoretical for a typical receiver as calculated above.



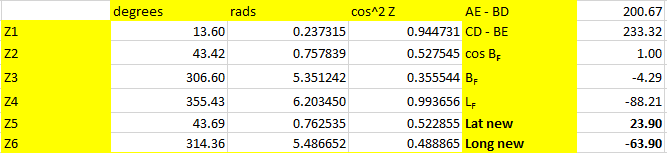
1. Using the attached spreadsheet “GPS computations4 highlighted,” and the attached week 519 GPS almanac files, guess an assumed position (i.e., 0 degrees lat, 0 degrees long) and use the spreadsheet to iterate until the actual position of 39 -77, for which your Hc values will agree with the Ho values from the GPS receiver, is derived. To accomplish this, you will need to examine the equations used in the spreadsheet and reverse engineer their function. Produce a block diagram that describes the algorithms represented in the spreadsheet.

The following shows the iterations using the “GPS computations4 highlighted” spreadsheet where the initial assumed position is {0°, 0°}. The resulting lat and long are then fed back into the input of the spreadsheet and recalculated. The result eventually converges onto the actual 39°N, 77°W position.

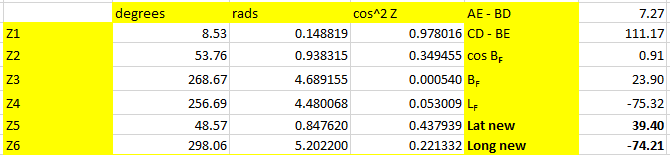
Iteration 1



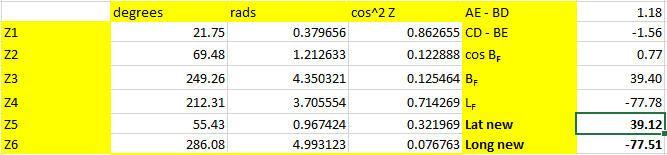
Iteration 2



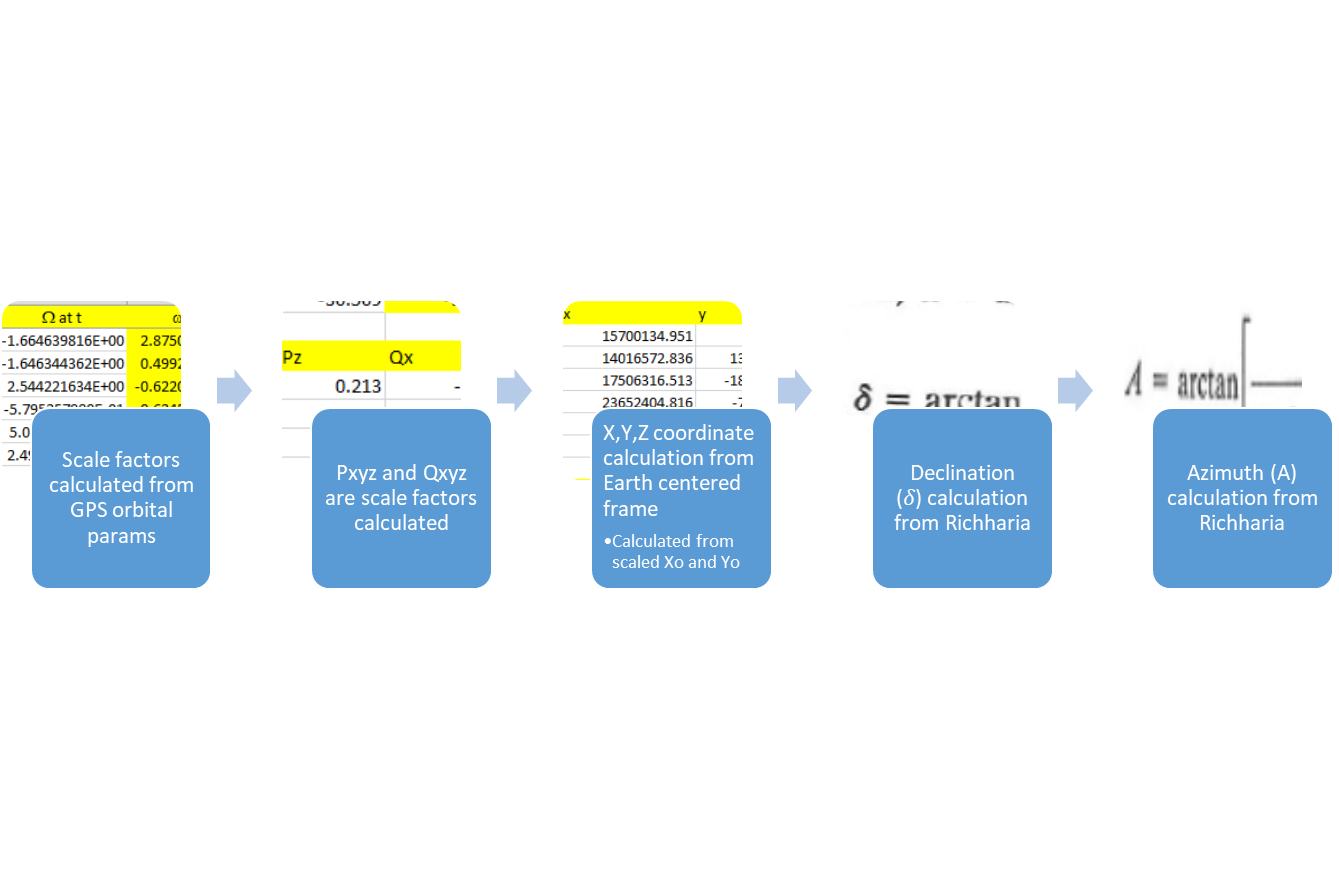
Iteration 3



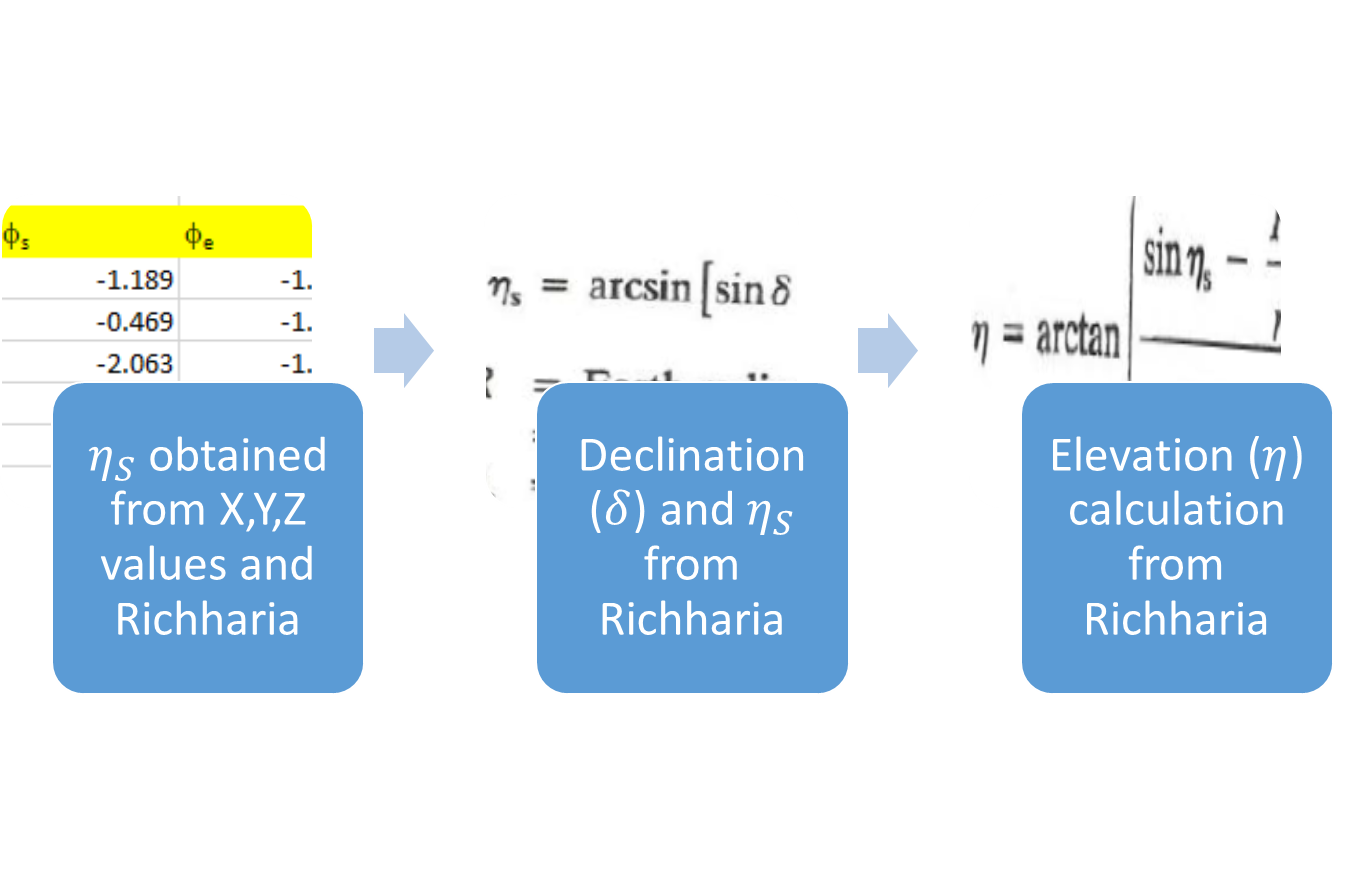
Iteration 4



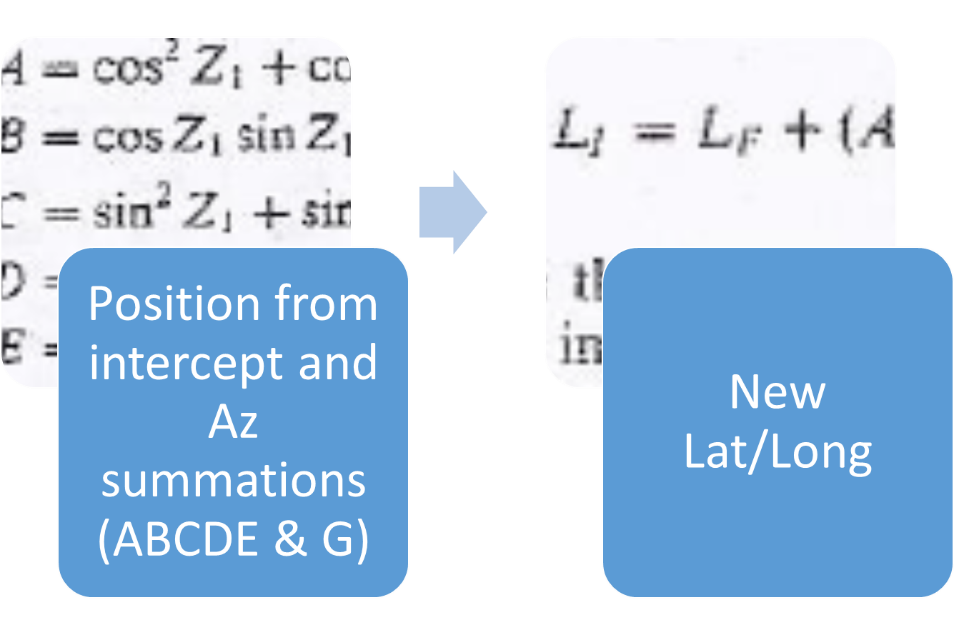
Using Excel’s “Trace Precedents” functions I was able to work backwards and deconstruct the steps to combining the orbital parameter data and the sight reductions algorithm to produce a resulting lat/long. The following is a block diagram that provides screenshots for the relevant equations and cells for each respective step in the spreadsheet.

**Parallel part 1**

**Parallel part 2**



**Combined final step**

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1. Experiment with the value of RA offset to see its effect on the accuracy of the solutions. Comment on your conclusions.

It appears that even with the slightest bit of modification to the Right Ascension offset cell causes the algorithm to converge a very different solution. Changing my RA reference offset to 2° (from 1.2524°) causes the algorithm to converge to 29° Lat and -124° long.

1. Comment on the relationship of your “exploration” of this problem to the method of lunars from Module 10.

The method of lunars that was covered in Module 10 was certainly a more simplistic approach to obtaining a position fix; however, using Newton’s method required many more iterations in a spreadsheet to eventually converge onto the correct answer of position. This method also used much less input data (two observations of the moon and sun) as oppose to that used in the GSP computation where you get input from many space vehicles.

I can understand why the GSP computation approach is a more practical solution for GSP technology. Although more memory is likely required to perform the computation (as a result of their being more steps and a larger set of input data) the convergence pattern of the algorithm saves a huge amount of processing time as opposed to the Newton’s method which could be much slower to obtain a position fix.